Sea Ice
Part II. Estimating the Full-Scale Tensile, Flexural, and Compressive Strength of First-Year Ice

Austin Kovacs

September 1996

\[
\sigma_C = 1970 \varepsilon^{0.32} \phi_B^{-0.95}
\]

\[r^2 \approx 0.941\]

\[S D = 0.25\]
Abstract: Sea-ice salinity, density, and temperature data were used to develop new methods for determining the bulk brine volume and porosity of sea-ice floes. Methods for estimating full-thickness ice sheet strength, based on large-scale field tests, are presented. The relationships among bulk sea-ice properties, strain rate, and strength are illustrated. A new constitutive equation was developed for predicting the full-thickness horizontal compressive strength $\sigma_c$ of first-year sea ice as a function of the applied strain rate and bulk porosity in the form $\sigma_c = B_2 \dot{\varepsilon}_B \phi_B^m$, where parameters $B_2$, $n$, and $m$ are about $2.7 \times 10^3$, 3, and –1, respectively, and $\dot{\varepsilon}_B$ and $\phi_B$ are the ice strain rate and ice floe bulk porosity of sea ice, respectively. An estimate of the horizontal force that may develop between first-year sea ice and a 90-m-wide structure is given. Estimating sea-ice strength based on remote ice conductivity measurements is also discussed conceptually.

Cover: Exxon ice floe $\sigma_c$ values vs. bulk porosity and strain rate.
PREFACE

This report was written by Austin Kovacs, Research Civil Engineer, Applied Research Division, Research and Engineering Directorate, U.S. Army Cold Regions Research and Engineering Laboratory (CRREL), Hanover, New Hampshire.

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INTRODUCTION

It is well known that the strength of sea ice is to a large extent dependent on brine volume, and that brine volume is a function of ice salinity and temperature. Brine volume may be estimated using the empirical expressions of Frankenstein and Garner (1967) or the theoretical density–volume fraction equations of Cox and Weeks (1983). The former is by far the simplest method, because the only inputs required in the brine volume equations are ice salinity and temperature. Since the latter approach is based on phase relationships, it allows for the calculation of the gas volume and thus the total ice porosity and the brine-free ice density. With this method, the bulk density of the sea ice is also needed and therefore must be determined. In principle this is a simple task, but it can be difficult to do in the field. Today, ice core weight is measured by electronic scales rather than triple beam balances, which are subject to uneven support surface and wind effect errors when used at unsheltered sites. Electronic scale weight measurements can be highly representative of the in-situ ice if brine drainage has not occurred during coring or after the ice core was removed from the bore hole. Determining ice core volume is a bit more troublesome. Ice core length should be measured along the center axis using a special caliper or gauging device (Kovacs 1993). A meter rule should not be used for this purpose (it often is), because the ends of the ice sample may not be parallel to each other. The more difficult measurement is ice core diameter. Again, a ruler should not be used. A pie tape (Kovacs 1993) or a caliper with long, wide tongs is generally satisfactory, provided the core is ridge-free and of uniform diameter. Where this is not the case, the diameter measured will not be accurate and the volume determination will be off. This is illustrated in Figure 1, which gives a comparison between brine volume determinations made on ~10-cm-long sections from several first-year Arctic sea-ice cores using the equations of Cox and Weeks and Frankenstein and Garner. The regression curve through the data in Figure 1 did not include the outlier (+). The outlier indicates where a measurement error occurred, perhaps because of a volume measurement error.

It is recommended that when the Cox and Weeks (1983) equations are used, and they must be for determining gas volume, total porosity, and
brine-free ice density, the Frankenstein and Garner (1967) equation also be used to estimate the ice brine volume. When the two brine-volume estimates are off by more than 2‰, a measurement or data logging error has occurred. If the problem cannot be resolved, the ice property values calculated with the Cox and Weeks equations should not be used. It should be noted that the Frankenstein and Garner equations for determining brine volume are not valid below –23°C. Only the Cox and Weeks equations should be used below this temperature.

