Research Report 196

TIME DEPENDENT DEFLECTION
OF A FLOATING ICE SHEET

by

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PREFACE

This report was prepared for USA CRREL by Mr. Nevel, Applied Research Branch (A. L. Wuori, Chief), Engineering Division (K. A. Linell, Chief).

The author wishes to thank G. Frankenstein, who kindly permitted the use of his data for this paper. The author is indebted to H. Forbes, who checked the development of the equations and helped with the editing of this paper and to A. D. Kerr for technical review.

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SUMMARY

A solution for a viscoelastic plate on an elastic foundation is presented for an infinite bulk modulus and a shear modulus which obeys Maxwell's model. Observed deflections of a floating ice sheet agree with this solution. A solution for a viscous shear modulus is also presented.
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INTRODUCTION

Floating ice sheets have been and are currently being used as airfields in Arctic areas. Present safe landing criteria are based upon the solution of an elastic plate on an elastic foundation, first solved by Hertz in 1884. It is well known that ice can be considered elastic for short durations of loading, but for long durations of loading ice will creep.

Lee (1955) presented a method for solving a certain class of problems in which the material is linearly viscoelastic. A linear viscoelastic material is one for which the stress-strain relation is given by

\[ \left( a_0 + a_1 \frac{\partial}{\partial t} + \ldots + a_n \frac{\partial^n}{\partial t^n} \right) \sigma = \left( b_0 + b_1 \frac{\partial}{\partial t} + \ldots + b_n \frac{\partial^n}{\partial t^n} \right) \epsilon \] (1)

where \( \sigma \) is the stress,
\( \epsilon \) is the strain,
\( t \) is the time, and the
\( a \)'s and \( b \)'s are material constants.

Using the above method, Hoskin and Lee (1959) solved the closely related problem of an elastic plate on a viscoelastic foundation.

Kheisin (1964) presented a solution by a different method of a viscoelastic plate on an elastic foundation. This work will be discussed in detail later.

A previous USA CRREL report (Kerr, 1959) also considered the time dependent deflection of a floating ice sheet. In this paper the mathematical model representing the ice sheet is not realistic, and the solution of this mathematical model did not meet boundary conditions.

STRESS-STRAIN RELATION FOR ICE

Jellinek and Brill (1956) showed that, for a simple tension test, the stress-strain relation for ice could be represented by the model shown in Figure 1, where the \( E \)'s are spring constants and the \( \eta \)'s are viscosity constants. For a tension test on an elastic material, we have

\[ \sigma = E \epsilon = \frac{9K(2G)}{6K + 2G} \epsilon \] (2)

where \( E \) is Young's modulus,
\( K \) is the bulk modulus, and
\( G \) is the shear modulus.

For ice the bulk modulus is a constant; hence the model proposed by Jellinek and Brill represents the shear modulus.
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Figure 1. Ice model. Figure 2. Maxwell model.

VISCOELASTIC ANALYSIS

The method of solution for a viscoelastic plate on an elastic 'foundation will be that given by E. H. Lee. For an elastic plate the differential equation is

$$\frac{Eh^3}{12(1-\mu^2)} \nabla^4 w + kw = q(r, \theta, t)$$

(3)

where $r$ and $\theta$ are polar coordinates,

- $w(r, \theta, t)$ is the deflection of the plate,
- $q(r, \theta, t)$ is the load density,
- $k$ is the foundation modulus,
- $h$ is the thickness of the plate, and
- $\mu$ is Poisson's ratio.

