Mechanics of cutting and boring
Part II: Kinematics of axial rotation machines
Cover: Claw marks made in Alaskan permafrost by the tunneling machine shown in Figure 1g.
Mechanics of cutting and boring
Part II: Kinematics of axial rotation machines

Malcolm Mellor

June 1976
The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.
This report, which is one of a series on the mechanics of cutting and boring in rock, deals with the kinematics of machines such as rotary drills, augers, tunnel boring machines, corers, and raise borers, in which the rotary cutting unit revolves about an axis that is parallel to the machine’s direction of advance. The discussion and analysis covers the geometry and motion of various components of the cutting system, including such topics as tool trajectories, tool speeds, motions of the more complicated mechanisms, chipping depth, penetration rates, production and clearance of cuttings, tool angles, and spatial distribution of cutters. Worked examples are given to illustrate the application of various equations to practical problems.
PREFACE

This report was prepared by Dr. Malcolm Mellor, Physical Scientist, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. The work was done under DA Project 4A762719AT42, Design, Construction and Operations Technology for Cold Regions, Technical Area 02, Soils and Foundations Technology, Work Unit 004, Excavation in Frozen Ground.

Paul V. Sellmann of USA CRREL and Dr. Ivor Hawkes provided technical reviews of the manuscript.

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SUMMARY

This report develops theoretical equations for the geometry and motion of axial rotation cutting and boring machines (e.g. rotary drills, tunnel boring machines, raise borers, machines for coring, trepanning and reaming). After terms used in the report are defined, the penetration trajectories of both fixed cutting tools and rolling cutters are analyzed and described by equations. Velocity equations are then derived for fixed cutters, rolling disc cutters, and wide roller cutters, and the effects of skidding are taken into account for roller tools. Planetary or epicyclic boring heads are analyzed; consideration is given to reaming machines using unpowered roller cutters, and to machines having independently powered planetary cutting heads. Equations relating tool chipping depth with machine penetration rate and rotational speed are given, both for flat-face and non-flat boring heads. Criteria are given for compatibility between cutting and clearing components where flight augers or screw conveyors are used. Kinematic factors that control requirements for relief angle and rake angle on fixed cutting tools are given, and distinctions are made between apparent and actual tool angles. The distribution and spacing of cutting tools on a boring head are considered, taking into account radial spacing, angular spacing, and spacing in the axial direction. A number of numerical examples are worked in order to demonstrate the application of various equations to practical problems of machine design or performance analysis.
## CONVERSION FACTORS FOR U.S. CUSTOMARY AND SI UNITS

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<th>To obtain</th>
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</thead>
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<td>Foot</td>
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<td>Meter</td>
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<td>Foot/minute</td>
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MECHANICS OF CUTTING AND BORING

FOREWORD

There are a multitude of tasks that involve the cutting, drilling, or excavating of natural ground materials and massive structural materials. The required technology varies with the properties of the materials and with the scale of operations, but a broad distinction can be made on the basis of the strength, cohesion, and ductility of the material that is to be worked. In weak materials that have little cohesion (e.g. typical soils) the forces and energy levels required for separation and disaggregation are often small compared with the forces and energy levels required for acceleration and transport, and materials handling technology dominates the consideration. By contrast, in strong materials that exhibit brittle fracture characteristics (e.g. rock, concrete, ice, frozen ground) the forces and energy levels required for cutting and breaking are high compared with those required for handling the broken material, and the technical emphasis is on cutting and breaking processes.

USA CRREL has long been concerned with excavating and drilling in ice and frozen ground, and over the past decade systematic research has been directed to this technical area. The research has covered a wide range of established technologies and novel concepts but, for short term applications, interest has necessarily centered on special developments of proven concepts. In particular, there has been considerable concern with direct mechanical cutting applied to excavation, cutting, and drilling of frozen soils, glacier ice, floating ice, and dense snow. During the course of this work, numerous analyses and design exercises have been undertaken, and an attempt is now being made to develop a systematic analytical scheme that can be used to facilitate future work on the mechanics of cutting and boring machines.

In the industrial sector, rock-cutting machines are usually designed by applying standard engineering methods in conjunction with experience gained during evolution of successive generations of machines. This is a very sound approach for gradual progressive development, but it may not be appropriate when there are requirements for rapid development involving radical departures from established performance characteristics, or for operations in unusual and unfamiliar materials. A distinct alternative is to design more or less from first principles by means of theoretical or experimental methods, but this alternative may not be practically feasible in its more extreme form.

There are numerous difficulties in attempting a strict scientific approach to the design of rock-cutting machines. The relevant theoretical rock mechanics is likely to involve controversial fracture theories and failure criteria, and to call for detailed material properties that are not normally available to a machine designer. Direct experiments are costly and time-consuming, and experimental data culled from the literature may be unsuitable for extrapolation, especially when (as is sometimes the case) they are described by
### MACHINE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Tool Action</th>
<th>Tool Action</th>
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</thead>
<tbody>
<tr>
<td>Transverse Rotation</td>
<td>Bucket-wheel trenchers, disc saws, excavators, pavement planers, rotary-drum graders, continuous miners, drum shooers, ripping booms, some tunnelers, rotary snowplows, dredge cutterheads</td>
</tr>
<tr>
<td>Axial Rotation</td>
<td>Rotary drills, augers, shaft sinks, raise borers, full-face tunnel borers, face miners, corers, rotary snowplows, trepanners</td>
</tr>
<tr>
<td>Continuous Belt</td>
<td>Chain-type trenchers, ladder dredges, coal saws, shale saws, etc.</td>
</tr>
</tbody>
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### TOOL ACTION

<table>
<thead>
<tr>
<th>Tool Action</th>
<th>Tool Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel Motion</td>
<td>Drag bits, picks, placing cutters, shearing blades, diamonds</td>
</tr>
<tr>
<td>Normal Indentation</td>
<td>Roller bits (with studs or teeth), disc cutters, impact and percussion tools</td>
</tr>
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Classification of machines and cutting tools for analytical purposes.

relationships that violate the basic physics of the problem. Comprehensive mechanical analyses for rock-cutting machines have not yet evolved, and while established design principles for metal-cutting machine tools may be helpful, they do not cover all pertinent aspects. For example, there are usually enormous differences in forces and power levels between machine tools and excavating machines, and force components that can be almost ignored in a relatively rigid machine tool may be crucial design factors for large mobile rock cutters that are highly compliant.

In dealing with cold regions problems where neither outright empiricism nor highly speculative theory seem appropriate, some compromise approaches have been adopted. While simple and practical, these methods have proved useful for analysis and design of cutting and boring machines working under a wide range of conditions in diverse materials, and it seems possible that they might form the basis for a general analytical scheme. The overall strategy is to examine the kinematics, dynamics and energetics for both the cutting tool and the complete machine according to a certain classification, adhering as far as possible to strict mechanical principles, but holding to a minimum the requirements for detailed information on the properties of the material to be cut.

**Kinematics** deals with the inherent relationships defined by the geometry and motion of the machine and its cutting tools, without much reference to the properties of the material being cut. **Dynamics** deals with forces acting on the machine and its cutting tools, taking into account machine characteristics, operating procedures, wear effects, and material properties. **Energetics** deals largely with specific energy relationships that are determined from power considerations involving forces and velocities in various parts of the system, taking into account properties of the materials that are being cut.

These mechanical principles are applied in accordance with a classification based on the characteristic motions of the major machine element and the actual cutting tools, as illustrated above. Machines are classified as **transverse rotation, axial rotation, or continuous belt**, while the action of cutting tools is divided into **parallel motion and normal indentation**.
Transverse rotation devices turn about an axis that is perpendicular to the direction of advance, as in circular saws. The category includes such things as bucket-wheel trenchers and excavators, pavement planers, rotary-drum graders, large disc saws for rock and concrete, certain types of tunneling machines, drum shearers, continuous miners, ripping booms, some rotary snowplows, some dredge cutterheads, and various special-purpose saws, millers and routers. Axial rotation devices turn about an axis that is parallel to the direction of advance, as in drills. The category includes such things as rotary drills, augers and shaft-sinking machines, raise borers, full-face tunnel boring machines, corers, trepanners, some face miners, and certain types of snowplows. Continuous belt machines represent a special form of transverse rotation device, in which the rotor has been changed to a linear element, as in a chain saw. The category includes “digger chain” trenchers, ladder dredges, coal saws, shale saws, and similar devices.

In tool action, parallel motion denotes an active stroke that is more or less parallel to the surface that is being advanced by the tool, i.e. a planing action. Tools working this way include drag bits for rotary drills and rock-cutting machines; picks for mining and tunneling machines; teeth for ditching and dredging buckets; trencher blades; shearing blades for rotary drills, surface planers, snowplows, etc.; diamond edges for drills and wheels; and other “abrasive” cutters. Normal indentation denotes an active stroke that is more or less normal to the surface that is being advanced, i.e. one which gives a pitting or cratering effect such as might be produced by a stone chisel driven perpendicular to the surface. Tools working this way include roller rock bits for drills, tunneling machines, raise borers, reamers, etc.; disc cutters for tunneling machines; and percussive bits for drills and impact breakers.

A few machines and operations do not fit neatly into this classification. For example, certain roadheaders and ripping booms used in mining sump-in by axial rotation and produce largely by transverse rotation, and there may be some question about the classification of tunnel reamers and tapered rock bits. However, the classification is very satisfactory for general mechanical analysis.

Complete treatment of the mechanics of cutting and boring is a lengthy task, and in order to expedite publication a series of reports dealing with various aspects of the problem will be printed as they are completed. The main topics to be covered in this series are:

1. Kinematics of transverse rotation machines (Special Report 226, May 1975)
2. Kinematics of axial rotation machines
3. Kinematics of continuous belt machines
4. Dynamics and energetics of parallel-motion tools
5. Dynamics and energetics of normal indentation tools
6. Dynamics and energetics of transverse rotation machines
7. Dynamics and energetics of axial rotation machines
8. Dynamics and energetics of continuous belt machines.
MECHANICS OF CUTTING AND BORING
PART 2: KINEMATICS OF AXIAL ROTATION MACHINES

by

Malcolm Mellor

Axial rotation machines for cutting and boring are devices that rotate the cutting head about the axis of penetration. In the drilling and excavating of rock and other earth materials, this category of machine includes such things as rotary drills, augers, corers, tunnel boring machines, mining machines, raise borers and some rotary snow plows (Fig. 1). In other technical fields (e.g. metal-working or woodworking) there are numerous devices, mostly drills, reamers, corers or trepanners, that conform to almost identical mechanical principles.

This report deals with the geometry and motion of the cutting elements on axial rotation machines, and to some extent with the geometry and motion of closely associated components that remove cuttings from the working surface. The treatment parallels that given for transverse rotation machines and continuous belt machines in separate reports (Mellor 1975, 1976), but there is an important difference in that axial rotation machines can utilize both parallel-motion (planing) tools and normal-indentation (crushing) tools, whereas the other machine types use only parallel-motion tools.

As in the other reports of this series, the intention is to provide a digest of theory and data, without describing or discussing the machining processes in much practical detail.

**Terminology**

Terms used to describe essentially similar components or functions of axial rotation machines may vary with the industry or country in which the machine is built or used, or with the technical background of the people using the terms. Some terminology adopted for the present work is given below.

