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DIGITAL FILTERS FOR EARTHQUAKE SITE STUDIES

by

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The objective of this study was to develop a computerized technique for processing strong-motion accelerograms from earthquakes in order that earthquake vulnerability for existing structures can be more fully investigated through the use of simplified base excitations. Through filter theory, the Wiener normal equations were used to reduce accelerogram pairs to basic time histories called operators. This approach isolates the differences contained within accelerogram pairs from project sites at which the data were obtained.
20. ABSTRACT (Continued).

The operators are used with mathematical shocks (such as half-sine pulses) as inputs to produce simplified responses. Variations in the simplified responses are obtained by changing the durations of the half-sine or other hypothetical wave form inputs and using the same operator for each variation. Each family of responses is then studied to determine its sensitivity to these changes. As an example of this theory, data from an earthquake of magnitude 4.4 were used to construct operators that related free-field motions to those on the main and auxiliary dams at the Isabella Reservoir near Bakersfield, California. Half-sine inputs of 0.1-g peak amplitude and 0.1- to 0.6-sec durations were used to study the sensitivity of the dam responses. The auxiliary dam had a maximum sensitivity to cross-dam excitation when the input duration was 0.4 sec. For this input duration, the response peak acceleration was 2.4 times as great as the input peak acceleration. The main dam showed maximum sensitivity to cross-dam excitation when the input duration was 0.3 sec; however, in this case the response peak acceleration was only 1.3 times as great as the input peak acceleration. Based on the results of this study, it is concluded that use of this technique should enable structural engineers to assess earthquake vulnerability for existing structures more fully, and that this in turn could lead to improvements in future designs.
PREFACE

This study was conducted during fiscal years 1973 and 1974 under the sponsorship of the Office, Chief of Engineers, U. S. Army, as part of the Civil Works Program Engineering Study 047, "Earthquake Effects on Concrete Structures." Much of the material discussed herein was presented at the Department of Defense 44th Shock and Vibration Symposium, 4-7 December 1973, under the title "Digital Filters for Shock Data Evaluation."

The work was conducted under the supervision of Mr. W. J. Flathau, Chief, Weapons Effects Laboratory (WEL), U. S. Army Engineer Waterways Experiment Station (WES), and Mr. L. F. Ingram, Chief, Phenomenology and Effects Division, WEL. Mr. H. D. Carleton of WEL was responsible for project development and documentation. Mr. H. W. Jones of the Computer Analysis Branch, Automatic Data Processing Center, WES, provided programming support for the project.

Directors of WES during the conduct of this study and the preparation and publication of this report were BG E. D. Peixotto, CE, and COL G. H. Hilt, CE. Technical Director was Mr. F. R. Brown.
CONTENTS

PREFACE ....................................................... 1

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)
UNITs OF MEASUREMENT ...................................... 3

PART I: INTRODUCTION ........................................... 4
   Background ............................................... 4
   Objective and Scope ..................................... 4
   Theory .................................................. 7

PART II: TIME DOMAIN TRANSFER .............................. 8
   Time Series Notation .................................... 8
   Convolution ............................................. 8
   Crosscorrelation ....................................... 9
   Autocorrelation ....................................... 10
   The Wiener Normal Equations .......................... 11
   Computation of a Wiener Operator ................... 11

PART III: EXAMPLES OF TRANSFER CHARACTERIZATION IN THE
   TIME DOMAIN .............................................. 15
   Three Linear Process Characterizations ............... 15
   Operator Determinations for an Arbitrarily Selected
   Wave Form Pair ....................................... 16

PART IV: APPLICATION OF TIME DOMAIN TRANSFER CHARACTERIZATIONS
   TO THE ISABELLA EARTHQUAKE OF 8 MARCH 1971 .......... 19
   Isabella Earthquake of 8 March 1971 .................... 19
   Site Difference Operator Determinations ............... 20
   Shock Pulse Response Site Characterizations .......... 22

PART V: CONCLUSIONS AND RECOMMENDATIONS ............... 27
   Conclusions ............................................ 27
   Recommendations ....................................... 27