In order to simplify the analysis, we will assume that ice is incompressible under hydrostatic stress and that it obeys Maxwell's model as shown in Figure 2 for deviatoric stress and strain. Hence from Bland (1960) we have $3K = \infty$ and $2G = (E_1 \eta_1 \theta/\partial t)/(E_1 + \eta_1 \theta/\partial t)$. With these assumptions eq 3 becomes

$$\frac{h^3}{6} E_1 \eta_1 \frac{\partial}{\partial \tau} \nabla^4 w + (E_1 + \eta_1 \frac{\partial}{\partial \tau}) kw = (E_1 + \eta_1 \frac{\partial}{\partial \tau}) q(r, \theta, t).$$

(4)

If in eq 4 we let $q(r, \theta, t) = q(r, \theta) H(t)$ where $H(t)$ is Heaviside's step function, and take the bilateral Laplace transform (Van der Pol and Bremmer, 1959), we obtain

$$\frac{E_1 h^3 s}{6(s + E_1 / \eta_1)} \nabla^4 \bar{w} + k\bar{w} = \frac{q(r, \theta)}{s}$$

(5)

where $\bar{w}(r, \theta, s) = \int_{-\infty}^{\infty} w(r, \theta, t) e^{-st} dt$.

A solution of eq 5 is given by Wyman (1950) when $q(r, \theta)$ is a uniform load distributed over a circular area of radius $a$. If we consider only the maximum deflection, which occurs at $r = 0$, we have from Wyman

$$\bar{w} = \frac{P}{\pi k a^2 s} \left[ 1 + \frac{a}{\lambda} \text{ker}^1 \left( \frac{a}{\lambda} \right) \right]$$

(6)

where $a = a/r$, $t^4 = E_1 h^3 / 6k$, $\lambda^4 = s/(s + E_1 / \eta_1)$, and $P$ is the total load. If we expand $\text{ker}^1 (a/\lambda)$ into a series (McLachlan, 1941), eq 6 becomes
The inverse bilateral Laplace transform (App. A) of eq 7 we get

\[ w = \frac{P H(t)}{8k t^2} \left\{ e^{-\xi/2} \left[ (1 + \xi) I_0 (\xi/2) + \xi I_1 (\xi/2) \right] + \right. \\
+ \frac{a^2}{8\pi} \left[ (1 + \xi) \left( -5 + E_1 (\xi) + \ln \frac{5a^4 \xi}{16} \right) + 1 - \xi - e^{-\xi} \right] + R_1 \right\} \]

where \( \xi = E_1 t/\eta_1 \), \( E_1 (\xi) \) is the exponential integral (Abramowitz and Stegun, 1964), not to be confused with the constant \( E_1 \), and \( R_1 \) is the inverse of \( R \).

The maximum moment, which occurs at \( r = 0 \), was obtained from Wyman as

\[ \bar{m} = \frac{p}{s} \left( \frac{1 + \mu}{2} \right) \frac{\lambda}{a} \text{kei}' \left( \frac{a}{\lambda} \right). \]

But in our case \( \mu = \frac{1}{2} \), since \( 3K = \infty \). If we expand \( \text{kei}' \left( \frac{a}{\lambda} \right) \) into a series, eq 9 becomes

\[ \bar{m} = \frac{3p}{32\pi s} \left[ 2 - 4 \ln \frac{a}{2\lambda} + \frac{\pi}{8} \left( \frac{a}{\lambda} \right)^2 + R \right] \]

where \( |R| \leq \frac{1}{48} \left( \frac{a}{\lambda} \right)^4 \left[ -\frac{5}{3} + \ln \frac{a}{2\lambda} \right] \).

The inverse bilateral transform of eq 10 gives

\[ m = \frac{3p H(t)}{32\pi} \left\{ 2 - E_1 (\xi) - \ln \frac{5a^4 \xi}{16} + \right. \\
+ \frac{\pi a^2}{8} e^{-\xi/2} \left[ (1 + \xi) I_0 (\xi/2) + \xi I_1 (\xi/2) \right] + R_1 \right\}. \]

If in eq 4 we let \( q(r, \theta, t) = \dot{q}(r, \theta) t H(t) \), a constant load rate applied at time zero, and take the bilateral Laplace transform, we obtain

\[ \frac{E_1 h^3 s}{6(s + E_1/\eta_1)} \quad \nabla^4 \bar{w} + k \bar{w} = \frac{\dot{q}(r, \theta)}{s^2}. \]

