THE EFFECT OF A LOW VISCOSITY LAYER ON CONVECTION IN THE MANTLE

by

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PREFACE

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SUMMARY

It appears reasonable to expect that, if thermal convection occurs in the Earth's mantle, it also may occur within the Moon and Mars. The dimensions of these latter two bodies are comparable to the thickness of the Earth's mantle. Presumably the amount of radioactive heat generated per unit mass is similar in all three bodies. Yet the surface morphology of the Earth, which many scientists believe arises ultimately from mantle convection, differs markedly from that of the Moon or Mars. The explanation advanced here for this difference is based on the effect produced on convection in the mantle by the presence of a low "viscosity" or low creep strength layer. It is assumed that the low velocity layer of the mantle is such a low creep strength layer. A low viscosity layer changes the amount of "coupling" between the outer crust and mantle convection. (The crust is "coupled" if mantle convection produces stresses in the crust which are large enough to deform it plastically. The crust is "decoupled" from the mantle if these stresses are insufficient to produce plastic deformation.) The theory assumes that the viscosity or creep strength is essentially zero in the low viscosity layer. The analysis is similar to that developed earlier for the calculation of stresses within the mantle. We find that the deeper the low viscosity layer lies within the mantle the greater is the coupling of the outer crust to the mantle convection currents. If the low creep strength layer lies close to the surface the outer crust is decoupled from the interior. According to the literature the depth of the low velocity layer is determined by the temperature and pressure profiles within the mantle.
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INTRODUCTION

In this paper we attempt to find a solution to the following problem. It appears reasonable to expect that, if thermal convection occurs in the Earth’s mantle, it also may occur within the Moon and Mars. The dimensions of these latter two bodies are comparable to the thickness of the Earth’s mantle. Presumably the amount of radioactive heat generated per unit mass is similar in all three bodies. Yet the surface morphology of the Earth, which many scientists believe arises ultimately from mantle convection, differs markedly from that of the Moon or Mars. The surfaces of Mars and the Moon present a serious obstacle to the acceptance of the importance, and indeed the existence, of thermal convection currents within the Earth’s mantle.

We shall attempt to show in this paper that the difference between the surface morphology of the Earth and that of the Moon and Mars does not necessarily disprove the existence of convection within all three bodies. In our theory we shall use an approximate "block" model to analyze thermal convection. The basis of this analysis will be described in the next section. We wish to make clear now that there is an important difference between our treatment of convection in the mantle and that usually discussed in the literature (i.e., Rayleigh's classical theory: Knopoff, 1964, Appendix I, II, III; Heiskanen and Vening Meinesz, 1958, p. 408; Chamalaun and Roberts, 1962). The conventional treatment of mantle convection makes use of an approximation that linearizes the basic equations. Thus the physical situation treated by this approximation is a convective motion in which the major amount of net heat transport between two surfaces occurs by conduction. The heat transported by convective motion is only a minor perturbation of the total. When it is remembered that in the case of mantle convection the major portion of the heat is transported by means of convective motion and only a minor portion by conduction, it can be appreciated that the usual treatment of convection in the mantle makes use of the worst possible limiting approximation.

BASIS OF THEORY

Only two-dimensional problems will be considered in this paper. For simplicity, mantle material will be treated as incompressible.*

Convection in the mantle is approximately similar to convection between two flat rigid surfaces such as shown in Figure 1. It is assumed that the rigid surfaces are lubricated so that no shear stress is transmitted across them. The top surface is maintained at the temperature $T_0$ and the bottom surface at the higher temperature $T_0 + \Delta T$. If no convection occurs, the

*An analysis could be developed for compressible matter. In this situation the adiabatic temperature profile must be considered.
THERMAL CONVECTION IN THE MANTLE

Figure 1. Thermal convection of the mantle in the plane layer approximation.

Figure 2. Thermal convection of the mantle when the top and bottom surfaces are free to assume any shape.

equilibrium temperature profile \( T \) is simply \( T = T_0 + \Delta T(1-z/d) \), where \( d \) is the thickness of the mantle and \( z \) is vertical distance. In this situation the total heat transport between \( z = 0 \) and \( z = d \) occurs by conduction.

Figure 1 shows the particle motion for the fundamental mode of convective motion. If the convective motion is very slow the temperature profile is perturbed only slightly from the value just given. It is possible to obtain an almost exact solution of this problem if it is assumed that the mantle material is a linear viscous solid and that the viscosity is constant and independent of depth and temperature (Knopoff, 1964).

In Figure 1 the top and bottom of the mantle are constrained to be planar surfaces. A more realistic picture of mantle convection is shown in Figure 2. The top and bottom surfaces are free to assume any shape.

The velocities, strain rates, stresses, etc., for the situations pictured in Figures 1 and 2 obviously are almost the same. These velocities, stresses, and strain rates, as well as the temperature, are smoothly varying functions of \( z \) and the horizontal distance \( x \). The regions of compressive and tensile deformation are indicated in Figure 2.

We make the basic assumption that the convection shown in Figure 2 is approximated to a reasonable accuracy by the situation shown in Figure 3. In this figure the longitudinal strain rate \( \dot{\epsilon} \) is tensile and constant throughout the dashed blocks I and III; in blocks II and IV the strain rate \( \dot{\epsilon} \) is compressive, constant, and equal in magnitude to the tensile strain rate. The shear strain rate \( \dot{\epsilon}_{xz} \) is taken to be zero within all four blocks (but not necessarily along their boundaries). It further is assumed that the horizontal derivative of temperature \( \partial T/\partial x \) is zero in all blocks. The boundaries of the blocks, of course, represent singularities where strain rates, temperature, etc. change discontinuously. The model will be refined later in the paper by relaxing some of these assumptions.