REFERENCES .................................................. 28

PLATES 1-9

APPENDIX A: NOTATION

2
U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

<table>
<thead>
<tr>
<th>Multiply</th>
<th>By</th>
<th>To Obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>feet</td>
<td>0.3048</td>
<td>metres</td>
</tr>
<tr>
<td>miles (U. S. statute)</td>
<td>1.609344</td>
<td>kilometres</td>
</tr>
<tr>
<td>acre-feet</td>
<td>1233.482</td>
<td>cubic metres</td>
</tr>
<tr>
<td>feet per second</td>
<td>0.3048</td>
<td>metres per second</td>
</tr>
<tr>
<td>cubic feet per second</td>
<td>0.02831685</td>
<td>cubic metres per second</td>
</tr>
</tbody>
</table>
1. A basic problem in the field of earthquake engineering is that of characterizing strong ground motions in a practical way. There is a considerable hazard in basing structural designs solely upon damage potential zonation, since ground motion amplitudes and durations may vary widely from site to site, even within a given zone. Strong-motion accelerograms, on the other hand, provide detailed information about specific earthquakes and particular sites. The character of the recorded motions is influenced by source motion, propagation path, and local geologic conditions. Where ground and structural motions have been recorded at an earthquake site, a potential exists for the use of these records and appropriate mathematical procedures to isolate the differences between base excitations and structural responses. The requirement for the study described herein grew out of the necessity for structural engineers to be able to examine structural response characteristics in a simplified form.

2. The objective of this study was to develop a computerized technique for processing strong-motion accelerograms from earthquakes in order that earthquake vulnerability for existing structures can be investigated more fully through the use of simplified base excitations. This empirical technique was to provide structural engineers with a readily implementable means of comparing structural responses at widely separated locations through its ability to use any given base excitation as a standard for studies at all locations. (See Figure 1 for a graphic portrayal of this concept.) The technique was to be developed from existing mathematical procedures which would be applicable to the
Figure 1. Concept for the use of a mathematical technique to provide a standard for the comparison of earthquake responses at widely separated locations.
Figure 2. Filter terminology relationships
above-stated objective, in order to place a usable product in the hands of field engineers at the earliest possible time.

Theory

3. A Wiener filter is a mathematical operator designed to convert a given wave form (the filter's input) into another wave form (the filter's output) which is as similar as possible, in the least-squares sense, to a third wave form (the desired output). If the fit of the Wiener filter's output to the desired output is close, the operator can be regarded as a measure of the difference between the filter's input and the desired output.

4. For this study, filter theory and the concept of Figure 1 were used to reduce accelerogram pairs to mathematical operators. The operators were then used with mathematical shocks (such as the half-sine pulses of Figure 1) as inputs to produce simplified responses. Variations in the simplified responses were then obtained by changing the durations of the half-sine or other hypothetical wave form inputs and using the same operator for each variation. Each family of responses was then studied to determine its sensitivity to these changes.

5. Because filter theory is reviewed in this paper, Figure 2 has been included to enable those who are not familiar with the terminology of the field to better visualize the concepts which will be discussed. Processing with filters may be accomplished in either the frequency domain (represented in the left column in Figure 2) or the time domain (represented in the right column in Figure 2). Reference 3 is a basic handbook which discusses (among other things) conversion between these two domains by use of the mathematical process of Fourier transformation. The operations discussed in the present paper involve comparisons of data in the time domain. However, at each step in the development of the basic theory, equivalents in the frequency domain are discussed for the convenience of readers who are more familiar with this domain.
6. If the displacement of a continuous wave form is sampled periodically, the resulting time sequence of equally spaced observations is said to be a "discrete time series." The time represented by any given sample in this series would be \( t = nT \), where \( n \) is the sample number (counting by units from sample number 0 at \( t = 0 \)) and \( T \) is the sampling period (a constant, commonly in seconds, that is the reciprocal of the sampling frequency, commonly in Hertz). If \( T \) is defined as one unit of time, \( t = nT \) becomes \( t = n \). Thus, a periodically sampled continuous wave is converted into a sequence of numbers

\[
(x_0, x_1, x_2, \ldots, x_m)
\]

where \( m \) is a parameter (maximum \( n \)) that defines the sampled data duration, \( x_0 \) is the value of (and continuous wave amplitude at) sample number zero (zero time), \( x_1 \) is the value of sample number one, and so forth to \( x_m \), which is the value of the last sample in the series. Sample values outside the time range of the sampled portion of the wave are defined to be zero. Then, for the time series in Expression 1

\[
x_n = 0 \quad \text{for} \quad n < 0 \quad \text{and} \quad n > m
\]

An \( n \)-subscripted \( x \) is used to represent Expression 1 in Equations 2, 3, and 4.