The physical and electromagnetic properties of 21 winter Beaufort Sea ice cores, obtained by the author on various field programs, were determined for each 10-cm increment of ice core. Physical property calculations were made using the mathematical expressions of Cox and Weeks (1983). The electromagnetic properties of the ice were determined using the procedures of Kovacs et al. (1987) as modified by Kovacs and Morey (1987). Examples of the calculations are given in Tables 1 and 2. Also assembled were the authors’ data from an additional 23 Beaufort Sea ice cores, where only the salinity and temperature of the winter ice were determined for each 10-cm increment of core. For these data, only the brine volume of the ice could be calculated using the equations of Frankenstein and Garner (1967).

From the above data, a number of winter ice floe property trends can be seen. Ice floe bulk density vs. thickness is shown in Figure 2. The bulk density is seen to decrease with ice floe thickness. For the first-year sea ice, the decrease is associated with brine drainage and growth rate processes, which reduce the volume fraction of the heavier brine entrained within the ice (Kovacs 1996). The lower multiyear ice densities are the result of this ice containing proportionally less brine and more gas, especially in the freeboard portion, which is for all intents and purposes low-density fresh ice. While density is the property that a material’s strength is often related to, as with the strength of snow (Kovacs et al. 1969), it is not used for this purpose in sea-ice mechanics. Traditionally, sea-ice failure strength, elastic modulus, and such have been referenced to the brine volume of the ice. This brine volume referencing is still the norm for sea-ice tensile or flexural strength test presentations. The reasoning is that the strength of a material is a function of its solidity. Therefore, the more brine volume there is, the less solid the ice, and the weaker it will be. This method, of course, does not take into account the added porosity associated with the small gas inclusions. Since brine volume, salinity, and temperature are interrelated, brine volume needs to be correlated with these two parameters, as shown in Figure 3. This figure clearly illustrates that, at any given bulk salinity $S_B$, the bulk brine volume $v_B$ of an ice sheet is dependent on the average ice sheet temperature $T_A$. While this presentation is enlightening, it does not serve the user very well. More useful is the presentation in Figure 4, where the relationship provided allows for the determination of the bulk brine volume

![Figure 2. Sea-ice bulk density and conductivity vs. floe thickness.](image-url)

![Figure 3. Sea-ice bulk brine volume vs. bulk salinity with average ice floe temperature as a parameter.](image-url)
Table 2. Physical and electromagnetic properties of 1.93-m-thick sea ice at 100 MHz.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Core diam. (cm)</th>
<th>Core length (cm)</th>
<th>Core weight (gr)</th>
<th>Core temp. (°C)</th>
<th>Core density (Mg/m³)</th>
<th>Core salinity (‰)</th>
<th>Air porosity (%)</th>
<th>Ice conductivity (µS/m)</th>
<th>Ice loss (‰)</th>
<th>Ice phase</th>
<th>Ice loss (±)</th>
<th>Ice dielectric const. at 100 MHz (m)</th>
<th>Ice tangent loss (dB/m)</th>
<th>Ice velocity (m/ns)</th>
</tr>
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<td>9.79</td>
<td>3.83</td>
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C-W = Cox and Weeks (1983) equations
F-G = Frankenstein and Garner (1967) equations
* In the electromagnetic equations (Kovacs and Morey 1987), parameters n and m were 0.1 and 1.65, respectively.

Table 1. Physical and electromagnetic properties of 1.23-m-thick sea ice at 100 MHz.

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F-G = Frankenstein and Garner (1967) equations
* In the electromagnetic equations (Kovacs and Morey 1987), parameters n and m were 0.1 and 1.65, respectively.
The equation for predicting the bulk brine volume \( \nu_B \) in ‰ as a function of \( S_B \) and the absolute value of \( T_A \) (Fig. 4),

\[
\nu_B = 41.64 \cdot S_B^{0.88} \cdot T_A^{-0.67},
\]

along with the expressions of Kovacs (1996) for estimating the bulk salinity (‰) of first-year sea ice vs. ice floe thickness \( T_F \) (cm) (Fig. 5),

\[
S_B = 4.606 + 91.603 / T_F,
\]

offer the opportunity to make other ice property assessments, which in turn can be used to estimate ice sheet strength.