For the following reasons we feel that the "block" model of Figure 3 is not too great a departure from physical reality. (1) The low velocity solution of Figure 1 (Knopoff, 1964; Heiskanen and Vening Meinesz, 1958) shows that velocities, strain rates, etc., depend sinusoidally on \( x \) and \( z \) with a wave length \( d \) in the vertical direction and \( \lambda \) in the horizontal direction, where \( \lambda = 2 \sqrt{2} d \). Figure 3 represents a square wave dependence on these quantities, which is a rough approximation to a sinusoidal solution. (2) The critical Rayleigh number for Figure 3, which will be obtained shortly,

*Throughout this paper the strain rates and stresses which appear in the equations are considered to be positive quantities whether they are tensile or compressive in character. The nature of the various stresses and strain rates will be obvious from the context.
is about the same as that of Figure 1. (3) If the (deviator) stresses acting in the mantle are larger than the critical resolved shear stress of mantle rock (which at high temperatures presumably is of the order of 1 to 10 bars) plastic deformation will take place by dislocation motion. The high temperature steady-state creep rate \( \dot{\varepsilon} \) resulting from dislocation motion in crystalline material usually is of the form (Kennedy, 1963; Garofalo, 1965; Weertman and Weertman, 1965)

\[
\dot{\varepsilon} = B \sigma^n \quad (1)
\]

Here \( n \) is a constant (\( n \approx 3 \) to 5) and \( B = B_0 \exp(-Q/kT) \), where \( B_0 \) is another constant and \( Q \) is a diffusion activation energy. The activation energy \( Q \) is pressure dependent. (If the stresses in the mantle fall below the critical resolved shear stress, creep deformation arising from dislocation motion should be relatively unimportant. Gordon (1965) has shown that for this situation Herring-Nabarro diffusional creep is likely to control the creep flow in the mantle. The creep rate of diffusional creep is

\[
\dot{\varepsilon} = A \sigma = \sigma/n \quad (2)
\]

where \( A = A_0 \exp(-Q/kT) \). The quantity \( A_0 \) is a constant, \( Q \) is the same, or approximately the same, activation energy mentioned before, and \( n \) is the coefficient of viscosity.) Equation 1 predicts a much more rapid increase of creep rate with stress than is found for a perfectly viscous solid whose creep rate is given by eq 2. In fact eq 1 approximates the behavior of a perfectly plastic solid, that is, a material for which \( n \) approaches infinity. Plastic deformation occurs in a perfectly plastic solid at a constant stress level. Since blocks I through IV are being deformed at a constant stress level, their physical state is similar to that described by eq 1.

**Calculation of stresses of Figure 3**

Blocks I and II of Figure 3 are in upwelling convective motion and have higher than average temperatures. The reverse holds true for blocks III and IV. If \( \rho \) is the average density of the mantle, the average density of blocks I and II is \( \rho - \Delta \rho \) and of blocks III and IV is \( \rho + \Delta \rho \), where \( 2 \Delta \rho / \rho = a \Delta \theta \). Here \( a \) is the coefficient of thermal expansion and \( \Delta \theta \) is the average temperature difference between blocks I and II and blocks III and IV.

The mantle of the model in Figure 3 is floating on a liquid core. Since the density \( \rho_L \) of the liquid core is approximately twice that of the mantle, for convenience we shall set \( \rho_L = 2\rho \). For this value of \( \rho_L \) the total thickness...
of blocks I and II is equal to that of III and IV but they are displaced vertically from each other by an amount $\Delta d$ given by

$$\Delta d = d(\Delta \rho / \rho).$$

(3)

This equation is derived easily. Let the thickness of blocks I and II be set equal to $d'$, where $d'$ is not necessarily equal to $d$. In Figure 3 the pressure $P$ at a depth equal to the level of the base of III must be the same for all values of $x$. Below blocks III and IV the pressure $P$ equals $(\rho + \Delta \rho)gd$, whereas below I and II it is $(\rho - \Delta \rho)gd' + \Delta d \rho L = (\rho - \Delta \rho)gd' + 2\rho \Delta d$. The two pressures are the same if $d' = d$ and $\Delta d$ is given by eq 3.

It also must be shown that the total force exerted across section 00 of Figure 3 above the level equal to the base of block III is identical to the force across section 0'0'. If these forces were unequal, Figure 3 would not represent an equilibrium situation in which the dimensions of the blocks remain constant with time. If it is assumed that the density in blocks I and II (or III and IV) is uniform and equal to the average density within the two blocks, the total force acting across 0'0' is simply $\frac{1}{2}(\rho + \Delta \rho)gd^2$; across 00 it is $\frac{1}{2}(\rho - \Delta \rho)gd'^2 + \frac{1}{2} \rho Lg(\Delta d)^2 + (\rho - \Delta \rho)gd' \Delta d$. (It should be noted that by symmetry the deviator stresses acting across 00 or 0'0' cancel out since one block is extending and its counterpart is compressing.) These forces equal each other if $d' = d$, $\Delta d$ is given by eq 3 and terms in $(\Delta d)^2$ are dropped.

The deviator stresses acting across the faces 00 and 0'0' can be estimated by calculating the forces on blocks I and IV. Consider Figure 4. The total force acting on blocks I and IV arises from the hydrostatic pressure, the tensile deviator stress $\sigma$ causing extension of block I, the compressive deviator stress $\sigma$ causing compression in block IV, and the shear stress $\sigma_{XZ}$ exerted at the bottom surface. If the stresses within the blocks of Figures 3 and 4 were smoothly varying functions of position as they are in the convection pictured in Figures 1 and 2, it would be reasonable to expect that $\sigma_{XZ} \approx \sigma$. Therefore let $\sigma_{XZ} \approx \sigma$. Further, assume that the wave length $\lambda$ of Figure 3 is approximately $2d$. The total force (directed to the left) acting on blocks I and IV arising from $\sigma$ and $\sigma_{XZ}$ is approximately $3\sigma d/2$. The force due to hydrostatic pressure $P_1$ on block I is $\frac{1}{2}(\rho - \Delta \rho)gd$ whereas the total force due to pressures $P_2$ and $P_3$ is $\frac{1}{4}(\rho + \Delta \rho)gd - \frac{1}{2}\rho d \Delta d$. The pressures $P_1$, $P_2$, $P_3$ are indicated in Figure 4. Setting the sum of the forces equal to zero results in

$$\sigma \approx gd \Delta \rho / 8 = \rho gd a \Delta \rho / 16.$$  

(4)

The creep rate in blocks I through IV is found by placing this stress into either eq 1 or 2. (According to eq 1 and 2 the creep rate is both temperature and pressure dependent. This dependence will be ignored throughout our paper.)
Calculation of average temperature difference $\Delta \theta$.

