ISUA, GREENLAND: CALCULATIONS OF GLACIER FLOW FOR AN OPEN-PIT MINE

Samuel C. Colbeck

July 1973

CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.
PREFACE

This report describes the calculations of ice flow into a proposed open-pit development at the edge of the Greenland Ice Cap. The work was done by Dr. S.C. Colbeck, Research Geophysicist, of the Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory (USACRREL).

The work was supported by the Marcona Corporation as part of a continuing program of glaciological studies associated with the feasibility of developing this mineral resource.

The author gratefully acknowledges the interest and support of Dr. C.C. Langway, Jr., Chief of the Snow and Ice Branch. Special recognition is given to Mr. Steven J. Mock for his involvement in this study. Mr. B. Lyle Hansen and Mr. Mock made arrangements with the Marcona Corporation to support this study and then reviewed this manuscript, making many useful suggestions for improvement.

The contents of this report are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such commercial products.
CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ii</td>
</tr>
<tr>
<td>Symbols</td>
<td>v</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Physical setting</td>
<td>1</td>
</tr>
<tr>
<td>Flow model</td>
<td>5</td>
</tr>
<tr>
<td>Calculated flow</td>
<td>9</td>
</tr>
<tr>
<td>Existing profile 4</td>
<td>9</td>
</tr>
<tr>
<td>Proposed profiles</td>
<td></td>
</tr>
<tr>
<td>Profile 1</td>
<td>11</td>
</tr>
<tr>
<td>Profile 2</td>
<td>11</td>
</tr>
<tr>
<td>Profile 3</td>
<td>11</td>
</tr>
<tr>
<td>Profile 4</td>
<td>15</td>
</tr>
<tr>
<td>Profile 5</td>
<td>15</td>
</tr>
<tr>
<td>Profile 6</td>
<td>15</td>
</tr>
<tr>
<td>Profile 7</td>
<td>15</td>
</tr>
<tr>
<td>Total excavation</td>
<td>18</td>
</tr>
<tr>
<td>Interpretation and conclusions</td>
<td>18</td>
</tr>
<tr>
<td>Sources of error</td>
<td>19</td>
</tr>
<tr>
<td>Recommendations</td>
<td>21</td>
</tr>
<tr>
<td>Literature cited</td>
<td>21</td>
</tr>
<tr>
<td>Appendix A</td>
<td>23</td>
</tr>
<tr>
<td>Abstract</td>
<td>25</td>
</tr>
</tbody>
</table>

ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Map of Greenland showing location of Isua</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Location of the ore deposit at the edge of the ice cap</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Directions of surface flow in the area of the ore body</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Base map showing surface contours, bedrock contours and locations of the</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>cross sections</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Cross-sectional view of the Greenland Ice Cap upstream from Isua</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>Idealized view of the proposed glacier profiles</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>Calculated strain rate, velocity and ablation for the existing profile at</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>cross section 4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Calculated strain rate, velocity and ablation for the proposed profile at</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>cross section 1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Calculated strain rate, velocity and ablation for the proposed profile at</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>cross section 2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Calculated strain rate, velocity and ablation for the proposed profile at</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>cross section 3</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Calculated strain rate, velocity and ablation for the proposed profile at</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>cross section 4</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Calculated strain rate, velocity and ablation for the proposed profile at</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>cross section 5</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Calculated strain rate, velocity and ablation for the proposed profile at</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>cross section 6</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Calculated strain rate, velocity and ablation for the proposed profile at</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>cross section 7</td>
<td></td>
</tr>
</tbody>
</table>
### CONTENTS (Cont'd.)

**TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Input data and results</td>
<td>10</td>
</tr>
</tbody>
</table>
SYMBOLS

\( a \)  
year

\( \dot{a} \)  
ablation plus excavation (m a\(^{-1}\))

\( B \)  
temperature dependent constant in flow law (bar a\(^{1/n}\))

\( \dot{\varepsilon}_x \)  
longitudinal-strain rate, \( \partial u / \partial x \) (a\(^{-1}\))

\( \dot{\varepsilon}_L \)  
longitudinal-strain rate at \( x = L \) (a\(^{-1}\))

\( g \)  
acceleration due to gravity (9.8 m s\(^{-2}\))

\( h \)  
dimension of glacier (m)

\( h_L \)  
value of \( h \) at \( x = L \) (m)

\( L \)  
reference position on \( x \) axis (m)

\( m \)  
profile exponent

\( n \)  
stress dependent exponent in flow law

\( q \)  
volume flux of ice per unit width (m\(^2\) a\(^{-1}\))

\( t \)  
time (s)

\( u \)  
flow rate averaged over thickness (m a\(^{-1}\))

\( u_L \)  
flow rate at \( x = L \) (m a\(^{-1}\))

\( x \)  
coordinate axis along bedrock (m)

\( z \)  
coordinate axis perpendicular to bedrock (m)

\( Z \)  
surface value of \( z \) (m)

\( Z_L \)  
value of \( Z \) at \( x = L \) (m)

\( \beta \)  
angle of inclination of bedrock (rad)

\( \rho \)  
density of glacier ice (kg m\(^{-3}\))

\( \sigma_x \)  
normal stress in \( x \) direction (bar)

\( \sigma_{xz} \)  
shear stress in \( x - z \) plane (bar)

\( \sigma_z \)  
normal stress in \( z \) direction (bar)

\( \bar{\sigma} \)  
averaged value of stress (bar)

\( \sigma' \)  
deviatoric stress (bar)
ISUA, GREENLAND: CALCULATIONS OF GLACIER FLOW
FOR AN OPEN-PIT MINE
by
S.C. Colbeck

INTRODUCTION

A large mineral deposit has been discovered at Isua, in southwest Greenland (see Fig. 1). In addition to the usual problems of mining in the subarctic regions, the exploitation of this mineral resource is complicated by the fact that it lies at the edge of the Greenland Ice Cap. Most of the ore is covered by ice (up to 150 m thick) and, as the ice is stripped away to expose the ore, additional ice flow into the area of the pit is expected.