For example, the horizontal uniaxial tensile strength \( \sigma_t \) of an ice sheet could be estimated using the empirical equation of Dykins (1970):

\[
\sigma_t = 0.816 - 0.069 \cdot \nu_B^{0.5}.
\]

Similarly, ice floe flexural strength \( \sigma_f \) may be estimated using the empirical equation (Timco and O’Brien 1994)

\[
\sigma_f = 1.76e^{-5.88 \cdot \nu_B}.
\]

These expressions could be used as follows. Ice thickness is determined by drill-hole measurement. Then from eq 2 the bulk salinity of the ice is estimated. Ice temperature is determined next at 20–25 cm below the ice surface. A linear temperature gradient to the ice/water interface (~1.7°C) is assumed, and the average ice sheet temperature is calculated. The bulk salinity and average temperature values are then used in eq 1 to determine the bulk brine volume. This value can now be used in eq 3 or 4 to determine \( \sigma_t \) or \( \sigma_f \), respectively. It should be noted that Gavrilo et al. (1995) compared various published expressions for determining the flexural strength of sea ice and found that eq 4 gave \( \sigma_f \) values 2 to 4 times higher than other research results. Further evaluation of the various \( \sigma_f \) equations may be in order.

The above procedure still requires a drill-hole thickness and a near-surface ice temperature measurement. These measurements may not be necessary. Using the formulations for determining the electromagnetic properties of sea ice (Kovacs and Morey 1987), the low-frequency (DC to ~100 MHz) bulk conductivity* \( \sigma_B \) of a model sea-ice
sheet was determined along with its brine volume \( \nuB \) vs. ice temperature and salinity. An ice sheet bulk density of 0.911 \( \text{Mg/m}^3 \) was used in the determinations. The calculated results are graphically shown in Figure 6. There is a decrease in brine volume with decreasing ice temperature. In addition to ice conductivity, Figure 6 indicates that either ice salinity or temperature is required to estimate \( \nuB \).

The complex interrelationship between the model sea-ice temperature, salinity, brine volume, and conductivity is well revealed in Figure 6. The results suggest that determining \( \sigmaB \) of natural sea ice remotely will not, of itself, allow \( \nuB \) to be estimated. Not revealed is the effect of density, which was fixed at 0.911 \( \text{Mg/m}^3 \) for this example. The decrease in the bulk density and conductivity with increasing ice floe thickness as revealed in Figure 2 affects all other ice properties. It turns out that this effect is favorable to the remote sounding of sea ice. For example, Kovacs and Morey (1991) and Kovacs et al. (1996) have shown that a portable electromagnetic induction sounding system can be used to measure remotely an apparent bulk conductivity from which an estimate of winter Beaufort Sea ice thickness can be made (Fig. 7). When the \( \nuB \) vs. \( \sigmaB \) at DC values listed in Tables 1 and 2 and from 20 other ice floes were compared, it was found that the calculated bulk conductivity of Beaufort Sea ice was directly related to \( \nuB \), as shown in Figure 8. Therefore, in principle, it should be possible to estimate both undeformed winter Arctic ice floe thickness and bulk brine volume remotely using a conductivity measurement system. Before the latter is possible, the relation between the field-measured apparent bulk conductivity, which is highly affected by the underlying seawater, and the bulk brine volume of sea ice floes of various thicknesses will need to be determined. Conductivity measurement at a frequency well above 50 kHz is likely to be necessary to separate the sea-ice conductivity con-

![Figure 6. 3-D presentation of sea-ice brine volume vs. DC conductivity and temperature with salinity as a parameter for an ice density of 0.911 \( \text{Mg/m}^3 \) (a) and a boresight view from the above temperature window (b).](image-url)

![Figure 7. EM-31 instrument conductivity reading at ~10 kHz vs. ice floe snow plus ice thickness (from Kovacs et al. 1996).](image-url)
tribution from the seawater conductivity contribution. A dual high and low frequency system or a wideband measurement system could be used.

In the interim, an off-the-shelf electromagnetic induction system operating at 10 kHz can be used to estimate ice thickness (Fig. 8), and the bulk salinity of the ice is estimated using eq 2. After determining the average ice sheet temperature, as previously described, eq 1 can be used to calculate the bulk brine volume. Ice strength may then be calculated from eq 3 or 4.

The relation between sea-ice tensile or flexural strength, brine volume, and the calculated bulk DC conductivity of natural sea ice is shown in Figures 9 and 10. These figures are the result of using the equation in Figure 6 and eq 3 and 4, respectively. In the future, using similar equations, an algorithm could be added to a portable electromagnetic induction sounding system that would provide both ice thickness and strength from the apparent sea-ice conductivity measurements.

