Analysis of explosively generated ground motions using Fourier techniques
Analysis of explosively generated ground motions using Fourier techniques

Scott E. Blouin and Stephen H. Wolfe

August 1976
# ANALYSIS OF EXPLOSIVELY GENERATED GROUND MOTIONS USING FOURIER TECHNIQUES

**Authors:** Scott E. Blouin and Stephen H. Wolfe

**Performing Organization:**
U.S. Army Cold Regions Research and Engineering Laboratory
Hanover, New Hampshire

**Contract or Grant Number(s):** Air Force Weapons Laboratory Project Order 74-173

**Program Element, Project, Task Area & Work Unit Numbers:**
CCC: 626710 DC: SCA
Job Order No. WDNS 3317, Defense Nuclear Agency MIPR No. 74-551

**Report Date:** August 1976

**Number of Pages:** 91

**Distribution Statement (of this report):** Approved for public release; distribution unlimited.

**Abstract:**
Fourier transforms of selected ground-motion time histories from five underground high-explosive and nuclear detonations are used to define the transmission properties (transfer functions) of three rock types. Absorption, a measure of a rock's energy dissipating characteristics, is expressed for each of the tests as a function of the frequency of transmission. Dispersion results from a variation in transmission velocity with frequency and is described for each test by a phase velocity spectrum. The transmission properties from one of the sites are used to predict a ground-motion time history at that site from another nuclear event. The potential use of Fourier techniques to make ground-motion predictions and to measure in-situ material properties is discussed.
PREFACE

This report was prepared by Scott E. Blouin, Research Civil Engineer, of the Foundations and Materials Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory, and by Stephen H. Wolfe, formerly a Research Civil Engineer at CRREL.

The work was sponsored by the U.S. Air Force Weapons Laboratory under Air Force Weapons Laboratory Project Order 74-173.

Dr. Malcolm Mellor technically reviewed the report.

The authors gratefully acknowledge the assistance of: Charles N. Hale of AFWL, who provided the bulk of the data records and who corrected and ran the AFWL transform program; James L. Drake of the U.S. Army Engineer Waterways Experiment Station, who gave his guidance and advice throughout the program; and Truman Brogan of WES, who furnished the MINERAL LODE data tapes.

The contents of this report are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of such commercial products.
CONTENTS

Section I: Introduction ........................................................................................................... 1
Section II: Analytical procedures .......................................................................................... 3
  1. Fourier transforms ............................................................................................................. 3
  2. Basic transform properties ............................................................................................... 4
  3. Transform techniques ....................................................................................................... 7
  4. The transfer function ...................................................................................................... 12
Section III: Parameter studies and program evaluations ....................................................... 15
  1. Analytical parameter study ............................................................................................. 15
  2. I-TRAIN evaluation ........................................................................................................ 18
  3. S-TRAIN evaluation ........................................................................................................ 22
Section IV: Data ..................................................................................................................... 24
  1. DATEX I .......................................................................................................................... 24
  2. DATEX II ....................................................................................................................... 27
  3. STARMET ....................................................................................................................... 31
  4. MINERAL LODE ............................................................................................................ 31
  5. HARD HAT and PILEDRIVER ....................................................................................... 35
Section V: Analysis of transformed data ................................................................................. 40
  1. Absorption ...................................................................................................................... 40
  2. Dispersion ...................................................................................................................... 48
  3. Transfer function ........................................................................................................... 59
Section VI: Fourier synthesis .................................................................................................. 60
Section VII: Discussion .......................................................................................................... 66
Literature cited ....................................................................................................................... 68
Appendix A: The Fourier integral and series ......................................................................... 69
Appendix B: Subroutine FOURT used in analysis ................................................................. 75
Appendix C: I-TRAIN and S-TRAIN programs ..................................................................... 81

ILLUSTRATIONS

Figure
  1. Complex plane representation of $F(\omega)$ ................................................................ 3
  2. Breakdown of a real-time function into even and odd parts .................................... 6
  3. Typical data irregularities ............................................................................................. 8
  4. Fourier transforms of the delta function .................................................................... 10
  5. Development of the impulse train for a real-time function .................................... 10
  6. Exponentially damped sine wave ............................................................................. 15
  7. Amplitude spectra of exponentially damped sine wave ........................................ 17
  8. Phase spectra of exponentially damped sine wave .................................................... 17
  9. Amplitude spectra of exponentially damped sine wave — logarithmic presentation 18
 10. Phase spectra of exponentially damped sine wave — logarithmic presentation 18
 11. Comparison of channel 018, DATEX I, and exponentially decaying sine function 19
 12. Effect of time shift on phase spectrum for exponentially decaying sine function 19
 13. Phase folding caused by inverse tangent function .................................................... 20
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.</td>
<td>Evaluation of I-TRAIN transform of exponentially decaying sine wave</td>
<td>21</td>
</tr>
<tr>
<td>15.</td>
<td>Transforms of Channel 018, DATEX I, and exponentially decaying sine wave</td>
<td>21</td>
</tr>
<tr>
<td>16.</td>
<td>Real part of Fourier transform of $f(t) = 50 e^{-100t} \sin 100t$</td>
<td>22</td>
</tr>
<tr>
<td>17.</td>
<td>Fourier synthesis of $f(t) = 50 e^{-100t} \sin 100t$</td>
<td>22</td>
</tr>
<tr>
<td>18.</td>
<td>A triangular time history and its Fourier synthesis</td>
<td>23</td>
</tr>
<tr>
<td>19.</td>
<td>Real part of the Fourier transform of triangular time history</td>
<td>23</td>
</tr>
<tr>
<td>20.</td>
<td>Plan and section of DATEX I</td>
<td>24</td>
</tr>
<tr>
<td>21.</td>
<td>Uncorrected velocity-time histories from DATEX I</td>
<td>25</td>
</tr>
<tr>
<td>22.</td>
<td>Velocity vs time for DATEX I</td>
<td>25</td>
</tr>
<tr>
<td>23.</td>
<td>Amplitude spectra from DATEX I</td>
<td>26</td>
</tr>
<tr>
<td>24.</td>
<td>Phase spectra from DATEX I</td>
<td>26</td>
</tr>
<tr>
<td>25.</td>
<td>Plan and section of DATEX II</td>
<td>27</td>
</tr>
<tr>
<td>26.</td>
<td>Uncorrected velocity-time histories from DATEX II</td>
<td>28</td>
</tr>
<tr>
<td>27.</td>
<td>Positive phase estimate points from DATEX II data</td>
<td>28</td>
</tr>
<tr>
<td>28.</td>
<td>Estimated ratio of negative to positive phase duration from DATEX II</td>
<td>28</td>
</tr>
<tr>
<td>29.</td>
<td>Velocity-time history straight-line approximations from DATEX II</td>
<td>29</td>
</tr>
<tr>
<td>30.</td>
<td>Amplitude spectra from DATEX II</td>
<td>29</td>
</tr>
<tr>
<td>31.</td>
<td>Phase spectra from DATEX II</td>
<td>29</td>
</tr>
<tr>
<td>32.</td>
<td>Plan view of STARMET</td>
<td>30</td>
</tr>
<tr>
<td>33.</td>
<td>Uncorrected velocity-time histories from STARMET</td>
<td>30</td>
</tr>
<tr>
<td>34.</td>
<td>Velocity-time history straight-line approximations from STARMET</td>
<td>32</td>
</tr>
<tr>
<td>35.</td>
<td>Amplitude spectra from STARMET</td>
<td>33</td>
</tr>
<tr>
<td>36.</td>
<td>Phase spectra from STARMET</td>
<td>33</td>
</tr>
<tr>
<td>37.</td>
<td>Plan view of MINERAL LODE</td>
<td>33</td>
</tr>
<tr>
<td>38.</td>
<td>Uncorrected velocity-time histories from MINERAL LODE</td>
<td>34</td>
</tr>
<tr>
<td>39.</td>
<td>Velocity-time history straight-line approximations from MINERAL LODE</td>
<td>34</td>
</tr>
<tr>
<td>40.</td>
<td>Amplitude spectra from MINERAL LODE</td>
<td>35</td>
</tr>
<tr>
<td>41.</td>
<td>Phase spectra from MINERAL LODE</td>
<td>35</td>
</tr>
<tr>
<td>42.</td>
<td>Corrected velocity-time histories from HARD HAT</td>
<td>36</td>
</tr>
<tr>
<td>43.</td>
<td>Velocity-time history straight-line approximations from HARD HAT</td>
<td>36</td>
</tr>
<tr>
<td>44.</td>
<td>Amplitude spectra from HARD HAT</td>
<td>37</td>
</tr>
<tr>
<td>45.</td>
<td>Phase spectra from HARD HAT</td>
<td>37</td>
</tr>
<tr>
<td>46.</td>
<td>Velocity-time histories from PILEDIVER</td>
<td>38</td>
</tr>
<tr>
<td>47.</td>
<td>Velocity-time history straight-line approximations from PILEDIVER</td>
<td>38</td>
</tr>
<tr>
<td>48.</td>
<td>Amplitude spectrum from PILEDIVER</td>
<td>38</td>
</tr>
<tr>
<td>49.</td>
<td>Phase spectrum from PILEDIVER</td>
<td>39</td>
</tr>
<tr>
<td>50.</td>
<td>Amplitude spectra corrections for MINERAL LODE data</td>
<td>41</td>
</tr>
<tr>
<td>51.</td>
<td>Adjusted amplitude spectra from MINERAL LODE</td>
<td>41</td>
</tr>
<tr>
<td>52.</td>
<td>Amplitude attenuation from MINERAL LODE</td>
<td>42</td>
</tr>
<tr>
<td>53.</td>
<td>Attenuation spectrum from MINERAL LODE</td>
<td>42</td>
</tr>
<tr>
<td>54.</td>
<td>Amplitude corrections for HARD HAT data</td>
<td>43</td>
</tr>
<tr>
<td>55.</td>
<td>Adjusted amplitude spectra from HARD HAT</td>
<td>43</td>
</tr>
<tr>
<td>56.</td>
<td>Adjusted amplitude attenuation from HARD HAT</td>
<td>44</td>
</tr>
<tr>
<td>57.</td>
<td>Attenuation spectrum from HARD HAT</td>
<td>44</td>
</tr>
<tr>
<td>58.</td>
<td>Range variable for a planar array</td>
<td>44</td>
</tr>
<tr>
<td>59.</td>
<td>Amplitude correction for DATEX I data</td>
<td>45</td>
</tr>
<tr>
<td>60.</td>
<td>Adjusted amplitude spectra from DATEX I</td>
<td>45</td>
</tr>
<tr>
<td>61.</td>
<td>Adjusted amplitude attenuation from DATEX I</td>
<td>45</td>
</tr>
<tr>
<td>62.</td>
<td>Attenuation spectrum from DATEX I</td>
<td>45</td>
</tr>
</tbody>
</table>
Figure

63. Corrected amplitude from DATEX II ........................................... 46
64. Adjusted amplitude spectra from DATEX II ................................... 46
65. Adjusted amplitude attenuation from DATEX II ............................. 46
66. Attenuation spectrum from DATEX II ........................................... 46
67. Amplitude corrections for STARMET data .................................... 47
68. Adjusted amplitude spectra from STARMET data ......................... 47
69. Adjusted amplitude attenuation from STARMET .......................... 47
70. Attenuation spectrum from STARMET data .................................. 47
71. Summary, attenuation spectra from MINERAL LODE, DATEX I and II, STARMET and HARD HAT ........................................... 47
72. Relationship between phase and phase velocity ............................ 49
73. Smoothed phase spectra from MINERAL LODE ............................ 51
74. Phase delay times from MINERAL LODE ...................................... 51
75. Phase velocity as a function of frequency from MINERAL LODE ........ 52
76. Phase velocity spectrum from MINERAL LODE ............................ 52
77. Smoothed phase spectra from HARD HAT ..................................... 52
78. Phase delay times from HARD HAT ............................................. 52
79. Phase velocity spectrum from HARD HAT ..................................... 53
80. Smoothed phase spectra from DATEX I ........................................ 54
81. Phase delay times from DATEX I ................................................. 54
82. Phase velocity spectrum from DATEX I ........................................ 55
83. Smoothed phase spectra from DATEX II ....................................... 55
84. Phase delay times from DATEX II ................................................. 55
85. Phase velocity spectrum from DATEX II ....................................... 55
86. Smoothed phase spectra from STARMET ..................................... 56
87. Phase delay times from STARMET ............................................... 56
88. Phase velocity spectrum from STARMET ..................................... 56
89. Summary, phase velocity spectra from STARMET, HARD HAT, DATEX I and II, and MINERAL LODE ........................................... 57
90. Use of phase velocity to predict phase spectrum ......................... 58
91. Fit to amplitude spectrum for 50-ft MINERAL LODE gage ............... 61
92. Real part of Fourier transform of 50-ft gage for MINERAL LODE ..... 61
93. Fourier synthesis of 50-ft MINERAL LODE time history from smoothed spectra .... 61
94. Predicted MINERAL LODE amplitude spectra ................................ 62
95. Predicted MINERAL LODE phase spectra .................................. 62
96. Predicted MINERAL LODE Fourier transforms ................................ 62
97. Fourier synthesis “prediction” of MINERAL LODE radial velocities .... 63
98. Predicted PILEDRIVER amplitude and phase spectra ..................... 64
99. Real part of predicted Fourier transform of PILEDRIVER at 1,543-ft range .... 65
100. Fourier synthesis of PILEDRIVER data using HARD HAT material model .... 65

TABLES

Table

1. Phase delay times from smoothed phase spectra – MINERAL LODE .......... 51
CONVERSION FACTORS: U.S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

These conversion factors include all the significant digits given in the conversion tables in the ASTM Metric Practice Guide (E 380), which has been approved for use by the Department of Defense. Converted values should be rounded to have the same precision as the original (see E 380).

<table>
<thead>
<tr>
<th>Multiply</th>
<th>By</th>
<th>To obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>inch</td>
<td>25.4*</td>
<td>millimeter</td>
</tr>
<tr>
<td>foot</td>
<td>0.3048*</td>
<td>meter</td>
</tr>
<tr>
<td>foot²</td>
<td>0.09290304</td>
<td>meter²</td>
</tr>
<tr>
<td>foot/second</td>
<td>0.3048*</td>
<td>meter/second</td>
</tr>
<tr>
<td>cycle/second</td>
<td>6.28318</td>
<td>radian/second</td>
</tr>
<tr>
<td>pound mass</td>
<td>0.4535924</td>
<td>kilogram</td>
</tr>
<tr>
<td>ton</td>
<td>907.1847</td>
<td>kilogram</td>
</tr>
<tr>
<td>pound/foot³</td>
<td>16.01846</td>
<td>kilogram/meter³</td>
</tr>
<tr>
<td>psi</td>
<td>6894.757</td>
<td>pascal</td>
</tr>
</tbody>
</table>

* Exact.
ANALYSIS OF EXPLOSIVELY GENERATED
GROUND MOTIONS USING FOURIER TECHNIQUES
Scott E. Blouin and Stephen H. Wolfe

SECTION I
INTRODUCTION

Geophysicists have long recognized that stress-wave energy in the earth's crust is dissipated more rapidly than can be explained by geometric factors alone. They have found that energy dissipation, generally termed absorption or attenuation, is dependent on both the material through which the wave is transmitted and the frequency or frequencies of transmission. Since stress waves normally consist of combinations of many frequencies, they must be broken down into their component frequencies before material absorption characteristics can be derived. Fourier transforms are generally used to shift ground-motion time-history data into the frequency domain. A transformed real-time signal is always a complex function of frequency and can be conveniently expressed as associated amplitude and phase spectra. When data are available from two, or preferably more, locations along a common line, the amplitude spectra can be used to determine the absorption properties of the medium as a function of frequency.

A second phenomenon, known as dispersion, tends to reshape a stress pulse as it progresses. Dispersion, which has been studied for many years by earthquake seismologists, is also dependent on both the material through which the wave is transmitted and the frequency of transmission. This results from the fact that propagation velocity in earth materials is a function of frequency. Thus, as a stress pulse travels, frequency components that travel faster move toward the front of the pulse, while those that travel more slowly move toward the rear of the pulse. This process tends to lengthen or spread the pulse, thus the term dispersion. At great distances from a seismic disturbance, such as many miles from an earthquake epicenter, individual frequency components can be visually identified near the pulse front. More recently, geophysicists have determined that dispersion also influences the shape of stress pulses relatively near explosive disturbances. Further, Futterman (ref. 1) shows that a material which is absorptive is also necessarily dispersive. The propagation velocity as a function of frequency can be obtained from a series of phase spectra from transformed time histories along a common line.

The usefulness of the absorption and dispersion properties of a material can be realized if a ground-motion time history can be obtained at any point in the medium. For instance, a reasonable time history might be calculated close to an explosive source. The transform of this time history can then be used along with the absorption and dispersion characteristics of the material to obtain a Fourier transform of the ground motion at any point in the medium. Next, an inverse Fourier transform can be applied to the calculated transform to obtain the corresponding time history. Thus, if a credible time history or predicted time history is available at any point in a medium, the relevant absorption and dispersion relations may be used to make time-history predictions at any other point in the medium.

Since defense planners began to design systems to resist nuclear attack, there has been a concerted, continuous effort to improve the understanding of the effects of nuclear weapons. A major portion of this effort has been in the field of blast effects. A particularly difficult problem in this area is predicting the ground motions that would result from a nearby detonation of a nuclear weapon in order to evaluate the survivability of hardened structures such as silos, command and control centers, and communications facilities. Over the past 20 years, an impressive series of experiments, including underground nuclear and underground and aboveground high-explosive shots of various geometric configurations, have been fielded to evaluate the survivability of some of these structures, to improve understanding of ground motion phenomenology and in-situ material properties and to upgrade ground-motion prediction capabilities.

Along with the field experiments, programs to evaluate material properties in both the laboratory and the field and to calculate ground motions have been undertaken. But, throughout these programs, the use of Fourier transforms for evaluating ground-motion data in the frequency domain and for extracting in-situ transmission properties as functions of frequency has been largely ignored.
The present report describes an attempt to apply Fourier techniques to a small sample of these data and, from the results, to obtain in-situ material properties as functions of frequency for several test locations.