The **boring head** is the complete rotor which revolves about the central axis of the hole, shaft or tunnel that is being bored. Its diameter corresponds to that of the bore. On a small drill it is simply the **bit**; on a full-face tunnel boring machine it is the face plate that carries the cutters; on a “selective” tunnel borer it is the rotor that carries subsidiary powered drums or unpowered cutters. Some mining machines have twin boring heads.

**Reamers** are devices that increase the diameter of an existing pilot hole. They may follow the principle of the standard metal-working reamer, using a tapered boring head to attack the hole walls continuously, or they may consist of a series of annular boring heads that cut out a set of discrete steps, each larger in diameter than the preceding one. Some raise borers and tunnel boring machines fall into this category.
a. Assortment of small hand-held augers.

Figure 1. Axial-rotation machines.
b. Large auger head for work in "soft ground" and weak rocks. (Courtesy of Calweld Division of Smith International Inc.)

c. Typical "tricone" roller rock bit for rotary drilling. (Courtesy of Reed Tool Company.)
Cutters, or cutting tools, are the actual cutting elements (usually consumable and replaceable) fitted to the main boring head. They are broadly subdivided into parallel-motion tools and normal-indentation tools.

Parallel-motion tools operate with a planing action which moves the cutter parallel to the surface that is being advanced. This category includes such things as carbide-tipped drag bits (as used on drills and mining machines), hardfaced teeth (as used on large augers and soft-ground tunnel borers), ground steel cutting blades (as used on ice drills), and diamond tools (as used in rotary drilling and coring).

Normal-indentation tools in the present context are limited to the various types of roller cutters that are thrust into the advancing surface by high normal forces. More generally, the category would also include percussive tools.

The term roller cutter is used here for all types of unpowered rotating cutters that work primarily by indentation (like a glasscutter wheel). Included under this term are beveled-rim disc cutters, disc cutters with hard indenter studs (or “buttons”) around the rim, gear tooth cutters and starwheels, as well as various types of wide roller cones (with studs or multiple discs), center cones (tricones), and crossrollers.
e. Raise-boring heads. (Courtesy of Mining Services and Equipment Division, Dresser Industries Inc.)
Typical “full-face” tunnel boring machines for medium-strength rocks (disc type), and for hard rocks (studded roller type). (Courtesy of Calweld Division of Smith International Inc., and James S. Robbins and Associates Inc.)

Figure 1 (cont’d). Axial-rotation machines.

Gauge cutters are the cutters set at the full radius of the bore. They have to cut the corner or angle that marks the transition from face to hole wall.

A tracking cutter follows one or more identical cutters set at the same radius on the boring head. If there are $n$ tracking cutters at a given radius, they would normally be uniformly spaced with an angle $2\pi/n$ between their positions.
PART 2: KINEMATICS OF AXIAL ROTATION MACHINES

\( f(\text{cont'd}). \)
g. Twin-head mining machine formerly used for tunneling in permafrost.

Figure 1 (cont'd). Axial-rotation machines.

The chipping depth is the depth of penetration of a cutter into the work, measuring either normal to the advancing surface (\(d\)) or parallel to the main boring axis (\(d_b\)). It is equivalent to “uncut chip thickness” of machine tool terminology.

The pitch \(P\) of any helix (as described by an auger flight or by a cutting track) is the axial advance for one complete revolution (Fig. 2).

The helix angle \(\alpha\) at any given radius \(r\) on a helical cutting track or a helical auger flight is the slope angle defined by \(\alpha = \tan^{-1} \left( \frac{P}{2\pi r} \right)\), where \(P\) is the pitch (Fig. 3).

The penetration axis is the central axis of the hole that is being bored, and the axis about which the boring head rotates.

The rotational velocity of the boring head is its angular velocity \(\omega\) (radians per unit time), but it is often expressed as an angular frequency \(f\) (revolutions per unit time). It is commonly referred to as “head speed.”

The penetration rate \(U\) is the speed at which the boring head advances in the axial direction.

The trajectory of a fixed cutting tool, or the trajectory of fixed parts of a roller cutter, is the path traced relative to fixed axes (relative to the rock) as the boring machine operates; it is usually a helix on a simple machine. The cutting trajectory for a roller cutter is the path traced relative to the rock by a given point on the rim of the roller; it approximates to a cycloid or epicycloid superimposed on a helix for a simple machine.
The absolute tool speed $u$ for a given point on the tip or rim of a cutter is the velocity of that point relative to the work, taking into account the components of motion due to both rotation and penetration, i.e. it is the time derivative of the tool trajectory. In the case of a fixed (drag bit) tool, it is equivalent to the “surface feet per minute” of machine shop terminology. Tool speed varies with the radius of the tool on the boring head; at the outer edges of the head, where speeds are highest, tangential velocity derived from rotation alone ($u_r$) is usually a good approximation to $u$. In the case of roller cutters, the velocity of the center of the roller is often taken as “tool speed,” although indentation velocity of the rim is more directly relevant to cutting.

The apparent rake angle $\beta_1$ is the angle between the leading face of a fixed cutting tool and the main axial direction, as shown in Figures 18 and 19. It is a constant angle for a given fixed tool.

The actual rake angle is the angle between the leading face of a fixed cutting tool and a normal to the helical penetration path. It varies with the operating conditions (Fig. 19d).

The apparent relief angle (or primary relief angle) $\beta_2$ is the angle between the flank or shoulder of a fixed cutting tool and a plane normal to the penetration axis, as shown in Figures 18 and 19. It is a constant angle for a given fixed tool.

The actual relief angle is the angle between the flank or shoulder of a fixed cutting tool and the helical penetration path (Fig. 19d). It varies with operating conditions.

The effective relief angle, or effective clearance angle, is illustrated in Fig. 19c. It defines the maximum helix angle that the tool can follow when there is a limitation imposed by some secondary projection of the flank or shoulder profile.
The included angle of a fixed cutting tool $\beta_3$ is the angle between the leading face and the shoulder, measured in a plane that is parallel to the penetration axis and tangential to the rotation, i.e.

$$\beta_3 = 90^\circ - (\beta_1 + \beta_2).$$

The edge angle of a beveled-rim disc cutter ($2\psi$) is the complete angle of the V-edge measured in a diametral plane (Fig. 21).

**Trajectories of Fixed Tools**

As a boring head rotates at constant angular frequency $f$ and simultaneously penetrates at constant axial speed $U$, any fixed point on the boring head at a given radius $r$ will follow a helical path around a circular cylindrical surface of radius $r$ (Fig. 2). The Cartesian description of the helix is usually given in parametric form as

$$x = a \cos \theta \quad y = a \sin \theta \quad z = b \theta$$

where $a$ and $b$ are constants and the angle $\theta$ is the parameter. In terms of the boring head, this translates as

$$x = r \cos \theta \quad y = r \sin \theta \quad z = Ut = \frac{U}{\omega} \theta = \frac{U}{\omega} \theta = \frac{U}{2\pi f} \theta$$

where $t$ is time, $\omega$ or $\dot{\theta}$ is angular velocity ($\omega = 2\pi f$), and $\theta$ is the total angle of rotation for the boring head.

The pitch, or penetration per revolution $P$ is

$$P = \frac{U}{f} = \frac{2\pi U}{\theta}$$

and the length of the helical path $s$ is

$$s = \theta \left[ (2\pi)^2 + \left( \frac{U}{f} \right)^2 \right]^{1/2} = \theta \left[ 1 + \left( \frac{U}{2\pi f} \right)^2 \right]^{1/2}$$

$$= \theta \left[ 1 + \left( \frac{U}{ru_t} \right)^2 \right]^{1/2} = \theta \left[ 1 + \left( \frac{U}{u_t} \right)^2 \right]^{1/2}$$

where $u_t$ is the tangential velocity of a point at radius $r$ when $U = 0$. The helix angle $\alpha$, defined at a given point as the angle between the tangent to the helix of radius $r$ and the tangent to the concentric circle of radius $r$ passing through the same point (Fig. 3), is

$$\alpha = \tan^{-1} \left( \frac{U}{2\pi rf} \right) = \tan^{-1} \left( \frac{U}{ru_t} \right) = \tan^{-1} \left( \frac{U}{u_t} \right).$$
Figure 4. Helix angle as a function of normalized radius.
These relations describe the motion of fixed cutting tools, or the motion of the bearing centers for roller cutters. They illustrate one of the major design problems for axial rotation devices:

$$\alpha \to 90^\circ \text{ as } (U/U_r) \to 0$$

or, since $f$ is finite,

$$\alpha = 90^\circ \text{ at } r = 0.$$  

In other words, a cutter at the center of a boring head has to progress directly into the work in the axial direction.

Figure 4 shows the helix angle $\alpha$ as a function of relative radius $r/R$, with $(U/2\pi R f)$ as parameter, $R$ being the maximum radius. The plot on linear scales illustrates that under typical conditions, where $U << 2\pi R f$, the abrupt increase of the helix angle does not occur until $r < 0.1 R$.

**Trajectories of Roller Cutters and Indenters**

The cutting track of a roller cutter traces out a helical path in the rock. That path is described by eq 1-5. If the cutter is a symmetrical disc set with its bearing axis along a radial of the main drum, the center of the bearing also traces out a similar helical path, but a point on the periphery of a non-skidding cutter describes a cycloidal trajectory relative to the helical track in the rock surface.

Consider a single knife-edged disc cutter mounted so that its axis of rotation is along a radial of the main boring head and perpendicular to the main penetration axis of the machine (i.e. with zero skew). Assume that the cutter mounting is "stiff," so that the disc cuts a groove of fixed depth $\ell$ without riding up between chipping stages. If the helical cutting track in the rock is developed into a plane, and $x$ and $y$ axes are taken from an arbitrary origin on the track, with $x$ and $y$ directions tangential and normal to the track respectively, then a particular point on the periphery of the disc cutter describes a regular cycloid whose equation is

$$\begin{align*}
    x &= R_c (\phi - \sin \phi) \\ 
    y &= R_c (1 - \cos \phi)
\end{align*}$$

where $R_c$ is the disc radius, and $\phi$ is the angle of rotation measured from an initial condition of $x = 0, y = 0, \phi = 0$ (Fig. 5). An alternative expression is

$$x = R_c \left\{ \cos^{-1} \left(1 - \frac{y}{R_c}\right) + \frac{1}{2} \left[ 2y/R_c - (y/R_c)^2 \right]^{1/2} \right\}.$$  

For one complete revolution of the disc, $x = 2\pi R_c$ and the length of the arc is $8R_c$.

It is often assumed for simplicity that a rolling indenter penetrates the rock normally, but this is not strictly true. Any point on the rim of a non-skidding indenting cutter will penetrate the rock along part of a cycloidal path, traveling forward as well as downward (Fig. 6). If the chipping depth measured normal to the helical path is $\ell$, then the forward travel of a rim point during indentation ($\Delta x$) is
Figure 5. Regular cycloid.