**COMPRESSIVE STRENGTH**

Estimating the compressive strength of sea ice is different. While the uniaxial tensile or flexural strength of sea ice does not reveal a significant strain rate dependence, at the strain rates of interest for most ice engineering problems, the compressive strength does. In addition, in reporting sea-ice compression test results, the recent, and correct, method is to relate strength to total porosity: brine volume plus gas volume. Recently, Timco and Frederking (1990) developed empirical equations for predicting the large-scale uniaxial unconfined compressive strength of Arctic sea ice. Their equations are based on small-scale strength tests and the various controlling factors, such as ice structure, brine volume, porosity, loading direction (horizontal–vertical), and strain rate. Their strength equation for horizontally loaded columnar sea ice is

$$\sigma_c = 37 \epsilon^{0.22} \left[1 - (\phi_B / 270)^{0.5}\right], \quad (5)$$
where $\sigma_c$ = horizontal uniaxial unconfined compressive strength (MPa)
$\dot{\varepsilon}$ = strain rate, s$^{-1}$
$\phi_B$ = bulk ice porosity (‰).

Timco and Frederking compared $\sigma_c$ values determined from eq 5 with in-situ, full ice sheet thickness, horizontal, uniaxial unconfined compression tests performed by Exxon on Beaufort Sea ice. The Exxon large-scale test data comprised ice thickness, average ice temperature, loading strain rate, and the measured strength (Table 3). Since the bulk porosity of the ice sheets tested was not provided, the authors had to make these estimates. To do this, they first included additional data in Figure 11 and, following the same approach as Cox and Weeks (1974), ran two linear regression curves through the data. Only slightly different from the Cox and Weeks equations (Fig. 11), the new equations are:

$$S_B = 13.4 - 0.174 T_F$$  \hspace{1cm} (6)

for $T_F \leq 0.34$ cm; and

$$S_B = 8.0 - 0.016 T_F$$  \hspace{1cm} (7)

for $T_F > 0.34$ cm.

From these equations, Timco and Frederking determined $S_B$ for each ice sheet tested (Table 3). Then the bulk density of the sea ice was selected to be 0.907 Mg/m$^3$. With these determinations and the average ice floe temperature given in Table 3, the expressions provided by Cox and Weeks (1983) were used to calculate the bulk porosity of each ice sheet. Note that these values were not provided by Timco and Frederking in their paper. The missing bulk porosity values were recalculated along with the bulk brine volume for each ice sheet. The values are listed in the left column of the brine volume and porosity data columns in Table 3. The $\sigma_c$ strengths measured by Exxon and calculated by Timco and Frederking are compared in Table 3 and shown graphically in Figure 12. The regression curve in Figure 13 shows that the calculated values are generally higher than the measured ones. This offset may be due to small- to large-scale scaling ambiguities.

To bring eq 5 into better agreement with the Exxon test results, the bulk salinity and brine volume of each ice sheet were determined using eq 2 and 1, respectively. Next, the $S_B$, $T_A$, and $\phi_B$ data from Tables 1 and 2 and others referenced were plotted as shown in Figure 13a. A boresight view of the data as seen looking into the salinity “window” is shown in Figure 13b. This view clearly...
reveals the interrelationship of average ice sheet temperature and bulk salinity with the bulk porosity.

Using the $S_B$ values determined from eq 2, the average ice floe temperature listed in Table 3, and the equation provided in Figure 13, $\phi_B$ was estimated for each of the Exxon ice sheets. These $\nu_B$, $S_B$, and $\phi_B$ values are listed in Table 3 as the right column under each respective heading. Using the new $\phi_B$ values and the related strain rates in Table 3, $\sigma_c$ was recalculated using eq 5. These so-called modified $\sigma_c$ values were then compared (Fig. 14) with the $\sigma_c$ estimates made by Timco and Frederking (Table 3). As can be seen in Figure 14, using the above $\phi_B$ estimates in eq 5 gives $\sigma_c$ values that are comparable with those of Timco and Frederking. However, the modified method for determining $\sigma_c$ gives values that are still offset from the Exxon field test results (Fig. 15). To alleviate this offset, eq 5 was revised to read

$$\sigma_c = 37 \varepsilon^{0.22} \left[ 1 - \left( \frac{\phi_B^{0.93}}{270} \right)^{0.5} \right] - 0.65. \quad (8)$$

This revision brings the $\sigma_c$ values (Table 3) into better agreement with the field determinations, as shown in Figure 16.

