Flexural Strength of Clear Lake Ice

by Robert D. Hitch
PREFACE

This investigation was carried out for USA SIPRE by Mr. Hitch, Michigan College of Mining and Technology, under SIPRE Proj 22.1-8 Mechanics of plastic deformation of ice. The research work was done at the Keweenaw Field Station, Houghton, Michigan. The purpose of this investigation was to study the flexural strength properties of clear lake ice.

Professor W. C. Polkinghorne, Head of the Department of Civil Engineering, Michigan College of Mining and Technology, provided valuable advice during the testing at the Keweenaw Field Station and later during the preparation of the report. Mr. W. H. Parrott, Director of the Keweenaw Field Station, made this work possible by providing facilities, material, and consultation. Dr. J. K. Landauer and Dr. A. Assur of USA SIPRE made their valuable suggestions regarding the testing procedure and preparation of this thesis.

This investigation was under the supervision of Mr. J. A. Bender, chief, Basic Research Branch. The report was submitted to the Michigan College of Mining and Technology in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

This report has been reviewed and approved for publication by the office of the Chief of Engineers.

WALTER H. PARSONS, JR.
Colonel, Corps of Engineers
Director

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SUMMARY

Values for the modulus of elasticity and strength of fresh-water lake ice in flexure were determined by laboratory experiments on 63 beams tested under various rates of loading at -5C and -20C.

The results were obtained by measuring the center deflection of the test beam and the load simultaneously. Third point loading was used in order to develop a more favorable distribution of bending stresses in the middle third of the beam than with a simple beam.

The experiments indicate that properties of ice vary according to loading rate, temperature, and crystal size. Specifically the results show: (1) lower temperatures give higher values of the modulus of elasticity and strength, (2) ice with larger crystals will have a higher modulus of elasticity but little difference in strength, (3) the rate of loading increases the modulus of elasticity but has little effect on the strength except at low temperatures where higher strength values are obtained with a faster rate of loading.
FLEXURAL STRENGTH OF CLEAR LAKE ICE

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INTRODUCTION

This investigation is limited to the flexural properties of clear ice formed on a fresh-water lake. An extensive search of literature in the field revealed unsatisfactory values of the modulus of elasticity, modulus of rupture, or yield point as determined by flexure. A total of 63 beams were tested to obtain the values presented. The testing procedures were established to determine the effects of varying test conditions on the modulus of elasticity, yield point, and flexural strength. The effects of temperature variations, crystal size, loading rate, and age were studied. Reported results are based on average values.

PREPARATION OF TEST BEAMS

Ice was harvested from Lake Superior off Keweenaw Bay in an area where only clear ice existed. The ice sheet was about 14 in. thick and contained vertical needle-like air pockets in two layers, each layer about 2 in. thick. The top layer of air bubbles was 6 in. from the surface and the second layer 10 in. from the surface. A few more or less spherical air bubbles were distributed uniformly throughout the ice. Measurements of density gave an average value of 0.915 g/cm³.

An analysis of water from this area indicated the following characteristics:

- P. H. ----------------------------- 7.5
- Alkalinity hardness (CaCO₃ equivalent)----- 45 ppm
- Total hardness (CaCO₃ equivalent)------ 58 ppm
- Hardness elements------------------------ Calcium and magnesium

Large beams were cut from the ice sheet, then sawed horizontally and cut into two test beams. This provided beams of different ice-crystal size, as the ice crystals are small near the top and increase in cross sectional area with depth (Fig. 1).

The test beams were cut with a bandsaw to 24 in. long, 3 in. wide, and 4 in. deep, to an accuracy of ±1 mm. As each beam was cut it was numbered and its original location in the ice surface recorded, so that it could be tested in its original orientation.

TEST PROCEDURE

Testing was accomplished on a Young Hydraulic Testing Machine in a thermostatically controlled cold room.

The applied load was measured with a 2000-lb capacity Baldwin load cell bolted to the head of the testing machine. A third point loading frame was attached to the load cell and the load was recorded electrically. The beam supports were 22 in. apart and rested on the moving table of the testing machine. Support settlement was measured with a SR-4 strain gage resting on a clamp attached over the beam over the right support, and center deflection was measured with a similar gage resting on a clamp attached over the center of the beam. The support and center deflection measurements were recorded simultaneously with the load.