Figure 6. Cycloidal indentation path.

\[ \Delta x = R_c \left\{ \cos^{-1} \left( 1 - \frac{\ell}{R_c} \right) - \frac{2\ell}{R_c} - \left( \frac{\ell}{R_c} \right)^2 \right\}^{\frac{1}{2}} \]

\[ = R_c \cos^{-1} \left( 1 - \frac{\ell}{R_c} \right) - \ell \left( \frac{2R_c}{\ell} - 1 \right)^{\frac{1}{2}}. \quad (8) \]

With a 14-in.-diam disc chipping to depths of 0.5 in. and 1.0 in., \( \Delta x \) would be 0.064 in. and 0.182 in. respectively. Thus penetration is very close to being perpendicular to the surface of the work in most practical circumstances.

Combining the cycloidal and helical motions, the trajectory of a point on the rim of a radial-axis non-skidding disc cutter can be expressed in cylindrical coordinates as

\[ \begin{align*}
    r' &= r_c \\
    \theta' &= \frac{R_c}{r_c} \left( \phi - \sin \phi \right) \cos \alpha \\
    z' &= R_c \left\{ \left( \phi - \sin \phi \right) \sin \alpha - \left( 1 - \cos \phi \right) \cos \alpha \right\}
\end{align*} \]  

(9)
in which \( r_c \) is the radius at which the disc is set on the main boring head, and \( \alpha \) is the helix angle of the cutting track as given by eq 5. In Cartesian coordinates the combined motion is described by taking \( r \) and \( \theta \) from eq 9 and setting \( x' = r' \cos \theta', y' = r' \sin \theta' \).

When the disc cutter is rotating without slip, there is a simple relation between cutter rotation \( \phi \) and rotation of the main drum \( \theta \):

\[
R_c \phi = s = r_c \theta \left[ 1 + \left( \frac{U}{2\pi r_c f} \right)^2 \right]^{\frac{1}{2}} = r_c \theta \left[ 1 + \left( \frac{U}{r_c \theta} \right)^2 \right]^{\frac{1}{2}}. \tag{10}
\]

Except for locations very close to the center of the main boring head, \( U/(2\pi r_c f) \) is typically much less than unity, so that

\[
R_c \phi \approx r_c \theta. \tag{11}
\]

By substituting into eq 9 from eq 10 or 11, \( \theta' \) can be expressed in terms of \( \theta \), the rotation angle of the main boring head.

If, instead of a continuous disc, the roller cutter is a studded disc with sharp conical indenters, the trajectory of an indenter point will be the same as the trajectory of a point on a continuous disc as long as the machine is "stiff," chipping depth is less than indenter length, the half-angle of the indenter point is less than the overbreak angle of the rock, and the cutter does not skid. However, in the case of a roller cutter with hemispheric indenters, first contact between the indenter and the rock is made at a point \( A \) (Fig. 7), where \( A \) is off-center from the extreme tip of the indenter by an angle \( \delta \) that is given approximately by

\[
\delta \approx \cos^{-1} \left( 1 - \frac{\delta}{R_c} \right) \tag{12}
\]
again assuming that $\ell$ is less than the length of the indenter. In this case the effective point of thrust moves forward during the working stroke by a distance of approximately $[R_c \delta - (R_c - R_s) \sin \delta]$, where $R_c$ is the radius to the stud tip, $R_s$ is the radius of the indenter stud, and $\delta$ is given by eq 12.

With a studded disc of 12 in. diameter ($R_c = 6$ in. measured to the stud tips), $\delta$ would be $13^\circ$ with chipping depth $\ell = 0.15$ in., or $20^\circ$ with $\ell = 0.35$ in. If the radius of the stud $R_s$ is 0.35 in., then $[R_c \delta - (R_c - R_s) \sin \delta]$ is 0.089 in. with $\ell = 0.15$ in., and 0.162 in. with $\ell = 0.35$ in. The respective values of $\Delta x$ for a simple disc, or for a disc with sharp-tipped indents, are 0.011 in. and 0.040 in. Thus, under these circumstances, the rolling action of the stud relative to the rock contributes more forward component than does the cycloidal motion; i.e. it does more to move the effective path of indentation away from the normal direction.

**Speed of Fixed Cutting Tools**

The velocity components relative to the rock for fixed cutting tools can be obtained directly by differentiating eq 2 with respect to time:

\[
\begin{align*}
\dot{x} &= -r \dot{\theta} \sin \theta = -2\pi rf \sin \theta \\
\dot{y} &= r \dot{\theta} \cos \theta = 2\pi rf \cos \theta \\
\dot{z} &= U.
\end{align*}
\]

(13)

Alternatively, the absolute tool speed relative to the rock $u$, is given by the time derivative of eq 4:

\[
u = \dot{r} = [(r \dot{\theta})^2 + U^2]^{1/2} = [(2\pi rf)^2 + U^2]^{1/2}
= u_t [1 + (U/u_t)^2]^{1/2}.
\]

(14)

Figure 8 shows the ratio $u/u_t$ as a function of relative radius $r/R$ for various values of the parameter $U/2\pi Rf$ ($R$ is the maximum head radius). In typical practical situations, $U$ is often much smaller than $2\pi Rf$, so that $u \approx u_t$ when $r/R > 0.05$.

In Figure 9, rotational speed $f$ is plotted against maximum head diameter ($2R$) for a range of auger drills designed for boring in rock and soil. The proportionality lines represent values of the peripheral speed ($u_t$)$_{max}$ for $U = 0$. The values mostly fall within the range 100 to 1000 ft/min, which is very similar to the tool speeds adopted on other types of drag-bit cutting machines.

The tangential speed of bits set at radii less than $r = R$ is simply $(r/R)(u_t)_{max}$ when $U$ is small. This can occasionally create problems when boring in materials that exhibit ductile behavior at low cutting speeds. For example, a drag-bit tunnel boring machine driving in fine-grained permafrost soil gave tool speeds of $(u_t)_{max} = 220$ ft/min at the outer limits of the twin boring heads, but tool speeds just outside the pilot drills (at $r/R = 0.1$) of only 22 ft/min; this latter cutting speed was too low to induce brittle fracture.
Figure 8. Absolute tool speed relative to nominal tangential speed as a function of relative radius.

Figure 9. Rotational speed plotted logarithmically against head diameter for a) a range of rock and soil augers fitted with drag bits (the proportionality lines represent values of peripheral tool speed \( u_t / max \) for \( r = R \) and \( U = 0 \)), and b) a variety of drilling tools.
PART 2: KINEMATICS OF AXIAL ROTATION MACHINES

Figure 9 (cont'd).

Speed of Rolling Disc Cutters

The speed of the center of a roller cutter is given by eq 14, where \( r \) is the radius to the cutter center measured from the center of the main boring head. If the cutter is rolling without skidding, a given point on the rim of a disc has tangential and normal velocity components relative to the rock that are given by the time derivative of eq 6:

\[
\begin{align*}
\dot{x} &= R_c \dot{\phi} (1 - \cos \phi) \\
\dot{y} &= R_c \dot{\phi} \sin \phi \\
\end{align*}
\]  

(15)
where \( \dot{\phi} \), the angular velocity of the roller cutter relative to its own center, is related to the angular velocity of the main boring head (\( \dot{\theta} \)) by

\[
\dot{\phi} = \frac{r_c \dot{\theta}}{R_c} \left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{1/2}
\]

or

\[
\dot{\phi} = 2\pi f \left( \frac{r_c}{R_c} \right) \left[ 1 + \left( \frac{U}{2\pi r_c f} \right)^2 \right]^{1/2}.
\] (16)

During the working stroke of a given point on the cutter rim, \( \phi \) is in the range

\[
[2\pi - \cos^{-1} \left( 1 - \frac{e}{R_c} \right)] \leq \phi \leq 2\pi
\] (17)

where \( e \) is the chipping depth of the cutter.

The velocity of penetration of the cutter into the work \( u \) is

\[
u = (\dot{x}^2 + \dot{y}^2)^{1/2} = R_c \dot{\phi} \left[ (1 - \cos \phi)^2 + \sin^2 \phi \right]^{1/2}
\]

or

\[
u = R_c \dot{\phi} \left[ 2(1 - \cos \phi) \right]^{1/2}.
\] (18)

At the point of entry, the initial velocity \( u_0 \) is

\[
u_0 = R_c \dot{\phi} \sqrt{2e/R_c}.
\] (19)

Under typical practical conditions, \((U/2\pi r_c f)^2 << 1\) in the outer portions of the boring head (say \( r/R > 0.1 \)), so that

\[
\dot{\phi} \approx \frac{r_c \dot{\theta}}{R_c}
\]

or

\[
\dot{\phi} \approx 2\pi f \left( \frac{r_c}{R_c} \right)
\] (20)

and under these circumstances

\[
u \approx r_c \dot{\phi} \left[ 2(1 - \cos \phi) \right]^{1/2}
\]

or

\[
u \approx 2\pi f r_c \left[ 2(1 - \cos \phi) \right]^{1/2}.
\] (21)

The approximate expression for initial velocity at the point of entry is

\[
u_0 \approx r_c \dot{\phi} \sqrt{2e/R_c}
\]

or

\[
u_0 \approx 2\pi f r_c \sqrt{2e/R_c}.
\] (22)
At the outer perimeter of a rock-boring machine, where \( r_c = R \), the likely range of \( r_c \dot{\theta} \) (or \( u_t \)) might be 200 to 400 ft/min in rock of moderate strength, and perhaps as low as 100 ft/min in very strong rock (see Fig. 10). At smaller cutter radii, the values of \( r_c \dot{\theta} \) are proportionately less. The probable range of values for \( k/R_c \) might be about 0.015 to 0.15, giving a corresponding range for \( \sqrt{2k}/R_c \) of 0.173 to 0.548. Thus the typical range of indentation entry speeds is likely to be about 20 to 200 ft/min at the outer perimeter of the boring head, and proportionately less at smaller radii.

From the above estimates it can be seen that a roller rock bit used in fine-grained permafrost soils might indent too slowly to induce brittle fracture near the center of the head \((r/R \ll 0.1)\), where initial indentation speeds could be as low as 2 ft/min.

**Speed and Geometry of “Wide” Roller Cutters**

Roller cutters for large boring machines very often have finite thickness in the radial direction. The cutter may be a drum rather than a disc, or it may consist of several discs joined together on the same core or shaft. Because the whole cutter unit revolves with a single rotational speed \( f \), the diameter has to vary systematically if skidding is to be avoided. On a flat-face boring head, the wide cutter has to take the form of a frustum of a cone.