### Convolution

7. A discrete time series may be filtered by means of a moving

---

* For convenience, a list of symbols is presented in the Notation (Appendix A).
** Sampling frequency selection is discussed by the author in an earlier paper.²⁴
The summation called convolution. This operation may be defined as

\[ y_n = \sum_{i=0}^{k} r_i x_{n-i}, \quad n = 0, 1, 2, \ldots, (m + k) \quad (2) \]

where

- \( y_n \) = output data sample at time \( n \)
- \( k \) = impulse response duration in sampling units
- \( i \) = impulse response sample number (elapsed time in sampling units)
- \( r_i \) = impulse response sample at time \( i \)
- \( x_n \) = input data sample at time \( n \)

The \( m + 1 \) values of Expression 1 constitute a time history input to a filter, the \( m + k + 1 \) values to be calculated for \( y_n \) will constitute the filter's output time history, and the \( k + 1 \) values of \( r_i \) represent the time history of a linear system's response to a unit impulse.*

The time series represented by values of \( r_i \), then, is descriptive of a filter, and is known as that filter's operator. A discrete filter operator may represent the impulsive response of an equivalent electrical network.

8. The frequency domain operation which corresponds to convolution involves multiplication (frequency by frequency) of the amplitude spectrum of a filter's input by that of its transfer function and addition (frequency by frequency) of the phase spectrum of the filter's input to that of its transfer function to yield the amplitude and phase spectra of the filter's output (see Figure 2).

Crosscorrelation**

9. Two time histories can be compared mathematically by means of their crosscorrelation function

* The unit impulse is Kronecker's delta function, which is defined as \( \delta_n = 1 \) when \( n = 0 \) and \( \delta_n = 0 \) when \( n \neq 0 \).

** Anstey discusses correlations and their uses and provides an extensive bibliography.
\[ \phi_{xd}(\tau) = \sum_{n=0}^{m} x_n d_{n+\tau}, \quad \tau = -m, \ldots, -1, 0, 1, 2, \ldots, m \]  

(3)

where the \( m + 1 \) values of Expression 1 constitute one time history and the \( m + 1 \) values of the desired output data sample \( d_n \) constitute a compared time history. Values of \( \phi_{xd}(\tau) \), when plotted against the correlation time shift in sampling units \( \tau \), produce a graph with strongest maxima at time shifts for which the compared waveforms most nearly coincide.* This plot contains only those frequencies common to both of the original time histories and is generally not symmetrical about \( \tau = 0 \).

10. The frequency domain operation which corresponds to cross-correlation involves multiplication of the amplitude spectra of the compared waveforms and subtraction of one phase spectrum (that of the \( x_n \) time history) from the other (that of the \( d_n \) time history) to give the amplitude and phase spectra of the crosscorrelation function.

Autocorrelation

11. If a time history is crosscorrelated with itself, the resulting wave form is called an autocorrelation function \( \phi_{xx}(\tau) \). This function can be defined by

\[ \phi_{xx}(\tau) = \sum_{n=0}^{m} x_n x_{n+\tau}, \quad \tau = -m, \ldots, -1, 0, 1, 2, \ldots, m \]  

(4)

where the \( m + 1 \) values of Expression 1 constitute the only input time history. A plot of this function is always symmetrical about \( \tau = 0 \), and values of \( \phi_{xx}(\tau) \) for \( \tau \neq 0 \) never exceed the value of \( \phi_{xx}(\tau) \) at \( \tau = 0 \).

12. The frequency domain operation which corresponds to autocorrelation is the same as that which corresponds to crosscorrelation,

* Values of \( \phi_{xd}(\tau) \) are usually normalized for such a plot.
with the special condition that the compared spectra are identical. Thus, the input's amplitude spectrum is squared (producing its power spectrum), and its phase spectrum is zeroed as a result of subtraction from itself. All phase information is therefore lost in the process of taking the autocorrelation function of a time history.

The Wiener Normal Equations

13. A discrete Wiener filter is determined by means of the Wiener normal equations, defined by

$$\sum_{i=0}^{k} r_i \phi_{xx}(\tau - i) = \phi_{xd}(\tau), \quad \tau = 0, 1, 2, \ldots, k \quad (5)$$

where the \( k + 1 \) values of \( r_i \) (the filter's operator) are to be calculated, and the \( k + 1 \) necessary values for each of the two correlations have been previously determined from the input (\( x_n \) time series) and desired output (\( d_n \) time series) through the use of Equations 3 and 4.