The center deflection measurements were made along the center line of the top surface of the beam, eliminating most of the error produced by beams rotating under loading. Support settlement, however, was measured on the side of the beam, because the end of the beam rose after failure and might have damaged a strain gage placed there. This multiplied the effect of beam rotation and caused the support settlement curves to vary. For each test beam, applied load vs support settlement was plotted on the same sheet as the stress-strain curve (applied load vs center deflection) so the influence of support settlement could be taken into consideration. In each experiment the total center deflection was very small, averaging about 0.016 in. With such small deflections errors were easily noticeable at the times the beam slipped or rotated.
Experiments were performed at -5°C and -20°C for temperature comparison. All tests were graphed and enough reliable curves selected to produce nine summary curves. Each summary curve establishes a stress-strain relationship for a definite temperature, loading rate, and crystal size.

RESULTS

Figures 2a-c give the stress-strain diagrams for beams of small crystal size cut from the top part of the ice and tested at -5°C, under various rates of loading. Figure 2d gives a loading rate comparison for these data. The modulus of elasticity increases as the rate of loading increases while the flexural strength appears to be unaffected.

The results obtained with beams cut from the bottom part of the ice (large crystals) are presented in Figures 3a-c. The summary curves (Fig. 3d) show a small change in modulus of elasticity with different loading rate, again with no definite change in flexural strength.
**FLEXURAL STRENGTH OF CLEAR LAKE ICE**

![Graph of deflection at center of beam vs applied load](image1.png)

- **a.** Avg loading rate 0.0055 in/min.

![Graph of deflection at center of beam vs applied load](image2.png)

- **b.** Avg loading rate 0.012 in/min.

![Graph of deflection at center of beam vs applied load](image3.png)

- **c.** Avg loading rate 2.2 in/min.

![Graph of deflection at center of beam vs applied load](image4.png)

- **d.** Comparison of curves.

**Figure 2.** Applied load vs center deflection at different rates of loading, top beams. Tested at -5°C.

Figure 4 compares beams of different crystal size tested under similar conditions. The beams with larger crystals show a distinctly higher modulus of elasticity.

Data from tests performed at -20°C are shown in Figures 5a-c. Again the modulus of elasticity increases as the loading rate is increased (Fig. 5d). This series of tests indicates almost conclusively that the flexural strength increases as the rate of loading increases. Figure 6 shows that clear ice has a higher modulus of elasticity at -20°C than at -5°C.

The aging of the ice beams does not seem to affect the results although fresh cut beams gave more reproducible results than beams cut and stored for a month or more.
a. Avg loading rate 0.012 in/min.

b. Avg loading rate 0.029 in/min.

c. Loading rate 0.063 in/min.

d. Comparison of curves.

Figure 3. Applied load vs center deflection at different rates of loading, bottom beams: Tested at -5C.

Figure 4. Stress-strain curves for beams of different crystal size.
FLEXURAL STRENGTH OF CLEAR LAKE ICE

Figure 5. Applied load vs center deflection at different rates of loading. Tested at -20°C.

Figure 6. Stress-strain curves at different temperatures, top beams.
DISCUSSION OF RESULTS

Most materials are both elastic and plastic depending upon the stress range. In the elastic range the material deforms in direct relation to the stress applied; in the plastic range, the material continues to deform without additional stress and will not return to its original shape when the stress is removed. The stress distribution in a plastic material is uncertain. A repeated load test proves that ice has both elastic and plastic properties (Fig. 7), but for this preliminary analysis the ice was assumed to be perfectly elastic and the results are based on this assumption.

The stress-strain relationship in bending cannot be determined without some arrangement for measuring the strain throughout the section. For this purpose the extreme fibers on the top and bottom of the beam were assumed to be equal in tension and compression and the neutral plane is therefore assumed to be located at one-half the depth.

The cross section of each beam is 3 in. wide by 4 in. deep. The moment of inertia is then

\[ I = \frac{bd^3}{12} = 16. \]

The modulus of elasticity \( E \) was determined from

\[ \Delta = \frac{PL^2}{36EI} \left\{ \frac{2L}{9} + \frac{5L}{12} \right\} \]

where \( \Delta \) is the deflection at the center of the beam. For beam dimensions,

\[ E = 11.8P/\Delta. \]

Using this equation, the modulus of elasticity for each group of tests was determined (Table I). These values do not take support settlement into account.

