The prediction of unfrozen water contents in frozen soils from liquid limit determinations
Cover: Sixteen-centimeter core of frozen silt from Barrow, Alaska, showing gradation in frozen mineral soil and ice content due to differential freezing rates and water supply. Note voids with hoarfrost deposition in upper section. (Photograph courtesy of J. O'Sullivan and J. Brown.)
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THE PREDICTION OF UNFROZEN WATER CONTENTS IN FROZEN SOILS FROM LIQUID LIMIT DETERMINATIONS

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Frozen soils
Liquid limit
Permafrost
Soil mechanics
Unfrozen water

During the past decade a number of methods for measuring the amount of unfrozen water in partially frozen ground have emerged. Means of quickly and simply predicting unfrozen water contents in clay have become increasingly important with the growth of interest in encapsulating clay soils compacted at low water contents to serve as base courses for roads. Unfortunately the measurements require sophisticated equipment and, in most instances, specially trained operators. In an effort to simplify the task of obtaining water-ice phase composition data, methods of calculating phase composition curves from other, simpler measurements on soils have been sought. In this paper we present a method of deriving the measurement of unfrozen water contents at various temperatures from liquid
20. Abstract (cont’d)

limit determinations. Previous studies have indicated that phase composition curves can be well represented by a simple power equation, \( W_u = a \theta^\beta \), where \( W_u \) is the unfrozen water content in g H\(_2\)O/g soil, \( \theta \) is the temperature in degrees below freezing and \( a \) and \( \beta \) are empirical constants characteristic of a given soil. When the liquid limits of a large group of soils encompassing a wide range of textures were regressed against values of \( a \), the correlation was found to be remarkably good. This has permitted the development of a prediction equation of sufficient accuracy for general engineering use.
PREFACE

This report was prepared by A.R. Tice, Physical Science Technician, and Dr. D.M. Anderson, formerly Chief, Earth Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory, and by Dr. A. Banin, Professor of Soil Science, Department of Soil and Water Sciences, Hebrew University, Rehovot, Israel. The report is funded under DA Project 4A161102AT24, Research in Snow, Ice and Frozen Ground, Task A1, Properties of Cold Regions Materials, Work Unit 002, Properties of Frozen Soil.

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CONVERSION FACTORS FOR U.S. CUSTOMARY AND SI UNITS

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SUMMARY

The importance of knowing the amount of unfrozen water in frozen soils has been realized for many years. Following the discovery of vast quantities of oil in Alaska and the subsequent construction of the Trans-Alaskan Pipeline through areas of perennially frozen ground, the ability to quickly make these calculations has become increasingly important. For example, where a pipeline is constructed in areas containing temperate permafrost, it might become necessary to refrigerate the supporting members. The effective design of the refrigeration system is based on the knowledge of how much water is to be frozen and hence on the amount of unfrozen water in a particular soil type.

A number of reliable methods have been developed within the past several decades to measure the amount of unfrozen water in partially frozen ground. In all instances, these determinations require sophisticated equipment not available to field engineers, and the time required to make the measurements further restricts their use.

To circumvent the above restrictions, methods of empirically calculating the relative amounts of unfrozen water from physical characteristics were sought. Previous studies have indicated that phase composition curves can be well represented by a simple power equation, \( W_u = \alpha \theta^\beta \), where \( W_u \) is the unfrozen water content in g H\(_2\)O/g soil, \( \theta \) is the temperature in degrees Celsius below freezing, and \( \alpha \) and \( \beta \) are empirical constants characteristic of a given soil.

In this investigation, the possibility of using one of the indices most common to field engineers (the liquid limit) to predict unfrozen water contents was examined. When the liquid limits of a large group of soils encompassing a wide range of textures were regressed against values of \( \alpha \), the correlation was found to be remarkably good. This has permitted the development of a prediction equation of sufficient accuracy for general engineering use, provided that the soils being investigated do not contain excessive amounts of soluble salts.
THE PREDICTION OF UNFROZEN WATER CONTENTS IN FROZEN SOILS FROM LIQUID LIMIT DETERMINATIONS

by

A.R. Tice, D.M. Anderson and A. Banin

INTRODUCTION

The importance of knowing the amount of unfrozen water remaining in frozen soils has been realized for many years. Recent developments in road and airfield construction techniques and the exploitation of arctic and subarctic regions have placed added emphasis on the ability to quickly and accurately calculate the relative amounts of ice and water in frozen soil systems. For example: when construction takes place in areas where permafrost exists, the thermal regime will be changed, resulting in degradation of the permafrost table. It is important, therefore, to be able to furnish an estimate of the amount of unfrozen water, as a function of temperature, for insertion into an appropriate heat balance equation to assess the impact of local disturbances.