14. The combined effect of the two correlations and the Wiener normal equations is to produce a filter's operator, given only that filter's input and output time histories. As shown in Figure 2, a filter's operator is equivalent to its transfer function. Frequency domain production of the transfer function involves division of the output's amplitude spectrum by the input's amplitude spectrum, and subtraction of the input's phase spectrum from the output's phase spectrum.

Computation of a Wiener Operator

15. As an example, let a two-sample Wiener filter be constructed to show the relationship between the following discrete time series:

<table>
<thead>
<tr>
<th>Input (( x_n ) time series)</th>
<th>2, 1, 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired output (( d_n ) time series)</td>
<td>0, -4, -2</td>
</tr>
</tbody>
</table>
Since a two-sample operator is specified, \( k = 1 \). Because the compared time histories consist of three samples each, \( m = 2 \).

16. The necessary values of the input autocorrelation will be determined from:

\[
\phi_{xx}(\tau) = \sum_{n=0}^{2} x_{n}x_{n+\tau}, \quad \tau = -1, 0, 1
\]

Note that the value of \( k \) establishes the range of \( \tau \) necessary for filter construction. Substituting in the above equation:

\[
\phi_{xx}(-1) = x_{0}x_{-1} + x_{1}x_{0} + x_{2}x_{1} = (2 \times 0) + (1 \times 2) + (0 \times 1) = 2
\]

\[
\phi_{xx}(0) = x_{0}^{2} + x_{1}^{2} + x_{2}^{2} = (2^{2}) + (1^{2}) + (0^{2}) = 5
\]

\[
\phi_{xx}(1) = x_{0}x_{1} + x_{1}x_{2} + x_{2}x_{3} = (2 \times 1) + (1 \times 0) + (0 \times 0) = 2
\]

17. The necessary values of the desired output/input cross-correlation will be determined from:

\[
\phi_{xd}(\tau) = \sum_{n=0}^{2} x_{n}d_{n+\tau}, \quad \tau = 0, 1
\]

This correlation is calculated only for positive values of \( \tau \), and maximum \( \tau \) is again equal to \( k \). Substituting:

\[
\phi_{xd}(0) = x_{0}d_{0} + x_{1}d_{1} + x_{2}d_{2} = (2 \times 0) + (1 \times -4) + (0 \times -2) = -4
\]

\[
\phi_{xd}(1) = x_{0}d_{1} + x_{1}d_{2} + x_{2}d_{3} = (2 \times -4) + (1 \times -2) + (0 \times 0) = -10
\]

18. The two-sample Wiener operator is now determined from the Wiener normal equations:

\[
\sum_{i=0}^{1} r_{i}\phi_{xx}(\tau - i) = \phi_{xd}(\tau), \quad \tau = 0, 1
\]
For the present example, this sets up two equations in two unknowns:

\[ r_0 \phi_{xx}(0) + r_1 \phi_{xx}(-1) = 5r_0 + 2r_1 = \phi_{xd}(0) = -4 \]

\[ r_0 \phi_{xx}(1) + r_1 \phi_{xx}(0) = 2r_0 + 5r_1 = \phi_{xd}(1) = -10 \]

Solving by the method of subtraction:

\[
\begin{align*}
5r_0 + 2r_1 &= -4 \\
-5r_0 - 12.5r_1 &= +25
\end{align*}
\]

\[ -10.5r_1 = +21 \]

\[ r_1 = - \frac{21}{10.5} = -2 \]

Substituting:

\[ 5r_0 + (2 \times -2) = -4 \]

\[ r_0 = \frac{0}{5} = 0 \]

The required two-sample operator \((r_i \text{ time series})\) is: \(0, -2\).