This report presents the results of a study sponsored by the Marcona Corporation to calculate the most favorable profile for the ice surface next to the open pit. Seven cross sections of flow were considered and in each case the flow characteristics of the most favorable profile were calculated. The results of these cross sections were averaged around the upstream edge of the pit in order to calculate the excavation necessary to establish a favorable profile and the yearly excavation necessary to maintain that profile. For the purposes of this study it was assumed that the ore will be completely removed down to the elevation of 885 m and that a 60° hanging wall will be established around the perimeter of the ore at that elevation.

PHYSICAL SETTING

The location of the ore deposit at the edge of the ice cap is shown in Figure 2. The deposit is crescent shaped, about 2 km long and 250 m to 450 m wide, with its long axis oriented approximately north-south. Glacial erosion in this area has left a ridge of rock standing 60 m above the ice surface; hence about 15% of the surface area of the ore is exposed above the ice. The existence of this ridge is very important in the present discussion of ice flow in that it controls the character of the flow in the vicinity of the ore body.

The general pattern of flow in this region (as determined from the surface contours) is downslope from the east-northeast. Ice flow is diverted to the north and south at Isua, causing decreased flow at the ore body and increased flow to the north and south. On the north side of the ore body the ice flows downslope to a lake at the edge of the glacier. The highest flow rates occur somewhat north of the ore body, but flow rates over the northern tip of the ore body itself are substantial (16 m a⁻¹; see Fig. 3 and cross sections 1 and 2; the location of each cross section is shown in Fig. 4).

The flow into the center of the ore body is small due to the blocking and diversion caused by the rock ridge. The glacier surface is nearly flat (see cross sections 2 and 4, Fig. 4), and the small flow is balanced by surface ablation. Ice flows over the southern section of the ore body and down the other side (see cross sections 5, 6 and 7). Flow rates are somewhat higher than in the
ISUA, GREENLAND: CALCULATIONS OF GLACIER FLOW FOR AN OPEN-PIT MINE

Figure 1. Map of Greenland showing location of Isua.

Figure 2. Location of the ore deposit at the edge of the ice cap. Regional velocities are shown.
Figure 3. Directions of surface flow in the area of the ore body.
center of the ore body but significantly less than on the northern tip. The presence of the ore body and ridge is indicated by an abrupt change in surface slope of the ice as it flows over the ridge (see Fig. 4). This abrupt change in surface slope continues for a short distance to the south of the deposit, suggesting that the ridge also continues for some distance to the south. About 1 km south of the deposit, a small discharge glacier drains the ice cap with flow rates (velocities) which are probably as high as those on the north.

The ridge at Isua is a major factor in the local pattern of flow, and once the ice surface is lowered and the ridge is partially removed, its blocking effect will be decreased and an increased amount of direct flow into the area is expected. At both ends, flow into the proposed pit will occur from three directions – upslope, downslope and laterally. Flow from the downhill side, however, will be insignificant compared with that from the upstream and lateral directions and no analysis of this upslope movement is made.

In Figure 5, which shows the configuration of the Greenland Ice Cap upstream from Isua, note that uphill flow occurs from the center of the ice cap. The average bedrock slope is only 0.36% over the last 150 km; however, because of the presence of the rock ridge, the bedrock slope at Isua is as much as 50%. Because of the lack of more detailed regional information, the input data for this analysis are estimated from the available data, maps and photographs. The necessity of collecting more field data is discussed later.
FLOW MODEL

Ice flow into the proposed pit will be uphill on all sides of the pit—a very favorable situation. Unfortunately, no study of glacier flow under similar circumstances has been made but the principles used in the analysis of conventional flow apply. As usual, the flow will be determined by the surface slope and thickness of the ice.

The two-dimensional profile proposed for the seven cross sections is idealized in Figure 6. The individual profiles are given in the Appendix. The approximation of a planar bed is not accurate for all of the cross sections although it is a good approximation for most. For convenience during open-pit development, a simple relationship between $h$ and $x$ must be assumed. Accordingly, we take the parabolic profile

$$\left(\frac{h}{h_L}\right)^n = \frac{x}{L}. \quad (1)$$

Although the parabolic profile is a well known glacial feature, this study is novel in that the profile will be imposed on the glacier by excavation.