The stress given by eq 4 depends on $\Delta \rho$ and hence on the average temperature difference between the upwelling and sinking convective currents. This temperature difference can be found from the equation of heat flow. If heat flow in the horizontal direction is neglected the equation of heat conduction in a moving medium under steady state conditions ($\partial T/\partial t = 0$) reduces to

$$K \left( \frac{d^2 T}{dz^2} \right) - w \frac{dT}{dz} = 0$$

where $K$ is the thermal diffusivity and $w$ is the velocity of the medium in the vertical direction. If the origin ($z = 0$) is situated at the bottom of block II for the upwelling case or at the bottom of block III for the sinking case velocity $w$ is given by: $w = \dot{\varepsilon} z$ in block II; $w = \dot{\varepsilon} (d-z)$ in block I; $w = -\dot{\varepsilon} z$ in block III; and $w = -\dot{\varepsilon} (d-z)$ in block IV. The creep rate in these equations always is considered to be a positive quantity. The solution of eq 5 subject to the boundary condition that $T = T_0 + \Delta T$ at $z = 0$ and $T = T_0$ at $z = d$ is

$$T_{\text{III}} = T_0 + \Delta T - \left( \frac{\Delta T}{\gamma} \right) \int_0^z \exp \left( -\dot{\varepsilon} z^2 / 2K \right) dz$$

in block III where

$$\gamma = \int_0^{d/2} \left[ \exp (-\dot{\varepsilon} z^2 / 2K) + \exp (-\dot{\varepsilon} (d^2 - z^2) / 2K) \right] dz$$

and

$$T_{\text{IV}} = T_0 + \Delta T - \left( \frac{\Delta T}{\gamma} \right) \int_0^{d/2} \exp (-\dot{\varepsilon} z^2 / 2K) dz - \left( \frac{\Delta T}{\gamma} \right) \exp (-\dot{\varepsilon} d^2 / 4K) \int_0^z \exp \left( \dot{\varepsilon} (d-z)^2 / 2K \right) dz$$

in block IV. The temperature $T_{\text{II}}$ in block II is found by substituting $-\dot{\varepsilon}$ for $\dot{\varepsilon}$ in eq 6a and 6b; similarly the temperature $T_{\text{I}}$ is found by making the same substitution in eq 6c and 6b.

For small creep rates these equations reduce to

$$\Theta_{\text{II}} = (\dot{\varepsilon} \Delta T z / K d) \left( \left[ \frac{d^2}{8} \right] - \left[ \frac{z^2}{6} \right] \right) = -\Theta_{\text{III}}$$

$$\Theta_{\text{I}} = (\dot{\varepsilon} \Delta T / K d)(d-z) \left( \left[ \frac{d^2}{8} \right] - \left[ \frac{(d-z)^2}{6} \right] \right) = -\Theta_{\text{IV}}$$

where the temperature $T = T_0 + \Delta T(1-z/d) + \Theta$ and $\Theta$ is a perturbation of the temperature.

The average temperature difference $\Delta \Theta$ is

$$\Delta \Theta = 5 \dot{\varepsilon} \Delta T d^2 / 96$$
when the creep rate is small. The stress resulting from this temperature difference is found by inserting eq 8 into eq 4. If it is assumed that the mantle is a perfectly viscous solid, eq 4 in turn may be inserted into eq 2. The resultant equation is

\[ R = \rho gd^3 \alpha \Delta T/\eta \kappa = 96 \times 16/5 = 300 \]  

(9)

where \( R \) is the dimensionless Rayleigh number.

In the (almost) exact solution of convection given in Figure 1 it is found (Knopoff, 1964) that the Rayleigh number equals 657. Our very crude analysis thus leads to a not unreasonable estimate of the Rayleigh number. (The two values could be brought into closer agreement by taking \( \lambda = 2d \) instead of \( \lambda \). Also it should be realized that the exact solution of Figure 2 in which there are no constraints to vertical displacement at the upper and lower surfaces probably would result in a Rayleigh number somewhat lower than 657.) This result gives us confidence that the crude block model of convection is physically plausible.

If the creep rate is large the temperature profiles \( T_I, T_{II}, \) etc. in blocks I, II, etc., reduce to

\[ T_{II} \approx T_I \approx T_0 + \Delta T \]  

(10a)

for all values of \( z \) in the range \( 0 \leq z \leq d - z' \), where \( z' \approx (2K/\dot{\varepsilon})^{1/2} \); for \( d - z' \leq z \leq d \)

\[ T_I \approx T_0 + (z' \Delta T/\gamma) (\dot{\varepsilon}/K)^{1/2} (1 - \dot{\varepsilon} z'^2 /4K + \dot{\varepsilon}^2 z'^4 /60K^2) \]  

(10b)

where \( z' = d - z' \);

\[ T_{III} \approx T_{IV} \approx T_0 \]  

(10c)

for \( z' \leq z \leq d \); and

\[ T_{III} \approx T_0 + \Delta T - (z' \Delta T/\gamma)(\dot{\varepsilon}/K)^{1/2} (1 - \dot{\varepsilon} z'^2 /4K + \dot{\varepsilon}^2 z'^4 /60K^2) \]  

(10d)

In all these equations \( \gamma \approx \frac{1}{2}(2\pi K/\dot{\varepsilon})^{1/2} + \frac{1}{2}(2K/d\dot{\varepsilon}) - (1/4d) (2K/d\dot{\varepsilon})^2 \).

It can be seen that the average temperature difference now is

\[ \Delta \theta \approx \Delta T. \]  

(11)

It is virtually independent of the strain rate. The creep rate \( \dot{\varepsilon}_c \) above which eq 11 is valid and below which eq 8 is valid is approximately

\[ \dot{\varepsilon}_c \approx 2K/d^2. \]  

(12)

For creep rates below this critical value the major portion of heat transport is by conduction. At creep rates above \( \dot{\varepsilon}_c \) convective heat transport predominates.

The stress arising from the temperature difference may be found by substituting eq 8 and 11 into eq 4. Let this "thermal" stress be labeled \( \sigma_T \). Figure 5 shows schematically the dependence of \( \sigma_T \) on creep rate for various values of \( \Delta T \). Also shown in this figure is the relationship between stress and creep rate given by the power law of eq 1. This stress is labeled \( \sigma_n \). (The stress (\( \sigma_n \)) given by the viscous creep equation (2) also is plotted.) The points at which the curves cross or are tangent to each other represent physically possible convective states. At these intersections or
tangencies the stress required to maintain a given creep rate is equal to the thermal stress arising from the creep rate. Points B and C of Figure 5 represent states of stable steady-state convective motion. If the creep rates were increased at either of these points the stress required to drive convection would be greater than the thermal stress available. If the creep rates were reduced an excess thermal stress would be available to speed up the convective motion. By the same argument point A represents unstable steady-state convection. The points at which the curve of \( \sigma_\eta \) is tangent to \( \sigma_T \) (for \( \dot{\varepsilon} < \dot{\varepsilon}_c \)) correspond to a condition of marginal (or neutral) stability (Knopoff, 1964). The intersections to the right of \( \dot{\varepsilon}_c (\dot{\varepsilon} > \dot{\varepsilon}_c) \) represent convective flow in the mantle. 