19. Referring to Equation 2 and using the original input time series \((2, 1, 0)\) and the calculated operator \((0, -2)\), this filter's output is determined as follows:

\[ y_0 = r_0 x_0 + r_1 x_{-1} = (0 \times 2) + (-2 \times 0) = 0 \]

\[ y_1 = r_0 x_1 + r_1 x_0 = (0 \times 1) + (-2 \times 2) = -4 \]

\[ y_2 = r_0 x_2 + r_1 x_1 = (0 \times 0) + (-2 \times 1) = -2 \]

\[ y_3 = r_0 x_3 + r_1 x_2 = (0 \times 0) + (-2 \times 0) = 0 \]

The output \((y_n \text{ time series})\) is 0, -4, -2, 0. This exactly corresponds to the desired output \((d_n \text{ time series})\). In this particular very simple
case, it has been possible to construct a filter that shows the exact relationship between an input and a desired output. The effect of the required filter is a delay of one sampling unit, a change in polarity, and 2X amplification, i.e., the filter's unit impulse response is a negative impulse of amplitude two at unit delay.

20. Moving from two-sample operators (which are convenient for use in theoretical discussions) to the more practical problem of processing real-world accelerograms, it will be found that operators can be hundreds of samples in length. Computer solutions make use of recursion, a mathematical shortcut which uses previously computed output samples as well as the usual input samples, in determining later output sample values. Such solutions run very quickly, and at minimal cost. Recursion and the Wiener matrix manipulation are discussed by Treitel and Robinson. Shanks discusses recursive alternatives to convolution.
PART III: EXAMPLES OF TRANSFER CHARACTERIZATION
IN THE TIME DOMAIN

21. The purpose of this part of the report is to demonstrate, by examples, the operation of a computer program based upon the theory discussed in Part II. Selected wave form pairs are reduced to single-operator wave forms that best represent the differences between the original paired wave forms. Each operator wave form is evaluated regarding its suitability for such representation in order to demonstrate the criteria for this determination.

Three Linear Process Characterizations

22. A single arbitrarily selected earthquake accelerogram has been processed by standard linear methods to produce the wave form pairs to be used in the three examples given in this section. This has been done to show that in certain idealized cases, the differences between chosen wave forms can be characterized in a condensed form by a single-operator wave form that describes the transfer involved. Amplitude, polarity, and time position changes

23. In Plate 1, the input wave form is the above-mentioned earthquake accelerogram. The desired output wave form is this same data lagged 0.1 sec in time, reversed in polarity, and doubled in amplitude. The computer-calculated Wiener operator that describes the relationship between this pair contains only one significant nonzero sample amplitude. This amplitude is 2.000, is negative, and occurs precisely at the 0.1-sec time position on the operator trace. The unit impulse response of the "system" defined by the input and desired output of Plate 1 is essentially a negative impulse of amplitude 2 at a 0.1-sec delay. If this operator is a good one for the definition of the differences in the indicated wave form pair, its use as a filter for the Plate 1 input should yield as an output a reasonable approximation of the desired output. The actual output from this convolution (shown at
24. In Plate 2, the input wave form is identical with that in Plate 1. The desired output wave form is the result of a trapezoidal integration of the input accelerograph trace. The operator constructed to define the differences in this pair of wave forms produces, upon convolution with the input, an output (shown at the right in Plate 2) that is a perfect reconstruction of the desired output. Thus, this operator may be said to be a unit impulse response that characterizes integration within the band of frequencies represented by the basic data.

Differentiation

25. In Plate 3, the input wave form is identical with the desired output in Plate 2. The Plate 3 desired output wave form is the Plates 1 and 2 input with a time delay of 0.4 sec introduced. The operator constructed to define the differences in this pair of wave forms produces, upon convolution with the input, an output (shown at the right in Plate 3) that is a perfect reconstruction of the desired output. This operator may be said to be a unit impulse response that characterizes time-lagged differentiation within the band of frequencies represented by the basic data.

Operator Determinations for an Arbitrarily Selected Wave Form Pair

26. A single pair of wave forms will be used in three different ways for the three examples given in this section. This will be done to demonstrate to the reader certain of the more common problems that will be encountered in the processing of arbitrarily selected wave form pairs.

A satisfactory operator determination

27. The input and desired output wavelets in Plate 4 are radial
velocity gage data from an explosion effects test at 100-ft* depth in rock. Both represent data taken from stations 100 ft from the shot point. However, the input data are from a location south of the explosion, while the desired output data are from a location west of the explosion. In each case, indicated zero time is 7 msec later than actual zero time, but the relative time positions of the wave forms are unchanged. The resulting operator's actual output (shown at the right in Plate 4) is an excellent reconstruction of the desired output. The calculated operator for this wavelet pair in its proper time relationship is a good one because its use as a filter for the Plate 4 input closely reproduces the desired output. However, the form of this operator is extremely complex, and it would be of little use as a visual characterization of the differences between the original pair of wave forms.