If the DC conductivity of the ice could be determined remotely along with ice sheet thickness, then $S_B$ could be determined from eq 2 and $\nu_B$ estimated from an expression similar to that given in Figure 6 and $\phi_B$ from Figure 17. The data in
analyzed using a two-parameter power function. The relationship is a further development of the Norton (1929) secondary creep rate equation containing a power function of stress:

\[ \dot{\varepsilon} = B\sigma^n, \]

where \( B \) is an empirical parameter that takes into account ice structure, temperature, activation energy, and other factors, \( \sigma \) is the applied stress, and the empirical exponent \( n \) is generally found to be about 3 for ice. Rearranging for \( \sigma \), the above equation becomes

\[ \sigma = B_1 \dot{\varepsilon}^{1/n}, \]

where \( B_1 = B^{-1/n} \). The former expression was used by Glen (1955, 1958) to evaluate the creep behavior of solid, not porous, freshwater ice. Since the ice porosity has a profound effect on \( \sigma \), the above equation must be modified to include a porosity parameter \( \phi \) as follows:

\[ \sigma = B_2 \dot{\varepsilon}^{1/n} \phi^m, \quad (9) \]

where \( B_2 = B_1 \phi^m \) and \( m \) is an empirical exponent.

Equation 9 was evaluated by substituting the \( \sigma_c \) values obtained from eq 8 for \( \sigma \) in eq 9 and using the related controlling parameters \( \phi_B \) and \( \dot{\varepsilon} \) given in Table 3. The result is shown in Figure 18. Equation 9 is shown to statistically fit the data extremely well with an \( r^2 \) value of 0.993.

As more data become available, the failure surface and therefore the variables in eq 9 may be better determined. Even though the failure surface fits the data with a high \( r^2 \) value of 0.993, the data suggest that this surface is not a straight line as viewed from the strain rate window in

Figure 13. Ice floe bulk porosity vs. average temperature and bulk salinity (a) and boresight view from the above bulk salinity window (b).

Figure 14. Modified Timco–Frederking ice floe \( \sigma_c \) strengths vs. their original values.
Figure 15. Modified Timco–Frederking ice floe $\sigma_c$ strengths vs. Exxon’s field-measured $\sigma_c$ values.

Figure 16. Ice floe $\sigma_c$ strengths calculated using eq 8 vs. Exxon’s field-measured $\sigma_c$ values.

Figure 17. Sea-ice floe bulk porosity vs. bulk brine volume.
18b. The three data points at a strain rate of about $10^{-6}$ are shown to be offset from their predicted failure surface position at the tip of their vertical tails, suggesting that the failure surface curves downward. This is not surprising if $B$ contains a nonlinear viscosity component. In this case, while stress is still a function of strain rate, it is not directly proportional (Mellor 1986), at least at strain rates between $10^{-5}$ and $10^{-3}$ as indicated by the stress vs. strain rate curves for fresh and saline ice given in Sanderson (1988). The Exxon data fall in the ductile-to-brittle strain rate transition region where the creep power-law may not be valid, that is, $\dot{\epsilon} \neq B\sigma^n$ because $n$ is no longer a constant.

A statistically more representative failure surface for the data is shown in Figure 19a. This curved surface fits the data with a correlation coefficient of $r^2 = 1.000$. A boresight view of the data from the strain rate window in Figure 19a is shown in Figure 19b. Based on the data presented by Sanderson (1988), below a strain rate of about $10^{-6}$ the failure surface should be linear, $a = 3$. However, the existing data are insufficient for developing this trend. In the interim, for strain rates between $10^{-4}$ and $10^{-3}$, the equation given in Figure 18 is preferred because it does not contain a second constant. The equation for the failure surface when $v_B$ is substituted for $\phi_B$ in Figure 18 is $\sigma_c = 150.75 \dot{\epsilon}^{0.31} v_B^{-0.36}$. In this expression, $v_B$ implicitly accounts for the bulk gas volume or the porosity. In using either $\sigma_c$ expression, the only parameters needed are the loading strain rate and the bulk ice porosity or brine volume. The latter may one day be estimated from remote conductivity measurements along with ice thickness using a helicopter-borne electromagnetic induction sounding system (Kovacs and Holladay 1990).