A series of five experiments in rock were chosen for evaluation. These included one contained (underground) nuclear detonation in granite, HARD HAT; one contained spherical high-explosive shot in granodiorite, MINERAL LODE; and three contained high-explosive shots in a planar geometry designed to simulate the ground motions induced by cratering resulting from a nuclear surface burst. Of the last three, DATEX I and II were sited in granodiorite and STARMET was sited in granite. Selected data from these experiments were transformed into the frequency domain, and from these transforms frequency-dependent absorption and dispersion characteristics were derived for each site. Finally, the derived absorption and dispersion properties of the HARD HAT granite were used to predict a ground-mOTION time history from a much larger contained nuclear burst, PILEDRIVER, in the same granite.

Section I of this report gives an overview of the frequency-based analysis techniques. Section II reviews the mathematics used in the analysis and in the computer programs used to take the Fourier transforms and inverse transforms. Section III evaluates the computer programs and some of the analysis techniques by applying them to an analytical time history with a known transform. In addition, a parameter study is performed on this function to give a qualitative feeling for the behavior of the techniques over a range of time-history signatures. Section IV develops the Fourier transform of selected data from the five tests, HARD HAT, MINERAL LODE, DATEX I and II, and STARMET. Section V derives the material-related transmission properties from each of the five sets of data, and Section VI evaluates the Fourier synthesis technique and predicts a PILEDRIVER velocity-time history using the material properties derived from the HARD-HAT data. Section VII discusses some of the potential benefits and difficulties of applying Fourier techniques to real-world problems.
SECTION II

ANALYTICAL PROCEDURES

1. FOURIER TRANSFORMS

Any real-time history of a variable which has a non-infinite integral and which does not have an infinite number of discontinuities can be represented by a summation over all frequencies of simple sinusoidal functions of that variable. This well-known Fourier decomposition is extremely useful because signals that appear random in nature can be resolved into a summation of single frequency components. Decomposition into a single frequency component at several points along the signal transmission path permits ready definition of the transmission system characteristics as functions of frequency and range, thus allowing prediction of the transmission system response to any set of inputs.

Two basic techniques are used to transform data into the frequency domain, the Fourier series and the Fourier integral. Nonperiodic functions, such as ground-motion time histories resulting from explosive loadings, are best suited to analysis by the Fourier integral. The integral transform of some function of time \( f(t) \) is given as a function of frequency \( \omega \) by

\[
F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt
\]  

where \( F(\omega) \) is called the Fourier transform of \( f(t) \) and \( j \) is defined as \( \sqrt{-1} \). Similarly, if the transform is known, the time history can be recovered by taking the inverse transform of \( F(\omega) \):

\[
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega
\]  

Thus, \( f(t) \) is called the inverse Fourier transform of \( F(\omega) \). \( F(\omega) \) and \( f(t) \) are termed the transform pair. The first equation transforms a function of time \( f(t) \) into its equivalent frequency function \( F(\omega) \), and the second equation transforms a function of frequency into its equivalent time function. The first operation is referred to as a Fourier analysis and the second as a Fourier synthesis.

The Fourier transform of a function of time \( f(t) \), is, in general, a complex function \( F(\omega) \):

\[
F(\omega) = A(\omega)e^{j\theta(\omega)}
\]

\[
A(\omega) = \left[F_1^2(\omega) + F_2^2(\omega)\right]^{1/2}
\]

\[
\theta(\omega) = \tan^{-1}\left[\frac{F_2(\omega)}{F_1(\omega)}\right]
\]

\[
F_1(\omega) = A(\omega)\cos\theta(\omega)
\]

\[
F_2(\omega) = A(\omega)\sin\theta(\omega)
\]

Figure 1. Complex plane representation of \( F(\omega) \).
\[ \theta(\omega) = \tan^{-1} \frac{F_2(\omega)}{F_1(\omega)} \]  
\[ \text{(5)} \]

Also,

\[ F(\omega) = A(\omega) \cos \theta(\omega) + jA(\omega) \sin \theta(\omega) \]
\[ = A(\omega)[\cos \theta(\omega) + j\sin \theta(\omega)] \]  
\[ \text{(6)} \]

According to the Euler formula,

\[ e^{j\theta(\omega)} = \cos \theta(\omega) + j\sin \theta(\omega) \]  
\[ \text{(7)} \]

which, substituted into equation 6, yields \( F(\omega) \) in polar form:

\[ F(\omega) = A(\omega)e^{j\theta(\omega)} \]  
\[ \text{(8)} \]

2. BASIC TRANSFORM PROPERTIES

There are several basic properties of Fourier transform pairs which are important to the development of the computer programs used in this analysis. The development of these properties as outlined here is after Guillemin (ref. 2).

Recalling that an even function has the property

\[ f(-t) = f(t) \]  
\[ \text{(9)} \]

and that an odd function has the property

\[ f(-t) = -f(t) \]  
\[ \text{(10)} \]

any function of time can be divided into its even and odd parts, \( f_e(t) \) and \( f_o(t) \), respectively:

\[ f(t) = f_e(t) + f_o(t) \]  
\[ \text{(11)} \]

The integral transform in equation 1 can be divided into its even and odd parts

\[ F(\omega) = \int_{-\infty}^{\infty} f_e(t)e^{-j\omega t}dt + \int_{-\infty}^{\infty} f_o(t)e^{-j\omega t}dt \]  
\[ \text{(12)} \]

Substituting the sine-cosine equivalent of \( e^{-j\omega t} \) into equation 12 yields

\[ F(\omega) = \int_{-\infty}^{\infty} f_e(t)(\cos \omega t - j\sin \omega t)dt \]
\[ + \int_{-\infty}^{\infty} f_o(t)(\cos \omega t - j\sin \omega t)dt \]  
\[ \text{(13)} \]
Since
\[ \sin(-t) = -\sin t \] (14)
and
\[ \cos(-t) = \cos t \] (15)
sin is an odd function and cos is an even function of \( t \). The integrals of the odd functions multiplied by the even functions in equation 13 taken over the limits of \(-\infty\) to \(+\infty\) are equal to zero; thus equation 13 reduces to
\[
F(\omega) = \int_{-\infty}^{\infty} f_e(t) \cos \omega t dt - i \int_{-\infty}^{\infty} f_o(t) \sin \omega t dt
\] (16)

Therefore, the two integrals in equation 16 represent the real \( F_1(\omega) \) and imaginary \( F_2(\omega) \) parts of \( F(\omega) \), respectively.

Since
\[
F(\omega) = F_1(\omega) + jF_2(\omega)
\] (17)
it follows that the real part of \( F(\omega) \) is
\[
F_1(\omega) = \int_{-\infty}^{\infty} f_e(t) \cos \omega t dt = 2 \int_{0}^{\infty} f_e(t) \cos \omega t dt
\] (18)
and the imaginary part of \( F(\omega) \) is
\[
F_2(\omega) = -\int_{-\infty}^{\infty} f_o(t) \sin \omega t dt = -2 \int_{0}^{\infty} f_o(t) \sin \omega t dt
\] (19)

The real part of the Fourier transform is even and is based solely on the even part of \( f(t) \), and the imaginary part of the transform is odd and is based solely on the odd part of \( f(t) \).

Following a similar line of reasoning, it can be shown that the even and odd parts of the time function are dependent on only the even and odd parts of the transform, respectively, as shown in equations 20 and 21:

\[
f_e(t) = \frac{1}{\pi} \int_{0}^{\infty} F_1(\omega) \cos \omega t d\omega
\] (20)

\[
f_o(t) = -\frac{1}{\pi} \int_{0}^{\infty} F_2(\omega) \sin \omega t d\omega
\] (21)

Any real time function, as shown graphically in Figure 2, is made up of an odd and even function of time. At time less than zero, the odd and even parts must cancel one another, and are thus equal and opposite. At time greater than zero, they must add and must be equal:

\[ f_e(t) = f_o(t) \quad \text{(for } t > 0) \] (22)
Figure 2. Breakdown of a real-time function into even and odd parts.

\[ f(t) = f_e(t) + f_o(t) \]
\[ f(t) = 2f_e(t) = 2f_o(t) \quad \text{for } t > 0 \]

This effect, termed real-part sufficiency, substantially simplifies the calculations involved in a Fourier synthesis.
3. TRANSFORM TECHNIQUES

Our initial analytic work was done using a Fourier analysis of several time histories based on a Fourier series representation. The Fourier series is shown to be equivalent to the Fourier integral in Appendix A. A Fourier analysis was run on three velocity-time histories from both DATEX I and DATEX II. They were analyzed on USA CRREL's Honeywell DDP-24 computer using 100- to 200-point hand digitizations of the data. The original program was essentially a summation representation of the real and imaginary parts of the Fourier transform of \( f(t) \) given by the integrals in equations 18 and 19. These were substituted into equations 4 and 5 to give the amplitude spectra \( A(\omega) \) and phase spectra \( \theta(\omega) \), respectively.

This technique was used for the original exploratory analysis only. It was impractical for use on a production basis because digitization had to be done by hand at equal time intervals, which in itself was very time consuming, and because the computer could not handle runs of over 200 points. Initial plans called for all production analysis to be performed on the Air Force Weapon Laboratory's (AFWL's) Control Data Corporation 6600 computer using the original digitized data tapes from the various experiments. Fourier analysis was to be performed using the Power Spectral Density (PSD) program. The actual Fourier decomposition is performed in the subroutine FOURT, which uses a Cooley-Tukey (ref. 3) fast Fourier transform. Using PSD, a typical Fourier analysis of a 1000- to 2000-point digitized record can be run in a few seconds.

After an extensive period of debugging, the program PSD was finally operational and was used to obtain a Fourier analysis of selected velocity-time histories from DATEX I and II and STARMET. During this time the program was undergoing continuous modification by other users; this caused many delays in our analysis. For the convenience of future users, a copy of the subroutine FOURT as used in this analysis is included in Appendix B.

Both day-to-day operational problems with the program PSD and other technical difficulties finally forced development of a second transform program at CRREL. As mentioned, PSD frequently failed to run because of changes by other users. AFWL personnel were often diverted to other higher priority work. Old data tapes had to be located and sometimes redigitized and/or converted for use on the CDC 6600 computer.

Inspection of transformed channels also revealed technical problems. Almost all velocity-time histories contained either baseline shifts, sharp truncations caused by cable breaks or other electrical failures, miscellaneous electrical noise, or combinations of two or more of these factors. Inclusion of these in the frequency analysis resulted in serious distortion of both the amplitude and phase spectra. Examples of these irregularities are shown in Figure 3. It became clear that a method of correcting the data was needed. At the time, AFWL could not make quick and easy corrections to the digitized tapes (it has since acquired an electronic display unit which permits such corrections). Thus, it was decided to write a second transform program at CRREL which would allow corrected data to be input and provide information over a wider frequency band than was possible using the original 200-point limitation.

Two simple transform programs were written using the impulse train technique (ref. 2 and ref. 4): I TRAIN for doing the Fourier analysis and S TRAIN for doing the Fourier synthesis. The impulse train technique* uses a straight-line approximation of the function to be transformed and thus requires digitization only in areas of changing slope. It is ideally suited to a small computer such as the Digital Equipment Corporation PDP-8 used at CRREL.

The first key point in the development of the impulse train technique is the use of the second derivative of \( f(t) \) and \( F_1(\omega) \) within the integrals in equations 24-26. Using a straight-line approximation of \( f(t) \) or \( F_1(\omega) \), the second derivative is then a series, or train, of impulses located at the junctions of the straight-line approximations. The integrals can be treated as sums of the impulses from all junction locations, which are simply represented in a computer program. The transform of the second derivative of \( f(t) \), represented as \( f''(t) \), can be obtained by using the formula for integration by parts, which reads

\[
\int u dv = uv - \int v du
\]

* This technique was first recommended by J.L. Drake of the U.S. Army Engineer Waterways Experiment Station.
Applying this formula to equation 25 for the real part of the transform $F_1(\omega)$, let $u = f(t)$ and $dv = \cos \omega td\tau$; then $du = f'(t)dt$ and $v = \frac{1}{\omega} \sin \omega t$. Substituting and combining equations 25 and 27

$$F_1(\omega) = f(t) \left( \frac{1}{\omega} \sin \omega t \right) \left|_0^\infty \right. - \frac{1}{\omega} \int_0^\infty f'(t) \sin \omega td\tau \tag{28}$$

Since we are treating an integratable real-time function, $f(t)$ equals zero at $t = 0$ and at $t = \infty$; and equation 28 reduces to

$$F_1(\omega) = -\frac{1}{\omega} \int_0^\infty f'(t) \sin \omega td\tau \tag{29}$$

Applying the formula for integration by parts a second time, with $u = f'(t)$, $dv = \sin \omega td\tau$, $du = f''(t)dt$, and $v = -(1/\omega) \cos \omega t$ yields

$$F_1(\omega) = -\frac{1}{\omega} \left[ f'(t) \left( \frac{1}{\omega} \cos \omega t \right) \right|_0^\infty + \frac{1}{\omega} \int_0^\infty f''(t) \cos \omega td\tau \tag{30}$$

The first term is again zero and the equation reduces to

$$F_1(\omega) = -\frac{1}{\omega^2} \int_0^\infty f''(t) \cos \omega td\tau \tag{31}$$

Figure 3. Typical data irregularities.
The same technique applied to equation 26 yields a similar expression for the imaginary part of the transform as a function of the second derivative of the time function:

$$F_2(\omega) = -\frac{1}{\omega^2} \int_0^\infty f''(t) \sin \omega t \, dt$$

(32)

A similar procedure applied to equation 24 gives \(f(t)\) as a function of the second derivative of the real part of \(F(\omega)\) as

$$f(t) = -\frac{2}{\pi t^2} \int_0^\infty F''(\omega) \cos \omega t \, dt$$

(33)

The second key point in the development of the impulse-train technique comes in the mathematics of handling the impulses expressed by the second derivatives in equations 31-33. This is accomplished by using the unit impulse, often termed the delta function \(\delta(t)\), which is interpreted in slightly different ways in the literature (ref. 2, 5 and 6). For our purposes, \(\delta(t)\) can be defined as the derivative of a unit step at time zero. \(\delta(t)\) has properties such that

$$\delta(t) = 0 \quad \text{for } t \neq 0$$

(34)

and

$$\int_{-\infty}^{\infty} \delta(t) \, dt = 1$$

(35)

Mathematically, \(\delta(t)\) is not a proper function. Physically, it has no value at times other than zero and its integral returns the unit step at time zero. For a step of magnitude \(a\), the derivative is \(a\delta(t)\) and

$$\int_{-\infty}^{\infty} a\delta(t) \, dt = a$$

(36)

For a unit step occurring at time \(t_0\), the delta function is also displaced \(t_0\). It is expressed as \(\delta(t-t_0)\) and is defined such that

$$\delta(t-t_0) = 0 \quad \text{for } t \neq t_0$$

(37)

and

$$\int_{-\infty}^{\infty} \delta(t-t_0) \, dt = 1$$

(38)

The Fourier transform of the delta function has unique properties. The transforms of \(\delta(t)\) and \(\delta(t-t_0)\) are shown graphically in Figure 4. Substituting \(\delta(t)\) for \(f(t)\) in equation 1 yields

$$F(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} \, dt$$

(39)
Delta Function \( f(t) = \delta(t - t_0) \)

Amplitude Spectrum

\[
\begin{array}{c|c}
\text{Amplitude} & A(\omega) \\
\hline
0 & 1.5 + A(\omega) \\
0.5 & 1.0 \\
1.0 & 0.5 \\
\end{array}
\]

Phase Spectrum

\[ \theta(\omega) = 0 \]

Delta Function Time Shifted

Amplitude Spectrum

\[
\begin{array}{c|c}
\text{Amplitude} & A(\omega) \\
\hline
0 & 1.5 + A(\omega) \\
0.5 & 1.0 \\
1.0 & 0.5 \\
\end{array}
\]

Phase Spectrum

\[ \theta(\omega) = \omega t_0 \]

Figure 4. Fourier transforms of the delta function.

Figure 5. Development of the impulse train for a real-time function.
Since $\delta(t)$ is zero except at time zero where its integral equals 1,

$$F(\omega) = 1$$  \hfill (40)

Likewise, the transform of an impulse at time $t_0$ is given by

$$F(\omega) = \int_{-\infty}^{\infty} \delta(t-t_0) e^{i\omega t} dt$$  \hfill (41)

Again, $\delta(t-t_0)$ is zero everywhere except at $t_0$, where its integral equals 1. Equation 41 then becomes

$$F(\omega) = e^{-i\omega t_0}$$  \hfill (42)

Recalling equation 8, it is obvious that the transform of $\delta(t)$ has an amplitude spectrum of 1 over all frequencies and a phase spectrum of zero. The transform of $\delta(t-t_0)$ also has an amplitude spectrum of 1, but has a phase spectrum of $\omega t_0$, in proportion to its displacement from time zero.

Referring to the continuous function of time $f(t)$ shown in Figure 5, we see that $f(t)$ is approximated by $n$ straight-line segments. Its derivative $f'(t)$ is represented by a series of steps equal in magnitude to the slopes of corresponding segments in $f(t)$. Its second derivative $f''(t)$ is zero everywhere but at intersection points of the straight-line approximation to $f(t)$. At these points, represented by time $t_i$ where $i$ runs from 1 to $n$, the derivative is represented by a series of impulses. Using the delta function notation, the impulses are represented by an amplitude $a_i$ multiplied by the delta function $\delta(t-t_i)$. The amplitude $a_i$ is the magnitude of the corresponding step in the first derivative, which is equal to the change in slope at the intersection point:

$$a_i = \left( \frac{f_{i+1}(t)-f_i(t)}{t_{i+1} - t_i} \right) - \left( \frac{f_i(t)-f_{i-1}(t)}{t_i - t_{i-1}} \right)$$  \hfill (43)

The real part of the transform of the straight-line approximation of $f(t)$ can be obtained by substituting $a_i \delta(t-t_i)$ for $f''(t)$ in equation 31 and changing the integral to a sum, since the second derivative is zero except at the points $t_i$. Equation 31 then becomes

$$F_1(\omega) = -\frac{1}{\omega^2} \sum_{i=1}^{n} a_i \cos \omega t_i$$  \hfill (44)

The same process applied to equation 32 for the imaginary part of the transform yields

$$F_2(\omega) = \frac{1}{\omega^2} \sum_{i=1}^{n} a_i \sin \omega t_i$$  \hfill (45)

Equations 44 and 45 can be easily programmed, with only the digitized linear approximation represented by the pairs $f_i(t)$ and $t_i$ of equation 43 required. The time increment for digitizing the waveform does not need to be constant or even small for excellent accuracy at high frequencies. A good linear approximation of the time function yields a good approximation of the transform as shown in Section III. The transform of a linear function of time is given exactly by this method. For example, the transform of a triangular time pulse is given exactly by a three-point representation of the time function at its beginning, $v$, $x$, and end. The amplitude and phase spectra
are obtained by substituting \( F_1(\omega) \) and \( F_2(\omega) \) from equations 44 and 45 into equations 4 and 5 for \( A(\omega) \) and \( \theta(\omega) \). A copy of the transform program I TRAIN is included in Appendix C.