The flow along each cross section is considered to be two-dimensional flow as described by Budd (1969, p. 112). For significant values of $\beta$, the equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} + \rho g \sin \beta = 0 \quad (2)$$

and

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} - \rho g \cos \beta = 0. \quad (3)$$
Since the longitudinal stress gradients must be retained in these equations, the simplified case of laminar flow (Nye 1952) cannot be adopted. However, we can treat these equations by averaging the stresses over the thickness of the glacier and then integrating the equations in such a way that they can be solved for the average deviatoric stress in the longitudinal direction. This allows us to solve for this stress in terms of the glacier profile and, since the deviatoric stress is related to the longitudinal strain rate, ultimately we can relate the geometry of the glacier to its flow field. The stresses are averaged over the thickness of the glacier, or

\[
\bar{\sigma}_{ij} = \frac{1}{Z} \int_{0}^{Z} \sigma_{ij} \, dz. \tag{4}
\]

Equation 2 is averaged in this manner and, where the shear stress at the surface \( \sigma_{xz}(x, Z) \) is zero

\[
\frac{\partial}{\partial x} \left( Z \, \bar{\sigma}_x \right) \right|_{0}^{x} = -\rho g \, Z \sin \beta + \sigma_{xz}(x, 0). \tag{5}
\]

By integrating from the terminus to any position \( x \)

\[
Z \bar{\sigma}_x = -\rho g \, \sin \beta \int_{0}^{x} Z \, dx + \int_{0}^{x} \sigma_{xz}(x, 0) \, dx. \tag{6}
\]

The stresses in eq 3 are averaged in a similar manner and then, integrating from any position \( z \) to the surface where \( \sigma_z(x, Z) \) is zero

\[
Z \bar{\sigma}_z = -0.5 \rho g \, Z^2 \cos \beta - \frac{\partial}{\partial x} \int_{0}^{z} \int_{Z}^{z} \sigma_{xz} \, dz \, dx. \tag{7}
\]

In two-dimensional flow we take the component of the deviatoric stress in the y-direction \( \sigma_y^r \) as zero, whence it follows that the average value of the deviatoric stress in the x-direction is related to the averaged values of the normal stresses by

\[
2 \bar{\sigma}_x^r = \bar{\sigma}_x - \bar{\sigma}_z. \tag{8}
\]
Combining eq 6, 7 and 8 gives an equation describing $\sigma_x^*$

$$2Z\sigma_x^* = -\rho g \sin \beta \int_0^x Z dx + \int_0^x \sigma_{xz} (x, 0) dx +$$

$$+ 0.5 \rho g \cos \beta Z^2 + \frac{\partial}{\partial x} \int_0^Z \int_0^Z \sigma_{xz} dz dz. \quad (9)$$

The use of this technique for resolving the equations has precluded their solution in terms of the internal deformations as functions of position. However, for our purposes in this applied problem, most of these deformations are of little interest and only the average motion in the longitudinal direction is needed to calculate the flow of ice toward the pit. The terms in this equation do have physical significance in terms of the forces operating on the ice mass. The term on the left-hand side is twice the total force (less hydrostatic pressure) acting across a vertical line. The first term on the right-hand side is the wedging stress due to uphill movement toward the pit, the second term represents the basal drag due to the shear stress at the bed, the third term represents the stress generated by the downhill surface slope, and the fourth term describes the longitudinal variations in the flow.

From Glen's (1955) experimental results, we adapt the flow law for ice in the form

$$\dot{e}_x = (\sigma_x^*/B)^n \quad (10)$$

where $\dot{e}_x$ is the longitudinal strain rate, $B$ is a temperature-sensitive coefficient and the exponent $n$ increases with the octahedral shear stress.

An expedient method of solution of eq 9 is adapted since we cannot measure the shear stress at the bed of the glacier. It is assumed that the shear stress at the bed is independent of position. There is considerable justification for adopting this assumption since calculated values of basal shear stress are invariably within 50% of 1 bar (Kamb 1964). This probably occurs because ice has an "effective yield stress" of 1 bar and, in fact, glacier ice has been modeled as a plastic material (Nye 1951). If the mode of flow at Isua differs from the normal flow where most of the deformation occurs near the bed, this assumption could be invalid. It might be expected that in flow up steeply inclined bedrock an area of "dead material" would exist over which the upper sections of the glacier would move. At Isua, however, the observed deformation of drill holes shows that the majority of the internal deformation occurs near the bed (Morey, personal communication, 1972).

Using this assumption the basal shear stress at $L$ has been derived and, upon substituting this expression into eq 9, it is found that

$$2Z\sigma_x^* = -\rho g \sin \beta \int_0^x Z dx + 2\sigma_{x,L}^* Z_L x L^{-1} + \rho g \sin \beta L^{-1} x \int_0^L \int_0^Z \sigma_{xz} dz dz -$$

$$- 0.5 \rho g \cos \beta Z^2 x L^{-1} + 0.5 \rho g \cos \beta Z^2 -$$

$$- x L^{-1} \frac{\partial}{\partial x} \int_0^Z \int_0^Z \sigma_{xz} dz dz \bigg|_L + \frac{\partial}{\partial x} \int_0^Z \int_0^Z \sigma_{xz} dz dz. \quad (11)$$
Budd (1969, p. 117) showed that the longitudinal variations are small over large distances but may be important when the horizontal scale is about equal to the thickness of the ice. These terms are neglected here although the computed flow at the terminus may be affected. The possible errors resulting from neglecting this term are later shown to be small.

The total thickness of ice is approximately given by

$$Z = h_L \sec \beta \left(\frac{x}{L}\right)^{1/m} + x \tan \beta . \hspace{1cm} (12)$$

By combining eq 10, 11 and 12

$$\dot{e}_x^{1/m} = 0.5 \ B^{-1} \ \rho g \ (h_L L^{-1/m} \ x^{1/m-1} + \sin \beta)^{-1}$$

$$\sin \beta \left[ h_L L^{-1/m} \ m(m+1)^{-1} \ (L^{1/m} - x^{1/m}) + 0.5 \ \sin \beta (L - x) \right] +$$

$$+ 0.5 \ x \ (h_L L^{-1/m} \ x^{1/m-1} + \sin \beta)^2 + \left[ 2 \rho^{-1} \ g^{-1} \ \bar{\sigma}_x \ (x, L) \ Z_L L^{-1} \right.$$  

$$- 0.5 \ \cos \beta \ Z_L^2 L^{-1} \] \cos \beta \} . \hspace{1cm} (13)$$

As a check on the validity of eq 13, the case of laminar flow over a level bed ($\bar{\sigma}_x = \beta = 0$) is examined. The solution of eq 13 gives $m = 2$, the classical profile for laminar flow (Nye 1952). Therefore, the model is consistent with known properties of glaciers in the simplest case.