*Figure 5 is altered as follows when convection is in a higher mode than the fundamental mode. Let \( M = 1, 2, 3, \ldots \) represent the mode number. The fundamental mode is \( M = 1 \). The curves of \( \sigma_n \) and \( \sigma_\eta \) are unaltered by increasing \( M \) [i.e., \( (\sigma_n)_M = (\sigma_n)_{M=1} \) and \( (\sigma_\eta)_M = (\sigma_\eta)_{M=1} \)]. The critical creep rate \( \dot{\varepsilon}_c \) is increased by a factor of \( M^2 \) [i.e., \( (\dot{\varepsilon}_c)_M = M^2 (\dot{\varepsilon}_c)_{M=1} \)]. The initial slope \( \sigma'_T = d\sigma_T/d\varepsilon \) of the thermal stress curve in the region to the left of \( \dot{\varepsilon}_c \) is reduced by a factor \( M^4 \). The same reduction occurs for the initial slope of the stress \( \sigma_H \) associated with constant heat transport. Thus \( (\sigma'_T)_M = M^{-4} (\sigma'_T)_{M=1} \) and \( (\sigma'_H)_M = M^{-4} (\sigma'_H)_{M=1} \). In the region to the right of \( \dot{\varepsilon}_c \) the stresses \( \sigma_T \) and \( \sigma_H \) are reduced by a factor of \( M^2 \) [i.e., \( (\sigma_T)_M = M^{-2} (\sigma_T)_{M=1} \) and \( (\sigma_H)_M = M^{-2} (\sigma_H)_{M=1} \). (Cont'd on page 8)
Under steady-state conditions the heat transported per unit time between the surfaces \( z = 0 \) and \( z = d \) must equal the total heat generated per unit time. Radioactive sources of heat are distributed throughout the mantle and, of course, this fact should have been taken into account in eq 5. We shall assume for simplicity that the heat source resides at the core-mantle boundary rather than being distributed throughout the mantle.

Refinement of the model

There is one very undesirable feature of the convective model of Figures 3 and 4. In this model the temperature of the material moving from block I into block IV or from block III into block II must change instantaneously. This can only happen if heat is added or removed through some external agency at the boundary. For the situation in which the major amount of heat is transported by conduction this physically unrealistic feature of the model introduces no serious error. However when heat is transported primarily by convection the error is serious. (In fact, the model is a perpetual motion machine.) We wish to show how the model may be refined to make it physically realistic. To do so we shall have to take into account temperature variations in the horizontal direction.

Assume that the Earth at a given instant in time is in the state pictured in Figures 3 and 4. Let the creep rate \( \dot{\epsilon} \) be much larger than the critical value \( \dot{\epsilon}_c \). The characteristic distance \( (2K/\dot{\epsilon})^{1/2} \) thus is much smaller than the dimensions of the mantle. Now assume that there is no temperature discontinuity between blocks I and IV (and between II and III). No heat is to be supplied or taken away at these boundaries. The initial temperature distribution will be that shown in Figure 6a. The shaded area indicates regions where there is an appreciable temperature gradient. The thickness of these regions is of the order of \( (2K/\dot{\epsilon})^{1/2} \).

Because so little heat can be transported by conduction, after a time the boundary separating regions of temperature \( T_0 + \Delta T \) and \( T_0 \) must take the steady-state form shown in Figure 6b. The temperature gradient is essentially zero everywhere except in the shaded zone whose thickness is of the order of \( (2K/\dot{\epsilon})^{1/2} \).

Figure 6c shows a more realistic "block" model for convection when conduction is not the major heat transfer mechanism. This model is based on

Suppose convection is the major heat transport mechanism. By combining eq 14 (after the right-hand side is divided by \( M^2 \)), 15 or 16 one can obtain the relationship \( \Delta T = M^2 (8/\rho dc)(2H\eta/\rho \gamma g) 1/2 \) for a viscous solid and \( \Delta T = M^2 (8H/\rho dc)(2c/\rho \gamma H) n/(n+1) \) for a solid obeying creep equation 1. In these expressions \( H \) is heat transported. If \( H \) is held constant and the convecting system is free to adjust the value of \( \Delta T \), the most likely value of \( M \) is 1. In the fundamental mode the temperature difference is minimized. It is to be expected therefore that convection within the Earth takes place in the fundamental mode. On the other hand if both \( H \) and \( \Delta T \) are held constant, in general, \( M \) will be greater than 1 in a convecting system.

If conduction is the major heat transport mechanism \( \Delta T \) is not altered appreciably by convection. For a viscous solid \( M \) will assume that value for which the initial slopes of the \( \sigma_T \) and \( \sigma_H \) curves are almost the same as the slope of the \( \sigma_\eta \) curve.

Steady-state convection cannot occur if the \( \sigma_\eta \) or the \( \sigma_\eta \) curve lies entirely above the \( (\sigma_H)_{M=1} \) curve of Figure 5.
Figure 6. Block model of convection when convective motion is the major mechanism of heat transport. (a) Temperature distribution of Figure 3. (b) Steady-state temperature distribution. (c) Block model and stresses within blocks corresponding to (b).

Figure 6b. Again we have blocks I and III in which tensile flow occurs and blocks II and IV in which compressive flow occurs. These blocks are separated by two "buffer" blocks V and VI in which there is neither compression nor tension but only a shear deformation. The dimensions of these buffer blocks are comparable to those of the other blocks. The calculation of the tensile and compressive deviator stresses in blocks I through IV is identical to that for Figures 3 and 4 and gives identical results. The refined model leads to essentially the same results as the cruder model.

Heat transport

We are now in a position to calculate the heat transported by a convective cycle under steady-state conditions. Let $H$ represent the heat generated per unit time and per unit horizontal distance at the bottom of the mantle.

When $\dot{\varepsilon} < \dot{\varepsilon}_c$ almost all of the heat is transported by conduction. Therefore

$$H = K \frac{\Delta T}{d} \tag{13}$$

where $K$ is the thermal conductivity.