If a conical roller cutter (Fig. 11) is set with its axis radial to the main boring head (but not necessarily exactly normal to the penetration axis), the required cone diameters at the inner and outer ends, \( D_i \) and \( D_o \), can be related to the radial distances of the cone ends on the main boring head \((r_i \text{ and } r_o)\) by substitution into eq 16:
In most practical cases, the square root term is very close to unity, so that

\[
\frac{D_0}{D_1} \approx \frac{r_0}{r_1}
\]

or

\[
\frac{D_0}{D_1} \approx 1 + \frac{W}{r_1}
\]

where \( W \) is the slant width of the cutter measured radially on the main boring head (Fig. 11 and 12). The half-angle of the cone \( \gamma \) is

\[
\gamma \approx \tan^{-1}\left(\frac{D_1}{2r_1}\right) \approx \tan^{-1}\left(\frac{D_0}{2r_0}\right).
\]

Equation 24 can be used to calculate the best position on the head for a cutter of given dimensions. For this purpose it is rewritten as

\[
(r_1)_{\text{opt}} = \frac{W}{(D_0/D_1 - 1)}
\]

where \((r_1)_{\text{opt}}\) is the optimum radius on the main head.
From the above relations it can be seen that "scrubbing" of the cutters is unavoidable when non-skewed roller-cones of standard design are fitted to a flat-face boring head at different radii. However, if the working face of the cone is angled relative to the main penetration axis by an angle \( \alpha \) (Fig. 12), then in principle it may be possible to avoid scrubbing while using cutters of standard design. Equation 24 can be rewritten as

\[
\frac{D_0}{D_i} \approx 1 + \frac{W \sin \alpha}{r_i}
\]

and the condition for non-skid operation of a standard cutter is obtained as

\[
\sin \frac{\alpha}{r_i} = \frac{1}{W} \left( \frac{D_0}{D_i} - 1 \right).
\]

In other words, \( \sin \alpha \) has to be proportional to the setting radius (measured normal to the penetration axis). Substituting into eq 28 from eq 26, the non-skid condition can also be written as

\[
\sin \alpha = \frac{r_i}{(r_i)_{opt}}
\]

from which it can be seen that, while a roller cone optimized for use at a large radius can be adjusted for use at a smaller radius, the converse is not true.

**Example 1.** A 9-ft-diam tunneling machine of typical full-face design is to be fitted with roller cone cutters of a standard design. Diameters of the large and small ends are 11 in. and 9 in. respectively, and the length of the cone is 10 in. Calculate the optimum setting position for non-skid operation of these cutters, and consider the feasibility of shaping the face of the boring head so as to permit non-skid operation of the same cutters at other face positions.

Slant length of the cone \( W = 10.05 \) in., \( D_0 = 11 \) in., \( D_1 = 9 \) in., and therefore the optimum radius to the inside end of the cone \((r_i)_{opt}\) is

\[
(r_i)_{opt} = \frac{10.05}{(11/9 - 1)} = 45.2 \text{ in.}
\]

This is the optimum value when \( \alpha = 90^\circ \), i.e., for "flat-face" operation. Adding the slant length \( W \), the outer end of the cone in this optimum setting would extend to a radius of 55.25 in., which is almost exactly the radius of the tunnel. Thus the cutters are ideally suited as gauge cutters with \( \alpha = 90^\circ \).

For radii less than the gauge radius, the standard roller cone can be made to run without skidding by inclining its axis so that the small end leads the large end. At any radius \( r_i \), the angle \( \alpha \) that is required to prevent skidding is

\[
\alpha = \sin^{-1} \left[ \frac{r_i}{(r_i)_{opt}} \right] = \sin^{-1} \left( \frac{r_i}{45.2} \right)
\]

where \( r_i \) is in inches. The required value of \( \alpha \) would be 45° at \( r_i = 32 \) in., and 30° at \( r_i = 22.6 \) in. It therefore seems likely that a different cone design would be required for the central part of the boring head, since the face of the machine would have to be shaped into a rather extreme point or prow in order to utilize the standard cutters at small radii.
Skidding Rollers

Roller cutters sometimes skid, perhaps because a bearing is seizing up or, more likely, because a wide roller has been mispositioned, either deliberately or inadvertently. Skidding gives the indenter point a tangential motion relative to the rock surface, and thus it can have some of the action of a drag bit. In some circumstances this could improve cutting effectiveness, although it also produces more rapid wear of the tool.

The motion of the indenter point relative to the rock is determined by the rotary motion of the cutter and the motion of the center of the cutter as described earlier. Developing the helical cutting path in the rock into a plane, and taking \( x \) and \( y \) axes tangential and normal to the work surface as before, the trajectory of a rim point is described by

\[
x = r_c \dot{\theta} \left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{1/2} - R_c \sin \phi
\]

\[
y = R_c (1 - \cos \phi).
\]

The cumulative slip between the disc and the work \( \Sigma \) is

\[
\Sigma = r_c \dot{\theta} \left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{1/2} - R_c \phi.
\]

The velocity components of an indenting point relative to the rock are

\[
\dot{x} = r_c \dot{\theta} \left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{1/2} - R_c \dot{\phi} \cos \phi
\]

\[
\dot{y} = R_c \dot{\phi} \sin \phi
\]

and for practical purposes a "skidding velocity" \( u_s \) can be defined as \( \dot{x} \) at the bottom of the indentation stroke, i.e. at \( \phi = 2\pi \):

\[
u_s = r_c \dot{\theta} \left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{1/2} - R_c \dot{\phi}.
\]

Under the common practical conditions where \( (U/r_c \dot{\theta})^2 \ll 1 \), eq 33 can be simplified to

\[
u_s \approx r_c \dot{\theta} - R_c \dot{\phi}.
\]

Skidding changes the indentation trajectory. During the indenting stroke of a point on the rim of a plain disc, the point moves forward a distance \( \Delta x \) relative to the rock:

\[
\Delta x = r_c \Delta \theta \left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{1/2} - \left( 2 \frac{R_c}{\dot{\chi}} - 1 \right)^{1/2}
\]
where \( \xi \) is the chipping depth and \( \Delta \theta \) is the angle turned by the main boring head while the disc is rotating through an angle of \( \cos^{-1} \left( 1 - \frac{\xi}{R_c} \right) \), i.e. while the point is indenting through a normal distance of \( y = \xi \). Writing the relative rates of rotation for the main head and the disc as \( d\theta/d\phi \),

\[
\Delta x = r_c \cos^{-1} \left( 1 - \frac{\xi}{R_c} \right) \frac{d\theta}{d\phi} \left[ \frac{1}{1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2} \right]^{\frac{1}{2}} - \xi \left( 2 \frac{R_c}{\xi} - 1 \right)^{\frac{1}{2}}.
\]  

(36)

The difference in forward thrust between the skidding and non-skidding situations is

\[
(\Delta x)_{\text{skid}} - (\Delta x)_{\text{roll}} = \cos^{-1} \left( 1 - \frac{\xi}{R_c} \right) \left[ r_c \frac{d\theta}{d\phi} \left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{\frac{1}{2}} - R_c \right].
\]  

(37)

This is also the total slip between disc and rock for the duration of the working stroke, as can be seen by rewriting eq 31 in appropriate form.

When a machine is functioning properly, the most likely cause of skidding in roller cutters is use of a "wide" roller that has insufficient taper for its position on the boring head. A special case is the crossroller bit used for rotary drilling. With an insufficiently tapered roller, the balance of tangential cutting forces will tend to make the outer end skid in a positive sense (i.e. not rotate fast enough to keep up with the travel of the axis, as when a car brakes hard), and the inner end skid in a negative sense (i.e. roll too fast for the travel of the axis, as when a car accelerates violently). The outer end will then be tending to behave with a slight drag bit action or with the action of a very slowly up-milling disc. The inner end will tend to thrust out to its rear, with the action of a very slow climb-milling disc. At some section of the roller, there will be non-skidding rotation.

If the roller radius at the non-skidding section is \( \overline{R}_c \), and the radius of action on the main boring head is \( \overline{r}_c \), then from eq 10

\[
\theta = \frac{\overline{R}_c}{\overline{r}_c} \frac{\phi}{\left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{\frac{1}{2}}}
\]  

(38)

and

\[
\frac{d\theta}{d\phi} = \frac{\overline{R}_c/\overline{r}_c}{\left[ 1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2 \right]^{\frac{1}{2}}}.
\]  

(39)

At any section other than the neutral section, the forward thrust \( \Delta x \), given by eq 35 and 39 is

\[
\Delta x = r_c \frac{\overline{R}_c}{\overline{r}_c} \cos^{-1} \left( 1 - \frac{\xi}{R_c} \right) \left[ \frac{1}{1 + \left( \frac{U}{r_c \dot{\theta}} \right)^2} \right]^{\frac{1}{2}} - \xi \left( 2 \frac{R_c}{\xi} - 1 \right)^{\frac{1}{2}}
\]  

(40)

\[
\approx r_c \frac{\overline{R}_c}{\overline{r}_c} \cos^{-1} \left( 1 - \frac{\xi}{R_c} \right) - \xi \left( 2 \frac{R_c}{\xi} - 1 \right)^{\frac{1}{2}}.
\]  

(41)

The approximate form, eq 41, is adequate wherever \( (U/r_c \dot{\theta})^2 \ll 1 \), or where \( \overline{r}_c/\overline{r}_c \approx 1 \) (this rules out positions near the center of the boring head, where \( r_c \rightarrow 0 \)). The corresponding difference in forward thrust between a free rolling situation and the forced skidding situation is
\[(\Delta x)_{\text{skid}} - (\Delta x)_{\text{roll}} = \cos^{-1}\left(1 - \frac{\varphi}{R_c}\right) \left[r_c \frac{\overline{R}_c}{r_c} \left[1 + \left(\frac{U/r_c \delta}{\overline{R}_c}\right)^2\right]^{1/2} - R_c\right]
\] (42)

\[\approx R_c \cos^{-1}\left(1 - \frac{\varphi}{R_c}\right) \left[r_c \frac{\overline{R}_c}{r_c} - 1\right].\] (43)

Equations 42 and 43 also give the total slip between disc and rock for the duration of the working stroke.

The skidding velocity \(u_s\) is given by eq 16 and eq 33:

\[\dot{\phi} = \frac{\overline{r}_c}{r_c} \dot{\theta} \left[1 + \left(\frac{U}{r_c \delta}\right)^2\right]^{1/2}\] (44)

\[u_s = r_c \delta \left[1 + \left(\frac{U}{r_c \delta}\right)^2\right]^{1/2} - \frac{\overline{r}_c}{r_c} \frac{R_c}{r_c} \left[1 + \left(\frac{U}{r_c \delta}\right)^2\right]^{1/2} \right] \]. (45)

When \((U/r_c \delta)^2\) and \((U/\overline{r}_c \delta)^2\) are both much less than unity, as is commonly the case,

\[u_s \approx r_c \delta \left[1 - \left(\frac{R_c}{\overline{R}_c}\right) \left(\frac{\overline{r}_c}{r_c}\right)\right].\] (46)

In the case of a nontapered crossroller, such as is used in certain types of rotary rock bits, \(R_c/\overline{R}_c = 1\).

An appreciation of the magnitudes involved in these quantities is best gained from numerical examples.

**Example 2.** Reconsider *Example 1*, taking the head speed of the tunneling machine as 12 rev/min. Assume that driving rates will be of the order of 7 ft/hr and that chipping depths for the cutters will be approximately 0.1 in. Calculate the effects of skidding when the standard roller cutter of *Example 1* is used in a “flat-face” configuration at inside radii of 2.5 ft and 0.75 ft.

With a head speed of 12 rev/min, \(\dot{\theta} = 75.4\) rad/min. At the smallest radius to be considered \((r_c = 0.75\) ft), \(r_c \delta = 56.5\) ft/min. The axial penetration rate \(U\) is of the order of 7 ft/hr, or 0.117 ft/min. Thus \((U/r_c \delta)^2\) will never be more than about \(4 \times 10^{-6}\), and approximate versions of the relevant equations are completely adequate.