When values of longitudinal velocity $u$ and stress $\bar{\sigma}_x$ can be assumed for the reference position $L$, the values of $\dot{e}_x$ and therefore of $u$ can be calculated for the rest of the profile. In this way eq 13 is used to construct the entire flow field over the area of excavation. The accuracy of the calculations is limited by the input information at the reference position $L$ and this information in turn is limited by the lack of knowledge about upstream flow — a situation which must be corrected by collecting more data. Note that if the chosen values at the reference position are grossly in error, the computed values of $\dot{e}_x$ immediately change to be compatible with the chosen profile. Once satisfactory values have been found, an internally consistent model of flow is obtained.

Another constraint can be placed on the chosen values of $\dot{e}_x$ and $u$ by noting that the matter is neither created nor destroyed by the process of flow. The equation expressing this concept is

$$\dot{a} = \frac{\partial q}{\partial x} + \frac{\partial Z}{\partial t} \hspace{1cm} (14)$$

where the total ice flux $q$ past any position $x$ is

$$q = uZ . \hspace{1cm} (15)$$

At Isua we are imposing the profile on the glacier; hence no variations in thickness can occur and all time-dependent derivatives disappear. Equation 14 reduces to

$$\dot{a} = \dot{e}_x Z + u \ \frac{\partial Z}{\partial x} . \hspace{1cm} (16)$$

It is necessary to assume that the ice thickness at the reference position $L$ remains undisturbed by the excavation of the pit (when more upstream data are available the reference position may have to be relocated). An ablation of 1 m $a^{-1}$ has been measured in this area (Morey, personal communication, 1972), whence at the reference position $L$
\[-1 = \dot{e}_L Z_L + u_L \frac{\partial Z}{\partial x} \bigg|_L. \tag{17}\]

For any assumed profile, we know \(h_L\) and \(\partial Z/\partial x\big|_L\) and the choice of values of \(\dot{e}_L\) and \(u_L\) must satisfy eq 17. With these limitations on the choice of initial values, the assumed values cannot be too wrong unless substantial surface lowering occurs at the reference position.

**CALCULATED FLOW**

The seven profiles were chosen such that each profile lies along an expected direction of principal flow; this is required in order to use the approximation of two-dimensional flow. Each profile is representative of the flow for some area along the margin of the proposed pit and, when the profiles are joined together, they represent the entire flow around the pit.

For each cross section a reference position \(L\) is generally chosen as far back from the pit as information is available. In some cases a larger distance is desirable but the necessary input data are lacking and, as a practical constraint, only a certain distance can be economically excavated. Then a value of \(m\) is chosen which determines the profile of the ice surface. Next a value of velocity is chosen using whatever information is available. Generally, velocities as high as 15 m a\(^{-1}\) occur at the northern end and in the thick center section of the ore body. Next \(\dot{e}_L\) is calculated from eq 17 and a check on the internal consistency is made; the values of \(\dot{e}_x\) computed downstream from \(L\) must not suddenly change. In cases where these values do undergo a rapid change, the estimate of \(\dot{e}_L\) is faulty and another attempt is made to calculate reasonable values.

With a satisfactory choice of \(\dot{e}_L\) and \(u_L\), \(\ddot{a}\) is calculated along the profile. These values represent the amount of material which must be removed by a combination of ablation and excavation in order to maintain the proposed profile. Values of \(\ddot{a}\) less than \(-1\) m a\(^{-1}\) indicate the amount of yearly ice excavation required to maintain the desired profile. If this excavation is not done, the glacier will thicken and then advance into the open pit.

**Existing Profile 4**

Prior to analyzing the proposed profiles, the existing configuration at cross section 4 is analyzed as a check on the validity of the model. A coordinate system with the origin at the existing ice margin and the \(x^*\)-axis corresponding to the approximately planar bedrock surface is established (see cross section 4, App.). Measured ice velocities (2 m a\(^{-1}\)) and strain rates (\(-0\) a\(^{-1}\)) along this cross section satisfy the constraints of eq 17, thus verifying the measured ablation rate of 1 m a\(^{-1}\).

Recent temperature measurements show \(-2^\circ\)C at a depth of 100 m and indicate that temperatures at a 200-m depth should approach \(0^\circ\). Accordingly, the value of \(-1^\circ\)C is used for determining \(B\). The overall level of effective stress is estimated for the reference position and then appropriate values of \(B\) and \(n\) are determined from Budd’s (1969, p. 23) compilation of laboratory and field measurements. The values used are given in Table I. A further assumption is necessary; namely, that the ice is randomly oriented and polycrystalline. The implications of these assumptions are discussed later.

The calculated strain rate, velocity and ablation rate for existing profile 4 are shown in Figure 7. The calculated values of strain rate are about zero to within a few meters of the margin. Also, the calculated value of velocity is \(-2\) m a\(^{-1}\) over the entire profile except at the margin.
Table I. Input data and results.