When $\dot{\varepsilon} > \dot{\varepsilon}_c$ almost all the heat is transported by convective motion. The amount transported can be estimated with the aid of Figures 6b and 6c. The net amount of heat transported upwards through the I-II and the III-IV interfaces by vertical motion of matter is approximately $\rho \ c \ \dot{\varepsilon} \ d \Delta T \lambda / 8$ where $c$ is the specific heat. Since block V is at a higher temperature than block VI there will be heat conducted downward across their interface.
Heat will be conducted (there is no vertical motion of matter across this interface) by the temperature gradient existing at the V-VI interface. This gradient can be expected to be of the order of the gradient at the top surfaces of blocks I and V or the bottom surfaces of blocks III and VI. Hence the amount of heat conducted downward across the interface V-VI will be of the order of $H\lambda/2$ under steady-state conditions. The total amount of heat transported thus is $(\rho c \dot{\varepsilon} \Delta T \lambda/8) - (H\lambda/2)$. This quantity must equal the total supply of heat $H\lambda/2$ at the bottom surface. Hence

$$H = \rho c \dot{\varepsilon} \Delta T / 8. \hspace{1cm} (14)$$

When the heat supply is constant and is limited, heat transfer by conduction or convection will adjust the temperature difference $\Delta T$ existing between the top and bottom surfaces to that value where the amount of heat transported just balances the amount created. For example, if the convection motion speeds up more heat is transported and $\Delta T$ will decrease. The steady-state convective motion that ultimately is established is that for which $\Delta T$ and the creep rate $\dot{\varepsilon}$ satisfy eq 13 or 14. These values of $\dot{\varepsilon}$ and $\Delta T$ also must be compatible with eq 8 or 11 and eq 1 and 2 as well as eq 4.

Consider Figure 5. The dashed curve in this figure represents a thermal stress $\sigma_T$ corresponding to values of $\dot{\varepsilon}$ and $\Delta T$ which lead to a constant value of $H$ in eq 13 and 14. Let this stress be labeled $\sigma_H$. The steady-state convective state is that state for which the curves of $\sigma_H$, $\sigma_T$, and $\sigma_n$ (or $\sigma_0$) all intersect at the same point. By combining eq 1, 4, 11 and 14 the following equation is found for the case of $\dot{\varepsilon} > \dot{\varepsilon}_c$:

$$\sigma \dot{\varepsilon} = \dot{\varepsilon}(\dot{\varepsilon}/B)^{1/n} = a g H/2 \c.$$  \hspace{1cm} (15)

When eq 2 describes the creep flow of mantle material this equation is replaced by

$$\dot{\varepsilon} = (a g H/2 \eta \c) \frac{1}{2}. \hspace{1cm} (16)$$

It is interesting to note that the thickness $d$ of the mantle has dropped out of these equations.

Equations 15 and 16 give reasonable values for $\dot{\varepsilon}$ and $\sigma$ if it is assumed that $H$ equals the geothermal heat escaping at the Earth's surface ($H \approx 40 \text{ cal/cm}^2/\text{yr}$). If we take $a = 2 \times 10^{-5} \degree \text{C}$ and $c = 5 \times 10^7 \text{ergs/cm}^3/\degree \text{deg}$ (Knopoff, 1964) and assume that $g$ is a constant throughout the mantle ($g \approx 10^3 \text{ cgs}$) we find that the product $\sigma \dot{\varepsilon}$ is approximately equal to 0.4 dynes/cm$^2$ -yr. This product is equivalent to a 40-bar stress and a creep rate of $10^{-8}/\text{yr}$. These are reasonable values. A viscosity† of $10^{23}$ cgs (which is in the range usually quoted for the Earth) substituted in eq 16 predicts a creep rate of $10^{-8}/\text{yr}$.

*See appendix for a more accurate treatment of the heat transport at large strain rates.

†MacDonald (1963) has pointed out that the viscosity of the Earth (as estimated from the time lag of the Earth's flattening) is so large ($10^{26}$ cgs) that convection cannot occur in the mantle. This objection can be overcome with the use of the power law creep equation (1). It gives an "apparent" viscosity equal to $B^{-1} \sigma^{-(n-1)}$. Thus the apparent viscosity estimated from different physical effects such as the rate of flattening will depend on the stress level involved. The smaller the stress level the greater will be the apparent viscosity measured.
If eq 15 and 16 are applied to the Moon and Mars it is seen that, other factors being equal, the creep rate \( \dot{\epsilon} \) is reduced because \( g \) is smaller for these bodies. At the surface of the Moon \( g \) is smaller than at the surface of the Earth by a factor of 6 and in the case of Mars \( g \) is smaller by a factor of 10/4. Smaller values of \( g \) make it less likely that appreciable convection occurs provided that the creep strengths in all three bodies are comparable. However, since creep strength depends on both pressure and temperature the creep strengths of the Moon and Mars will be lower than that of the Earth if the temperatures in all three are similar. The hydrostatic pressure within the Moon and Mars is less than within the Earth.

**EFFECT OF A LOW STRENGTH LAYER**

It has been postulated in the past that the low velocity layer of seismology is a region of low strength (Anderson, 1962) occurring at a depth of the order of 100 to 200 km. The internal friction or damping of the mantle is much higher in this region than deeper in the mantle. The rate of earthquake energy release seems to go through a minimum here.

Gordon (1965) has shown very clearly how it is possible for the creep strength of the mantle material to go through a minimum as the depth increases. The increase in temperature with depth causes a decrease in creep strength. However the increase in pressure with depth causes an increase in the creep strength. A suitable pressure-temperature profile permits the existence of a region with very low creep strength.

Gordon's analysis was applied under the assumption that Herring-Nabarro diffusional creep is the dominant flow mechanism in the mantle. His analysis applied to creep controlled by dislocation motion leads to the identical prediction of a low creep strength layer. This similarity arises from the fact that both Herring-Nabarro creep and the movement of dislocations at relatively high temperatures are controlled by the same mechanism, the diffusion of atoms. The effect of temperature and pressure is identical for both types of creep.

If the low strength layer occurs at the center of the mantle \( z = \frac{1}{2}d \) it is easy to predict its effect on convective flow in Figures 3 and 4. In this case the shear stress corresponding to \( \sigma_{xz} \) of Figure 4 is zero. Thus the stress given by eq 4 will be twice as big; the Rayleigh number of eq 9 will be halved; the creep rates given by eq 15 and 16 will be increased; and the temperature difference \( \Delta T \) required to maintain convection will be decreased. In other words; it is easier to establish convection.

