Effect of water content on the compressibility of snow-water mixtures
EFFECT OF WATER CONTENT ON THE COMPRESSIBILITY OF SNOW-WATER MIXTURES

Gunars Abele and F. Donald Haynes

INTRODUCTION

The addition of water to snow to increase its density, and therefore its ultimate strength, has been widely used in snow road and runway construction and foundation preparation in deep snow areas. The compressibility characteristics of dry snow over a wide range of densities and temperatures have been studied by Abele and Gow (1975, 1976). Colbeck (1973, 1976, 1978) has conducted extensive theoretical and experimental studies on the behavior of wet snow and has analyzed the mechanisms of densification under a constant load, particularly in relation to the thermodynamic principles that affect the deformation of snow containing liquid water.

Hanamoto et al. (1976) conducted a study on the feasibility of using a snow-water slurry for backfilling truss enclosures at the DYE radar sites in Greenland. In support of this study, tests were conducted to investigate the effect of water content on the compressibility of confined snow-water mixtures under load applied at constant rates of deformation.

DESCRIPTION OF STUDY

Sample preparation

The snow used in these tests had been stored in sealed plastic bags in a coldroom at a temperature of $-35^\circ C$ for a period of a few months. Several days before the tests, the snow bags were placed in the sample preparation room at a temperature of $-3^\circ C$.

The sample containers were aluminum cylinders with an inside diameter of 20.3 cm and a height of 5.0 cm, coated with Teflon on the inside walls and on the removable baseplate. Snow was sieved into the containers by rubbing snow blocks through a no. 20 sieve (0.84-mm mesh size) and then leveled even with the top of the container. The resulting snow density for this procedure was approximately $0.4 \text{ g cm}^{-3}$. To obtain densities around $0.5 \text{ g cm}^{-3}$, manual compaction with a thick metal plate was required. A Materials Testing System (MTS) machine was used to precompact the snow to approximately a $0.6 \text{ g cm}^{-3}$ density.

A predetermined amount of $0^\circ C$ water from an "ice bath" was poured slowly, and as evenly as possible, into the snow sample, which was at a temperature of $-3^\circ C$. The sample was immediately placed in an insulated Styrofoam container, carried to the test room, removed from the container, and placed in the cold chamber ($-3^\circ C$) of the test apparatus. The elapsed time between the addition of water and the start of the compression test was approximately 2 min.

Test equipment and procedure

The compression tests were conducted with the 10,000-kg load capacity servocontrolled MTS machine equipped with an environmental test chamber (temperature controllable to $-50^\circ C$) and a calibrated ram speed control; the MTS is
The capable equipment in the test setup. The oscilloscope trace obtained from the photographs test was checked for the rate, and the temperature in the chamber position. The cell was moved down to the top of the test chamber, the air plate drilled through the load plate to permit air and water escape from the sample during compression. A fitted sheet of Pellon, a material easily permeable to air and water but not to snow particles, was placed between the surface of the snow sample and the load plate.

All tests were performed to the maximum load capability of the testing system. The maximum final stress on the 20.3-cm-diam sample was approximately 2.5 MPa (25 bars).

Three rates of deformation were used: 0.042, 0.42 and 4.2 cm s\(^{-1}\). The maximum available rate of 42 cm s\(^{-1}\) caused a considerable splash of water within the test chamber, implying that deformation rates of this magnitude may be less effective for compacting snow-water mixtures than slower rates.

After removal of the sample from the test chamber, the excess water, which had been squeezed out during the test, was poured off and the remaining sample returned to the coldroom where it was weighed and measured. The density of the sample at any stress during the test was computed from the deformation data obtained from the oscilloscope photographs.

A total of 62 tests were performed. The first 12 were trial tests to get a feel for the behavior of snow with various water contents under various compaction rates and to establish a suitable test procedure.

**Test sample characteristics**

The characteristics of the test samples (initial snow density, density of the snow-water mixture and water content) and the rate of deformation are shown in Table I.

Figure 2 shows the relationship between snow density, water content, saturation, and the resulting density of the snow-water mixture. The equations for these relationships are shown in the Nomenclature.

The amount of water in each sample was originally computed in terms of water content by weight (weight of water/weight of snow). To make the equations and relationships numerically correct, the water content (and saturation) figures used in this report are in terms of their actual numerical values (ratios), instead of the usual percentage values used in soil mechanics.