The amount of slippage between the rim of the cutter and the rock is given by eq 43. Considering the inner end of the roller when it is set at \(r_c = 2.5\) ft, the chipping depth \(\xi\) is taken as 0.1 in. and \(R_c\) is 4.5 in. Thus \(R_c \cos^{-1}\left(1 - \varphi/R_c\right) = 0.95\) in. In order to estimate the position and working radius of the neutral section, it is probably permissible to assume here that the neutral section is midway along the roller, guessing that the integrated resistances to positive and negative skidding will roughly balance out across the roller for the small chipping depth involved. With this assumption, \(r_c \approx (r_c + 5)\) in., and \(\overline{R}_c \approx 5.0\) in. This gives a slippage of approximately 0.045 in., which can be looked upon as a wear distance per revolution for each point on the rim, or alternatively as the tangential displacement involved in a drag bit effect. The skidding velocity \(u_s\), given by eq 46, is approximately 9 ft/min.

Similarly, when the inner end of the roller is at \(r_c = 0.75\) ft, the slippage at the inner end is 0.27 in., and the skidding velocity is approximately 16 ft/min.
Some types of roller cutters have long projections that indent deeply into soft rock and allow the disc to effectively roll on a rim that is at a smaller radius than the radius to the tips of the indenters. If the disc rolls without skidding, the tip of the indenter follows a path that loops back upon itself, giving a scooping action.

For a non-skidding “spiked” roller of the type just described, the idealized trajectory of the indenter tip (relative to the developed cutting track in the rock) is a prolate cycloid (Fig. 13). The equation for this kind of trajectory is

\[ \begin{align*}
    x &= r_w \phi - r_s \sin \phi \\
    y &= r_w - r_s \cos \phi
\end{align*} \]

where \( r_w \) is the radius to the rolling rim, \( r_s \) is the radius to the indenter tip, and the rolling is assumed to take place on the uncut surface at \( y = 0 \). As the indenter tip passes through the uncut surface \( y = 0 \), where \( r_w = r_s \cos \phi \), the slope of its trajectory \( dy/dx \) is

\[ \frac{dy}{dx} = \frac{r_s}{r_w - r_s \cos \phi} = \infty \]

i.e. the indenter tip enters and leaves the level of the uncut surface at right angles. During the time the indenter tip is in the work, it moves backward relative to the direction of travel of the roller by a distance \( \Delta x \) that is

\[ \Delta x = 2 \left[ r_w \cos^{-1} \left( \frac{r_w}{r_s} \right) - r_s \left[ 1 - \left( \frac{r_w}{r_s} \right)^2 \right]^{1/2} \right] . \]

This type of trajectory is somewhat idealized, but it is probably fairly realistic for an aggressive indenter thrusting into soft material. The equations afford a means of estimating the magnitude of the scooping action, and of assessing the effects of changes in cutter design and operating procedure.
Figure 14. Inside epicycloid, or hypocycloid.

Epicyclic or Planetary Mechanisms

There are a number of boring machines which carry one or more subsidiary rotors that have the axis of rotation more or less parallel to the axis of the main boring head and the axis of overall penetration. A simple example is a roller reamer, which rolls disc or drum cutters approximately normal to the hole wall, imparting a hypocycloid (inside epicycloid) motion to a point on the rim of a disc. A more complex example is the tunnel boring machine conceived by Wohlmeyer and subsequently developed by Krupp, Habegger and Atlas-Copco; this has multiple independently powered rotary cutting heads which are fitted to a main boring head that itself rotates. Planetary motion can also be produced by beam-type machines such as the Greenside-McAlpine heading machines.

When a reaming roller is operating, its center follows a helical path and it cuts a helical path in the rock. These paths are described by eq 2 when appropriate radii are inserted. Under typical rock-cutting conditions, the axial penetration rate $U$ is small compared with tangential speed $2 \pi R f$, and each revolution of the helical cutting path closely approximates a circle. When this situation prevails, a point on the rim of a non-skidding roller follows a trajectory that is an inside epicycloid, or hypocycloid (Fig. 14), described by the equations

$$
\begin{align}
\frac{R}{R_c} &= 1 + \frac{r_c}{R_c} \frac{\phi}{\theta} \\
\frac{r_c}{R_c} &= \delta \\
\frac{R}{R_c} &= \theta \\
\phi &= \theta \\
\end{align}
$$

$$
\begin{align}
x &= (R - R_c) \cos \theta + R_c \cos \left( \left( \frac{R - R_c}{R_c} \right) \theta \right) \\
y &= (R - R_c) \sin \theta - R_c \sin \left( \left( \frac{R - R_c}{R_c} \right) \theta \right)
\end{align}
$$

(50)
where the $x$ and $y$ axes are taken as shown in Figure 14, $R$ is the gauge radius of the cutting track (measured from the axis of the hole or tunnel), $R_c$ is the radius of the subsidiary roller cutter of the reaming machine, and $r_c$ is the radius from the center of the main boring head to the center of the subsidiary cutter.

With an independently driven planetary cutting head, cutter motion is more complicated. In this case, the subsidiary cutting heads approximate to axial-rotation drums fitted with peripheral drag bits (they can also have bits on the front face). As a sub-drum rotates at relatively high speed, it is carried slowly around a circular (or helical) path by the main drum or by a rotating arm, so that it mills a circular swath.* This type of motion is shown schematically in Figure 15. If the axis of the sub-drum is parallel to the main boring axis, then the trajectory of a cutting tool on the sub-drum is given by

$$\begin{align*}
    x &= r_c \cos \theta + R_c \cos \phi \\
    y &= r_c \sin \theta + R_c \sin \phi \\
    z &= \frac{Ut}{2\pi f}
\end{align*}$$

where $r_c$ is the radius from the center of the bore to the center of the sub-drum, $R_c$ is the radius of the sub-drum itself, $\theta$ is the angular displacement of the sub-drum center from the $x$-direction, and $\phi$ is the angular displacement of the sub-drum cutter from the $x$-direction. $\theta$ and $\phi$ are measured for the same time interval, and are measured in the same sense. If the main drum and the sub-drum are contrarotating, as in Figure 15b, $\phi$ takes a negative sign, so that, in eq 51, $\cos \phi$ remains positive while $\sin \phi$ becomes negative.

The velocity components for a tool on the sub-drum are

$$\begin{align*}
    \dot{x} &= -r_c \dot{\theta} \sin \theta - R_c \dot{\phi} \sin \phi \\
    &= -\dot{\theta} (r_c \sin \theta + NR_c \sin N\theta) \\
    \dot{y} &= r_c \dot{\theta} \cos \theta + R_c \dot{\phi} \cos \phi \\
    &= \dot{\theta} (r_c \cos \theta + NR_c \cos N\theta) \\
    \dot{z} &= U
\end{align*}$$

where the constant angular velocities $\dot{\theta}$ and $\dot{\phi}$ are linearly related by $\dot{\phi} = N\dot{\theta}$, and the main drum and the sub-drum are rotating in the same sense. For contrarotating drums, where $\dot{\phi} = -N\dot{\theta}$,

$$\begin{align*}
    \dot{x} &= -r_c \sin \theta - R_c \dot{\phi} \sin \phi \\
    &= -\dot{\theta} (r_c \sin \theta + NR_c \sin N\theta) \\
    \dot{y} &= r_c \dot{\theta} \cos \theta - R_c \dot{\phi} \cos \phi \\
    &= \dot{\theta} (r_c \cos \theta + NR_c \cos N\theta) \\
    \dot{z} &= U
\end{align*}$$

(note that in numerical calculation for this case, the two terms of $\dot{y}$ are of different sign).

* Straight-line slot milling is treated separately (see Mellor 1975).
Figure 15. Epicyclic motion of independently driven planetary cutting heads.
The absolute tool velocity \( u \) is \( [x^2 + y^2 + z^2]^{1/2} \). Differentiating \( u \) with respect to \( \theta \) and equating to zero gives the positions at which tool velocity \( u \) is a maximum or minimum; these positions are \( \phi = \theta, \theta + \pi, \theta + 2\pi, \) etc. Tool velocities at these positions \( (u') \) are
\[
\dot{u}' = [(R_c \dot{\phi} \pm r_c \dot{\theta})^2 + U^2]^{1/2}.
\]

For the case where the main drum and the sub-drum rotate in the same sense, maximum tool velocity is reached at the gauge radius \( r = (r_c + R_c) \), while minimum velocity is reached at radius \( r = (r_c - R_c) \). For contrarotation of main drum and sub-drum, the converse is true.

At any radius \( r \) from the center of the bore, the cutting track in the rock is helical, with a helix angle of \( \alpha = \tan^{-1} (U/r\dot{\theta}) \). If the sub-drum is armed only with peripheral cutting tools, then obviously it must have its axis inclined to the direction of the main boring axis if it is to operate properly. Failure to provide appropriate inclination would result in the trailing part of the drum face scraping against uncut rock. To provide the necessary clearance, or relief angle, the axis of the sub-drum has to be inclined in the tangential \( \theta-z \) plane such that the leading edge of the sub-drum is farther advanced in the \( x \) direction than is the trailing edge. Inclination of the sub-drum axis in the radial \( r-z \) plane makes no contribution to the relief angle under discussion, although a designer may have other reasons for providing some inclination in this place. The critical main drum radius for determination of sub-drum relief angle is \( r = (r_c - R_c) \), since the helix angle \( \alpha \) for points under the sub-drum reaches its maximum value there. Thus the minimum requirement for inclination of the sub-drum in the \( \theta-z \) plane \( (\chi) \) is
\[
\chi > \tan^{-1} \left( \frac{U}{(r_c - R_c)\dot{\theta}} \right).
\]

**Chipping Depth and Penetration Rate**

On simple axial rotation machines, such as bladed rotary drills or drag-bit full-face tunnel borers, all points on the cutting head advance axially at the same penetration rate \( U \) (length per unit time), and revolve about the center at the same angular velocity \( \omega \) (radians per unit time) or at the same rotational frequency \( f \) (revolutions per unit time). Thus the axial penetration of a single independent cutter into uncut material \( q_a \) is the same for all positions on the boring head:
\[
q_a = \frac{U}{fn}
\]

where \( n \) is the number of tracking cutters on the boring head (i.e. the number of cutter repetitions at any given radius \( r \) for a 360° sweep), and \( n \) is the same for all radii.

If all the cutters lie on a single plane that is normal to the penetration axis, then the advancing surface is a helical ramp defined by eq 2 for all values of \( r \), from \( r = R_1 \) to \( r = R_0 \). Under these circumstances, the chipping depth measured normal to the advancing surface \( \delta \) is

\[\text{Figure 16. Helical chipping track.}\]
$\ell = \ell_a \cos \alpha$

$$= \frac{U}{f n} \cos \left[ \tan^{-1} \left( \frac{U}{2 \pi r f} \right) \right]$$

$$= \frac{U}{f n} \left[ 1 + \left( \frac{U}{2 \pi r f} \right)^2 \right]^{-\frac{1}{2}}$$

$$= \frac{U}{f n} \left[ 1 + \left( \frac{U}{u_t} \right)^2 \right]^{-\frac{1}{2}}. \quad (57)$$

Whereas $\ell_a$ is the same for all positions on the boring head, $\ell$ varies with $r$, going to zero at $r = 0$.

When individual cutters are stepped relative to each other in the axial direction, but still have their planing edges normal to the penetration axis, the advancing surface is a radially stepped set of concentric helices. However, the chipping depth $\ell$ remains as given by eq 57.