A comparison of the $\sigma_c$ data derived from using eq 8 with the horizontal uniaxial unconfined compressive strength data of others should be of interest. The data compiled by Sanderson (1988; see his Fig. 4.8) are shown in Figure 20 for the strain rate range of our data. In this standard
presentation, no corrections have been made to the data to account for differences in test technique, ice porosity, grain size, or crystal orientation. This is not unusual since some of these parameters were not measured or provided in the original source material, as is the case for the Exxon data used in this report. For this reason there is considerable scatter in the data. The upper regression curve shown in Figure 20 fits the data from Sanderson; the lower curve represents the data generated from eq 8. The small amount of data from this study coupled with the unknowns mentioned above does not allow a definitive statement to be made on the difference between the two lines. Clearly, the data from this study fit well within the scatter of the data from Sanderson. Both sets of data are statistically better represented by the nonlinear curves shown in Figure 21. Here again, the failure surface appears to be a non-power-law function of the strain rate.

The Timco and Frederking (1990) study, which resulted in the formulation of eq 6, is based on small-scale test results. How applicable this equation is to full-scale ice force assessments can be questioned on the basis of small- to large-scale scaling uncertainties. To avoid this problem, the full-scale Exxon $\sigma_c$ data were directly analyzed using the values listed in the right-hand bulk porosity column in Table 3. Applying eq 9 to analyze the $\dot{\varepsilon}$ and $\sigma_c$ data of Exxon vs. the above-related $\phi_B$ values (Fig. 22a) gives the following expression for the failure surface:

$$
\sigma_c = 1970 \dot{\varepsilon}^{0.32} \phi_B^{-0.95}.
$$

In the boresight view of the data from the strain rate window in Figure 22b, the “outlier” with the long offset tail at a strain rate of $7.9 \times 10^{-7}$ s$^{-1}$ is very apparent. However, removing this data point and reanalyzing the remaining data gives a new equation whose calculated $\sigma_c$ values do not vary by more than 0.04 MPa from those calculated by eq 10.

The form of the expression for the curved failure surface shown in Figure 17 does not fit the new data set as well as the power type equation in Figure 22. This may be seen by comparing the statistics listed in Figure 22 with those in Figure 23. The boresight view of the failure plane from the strain rate window (Fig. 23b) also indicates no fit improvement for the outlier over the failure surface shown in Figure 22b.

Substituting the new $\nu_B$ data for the $\phi_B$ data and reanalyzing as before gives the results shown in Figure 24. The expression for the failure surface is now

$$
\sigma_c = 523.24 \dot{\varepsilon}^{0.32} \nu_B^{-0.67}.
$$

As with the bulk porosity data, applying a curved failure surface through the $\sigma_c$ vs. bulk brine volume and $\dot{\varepsilon}$ data does not result in a statistically better curve fit.

It is interesting that the $\sigma_c$ vs. $\dot{\varepsilon}$ trend, as obtained with eq 8 and from Sanderson’s data, ap-
\( r = 0.9412 \)

\( S_D = 0.25 \)

\( s = -0.58 + 271.20 \cdot \phi_B - 0.63 \cdot 0.22 \)

\( \sigma_C = 1970e^{0.32} \phi_B^{0.95} \)

\( r^2 = 0.941 \)

\( S_D = 0.25 \)

\( \sigma_C = -0.58 + 271.20e^{0.32} \phi_B^{0.53} \)

\( r^2 = 0.929 \)

\( S_D = 0.29 \)

\( \sigma_C = 523.24e^{0.32} \phi_B^{0.67} \)

\( r^2 = 0.938 \)

\( S_D = 0.26 \)

\( \sigma_C = 523.24e^{0.32} \phi_B^{0.67} \)

\( r^2 = 0.938 \)

\( S_D = 0.26 \)

Figure 22. Exxon ice floe \( \sigma_c \) values vs. bulk porosity and strain rate (a) and the boresight view of the above from the strain rate window (b). Data were fitted with a two-term power function, i.e., eq 9.

Figure 23. Ice floe \( \sigma_c \) values vs. bulk porosity and strain rate (a) and the boresight view of the above from the strain rate window (b). Data were fitted with a non-log-log power equation.