To conveniently illustrate the range of the test sample properties, a graphical presentation of all the samples, arranged according to their density characteristics, is shown in Figure 3. Figure 4 shows the test samples arranged according to their snow density and water content characteristics.
Table I. Test sample characteristics.

<table>
<thead>
<tr>
<th>Test</th>
<th>Rate of deform $v$ (cm s$^{-1}$)</th>
<th>Density Snow $\rho_{s|}$ (g cm$^{-3}$)</th>
<th>Mixture $\rho_{m|}$ (g cm$^{-3}$)</th>
<th>Water content $w$ (ratio)</th>
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Figure 3. Density range of test samples.

Figure 4. Snow density and water content range of test samples.
Figure 5. Major principal stress vs density for various water contents ($Q_{0(s)} \approx 0.4 \text{ g cm}^{-1}$, $v = 0.042 \text{ cm s}^{-1}$).

Figure 6. Major principal stress vs density for various water contents ($Q_{0(s)} \approx 0.4 \text{ cm}^{-3}$, $v = 0.42 \text{ cm s}^{-1}$).

Figure 7. Major principal stress vs density for various water contents ($Q_{0(s)} \approx 0.4 \text{ g cm}^{-1}$, $v = 4.2 \text{ cm s}^{-1}$).

Figure 8. Major principal stress vs density for various water contents ($Q_{0(s)} \approx 0.5 \text{ g cm}^{-1}$, $v = 0.042 \text{ cm s}^{-1}$).
Figure 9. Major principal stress vs density for various water contents ($Q_{0,1} \approx 0.5$ g cm$^{-3}$, $v = 0.42$ cm s$^{-1}$).

Figure 10. Major principal stress vs density for various water contents ($Q_{0,1} \approx 0.5$ g cm$^{-3}$, $v = 4.2$ cm s$^{-1}$).

Figure 11. Major principal stress vs density for various water contents ($Q_{0,1} \approx 0.6$ g cm$^{-3}$, $v = 0.042$ cm s$^{-1}$).

Figure 12. Major principal stress vs density for various water contents ($Q_{0,1} \approx 0.6$ g cm$^{-3}$, $v = 0.42$ cm s$^{-1}$).
DISCUSSION OF RESULTS

Data presentation format

The stress-density test data are shown in the following graphs. The format of all the graphs (except Fig. 24 and 29) is the same: the major principal stress $\sigma_1$ is plotted vs the density of the sample $q$ at any point during the compression test.

Below each graph, the sample properties (snow density, water content and the resulting mixture density) are shown in an auxiliary graph. This seemed to be a more meaningful method of identifying the initial sample properties than to indicate the three properties numerically on each curve. This method also permits a convenient comparison of the similarities or differences between the samples in a specific group shown on each graph. The samples are identified by test number for cross-referencing. (Test data of some samples with a high snow-water mixture density were below the range of the stress scale on the $y$ axis and, therefore, do not appear on the stress vs density plot.)

Effect of water content and mixture density

Figures 5-13 show the influence of water content on the compressibility of snow-water mixtures with approximately the same initial snow density and at the same rate of deformation. That is, on each figure showing the stress-density relationship, the initial snow density $q_{0(i)}$ and rate of deformation $v$ are constant, with the water content $w$ (and thus the mixture density $q_{0(m)}$) being the variable. The samples were divided into three groups with snow densities of approximately 0.4, 0.5, and 0.6 g cm$^{-3}$. Three rates of deformation (0.042, 0.42, and 4.2 cm s$^{-1}$) were used for each density group (refer to Table I).

It is quite evident that an increase in the water content increases the compressibility of the snow-water mixture. That is, an increase in the water content decreases the stress required to reach a particular density of the mixture, as has been also demonstrated by Colbeck et al. (1978). During rapid deformation of a snow-water mixture no shear forces can be supported by the interparticle contact, because the ice surfaces are separated by a water film of sufficient thickness to prevent ice-to-ice contact (Colbeck 1976).