The inverse transform for synthesizing \( f(t) \) from the amplitude and phase spectra is done in a manner similar to that just described for the transform. Equation 33 is used, with the real part of the transform \( F_1(\omega) \) represented by a straight-line approximation. The real part of the transform, obtained from the real part of equation 6 is

\[
F_1(\omega) = A(\omega) \cos \theta(\omega)
\]

This is simply the amplitude multiplied by the cosine of the phase. The second derivative of the straight-line approximation of \( F_1(\omega) \) is a series of delta functions at frequencies specified by \( \omega_i \), where \( i \) runs from 1 to \( n \), multiplied by their appropriate amplitudes \( b_i \). In this case, \( b_i \) is the change in slope of the \( F_i(\omega) \) versus \( \omega \) straight-line approximation at the line junctions:

\[
b_i = \left( \frac{F_{i+1}(\omega) - F_i(\omega)}{\omega_{i+1} - \omega_i} \right) - \left( \frac{F_i(\omega) - F_{i-1}(\omega)}{\omega_i - \omega_{i-1}} \right)
\]

Transforming the delta function representation of the second derivative of \( F_1(\omega) \) in equation 33 and replacing the integral with a summation of all the impulses yield the equation for the impulse train inverse transform

\[
f(t) = -\frac{2}{\pi t^2} \left( \frac{b_1}{2} + \sum_{i=2}^{n} b_i \cos \omega_i t \right)
\]

The first impulse in the series occurs at \( \omega_1 = 0 \) and must be halved because the summation is only over positive frequencies, although \( F_1(\omega) \) is defined over both positive and negative frequencies. Thus,

\[
b_1 = \frac{F_2(\omega) - F_1(\omega)}{\omega_2 - \omega_1}
\]

As in the case of the direct transform, \( F_1(\omega) \) may be digitized into nonuniform increments of whatever lengths are convenient for making a good straight-line approximation of \( F_1(\omega) \). A copy of the synthesis program S TRAIN is included in Appendix C.

4. THE TRANSFER FUNCTION

Probably the most significant potential use of Fourier transformed data is to determine transmission characteristics of the soil or rock through which stress waves propagate. For a physical model in which energy is transmitted over a single path between motion monitoring instrumentation, changes in the signal depend only on the properties of the medium along the transmission path between the instrumentation locations. For such a system, a system function or transfer function \( H(\omega) \) can be derived which defines the transmission characteristics of the medium in terms of amplitude and phase and thus allows prediction of system response at other locations or from different inputs.

Assume that \( f(t) \) is a real-time signal and that \( g(t) \) is that same signal after traveling outwardly along a transmission path between the signal source and the signal detector. If the transmission path is assumed to be linear and causal, then a function \( h(t) \) exists, which describes the system transmission properties such that

\[
g(t) = \int_{-\infty}^{\infty} f(t-\tau)h(\tau)d\tau
\]
where $\tau$ is time shift. The integral expresses the convolution of the functions $f(t)$ and $h(t)$. In practice, it is difficult to work with equation 50. However, it happens to be quite easy to work with this equation expressed in the frequency domain rather than in the time domain.

Guillemin (ref. 2) demonstrates that, since every operation in the time domain has a corresponding operation in the frequency domain, equation 50 can be expressed in its frequency domain equivalent. According to equation 1, the transform of $g(t)$ is

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$  (51)

Substituting equation 50 into equation 51 yields

$$G(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} \left[ \int_{-\infty}^{\infty} f(t-\tau) h(\tau) d\tau \right] dt$$  (52)

Assuming absolute convergence of the integrals and interchanging the order of integration

$$G(\omega) = \int_{-\infty}^{\infty} h(\tau) \left[ \int_{-\infty}^{\infty} f(t-\tau) e^{-j\omega t} d\tau \right] d\tau$$  (53)

The integral in brackets is the transform of $f(t)$ shifted in time by $\tau$. Taking the transform of $f(t)$ with $t = t-\tau$, according to equation 1, yields

$$F(\omega) = \int_{-\infty}^{\infty} f(t-\tau) e^{-j\omega(t-\tau)} d\tau$$  (54)

or

$$F(\omega) e^{-j\omega \tau} = \int_{-\infty}^{\infty} f(t-\tau) e^{-j\omega t} dt$$  (55)

which shows that the transform of a time-shifted function equals the transform of the function multiplied by $e^{-j\omega \tau}$ where $\tau$ is the time shift. Replacing the bracketed integral in equation 53 with $F(\omega)e^{-j\omega \tau}$ yields

$$G(\omega) = F(\omega) \int_{-\infty}^{\infty} h(\omega) e^{-j\omega t} d\tau$$  (56)

The integral is simply the transform of $h(\omega)$, expressed as $H(\omega)$, so

$$G(\omega) = F(\omega)H(\omega)$$  (57)

Equations 50 and 57 demonstrate the convolution theorem by showing that convolution in the time domain corresponds to multiplication in the frequency domain. In this instance, a simple process in the frequency domain can be employed in place of an obscure concept in the time domain. As noted previously, $H(\omega)$ is referred to as the transfer function.
Referring to equation 8, equation 57 can be rewritten in polar form as

\[ A_G(\omega)e^{j\theta_G(\omega)} = A_F(\omega)A_H(\omega)e^{j[\theta_F(\omega)+\theta_H(\omega)]} \]  

(58)

From this equation, it can be seen that if the transforms of a signal at different locations along its transmission path are known, they can be used to obtain the transfer function

\[ H(\omega) = A_H(\omega)e^{j\theta_H(\omega)} \]  

(59)

Once the transfer function is known for any medium, it can be used to predict ground motions at any point in that medium from any input provided that the input motion at some point can be measured or predicted. The transform of the known or predicted motion is taken and is multiplied by the transfer function as shown in equation 57 to obtain the transform of the desired motion time history \( G(\omega) \). The time history can then be recovered by taking the inverse transform of \( G(\omega) \).
1. ANALYTICAL PARAMETER STUDY

In order to evaluate the transform and inverse transform programs and to gain a better intuitive feeling for Fourier transforms, a series of parameter studies and evaluations were performed. An exponentially decaying sine function was chosen for analysis:

\[ f(t) = A e^{-at} \sin bt \quad \text{where } t > 0 \quad (60) \]

This function is shown graphically in Figure 6 over a range of \( a/b \) from 1/5 to 5. As \( b \) becomes larger than \( a \), the sine term dominates; and as \( a \) becomes larger than \( b \), the exponential term dominates. The plots with \( a/b \) from 1/2 to 5 closely resemble typical velocity-time histories. The shape of the plot for \( a/b = 5 \) is representative of data close to an explosive source, while that for \( a/b = 1/2 \) is representative of data farther from the source.

The transform of \( f(t) \) can be derived by substituting the expression for \( f(t) \) from equation 60 into equation 1 with limits from 0 to \( \infty \), since \( f(t) \) is a real time function, to obtain

\[ F(\omega) = A \int_{0}^{\infty} e^{-(a+\omega)t} \sin bt \, dt \quad (61) \]
This is similar in form to the definite integral
\[
\int_0^e e^{cx} \sin mx \, dx = \frac{m}{c^2 + m^2} \quad \text{for } c > 0
\] (62)
given in standard mathematics tables. Substituting \((a + j\omega) = c, t = x, \) and \(b = m\) gives
\[
F(\omega) = \frac{Ab}{(a + j\omega)^2 + b^2} = \frac{Ab}{a^2 + b^2 - \omega^2 + 2ja\omega}
\]
\[
= \frac{Ab(a^2 + b^2 - \omega^2 - 2ja\omega)}{[a^2 + (b - \omega)^2] [a^2 + (b + \omega)^2]}
\] (63)
The real part of \(F(\omega)\) is given by
\[
F_1(\omega) = \frac{Ab(a^2 + b^2 - \omega^2)}{[a^2 + (b - \omega)^2] [a^2 + (b + \omega)^2]}
\] (64)
and the imaginary part by
\[
F_2(\omega) = \frac{-2Ab\omega}{[a^2 + (b - \omega)^2] [a^2 + (b + \omega)^2]}
\] (65)
The amplitude spectrum and phase spectrum can now be obtained by substitution into equations 4 and 5
\[
A(\omega) = \sqrt{F_1^2(\omega) + F_2^2(\omega)}
\] (4)
\[
\theta(\omega) = \tan^{-1}\left(\frac{F_2(\omega)}{F_1(\omega)}\right)
\] (5)
Algebraic manipulation yields
\[
A(\omega) = \frac{Ab}{\left[\sqrt{a^2 + (b - \omega)^2} \sqrt{a^2 + (b + \omega)^2}\right]^{\frac{1}{2}}}
\] (66)
and
\[
\theta(\omega) = \tan^{-1}\left(\frac{-2a\omega}{a^2 + b^2 - \omega^2}\right)
\] (67)
Two basic points are noted: the magnitude of the amplitude spectrum is directly proportional to the magnitude of the time function; and the phase spectrum is dependent only on the shape of the time function and is independent of its magnitude.

The amplitude and phase spectra for \(a/b\) ranging from 1/5 to 5 are plotted in Figures 7 and 8, respectively; they are replotted on logarithmic paper in Figures 9 and 10. Note that the spectra are defined for both positive and negative values of frequency, but that only positive values are shown. Amplitude spectra are even functions and hence
\[ A(\omega) = \frac{Ab}{[a^2+(b-\omega)^2]^{1/2} [a^2+(b+\omega)^2]^{1/2}} \]

for \(a/b\) from 1/5 to 5

Figure 7. Amplitude spectra of exponentially damped sine wave.

\[ \theta(\omega) = \tan^{-1} \frac{-2a\omega}{a^2+b^2-\omega^2} \]

for \(a/b\) from 1/5 to 5

Figure 8. Phase spectra of exponentially damped sine wave.

are symmetric about the vertical axis, while phase spectra are odd functions. Also, the phase spectrum is negative for positive frequencies but is plotted as positive throughout this report to be compatible with the AFWL computer output. The plot of the amplitude spectra in Figure 7 shows that for small values of \(a/b\), where the sine term dominates, peak amplitude occurs at the frequency of the sine wave, \(b\). As the exponential term gains dominance (for increasing values of \(a/b\)), the well-defined amplitude peak shifts toward zero frequency and finally disappears at values of \(a/b\) equal to or greater than 1. Peak amplitude at values of \(a/b\) greater than 1 occurs at zero frequency. The zero frequency amplitude is known as the steady-state amplitude and is equal to the integration of the time history. Recalling equations 25 and 26 for the real and imaginary parts of the transform \(F(\omega)\),
Figure 9. Amplitude spectra of exponentially damped sine wave – logarithmic presentation.

Figure 10. Phase spectra of exponentially damped sine wave – logarithmic presentation.

\[ F_1(\omega) = \int_{0}^{\infty} f(t) \cos \omega t \, dt \]  
(25)

\[ F_2(\omega) = -\int_{0}^{\infty} f(t) \sin \omega t \, dt \]  
(26)

for \( \omega = 0, F_2(\omega) = 0 \) and

\[ F_1(\omega) = \int_{0}^{\infty} f(t) \, dt = A(\omega) \]  
(68)

In the case of a velocity-time history, the steady-state amplitude equals the permanent displacement. As shown in later sections, the shapes of these amplitude spectra are typical of those from the field data.

The phase spectra shown in Figure 8 are asymptotic to \( \pi \) at large values of frequency. In general, the lower the ratio of \( a/b \) the more rapid the increase in phase with frequency. At frequencies lower than \( b \), however, this does not hold true. The initial phase increase at low frequency is most rapid for \( a/b = 1 \). As can be seen in Figure 10, the initial low-frequency phase increases for \( a/b \) ratios of 1/2 and 2 are identical, as are those for \( a/b \) ratios of 1/5 and 5. These begin to diverge rapidly at frequencies greater than 0.1b. The shapes and trends of these frequency spectra are also typical of those from the field data.

2. I-TRAIN EVALUATION

I-TRAIN, the program written to take Fourier transforms on CRREL's PDP-8 computer, and listed in Appendix C, was first evaluated by taking the transform of the exponentially decaying sine wave defined in equation 60. The results were compared with the analytic solution expressed by equations 66 and 67. The following values were substituted for the parameters in equation 60: \( A = 50 \) ft/sec and \( a = b = 100 \) radians/sec, yielding
$f(t) = 50e^{-100t}\sin 100t$ (69)

These values were chosen so that $f(t)$ resembled a typical velocity-time history. Channel 018 from DATEX I was used as a reference. A comparison plot of equation 69 and Channel 018 is shown in Figure 11.

The DATEX I time history was shifted in time so that the time of arrival coincides with that of $f(t)$ at the origin. There is a significant advantage in shifting the time history so that it starts at time zero. As shown in equations 41 and 42, shifting a function of time (in this case the delta function) from time zero to $t_0$ results in an increase in phase of $\omega t_0$. This effect is shown graphically in Figure 12 for the transform of the exponentially decaying sine wave of equation 60 with $a = b$. For the function beginning at the origin, the phase is given by equation 67. If the function is delayed by shifting it along the time axis $t_0$, equation 67 becomes
As shown in Figure 12, this results in a linear increase in phase, in proportion to the delay time, or arrival time \( t_0 \). Stated another way, the slope of the phase spectrum is everywhere increased by \( t_0 \).

This creates a real problem in obtaining the phase spectrum, since it involves taking the inverse tangent as expressed in equation 5

\[
\theta(\omega) = \tan^{-1}\left(\frac{F_2(\omega)}{F_1(\omega)}\right)
\]

Referring to the complex plane representation of \( F(\omega) \) given in Figure 1, as the vector \( F(\omega) \) rotates, the computer cannot keep track of \( \theta(\omega) \) for more than \( \pm \frac{\pi}{2} \) or 1 revolution, depending on the inverse tangent function of the particular computer. The CRREL PDP-8 computer gives values of \( \tan^{-1} \) between \( \pm \frac{\pi}{2} \), as shown in Figure 13. As \( \theta(\omega) \) exceeds \( \frac{\pi}{2} \), the inverse tangent changes to a negative sign and begins decreasing in absolute magnitude. It reaches zero at \( \theta(\omega) = \pi \), then becomes positive and increases in magnitude until \( \theta(\omega) \) reaches \( 3\pi/2 \), when the inverse tangent again equals \( \frac{\pi}{2} \). The AFWL CDC 6600 computer has a more sophisticated inverse tangent function, giving values of \( \tan^{-1} \) between \( \pm \pi \). Only one change in sign occurs per revolution, at values of \( \theta(\omega) = n\pi \). But for either inverse tangent function, it is necessary to “unfold” the phase spectrum from the computer output, as shown in Figure 13. If \( \theta(\omega) \) is rapidly increasing, as would be the case for a time history delayed in time \( t_0 \), it makes unfolding of the phase spectrum very difficult or impossible because of the confusion created by the rapid sign changes. Therefore, all data were shifted to start at time zero to facilitate unfolding of the phase spectra.

To eliminate confusion, the phase spectra derived from the time-shifted data are represented by \( \phi(\omega) \). The actual phase spectra can then be easily recovered by adding \( \omega t_0 \) to them so that

\[
\theta(\omega) = \phi(\omega) + \omega t_0
\]

In practice, it is easier to work with the phase spectra from the zero shifted time histories, correcting them by adding \( \omega t_0 \) only when necessary (for instance, when calculating phase velocities). Shifting the data in time has no effect on the amplitude spectrum, as is demonstrated in Figure 12.

A 28-point approximation of equation 69 was used to evaluate I-TRAIN. The I-TRAIN transform of the straight-line approximation is compared with the actual transform in Figure 14. Representative data points from the output are plotted on the actual amplitude and phase spectra obtained by substituting the relevant values of the constants into equations 66 and 67. Overall agreement between I-TRAIN and actual spectra is excellent. There is a small cyclic aberration above 100 cps in the phase spectrum; elsewhere there are no significant deviations.

The transform of Channel 018 from DATEX I was taken for comparison with that of the function given in equation 69. A 24-point approximation was used. The amplitude and phase spectra are shown in Figure 15, along with those obtained analytically for equation 69. The spectra display some similarities, but also show significant differences. The amplitude spectrum more nearly resembles the spectrum for the analytic function with an \( a/b \) ratio of 1/2 as
Actual amplitude spectrum
\[ A(\omega) = \frac{Ab}{\sqrt{(a^2+(b-\omega)^2)(a^2+(b+\omega)^2)}} \]
where \( A = 50 \text{ ft/sec} \), \( a, b = 100 \text{ rad/sec} \)

Figure 14. Evaluation of I-TRAIN transform of exponentially decaying sine wave.
\[ F_1(\omega) = \frac{(a^2 + b^2 - \omega^2)(ab)}{[a^2 + (b - \omega)^2] [a^2 + (b + \omega)^2]} \]

with \( A = 50 \text{ ft/sec} \), \( a = b = 100 \text{ radians/sec} \)

Figure 16. Real part of Fourier transform of \( f(t) = 50 e^{-100t} \sin 100t \).

Figure 17. Fourier synthesis of \( f(t) = 50 e^{-100t} \sin 100t \).

shown in Figure 7. It shows a distinct peak at about 11 cps before tailing off quite like the analytic spectrum. The phase spectra are more irregular than the analytic phase spectra with a considerable disparity over the middle frequencies. There is also a slight delay between 0 and 5 cps similar to that for low \( a/b \) ratios, as shown in Figure 8.

3. S-TRAIN EVALUATION

S-TRAIN, the program which performs the Fourier synthesis, also listed in Appendix C, was evaluated using an 18-point approximation of the real part of the transform of equation 69. \( F_1(\omega) \) is given by equation 64. Using the values of \( A, a \) and \( b \) given for equation 69, \( F_1(\omega) \) is plotted in Figure 16. The 18-point approximation of \( F_1(\omega) \) ended with the last point \( [F_1(\omega) = 0] \) at 300 cps. The synthesized time history is compared with the actual time history in Figure 17. The two time histories are virtually identical.