Two imperfect operator determinations

28. If the input in Plate 4 is made the desired output, and the Plate 4 desired output becomes the input, two problems can be anticipated, one relative to time positions and another concerned with wavelet frequency content. From Equation 2, it is apparent that the duration of a convolution's output is equal to the sum of the durations of its input and its operator. The first portion of a convolution's output, with a duration equivalent to that of the input, correlates to the desired output wave form. The final portion represents system settling time, usually appearing as a "noisy" trace if plotted. The best operator for a given wavelet pair will result from the positioning of the desired output's energy** relative to the anticipated convolution output's duration approximately as the input's energy is positioned relative to its own duration. Plate 5 shows the input/desired output relationship of Plate 4 reversed, with a relative lag of 4.5 msec assigned

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* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 3.
** A wavelet's energy is defined as the sum of the squares of its sample amplitudes. Treitel and Robinson\textsuperscript{2,6} discuss wavelet energy and time positions.
to the new desired output. The actual output for this data set (at the right in Plate 5) shows that the operator produced from this relationship is not good; its reconstruction of the desired output includes a rise time that is a very poor approximation of that for the gage data. This is an indication that the desired output has not been lagged enough. Plate 6 shows the same gage pair again, but with the desired output lagged 12 msec relative to the true zero time relationship. The actual output in this case (at the right in Plate 6) is reasonably good but not perfect, in spite of the fact that the 12-msec time lag is optimum for the wavelet pair being considered. The shorter pulse duration of the desired output wavelet in Plate 6 suggests that the frequency band width contained in this signal is wider than that for the broader pulse of the input. (The low-amplitude, high-frequency noise that "rides" the input is not considered in this comparison.) Since a linear system's output can contain only those frequencies present in its input, it is reasonable to expect the reconstruction to be less than perfect in this case.
Isabella Earthquake of 8 March 1971

29. Figure 3 is a topographic map of the Isabella Reservoir and vicinity, this area being entirely within Kern County, California, and approximately 40 miles northeast of Bakersfield, California. The Isabella Dam Project includes a 185-ft-high earth-fill dam with an ungated
concrete spillway in the left abutment (discharge capacity, 53,000 cfs) and a 100-ft-high earth-fill auxiliary dam as shown in Figure 4. Isabella Reservoir has a gross storage capacity of 570,000 acre-ft, for flood control, irrigation, and recreational purposes. 

30. On 8 March 1971, an earth tremor with a magnitude of approximately 4.4 was felt with intensity V effects over a wide area including most of that shown in Figure 3. Its focus was approximately 3.7 miles beneath the epicenter shown in Figure 3; slant range from this focus to the Isabella Dam Project was approximately 5.7 miles. The disturbance triggered strong-motion accelerographs that had been installed in early 1968 on both dams and on the right abutment of the auxiliary dam.

Site Difference Operator Determinations

31. In Plates 7, 8, and 9, five accelerograms from three locations (shown in Figure 4) have been paired and processed to produce operators representative of the differences between abutment and dam crest motions. Plate 7 compares N76°W horizontal (along-dam) accelerations on the crest of the auxiliary dam to N76°W accelerations on this dam’s right abutment. Plate 8 compares N14°E horizontal (cross-dam) accelerations on the crest of the auxiliary dam to N14°E accelerations on the same dam’s right abutment. Plate 9 compares N14°E (cross-dam) accelerations on the crest of the main dam to N14°E accelerations on the right abutment of the auxiliary dam. In each plate, the desired output wave form has been lagged 0.4 sec relative to the input wave form to optimize the operator construction by the adoption of the best possible time relationship (as discussed in paragraph 28).

32. A visual examination of the operators in Plates 7-9 certainly does not simplify the problem of understanding the site differences involved. These operators may nevertheless be considered effective transfer means, since the outputs of their input convolutions (at the right in each plate) are reasonably good reproductions of the desired outputs. The fact that the Plate 9 operator is the least effective of the three is consistent with the fact that this transfer represents
Figure 4. Isabella Dam Project, showing locations of strong-motion accelerographs
a greater distance between accelerograph stations. Obviously, the influence of a greater difference in propagation paths is also represented in the Plate 9 operator. Operators will best represent site differences if the distances between represented stations are very small compared with their average distance from a mutually recorded earthquake's focus.