<table>
<thead>
<tr>
<th>Profile no.</th>
<th>B (1/a) (bar)</th>
<th>(\beta_0) (rad)</th>
<th>(\alpha_L) (bar)</th>
<th>L (m)</th>
<th>Z_L (m)</th>
<th>n (m/a(^{-1}))</th>
<th>0 (\int_0^L \hat{a} dx) (m(^2)/a(^{-1}))</th>
<th>(u_L Z_L) (m(^3)/m)</th>
<th>Initial excavation (m(^3)/m)</th>
<th>Width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing no. 4</td>
<td>1.88</td>
<td>0.436</td>
<td>0</td>
<td>∞</td>
<td>700</td>
<td>325</td>
<td>2</td>
<td>-2</td>
<td>651</td>
<td>650</td>
</tr>
<tr>
<td>1</td>
<td>1.88</td>
<td>0.151</td>
<td>0.163</td>
<td>2.5</td>
<td>500</td>
<td>180</td>
<td>2</td>
<td>-10</td>
<td>2,201</td>
<td>1,800</td>
</tr>
<tr>
<td>2</td>
<td>1.88</td>
<td>0.18</td>
<td>0</td>
<td>2.4</td>
<td>225</td>
<td>225</td>
<td>2</td>
<td>-5</td>
<td>1,331</td>
<td>1,125</td>
</tr>
<tr>
<td>3</td>
<td>1.69</td>
<td>0.211</td>
<td>0.64</td>
<td>2.8</td>
<td>350</td>
<td>195</td>
<td>4</td>
<td>-15</td>
<td>3,633</td>
<td>2,925</td>
</tr>
<tr>
<td>4</td>
<td>1.77</td>
<td>0.242</td>
<td>0.39</td>
<td>2.7</td>
<td>700</td>
<td>360</td>
<td>2</td>
<td>-15</td>
<td>7,034</td>
<td>5,400</td>
</tr>
<tr>
<td>5</td>
<td>1.77</td>
<td>0.234</td>
<td>0.419</td>
<td>2.5</td>
<td>600</td>
<td>248</td>
<td>3</td>
<td>-15</td>
<td>6,813</td>
<td>5,370</td>
</tr>
<tr>
<td>6</td>
<td>1.88</td>
<td>0.265</td>
<td>0.109</td>
<td>2.5</td>
<td>700</td>
<td>365</td>
<td>2</td>
<td>-6</td>
<td>2,750</td>
<td>2,190</td>
</tr>
<tr>
<td>7</td>
<td>1.88</td>
<td>0.01</td>
<td>0</td>
<td>2</td>
<td>500</td>
<td>135</td>
<td>2</td>
<td>-10</td>
<td>1,463</td>
<td>1,350</td>
</tr>
</tbody>
</table>

Figure 7. Calculated strain rate, velocity and ablation for the existing profile at cross section 4.
This result agrees well with the measured values of velocity over this area. The calculated ablation rates are also constant at \(-0.93\) m a\(^{-1}\), a result which closely agrees with the measured value of about \(-1\) m a\(^{-1}\) and is encouraging since the mass balance should be more or less constant over this area. The successful calculation of the ablation rates along this existing profile increases our faith in this procedure for predicting the behavior around the proposed pit. The analysis fails within a few meters of the margin, indicating that some caution must be used in interpreting the results at the margin.

**Proposed Profiles**

The analysis of each profile is discussed separately below. For all of the proposed profiles, the input data, the required ablation each year, and volume of excavation required to establish the initial profile are recorded in Table I. The volume of excavation calculated here is in addition to that required to excavate a 60° pit wall.

**Profile 1**

The reference location is taken as 500 m for the convenience of excavation, although, for this profile only, more topographic information is available. The flow rate \(u\) at 500 m is taken as 10 m a\(^{-1}\) due to the high flow rates on the northern end, and the longitudinal stress deviator \(\sigma_0\) as 0.163. The exponent \(n\) is taken as 2 because of the low level of stress. The expected mode of flow is then extending (i.e., tensile stresses in flow direction) flow over the entire profile with a maximum extensional-strain rate of \(0.0138\) a\(^{-1}\) at 37 m from the terminus (see Fig. 8). This strain rate is sufficiently large to cause crevasse formation since, as measured by Meier (1958), crevasse formation occurs when the principal strain rates exceed \(0.01\) a\(^{-1}\). Some serac (seracs are large blocks of ice which tend to rotate and collapse) formation might be expected near the margin but should not be a problem. If anything, the formation of crevasses may be favorable since less blasting will be required. The calculated ablation rate \(a\) required to maintain the proposed profile increases toward the margin where it reaches \(-53\) m a\(^{-1}\). Most of the required excavation will occur within the zone of crevasse formation.

**Profile 2**

The flow rates at the reference position (625 m) are assumed to be low (5 m a\(^{-1}\)) because this area is protected by the rock ridge which runs east-southeast from this position and partially blocks the regional flow from the east-northeast. The longitudinal strain rate is assumed to be zero at 625 m. This is confirmed by the calculated values which decrease to \(-6.4 \cdot 10^{-6}\) at 487 m and then increase rapidly near the margin. The extending strain rates never exceed \(0.01\) a\(^{-1}\), however, so no crevassing is expected. The calculated values of \(\dot{a}\), which are most negative around the perimeter of the proposed pit, are shown in Figure 9.