The effect of lopping off the high-frequency components of the real part of the transform was examined by terminating \( F_1(\omega) \) at 50 cps as shown in Figure 16. The first 13 points of the original approximation were used, the 13th point being the value of \( F_1(\omega) \) at 40 cps. The last point of the new approximation was \( F_1(\omega) = 0 \) at 50 cps. Thus, the new approximation had only 14 points, was identical to the original approximation out to 40 cps, but then made a ramp return to zero between 40 and 50 cps. The synthesized velocity-time history is plotted in Figure 17. Agreement with the actual time function is poor at early times but becomes reasonably satisfactory beyond 12 msec.
A second evaluation of S TRAIN was run using the triangular time history shown in Figure 18. The triangle was chosen because it provided a difficult test of the synthesis technique, even though it was not typical of field data. The real part of the Fourier transform is given by Guillemin (ref. 2) as

\[ F_1(\omega) = \left( \frac{\sin \omega/2}{\omega/2} \right)^2 \cos \omega \]  

and is plotted in Figure 19. A 24-point straight-line approximation of \( F_1(\omega) \) was used to obtain the synthesized time history shown in Figure 18. This is a reasonable match to the triangular time history, although the corners are rounded because some frequency information is distorted in the straight-line approximation of the transform. Overall, the impulse train technique appears to be a very satisfactory method of synthesizing time histories.

Finally, synthesizing a time history from a transform which has the \( \omega t_0 \) of equation 70 deleted, by definition, yields a time history that begins at time zero. The correct time history is obtained by simply displacing the synthesized time history \( t_0 \) in time. Since it is more convenient to work with transforms of time histories shifted to start at time zero, it is also easier to synthesize these transforms directly and to shift the time histories by the appropriate values of \( t_0 \) once they have been synthesized.
SECTION IV

DATA

A brief description of each experiment is included in this section along with an explanation of how and why particular data channels were chosen for analysis. As illustrated in Section II, nearly all the data had problems of one form or another which would tend to seriously distort transforms of them. For this reason, considerable liberty was taken, both in choosing the channels for analysis (i.e., all the gages may not lie on a single line radiating from the explosive) and in manipulating the data to correct for abnormalities which are not a part of the ground-motion time history.

1. DATEX I

DATEX I is the first of three Direct Induced High-Explosive Simulation Technique (DIHEST) shots used in the Fourier analysis. All DIHEST experiments used high explosives buried in a geometric array to produce a desired particle velocity-time history at a given range from the array (ref. 7). Specification of the time histories was based on predicted cratering-induced ground motions from nuclear surface bursts. The three DIHEST experiments discussed in this analysis used rectangular, planar, and vertical arrays of explosives.

The DATEX I explosive array (ref. 8) consisted of 110 standard 40-lb ammonium nitrate canisters (2.2 tons total) grouted in eleven 9-in. diameter holes. As shown in Figure 20, the explosives were contained between depths from 12 to 50 ft to form an array 100 ft long and 38 ft high centered at a depth of 31 ft.

Figure 20. Plan and section of DATEX I.
The experiment was fielded near Cedar City, Utah, in an outcropping of moderately weathered granodiorite. Joint spacing averaged about 4 ft, unconfined compressive strength about 8,000 psi, and unit weight about 157 lb/ft³. Laboratory Young's modulus at 50% of ultimate strength averaged $1.5 \times 10^6$ psi.

Motion monitoring instrumentation was located in 15 holes between 20 and 120 ft from the explosive array, as shown in Figure 20. The preponderance of instrumentation used in these experiments, and that generally producing the most reliable ground-motion data, consisted of velocity gages. It was desired to pick motion measurements from a line perpendicular to the center of the explosive array. Therefore, the velocity records from the 31-ft depth in holes 2, 3, 6, 9, 10 and 11 were examined. The 31-ft-depth time histories from holes 2, 6 and 10 were rejected for several reasons. The gage at the 31-ft depth in hole 2 was within the crater; however, the gage at the 43-ft depth was below the crater and therefore was substituted. The data record for hole 6 had excessive electrical noise; so, again, the gage at the 43-ft depth was substituted. The waveform for hole 10 was irregular and its integration showed a substantial net displacement toward the explosive array. However, since this was the only instrument in hole 10, no substitution could be made.

Thus, the five velocity-time histories shown in Figure 21 were used in the analysis. Corrections were made to two of the five time histories: the cable break was eliminated from Channel 012 (hole 2) by terminating the record at 94 msec, and a linear ramp shift was applied to Channel 018 (hole 3) to return it to zero at 115 msec. The data were time-shifted to start at time zero, and the linear approximations, shown in Figure 22, were made. Between 20 and 32 points were used in the approximations, depending on the shape of the data record.
Figure 23. Amplitude spectra from DATEX I.

Figure 24. Phase spectra from DATEX I.
Fourier transforms of the five records shown in Figure 22 were taken using the I-TRAIN program. The amplitude spectra are shown in Figure 23 and the phase spectra in Figure 24.

2. DATEX II

DATEX II is the second of three DIHEST shots analyzed. It was also sited at Cedar City, Utah, within a few hundred yards of the DATEX I experiment. Although no laboratory tests were performed on samples from this site, the drill crew superintendent indicated that the rock was somewhat "softer" than that at the DATEX I site.

The DATEX II explosive array (ref. 9) consisted of 41 tons of aluminized slurry explosive pumped into 29 12-in.-diameter holes. The slurry was contained between 29 and 65 ft in depth to form an explosive array 200 ft long by 36 ft high centered at a depth of 47 ft. A plan view of the test layout is shown in Figure 25. Again, it was desired to analyze ground motions along a perpendicular line emanating from the center of the explosive array.

After careful consideration, six data records were chosen as representative of the data on or as near as possible to the instrumentation line leading from the center of the explosive array. The gages were located in holes 17, 2, 5, 9, 14 and 16, respectively. All records were from a depth of 40 ft except measurement 005 (hole 2), which was from a depth of 16 ft. The uncorrected velocity-time histories are shown in Figure 26. The severe differential displacements which occurred in the DATEX II testbed caused premature cable failures on all instruments close to the explosive array, three of which are evident on the three closest data records. Use of these three records was essential to provide data over a sufficient range to enable determination of transmission properties of the testbed. Therefore, to remove the cable breaks, some rather gross liberties were taken, especially with the 50-ft record.

The cable breaks were eliminated from the three records and estimated completions of the time histories were substituted. Extrapolations from other data, at a greater range, were used to estimate the positive and negative phase durations of the three records. The positive phase duration for the 50-ft record was taken from the extrapolation shown in Figure 27. The negative phase durations were obtained by multiplying the positive phase durations by the ratios shown in Figure 28. The curve shapes were estimated by scaling the appropriate portions from the nearest existing data. The result of this exercise is shown in Figure 29, which is a plot of the straight-line velocity-time history.
Figure 26. Uncorrected velocity-time histories from DATEX II.

Figure 27. Positive phase estimate (50-ft record) points from DATEX II data.

Figure 28. Estimated ratio of negative to positive phase duration from DATEX II.
Figure 29. Velocity-time history straight-line approximations from DATEX II.

Figure 30. Amplitude spectra from DATEX II.

Figure 31. Phase spectra from DATEX II.
Figure 32. Plan view of STARMET.

Figure 33. Uncorrected velocity-time histories from STARMET.
approximations. The approximations range from 30 to 48 points, depending on the irregularities in the data records. They have been shifted to start at time zero. The Fourier transforms were taken using I TRAIN; the amplitude spectra are shown in Figure 30 and the phase spectra in Figure 31.

3. STARMET

STARMET is the last of the three DIHEST experiments examined in this study. It was essentially a duplicate of the DATEX I experiment, but was fielded in a much harder rock. The test site was located in the Pedernal Hills, west of Encino, New Mexico. The rock was a highly-jointed (joint spacing averaged 6 in.) metamorphosed granite of Precambrian origin (ref. 10). Laboratory unconfined compressive strength averaged higher than 30,000 psi and Young's modulus at 50% of ultimate strength averaged between 9 and 10 x 10⁶ psi.

A plan view of the test layout is shown in Figure 32. It was desired to use data records from the west centerline for analysis, but, as in the other tests, some compromises had to be made. STARMET instrumentation performance and ground motions were complicated by the upthrust of a large block of rock on the west side of the explosive array. The boundaries of this block are sketched on the plan view of Figure 32. The west boundary of the block was formed by a joint which dipped toward the explosive array at an angle of 67° and intersected all the instrumentation holes to the east of its intersection with the testbed surface. Instrumentation within the block generally survived, but recorded the 3-ft westward horizontal displacement of the block. Records from instrumentation located below the block were terminated prematurely by cable cuts resulting from relative motion along the boundary joint.

One data record from within the thrust block was analyzed, from a depth of 15 ft in hole 6. All records in hole 7 received cable cuts, so the 30-ft record from hole 2 was substituted. The 30-ft record from hole 8 was acceptable, but the corresponding records from holes 9 and 10 were noisy, necessitating substitution of the 45-ft records. Thus a total of five STARMET records were analyzed; however, the data from the closest gage were severely influenced by the motion of the thrust block. These data are shown in Figure 33. The hole 6 gage is easily distinguished by its positive phase duration of almost 1 sec as opposed to typical durations of 20 msec for gages outside the thrust block.

The straight-line data approximations, time-shifted to zero, are shown in Figure 34. The transforms were taken using I TRAIN. The resulting amplitude spectra are shown in Figure 35 and the phase spectra in Figure 36. The effects of the long positive phase duration of the 30-ft time history are easily seen in both the amplitude and phase spectra. The low-frequency portions of the 30-ft spectra are significantly higher in both amplitude and phase. However, above 10 cps there appears to be little difference between the 30-ft spectra and the others.

4. MINERAL LODE

MINERAL LODE was sited at Cedar City, Utah, although the site was located several miles to the north of the DATEX sites. The test comprised a 16-ton spherical charge of ammonium nitrate slurry contained at a depth of 100 ft (ref. 11). Two radial lines of instrumentation extended horizontally outward from the charge, as shown in Figure 37. The west line, because it was the longer of the two, was chosen for analysis.

The uncorrected data are shown in Figure 38. Early cable cuts prevented use of data from the 40-ft range. The only time history available at 50 ft was the integrated acceleration in Figure 38. This was returned to zero at 67 msec using a linear baseline shift as shown in Figure 39. One velocity gage-time history was also corrected; the 60-ft gage was returned to zero at 72 msec. The remaining four time histories, at 75, 85, 100 and 150 ft, were run with only minor corrections.

Figure 39 shows the time-shifted straight-line representations of the MINERAL LODE data. Between 28 and 55 points were used to approximate the data. Fifty-five points were needed for the integrated accelerometer because of the late time fluctuations in the time history. The next highest number of points used was 37 for the velocity gage record from 150 ft. The transforms were taken using I TRAIN. The amplitude spectra are shown in Figure 40 and the phase spectra in Figure 41.
Figure 34. Velocity-time history straight-line approximations from STARMET.
Figure 35. Amplitude spectra from STARMET.

Figure 36. Phase spectra from STARMET.

Figure 37. Plan view of MINERAL LODE.

Explosive and instrumentation placed at 100 ft depth.
Figure 38. Uncorrected velocity-time histories from MINERAL LODE.

Figure 39. Velocity-time history straight-line approximations from MINERAL LODE.
HARD HAT and PILEDRIVER were both underground nuclear detonations in a granite stock in area 15 of the Nevada Test Site. PILEDRIVER was shot at a depth of 1518 ft and had an effective yield of approximately 61 kiloton (kt) (ref. 12). HARD HAT was shot at a depth of 950 ft and had a yield of 5.9 kt (ref. 13). The rock was an equigranular granodiorite and porphyritic quartz monzonite with an average density of 168 lb/ft³ and unconfined compressive strength varying from 6,700 to 24,000 psi. Instrumentation extended radially outward at shot level.

Data were selected from HARD HAT at ranges from 306 to 784 ft. The radial velocity-time histories from five gage locations are shown in Figure 42. All data are from integrated accelerometers, except for the plot at the 784-ft range, which is from a velocity gage. The data had been baseline corrected previously. It appears that the 306-ft and the 505-ft channels were terminated prematurely, probably because of cable breaks. These data were transformed as is; no attempt was made to extend them. The straight-line approximations of between 37 and 67 points are shown in Figure 43. The transforms were taken using I TRAIN. The amplitude spectra are shown in Figure 44 and the phase spectra in Figure 45.

Only two channels of data were used from PILEDRIVER. These were radial velocities from ranges of 668 ft and 1543 ft and are shown in Figure 46. The transform at the 668-ft range was taken and was used to predict the transform at the 1543-ft range using the transmission properties of the rock obtained from the HARD-HAT transforms. A velocity-time history was then synthesized from the predicted transform using S TRAIN and compared with the actual 1543-ft-range velocity-time history. The 39-point approximation of the 668-ft-range record is shown in Figure 47 and the corresponding amplitude and phase spectra are shown in Figures 48 and 49, respectively.
Figure 42. Corrected velocity-time histories from HARD HAT.

Figure 43. Velocity-time history straight-line approximations from HARD HAT.
Figure 43 (Cont'd).

Figure 44. Amplitude spectra from HARD HAT.

Figure 45. Phase spectra from HARD HAT.
Figure 46. Velocity-time histories from PILEDRIVER.

Figure 47. Velocity-time history straight-line approximations from PILEDRIVER.

Figure 48. Amplitude spectrum from PILEDRIVER.
Figure 49. Phase spectrum from PILEDRI\'VER.
The principal objective in taking Fourier transforms of the ground-motion data in Section IV was to develop a means of specifying the transmission characteristics of the different rock types. Once the transmission characteristics are described, they can be used to predict ground motions at any range in the testbed; or, they can be used to predict ground motions from any explosive input, as long as a motion-time history can be measured, predicted or otherwise defined at some point in the testbed. They can also be used to compare different rock types; for instance, siting requirements for a weapons system or a command and control center might indicate that rock with high dispersion and attenuation properties is desirable. The transmission characteristics of the material provide the most logical means of comparing sites which appear favorable.

As noted in the last part of Section II, the transfer function $H(\omega)$ is used to describe the transmission characteristics. In practice, a number of problems are encountered in defining a transfer function. Some of these are a result of the geometry of the test and others are caused by peculiarities and inconsistencies in the data. Most of these problems will be discussed as they arise in the analysis.

1. **ABSORPTION**

As discussed above and as described in equation 57,

$$G(\omega) = F(\omega)H(\omega)$$  \hspace{1cm} (57)

The transform at any location $G(\omega)$, given the transform at some other location $F(\omega)$ and the transfer function $H(\omega)$, can be obtained. Consequently, if the transforms are specified at two or more locations, they can be used to obtain the transfer function.

Equation 58, the polar representation of equation 57, can be rewritten as

$$A_G(\omega)e^{j\theta_G(\omega)} = A_F(\omega)e^{j\theta_F(\omega)}A_H(\omega)e^{j\theta_H(\omega)}$$  \hspace{1cm} (73)

McDonal et al. (ref. 14), in analyzing motion-time histories in Pierre shale, treated absorption separately from dispersion. Examining equation 73 and equations 4, 5 and 8 shows that absorption results from a change in amplitude $A(\omega)$, while dispersion results from a change in phase $\theta(\omega)$. This is in keeping with the definition of the two. Absorption is the process of energy dissipation in the material through which a disturbance is transmitted. Dispersion is the variation in propagation velocity with frequency which results in a reshaping of the disturbance as it propagates. Thus, the amplitude spectra and phase spectra may be used to define absorption and dispersion, respectively, although, as Futterman (ref. 1) showed, they are not mutually independent. Wuenschel (ref. 15), using a model which considered both absorption and dispersion, was able to make an excellent approximation of McDonal's data, better, in fact, than was achieved using the absorption model alone.

Before obtaining the absorption from a series of amplitude spectra taken at different ranges along a radial line from an explosive input, the spectra must be corrected for distortion due to the geometry of the experiment. McDonal et al.  

40
(ref. 14) made this correction for a point charge by multiplying each velocity amplitude spectrum by its range from the source. Since peak particle velocity varies with the inverse of the range for a spherical wave in an elastic medium, this correction removes effects of the spherical geometry.

It was decided to use the MINERAL LODE data to examine methods of obtaining the absorption and dispersion characteristics. As discussed in Section IV, six velocity-time histories were transformed. Since only six data records were used, there is no guarantee that their amplitude spectra are typical of MINERAL LODE data. For the derived absorption properties to be representative of the MINERAL LODE site, the amplitude spectra from which they are derived must also be representative. As shown in Section III, the magnitude of a velocity amplitude spectrum is directly proportional to the magnitude of the velocity-time history from which it is derived. In trying to use only a few amplitude spectra to represent data from a test, a problem arises because they are taken from velocity-time histories higher or lower in magnitude than the average velocities at the locations chosen. Cooper and Blouin (ref. 7) point out that peak particle velocities commonly vary by a factor of 3 at any given range. Therefore, the MINERAL LODE amplitude spectra were corrected to match more closely what were felt to be average MINERAL LODE spectra at the gage locations selected.

These spectra were corrected by adjusting the magnitude of the velocity-time histories so that peak velocities matched the least squares fit to the data given by Murrell and Carleton (ref. 11). Their fit is shown in Figure 50 along with the uncorrected peak particle velocities from the six time histories used in the analysis. Some of the data fall above the fit and some below. The peak velocities were adjusted to fall on the least squares fit by multiplying them by the correction factors listed in Figure 50. The amplitude spectra were also adjusted by multiplying them by the same correction factors. In addition, the spectra were also corrected for geometry by multiplying by the range, as discussed previously. The adjusted MINERAL LODE amplitude spectra, after multiplication by the correction factors listed in Figure 50 and by their range, are shown in Figure 51. In this manner, allowance was made both for the geometry of the experiment and for the data scatter common to all such tests.
To determine the absorption characteristics, the amplitude spectra in Figure 51 had to be presented in a way which showed more clearly the effects of frequency and range. After experimenting with several modes of presentation, we decided that the method used by both McDonal et al. (ref. 14) and Wuenschel (ref. 15) also gave the best fits to these data. Thus, the log of adjusted amplitude is plotted as a function of range for a series of frequencies between 10 and 200 cycles/sec in Figure 52. Straight-line fits to the data are shown, implying that the log of amplitude at any given frequency varies linearly with range. Also, it is apparent that the higher the frequency, the sharper the drop in amplitude.