Shock Pulse Response Site Characterizations

33. The operator which properly represents the difference between two complicated wave forms can be applied as a filter to simplified mathematical shock pulses (such as the half-sine) to produce simplified outputs (shock pulse responses). In Figure 5, the data in Plate 7 are presented in this form. The "inputs" at the top of Figure 5 are a series of half-sine pulses. Each of these pulses peaks at 0.1 g, and their durations range from 0.1 to 0.6 sec in 0.1-sec increments. The "responses" arrayed in the lower part of Figure 5 are acceleration-time histories that correspond to the inputs in the upper part of the figure. Thus, the upper response wave form in Figure 5 shows the along-dam motion that could have occurred at the crest of the auxiliary dam if the 0.1-sec duration half-sine pulse had been observed for the same component of motion on the auxiliary dam's right abutment. In like manner, Figure 6 presents the transfer information of Plates 8 and 9, except that the half-sine input array is not repeated. The lower response array in Figure 6 represents data from Plate 8, while the upper response array represents data from Plate 9. Note that while the Figure 5 input pulses apply to all three response arrays, they represent the along-dam component of motion for the Figure 5 responses and the cross-dam component of motion for the Figure 6 responses.

34. Figure 7 summarizes peak dam response information from Figures 5 and 6. Each response wave form's peak amplitude is plotted against the duration of the 0.1-g amplitude half-sine pulse that caused it. It can be readily seen from this plot that certain input pulse durations cause greater accelerations at the dams' crests than do others. The auxiliary dam curves show a stronger tendency to peak at
one point than does that for the main dam. The cross-dam crest responses for both dams, as might be expected, amplify the motion apparent at the auxiliary dam's right abutment. The along-dam motion on the auxiliary dam's crest, on the other hand, is diminished from that at this dam's right abutment, at least insofar as peak motion is concerned. Worthy of special note is the auxiliary dam's sensitivity to the 0.4-sec input situation for the cross-dam component of motion. For this input duration, the response peak acceleration was 2.4 times the input peak acceleration. The main dam showed maximum sensitivity to cross-dam excitation when the input duration was 0.3 sec; however, in this case, the response peak acceleration was only 1.3 times the input peak acceleration.
Figure 5. Characterization of along-dam motions for Isabella Auxiliary Dam (from earthquake of 8 March 1971)
Figure 6. Characterization of cross-dam motions for Isabella Dams (from earthquake of 8 March 1971)
Figure 7. Peak dam responses for half-sine pulses at abutment
PART V: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

35. Mathematical operators can be used to define, accurately and economically, the differences between earthquake accelerograms representing base excitations and structural motions at Corps of Engineers project sites. When used with simplified base excitations (such as the half-sine pulse series in Figure 5), these operators should enable structural engineers to assess earthquake vulnerability for existing structures more fully. This, in turn, could lead to improvement in future designs.

36. Where mathematical operators have been used to define the differences between base excitations and structural motions at a number of project sites, use of a standardized set of simplified base excitations should permit comparison of structural response characteristics at widely separated locations. (For concept explanation, see Figure 1.) When used with engineering judgment, such comparisons could reveal important relationships between project conditions and identifiable patterns of motion.

Recommendations

37. As base excitation/structural motion earthquake accelerogram data become available for Corps of Engineers projects, an inventory of project studies by this technique should be assembled to support structural engineers in their earthquake vulnerability assessments.

38. The variety of seismic applications being reported for information theory techniques (e.g., References 9 and 10) indicates a great flexibility in these methods. More powerful basic tools are evolving, and these tools should be investigated to determine their applicability to the problems of earthquake engineering. Earthquake site characterization and earthquake time history prediction are, of course, very much related problems, and they should yield in time to an appropriate use of information theory techniques.
REFERENCES


INPUT: ACCELEROGRAM.

DESIRED OUTPUT: INPUT ACCELEROGRAM WITH 0.1-SEC TIME LAG, AMPLITUDE DOUBLED, AND POLARITY REVERSED.

ACTUAL OUTPUT: EXCELLENT OPERATOR CHECKOUT.

OPERATOR: FOR TIME POSITION, AMPLITUDE, AND POLARITY DIFFERENCES BETWEEN INPUT AND DESIRED OUTPUT.
INPUT: ACCELEROMETER.