**Profile 3**

The analysis of this profile is difficult because, at present, no lateral flow occurs in the direction of the proposed pit, but clearly much flow will occur when the excavation is completed. The flow rates at the reference position (350 m) are presently about 25 m a\(^{-1}\). The flow rates into the proposed pit will not be that high because the ice surface will not be steeply inclined toward the pit and flow toward the pit will be uphill flow (see cross section 3, App.). Accordingly, a flow rate of 15 m a\(^{-1}\) is assumed. Because of the large flow rates in two directions, a large value of \(n\) is used.
Figure 8. Calculated strain rate, velocity and ablation for the proposed profile at cross section 1.

Figure 9. Calculated strain rate, velocity and ablation for the proposed profile at cross section 2.
The suggested profile is described later. It is instructive at this point to consider the effect of various profiles. For this purpose, the results for three values of \( m \) are shown in Figure 10. For higher values of \( m \), the profile of the ice surface is steeper toward the margin and the ice is generally thicker. The calculated results clearly show that flow velocities near the margin are greater for higher values of \( m \) and, accordingly, the calculated values of \( \dot{a} \) are greater. Therefore, there is a tradeoff between the initial excavation necessary to achieve a profile and the yearly excavation required to maintain the profile. For higher values of \( m \), less initial excavation is necessary but the yearly requirement increases sharply. For example, the difference between the initial excavations for the profiles shown in Figures 8 and 10a is only about 1200 \( \text{m}^3 \) per meter of margin but the difference between the yearly excavations is 890 \( \text{m}^3 \) per meter of margin. Also, for thicker profiles, smaller strain rates (and possibly flow rates) are expected at the reference position but large flow rates occur near the margin. For \( m = 3 \) (Fig. 10c), for example, strain rates increase to 0.032 \( \text{a}^{-1} \) at 86 m, which indicates that very large crevasses should occur, and, since the surface profile is quite steep near the margin, seracs would be likely to break off and avalanche into the open pit. For \( m = 2.5 \) (Fig. 10a) much lower strain rates are calculated near the margin but this model cannot be considered to be internally consistent since the value of \( \dot{e}_x \) changes very rapidly near the reference position.

The suggested profile with \( m = 2.8 \) (Fig. 10b) shows a more stable configuration where the longitudinal strain rate increases slowly to a maximum value of 0.022 \( \text{a}^{-1} \) at 153 m. Strain rates are large throughout this area and crevassing should be a problem everywhere. It is possible that the northern 100 or 200 m of the ore will have to be left in place because of the rapid ice flow over this area. This represents a very small part of the ore body, however, and does not constitute a large sacrifice.

Figure 10. Calculated strain rate, velocity and ablation for three profiles at cross section 3.
Figure 10 (Cont'd). Calculated strain rate, velocity and ablation for three profiles at cross section 3.
This discussion of the different flow regimes associated with different values of \( m \) illustrates the large changes which occur in the flow regime when small changes are made in surface slope and thickness. Most importantly, the steeper the surface slope near the edge of the pit the better the chances of extensive crevassing and serac formation. Because of the hazards involved in working in an 'icefall' region, a steep-sided pit wall must be avoided.

Profile 4

Large flow rates into the center section of the pit along about 800 m of the margin will occur because the edge of the proposed pit reaches its lowest elevation in this area and the proposed profiles have an effective thickness of over 200 m above the elevation of the terminus. Ice velocities at the reference position are assumed to be 15 m a\(^{-1}\) and extensional stresses are large, whence we take \( n = 3 \). The value of \(-1^\circ C\) is used for the temperature, although it is important to note that a linear extrapolation of the temperature profile from the surface suggests that ice temperatures are probably at melting at \( x = 700 \) m. If so, the flow could be increased by an order of magnitude above that calculated here (Colbeck and Evans in press). There is an immediate need for more measurements to determine the temperature at greater depths in the ice.

Once again, large strain rates occur near the margin and much crevassing and serac formation are expected. The values of \( a \) are quite large because of the large flow rates in the steep part of the profile (see Fig. 11).

Profile 5

This profile is very similar to profile 4 and gives similar results (see Fig. 12). Large flow rates, much crevassing and possible formation of seracs are predicted along the 800 m of the margin which are characterized by profiles 4 and 5. Excavation of the ice under these conditions may be impossible and it may be necessary to leave some of the ore along the eastern side of the deposit. By doing this, the effective thickness of the glacier can be reduced and the flow rates reduced accordingly. For this area, where flow velocities and extensional stresses are high, the flow rates decrease as the fourth power of the thickness and a significant reduction in flow could result from a small sacrifice of ore. The suggested sacrifice of ore is shown on Figure 4.

Profile 6

The flow over the southern end of the deposit is not as large as the flow over the northern end because of the aforementioned rock ridge. After the open pit has been excavated, the character of the flow will still be partially controlled by the ridge. Also, the upstream edge of the deposit is some 45 m higher in this area than at its lowest point between profiles 4 and 5. Currently flow rates over this section are 2 to 3 m a\(^{-1}\) over the ore itself and 8 to 14 m a\(^{-1}\) to the south.

The flow rate of 6 m a\(^{-1}\) is assumed for profile 6. The strain rate increases to 0.017 a\(^{-1}\) at 16 m from the margin (see Fig. 13) and some crevassing is expected. Again, if necessary, the flow into the pit can be reduced by leaving a small amount of ore in place on the upstream edge of the pit.