Figure 24. Ice floe \( \sigma_c \) values vs. bulk brine volume and strain rate (a) and the boresight view of the above from the strain rate window (b).
pears to favor a curved failure surface (Fig. 17 and 19), while the trend resulting from the analysis of the Exxon $\sigma_c$ vs. $\dot{\varepsilon}$ data does not show a strong preference. Additional full ice sheet tests should help to further define the failure surface, especially outside the area of the current data.

Within the range of the current data, $10^{-6}s^{-1} < \dot{\varepsilon} < 10^{-3}s^{-1}$ and $-25\% < \nu_B < 80\%$, eq 11 well represents the $\sigma_c$ strength of sea ice as determined from full-scale in-situ tests. This is equally true for eq 10 and its counterpart in Figure 23a, in which $\phi_B$ is a parameter. Nevertheless, eq 10 is preferred because of its simpler form; it does not contain a secondary constant. In addition, outside the range of this field data, the latter equation predicts an unrealistic zero ice strength between strain rates of about $10^{-7}$ and $10^{-8}s^{-1}$.

The relationship of bulk porosity and strain rate to sea ice strength is clearly revealed in Figure 25. The effect of $\phi_B$ on $\sigma_c$ is shown to be significantly greater at the higher strain rates. As would be expected, the same trend is maintained when $\nu_B$ is substituted for $\phi_B$ (Fig. 26).

If one assumes that $n$ and $m$ are 3 and $-1$ respectively, then eq 10 can be simplified to

$$\sigma_c = 2.7 \times 10^3 \dot{\varepsilon}^{1/3} \phi_B^{-1}. \quad (12)$$

$\sigma_c$ values calculated from eqs 10 and 12 were compared and found to cross-correlate with an $r^2 = 1.000$ and a standard deviation of 0.012 over the range of the Exxon full-scale test data.

By rearranging eq 9, one obtains an equation for the secondary creep of sea ice:

$$\dot{\varepsilon} = B_3 \sigma^m \phi^w, \quad (13)$$

where $B_3 = B_2^{-n}$ and $w = -mn$.

To take into account the creep process of ice with time, eq 13 can be modified to describe primary creep as follows:

$$\dot{\varepsilon} = B_4 \sigma^m \phi_B^w \lambda, \quad (14)$$

where parameter $B_4$ will be (and the magnitude of exponents $n$ and $m$ may be) different from those in eq 13, and $\lambda$ is an exponent. For a more complete description of the creep process of ice under a complex stress–strain state, the reader is referred to the recent work of Fish (1993).

**ICE–STRUCTURE INTERACTION FORCE**

As is well recognized by engineers engaged in estimating sea-ice forces on offshore structures, many variables enter into the calculations. In the following, an estimate is made of the horizontal force that could develop between a cold first-year sea-ice sheet and an offshore structure in the southern Beaufort Sea. In this scenario only the structure width (90 m) and the degree-days of seawater freezing since a stable ice cover formed (5000) are known. From the work of Anderson (1961), Wen et al. (1991), and R. Lewellen (pers. comm.), the relationship between Arctic ice floe thickness and the cumulative freezing degree-days since a stable ice cover formed is known. From the equation for the curve in Figure 27, the expected ice thickness for 5000 freezing degree-days is 176 cm, and from eq 2 the bulk salinity of the ice
sheet is estimated to be 5.15‰. We assume the ice sheet has an average temperature of −10°C. This is a reasonable value for cold, late-winter first-year sea ice along the Alaska Beaufort Sea coast. The bulk porosity is found to be 52.20‰ from the equation, \( \phi_B = 19.37 + 36.18 \frac{S}{\phi_B} - 0.69 \), given in Figure 13a.

The peak failure stress in sea ice occurs at a strain rate of about \( 10^{-3}\text{s}^{-1} \) for small-scale ice samples with an aspect ratio less than 2. As the sample size or the volume of ice under load increases, the peak stress occurs at a lower strain rate (Sanderson 1988, Gavrilo 1995). At full-scale ice structure interaction when the aspect ratio is greater than 50, the peak stress appears to occur at an effective strain rate \( \dot{\varepsilon}_e \) around \( 10^{-5}\text{s}^{-1} \). Since the structure width \( d \) is very much greater than \( T_F \), plane stress conditions apply and the apparent ice sheet velocity \( V \) at this strain rate is (Sanderson 1988)

\[
V = \dot{\varepsilon}_e 0.44d
\]

or 1.4 m/h. Using 52.2‰ for \( \phi_B \) and \( 10^{-5}\text{s}^{-1} \) for \( \dot{\varepsilon} \) in eq 5 gives 1.6 MPa for the ice sheet peak horizontal uniaxial unconfined compressive strength. For plane stress conditions, the indentation force \( F \) between a vertical structure and the level ice sheet may be estimated using the reference stress expression (Sanderson 1988)

\[
F = 1.15 dT_F \sigma_c.
\]