The slope of the amplitude curves, known as the absorption or attenuation factor \( \alpha(\omega) \), can be expressed as

\[
\alpha(\omega) = \frac{\log A_1(\omega) - \log A_2(\omega)}{\Delta x} = \frac{\log A_1(\omega) - \log A_2(\omega)}{x_1 - x_2}
\]

where \( \Delta x \) is the distance between the adjusted amplitude measurements \( A_1(\omega) \) and \( A_2(\omega) \). For convenience, equation 74 can be rewritten as

\[
\alpha(\omega) = \frac{\log[ A_1(\omega)/A_2(\omega) ]}{\Delta x}
\]

The attenuation factor \( \alpha(\omega) \) is plotted as a function of frequency in Figure 53. The slopes were calculated for a \( \Delta x \) of 100 ft. The curve fitted to the data suggests that the attenuation factor begins at or near zero and increases smoothly with increasing frequency.
By using the notation of equation 73, if an amplitude spectrum $A_F(\omega)$ is known and the attenuation curve for the medium is defined as in Figure 53, we can obtain the amplitude spectrum at any other location $A_G(\omega)$ by manipulating equation 75 to give

$$A_G(\omega) = f_g A_F(\omega) 10^{q(\omega)\Delta x} = f_g A_F(\omega) 10^{q(\omega)(x_G-x_F)}$$

(76)

where $\Delta x$ is in hundreds of feet and $f_g$ is a geometric factor. In the case of a plane wave, $f_g = 1$, and in the case of a spherical wave, $f_g = x_F/x_G$, where $x_F$ and $x_G$ are the ranges of the known and desired amplitude locations, respectively.

Referring to equations 73 and 76, it is apparent that the amplitude portion of the transfer function is given by

$$A_H(\omega) = f_g 10^{q(\omega)\Delta x}$$

(77)

and that $A_H(\omega)$ is a function of both frequency and range.

The procedure to obtain the attenuation spectrum for HARD HAT was identical to that used in MINERAL LODE. As shown in Figure 54, we adjusted velocity-time histories to coincide with the fit to peak data given by Hoffman and Sauer (ref. 12). Figure 55 shows the amplitude spectra, corrected for both geometry and scatter. Figure 56 gives the semilog presentation of adjusted amplitude as a function of frequency and range, and Figure 57 shows the plot of attenuation factor as a function of frequency derived from Figure 56.

Derivation of the absorption properties for the three DIHEST sites was complicated by the explosive geometries' being planar rather than spherical. Thus, the method of correcting the data for geometric related attenuation had to be modified.

Since the amplitude spectra from velocity-time histories are proportional to the magnitude of the velocity, it was decided to correct the spectra using a scaling procedure originally developed for removing geometric effects from velocity-time histories to allow comparison of data from various DIHEST experiments. R.J. Port of the Air Force Weapons Laboratory suggested this method (ref. 16).

To compare the data of different DIHEST arrays and rock types we reduced the data to a common scaling base using similitude concepts from the theory of explosions in high density gases (ref. 17). As indicated in Figure 58, the attenuation of peak velocities at a shock front generated by a planar explosive array can be divided into three regions.
Near the array (within less than half its shortest dimension), there are no apparent edge effects and the wave attenuates as a plane wave. Beyond a distance equal to half the array's longest dimension, the attenuation properties are the same as for a spherical source equal in yield to the array yield. Between these two ranges, the attenuation is like that for a cylindrical source. A logarithmic plot of peak velocity versus range would show three attenuation regions distinguished by the changes in slope indicated in Figure 58. For an array whose length \( l \) exceeds the height \( h \), the geometric factor can be folded into the data presentation by use of a normalized range variable \( \rho \) defined by

\[
\rho = \left( \frac{h}{2} \frac{l}{2} x \right)^{\frac{1}{3}} \quad 0 \leq x < \frac{h}{2}
\]

(78)

\[
\rho = \left( \frac{l}{2} x^2 \right)^{\frac{1}{3}} \quad \frac{h}{2} \leq x < \frac{l}{2}
\]

(79)

\[
\rho = x \quad x \geq \frac{l}{2}
\]

(80)

where \( x \) is the range from the array. A logarithmic plot of peak velocity vs \( \rho \) results in a straight-line plot, as shown in Figure 58, thus eliminating the geometric effects of the array.

The technique for eliminating geometric effects from the DIHEST amplitude spectra was modified by multiplying the spectra by the range variable \( \rho \) instead of the range \( x \) as was done in the spherical case. The range variable \( \rho \) was
also used in place of range when correcting the spectra for data scatter. Thus, the ranges for the three closest data records from DATEX I have been altered slightly in Figure 59 to correct the data for scatter. The corrected amplitude spectra were multiplied by $\rho$; the correction factors are shown in Figure 59, and the corrected amplitude spectra are shown in Figure 60. Figure 61 shows the semilogarithmic plot of adjusted amplitude as a function of range and frequency. Again, the range variable $\rho$ is used in place of actual range. The attenuation spectrum of Figure 62 is developed in the usual way, by taking the slopes of the fits to the amplitude versus adjusted range data and plotting them...
as a function of frequency. The geometric factor \( f_g \) in equations 76 and 77 becomes the ratio of the range variables \( \rho_F/\rho_G \) at the known and unknown transform locations.

The development of the attenuation spectrum for DATEX II was the same as for DATEX I, with the range variable \( \rho \) substituted for actual range. The corrections for data scatter are shown in Figure 63 along with the combined amplitude correction factors. The corrected amplitude spectra are shown in Figure 64, the semi-logarithmic presentation in Figure 65, and the attenuation spectrum in Figure 66.
The STARMET attenuation spectrum was complicated somewhat by the fact that the 30-ft velocity gage was within the thrust block. As noted in Section IV, the positive phase duration of this data record was many times longer than the durations from greater ranges, producing a very large component of low-frequency amplitude (Fig. 35). However, since the low-frequency component appeared to be predominantly below a frequency of 10 cycles/sec and the amplitude spectrum above 10 cps seemed to closely resemble the other spectra, we decided to ignore the amplitude spectrum below 10 cycles/sec and use the spectrum above 10 cycles/sec without further modification.

The five velocity peaks are compared with the fit to all the STARMET data (ref. 10) in Figure 67. As can be verified from the velocity-time history in Figure 33, the 30-ft peak occurs very early in the record and is not influenced by the later time block motion. The scatter correction factors are shown in Figure 67. The corrected amplitude spectra are shown in Figure 68, the fits to the log amplitude as a function of frequency and range data in Figure 69, and the resulting attenuation spectrum in Figure 70.
The attenuation spectra from all five tests are plotted for comparison in Figure 71. The attenuation in the Cedar City granodiorite is markedly higher than in either the Pedernal Hills granite or the Nevada Test Site granite. Attenuation spectra from the two DIHEST shots at Cedar City, DATEX I and II, are nearly identical. They are, however, only about half as high as the attenuation measured from MINERAL LODE. There are two possible explanations for this discrepancy. First, the MINERAL LODE site was several miles from the DATEX sites, and while all three tests were in the same rock type, local differences could account for the change in attenuation from one location to the other. Second, the attenuation differences are due to the different test geometries rather than to measures of a real attenuation difference in the rock itself. As noted in the discussion of the transfer function in Section II, a basic assumption involved in the development of the analysis technique employed in this study is that energy is transmitted along a single path between sensors and changes in the signal are due to effects of the material on that signal.

Inclusion of a free surface in the model generates reflections and subsequently there is no longer a single transmission path between sensors. In addition, the block-like, jointed nature of the in-situ rock mass makes these boundary properties very difficult to define. Heuristically, it can be argued that the farther from the free surface an experiment is, the less noticeable are the effects of the free surface. Thus, the relatively shallow DIHEST experiments are probably influenced more by the ground surface than the deeper contained shots. In fact, in the DIHEST experiments, energy may be channeled along near the ground surface in a surface wave effect, thereby reducing the apparent material attenuation.

Thus, the Cedar City data indicate that free surface effects might cause an apparent reduction in attenuation measured in a near-surface experiment. Removing the surface effects may be possible, although this would probably be difficult because of the physical uncertainties associated with the free surface.

The HARD-HAT attenuation is only a fraction of that measured at Cedar City. While most of this difference presumably is due to rock quality, a portion of it may result from higher lithostatic stresses caused by the much deeper burial of the nuclear device.

The STARMET attenuation spectrum is distinctive in that it shows little change with frequency. At higher frequencies, it is close to the attenuation spectrum derived for the HARD HAT granite; however, since it does not decrease with frequency, it is comparable to the Cedar City attenuation spectrum at low frequencies. Examination of Figure 70 shows that the STARMET attenuation may be starting to decrease at about 10 cycles/sec.

2. DISPERSION

As noted previously, dispersion is the spreading of a stress pulse as it propagates. Dispersion results from the fact that propagation velocity generally varies with frequency in earth materials. Propagation velocity defined as a function of frequency is known as the phase velocity and is related to the phase spectrum. If phase spectra are available from two or more stations along a transmission path, average phase velocities can be determined between the different stations as functions of frequency. This phase velocity spectrum can then be used to obtain the phase portion of the transfer function which, combined with the amplitude portion discussed in the previous section, completes the derivation of the transfer function.

We can derive the relationship between phase velocity and phase angle in a simple example. Figure 72 shows a velocity-time history in the form of a cosine wave of frequency $\omega_1$. The wave is positioned so that at time zero, a crest is just crossing the origin. Since all the energy is transmitted at a single frequency, the Fourier amplitude and phase spectra for this signal are defined only at that frequency, $\omega_1$, and have values of $\delta(\omega_1)$ and 0 radians, as shown in Figure 72.

If the same cosine time history is observed at a second point, at a distance $x_1$ beyond the first observation point, the crest observed at the origin will not arrive at $x_1$ until time $t_1$. Thus, the velocity of the wave $v_1$, in this case the phase velocity because only a single frequency is involved, is
\[ v(t) = v_0 \cos(\omega_1 t) \]

at \( x = 0 \)

\[ v(t) = v_0 \cos \left( \frac{\omega_1}{v_1} x_1 \right) \]

at \( x = x_1 > 0 \)

\[ t_1 = \frac{x_1}{v_1} \]

Figure 72. Relationship between phase and phase velocity.

As shown in Figure 72, the delay associated with the arrival of the crest at \( x_1 \) is also equivalent to a shift in phase of the cosine wave by \( \omega_1 t_1 \) radians. The phase spectrum for the signal at \( x_1 \) is still defined only at \( \omega_1 \), but now has a value given by

\[ \theta(\omega_1) = \omega_1 t_1 \]

The phase delay time is obviously

\[ t_1 = \frac{\theta(\omega_1)}{\omega_1} \]

The amplitude spectrum remains an impulse at \( \omega_1 \).

In the general case, where any real-time history can be broken down into a summation of cosine waves over all frequencies, the phase delay time as a function of frequency is given by the analogous equation

\[ t_{\theta(\omega)} = \frac{\theta(\omega)}{\omega} \]

Likewise, phase velocity is given by

\[ v(\omega) = \frac{x}{t_{\theta(\omega)}} \]
When a number of transforms are available along a transmission path, the phase velocity at any frequency may be obtained by plotting the phase delay times obtained from equation 84 as a function of range. As indicated by equation 85, the phase velocity at that frequency is given by the slope of the fit to the delay-time-versus-range data. By repeating this process over a range of frequencies, phase velocity as a function of frequency can be determined.

Applying this technique to the phase spectra given in Section IV requires one further modification because the time histories were all shifted to begin at zero time before taking the transforms. As shown in Figure 12, the phase spectrum obtained from the zeroed data given by I TRAIN, \( \phi(\omega) \), must be added to \( \omega t_0 \), where \( t_0 \) is the time shift applied to the time history. The phase is given by equation 71

\[
\theta(\omega) = \phi(\omega) + \omega t_0
\]  

(71)

Substitution of equation 71 into equation 84 gives phase delay times in terms of the I TRAIN phase spectrum and the time shift

\[
t_\theta(\omega) = \frac{\phi(\omega) + \omega t_0}{\omega} = \frac{\phi(\omega)}{\omega} + t_0
\]  

(86)

Since the phase spectra frequencies \( f \) are expressed in cycles per second, equation 86 can be rewritten as

\[
t_\theta(\omega) = \frac{\phi(\omega) + 2\pi f t_0}{2\pi f} = \frac{\phi(\omega)}{2\pi f} + t_0
\]  

(87)

MINERAL LODE data are again used to demonstrate the techniques and assumptions used to obtain the phase velocity spectra for the various tests. The phase spectra obtained from the I TRAIN transform of the MINERAL LODE time histories are shown in Figure 41. There are several peculiarities evident which are also noticeable on the spectra from most of the other tests. Some of the spectra have a rather pronounced cyclical variation; examples are those for the 50- and 100-ft ranges. Other spectra, such as those from the 85- and 60-ft ranges, do not exhibit this behavior. It is felt that these variations are associated with peculiarities in a particular measurement and are not characteristic of the average ground motion.

Another peculiarity is the behavior of the spectra at low frequency. Some, such as the 60-, 75- and 100-ft spectra, begin at zero and diverge in a smooth function toward their high-frequency values, while others, such as the 50-, 85- and 150-ft spectra, make a small negative excursion before settling into the pattern of the other data. The small negative loops at low frequency are not understood at this time, but they are not believed to be of major significance. It was noticed in experimenting with the transform of Channel 018 of DATEX I that a minor increase in the digitization accuracy around the velocity peak resulted in the virtual elimination of a negative excursion similar to the excursions in Figure 41. No significant change was noticed in the amplitude spectrum or in the phase spectrum at higher frequencies.

With these assumptions in mind, we decided to apply what amounted to, in some cases, coarse smoothing of the phase spectra. Regardless of the rationalizations prompting the smoothings, the real test of these assumptions comes in the synthesis of the MINERAL LODE data in Section VI, using spectra predicted from the model based on the smoothed data. We feel that the synthesized MINERAL LODE time histories do justify the use of the smoothed phase spectra. The smoothed MINERAL LODE phase spectra are shown in Figure 73. The initial negative loops have been ignored and the cyclical variations averaged out.

Equation 87 was used to obtain phase delay times as a function of frequency and range. These are shown in Table 1. Since the 50- and 60-ft and 75- and 85-ft-range spectra are very similar and at nearly the same ranges, only the 50- and 75-ft ranges were used in the calculations. The data are plotted in Figure 74. Straight-line fits were made, and the slopes, or phase velocities, listed in the figure. As is common in earth materials, phase velocity varies with
Figure 73. Smoothed phase spectra from MINERAL LODE.

Figure 74. Phase delay times from MINERAL LODE.

Table 1.

PHASE DELAY TIMES FROM SMOOTHED PHASE SPECTRA – MINERAL LODE

\[ t_\theta(\omega) = \frac{\phi(\omega) + 2\pi ft_0}{2\pi f} \]

<table>
<thead>
<tr>
<th>Line</th>
<th>( v(\omega) ) (cps)</th>
<th>( \omega )</th>
<th>( \phi(\omega) )</th>
<th>( t(\omega) )</th>
<th>( t(\omega)^2 )</th>
<th>( t(\omega)^3 )</th>
<th>( t(\omega)^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>3760</td>
<td>0.20</td>
<td>0.0107</td>
<td>1.39</td>
<td>0.0193</td>
<td>0.53</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>4070</td>
<td>0.37</td>
<td>0.0102</td>
<td>0.66</td>
<td>0.0174</td>
<td>0.91</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>4760</td>
<td>0.67</td>
<td>0.0096</td>
<td>1.06</td>
<td>0.0153</td>
<td>1.42</td>
</tr>
<tr>
<td>D</td>
<td>50</td>
<td>5830</td>
<td>1.27</td>
<td>0.0083</td>
<td>1.86</td>
<td>0.0128</td>
<td>2.31</td>
</tr>
<tr>
<td>E</td>
<td>100</td>
<td>6810</td>
<td>1.96</td>
<td>0.0074</td>
<td>2.65</td>
<td>0.0111</td>
<td>3.22</td>
</tr>
<tr>
<td>F</td>
<td>200</td>
<td>7450</td>
<td>2.49</td>
<td>0.0069</td>
<td>3.23</td>
<td>0.0103</td>
<td>3.92</td>
</tr>
<tr>
<td>G</td>
<td>TOA</td>
<td>9670</td>
<td>2.94</td>
<td>0.0066</td>
<td>3.70</td>
<td>0.0098</td>
<td>4.47</td>
</tr>
</tbody>
</table>

frequency, the low-frequency components having a lower velocity than the high-frequency components. The first arrival times are also plotted in Figure 74. These are evidently a result of high-frequency arrivals. Experience has shown that the transmission velocity calculated from them is generally equivalent to the seismic compressional wave velocity, measured using seismic cross-hole techniques with small explosive charges. The first arrival velocity \( v_0 \) calculated from the fit to the time of arrival data in Figure 74 was 9,670 ft/sec. This serves as an upper bound to the phase velocities.
Figure 75. Phase velocity as a function of frequency from MINERAL LODE.

Figure 76. Phase velocity spectrum from MINERAL LODE.

Figure 77. Smoothed phase spectra from HARD HAT.

Figure 78. Phase delay times from HARD HAT.
The phase velocities from Figure 74 are plotted as a function of frequency in Figure 75. The shape of the phase velocity spectrum is suggestive of an exponential function, so the same data are replotted in Figure 76 as log phase velocity versus log frequency. The data fall nearly on a straight line, which can be expressed analytically as

\[ v(\omega) = 2,690 f^{0.20} \]  

(88)

where frequency \( f \) is in cycles per second. This function reaches the upper limit of 9,670 ft/sec at approximately 600 cycles/sec. Futterman (ref. 1) suggests that there is also a lower bound to phase velocity, below which frequency there is no longer any amplitude attenuation. Examination of the attenuation factors in Figure 71 along with the phase velocity spectrum of Figure 75 suggests that a lower cutoff frequency of 1 cycle/sec might be suitable for these data. Thus, equation 88 gives a representation of phase velocity between frequencies of 1 and 600 cycles/sec.

Phase velocity spectra were derived from the other four tests in the same manner. The smoothed HARD HAT phase spectra are shown in Figure 77. Phase delay times as a function of frequency and range are shown in Figure 78, and the derived phase velocity spectrum in Figure 79. HARD-HAT phase velocity is given by

\[ v(\omega) = 11,200 f^{0.0695} \quad \text{for} \ 1 < f < 2,000 \]  

(89)

The corresponding DATEX I data are shown in Figures 80-82. DATEX I phase velocity is given by

\[ v(\omega) = 3,180 f^{0.20} \quad \text{for} \ 1 < f < 320 \]  

(90)

The DATEX II phase velocity spectrum is markedly inconsistent with the other Cedar City spectra. The DATEX II smoothed spectra are shown in Figure 83. The phase delay times are plotted as a function of range in Figure 84 and the derived phase velocity spectra in Figure 85. It appears that the phase velocity for DATEX II is independent of frequency, being constant at about 5,900 ft/sec over all frequencies. Thus, there would be no dispersion. This is inconsistent with Futterman's model, since absorption is very definitely present. Also, the first arrival velocity (Fig. 84) of 8,970 ft/sec is considerably higher than the 5,900 ft/sec figure, indicating that some frequencies must travel faster than others. A possible explanation of this DATEX II anomaly is that considerable portions of the late time velocity data were estimated and may have resulted in distortion of the phase spectra. However, since this would have little effect on the high frequency data, this does not appear to be a very satisfactory explanation.