We will consider the same loading distribution as before. A comparison of eq 5 and 12 shows that eq 7 is the solution of eq 12 provided \( P \) is replaced by \( \dot{P}/s \). The constant \( \dot{P} \) is the total load rate. Taking the inverse transform we get

\[ w = \frac{\dot{P} t H(t)}{8k t^2} \left\{ e^{-\xi/2} \left[ (3 + 2\xi) I_0 (\xi/2) + (1 + 2\xi) I_1 (\xi/2) \right] + \right. \\
+ \frac{a^2}{16\pi} \left[ (2 + \xi) \left( -5 + E_1 (\xi) + \ln \frac{5a^4 \xi}{16} \right) + \frac{1}{3} - e^{-\xi} - \frac{3\xi}{2} \right] + R_1 \right\}. \]

To obtain the moment, we replace \( P \) with \( \dot{P}/s \) in eq 10, and the inverse transform gives
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\[
m = \frac{3 \dot{P} t H(t)}{32 \pi} \left\{ 3 - E_1(\xi) - \ln \frac{\gamma a^4 \xi}{16} + \frac{e^{-\xi^2}}{\xi} + \right.
\]
\[
+ \frac{\pi a^2}{8} \frac{e^{-\xi^2/2}}{3} \left[ (3 + 2\xi) I_0(\xi/2) + (1 + 2\xi) I_1(\xi/2) \right] + R_1 \right\}. \tag{14}
\]

At \( t = 0 \), eq 8 and 11 reduce to

\[
w = \frac{P}{8k\ell^2} \left[ 1 + \frac{a^2}{2\pi} \left( -\frac{5}{4} + \ln \frac{\gamma a}{2} \right) \right] \tag{15}
\]

\[
m = \frac{3P}{8\pi} \left[ \frac{1}{2} - \ln \frac{\gamma a}{2} + \frac{\pi a^2}{32} \right], \tag{16}
\]

the case of an elastic plate with \( \mu = \frac{1}{2} \). If \( \dot{P} t \) is considered as the load, eq 13 and 14 also reduce to eq 15 and 16.

Kheisin (1964) gives another solution of eq 4 for a constant concentrated load applied at time zero and for a constant concentrated load rate applied at time zero. Essentially he takes a Fourier transform with respect to \( x \), followed by another Fourier transform with respect to \( y \), where \( x \) and \( y \) are Cartesian coordinates. He then transforms to polar coordinates and takes the inverse Fourier transform with respect to \( \theta \). He leaves the inverse Fourier transform with respect to \( r \) in integral form. For small values of \( \xi \) at \( r = 0 \), he integrates, obtaining

\[
w = \frac{P}{8k\ell^2} \left[ 1 + \frac{\xi}{2} - \frac{\xi^2}{16} + \ldots \right] \tag{17}
\]

for the constant load and

\[
w = \frac{\dot{P} t}{8k\ell^2} \left[ 1 + \frac{\xi}{4} - \frac{\xi^2}{48} + \frac{\xi^3}{384} + \ldots \right] \tag{18}
\]

for a constant load rate. For small values of \( \xi \) and for \( a = 0 \), eq 8 and 13 reduce to eq 17 and 18 respectively.

VISCOUS ANALYSIS

To obtain a simpler solution we will assume that ice is incompressible under hydrostatic stress and that it obeys Newton's law of viscosity for deviatoric stress and strain. Then \( 3K = \infty \) and \( 2G = \eta \partial \theta / \partial t \). Now eq 3 becomes

\[
\frac{h^3}{6} \eta \frac{\partial}{\partial t} \nabla^4 w + k w = q(r, \theta, t). \tag{19}
\]

If \( q(r, \theta, t) = q(r, \theta) H(t) \), as before, the bilateral Laplace transform of eq 19 yields

\[
\frac{\eta h^3 s}{6} \nabla^4 \tilde{w} + k \tilde{w} = \frac{q(r, \theta)}{s}. \tag{20}
\]