For \( d = 90 \text{ m} \), \( T_F = 1.76 \text{ m} \) and \( \sigma_c = 1.6 \text{ MPa} \) as given above, the global ice force against the struc-
ture is estimated to be 291 MN or \( = 3.24 \text{ MN/m} \). Since most estimates for the integrated limiting pack ice force lie between \( 10^4 \) and \( 10^5 \text{ N/m} \) (Parmeter and Coon 1973, Rothrock 1975, Hibler 1980, Nevel 1983 and Croasdale 1984), the pack ice driving force would seem to be the design limiting-force load. However, since the pack ice driving force can be distributed across a floe that is much wider than the structure, the concentrated force at the structure can easily exceed the stress required to fail the ice. Therefore, for this model and no other factors being considered, the design force would be the calculated value.

The constants given in eq 15 and 16 are best estimates and may need to be revised (Sanderson 1988). The estimated force of 291 MN is considered an upper limit for the conditions given. Indeed, for a similar ice sheet thickness and structure width, the ice force developed against the structure due to nonsimultaneous ice crushing failure appears to be less than half the above calculated indentation force (Wright and Timco 1994).

**SUMMARY**

A constitutive relationship, in the form \( \sigma_c = B \dot{\varepsilon}^{1/\mu} \phi_B^{\eta} \), was developed for predicting the unconfined compressive strength of first-year sea-ice floes as a function of only two parameters, the applied strain rate and the ice floe bulk porosity. This equation was evaluated using full sea-ice sheet thickness, horizontal unconfined compression test data, so no uncertain small- to large-scale scaling laws were involved. The magnitudes of the parameters in the equation were found to be \( \sigma_c = 2.7 \times 10^{-3} \dot{\varepsilon}^{1/3} \phi^{-1} \). Using this relationship, an example is given for estimating the design sea-ice crushing force against a 90-m-wide offshore structure.

Relationships are also provided for determining ice sheet tensile and flexural strength. The relation between sea-ice bulk conductivity and strength was explored. First-year sea-ice thickness can currently be measured using surface or airborne systems. It is predicted that one day ice sheet conductivity will also be remotely measured and that this will lead to the capability of remotely estimating sea-ice strength.

Two widely used methods for calculating the

---

*Figure 27. Sea-ice thickness vs. seawater freezing degree-days after a stable ice cover has formed.*

\[
\theta = \int (T_w - T_a) \, dt \\
\theta = \text{Cumulative } {^\circ\text{C}} \text{ Freezing Degree-days After Ice Formation} \\
T_w = \text{Seawater Freezing Temp.} \\
T_a = \text{Mean Daily Air Temp.} < T_w
\]
LITERATURE CITED


Sea Ice: Part II. Estimating the Full-Scale Tensile, Flexural, and Compressive Strength of First-Year Ice

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Sea-ice salinity, density, and temperature data were used to develop new methods for determining the bulk brine volume and porosity of sea-ice floes. Methods for estimating full-thickness ice sheet strength, based on large-scale field tests, are presented. The relationships among bulk sea-ice properties, strain rate, and strength are illustrated. A new constitutive equation was developed for predicting the full-thickness horizontal compressive strength $\sigma_c$ of first-year sea ice as a function of the applied strain rate and bulk porosity in the form $\sigma_c = B_2 \varepsilon^{1/n} \phi_B^2$, where parameters $B_2$, $n$, and $m$ are about $2.7 \times 10^3$, 3, and $-1$, respectively, and $\varepsilon$ and $\phi_B$ are the ice strain rate and ice floe bulk porosity of sea ice, respectively. An estimate of the horizontal force that may develop between first-year sea-ice and a 90-m-wide structure is given. Estimating sea-ice strength based on remote ice conductivity measurements is also discussed conceptually.