The dead load moment was not used in determining the modulus of elasticity because the deformation due to dead load is present before the gages are rested on the beam. For determining the unit stress, however, the dead load was included. With a dead load

\[ w = 0.397 \text{ lb/in.} \]

The dead load moment becomes

\[ M = wL^2/8 = 24.0 \text{ in-lb.} \]

The unit stress

\[ S = Mc/I = (M + Mc)/I \]

and

\[ I/c = 8. \]

Therefore,

\[ S = 3.00 + 0.458P. \]

This equation was used to calculate the minimum and maximum strength values given in Table I.

The values obtained are compared with values obtained by other investigators in Table II.

Figure 7. Rewloading test on a bottom beam. Loading rate approx 0.014 in/min. Temp -5C.
FLEXURAL STRENGTH OF CLEAR LAKE ICE

Table I. Mechanical properties of fresh-water lake ice in bending. Test beam size: 3" wide, 4" deep and 22" between supports. T = top beam; B = bottom beam.

<table>
<thead>
<tr>
<th>C</th>
<th>Loading rate</th>
<th>Minimum strength</th>
<th>Maximum strength</th>
<th>E, modulus of elasticity</th>
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<tbody>
<tr>
<td></td>
<td>(mm/sec)</td>
<td>(psi) (kg/cm²)</td>
<td>(psi) (kg/cm²)</td>
<td>(psi) (kg/cm²)</td>
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<tr>
<td>-5</td>
<td>0.0051 0.012</td>
<td>173 12.2 388,000</td>
<td>201 14.1 27,300</td>
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</tr>
<tr>
<td>-5</td>
<td>0.012 0.029</td>
<td>116 8.16 339,000</td>
<td>150 10.6 23,800</td>
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<tr>
<td>-5</td>
<td>0.027 0.063</td>
<td>171 12.0 373,000</td>
<td>190 13.3 26,200</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>0.0023 0.0055</td>
<td>165 11.6 223,000</td>
<td>169 11.9 15,700</td>
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<tr>
<td>-5</td>
<td>0.0051 0.012</td>
<td>145 10.2 273,000</td>
<td>225 15.8 19,200</td>
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</tr>
<tr>
<td>-5</td>
<td>0.93 2.2</td>
<td>129 9.70 487,000</td>
<td>157 11.0 34,300</td>
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</tr>
<tr>
<td>-20</td>
<td>0.0080 0.019</td>
<td>147 10.3 412,000</td>
<td>186 13.2 29,000</td>
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<tr>
<td>-20</td>
<td>0.0093 0.022</td>
<td>175 12.3 589,000</td>
<td>207 14.6 41,400</td>
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<tr>
<td>-5</td>
<td>1.7 4.0</td>
<td>189 13.3 912,000</td>
<td>243 17.1 64,100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reloading test</td>
<td>-</td>
<td>-</td>
<td>184 12.9 398,000</td>
</tr>
</tbody>
</table>

Table II. Comparison of strength values for fresh-water lake ice in bending.

<table>
<thead>
<tr>
<th>Investigator</th>
<th>No. of beams tested</th>
<th>Temp</th>
<th>Ultimate strength (psi)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>(F)</td>
<td>Min</td>
</tr>
<tr>
<td>Brown*</td>
<td>12</td>
<td>29</td>
<td>126</td>
</tr>
<tr>
<td>Horeth and Wilson**</td>
<td>6</td>
<td>21</td>
<td>151</td>
</tr>
<tr>
<td>Hitch</td>
<td>21</td>
<td>23</td>
<td>116</td>
</tr>
<tr>
<td>Brown*</td>
<td>12</td>
<td>15</td>
<td>177</td>
</tr>
<tr>
<td>Hitch</td>
<td>12</td>
<td>-4</td>
<td>147</td>
</tr>
</tbody>
</table>

CONCLUSIONS

Relatively consistent values for the modulus of elasticity were found under static conditions, with variable values for the ultimate strength. Factors affecting these values are crystal size, temperature, and rate of loading.

In general the results show that lowering the temperature increases the modulus of elasticity and strength. Ice with larger crystals will have a higher modulus of elasticity with little change in strength. Increased loading rates increase the modulus of elasticity but has little effect on the strength except at low temperatures where the faster rate of loading seems to increase the strength.

Laboratory technique is very important and precaution must be taken to ensure that each beam support surface is parallel and smooth.

* Tests for the St. Lawrence Seaway project on ice cut from the St. Lawrence River (1926).

** Tests for the United States Coast Guard on ice cut from northern Lake Michigan. Ice was tested aboard the Coast Guard Cutter Makinaw (1948).
REFERENCES