Profile 7

This is the only profile where uphill flow does not occur and the assumed flow rate is increased to 10 m a\(^{-1}\) for that reason. The strain rates are nearly zero (see Fig. 14) over the entire profile but the ablation rate increases near the margin because of the increased surface slope. Fortunately the ice is fairly thin over this area and recovery of the entire ore deposit at this location may be possible.
Figure 11. Calculated strain rate, velocity and ablation for the proposed profile at cross section 4.

Figure 12. Calculated strain rate, velocity and ablation for the proposed profile at cross section 5.
Figure 13. Calculated strain rate, velocity and ablation for the proposed profile at cross section 6.

Figure 14. Calculated strain rate, velocity and ablation for the proposed profile at cross section 7.
Total Excavation

The ablation rates necessary to maintain the proposed profiles are calculated using the model. These are found to be many times greater than the current ablation rates and a program of surface dusting and mechanical excavation is necessary to supplement the summer melt. Surface dusting may increase ablation by as much as 100% (see Slaughter 1969, for a review of albedo modification) but much of the increased volume of flow must be balanced by excavation.

The initial excavation necessary to establish a stable profile beyond the 60° hanging wall and the yearly ablation necessary to maintain that profile are calculated by averaging the seven profiles around the perimeter of the pit. The initial excavation is 66 million m³ and the yearly ablation for the proposed pit is 7.9 million m³. A successful program of surface dusting should remove 3 million m³/year over the excavated profile leaving about 5 million m³ to be removed by direct excavation each year.

There are four further considerations which are important here:

1. The costs of ice removal by surface dusting are small compared with those of direct excavation. Meiman (in press) reports that the most efficient application of carbon black is 90 kg/acre. This application should be made only once per year since the point of diminishing returns occurs when the layer of dusting material is more than one particle thick. Dusting from helicopters may be the most efficient method and the cost of this procedure can be estimated from information supplied by Meiman (personal communication, 1972).

2. Surface dusting should begin as soon as possible over an area of about 10 km². This is the approximate area of influence for flow along the margin of the proposed profile. Two hundred twenty metric tons of carbon black are required per year to cover this area. After the first few years it may be desirable to decrease the application to prevent excess accumulation of the carbon black. About 25 million metric tons of ice could be removed over this area each year and conceivably, by the time the mining operation has reached the 885-m elevation, a sufficient quantity of ice could be removed that ice flow would be only a minor problem.

3. The mining operation will begin at the surface and slowly proceed to lower elevations requiring an increasing interaction with the glacier each year. At first no glacier flow will interfere with mining but the shape of the excavated profile and the flow rates will change with each step. In particular, both the excavation necessary to establish a satisfactory profile and the flow rates will steadily increase. The larger flow rates will occur later in the project after much of the ice has been removed by many years of surface dusting.

4. Some ore should be sacrificed in order to reduce the flow into the open pit. The suggested outline of the limit of mining is shown in Figure 4. The reduced flow rates associated with this sacrifice have not been calculated but should significantly reduce both the total volume of excavation required and the crevassing problem in the center of the ore deposit.

INTERPRETATION AND CONCLUSIONS

The flow of ice into the vicinity of the ore body is controlled by the presence of the high ridge of rock associated with the ore. Once excavation of the open pit is completed, the elevation of the pit wall will be as low as 980 m and the character of flow in this area will be significantly altered. The flow will still be uphill toward the pit but the flow rates will largely be determined by the surface slope of the ice and the height of the ice surface above the terminus. By creating
a parabolic profile for the glacier surface, the flow at the terminus can be greatly reduced; however, because of practical limitations, the surface slope and thickness will still be sufficiently large to give large flow rates, crevassing near the margin, and possible formation of unstable ice blocks. These problems are worst near the center where the pit wall reaches its lowest elevation.

On the northern tip, flow velocities are large at present and will increase during the excavation. The ice cover is thin and uphill flow occurs, so influx will not be as large as might be expected. The existence of a rock ridge running east-southeast from the northern tip will prevent some influx of ice into the northern area.

Currently ice flows uphill and over the ore in the southern area. This uphill flow will continue after excavation but flow into the area will increase because of the increased surface slope and effective glacier thickness. Also, flow will occur into the southern portion of the pit from three directions. Upstream flow will be insignificant, however, and no calculation of it is made. The lateral flow into the pit and downstream flow will both be significant although probably not as large as the flow in the center section or the northern tip.

The flow into the proposed pit is modeled using standard glaciological techniques. The unusual aspects of this problem are twofold: 1) the surface profile is being imposed on the ice mass by excavation and, 2) a combination of uphill flow along the bedrock and downslope flow occur simultaneously. The model is capable of accurately representing this type of flow although the accuracy of the predictions is limited by the estimates of flow velocity at the reference position. To improve this estimate more flow data and topography information should be collected. Other data collection and errors are discussed later.

The most significant conclusions derived from this study are:

1. More field data must be collected to improve the input to the model.
2. A very large amount of ice will have to be removed during the lifetime of this project. One way of significantly reducing that amount would be to leave some of the ore in place. About 66 million m$^3$ of ice will have to be removed in order to establish the proposed profile. An additional 106 million m$^3$ will have to be removed to expose the ore.
3. About 3 million m$^3$ of ice can be removed each year over the excavated profile by modifying the albedo of the surface. An additional 5 million m$^3$ of ice will have to be removed every year by excavation when the mining has reached the 885-m height. Most of this excavation will be near the margin.
4. Even with this amount of excavation, a significant problem may develop because of excessive crevassing near the terminus. When large crevasses exist on a steep slope, individual ice blocks tend to form and an avalanche hazard develops.