The STARMET phase velocity spectrum also appears to be independent of frequency. This would seem much more plausible in this case, since the Pedernal Hills granite was extremely hard and, as shown in Figure 71, exhibited very little absorption. The smoothed phase spectra are shown in Figure 86, the phase delay times in Figure 87, and the phase velocity spectrum in Figure 88. Phase velocity is nearly constant, at about 13,800 ft/sec. Unlike the DATEX II data, the STARMET phase velocity is very close to the 14,800 ft/sec first-arrival velocity shown in Figure 87.
Figure 80. Smoothed phase spectra from DATEX I.

Figure 81. Phase delay times from DATEX I.
Figure 82. Phase velocity spectrum from DATEX I.

Figure 83. Smoothed phase spectra from DATEX II.

Figure 84. Phase delay times from DATEX II.

Figure 85. Phase velocity spectrum from DATEX II.
Figure 86. Smoothed phase spectra from STARMET.

Figure 87. Phase delay times from STARMET.

Figure 88. Phase velocity spectrum from STARMET.
A summary of the phase velocity spectra is given in Figure 89. The DATEX I and MINERAL LODE phase velocities are very similar, both showing a comparatively strong dependence on frequency. The anomalous DATEX II spectrum, while showing no dependence on frequency, does have a constant phase velocity about midway in the range covered by the DATEX I and MINERAL LODE phase velocities. The HARD HAT and STARMET velocities are considerably higher than those from the tests at Cedar City. Neither show a strong dependence on frequency. STARMET, as mentioned, is independent of frequency and HARD HAT varies with only the 0.0695 power of frequency. Thus, dispersion is quite high in the Cedar City granodiorite (DATEX II excepted), while the HARD HAT and Pedernal Hills granites show little or no dispersion.

In general, phase velocity appears to be an exponential function of frequency in the various rock types examined in this study. Dispersion, which is related to the velocity change with frequency, is highest in softer rock with lower propagation velocities. The modulus of in-situ rock, which is proportional to the propagation velocity squared, is obviously also a function of frequency. Because the velocity is squared, the in-situ modulus is more sensitive to frequency than is the phase velocity.

The first part of this section showed that the attenuation spectrum could be used to predict the amplitude spectrum at any range, given one amplitude spectrum at another range. Likewise, it is possible to predict the phase spectrum at some other location. Using the notation of equation 73, the known phase spectrum can be represented by $\theta_F(\omega)$ as shown in Figure 90. As noted earlier in this section, $\theta_F(\omega)$ consists of two parts: the phase spectrum $\phi_T(\omega)$, obtained from the time history shifted to begin at time zero; and the linear spectrum $\omega t_0$, which represents the portion of $\theta(\omega)$ due to the time delay $t_0$. Thus, equation 71 can be modified to give the known phase spectrum as

$$\theta_F(\omega) = \phi_T(\omega) + \omega t_0 \tag{91}$$

The unknown phase spectrum $\theta_G(\omega)$, also shown in Figure 90, is likewise given by

$$\theta_G(\omega) = \phi_G(\omega) + \omega t'_0 \tag{92}$$

where $t'_0$ is the arrival time at the second location. The arrival times are related to the propagation velocity of the first arrival $v_0$, which is also the upper limit of the phase velocity, and are given by

$$t_0 = \frac{x_F}{v_0} \tag{93}$$

and

$$t'_0 = \frac{x_G}{v_0} \tag{94}$$

Substituting these expressions for $t_0$ and $t'_0$ into equations 91 and 92, respectively, and subtracting equation 91 from equation 92 yields
Substituting equation 84 into equation 85 gives phase velocity as a function of the phase angle

$$v(\omega) = \frac{\omega x_F}{\theta_F(\omega)} = \frac{\omega x_G}{\theta_G(\omega)}$$

(96)

Solving for $\theta_F(\omega)$ and $\theta_G(\omega)$ and subtracting $\theta_F(\omega)$ from $\theta_G(\omega)$ give an alternate expression for the phase difference of

$$\theta_G(\omega) - \theta_F(\omega) = \frac{\omega}{v(\omega)} (x_G - x_F)$$

(97)

Since the phase velocity spectrum $v(\omega)$ is given, equation 97 can easily be solved for the unknown phase spectrum $\theta_G(\omega)$:

$$\theta_G(\omega) = \frac{\omega \Delta x}{v(\omega)} + \theta_F(\omega)$$

(98)
\[ \Delta x = x_G - x_F \] (99)

In order to work with the I-TRAIN time-shifted phase spectra, the right-hand portions of equation 95 and equation 97 can be equated and the unknown spectrum \( \phi_G(\omega) \) derived as

\[ \phi_G(\omega) = \phi_F(\omega) + \frac{1}{\nu(\omega)} \left( \frac{1}{\nu(\omega)} - \frac{1}{\nu_0} \right)^{-1} \] (100)

### 3. TRANSFER FUNCTION

By treating absorption and dispersion as separate but related entities, equation 77 for the amplitude of the transfer function was derived. This is rewritten here to the base e as

\[ A_H(\omega) = f_0 e^{2.303 \alpha(\omega) \Delta x} \] (101)

Using the same assumption, the phase portion of the transfer function can be obtained from equation 58 as

\[ \theta_H(\omega) = \theta_G(\omega) - \theta_F(\omega) \] (102)

Substituting the expression for \( \theta_G(\omega) \) from equation 98 into 102 gives

\[ \theta_H(\omega) = \frac{\omega \Delta x}{\nu(\omega)} \] (103)

Substituting these expressions for the amplitude and phase portions of the transfer function into equation 59 gives the equation for the transfer function itself:

\[ H(\omega) = f_0 e^{2.303 \alpha(\omega) \Delta x} e^{j [2.303 \alpha(\omega) + \omega / \nu(\omega)]} \] (104)

Thus, the transfer function can be obtained from the attenuation and phase velocity spectra. This, in turn, can be combined with a known or predicted transformed time history, as shown in equation 57, to obtain a transformed time history at any other range, a distance \( \Delta x \) from the known function. In practice, the amplitude and phase portions of this process are handled separately. The unknown amplitude spectrum is obtained using equation 76 and the unknown phase spectrum using equation 100.
SECTION VI
FOURIER SYNTHESIS

In Section V, methods for developing the attenuation and phase velocity spectra were developed and the equations for using them to predict Fourier transforms were derived. In this section the viability of the transform approach is examined by synthesizing time histories from transforms constructed using some of the attenuation and phase velocity spectra derived in Section V. Recall that a good deal of data smoothing and interpolation were performed to obtain these attenuation and phase velocity curves. While these simplifications were carried out in a rational way, it is difficult to predict the effects, in the time domain, of these changes in the frequency domain. They can only be justified if the data recovered through Fourier syntheses of the altered transforms are an acceptable duplication of the original time histories.

It was decided to use the MINERAL LODE data to perform such an evaluation. A good deal of smoothing and interpolation were involved in obtaining the attenuation and phase velocities for MINERAL LODE, and the 50-ft velocity-time history was obtained from an integrated accelerometer, whereas the other data were obtained from velocity gages. Therefore, it was felt that MINERAL LODE would provide a good proof test of the frequency-based analysis techniques. The transform of the 50-ft velocity-time history was combined with the attenuation and phase velocity spectra of MINERAL LODE derived in Section V to obtain predictions of the transforms at the 85- and 150-ft ranges. S-TRAIN was used to synthesize the velocity-time histories and these were compared with the original MINERAL LODE data.

The 50-ft amplitude spectrum from Figure 40 has been replotted to a linear scale in Figure 91. The strong cyclic variations were believed to be peculiar to this particular measurement and were eliminated by smoothing the data as shown. The smoothed 50-ft phase spectrum of Figure 73 was combined with the smoothed amplitude spectrum using a modification of equation 46 to obtain the real part of the transform:

\[ F_1(\omega) = 1.11 A(\omega) \cos \phi(\omega) \]  

(105)

where 1.11 is the data scatter correction given in Figure 50. \( F_1(\omega) \) is plotted in Figure 92. An extrapolation to return the transform to zero at 400 cycles/sec was added as indicated. The effects of smoothing both the amplitude and phase spectra were determined by taking the inverse transform using a 20-point straight-line approximation of \( F_1(\omega) \). The S-TRAIN output was divided by 1.11 to negate the scatter correction. The recovered velocity-time history is compared with the original in Figure 93. The agreement is considered to be excellent. Smoothing the amplitude and phase spectra also resulted in smoothing the time history. Evidently, the cyclic variations in the phase and amplitude spectra result from similar variations in the time history. Removing these from the transform, in turn, removes them from the time history.

Having demonstrated that the smoothed 50-ft amplitude and phase spectra gave a good synthesis of the 50-ft time history, these spectra were subsequently used to predict the spectra at the 85- and 150-ft ranges. The amplitude spectra, shown in Figure 94, were obtained by applying equation 76 to the 50-ft smoothed spectrum of Figure 91. The attenuation factor \( \alpha(\omega) \), used in equation 76, was taken from Figure 53. The phase spectra, shown in Figure 95, were obtained by applying equation 100 to the smoothed 50-ft phase spectrum of Figure 73. The phase spectra were combined, according to equation 46, to give the predicted real part of the transforms at 85 and 150 ft. These are plotted in Figure 96 along with \( F_1(\omega) \) for the 50-ft range.
Figure 91. Fit to amplitude spectrum for 50-ft MINERAL LODE gage.

Figure 92. Real part of Fourier transform of 50-ft gage for MINERAL LODE.

Figure 93. Fourier synthesis of 50-ft MINERAL LODE time history from smoothed spectra.
Figure 94. Predicted MINERAL LODE amplitude spectra.

\[ A(\omega) = \frac{50}{\lambda} A_{\infty}(\omega) 10^{\alpha(\omega)} \Delta x \]  

(From smoothed spectra)

Figure 95. Predicted MINERAL LODE phase spectra.

\[ \phi(\omega) = \omega \Delta x \left[ \frac{1}{\nu(\omega)} - \frac{1}{\nu_0} \right] \phi_{\infty}(\omega) \]  

(Smoothed Spectrum)

Figure 96. Predicted MINERAL LODE Fourier transforms.
The predicted velocity-time histories from these transforms were synthesized using S TRAIN. A 26-point straight-line approximation of the 85-ft transform and a 20-point approximation of the 150-ft transform were used. The predicted velocity-time histories are compared with the actual time histories in Figure 97. The predictions are reasonably good reproductions of the data. Since the synthesized data are "average" data in the sense that their magnitude matches the peak velocity fit of Figure 50, the agreement would be even better were they corrected by the appropriate scatter correction factors shown in Figure 50. It is felt that the agreement between the MINERAL LODE data and the time histories synthesized from the phase velocity and attenuation spectra developed in Section V justifies the assumptions and approximations made in the development of these spectra.

One of the most beneficial potential uses of Fourier transforms is to predict ground motions based on transmission properties defined by small-scale tests. In essence, the transfer function, expressed in terms of the phase velocity and attenuation spectra, can be defined for a particular site using a small, relatively inexpensive in-situ test. Ground-motion predictions can then be made at this site for any explosive input, provided a ground-motion prediction at some point (most likely close to the explosive device) can be made. The transform of the prediction is taken and combined with the transfer function to obtain the transforms at the desired locations. Synthesizing these predicted transforms gives the desired ground-motion predictions.

This process is demonstrated using the HARD-HAT transmission properties to predict ground motions from PILE-DRIVER. Only two complete credible velocity-time histories were obtained from the PILEDRIVER experiment, at ranges of 668 and 1,543 ft. The 668-ft record was used in combination with the phase velocity and attenuation spectra derived from the HARD HAT data in Section V to predict the 1,543-ft ground motion. The 668-ft velocity-time history is shown in Figure 46.
The amplitude and phase spectra obtained from a 39-point approximation of the velocity-time history are shown in Figures 48 and 49, respectively. These spectra were smoothed, as shown in Figure 98. They were then combined with the HARD HAT attenuation curve of Figure 57 and the phase velocity spectrum of Figure 79 using equations 76 and 100, respectively, to obtain the predicted spectra, also included in Figure 98. These predicted spectra were combined using equation 46 to give the real part of the 1,543-ft transform shown in Figure 99. The predicted 1,543-ft-range velocity-time history was synthesized from a 33-point approximation of $F_1(\omega)$ using S TRAIN. The prediction is compared with the actual velocity-time history in Figure 100. While the predicted time history has slightly longer positive and negative phase durations than the data, it is a reasonable approximation of it.
Figure 99. Real part of predicted Fourier transform of PILEDRIVER at 1,543-ft range.

Figure 100. Fourier synthesis of PILEDRIVER data using HARD HAT material model.
SECTION VII
DISCUSSION

Section VI demonstrated that Fourier techniques have potential application in describing the material transmission properties of in-situ earth media subjected to high-intensity dynamic loadings and in predicting the ground motions resulting from these loadings. However, a number of considerations that may affect the applicability of these techniques have not been discussed.

The application of Fourier techniques was pioneered by electrical engineers who found them of immense value in areas of electrical engineering. As is often the case, an electrical analogy is helpful in understanding the physical implications in the application of transform techniques to ground motion analysis.

The Fourier integral transform is frequently used to analyze electrical circuits. If the classic black box is used to mask the components of an electrical circuit, the characteristics of the circuit can still be described by obtaining its transfer function. A multiple frequency input, such as a step function, is applied to the black box and the output is measured. The transfer function of the box is obtained by substituting the transforms of the input and output in equation 58. Once the transfer function has been defined, the output of the black box can be predicted for a wide range of inputs. Thus, the transmission characteristics of a complicated circuit can be derived without our knowing the components that make up the circuit. The obvious advantage of this technique is that it eliminates an arduous circuit analysis, but not without payment of a penalty: the analyst is able to describe and predict the performance of the circuit, but he does not understand how the circuit works. His lack of understanding may or may not be a debility, depending on whether he will be required to alter or repair the circuit.

In our situation, the rock or soil is analogous to the black box. Basically, the transfer function of the earth material is obtained by substitution of transformed ground motions from two or more locations along a common transmission path into equation 58. As with the electrical analog, these transmission characteristics (represented by the transfer function) are themselves descriptive of the medium and allow ground motion predictions to be made over a wide range of inputs. However, the transmission properties do not describe the processes governing the behavior of the medium, nor is their relationship to standard descriptive material properties obvious. Since the attenuation and phase velocity spectra that describe the transfer function are not mutually independent, their relationship to material properties such as modulus and hysteresis is probably not straightforward. A change in modulus, for instance, might influence both the attenuation and phase velocity spectra, although the influence might not be shared equally.

Another consideration which has not been discussed is the relationship between the transfer function and stress level. Consider the electrical analog again. If the black box is excited by a voltage function high enough in magnitude to exceed the linear range of some of the circuit components, the predicted output will no longer match the actual output (assuming that the applied voltage used to obtain the transfer function is within the linear range of the components). As with the electronic analog, it would be imprudent to suggest that the transfer function for earth media developed over one stress range should apply at a completely different stress level. Ground motions examined in this study had peak particle velocities between 1 and 100 ft/sec, which correspond to peak stresses from several hundred to several tens of thousand psi. While this range of stresses was substantial, the transmission properties did not appear to vary with stress. Generally, however, peak velocities from any one test did not cover nearly as wide a range as indicated above.
Thus, the use of transfer functions derived from ground motion data at stress levels above or below those used in the derivation should be avoided until further work verifies or disproves their applicability in these stress regimes. Conversely, caution should be used in trying to develop material transmission properties from small-scale explosive tests. Seismic stress levels are generally well below those examined here, requiring an extrapolation to the stress levels of interest. Even when small-scale tests are instrumented at ranges close enough to produce the desired stress levels, the frequencies produced may be much higher than desired, requiring an extrapolation in frequency before high-yield predictions can be made.

A final consideration, discussed briefly in Section V, is the effect of geometric discontinuities on the transfer function. The derivation of the transfer function is based on the assumption that the stress pulse follows a single line of transmission between the sensor locations used in the derivation. Inclusion of discontinuities can result in multiple transmission paths due to reflections and refractions at the discontinuities. Examples of such discontinuities in a rock mass are the free surface, bedding planes, faults, intrusives, alteration zones and even joints, especially weathered joints. While the inclusion of all such discontinuities could lead to hopeless confusion, it is not clear that all or any of the discontinuities are important, particularly where large stresses and relatively long wavelengths are involved. The only effect of discontinuities noticed in this study was the possible influence of the free surface on the attenuation spectra from the DIHEST shots at Cedar City.

There appear to be two approaches to treating discontinuities: 1) they can be included in the transfer function, or 2) they can be handled separately with a model which includes the geometry of the experiment. In the first approach, the transfer function is developed for geometrically similar experiments (such as using the transfer function from one underground nuclear shot to predict the ground motions from another), and the consequences of geometry, if any, are assumed to be included in the derived transmission properties. In this case, caution must be exercised in going from one experimental geometry to another. The second approach is the more formal. Transmission properties are derived for all separate material units involved in the experiment. The transfer functions govern the ground motions within the various material units, but conventional mechanics govern the motions due to reflections and refractions at the discontinuities. The usual problem of modeling the discontinuity properties is not eliminated with this approach. Judgment as to the feasibility and practicality of the second approach is beyond the scope of this study.
LITERATURE CITED

APPENDIX A

THE FOURIER INTEGRAL AND SERIES

The Fourier integral is correctly applicable in the general case; however, the functions being dealt with are defined only over a finite time interval, are causal [i.e., \( f(t) = 0 \) for \( t < 0 \)], and are real. In this case, several simplifications in the analysis can be made.

Figure A1 defines a function that equals the original function in the interval \( 0 < t < T \) and for which \( f(t+nT) = f(t) \) outside that interval. For such a function (ref. 18),

\[
F(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \alpha_n \delta(\omega - n\omega_0)
\]

where \( \omega_0 = 2\pi/T \)

\[
\alpha_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j\omega_n t} dt
\]

and

\[
f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{j\omega_n t}
\]

\( (A1) \)

---

Figure A1.