DESIRED OUTPUT: VELOCITY-TIME HISTORY RESULTING FROM TRAPEZOIDAL INTEGRATION OF INPUT ACCELEROMETER.

OPERATOR: FOR INTEGRATION.

ACTUAL OUTPUT: EXCELLENT OPERATOR CHECKOUT.
INPUT: VELOCITY-TIME HISTORY.

DESIRED OUTPUT: ACCELEROMETER FROM WHICH THE INPUT VELOCITY-TIME HISTORY WAS PRODUCED BY TRAPEZOIDAL INTEGRATION (0.4-SEC LAG ON DESIRED OUTPUT).

ACTUAL OUTPUT: EXCELLENT OPERATOR CHECKOUT.

OPERATOR: FOR DIFFERENTIATION WITH 0.4-SEC TIME LAG INTRODUCED.
INPUT: GAGE 100 FT SOUTH OF SHOT POINT.

DESIRED OUTPUT: GAGE 100 FT WEST OF SHOT POINT.

OPERATOR: FOR REPRESENTATION OF THE DIFFERENCES BETWEEN THE ABOVE WAVE FORMS.

ACTUAL OUTPUT: SATISFACTORY OPERATOR CHECKOUT.
INPUT: GAGE 100 FT WEST OF SHOT POINT.

DESERVED OUTPUT: GAGE 100 FT SOUTH OF SHOT POINT (WITH 4.5-MSEC TIME LAG INTRODUCED).

ACTUAL OUTPUT: UNSATISFACTORY OPERATOR CHECKOUT DUE TO POOR TIME RELATIONSHIP.

OPERATOR

INPUT:

DESIRED OUTPUT:

ACTUAL OUTPUT:
INPUT: GAGE 100 FT WEST OF SHOT POINT.

DESIRED OUTPUT: GAGE 100 FT SOUTH OF SHOT POINT (WITH 12-MSEC TIME LAG INTRODUCED).

ACTUAL OUTPUT: IMPERFECT OPERATOR CHECKOUT DUE TO POOR FREQUENCY RELATIONSHIPS.
INPUT: ALONG-DAM ACCELERATION AT RIGHT ABUTMENT OF ISABELLA AUXILIARY DAM.

DESIRED OUTPUT: ALONG-DAM ACCELERATION AT CREST OF ISABELLA AUXILIARY DAM.

ACTUAL OUTPUT: (OPERATOR CHECKOUT).

OPERATOR: FOR REPRESENTATION OF THE DIFFERENCES BETWEEN THE ABOVE WAVE FORMS.
INPUT: CROSS-DAM ACCELERATION AT RIGHT ABUTMENT OF ISABELLA AUXILIARY DAM.

DESIRE OUTPUT: CROSS-DAM ACCELERATION AT CREST OF ISABELLA AUXILIARY DAM.

ACTUAL OUTPUT: (OPERATOR CHECKOUT).

OPERATOR: FOR REPRESENTATION OF THE DIFFERENCES BETWEEN THE ABOVE WAVE FORMS.
INPUT: CROSS-DAM ACCELERATION AT RIGHT ABUTMENT OF ISABELLA AUXILIARY DAM.

DESIRED OUTPUT: CROSS-DAM ACCELERATION AT CREST OF ISABELLA MAIN DAM.

ACTUAL OUTPUT: (OPERATOR CHECKOUT).

OPERATOR: FOR REPRESENTATION OF THE DIFFERENCES BETWEEN THE ABOVE WAVE FORMS.
APPENDIX A: NOTATION

**Time Values**

- i: Impulse response sample number (elapsed time in sampling units)
- k: Impulse response duration in sampling units
- m: Data duration in sampling units
- n: Data sample number (elapsed time in sampling units)
- t: Elapsed time
- T: Sampling period (constant that is the reciprocal of the sampling frequency)
- \( \tau \): Correlation shift sample number (time shift in sampling units)

**Amplitude Values**

- \( d_n \): Desired output data sample at time \( n \)
- \( r_i \): Impulse response sample at time \( i \)
- \( x_n \): Input data sample at time \( n \)
- \( y_n \): Output data sample at time \( n \)
- \( \phi_{xd}(\tau) \): Crosscorrelation function sample at time shift \( \tau \)
- \( \phi_{xx}(\tau) \): Autocorrelation function sample at time shift \( \tau \)
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