Effect of water content and initial snow density

The effect of water content can be better illustrated by treating the initial snow density $q_{0(i)}$ as a variable and the mixture density $q_{0(m)}$ as a constant, thus comparing the stress-density relationship of dry snow vs wet snow at the same density, as shown in Figures 14-21. The samples were again divided into three groups with mixture densities of approximately 0.5, 0.6, and 0.7 g cm$^{-3}$. (Only 24 out of 50 samples could be classified in these three groups.) For various snow-water mixtures of the same density, the one with the highest water content will require the least stress to reach a particular density, the dry snow being the most difficult to compact.

The effects of both the initial snow density $q_{0(i)}$ and water content $w$ on the stress-density relationship are summarized for the slow (0.042-cm s$^{-1}$) and fast (4.2-cm s$^{-1}$) deformation rates in Figures 22 and 23. Some of the curves were obtained by interpolation of data, since a complete set of samples with the exact density...
Figure 14. Major principal stress vs density for various water contents ($q_{0(m)} \approx 0.5$ g cm$^{-3}$, $v = 0.042$ cm s$^{-1}$).

Figure 15. Major principal stress vs density for various water contents ($q_{0(m)} \approx 0.5$ g cm$^{-3}$, $v = 0.42$ cm s$^{-1}$).

Figure 16. Major principal stress vs density for various water contents ($q_{0(m)} \approx 0.5$ g cm$^{-3}$, $v = 4.2$ cm s$^{-1}$).

Figure 17. Major principal stress vs density for various water contents ($q_{0(m)} \approx 0.6$ g cm$^{-3}$, $v = 0.042$ cm s$^{-1}$).
Figure 18. Major principal stress vs density for various water contents ($Q_{0(m)} \leq 0.6$ g cm$^{-3}$, $v = 0.42$ cm s$^{-1}$).

Figure 19. Major principal stress vs density for various water contents ($Q_{0(m)} \leq 0.6$ g cm$^{-3}$, $v = 4.2$ cm s$^{-1}$).

Figure 20. Major principal stress vs density for various water contents ($Q_{0(m)} \leq 0.7$ g cm$^{-3}$, $v = 0.42$ cm s$^{-1}$).

Figure 21. Major principal stress vs density for various water contents ($Q_{0(m)} \leq 0.7$ g cm$^{-3}$, $v = 4.2$ cm s$^{-1}$).
and water content characteristics shown was not available. These graphs give an indication of the decrease in stress with an increase in water content for any particular density condition.

Another method of illustrating this effect is shown in the example in Figure 24 where the stress required to obtain a density of 0.8 g cm⁻³ at the slow (0.042-cm s⁻¹) deformation rate is plotted vs the water content of the three snow density groups (from Fig. 22).

**Effect of deformation rate**

To observe the influence of the deformation rate on the stress−density relationship at a specific water content, one pair of samples (one at the 0.042-cm s⁻¹ rate, the other at 4.2-cm s⁻¹) was selected at a low initial density and another pair at a high initial density, both pairs having the same or similar water contents. The results for four groups of water contents (0, approx. 0.1, approx. 0.2, and approx. 0.4) are shown in Figures 25–28.

After a certain stress level, an increase in the rate of deformation increases the stress required to reach a specific density, the initial density and water content being the same. That is, a snow−water mixture can be compacted more easily at a slower compaction rate. The significance of this effect increases with an increase in water content.

Figure 29 shows the approximate stress vs water content envelope at which the difference between the slow (0.042-cm s⁻¹) and the fast (4.2-cm s⁻¹) deformation rates becomes noticeable. Below this envelope, the stress−density relationship is not influenced by the deformation rate. Above the envelope, the stress required to reach a certain density at a low deformation rate is lower than that required at a high deformation rate.
Figure 25. Major principal stress vs density for two rates of deformation ($w = 0; \dot{Q}_{0(t)} \approx 0.4$ and $0.6$ g cm$^{-3}$).

Figure 26. Major principal stress vs density for two rates of deformation ($w \approx 0.1; \dot{Q}_{0(t)} \approx 0.4$ and $0.6$ g cm$^{-3}$).

Figure 27. Major principal stress vs density for two rates of deformation ($w \approx 0.2; \dot{Q}_{0(t)} \approx 0.4$ and $0.6$ g cm$^{-3}$).

Figure 28. Major principal stress vs density for two rates of deformation ($w \approx 0.4; \dot{Q}_{0(t)} \approx 0.4$ and $0.5$ g cm$^{-3}$).
In previous studies (Abele and Gow 1975, 1976) on dry snow, the rate of deformation was found to have very little influence on the stress-density relationship.