For many years engineers have known that fine subgrade materials compacted at or slightly below their optimum water content will sustain very high loads. This has been substantiated by in-situ testing, e.g. plate bearing tests. As the water content is raised due to infiltration or capillarity, the strength is expected to decrease quite rapidly. In addition, in areas where freezing occurs, the formation of ice lenses and accompanying inflow of water cause the soil to expand. An additional reduction in strength can be expected as the soil is thawed. With this in mind it has been proposed that local materials, such as clays and silty clays, be compacted at low water contents and be surrounded with an impermeable membrane to eliminate the possibility of strength loss due to these effects. In such an approach, one of the important parameters which must be considered for any selected soil is the amount of water remaining unfrozen as a function of temperature.

In recent years a number of reliable methods have been devised to obtain phase composition curves, but invariably, sophisticated equipment not available to field engineers is required. In an effort to simplify the task of obtaining water-ice phase composition data and making such information available for wider field applications, indirect methods of estimating water-ice phase composition have been sought. One recent approach relied on the correlation between the specific surface area of a soil and its unfrozen water content at various temperatures (Anderson and Tice 1972). Earlier, Dillon and Andersland (1966) employed values of the specific surface area, Atterberg limits, freezing point depression of the pore water, clay mineral type and the activity ratio in a prediction equation. Although most of these soil parameters are familiar, they are not widely used by soil engineers. We have therefore explored correlations of unfrozen water contents of frozen soils with simpler, more widely used, indices. We have found that a useful correlation exists with liquid limits and herewith report these results.
LITERATURE REVIEW

When a soil-water system is cooled below 0°C only part of the water freezes (Bouyoucos 1917); some remains in the form of thin, liquid-like layers on particle surfaces. The most familiar method used to obtain phase composition data has been the use of the adiabatic calorimeter. Nersesova (1953) and Williams (1964), for example, have obtained good results employing this technique. The phase composition data that furnished the basis for our study were obtained by means of an isothermal calorimeter recently designed and assembled in this laboratory (Anderson and Tice 1973, Anderson et al. 1973). A number of other methods have been used from time to time. Extensive literature on this subject has been reviewed recently by Anderson and Morgenstern (1973).

The specific surface area of fine-grained mineral matter is one of its most fundamental properties (Banin and Amiel 1970); it frequently is the most important determinant in establishing the critical physical or chemical characteristics of soils. The most widely accepted method of measuring the surface area of soils is the equilibrium ethylene glycol retention method developed by Dyal and Hendricks (1950) and improved by Bower and Gschwend (1952) and Bower and Goertzen (1959). Recently a rapid method in which ethylene glycol monoethyl ether is substituted for ethylene glycol has been developed (Carter et al. 1965). All give results which, when put on equal terms, compare favorably with those obtained by nitrogen and water vapor adsorption (Brunauer et al. 1938). In making comparisons from one method to another it is important to realize that some soil constituents, notably the expanding lattice clay minerals, have both internal and external surfaces that are accessible to water, and that this must be suitably taken into account. Consequently, it is important that the method employed to evaluate specific surface areas be capable of measuring both internal and external areas so that an estimate of the total is obtained.

The limits tests devised by Atterberg (1911) originally were intended for the classification of agricultural soils. Soon afterward, Terzaghi (1926) demonstrated basic relationships between Atterberg's plastic limit and the critical bearing point of plastic soils and between the liquid limit and critical bearing point of friable soils.* Later Hogentogler et al. (1931) pointed out the significance of the Atterberg limit tests in disclosing certain important subgrade properties. At about the same time Casagrande (1932a) provided evidence of a relationship between the Atterberg limit tests and shear strength of soils. Somewhat later he introduced a widely accepted classification system for fine-grained soil in which the Atterberg limits are critical criteria (Casagrande 1948). Although many soil characteristics, such as permeability, compressibility, strength, etc., can be approximated from the Atterberg limits, in practice this is not often done, principally because these tests require that the samples be remolded. Casagrande (1932b, 1932c) has given a detailed discussion of the structure of undisturbed clays and the effects of remolding. At the present time, measurements of the Atterberg limits are performed principally to roughly characterize the general properties of soils.