**SOURCES OF ERROR**

The most serious source of error is the assumed location and velocity at each of the reference positions. It would be meaningless to place the reference position further upstream where no information is available and hence the selected distances are not extended. However, two possibilities exist:

1. The surface will drop at the reference position.
2. The flow rates will be larger than the assumed values.
More information is needed to determine to what extent the regional flow will be interrupted by the excavation. When more information is available the accuracy of the assumed flow rates can be reevaluated. Until then it must be assumed that the values used here are correct.

The ice is assumed to be isotropic although glacial ice is generally recrystallized into an anisotropic fabric. In the Antarctic ice cap, Cow et al. (1968) found ice with a vertically oriented c-axis. If this situation occurs at Isua, we might expect flow rates as much as two orders of magnitude greater than those calculated here. Fortunately, recent laboratory measurements (Budd, personal communication) show that flow rates are only enhanced by a factor of three by the preferential orientation of glacier-ice samples. Nevertheless, the ice fabrics at Isua should be determined by taking samples at depth. Also, the recrystallization of the ice should be monitored once flow accelerates.

The temperature of the ice is critical in assessing the rheological parameters. The closer the temperature is to 0°C the more critical this parameter becomes. Since most of the deformation occurs near the base of the ice sheet, the basal temperatures control the flow. Measured ice temperatures at Isua are about -4°C near the surface and -2°C at a depth of 100 m. A linear extrapolation shows that the temperature should reach 0°C near a depth of 200 m. Gudmandsen and Christensen (1972) suggest that the basal temperatures along the entire southwest edge of the ice cap are at 0°C except right at the margin where heat conduction is sufficient to reduce the temperatures. If so, flow rates could be significantly larger than those predicted here.

When liquid water exists at the bed, a glacier moves by internal deformation and basal sliding which can constitute as much as 90% of the total surface movement. Also, if the base is at 0°C in this area, internal deformation would be about 10 times as large as that predicted here. More temperature measurements are necessary to decide if a serious problem exists. Fortunately, after excavation has decreased the thickness of the ice sheet, the basal temperatures should decrease because of increased heat conduction to the surface. This benefit could be offset by either increased production of strain energy due to flow or recrystallization as the ice adopts a fabric more suitable for flow.

Other than the uncertain input data, the most serious limitation of this model is the analysis of flow near the margin. The longitudinal variations were neglected in eq 11, producing some error near the margin. These errors are most apparent in the analysis of the existing profile 4 where meaningless results were obtained for the last few meters near the margin. It is possible that the results of all seven of the proposed profiles are in error at the margin where the ablation rates invariably assume their largest values. These errors are not significant, however, because there is only a small difference between the values of ablation calculated by two separate techniques. First, the mass of ice flow past the reference position $L$ is exactly equal to $u_L Z L$. Second, we can integrate the calculated ablation $a$ over the length of the profile ($\int a \cdot dx$). These two values are given for each profile in Table 1. The mass flux into the profile is less than the integrated ablation by 1 to 30% for every profile. This discrepancy is not large (compared with reliability of the input data) and it shows that the model is basically consistent. For the purposes of calculating the total ablation necessary to maintain the desired profile, the total flux past the reference position is used.

Neglecting the errors in the input data, the results of calculations of glacier flow are generally accurate to within 50% (Budd, personal communication, 1972). The results given here should be considered within that range until more information is obtained about the current situation at Isua.
RECOMMENDATIONS

Five specific recommendations are made to the Marcona Corporation in order to improve upon this study:

1. Ice temperatures should be measured to bedrock at several locations upstream along cross section 5. If temperatures above -0.1°C are measured anywhere, the measurements should be extended 1 km upstream from that point (or until the 0°C isotherm is found). The pressure in the subglacial water should also be measured.

2. Ice cores should be taken to bedrock in the same boreholes. A petrofabric analysis should be made of these cores to decide whether a correction must be made to the present calculations to allow for anisotropy in the rheological properties of the ice.

3. The array of surface markers should be extended to measure the regional flow as much as 5 km upstream from the ore deposit. The system of surface markers around the deposit should be extended 1 km to the north and south.

4. Surface and bedrock topography should be mapped as far upstream as possible.

5. Experiments with surface dusting should be done. If ablation rates are significantly increased, a large-scale program of dusting should be begun immediately.

LITERATURE CITED

Budd, W.F. (1969) The dynamics of ice mass. ANARE Scientific Reports, Series A (IV) Glaciology, Publication no. 108, Antarctic Division, Department of Supply, Melbourne, Australia.


Gudmandsen, P. and E. Lintz Christensen (1972) Radioglaciology. Laboratory of Electromagnetic Theory, Technical University of Denmark.


APPENDIX A: CROSS SECTIONS

1. Existing Ice Surface
   - Proposed Pit
   - Proposed Ice Surface
   - Rock
   - Ice
   - 180m

2. Ice
   - Rock
   - 225m

3. Ice
   - Rock
   - 195m

4. Ice
   - Rock
   - 325m
   - 360m
The Marcona Corporation and Kryolitselskabet Øresund, A/S (a Danish corporation) are cooperatively investigating the possibility of developing an open-pit mine along the edge of the Greenland Ice Cap. The response of the glacier to a sudden change in surface slope and thickness is calculated. The existing flow is diverted away from the mineral deposit but will increase when the excavation begins. It is calculated that 66 million cubic meters of ice must be removed in order to establish a stable profile beyond the pit. An additional 7.9 million cubic meters of ice must be removed yearly in order to maintain the profile.