Suppose the low strength layer is not at the midpoint but is situated much closer to the upper surface*. This situation is shown in Figure 7. Two possible fundamental modes of convection can be imagined for this asymmetric situation. The deformation may proceed in a manner similar to that given in Figure 3 or 6. Again "blocks" must be considered but they are not of the same thickness. Tensile flow occurs in blocks I and III and compressive flow in II and IV (see Fig. 7). The low strength zone forms a boundary between the upper and lower blocks. The situation pictured in Figure 7 certainly would occur if the low strength zone were at the midpoint.

*Throughout this paper the upper mantle is defined to be that region of the mantle lying above the low strength layer.
If the low strength zone were placed slightly above the midpoint \( z > \frac{1}{2} d \) the physical state still would be described by Figure 7. However it might be expected that the model would break down if the low strength zone were placed close enough to the upper surface.

The low strength zone will be considered to be so weak that it is essentially a fluid. The hydrostatic pressure in this region is a constant for all values of \( x \). This fact determines the ratio \( D_I/D_{IV} \), where \( D_I \) and \( D_{IV} \) are the thicknesses of the different blocks.

Let \( \rho - \Delta \rho \) be the average density in block I; \( \rho + \Delta \rho \) the average density in block IV; and \( \rho \) the average density throughout the mantle. Similarly, let \( \rho - \Delta \rho \) be the average density in block II and \( \rho + \Delta \rho \) the average density in block III.

The thicknesses \( D_I \) and \( D_{IV} \) must satisfy the relationship

\[
D_I - D_{IV} \approx D_I \Delta \rho'/\rho = D_{IV} \Delta \rho'/\rho
\]

in order to insure that the pressure is constant within the low strength zone. (Second order terms have been dropped in this expression.) The tensile deviator stress \( \sigma_I \) acting across block I will have the same magnitude as the compressive deviator stress acting across II. From a balance of forces argument it is found that

\[
\sigma_I = \frac{1}{2} \Delta \rho' g D_I.
\]

The thickness of blocks II and III may be obtained from similar arguments:

\[
D_{III} - D_{II} \approx 2 \Delta \rho D_{II}/\rho \approx 2 \Delta \rho D_{III}/\rho.
\]

Again second order terms have been dropped and it is assumed that \( \rho_L \) equals \( 2\rho \). The deviator stresses in both blocks have the same magnitude and are equal to

\[
\sigma_{III} = \frac{1}{2} g \Delta \rho D_{III}.
\]

Under steady-state conditions the vertical velocity is zero at the lower and upper surfaces of the mantle. Hence the vertical velocity at the lower boundary of block I is \( \xi_I D_I \) and the velocity at the upper boundary of block II is \( \xi_{II} D_{II} \). These two velocities need not match each other. Mass balance can be maintained by flow of matter within the low strength zone, which has the properties of a fluid of low viscosity. (However, unlike a fluid, the low viscosity zone can not be "squeezed out" and thus eliminated by the flow of matter in it. As long as the flow does not change the temperature-pressure conditions unduly, the low strength zone will always regenerate itself.)

The flow of matter within the low strength layer will change the manner of heat transport. The heat transport equation (5) has to be solved taking into account this new boundary condition. To make the problem manageable...
we shall assume that per unit time a uniform layer of mantle material of thickness \((\epsilon_{II D_{II}} - \epsilon_I D_I)\) is removed between blocks I and II and inserted between blocks III and IV. (Of course, reverse flow also is possible. It will become evident later that this situation is unlikely to occur.) Material is removed from a region at a higher temperature to another region at a somewhat lower temperature. The low strength layer thus is a heat sink between I and II and a heat source between III and IV. Any change in the heat budget between blocks V and VI will be ignored.

Again let the temperature of the upper surface of the mantle be maintained at \(T_0\) and the lower surface at \(T_0 + \Delta T\). Let the temperature between blocks I and II be \(T_I\) and between III and IV be \(T_{IV}\). Consider now the temperature profile found from eq 5 for two limiting cases.

Large creep rates in both upper and lower mantle

Suppose that \(\epsilon_{II} > 2K/D_{II} \epsilon_I^2\) and that \(\epsilon_I > 2K/D_I \epsilon_I^2\). According to eq 12 we are in the region of high creep rates. Hence the temperature profile must resemble the profile shown in Figure 8a. In the rising convective current the temperature remains at \(T = T_0 + \Delta T\) to within a distance \(z' = z_{II} (2K/\epsilon_{II})^2 \) from the upper surface. The temperature \(T_I\) thus is \(T_0 + \Delta T\). In the descending current the temperature in block IV remains at \(T = T_0\) to within a distance \(z'_{III} = z_{III} (2K/\epsilon_{III})^2\) from the bottom surface, at which point the temperature rises to \(T_0 + \Delta T\). The temperature \(T_{IV}\) remains to be determined.

The average difference in temperature between blocks I and IV is simply \(\Delta T\). Equation 18 thus becomes

\[\sigma_I = \frac{1}{2} g D_I \rho a \Delta T.\]  
(21)

Similarly the stress \(\sigma_{III}\) is given by

\[\sigma_{III} = \frac{1}{2} g D_{III} \rho a \Delta T'.\]  
(22)

where \(\Delta T' = T_0 + \Delta T - T_{IV}\). Another expression for the stress can be found if the creep rate \(\epsilon_{III}\) is known. This creep rate must satisfy the modified version of eq 14 in which \(\epsilon_{III}\) is substituted for \(\epsilon\) and \(D_{III}\) for \(d/2\). Hence the stress \(\sigma_{III}\) is

\[\sigma_{III} = 4H \eta / \rho c D_{III} \Delta T'.\]  
(23a)

if the creep law is the purely viscous relationship given by eq 2. Otherwise the stress is

\[\sigma_{III} = (4H / \rho c D_{III} B \Delta T')^{1/n}.\]  
(23b)

The following equations for the stress may be obtained from eq 22 and 23 by eliminating the temperature difference \(\Delta T'\):

\[\sigma_{III} = (2 \eta H g a/c) \frac{1}{2}\]  
(24a)
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in the case of viscous creep and

\[ \sigma_{\text{III}} = (2Hg \alpha/cB)^{1/(n+1)} \]  

(24b)

for the other type of creep given by eq 1. An argument identical to that used to obtain eq 24 can be invoked to obtain an expression for the stress \( \sigma_1 \) in which the temperature difference \( \Delta T \) does not appear. One finds that \( \sigma_1 \approx \sigma_{\text{III}} \). Hence \( \Delta T'/\Delta T \approx D_1 / D_{\text{III}} \). (If either the coefficient of viscosity \( \eta \) or the constant \( B \) has a different value in the upper mantle than the lower mantle, the stress \( \sigma_1 \) still is given by eq 24 with \( \sigma_1 \) substituted for \( \sigma_{\text{III}} \). However, the values of \( \eta \) and \( B \) appropriate to the upper mantle must be placed in these equations. A higher creep strength in the upper mantle requires that \( \Delta T \) be larger.)