Within the past two decades it has been found that many of the physical and chemical properties of soils cross-correlate well. Mortland (1954) has shown a very highly significant correlation between cation exchange capacity and total specific surface and a correlation only somewhat less significant between clay content and surface area. Gill and Reaves (1951) have called attention to the relationships between the plastic index, cation exchange capacity, surface area and other physical properties (including the 15-bar moisture content for several soils). A high correlation between the total surface area, liquid limit and cation exchange capacity has been found by Farrar and Coleman (1967). In addition they also found, in agreement with the data of Warkentin (1968, 1972), that the correlation between the liquid limit and surface area was as high as or higher than the correlation between clay content and surface area. Seed et al. (1964a, 1964b) examined the physical significance of the Atterberg limits and the relationships between the liquid and plastic limits, clay content

*For medium and feebly plastic soils the critical bearing point is equal to, or somewhat lower than, the plastic limit. For friable soils the critical bearing point is approximately equal to 75% of the water content corresponding to the lower liquid limit.
and clay mineralogy. Banin and Amiel (1970) determined the relationships between the specific surface area and cation exchange capacity, hygroscopic moisture content, 1/3-bar and 15-bar moisture content and saturated paste moisture content. They concluded that "on theoretical as well as practical grounds it is suggested that the specific surface area of a soil can and shall be used as a key characteristic for its properties."

EXPERIMENTAL PROCEDURE

The liquid limit determinations were made in accord with the A.S.T.M. (American Society for Testing and Materials 1950) procedure with a slight variance in sample preparation. Distilled water was added to air-dry samples to form a thick paste. After the paste was thoroughly mixed, additional water was added to bring the sample water content to a point corresponding to about 8 blows with the liquid limit device. The samples were then sealed and stored for two weeks to equilibrate. The samples were exposed to the atmosphere to reduce the water content between successive determinations, then mixed and allowed to re-equilibrate for 45 min. As mentioned above, the unfrozen water contents were measured by the isothermal calorimetric method of Anderson and Tice (1973) and Anderson et al. (1973).

RESULTS

Figure 1 shows the liquid limit flow curves of the experimental data. The lines drawn through the data points were obtained by a linear regression, and in most cases the correlation coefficients exceed 0.99. Water contents at \( N = 25 \) (liquid limit) and \( N = 100 \) calculated from the regression equations are given in Table I. To provide an indication of the quality of the phase composition data used, previously published curves are collected and presented in Figures 2-5. The solid line is the best fit generated by regressing temperature against unfrozen water and the dashed line is the unfrozen water curve calculated from individual liquid limit flow curves. These data are well represented by the simple power curve

![Liquid limit flow curves](image-url)
Figure 2. Phase composition curves for two soils. Dashed lines represent values predicted from liquid limit determinations.

Figure 3. Phase composition curves for three soils and one soil constituent. Dashed lines represent values predicted from liquid limit data.

Figure 4. Phase composition curve for one soil constituent. Dashed line represents values predicted from liquid limit data.
where \( W_u = \text{unfrozen water in g H}_2\text{O/g soil}, \alpha \) and \( \beta \) are empirical constants characteristic of a given soil, and \( \theta \) is the temperature below \( 0^\circ\text{C} \). Previous studies have indicated that segments of two power curves are required to provide a "best fit" for kaolinite and rust (\( \text{Fe}_2\text{O}_3 \)). However, in the present work we elect, for the sake of simplicity, to employ a single power curve for all the soil materials. Thus, for present purposes unfrozen water contents at -1°C and -2°C are taken from these lines (Fig. 2-5).

Figure 6 illustrates the correlation between the amount of unfrozen water at -1°C and the liquid limit \( (N = 25) \). The regression equation is

\[
W_u(\theta=1) = 0.346 W_{(N=25)} - 3.01 \quad (r = 0.98)
\]

where \( W_u(\theta=1) \) is the amount of unfrozen water at -1°C, and \( W_{(N=25)} \) is water content at \( N = 25 \) (liquid limit). It seemed reasonable that if the amount of unfrozen water at -1°C correlated well with the liquid limit \( (N = 25) \), then similar correlations would be found between water contents
Figure 6. Line A is the regression of the amount of unfrozen water (from phase composition data) at $-1^\circ C$ vs the % $H_2O$ at the liquid limit for nine soils and soil constituents. Line B is the regression of the amount of unfrozen water (from phase composition data) at $-2^\circ C$ vs the % $H_2O$ at 100 blows for nine soils and soil constituents.