**Effect of water content and rate of deformation on specific energy**

To determine the energy required to compact snow with various water contents, the specific energy was computed for each test. The area under the load-deformation curve was calculated by using an Autech A-12 Planimeter with a 102-A Measurement Scanning Camera. The error involved in using this optical method for determining areas was within 2%. The energy corresponding to the area under the load-deformation curve was then divided by the initial snow volume to obtain the specific energy. Since this type of a compressive test does not result in what could be considered specimen failure, the term “specific energy” is used here to indicate the required energy at the maximum available compressive load, 8618 kgf.

Figure 30 shows the specific energy as a function of water content for three initial snow densities, 0.4, 0.5 and 0.6 g cm\(^{-3}\), at the three different rates of deformation (0.042, 0.42 and 4.2 cm s\(^{-1}\), with the corresponding approximate strain rates being 0.01, 0.1 and 1 s\(^{-1}\), respectively). The same data are replotted in Figure 31 with the rate of deformation (instead of initial density) being constant in each plot.

The following tentative observations can be made from the available data on how the
specific energy varies with water content in snow which is being subjected to a maximum stress of 25 bars:

1. The specific energy \( E \) required to compact dry snow \((w = 0)\) is sensitive to the rate of deformation \( v \) between 0.042 and 0.42 cm s\(^{-1}\), but not noticeably sensitive between 0.42 and 4.2 cm s\(^{-1}\) (Fig. 30), and it is insensitive to an initial snow density \( \rho_0 \) between 0.4 and 0.6 g cm\(^{-3}\) (Fig. 31). That is, for dry snow \( E \), decreases with an increase in \( v \) (or an increase in the strain rate), the initial snow density having no particular influence.

2. The specific energy for a snow-water mixture is sensitive to the rate of deformation below a water content of approximately 0.2 (20%). As \( w \) is increased to 0.2, \( E \) decreases for the low \( v \) (0.042 cm s\(^{-1}\)), but increases for the medium and high \( v \) (0.42 and 4.2 cm s\(^{-1}\)), this phenomenon occurs for all three initial snow densities (0.4, 0.5 and 0.6 g cm\(^{-3}\)). For water contents above 0.2, \( E \) decreases gradually with an increase in \( w \) at the higher \( v \) and \( \rho_0 \) values (Fig. 31b and 31c), or remains approximately the same in the case of the lower \( v \) and \( \rho_0 \) values (Fig. 31a and 31b).

It appears, therefore, that the addition of water to snow for the purpose of decreasing the energy required for compaction is of no benefit, except when a very low strain rate (approx. 0.01 s\(^{-1}\)) is used (Fig. 31a). And even in this case, the required energy level would not be decreased below that required for compacting dry snow at a higher strain rate (approx. 1 s\(^{-1}\), see Figs. 31b and 31c). However, the addition of water is certainly beneficial for increasing the density, and thus the ultimate strength, of the snow–ice mixture.

Considering the mechanisms involved in snow compaction, Colbeck* has indicated the importance of surface energy change. This concept involves the assumption of a spherical snow grain being transformed into a cubical shape to maximize the packing density. The change in energy per unit volume is

\[
\Delta E = \Delta A \sigma_s n
\]

where \( \Delta A = \text{change in surface area} \)

\( \sigma_s = \text{surface energy for the water–ice boundary} \)

\( n = \text{number of particles per unit volume} \)

* S. Colbeck, CRREL, personal communication, 1977

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Based on the estimated average grain size of 0.2 mm in the test samples used, the \( \Delta E \), value is 0.046 MJ m\(^{-3}\), which is only 1 to 2% of the specific energy for snow being subjected to a maximum stress of 25 bars and therefore resulting in a final density approaching that of ice. This indicates that other mechanisms such as regulation are apparently more important factors in the snow compaction mechanism.

**SUMMARY**

The general influence of water content and rate of deformation on the stress–density relationship of snow–water mixtures is summarized in Figure 32.

Figure 32a shows the general shape of the stress–density curves at two rates of deformation for two snow samples having the same initial snow density but different water contents, thus resulting in different mixture densities \( (\rho_{0(is)}) \).