If the cutting head is such that it traces out a conical surface when $U = 0$ (as in a metal-cutting twist drill), then the chipping depth normal to the advancing surface for appropriately set cutters ($\ell'$) is

$$\ell' = \ell \sin \psi$$

$$= \frac{U}{f n} \left[ 1 + \left( \frac{U}{2 \pi r f} \right)^2 \right]^{-\frac{1}{2}} \sin \psi \quad (58)$$

where $\psi$ is the half-angle of the conical tip of the boring head. In this case $\ell'$ tends to zero for all values of $r$ when $\psi$ tends to zero.

For other shapes of boring head (say hemispherical or otherwise domed), penetration normal to the advancing surface can be expressed by taking something equivalent to a local value of $\psi$ when cutters are set normal to the head profile. If the acute angle between the cutter axis and the penetration axis is $\chi$ (measuring in a radial plane through the cutter), then

$$\ell' = \frac{U}{f n} \left[ 1 + \left( \frac{U}{2 \pi r f} \right)^2 \right]^{-\frac{1}{2}} \cos \chi. \quad (59)$$

An interesting feature of eq 59 is that it offers the possibility of having the chipping depth $\ell'$ more nearly uniform over the advancing surface. The condition for complete uniformity of $\ell'$ with respect to $r$ would be inverse proportionality between $\cos \chi$ and $\cos \alpha$; this is unattainable, but approximate proportionality between $\cos \chi$ and $[1 + (U/u_t)^2]^{\frac{1}{2}}$ could be arranged over the main part of the cutting head.

If a machine is to operate effectively, then there obviously must be some practical limits to $\ell$. For example, the chipping depth of a drag bit should be significantly greater than the tool tip radius if the tool is not to just grind ineffectually against the rock, but it should not be so great that the tool plunges too deeply or produces chips that are too big for easy clearance in the annulus between the drill stem and the hole wall. Similarly, the chipping depth of a studded roller bit cannot usefully exceed the projecting length of the studs, and the chipping depth of a diamond bit cannot exceed the exposure of the stones. These considerations immediately narrow down the range of possibilities for rotational speed, as can be seen from a simple example.
Example 3. A 3.5-in.-diam ice auger is required to penetrate at a maximum rate of 15 ft/min. The bit has two symmetrical wings ground to give continuous cutting edges, and each cutting edge is inclined at an angle of 75° to the penetration axis (measuring in a radial plane). The cutting edges terminate at a radius of 0.3 in. from the center. The bit feeds cuttings to a flight that has a stem diameter of 0.9 in. What range of rotational speeds would be reasonable to consider, taking into account the possibility of the actual penetration rate dropping to one-third of the design value under adverse conditions?

From eq 58, the rotational speed \( f \) is

\[
f = U \left( \frac{\sin \psi}{n \sigma} \right)^2 - \left( \frac{1}{2\pi r} \right)^2 \]

in which maximum penetration rate \( U \) is 180 in./min, the number of tracking cutters \( n \) is 2, the angle \( \psi \) is 75°, and the chipping depth \( \ell' \) has to be estimated. If the bit is to cut chips of ice rather than to shave, abrade or polish, then \( \ell' \) should probably be not less than about 1 mm, or about 0.04 in. Since the width of the auger flight is 1.3 in. and chips of brittle ice could be as much as 3 \( \ell' \) in length, \( \ell' \) should certainly not be more than about 0.3 in. for easy transport, but power and torque considerations suggest a more conservative limit of 0.2 in. With \( 0.04 < \ell' < 0.2 \) in., the speed range indicated for penetration at the full design rate of 15 ft/min is \( 430 < f < 2170 \) rev/min (the effect of the term containing \( r \) is negligible for practical purposes in this example).

When penetration rate drops to one-third of the maximum rate, i.e. to \( U = 60 \) in./min, the indicated speed range is \( 140 < f < 720 \) rev/min.

Combining the two sets of limits for \( f \), attention would be directed to drive units giving speeds in the range 430 to 720 rev/min.

On machines fitted with rolling cutters (e.g. rotary drills with roller rock bits, or tunnel borers with disc cutters or studded rollers) the chipping depth relationships are virtually identical to the equations developed for drag bits. In the case of studded rollers, chipping depth can set very firm limits on the operating performance of a machine, as is illustrated in the following example.

Example 4. A hard-rock tunneling machine encounters a region of relatively weak rock, and the driving rate increases abruptly from the 9 ft/hr rate that prevailed previously in stronger rock. Using full thrust in an effort to gain maximum production, an advance speed of 20 ft/hr is achieved. However, the studded disc cutters begin to show unusually rapid wear; the tungsten carbide studs themselves are holding up well, but the steel rims into which they are set are being ground away. The machine has a typical full-face boring head that rotates at 9 rev/min, and the roller cutters are set on the flat face plate in the usual way, with their bearing axes radial to the main drum. The diameter of the boring head is 12.5 ft, and the innermost face cutter tracks at a radius of 1.2 ft (a separate center cutter is installed). For each circular cutting track there is only one disc on the boring head. The tungsten carbide studs project 0.375 in. from the rim of the disc. The problem is to determine the reason for the rapid wear of the cutters.

Applying eq 7, the chipping depth of the cutters \( \ell \) is

\[
\ell = \frac{U}{fn} \left[ 1 + \left( \frac{U}{2\pi rf} \right)^2 \right]^{-\frac{1}{2}} .
\]

As far as the main face cutters of this machine are concerned,
so that for practical purposes all face cutters can be taken as chipping to a uniform depth of

\[ \ell = \frac{U}{fn} \]

where \( U = 20 \text{ ft/hr (4 in./min)} \), \( f = 9 \text{ rev/min} \), and \( n = 1 \). Thus the cutters are penetrating the rock to a depth of 0.444 in. The carbide studs project only 0.375 in. from the rim of the disc, and therefore the rim will be thrust against uncut rock unless the overbreak angle of the chipping craters is large enough to provide clearance. The maximum driving speed that can be reached without the chipping depth exceeding the stud height is

\[ U = 0.375 \times 9 \times 1 \text{ in./min} = 16.9 \text{ ft/hr} \]

In the real situation from which this example was taken, the tunnel face bore clear imprints of the studs, indicating only small overbreak angles in the soft rock. Thus there was no reasonable doubt that the cutters were being overdriven.

**Chip Production and Cutting Removal Rate**

With penetration rate \( U \) in a hole of radius \( R \), the volumetric cutting rate \( \dot{V} \) is

\[ \dot{V} = \pi R^2 U. \]  

This represents “in place” volume, but the loose mass of chips produced by the cutting process occupies a larger volume. Introducing a bulking factor \( K_b \) (where \( K_b > 1 \)) to represent the decrease in bulk density and increase in porosity, the volumetric production rate for loose cuttings \( \dot{V}_c \) is

\[ \dot{V}_c = K_b \pi R^2 U. \]  

In another section of this work (Mellor 1975) a value of \( K_b = 1.85 \) was suggested for processes involving chipping of brittle materials. With this value, \( \dot{V}_c = 5.8 R^2 U \). A value of approximately 1.85 is probably appropriate for dry cuttings held in loose contact under gravity body forces (as in auger drills or tunnel boring machines), but higher values might be needed where inertial forces or suspensions are involved (as in high-speed rotary snow plows or fluid circulation drilling systems).

For continuous operation, \( \dot{V}_c \) is the rate at which cuttings have to be removed from the boring head by the clearing system (auger flights, screw conveyor, belt conveyor, fluid circulation, ejection chute, etc.).

If \( A = \pi R^2 \) is the cross-sectional area of the borehole and \( a \) is the effective cross section available for cutting removal, the axial clearing speed \( u_a \) must be

\[ u_a > K_b \frac{A}{a} U. \]  

While eq 60-62 may appear obvious, or even trivial, they are important in establishing compatibility between the penetrating system and the clearing system. It is by no means uncommon for axial rotation boring machines to stall or jam when they are overdriven to the point of overwhelming the clearing system.
Balancing Cutting and Clearing Rates in Flight Augers

An auger flight lifts cuttings by sliding them up an inclined plane. Motion up the inclined plane is resisted by friction and gravity forces, and if these resisting forces are to be overcome, then the cuttings have to be subjected to some reaction greater than the resistance to motion up the ramp. In an auger, the conveying resistance is determined largely by the frictional properties of the flight and stem, and by the steepness of the helix. The reaction forces that prevent cuttings from simply swirling around are provided by end conditions at the cutting head and by friction against the hole wall. They can also be supplemented by inertia when the auger accelerates. In the initial stage of conveyance, when cuttings are loading into the flight from the cutting head, the flight can be imagined as “shoveling” chips from a pile at the base of the hole. At a later stage of transport, however, cuttings may have to react against the hole wall if they are to progress up the flight. An ideal situation would provide hole wall friction that was high in the tangential direction and low in the axial direction.* The situation is quite similar for a screw conveyor, except for the fact that gravitational forces act at different angles to the screw axis.

On a continuous-operation machine, the clearing capability of the conveyor should obviously exceed the production capability of the cutting head. This has been stated in eq 62, which can be rewritten for augers as

$$u_a (R_h^2 - R_s^2) > K_b (1 - \frac{t_a}{P}) UR_h^2$$ for a plain auger (63)

or

$$u_a (R_h^2 - R_s^2) > K_b (1 - \frac{t_a}{P}) UR_h^2$$ for a coring auger (64)

where $R_h$, $R_s$, $R_c$ are the radii of the hole, the auger stem and the core, respectively (Fig. 17), $t_a$ is the flight thickness measured in the axial direction, and $P$ is the pitch distance (axial distance between successive wraps of the flight).

If there is perfect peripheral restraint of the cuttings (i.e. cuttings do not whirl around the hole), then the axial velocity $u_a$ is

$$u_a = 2\pi R_h f \tan \alpha_R = fP$$ (65)

where $\alpha_R$ is the outside helix angle of the flight. There will usually be some peripheral slip between the cuttings and the hole wall, and to allow for this, eq 65 can be written as

$$u_a = F_s fP$$ (66)

where $F_s$ is a dimensionless slip factor ranging from zero to unity.

Substituting from eq 66 into eq 63 and 64, the conditions for effective clearing are

* This may be practically feasible when drilling inside a casing, as the inside of the casing can have longitudinal ribs or flutings. J. Rand of CRREL has applied this concept successfully with bristle-edged auger flights running inside a casing that has a vertical rib.
Plain auger \( \frac{fP}{U} > \frac{K_b}{F_s} \frac{(1 - t_a/P)}{1 - (R_s/R_h)^2} \) \hspace{1cm} (67)

Coring auger \( \frac{fP}{U} > \frac{K_b}{F_s} \frac{(1 - t_a/P)}{1 - (R_c/R_h)^2} \) \hspace{1cm} (68)

With typical equipment, \((t_a/P) < 1\), \((R_s/R_h)^2 < 1\), and \((R_c/R_h)^2 < 1\), so that for most practical purposes the condition for effective clearing can be written as

\[ \frac{fP}{U} > \frac{K_b}{F_s} \] \hspace{1cm} (69)

The slip factor \( F_s \) is not likely to be known, but since it cannot be greater than unity, it is absolutely essential that \( fP/U \) should be greater than \( K_b \). The practical relevance of these conditions can be seen from a simple numerical example.