Figure 7. Log-log plot of unfrozen water content vs temperature for two soils.
selected from the flow curve at different numbers of blows and the unfrozen water contents at various other temperatures. After performing many tests, the water content at \( N = 100 \) yielded

\[
W_{u(\theta=2)} = 0.338 W_{(N=100)} - 3.72 \quad (r = 0.98)
\]  

(3)

where \( W_{u(\theta=2)} \) is the amount of unfrozen water at \(-2^\circ C\), and \( W_{(N=100)} \) is water content at \( N = 100 \). Equations 2 and 3 were retained for use in predicting unfrozen water contents in frozen soils from liquid limit data as follows.

The amount of unfrozen water at \(-1^\circ C\) and \(-2^\circ C\) is taken from lines A and B respectively in Figure 6 (or calculated from eq 2 and 3). The two \( W_u \) points obtained are plotted vs temperature on a log-log scale, and a straight line is passed through them. This has been done for two typical soils in Figure 7. This line then becomes the predicted phase-composition curve, equivalent to eq 1. Data to construct the phase-composition curve on a linear scale are then easily obtained.

**DISCUSSION**

The intent of this investigation was to determine if a useful relationship exists between experimentally derived values of unfrozen water and those that can be calculated from some simple engineering tests. The liquid limit determination was chosen because it is one of the most readily available tests for determining an on-site soil characteristic and because it has fundamental significance.

The observed values of unfrozen water content at \(-1^\circ C\) and \(-2^\circ C\) are plotted against the calculated values in Figure 8. The fact that, in most instances, the observed and predicted values of unfrozen water are in close agreement is apparent. However, deviations are observed in several soils and minerals (e.g. Fig. 2, and curve C in Fig. 3). These can be traced to various specific compositional properties of these soils or minerals.

The soils in this investigation were used without treatment. Some of them contained small amounts of soluble salts. Liquid limit determinations are not to be expected to be influenced greatly by moderate additions of salts, but the amount of unfrozen water, especially in the temperature range of \(-1^\circ C\) to \(-2^\circ C\), changes appreciably. For example, the amount of soluble salts extracted from the Suffield silt clay was considerable, compared to the Fairbanks silt and kaolinite which both contained lesser quantities of solute; this is reflected in the degree to which the calculated values depart from the observed data.

Figure 8. Plot of the amount of unfrozen water at \(-1^\circ C\) and \(-2^\circ C\) (from phase composition data) vs the amount of unfrozen water at \(-1^\circ C\) and \(-2^\circ C\) calculated from eq 2 and 3.

The difference was found to be largest for the Suffield clay, and smallest for the kaolinite (Fig. 2 and 3). Conversely, the nature of the exchangeable cations is expected to be reflected in the liquid limit determination, but it is not expected to greatly change the relative amounts of unfrozen water, particularly if the cations in question are the most commonly occurring ones, sodium and calcium. The mineralogy of the soil constituents should also be considered because of the difference in the observed and predicted curves of the rust and limonite, which are unlike any of the other natural...
soils. In addition, it seems that, in the case of swelling minerals with liquid limits in excess of 100, large deviations between predicted and observed values may be found. Preliminary data on montmorillonite, nontronite, hectorite, etc. (smectites) having liquid limits that range from 200-700 indicate that the amount of unfrozen water is really much lower than that predicted from eq 2 and 3. Consequently, eq 2 and 3 presently must be regarded as applying only to soils with liquid limits smaller than 100. Subsequent investigations will eventually reveal whether or not another simple relationship exists between liquid limit data and unfrozen water contents for soils composed primarily of clay minerals belonging to the smectite group.

CONCLUSIONS

Our results indicate that a high correlation exists between the amount of unfrozen water in frozen soils and water contents on liquid limit flow curves. As a result, water-ice phase composition curves of frozen soil can be predicted with sufficient accuracy for many engineering applications, provided that the soils being considered do not contain excessive amounts of soluble salts. In cases where soluble salts are present, a correction is required; the procedure for calculating and applying it has been outlined by Banin and Anderson (1974). Caution should be exercised in applying the prediction equations developed here for soils which have liquid limits in excess of 100.

LITERATURE CITED


Bouyoucos, G.J. (1917) Classification and measurement of the different forms of water in the soil by means of the dilatometer method. Michigan Agricultural Experimental Station Technical Bulletin no. 36.