The stresses found by using eq 24 are approximately the same as those given by eq 15 and 16. It should be noted that the temperature differences \( \Delta T' \) and \( \Delta T \) required to obtain a stress of the order of 40 bars are not excessive. For \( D_1 = 200 \text{ km}, \Delta T \approx 66^\circ \text{C} \) and \( \Delta T' \approx 5^\circ \text{C} \).

Small creep rates in upper mantle and large creep rates in lower mantle

If the thickness of the upper blocks of Figure 7 is made small enough, conduction becomes the predominant mechanism for the transport of heat through the blocks. Suppose that the strain rate in the lower mantle is high enough that convection is the predominant heat transport mechanism there. The resultant temperature profile is shown in Figure 8a. The same arguments used in the previous section lead to the conclusion that the average temperature difference \( \Delta T' \) in the lower mantle is the same as that previously calculated. The stress \( \sigma_{\text{III}} \) also is the same. The average temperature difference in the upper mantle is only about \( \frac{1}{3} \Delta T' \). Therefore the stress \( \sigma_1 \) (given by eq 21 with \( \Delta T' \) substituted for \( \Delta T \)) is small. One would not expect the upper mantle to suffer appreciable creep deformation.

The temperature difference \( \Delta T \) is given by eq 13 with \( D_1 \) substituted for \( d \). The critical value of \( D_1 \) at which heat transfer mechanism in the upper mantle changes from convection to conduction can be found from the characteristic diffusion distance \( (\varepsilon / 2K)^{1/2} \). If \( \varepsilon \) again is taken to be \( 10^{-8} / \text{yr} \) and \( K = 3 \times 10^{-2} \text{ cm}^2 / \text{sec} \) (Knopoff, 1964) this characteristic distance is 140 km. Thus if the low strength layer were at a much shallower depth the upper mantle would not deform plastically.

It might be argued that in the presence of a low velocity layer the convection loop would not be as shown in Figure 7 but rather would more closely resemble the model shown in Figure 9. This latter model is an approximation of Figure 6. In Figure 9 blocks II (compression), VI (shear), and III (tension) correspond to and have essentially the same dimensions as the analogous blocks of Figure 6c. Blocks I' (tension), V' (shear), and IV' (compression) likewise correspond to block I, V, and IV of Figure 6c. The upper mantle blocks I (tension), V (shear), and IV (compression) are merely perturbation on the overall convective flow pattern which is, essentially, the same as Figure 6c. If the heat transport through the upper mantle were by conduction the temperature profile would be similar whether Figure 9 or 7 described the convection in the lower mantle. However if the major amount of heat transport in the upper mantle were by convection, Figure 9 would represent an unlikely convection model. As we have just learned, in order that convection occur in the upper mantle the average temperature
Thermal Convection in the Mantle

To Low Strength Zone

Figure 8. Temperature distribution of the block model of Figure 7. (a) Large creep rates above and below the low strength layer. (b) Large creep rate below the low strength layer and small creep rate above it.

Figure 9. Block model corresponding to Figure 6c when convection currents approximate those shown in Figure 2 and there is little mass transport through the low strength layer.

difference there \((\Delta T)\) must be a factor \(D_II/D_I\) larger than the average temperature difference \((\Delta T')\) in the lower mantle. This much greater temperature difference arises from the transport of matter in the convective loop through the low strength layer. If this transport is minimized, as in the case in Figure 9, it would be impossible to establish that \(\Delta T > \Delta T'\). (However, if in Figure 9 block I were in compression and block II in tension, the average temperature difference in the upper mantle would be a factor \(D_II/D_I\) larger than in the lower mantle. In this situation the mass transport of matter in the low strength zone would be greater than in the model pictured in Figure 7.)

Discussion

The results of this paper show that convection can take place in the upper mantle even though there may be an underlying low strength layer which, over long periods, cannot transmit shear stresses. The resultant creep deformation in the upper mantle can lead to still higher stresses in the Earth's crust in the manner described earlier (Weertman, 1962, 1963).

If the thickness of the mantle above the low strength layer is small the stresses in this region are small and the upper mantle will not deform plastically. This latter result suggests a possible explanation for the lack of widespread deformation (other than impact craters) of the crust of the Moon and Mars. If these bodies contain low strength layers, and if these layers lie closer to the upper surface than the corresponding layer in the Earth, the crusts of the Moon and Mars cannot be deformed plastically by thermal convection currents.

Of course it could be denied that convection currents exist at all in Mars and the Moon. After all, it is still open to question whether convection even exists within the Earth's mantle. However, suppose one is a proponent of continental drift and therefore regards the existence of convection currents within the Earth as being almost certain. If convection currents exist within the Earth's mantle then arguments can be advanced to support the view that they may exist within the Moon and Mars. It is likely that the amount of
radioactive heat generated per unit mass is about the same for Mars and the Moon as it is for the Earth. If the Earth has been in a steady-state convective motion over the major portion of its history and the Moon and Mars have not, obviously the amount of radioactive heat retained in the interior of these latter bodies is much larger than that retained within the Earth. The internal temperatures of Mars and the Moon would be much higher than the Earth's internal temperature provided that the internal temperatures of all three bodies were similar at the time of their formation. The hydrostatic pressures within Mars and the Moon are smaller than within the Earth. Thus even if the internal temperatures of the Moon and Mars were somewhat lower than that of the Earth the viscosity or the creep strength of the Moon and Mars still could be much lower than that of the Earth because the reduced pressure permits their interiors to be closer to the melting point. (That is, the diffusion rate of atoms within a solid, and hence the viscosity or creep strength, depends exponentially on the proximity of a solid to its melting point. The diffusion rate is proportional to a factor \( \exp(-CT_m/T) \), where \( C \) is a constant and \( T_m \) is the melting temperature. The melting temperature of a normal solid is raised by increasing the pressure.) All these factors suggest that if the Earth is in a state of convection so should be the Moon and Mars. The surface morphology of the Moon and Mars thus may present a serious objection to the postulation of convection within the Earth's mantle.