**Example 5.** The ice auger considered in Example 3 has a pitch distance of 3 in. and its flight thickness (measured axially) is 0.125 in. Check for adequacy of cutting clearance at the maximum penetration rate with drive speeds at the extremes of the proposed range:

\[
\begin{align*}
&f = 430 \text{ - } 720 \text{ rev/min} \\
&U = 15 \text{ ft/min} = 180 \text{ in./min} \\
&P = 3 \text{ in.} \\
&t_a = 0.125 \text{ in.} \\
&R_h = 1.75 \text{ in.} \\
&R_s = 0.45 \text{ in.}
\end{align*}
\]

Taking a value of \( K_b = 1.85 \) and substituting into eq 67, the unknown slip factor \( F_s \) must be

\[
F_s > 0.265 \quad \text{for } f = 430 \text{ rev/min}
\]

\[
F_s > 0.158 \quad \text{for } f = 720 \text{ rev/min}.
\]

Thus the higher drive speeds provide a greater margin of safety for clearing cuttings.

**Tool Relief Angles – Kinematic Considerations**

When a cutting tool penetrates the work along an inclined path, its shoulder, or flank, has to be relieved, or inclined, to prevent scraping of that part of the tool which trails the cutting edge. The usual way of specifying this *relief angle* (Fig. 18 and 19) is with reference to directions on the boring machine. The *apparent* relief angle \( \beta_2 \) is the angle between the shoulder of the tool and a plane that is normal to the main boring axis, measuring in a direction that is tangential to the circle of rotation. The minimum required relief angle, or "kinematic" relief angle, \( \beta'_2 \) is set by the helix angle of the tool’s penetration path \( \alpha \), i.e.

\[
\beta'_2 \geq \tan^{-1} \left( \frac{U}{r \dot{\theta}} \right)
\]

or

\[
\beta'_2 \geq \tan^{-1} \left( \frac{U}{2\pi rf} \right).
\] \hspace{1cm} (70)
Figure 18. Reference directions for definition of tool angles.

Figure 19. Tool angles for drag bits on rotary boring heads.
Since \( U \) and \( f \) have uniform values for all points on a simple boring head, the kinematic relief angle \( \beta_2 \) has to increase as the radius to the tool position decreases, and theoretically it has to be 90° at the exact center of the boring head. If rotary speed \( f \) is fixed and penetration rate \( U \) varies, \( \beta_2 \) has to increase at all points as \( U \) increases. If penetration rate \( U \) is fixed and rotary speed \( f \) varies, then \( \beta_2 \) has to increase at all points as \( f \) decreases.

There are also dynamic considerations in the design of the complete relief angle \( \beta_2 \), since the tool has to be able to penetrate the work with a minimum of normal force. In practical terms, this usually means that 5° or more is added to the kinematic relief angle \( \beta_2 \) in order to obtain the final value of \( \beta_2 \).

In some types of tools, the dynamic and kinematic requirements can be separated by the tool geometry. A high primary relief angle can be provided to minimize thrust requirements, while a smaller effective relief, or clearance, angle can be set so as to limit chipping depth (see Fig. 19c). This may be done to avoid excessive force, torque, or power demand, or else to control the quality of cut, e.g. in coring work.

When tool relief angle is known, eq 70 can be used to calculate a practical limit of penetration rate for any given rotary speed, since thrust demands will rise sharply to high values when the helix angle reaches the same value as the relief angle or clearance angle. This can be illustrated by a numerical example.

**Example 6.** An ice coring drill has a cutting head that consists of a thick annulus pierced by two planing blades at opposite ends of a diameter (see diagram). Internal and external diameters of the cutting head are 3.0 in. and 4.3 in. respectively. Although the primary relief angle of each planing blade is a constant 20°, the effective clearance angle is determined by the projection of the blade from the face of the mounting ring (see Fig. 19c). This projection distance is 0.16 in., and the circular distance from the cutting edge of the tool to the opening in the mounting ring for the second cutter is 45% of the circumference. What is the kinematic limit of penetration rate at 300 rev/min, assuming that the auger flights can clear cuttings adequately?

With this tool geometry, the effective clearance angle \((\beta_2)_{\text{eff}}\) sets the kinematic limit of penetration rate. For the limit condition:

\[
(\beta_2)_{\text{eff}} = \tan^{-1} \left( \frac{0.16}{0.45 \times 2\pi r} \right) = \tan^{-1} \left( \frac{U_{\text{max}}}{2\pi f} \right)
\]

or,

\[
U_{\text{max}} = \frac{0.16 \times 300}{0.45} = 107 \text{ in./min}
\]

\[
= 8.89 \text{ ft/min}.
\]

Critical conditions are reached simultaneously at all radii in this case.

When a machine is boring at something less than the limiting penetration rate \( U_{\text{max}} \), the actual relief angle of the tool relative to the work is the difference between the apparent relief angle \( \beta_2 \) (as defined in Fig. 19a, b and c) and the helix angle of the tool's penetration path \( \alpha \) (as given by eq 5):

\[
(\beta_2)_{\text{actual}} = \beta_2 - \alpha. \tag{71}
\]

This is illustrated in Figure 19d.
Tool Rake Angles

As is the case for relief angle, the conventional way of specifying rake angle on an axial rotation machine is with reference to the machine itself. By this convention, the apparent rake angle $\beta_1$ is the angle between the leading face of the tool and the main boring direction, measuring at the cutting tip in a plane that is tangential to the tool’s circle of rotation (Fig. 19).

The principal significance of rake angle is its influence on cutting forces, which generally decrease with increasing positive rake. In this connection, it is the actual rake angle, i.e. the rake angle relative to the work, that matters. The actual rake angle (Fig. 19d) is the apparent rake angle $\beta_1$ plus the helix angle of the tool’s penetration path $\alpha$:

$$(\beta_1)_{\text{actual}} = \beta_1 + \alpha.$$  (72)

Since $\alpha$ approaches $90^\circ$ as radius tends to zero, it is possible for tools with large apparent negative rake to have actual rake angles that are positive.

Rake angle is often implicitly related to relief angle by virtue of the fact that the included angle $\beta_3$ has to stay within certain limits. This interrelationship can be seen in the following example.

**Example 7.** A small auger drill of 1.5 in. diameter is required for shallow-depth boring in fine-grained frozen soils and in ice. The drill is to be capable of penetration rates up to 12 ft/min with a 400 rev/min power drive, and up to 3 ft/min with 100 rev/min hand drive. Design the rake and relief angles so as to permit fabrication of a prototype bit, paying special attention to the center of the bit. A symmetrical 2-wing bit design is favored for smooth running. Cutter tips are to be of tungsten carbide, and $60^\circ$ is judged to be the minimum included angle for the required durability.

The helix angle at maximum design penetration rate ($\alpha_m$) is given by eq 5:

$$\alpha_m = \tan^{-1} \left( \frac{U_m}{2\pi r f} \right) = \tan^{-1} \left( \frac{12 \times 12}{2\pi \times 400} \right) = \tan^{-1} \left( \frac{3 \times 12}{2\pi \times 100} \right)$$

$$= \tan^{-1} \left( \frac{0.0573}{r} \right)$$

where radius $r$ is in inches. Values of $\alpha_m$ are given in the graph (p. 38).

The required minimum values of apparent relief angle $\beta_2$ are equal to the values of maximum helix angle $\alpha_m$ eq 70. However, from knowledge of the dynamics of drag bits, it is decided that actual relief angle should never be less than $5^\circ$, i.e. from eq 71:

$$5^\circ = \beta_2 - \alpha_m$$

or

$$\beta_2 = \alpha_m + 5^\circ.$$  

Values of $\beta_2$, which give actual setting angles to be used by the maker of the drill, are shown in the graph.

Apparent rake angle $\beta_1$ is determined by the apparent relief angle $\beta_2$ and the included angle $\beta_3$: 
\[
\beta_1 = 90° - (\beta_2 + \beta_3)
\]
\[
= 90° - \left[ \tan^{-1} \left( \frac{0.0573}{r} \right) + 5° \right] - 60°
\]
\[
= 25° - \tan^{-1} \left( \frac{0.0573}{r} \right).
\]

Values of \(\beta_1\) are given in the graph.

The actual rake angle is the apparent rake \(\beta_1\) plus the helix angle \(\alpha\) (eq 72). At the maximum penetration rate

\[
(\beta_1)_{\text{actual}} = \beta_1 + \alpha_m
\]
\[
= 90° - (\alpha_m + 5° + \beta_3) + \alpha_m
\]
\[
= 25°.
\]

The graph also gives values of actual rake angle at half of the maximum penetration rate.

Very close to the center of the bit there are problems. Apparent relief angle has to be large, actual rake angle becomes negative at penetration rates below the design maximum, and tool speed drops to small values. An arbitrary value of \(r/R = 0.1\) has previously been mentioned as a limit for difficult center conditions, and this can be taken as an inner termination radius for the wings of the bit. If the center section of the bit is left without cutters out to a radius of 0.075 in., the bit will tend to form a core of 0.15 in. diameter. If the center section is relieved so that a core can develop up to a length of 0.3 in. (length/diameter ratio of 2), then the core will easily break off and form fragments that can clear easily. The proposed inner cutoff is shown in the graph.

To build the bit, carbides are formed or ground to give a 60° cutting edge (\(\beta_3\)), and they are set on the wings (perhaps in milled pockets) at the angles \(\beta_1\) and \(\beta_2\) appropriate to their radii.
Distribution and Spacing of Cutting Tools

In general, only a very small proportion of the area of the advancing surface is in contact with cutting tools at any given instant. This is obviously a necessary condition if the cutting tools are to achieve adequate penetration under limited axial thrust; with too many cutters on the boring head, the pressure exerted by each cutting tip would drop below the pressure needed for indentation. Even if adequate normal penetration could be achieved for a large number of tools, the torque requirements for the boring head could become prohibitive. Thus there is a problem of spacing the tools and cutting edges to achieve efficient cutting and to avoid imbalance of forces and moments. This tends to become increasingly important as the size of the machine increases.

Radial tool spacing

If the cutting element of an axial-rotation boring head consists of one or more continuous radial blades, then there is complete cutting over the whole area of the advancing face. This is referred to as “100% coverage.” However, in rock-boring machines and similar devices, it is common to have discrete tools set at various radii in such a way that they sweep out a concentric set of cutting tracks (kerfs) that are separated by ribs of uncut material. The idea is that the uncut ribs will be removed indirectly, either by lateral overbreak to the sides of the cutters, or by eventual formation of an unstable rib after several cutter passes.

The determination of optimum spacing between kerfs is largely related to considerations of tool forces, total machine forces, and specific energy consumption, and it will be treated in detail in the sections of the work that deal with dynamics and energetics. However, since the geometric aspects are relevant to the kinematics of the machine, some discussion is included here. The question of lateral spacing for drag bits has already been broached in an earlier report (Mellor 1975), but a restatement is in order because of the difference in operating characteristics for transverse rotation and axial rotation machines.