Figure 32b shows the same type of relationship, except that the initial snow densities are different and appropriate amounts of water are added to produce the same mixture density.

Figure 32c shows the stress–density relationship for two samples with different densities but the same water content at two different rates of deformation.
In general, an increase in water content in snow at a particular density (and thus an increase in mixture density) or a decrease in the rate of deformation (or strain rate) decreases the stress required to reach a specific density of a snow-water mixture, the water acting as a lubricant between the ice particles.

The addition of water to snow does not help to decrease the specific energy required for compacting snow to a maximum stress of 25 bars, except for low deformation rates (strain rates in the order of magnitude of 0.01 s⁻¹) and water contents below 0.2 (20%). At higher deformation rates (strain rates on the order of 0.1 and 1 s⁻¹), the specific energy increases with an increase in water content of up to approximately 0.2, and then gradually decreases with further increase in water content.

**LITERATURE CITED**


APPENDIX A. DERIVATION OF EQUATIONS

\[ V_s \text{ total volume of snow sample} \]
\[ V_v \text{ volume of voids} \]
\[ V_i \text{ volume of ice} \]
\[ V_s = V_v + V_i \]
\[ V_w \text{ volume of water} \]
\[ V_a \text{ volume of air} \]
\[ V_s = V_w + V_a \]
\[ V_i = V_w + V_a + V_i \]

\[ V_m \text{ volume of snow-water mixture} \]
\[ V_m = V_w + V_a + V_i = V_s \]

\[ W_s \text{ weight of snow} \]
\[ W_i \text{ weight of ice} \]
\[ W_s = W_i \]

\[ W_w \text{ weight of water} \]
\[ W_m \text{ weight of snow-water mixture} \]
\[ W_m = W_i + W_w \]

\[ \rho_s \text{ density of snow} \]
\[ \rho_i \text{ density of ice} \]
\[ \rho_m \text{ density of snow-water mixture} \]
\[ \rho_w \text{ density of water} \]
\[ w \text{ water content} \]
\[ S \text{ saturation} \]

1. Maximum water content for snow at any density

\[ w = f(\rho) \]
\[ w = W_w/W_s \]  

\[ W_w = \rho_w + V_w \] for \( w(\text{max}) \): \( V_w = 0; V_w = V_s \)

\[ \rho_w = \rho_w V_s \]

\[ \rho_w = \rho_s V_s; W_s = \rho_i V_s \]
\[ w = V_s/V_w; V_s = \rho_i V_s \]

\[ V_s = V_I - V_i; V_i = W_i/\rho_i; W_s = W_i = \rho_i V_i \]

\[ w = (V_s - \rho_i V_i)/\rho_i V_i = (1 - \rho_i/\rho_i)\rho_w/\rho_w \]
\[ w(\text{max}) = [(1/\rho_i) - (1/\rho_i)]\rho_w \]

2. Water content at any saturation

\[ S = V_w/V_s \]

\[ V_w = SV_i; W_w = \rho_w V_w \]
\[ w = W_w/W_s = \rho_w SV_i/W_s = \rho_w SV_i/\rho_i V_i \]
\[ w = S[(1/\rho_i) - (1/\rho_i)]\rho_w \]
\[ w/S = [(1/\rho_i) - (1/\rho_i)]\rho_w \]

\[ S = 1 \text{ at } w(\text{max}) \]

3. Mixture density

\[ \rho_m = W_m/V_m = (W_w + W_i)/V_m \]
\[ W_w = w W_s = w \rho_w V_s \]
\[ W_s = \rho_i V_i \]
\[ W_w + W_s = \rho_i V_i (w + 1) \]
\[ V_m = V_s \]
\[ \rho_m = \rho_i V_i (w + 1)/V_i \]
\[ \rho_m = \rho_i (w + 1) \]
APPENDIX B. LOAD-DEFORMATION TRACES

Scale:

Sample diameter = 20.3 cm
Sample area = 323.5 cm²
Sample height = 5.0 cm

$\rho_{0,s} =$ initial density of snow (g cm$^{-3}$)

$w =$ water content (weight of water/weight of snow)

$\rho_{0,s-m} =$ initial density of snow-water mixture (g cm$^{-3}$)

$v =$ rate of deformation (cm s$^{-1}$)
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<td>0.41</td>
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<tr>
<td>$v$</td>
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<tbody>
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