This objection to convection within the Earth can be answered in two ways. It could be postulated that the interior temperatures at the time of the formation of the Earth were very much higher than the corresponding temperatures for the Moon and Mars. Thus these latter bodies may be so much colder than the Earth that their viscosities or creep strengths are too great to permit convection at the present time. Another answer to the objection is the analysis of this paper. Convection within Mars and the Moon need not lead to plastic deformation of their crusts if their low strength zones lie at shallower depths than the corresponding layer in the Earth. A theory such as Gordon's (1965) could be used to explain why the low strength zones may exist at shallower depths in the Moon and Mars.

LITERATURE CITED


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APPENDIX. HEAT TRANSPORT AT LARGE STRAIN RATES

At large strain rates \( \dot{\varepsilon} \gg \dot{\varepsilon}_c \) the heat transported by convection no longer is given by eq 14. The velocity of matter in the vertical direction is zero at the upper and lower surfaces of a convecting layer. Hence the heat transported by convection must equal the heat transported by conduction down the temperature gradient that exists at these surfaces. This temperature gradient is found from eq 6 and is equal to \( \Delta T \left( \frac{\dot{\varepsilon}}{2\kappa} \right)^{\frac{1}{2}} \). Thus \( H \), the heat transported by convection, is equal to

\[
H \approx K \Delta T \left( \frac{\dot{\varepsilon}}{2\kappa} \right)^{\frac{1}{2}}. \tag{14a'}
\]

In general, the heat transport given by this equation will not equal that given by eq 14. The magnitude of the difference between them will increase with larger \( \dot{\varepsilon} \).

The heat transported by convection also must equal an expression proportional to the temperature difference \( \Delta T \) between the rising and sinking convective currents, the average velocity of a convective current \( \sim \dot{\varepsilon}d \), and the heat capacity \( pc \). Equation 14 was obtained from such an analysis. However, in determining eq 14 it was assumed that blocks I through VI in Figure 6 were all comparable in size. This situation is true only if the strain rate \( \dot{\varepsilon} \) is of the same magnitude as the critical strain rate \( \dot{\varepsilon}_c \). At larger strain rates the horizontal dimensions of blocks I through IV will be much smaller than the horizontal dimensions of the buffer blocks V and VI. Thus the effective width of the rising and sinking convective currents decreases as \( \dot{\varepsilon} \) increases. Therefore they become less efficient in transporting heat.

Let \( \beta \) represent the width in the horizontal direction of block I divided by the width of block V. As \( \dot{\varepsilon} \) increases \( \beta \) will decrease. The heat \( H \) transported through convective motion of matter can be expected to be of the order of

\[
H \approx \beta pc \dot{\varepsilon} d \Delta T / 8. \tag{14b'}
\]

This equation is simply eq 14 of the text modified by the insertion of the factor \( \beta \). Since eq 14a' and 14b' must lead to identical heat transport the term \( \beta \) can be found by setting these equations equal to each other.

When \( \beta \) approaches zero the temperatures in Figure 6 are equal to \( T_0 + \Delta T \) almost everywhere in the top blocks and to \( T_0 \) almost everywhere in the bottom blocks. In other words, an almost stable situation is reached. Hot, light matter resides over cold, dense matter. Clearly this situation tends to reduce the stresses driving the convective motion. It is simple to show that eq 4, which gives the stress driving convection, is modified into the following one

\[
\sigma \approx \beta \rho g d \Delta T / 16. \tag{4'}
\]

(This last equation is obtained using an analysis similar to that employed for Figure 4. One first analyzes the forces acting on blocks I, IV, and V of Figure 6. These blocks are considered as one unit just as blocks I and IV in Figure 4 were considered as a unit. Then one analyzes the forces acting on block V alone. Equation 4' is the result of the analysis.)

For a viscous solid \( \sigma = \eta \dot{\varepsilon} \). If this expression is inserted into eq 4' and then this equation is combined with equations 14a' and 14b' one finds that the heat transport is given by

\[
H \approx \left( \frac{agK^4}{2c \eta k^2} \right)^{\frac{1}{3}} (\Delta T)^{\frac{1}{3}}. \tag{25}
\]
Thus the heat transport is proportional to $(\Delta T)^{\frac{3}{2}}$. This dependence agrees with that found in laboratory experiments on heat transport at fast rates of convection. (See, for example, Fig. 13, p. 68 of S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (1961), Clarendon Press, Oxford.)

When $\beta$ and $\Delta T$ are eliminated from eq. 4' through the insertion of equations 14a' and 14b' one finds that the stress is equal to

$$\sigma = \left( \frac{\dot{\varepsilon}}{B} \right)^{1/n} = (a g H B / 2 c)^{1/(n+1)}$$

(15')

for a non-viscous solid and

$$\sigma = \eta \dot{\varepsilon} = (a \eta g H / 2 c)^{1/3}$$

(16')

for a viscous solid. Surprisingly, these equations give stresses (as well as strain rates) that are identical to those determined from equations 15 and 16 of the text.
If thermal convection occurs in the Earth's mantle, it may also occur within the Moon and Mars. The dimensions of these planets are comparable to the thickness of the Earth's mantle. Presumably the amount of radioactive heat generated per unit mass is similar in all three bodies. However, the surface morphology of the Earth differs markedly from that of the Moon or Mars. The explanation for the difference is based on the effect produced on convection in the mantle by the presence of a low "viscosity" or low creep strength layer. A low viscosity layer changes the amount of "coupling" between the outer crust and mantle convection. The theory assumes that the viscosity or creep strength is essentially zero in the low viscosity layer. The analysis is similar to that developed earlier for the calculation of stresses within the mantle. The deeper the low viscosity layer lies within the mantle, the greater is the coupling of the outer crust of the mantle convection currents. If the low creep strength layer lies close to the surface the outer crust is decoupled from the interior. According to the literature, the depth of the low velocity layer is determined by the temperature and pressure profiles within the mantle.
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Geophysics --Earth (Planet)

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