Figure 20 represents a cross section of two radially adjacent cutting tracks that are being swept out by simple chisel-edge drag bits. The two bits are not necessarily set on the same radial of the boring head, but there is no other cutter that operates within the radial interval \( s \) between their tracks. If the “overbreak angle” \( \phi \) is a characteristic of the rock and the cutting process, as is often assumed,* the space between the cutters \( (s - w) \) is a function of the chipping depth \( \ell \) and the overbreak angle \( \phi \):

\[
s - w = x + 2\ell \tan \phi . \tag{73}
\]

The two adjacent cutting tracks will definitely begin to interact when \( x \) falls to zero, i.e. when

\[
\frac{s - w}{\ell} \leq 2 \tan \phi . \tag{74}
\]

In principle, interaction might occur with positive values of \( x \) because of stress field perturbation, but upon investigation this seems unlikely under real conditions. For typical rocks that “cut” by brittle fracture, the overbreak angle seems to be in the range 50° to 70°, so that 2 \( \tan \phi \) would be in the approximate range 2.5 to 5.5. Presumably the more ductile materials would extend this range to lower values, while highly friable materials could extend the range to higher values. These

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* This seems a questionable assumption; it is possible that the effective value of \( \phi \) will decrease with increase of \( \ell \) when a sufficiently wide range of \( \ell \) is considered.
values of $2 \tan \phi$ represent the critical lateral spacing, beyond which each cutter operates independently of its neighbors.

If the spacing drops below the limit given by eq 74, the actual chipping depth is less than the theoretical chipping depth $\xi$ that is given by eq 56 and 57, since each tool is sweeping along within the overbreak zone of its neighbors.

For optimum working conditions, where specific energy consumption is minimized, it is probably desirable to have the two side breaks overlapping, but not to the extent that the cutters themselves are overlapping. From simple geometrical considerations, one possible lower limit for advantageous interaction might be

$$\frac{s - w}{\xi} \geq \tan \phi.$$  \hspace{1cm} (75)

Under the idealized conditions represented in Figure 20, the width of the cutter $w$ is largely irrelevant in determining the required space between cutters for obvious reasons. Nevertheless, some practical recommendations for optimum spacing have been expressed in terms of $s/w$, without explicit reference to $\xi$. This is perhaps understandable in the case of transverse rotation machines, where $\xi$ usually varies continuously between zero and some maximum during the working sweep of the tool, but it could lead to a certain amount of confusion.

Roxborough (1973) and Roxborough and Rispin (1973a, 1973b) experimented on sandstone, limestone, anhydrite, dry chalk, and wet chalk. In most cases, direct experimental determination of the critical tool spacing (maximum for interaction) gave values close to the theoretical value expressed in eq 74. Optimum spacing, at which specific energy was minimized, was approximately half the critical spacing, or slightly less than half in some cases. Thus, eq 75 may not be too unrealistic.

Valantin et al. (1964) and Fourmaintraux (1972) suggested an upper limit of cutter spacing for efficient working as

$$\frac{s - w}{\xi} \leq 1.$$  \hspace{1cm} (76)

However, Barker (1964) obtained best results (minimum specific energy) at wider spacings:
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![Diagram of rolling disc cutters](image)

Figure 21. Radial spacing of rolling disc cutters.

chisel edge pick \( \left( \frac{s - w}{\ell} \right)_{opt} \approx 1.0 - 1.7 \)

pointed pick \( \left( \frac{s - w}{\ell} \right)_{opt} \approx 1.5 - 2.0 \) \( (77) \)

In very ductile materials, the overbreak angle \( \phi \) may have very small values, so that the cutter tends to carve a groove that conforms to its own cross section. Under these circumstances it may be advantageous to space the tools so that the uncut rib is an unstable cantilever that breaks easily across the base. The writer has made model tests to assess kerf-rib breakage in frozen silt, finding that the ribs break easily across the base when the ratio of rib height to rib width reaches about 1.5. This would suggest a spacing criterion for such material as

\[ \frac{s - w}{\ell} \approx 0.7. \] \( (78) \)

On axial rotation machines there is also the question of suitable lateral spacing for roller cutters. For present purposes it is convenient to base the discussion on a pair of simple disc cutters, as shown in Figure 21. The considerations are essentially similar for spacing of studded discs, and for the spacing of adjacent rings of studs on wide roller cones.

In Figure 21, \( s \) is the center-to-center spacing of two adjacent cutting tracks, but the disc cutters that produce them (shown in cross section) are not necessarily on a common radial. In this case, \( w \) is the effective contact width of the rim of the disc, controlled largely by the radius at the tip of the bevel and the crushing characteristics of the rock and debris at the base of the groove. The half-angle of the beveled rim is \( \psi \), which typically lies in the range 30° to 45°. The overbreak angle \( \phi \) will usually be greater than \( \psi \) in brittle materials, but it is not necessarily the same as the overbreak angle for drag bit cutting in the same material (nor is it necessarily the same for different disc cutters).* Under these conditions, the critical spacing for the limit of interaction should again be

\* Work by Morrell and Larson (1974) suggests that, for bevel-edge disc cutters, \( \phi \) increases as \( \psi \) decreases. The value of \( \tan \phi \) increased by a factor of 1.32 as \( \psi \) decreased from 45° to 30°.
given by eq 74, and eq 75 might still be considered a reasonable hypothesis for approximate optimum spacing.

Roxborough and Rispin (1973a) measured overbreak angle \( \phi \) for cuts made with discs of edge angle \( (= 2\psi) \) 60° and 90°. For wet chalk, \( \phi \) was in the range 72° to 74°, which was a good deal higher than the range 53° to 57° obtained for drag bit cutting in the same material. In dry chalk, \( \phi \) for disc cutting was 74° to 80°, compared with 67° to 70° for drag bit cutting in the same material. With these values of \( \phi \), 2 tan \( \phi \) is 6.2 to 7.0 for wet chalk and 7.0 to 11.3 for dry chalk. The direct experimental values for critical spacing, defined as \( s/\ell \) in this case, were 6 to 8 for wet chalk and 8 to 12 for dry chalk. Values of \( w \) were not measured, but with penetrations in the range 5 to 15 mm, it might be reasonable to guess that \( w/\ell \) was in the range 0.1 to 1.0. Thus, there seems to be very fair agreement between the calculated values of \( (s - w)/\ell \) from eq 74, and the measured values of \( s/\ell \) based on specific energy trends. For optimum disc spacing, eq 75 might suggest values of \( (s - w)/\ell \) as 3.1 to 3.5 for wet chalk and 3.5 to 5.7 for dry chalk. The directly measured values of \( s/\ell \) were 2 to 4 for wet chalk and 3 to 5 for dry chalk; i.e. the specific energy trend suggested a slightly closer optimum spacing than would have been deduced from eq 75.

Other published results for disc cutting experiments (Morrell and Larson 1974, Rad and Olson 1974, Rad 1975) are less easy to work with because of differences in experimental design and data handling. Whereas Roxborough and his associates imposed constant cut depth \( \ell \) while monitoring normal and tangential force components (which seems a realistic simulation for large multiple cutter machines), the other groups imposed constant normal force, measuring cut depth and tangential force. Rad (1975) tested marble, limestone, granite and quartzite, and gave absolute values of critical and optimum spacing. These values cannot be normalized with respect to cutting depth directly, since depth of cut varied with groove spacing. However, normalizing with respect to cut depth for an independent groove, \( s/\ell \) had critical values from 10 to 15 and optimum values from 6 to 7. Rad and Olson (1974) found that the ratio of optimum spacing to critical spacing was in the range 0.43 to 0.78 for their experiments, with no systematic trends evident.

**Angular tool spacing**

In planning the distribution of cutting tools across the projected area of the boring head, probably the major factors to be taken into account are 1) structural considerations (how to attach and support the tools) and 2) smooth running characteristics (elimination of transverse oscillations of the head). The first of these may require that cutters be well distributed over the available area, rather than concentrated along a few radials. The second demands that there should be no unbalanced moments at any point on the head.

In a simple arrangement where cutters are arrayed along two or more lines, as in Figure 22a, it is obviously desirable to have the radial arms at uniform angular spacing. On each of \( n \) arms there is a resultant of the tangential components of cutting force \( F_t \), acting at a radius \( r \), and the sum of the moments about the center of the head, \( nrF_t \), equals the applied torque \( T \). With equal angular spacing \( 2\pi/n \), the moments sum to zero for all points on the head, and there is no tendency for turning about any axis other than the central axis. With unequal spacing, this generally ceases to be the case, and there is a tendency for eccentric running. If simple radial lines of cutters are inconvenient for structural reasons, then the individual cutters can be dispersed across the head by displacing complete rings of cutters, such that angular spacing remains uniform at any given radius (Fig. 22b, c, d). This will not disturb the balance of moments.
In an arrangement where there is only one set of tracking cutters, the situation is not quite as straightforward. With a single radial line of cutters, such as is often employed with a single-start flight auger, there are obviously unbalanced moments that have to be resisted by some firm restraints, and cutting forces are concentrated on one part of the boring head. This can be tolerated on some pieces of equipment, but on others it may be necessary to disperse the cutters in such a way that moments about any off-center point sum to zero or to some acceptably small value. When there are many individual cutters, this is not very difficult. For example, the cutters can be arrayed along a complete diameter, with alternate cutters on one radial arm running in the spaces left by cutters on the other arm. When there are very many cutters, as on a typical tunnel boring machine, they can be dispersed over the head in an approximation to the type of arrangement illustrated in Figure 22c. More generally, if $N$ cutters are set out as shown in Figure 23, and each cutter is assumed to experience equal tangential resistance $f_t$, then a condition for balanced running can be obtained by taking moments about a point such as $A$:

$$
\sum_{n=1}^{n=N} f_t (R \cos \theta_n - r_n) + T = 0
$$

(79)

where $r$ is the radius of action of the cutter resistance $f_t$, $R$ is the head radius, $\theta$ is the angle between the radial through $A$ and the radial through the cutter, and $T$ is the applied torque. This matter will be discussed further in the section on dynamics of axial rotation machines.
In principle, angular spacing could also take into account the possibility of using a phase lag to optimize lateral step formation between adjacent cutting tracks, in a manner corresponding to the step-cutting that was considered for helical tool arrays on transverse-rotation machines. However, the effect is so slight compared to the radial stepping, which can be achieved from head profiles (as discussed below), that it is not worth considering.

**Spacing in the axial direction**

Cutters are often offset relative to each other in the axial direction, with the result that the face being cut has a non-flat profile. This can yield a number of advantages, including lateral stabilization of the head, the potential for easier cutting, special treatment of center conditions, and convenience in clearing cuttings.

The center of a boring head commonly has cutters in advance of the main head, e.g. a pilot bit or mandrel (Fig. 24a). An alternative, which actually may be more efficient, is to have the center of the head recessed, so that a small core can form (Fig. 24b).

Another fairly common arrangement for the outer parts of the head is to have the cutters stepping back as radius increases, so that each cutting track has a free surface to one side (Fig. 24). When this arrangement is adopted, it may be advantageous to leave an uncut rib between the kerf and the free edge, taking advantage of the tendency for breakout. No formal discussion of this has been found in the literature, but it might be desirable to have the height of the step at least as great as the width, and of course the step height should exceed the maximum anticipated chipping depth.

In the case of flight augers, stepping in the axial direction is usually achieved by setting cutters on the lip of the flight at different angular positions. Here the axial displacement depends on the radius of the cutter position, the helix angle, and the pitch of the flight, and it may be necessary to consider the conditions for balanced running, as discussed above.
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Literature Cited