---

69
This is a finite time, or periodic function simplification of the Fourier integral. Four points are noteworthy regarding this finite time calculation:

1. 

\[
\alpha_n = \frac{1}{T} \sum_{-T/2}^{T/2} f(t) e^{-j\omega_0 t} dt = \int_{-T/2}^{0} + \int_{0}^{T/2} + \int_{T/2}^{T} = \int_{0}^{T/2} + \int_{0}^{T/2}
\]

Since \( f(t) = f(t+nT) \), then \( f(0) = f(T) \) and \( f(-T/2) = f(-T/2+T) = f(T/2) \). Therefore,

\[
\alpha_n = \frac{1}{T} \int_{0}^{T} f(t) e^{-j\omega_0 t} dt
\]

(A2)

2. From equation A2,

\[
\alpha_n = \frac{1}{T} \int_{0}^{T} f(t) e^{-j\omega_0 t} dt
\]

\[
\alpha_n = \frac{1}{T} \int_{0}^{T} f(t)(\cos\omega_0 t - j\sin\omega_0 t) dt = \frac{A_n - jB_n}{2}
\]

and

\[
\alpha_{-n} = \frac{1}{T} \int_{0}^{T} f(t)(\cos\omega_0 t + j\sin\omega_0 t) dt = \frac{A_n + jB_n}{2}
\]

where \( A_n \) and \( B_n \) have been defined for convenience as

\[
A_n = \frac{2}{T} \int_{0}^{T} f(t) \cos\omega_0 t dt
\]

\[
B_n = \frac{2}{T} \int_{0}^{T} f(t) \sin\omega_0 t dt
\]

Since \( f(t) \) is a real function, both \( A_n \) and \( B_n \) are real. Note that the quantities \( A_n \) and \( B_n \) are formally identical to the Fourier series coefficients. Also,

\[
\phi(n) = \tan^{-1} \frac{\Im(\alpha_n)}{\Re(\alpha_n)} = \tan^{-1} \frac{-B_n}{A_n} = -\tan^{-1} \frac{B_n}{A_n}
\]

\[
\phi(-n) = \tan^{-1} \frac{B_n}{A_n}
\]

\[
\phi(-n) = -\phi(n)
\]
and

$$|\alpha_n| = |\alpha_{-n}|$$

We see that for periodic functions the phase angles of the negative frequency components are just the values of the positive components with the signs changed and the magnitude of positive and negative frequency components are equal.

3. From equation A1,

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{j\omega_0 t}$$

Equation A1 may be regrouped and rewritten as

$$f(t) = a_0 + \sum_{n=1}^{\infty} \alpha_n e^{j\omega_0 t} + \alpha_{-n} e^{-j\omega_0 t}$$

$$= a_0 + \sum_{n=1}^{\infty} A_n \left( \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) + B_n \left( \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \right)$$

$$= a_0 + \sum_{n=1}^{\infty} A_n \cos \omega_0 t + B_n \sin \omega_0 t$$

(A3)

where

$$a_0 = \frac{1}{T} \int_{0}^{T} f(t) dt$$

We can see that finite time periodic calculation of the Fourier integral simply reduces to the calculation of the Fourier series.

4. The particle velocity function depicted in Figure A1 has a finite energy which is proportional to

$$\int_{-\infty}^{\infty} |f(t)|^2 dt$$

The proportionality constant depends on the elastic modulus and the medium density. Parseval's theorem (eq 4) can be used to relate the energy to an integral over the frequency:

$$\text{Parseval's equality: } \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} A^2(\omega) d\omega$$

(A4)
Based on equation 5, it is customary to define an energy spectral density (ESD)

\[ ESD = \lim_{\Delta \omega \to 0} \frac{1}{\Delta \omega} \int_{\omega-\Delta \omega/2}^{\omega+\Delta \omega/2} A^2(\xi) d\xi = A^2(\omega) \]  

For periodic functions, a power spectrum \( S(\omega) \) is defined (ref. 18):

\[ S(\omega) = 2\pi \sum_{n=\infty}^{\infty} \left| \alpha_n \right|^2 \delta(\omega-n\omega_0) \]  

where \( \omega_0 = \frac{2\pi}{T} \)

A power spectral density (PSD) is also defined for periodic functions

\[ PSD = \frac{S(n\omega_0)}{(n+\frac{1}{2})\omega_0 - (n-\frac{1}{2})\omega_0} = \frac{2\pi}{\omega_0} \left| \alpha_n \right|^2 \]  

We can investigate the power spectral density of the function depicted in Figure A1b for very large \( T \). From equation A8 the limit in this case is:

\[ PSD(n\omega_0) = \lim_{T \to \infty} \left( \frac{2\pi}{\omega_0} \left| \alpha_n \right|^2 \right) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jnw_0 t} dt \]  

\[ = \frac{2\pi}{\omega_0} \left( \frac{1}{T} \right) \left| A(n\omega_0) \right|^2 = \frac{1}{T} A^2(n\omega_0) \]  

where \( A(\omega) \) is the amplitude spectrum of the function shown in Figure A1a. Therefore, for very large \( T \)

\[ PSD(n\omega_0) = ESD(n\omega_0)/T \]  

This result conforms with intuition in that one does not expect the spectral characteristics of Figure A1a and A1b to be different in the limit of large \( T \). Equation A10 shows that the essential difference in the periodic and nonperiodic spectral functions is that for periodic functions power is the appropriate quantity while for nonperiodic functions the energy is:

\[ \text{PSD} (n\omega_0) = \frac{2\pi}{\omega_0} \left| \alpha_n \right|^2 \]

\[ = \frac{2\pi}{\omega_0} \left( \frac{A_n - jB_n}{2} \right) \left( \frac{A_n + jB_n}{2} \right) = \frac{\pi}{\omega_0} \left( A_n^2 + B_n^2 \right) \]  

\[ (A11) \]
Equation A11 shows that the power spectral density function can be calculated from the summed squared coefficients of the Fourier series.

The foregoing comments demonstrate that a Fourier series calculation for a periodic function modeled after the original function provides an appropriate approximation of the Fourier integral.
APPENDIX B

SUBROUTINE FOURT USED IN ANALYSIS

SUBROUTINE FOURT (DATA1, DATA2, NN, NCIM, IFPWC, ICPLX, WORK1)

DIMENSION DATA1(1), DATA2(1), NN(1), WORK1(1), IFPWC(20)

DOUBLE PRECISION TWP, TMIN, TMAX, W, W2, W3, W4, W5, W6, W7, W8,

C TMIN, TMAX, W, WORK1, IFPWC, WORK2, WORK3, WORK4, WORK5

TH COOLEY-TUKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN

TH EVALUATES COMPLEX FOURIER SERIES FOR COMPLEX OR REAL FUNCTIONS.

TH.That is, IT COMPUTES

TH \sum \text{DATA}(I1, I2, \ldots) \times \text{W1} \times (I1-1) \times (J1-1) \times (W2-1) \times (J2-1) \times \ldots,

C THW1 = exp(2*PI*SORT1-1/NN(1)), W2 = exp(-2*PI*SORT1-1/NN(2)),

C AND I1 AND J1 RUN FROM 1 TO NN(1), I2 AND J2 RUN FROM 1 TO

C NN(2), ETC.

TH THERE IS NO LIMIT ON THE DIMENSIONALITY (NUMBER OF

C SUBSCRIPTS) OF THE ARRAY OF DATA. TH. PROGRAM WILL PERFORM

C A MULTIDIMENSIONAL FOURIER TRANSFORM AS EASILY AS A ONE-DIMENSIONAL ONE, THE IN A PROPORTIONALLY GREATER TIME. AN INVERSE

C TRANSFORM CAN BE PERFORMED, IN WHICH THE SIGN IN THE EXPONENTIALS

C IS +, INSTEAD OF --. IF AN INVERSE TRANSFORM IS PERFORMED UPON

C AN ARRAY OF TRANSFORMED DATA, THE ORIGINAL DATA WILL APPEAR,

C MULTIPLIED BY NN(1)*NN(2)*\ldots. THE ARRAY OF INPUT DATA MAY BE

C REAL OR COMPLEX, AT THE PROGRAMMERS OPTI9ON, WITH A SAVING OF

C ABOUT THIRTY PER CENT IN RUNNING TIME FOR REAL OVER COMPLEX,

C (FOR FASTEST TRANSFORM OF REAL DATA, NN(1) SHOULD BE EVEN.)

C TRANSFORM VALUES ARE ALWAYS COMPLEX, AND ARE RETURNED IN THE

C ORIGINAL ARRAY OF DATA, REPLACING THE INPUT DATA. THE LENGTH

C OF EACH DIMENSION OF THE DATA ARRAY MAY BE ANY INTEGER.

C THE PROGRAM RUNS FASTER ON COMPOSITE INTEGRALS THAN ON PRIMES, AND IS

C PARTICULARLY FAST ON NUMBERS RICH IN FACTORS OF TWO.

TH TIMING IS IN FACT GIVEN BY THE FOLLOWING FORMULA, LET NTOT BE THE

TH TOTAL NUMBER OF POINTS (REAL OR COMPLEX) IN THE DATA ARRAY, THAT

TH IS, NTOT = NN(1)*NN(2)*\ldots. THEN DECOMPOSE NTOT INTO ITS PRIME FACTORS,

TH SUCH AS 2**4 * 3**2 * 5**3 * 7**1 * \ldots

TH SUM2 BE THE SUM OF ALL

TH FACTORS OF TWO IN NTOT, THAT IS, SUM2 = 2**4. LET SUMF BE

TH SUM OF ALL OTHER FACTORS OF NTOT, THAT IS, SUMF = 3**2 + 5**3 + 7**1 + \ldots.

TH THEN, TAKEN BY A MULTIDIMENSIONAL TRANSFORM ON THESE NTOT DATA

TH IS T = T0 + T1*NTOT + T2*NTOT*SUM2 + T3*NTOT*SUMF.

TH FOR THE PARTICULAR IMPLEMENTATION FORTRAN 32 ON THE CDC 3300 (FLOATING POINT

TH ADDITION, \approx 6 MICROSECONDS),

TH T = 3.000 + 600*NTOT + 90*NTOT*SUM2 + 170*NTOT*SUMF MICROSECONDS

ON COMPLEX DATA.

TH IMPLEMENTATION OF THE DEFINITION BY SUMMATION WILL RUN IN A TIME

TH PROPORTIONAL TO NTOT**2. FOR HIGHLY COMPOSITE NTOT, THE SAVINGS

TH OFFERED BY COOLEY-TUKEY CAN BE DRAMATIC. A MATRIX 100 BY 100 WILL

TH BE TRANSFORMED IN TIME PROPORTIONAL TO 100**2*(2+2+2+5+5+5+5) =

TH \approx 280,000,000 (ASSUMING T2 AND T3 TO BE ROUGHLY COMPARABLE) VERSUS

TH 100**2*2 = 100,000,000 FOR THE STRAIGHT-FORWARD TECHNIQUE.

TH COOLEY-TUKEY ALGORITHM PLACES TWO RESTRICTIONS UPON THE

TH NATURE OF THE DATA BEYOND THE USUAL RESTRICTION THAT

TH DATA IS FROM ONE CYCLE OF A PERIODIC FUNCTION. THEY ARE:

TH 1) THE NUMBER OF TRANSFORM VALUES

TH MUST BE THE SAME.

TH 2) CONSIDERING THE DATA TO BE IN THE TIME DOMAIN,

TH THEY MUST BE EQUIDISTANT AT INTERVALS OF \pi.

TH FURTHER, THE TRANS-
SUBROUTINE FOUR:

CALL Routines of a FORTRAN IV Compiler

NEED TO ALLOW FOR REAL AND IMAGINARY PARTS OF THE INPUT DATA ON INPUT AND THE TRANSFORM VALUES ON OUTPUT.

THE CALLING SEQUENCE IS:

CALL FOURT(DATAR,DATAI,NN,N0DIM,IFRWO,ICPLX,HOR<R,WORKR,WORKI)

DATAR AND DATAI ARE ARRAYS USED TO HOLD THE REAL AND IMAGINARY PARTS OF THE INPUT DATA ON INPUT AND THE TRANSFORM VALUES ON OUTPUT.

THE AIR DIMENSIONALITY AND XXTENT OF THE INPUT DATA ARE GIVIN IN THE INTEGER ARRAY NN, OF LENGTH 0NDIM. THE 0NDIM IS THE DIMENSIONALITY OF THE ARRAYS DATAR AND DATAI.

IFRWO IS AN INTEGER USED TO INDICATE THE DIRECTION OF THE FOURIER TRANSFORM. IT IS NON-ZERO TO INDICATE A FORWARD TRANSFORM (EXPONENTIAL SIGN IS +) AND ZERO TO INDICATE AN INVERSE TRANSFORM (SIGN IS -).

ICPLX IS AN INTEGER TO INDICATE WHETHER THE DATA ARE REAL OR COMPLEX. IT IS NON-ZERO FOR COMPLEX, ZERO FOR REAL.

IF IT IS ZERO (REAL) THE CONTENTS OF ARRAY DATAI WILL BE ASSUMED TO BE ZERO, AND NOD NOT BE EXPLICITLY SET TO ZERO. FOR REAL ARRAYS, THE TRANSFORM RESULTS ARE ALWAYS REAL, AND STORED IN DATAR AND DATAI ON RETURN.

WORKR AND WORKI ARE ARRAYS USED FOR WORKING STORAGE. THEY ARE NOT NEEDED IF ALL THE DATA ARE POWERS OF TWO.

EXAMPLE 1: THREE-DIMENSIONAL FORWARD FOURIER TRANSFORM OF A COMPLEX ARRAY DIMENSION 100 BY 16 BY 13.

DIMENSION DATAR(100,16,13),DATAI(100,16,13),WORKR(100),WORKI(100)

CALL FOURT(DATAR,DATAI,NN,N0DIM,IFRWO,ICPLX,HOR<R,WORKR,WORKI)

EXAMPLE 2: ONE-DIMENSIONAL FORWARD FOURIER TRANSFORM OF A REAL ARRAY OF L.NOTH 64.

DIMENSION DATAR(64),DATAI(64)

CALL FOURT(DATAR,DATAI,64,1,1,1)

END
SUBROUTINE FOUR: 74/74 C8=1 F1N 4.2+74265 04/17/75 08.42.36, PAGE 4

C S.P.RAT: FOUR CASES--
C 1. COMPLEX TRANSFORM
C 2. REAL TRANSFORM FOR THE 2ND, 3RD, ETC. DIMENSION. METHOD--
C TRANSFORM HALF THE DATA, SUPPLYING THE OTHER HALF BY CONJUGATE SYMMETRY.
C 3. REAL TRANSFORM FOR THE 1ST DIMENSION, N ODD. METHOD--
C SET THE 1ST HAGULAR PARTS TO 2.RO.
C 4. REAL TRANSFORM FOR THE 1ST DIMENSION, N EVEN, METHOD--
C TRANSFORM A COMPLEX ARRAY OF LENGTH N/2 WHOSE REAL PARTS ARE THE EVEN NUMBERED REAL VALUES UNSCRAMBLED AND SUPPLY THE SECOND HALF BY CONJUGATE SYMMETRY.

ICASE=1
IF(INI=1)
IF(IP=1000,100,100)
70
ICASE=2
IF(IP=1172,72,100)
72
ICASE=3
IF(INTO=NP1,100,73)
73
ICASE=4
IF(NP2=NP2,2/2)
N=N/2
NP2=NP2/2
NTOT=NTOT/2
1=1
80
DO 80 J=1,NTOT
DA=AR(J)
DATAK(J)=DATAR(I)
DATAI(J)=DATAI(I+1)
80
1=1+2
C SHUFFL DATA BY BIT REVERSAL, SING: N=2**K AS THE SHUFFLING CAN BE DONE BY SIMPLE INTERCHANGE; NO scrambling ARRAY IS NEEDED.
C
100 IF(NP2P1=1101,101,200)
101 NP2P1=NP2/2
J=1
105 DO 125 I=1,NP2P1,NP2P1
110
121 T=MAX=12-NP2P1+1
DO 125 I=11,MAX
DO 125 I=11,MAX,
J,J=J+13-NP2P1
T=-MAX=T-1
125 DATAI(J)=T-MP1
130 M=M+NP2P1
140 IF(J=M)150,150,141
141 J=J-M
M=M+2
IF(NP1=NP1+150,140,141
150 J=J-M
C SHUFFL DATA BY DIGIT REVERSAL FOR GENERAL N
C
200 C0 270 I=1,NP1
205 DO 270 I=11,NTOT,NP2
J=1
210 IF(IP=270,111,NTOT,NP2
J=I+1
210 W=KR(I)=DATAR(I)
210 W=KI(I)=DATAI(I)
GO TO 240
220 W=KR(I)=DATAR(I)
W=KI(I)=DATAI(I)
240 IFNP2P2,MP2,IF
250 IF(IP=41P2P21FACT(IF)
J=I-41P2P2
255 J=J-41P2P2
IF(IFP2P2=NP2)
250 CONTINUE
12MAX=12-NP2P1
1=1
300 DO 300 I=1,MAX
305 IMIN=NP1+I1
ISTEP=2*NP1
GO TO 320
310 J=1
320 DO 320 I=IMIN,NTOT,ISTEP
T=MP1=DATAR(I)
T=MPI=DATAI(I)
330 DA=AR(I)=DATAR(I)+MP1
DA=AI(I)=DATAI(I)+MPI
DA=AI(I)=DATAI(I)+MPI
DA=AI(I)=DATAI(I)+MPI
320 J=J+1
310 IMAX=IMIN+IMIN-11
ISTEP=ISTEP+ISTEP
330 IF(IP=11,NTOT,NP2P1,IF
330 IF(IP=1,1,FLF,IF)
310 IFNP1P1=NP1,1,1111
331 IF(IN=3*NP1+11
SUBROUTINE FOURT 74/74 CPT=1 FTN 4.2+74265 04/17/75 08.42.36.

IST.P=4*NP1
GO TO 420

400 J=IMIN-IST.P/2
GO 410 I=IMIN,NTOT,IST.P
IF(IFRWD).NE.1,GO TO 401

411 T.MPR=DATA1(I)
T.MPI=DATA2(I)
GO TO 403

402 T.MPR=DATA1(I)
T.MPI=DATA2(I)
GO TO 403

403 DATA1(I)=DATA1(I)-TEMPK
DATA2(I)=DATA2(I)-TEMFP
DATA1(I)=DATA1(I)+TEMFR
DATA2(I)=DATA2(I)+TEMPI
GO TO 410

CONTINUE

C MAIN LOOP FOR FACTORS CF TWO, W=EXP (-2*° T* SQRT (-1)) AX 

TH.. TA= TWOPI/3
WSTPR=0.
WSTPI=-1.
IF(IFRWD) 502,501,502
TH.. TA=-THETA
WSTP I = 1.

MMAX=8*NP1
GO TO 540

WMII I R=DCOS (TH.. TA)
WMII I I=DSIN (TH.. TA)
WR=WMII I R
WI=WMII I I
MMI.. N=MMAX/2+NP1
IMAX=M+NP1
GO TO 540

DO 530 J1=1,IFP1,NP1
II AX=J1 + I1RNG-1
DO 525 II=J1,II MAX
I1=1
WR=WMII R
WI=WMII I
J2MAX=J1 + NP2-IFP2
DO 640 J2=13,J2 MAX,IFP1
JMIN=J1
J3MAX=J2rNP2-IFP2
DO 630 J3=J2,J3MAX,IFP2
J = Jf1N+1FP2-IFP2
SR=DATA1(J)
SI=DATA2(J)
OLDSR=S
OLL'SI=0.
J=J+IFP1
620 STMTPR=WR
STMTP I =SI
STWOKR=SR-OLDSR+DATA1(J)
STWOSI=SI-OLL'SI+DATA2(J)
OLDSR=SR
OLL'SI=SI
J=J+IFP1
IF(J=JMIN).NE.621,621,620

DO 600 J=J+1FP2,NTWO

CONTINUE

WSTPI=WMII I

C MAIN LOOP FOR FACTORS NOT EQUAL TO TWO

C W= XP ( 2*P1'3QFT(-1) /IFP2) TH TA=-THET A

IFPl=NTWO
IF INP2
IFP2=IFACTiIF)*IFP1
TH.. TA=-TWOP 1/FLOAT(IFACT(IF))

WSI..PR=DCOS(THETA)
WSI..PI=DSIN(THETA)
WMJ TR=DCOS(THETM)
W-ETI = DSI N(THET M)
WMII R=1.
WMII I =0.
DO 660 J1=1,IFP1,NP1
II AX=J1 + I1RNG-1
DO 650 J2=2,IFP2,IFP1
J = J+1FP2-IFP2

DO 640 J2=13,J2 MAX,IFP1
JMIN=J1
J3MAX=J2rNP2-IFP2
DO 630 J3=J2,J3MAX,IFP2
J = Jf1N+1FP2-IFP2
SR=DATA1(J)
SI=DATA2(J)
OLDSR=S
OLL'SI=0.
J=J+IFP1
620 STMTPR=WR
STMTP I =SI
STWOKR=SR-OLDSR+DATA1(J)
STWOSI=SI-OLL'SI+DATA2(J)
OLDSR=SR
OLL'SI=SI
J=J+IFP1
IF(J=JMIN).NE.621,621,620

DO 600 J=J+1FP2,NTWO

CONTINUE

WSTPI=WMII I

W=WI+*WSTPI+*WSTPI

W=WI+*WSTPI+WT*MP
WSTPR=WMII R
SUBROUTINE FOURT 74/74 CPT=1 FTN 4.2+74265. 04/17/75 38.42.36.

GO TO 755

750 DATA (J) = DATA (I)

755 I = I + 1

J = J - 1

IF (I .GE. JMAX) GO TO 750

DATA (I) = DATA (J)

DATA (J) = DATA (I)

GO TO 755

END
APPENDIX C

I-TRAIN AND S-TRAIN PROGRAMS

I-TRAIN PROGRAM

C IMPULSE TRAIN TRANSFORM — S BLOUIN

C

C DIMENSION F(100), T(100), A(100)

C READ DATA

READ(1,1) N
DO 200 I=1,N
READ(1,2) T(I)
200 READ(I,2) F(I)
1 FORMAT(‘N=’,I10)
2 FORMAT(E20.0)

F(N+1)=0
T(N+1)=T(N)+1.
A(1)=(F(2)-F(1))/(T(2)-T(1))
DO 240 I=2,N
240 A(I) = (F(I+1)-F(I))/(T(I+1)-T(I))-((F(I)-F(I-1))/(T(I)-T(I-1)))

500 WRITE(1,3)
OMEGA=0.
AO=0.
3 FORMAT(‘OM1, OM2, OM3, DEL1, DEL2, DEL3‘)
READ(1,2) OM1, OM2, OM3, DEL1, DEL2, DEL3
50 DO 70 I=2,N
70 AO=A0+(F(I)+F(I-1))*(T(I)-T(I-1))/2.
70 CONTINUE

WRITE(3,4) AO
4 FORMAT(‘AO=’,E15.5)

120 IF (OM1-OMEGA) 140, 140, 130
130 OMEGA=OMEGA+DEL1
GO TO 180
140 IF(OM2-OMEGA) 160, 160, 150
150 OMEGA=OMEGA+DEL2
GO TO 180
160 IF(OM3-OMEGA) 500, 500, 170
170 OMEGA=OMEGA+DEL3
180 RE=0
OMEGA=OMEGA*2. *3. 14159
XIM=0.
XIM=XIM+(A(I)*SIN(OMEGA*T(I)))
CONTINUE

RE=RE/OMEGA**2
XIM=XIM/OMEGA**2
AW=(RE**2+XIM**2)**. 5
XNORMAW=AW/OMEGA
PHI=ATAN(XIM/RE)
OMEGA=OMEGA/(2. *3. 14159)
WRITE(3,5) OMEGA, AW, XNORMAW, PHI
5 FORMAT(4E15.5)
GO TO 120
END
I-TRAIN Typical Input

DATEX 1012    23-Point Approximation

<table>
<thead>
<tr>
<th>N=23 Number of points in f(t) approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0004 Digitized pairs</td>
</tr>
<tr>
<td>0006 Time - seconds</td>
</tr>
<tr>
<td>0013 Velocity - ft/sec</td>
</tr>
</tbody>
</table>

Output parameters

- 0 to 20 cps in increments of 2 cps
- 20 to 100 cps in increments of 5 cps
- 100 to 200 cps in increments of 10 cps
### I-TRAIN Typical Output

<table>
<thead>
<tr>
<th>Freq (cps)</th>
<th>Amplitude</th>
<th>Normalized Amplitude</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.49415E+01</td>
<td>0.82162E-01</td>
<td>0.65382E-02</td>
<td>0.58447E+00</td>
</tr>
<tr>
<td>0.20000E+01</td>
<td>0.13362E+00</td>
<td>0.53166E-02</td>
<td>0.51421E+00</td>
</tr>
<tr>
<td>0.40000E+01</td>
<td>0.17656E+00</td>
<td>0.46833E-02</td>
<td>0.28167E+00</td>
</tr>
<tr>
<td>0.60000E+01</td>
<td>0.20380E+00</td>
<td>0.40545E-02</td>
<td>0.27096E+00</td>
</tr>
<tr>
<td>0.80000E+01</td>
<td>0.21324E+00</td>
<td>0.33939E-02</td>
<td>0.26724E+00</td>
</tr>
<tr>
<td>0.10000E+02</td>
<td>0.13151E+00</td>
<td>0.49831E+00</td>
<td>0.10570E+01</td>
</tr>
<tr>
<td>0.12000E+02</td>
<td>0.15901E+00</td>
<td>0.77581E-02</td>
<td>0.10700E+01</td>
</tr>
<tr>
<td>0.14000E+02</td>
<td>0.18608E+00</td>
<td>0.10958E+00</td>
<td>0.12961E+00</td>
</tr>
<tr>
<td>0.16000E+02</td>
<td>0.21324E+00</td>
<td>0.87198E-03</td>
<td>0.13252E+01</td>
</tr>
<tr>
<td>0.18000E+02</td>
<td>0.24139E-03</td>
<td>0.22115E-03</td>
<td>0.13351E+01</td>
</tr>
<tr>
<td>0.20000E+02</td>
<td>0.10958E+00</td>
<td>0.22115E-03</td>
<td>0.13351E+01</td>
</tr>
<tr>
<td>0.25000E+02</td>
<td>0.56717E-03</td>
<td>0.56717E-03</td>
<td>0.98733E+00</td>
</tr>
<tr>
<td>0.30000E+02</td>
<td>0.46499E-03</td>
<td>0.46499E-03</td>
<td>0.98296E+00</td>
</tr>
<tr>
<td>0.35000E+02</td>
<td>0.40979E-03</td>
<td>0.40979E-03</td>
<td>0.98296E+00</td>
</tr>
<tr>
<td>0.40000E+02</td>
<td>0.32391E-03</td>
<td>0.32391E-03</td>
<td>0.98296E+00</td>
</tr>
<tr>
<td>0.45000E+02</td>
<td>0.24139E-03</td>
<td>0.24139E-03</td>
<td>0.98296E+00</td>
</tr>
<tr>
<td>0.50000E+02</td>
<td>0.19777E-03</td>
<td>0.19777E-03</td>
<td>0.15059E+01</td>
</tr>
<tr>
<td>0.55000E+02</td>
<td>0.15693E-03</td>
<td>0.15693E-03</td>
<td>0.14861E+01</td>
</tr>
<tr>
<td>0.60000E+02</td>
<td>0.12125E-03</td>
<td>0.12125E-03</td>
<td>0.14559E+01</td>
</tr>
<tr>
<td>0.65000E+02</td>
<td>0.10710E-03</td>
<td>0.10710E-03</td>
<td>0.14559E+01</td>
</tr>
<tr>
<td>0.70000E+02</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>0.75000E+02</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>0.80000E+02</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>0.85000E+02</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>0.90000E+02</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>0.95000E+02</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.00000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.10000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.20000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.30000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.40000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.50000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.60000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.70000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.80000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>1.90000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
<tr>
<td>2.00000E+03</td>
<td>0.13252E+01</td>
<td>0.13252E+01</td>
<td>0.14216E+01</td>
</tr>
</tbody>
</table>
S-TRAIN PROGRAM

FOURIER SYNTHESIS - IMPULSE TRAIN - S BLOUIN

DIMENSION OM(100), F(100), A(100)

READ DATA
READ (1,1) N
DO 200 K=1, N
READ (1,2) OM(K)

OM(K)=OM(K)*2.*3.14159

1 FORMAT('N=',110)
2 FORMAT(E20.0)

F(N+1)=0.
OM(N+1)=OM(N)+1.
A(1)=2.*((F(2)-F(1))/(OM(2)-OM(1)))

DO 240 K=2, N

A(K)= (F(K+1)-F(K))/(OM(K+1)-OM(K))- (F(K)-F(K-1))/(OM(K)-OM(K-1))

WRITE (1,3)
T=0.

3 FORMAT('T1, T2, T3, DEL1, DEL2, DEL3' )
READ (1,2) T1, T2, T3, DEL1, DEL2, DEL3

120 IF(T1-T) 140, 140, 130
130 T=T+DEL1
GO TO 180
140 IF(T2-T) 160, 160, 150
150 T=T+DEL2
GO TO 180
160 IF(T3-T) 500, 500, 170
170 T=T+DEL3
180 SUM=0.

DO 250 K=2, N
SUM=SUM+(A(K)*COS(OM(K)*T))

250 CONTINUE
FUNCT=(-2./(3.14159*(T**2.))(*(A(1)/2.+SUM))
WRITE(3,5) T, FUNCT

5 FORMAT(2E15.5)
GO TO 120
END
S-TRAIN Typical Input - MINERAL LODE 150 ft 20 points

```fortran
P FORT
*STRAIN/G/$ILLEGAL SYNTAX
*STRAIN/G/US
0989
0921
0936
N=27 Number of points in \( F_1(\omega) \) approximation
9.
6551 Digitized pairs
1.
3547
3. Frequency - cps
8464 \( F_1(\omega) \) - ft
5.
0389
8.
0255
10.
0174
11.
0136
13.
0057
17.
-0052
27.5
-0128
35.
-0112
45.
-0073
57.5
-0037
57.5
-00192
62.5
-00955
160.
-00605.
125.
-00117
160.
-00069
200.
-00003
400.
5.
0.010 sec in increments of 0.001 sec
3.45 0.010 to 0.040 sec in increments of 0.002 sec
25 0.04 to 0.25 sec in increments of 0.005 sec
061
062
065
T1, T2, T3, DEL1, DEL2, DEL3
```

Output parameters

85
<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000E+00</td>
<td>0.15274E+00</td>
</tr>
<tr>
<td>0.2000E+00</td>
<td>0.26842E+00</td>
</tr>
<tr>
<td>0.3000E+00</td>
<td>0.42681E+00</td>
</tr>
<tr>
<td>0.5000E+00</td>
<td>0.60009E+00</td>
</tr>
<tr>
<td>0.6000E+00</td>
<td>0.11404E+00</td>
</tr>
<tr>
<td>0.7000E+00</td>
<td>0.14779E+01</td>
</tr>
<tr>
<td>0.8000E+00</td>
<td>0.17763E+01</td>
</tr>
<tr>
<td>0.9000E+00</td>
<td>0.21007E+01</td>
</tr>
<tr>
<td>1.0000E+01</td>
<td>0.22746E+01</td>
</tr>
<tr>
<td>1.1000E+01</td>
<td>0.23187E+01</td>
</tr>
<tr>
<td>1.3000E+01</td>
<td>0.22790E+01</td>
</tr>
<tr>
<td>1.5000E+01</td>
<td>0.21262E+01</td>
</tr>
<tr>
<td>1.7000E+01</td>
<td>0.19190E+01</td>
</tr>
<tr>
<td>1.9000E+01</td>
<td>0.16813E+01</td>
</tr>
<tr>
<td>2.1000E+01</td>
<td>0.14685E+01</td>
</tr>
<tr>
<td>2.3000E+01</td>
<td>0.12613E+01</td>
</tr>
<tr>
<td>2.5000E+01</td>
<td>0.10767E+01</td>
</tr>
<tr>
<td>2.7000E+01</td>
<td>0.91936E+00</td>
</tr>
<tr>
<td>3.2000E+01</td>
<td>0.58197E+00</td>
</tr>
<tr>
<td>3.5000E+01</td>
<td>0.30253E+00</td>
</tr>
<tr>
<td>3.7000E+01</td>
<td>0.48655E+00</td>
</tr>
<tr>
<td>3.8000E+01</td>
<td>0.36529E+00</td>
</tr>
<tr>
<td>4.1000E+01</td>
<td>0.32692E+00</td>
</tr>
<tr>
<td>4.6000E+01</td>
<td>0.25025E+00</td>
</tr>
<tr>
<td>5.1000E+01</td>
<td>0.17126E+00</td>
</tr>
<tr>
<td>5.6000E+01</td>
<td>0.10438E+00</td>
</tr>
<tr>
<td>6.1000E+01</td>
<td>0.65661E-01</td>
</tr>
<tr>
<td>6.6000E+01</td>
<td>0.41818E-01</td>
</tr>
<tr>
<td>7.1000E+01</td>
<td>0.39238E-01</td>
</tr>
<tr>
<td>7.6000E+01</td>
<td>0.47526E-01</td>
</tr>
<tr>
<td>8.1000E+01</td>
<td>0.62207E-01</td>
</tr>
<tr>
<td>8.6000E+01</td>
<td>0.79586E-01</td>
</tr>
<tr>
<td>9.1000E+01</td>
<td>0.76359E-01</td>
</tr>
<tr>
<td>9.6000E+01</td>
<td>0.63384E-01</td>
</tr>
<tr>
<td>1.0100E+00</td>
<td>0.46751E-01</td>
</tr>
<tr>
<td>1.0600E+00</td>
<td>0.30485E-01</td>
</tr>
<tr>
<td>1.1100E+00</td>
<td>0.19171E-01</td>
</tr>
<tr>
<td>1.1600E+00</td>
<td>0.17817E-01</td>
</tr>
<tr>
<td>1.2100E+00</td>
<td>0.24031E-01</td>
</tr>
<tr>
<td>1.2600E+00</td>
<td>0.26184E-01</td>
</tr>
<tr>
<td>1.3100E+00</td>
<td>0.28094E-01</td>
</tr>
<tr>
<td>1.3600E+00</td>
<td>0.25177E-01</td>
</tr>
<tr>
<td>1.4100E+00</td>
<td>0.19289E-01</td>
</tr>
<tr>
<td>1.4600E+00</td>
<td>0.17697E-01</td>
</tr>
<tr>
<td>1.5100E+00</td>
<td>0.17991E-01</td>
</tr>
<tr>
<td>1.5600E+00</td>
<td>0.18513E-01</td>
</tr>
<tr>
<td>1.6100E+00</td>
<td>0.16232E-01</td>
</tr>
<tr>
<td>1.6600E+00</td>
<td>0.11016E-01</td>
</tr>
<tr>
<td>1.7100E+00</td>
<td>0.74828E-02</td>
</tr>
<tr>
<td>1.7600E+00</td>
<td>0.56775E-02</td>
</tr>
<tr>
<td>1.8100E+00</td>
<td>0.41620E-02</td>
</tr>
<tr>
<td>1.8600E+00</td>
<td>0.26714E-02</td>
</tr>
<tr>
<td>1.9100E+00</td>
<td>0.15312E-02</td>
</tr>
<tr>
<td>1.9600E+00</td>
<td>0.10679E-02</td>
</tr>
<tr>
<td>2.0100E+00</td>
<td>0.16907E-02</td>
</tr>
<tr>
<td>2.0600E+00</td>
<td>0.15471E-02</td>
</tr>
<tr>
<td>2.1100E+00</td>
<td>0.11035E-02</td>
</tr>
<tr>
<td>2.1600E+00</td>
<td>0.62784E-02</td>
</tr>
<tr>
<td>2.2100E+00</td>
<td>0.34593E-02</td>
</tr>
<tr>
<td>2.2600E+00</td>
<td>0.16036E-02</td>
</tr>
<tr>
<td>2.3100E+00</td>
<td>0.11926E-02</td>
</tr>
<tr>
<td>2.3600E+00</td>
<td>0.12876E-02</td>
</tr>
<tr>
<td>2.4100E+00</td>
<td>0.12736E-02</td>
</tr>
<tr>
<td>2.4600E+00</td>
<td>0.10764E-02</td>
</tr>
<tr>
<td>2.5100E+00</td>
<td>0.83136E-02</td>
</tr>
</tbody>
</table>