With the same load distribution as before, the solution of eq 20 at \( r = 0 \) is eq 7 and the moment is given by eq 10 provided that now \( l^4 = \eta h^3 / 6k \) and \( \lambda^4 = s \). The inverse transforms yield
\[ w = \frac{P}{8kl^2} \frac{\sqrt{4t}}{\pi} \left[ 1 + \frac{1}{16} \sqrt{\frac{\alpha_0^4 t}{\pi}} \left(-6 + \ln \frac{\sqrt{\alpha_0^4 t}}{16} \right) + R_1 \right] \]  

(21)

and

\[ m = \frac{PH(t)}{32} \frac{\sqrt{4t}}{\pi} \left[ 2 - \ln \frac{\sqrt{a_0^4 t}}{16} + \sqrt{\frac{\pi a_0^4 t}{4}} + R_1 \right] \]  

(22)

where \( \alpha_0 = a/l_0 \) and \( l_0^4 = \eta h^3 / 6k \).

If \( q(r, \theta, t) = \dot{q}(r, \theta) t H(t) \), the transform of eq 19 yields

\[ \frac{\eta h^3 s}{6} \nabla^4 \dot{w} + k \dot{w} = \frac{\dot{q}(r, \theta)}{s^2} \]  

(23)

The solution of eq 23 is eq 7 and the moment is given by eq 10, provided \( P = \dot{P}/s \), \( t^4 = \eta h^3 / 6k \), and \( \lambda^4 = s \). The inverse transforms yield

\[ w = \frac{\dot{P}t H(t)}{8kl^2} \frac{2}{3} \frac{\sqrt{4t}}{\pi} \left[ 1 + \frac{3}{64} \sqrt{\frac{\alpha_0^4 t}{\pi}} \left(-\frac{13}{2} + \ln \frac{\sqrt{\alpha_0^4 t}}{16} \right) + R_1 \right] \]  

(24)

and

\[ m = \frac{\dot{P}t H(t)}{32} \frac{3}{\pi} \frac{\sqrt{4t}}{\pi} \left[ 3 - \ln \frac{\sqrt{a_0^4 t}}{16} + \sqrt{\frac{\pi a_0^4 t}{6}} + R_1 \right] . \]  

(25)

**RESULTS**

\( \alpha_0^4 t \) in the viscous solution is equal to \( \alpha^4 \xi \) in the viscoelastic solution. For large values of \( \xi \), eq 8, 11, 13 and 14 reduce to eq 21, 22, 24, and 25 respectively. Since the viscoelastic solution for large \( \xi \) and the viscous solution for all \( \xi \) depend only on \( \alpha^4 \xi \) it seems reasonable to say that the solutions are valid for small \( \alpha^4 \xi \) when \( R_1 \) is neglected. Equations 8 and 21 are shown in Figure 3 and eq 13 and 24 are shown in Figure 4. In all cases, \( R_1 \) is neglected. From these figures it can be seen that for small \( \alpha \) the deflections do not significantly depend on \( \alpha \). In most applications \( \alpha \) will be small. For small \( \alpha \), large \( \xi \), and a small \( \alpha^4 \xi \), eq 8 and 13 become respectively

\[ \frac{wh^{3/2}}{P \sqrt{t}} = \frac{1}{4} \sqrt{\frac{6}{\pi k \eta_1}} \left[ 1 + \frac{\eta_1}{4E_1 t} \right] \]  

(26)

and

\[ \frac{wh^{3/2}}{\dot{P}t \sqrt{t}} = \frac{1}{\sqrt{6\pi k \eta_1}} \left[ 1 + \frac{3\eta_1}{4E_1 t} \right] . \]  

(27)

G. Frankenstein (In prep.) has performed constant load rate tests on fresh water lake ice at Point Barrow, Alaska. As an example we shall use his results from test no. 8, Oct 28, 1961, for which \( \dot{P} = 2712 \text{ lb/min} \), \( h = 13.5 \text{ in.} \), and \( a = 1 \text{ ft} \). The test results plotted in Figure 5 agree in form with the viscoelastic solution of Figure 4 except near the time of failure.

The constants \( E_1 = 9.445 \times 10^6 \text{ lb/ft}^2 \) and \( \eta_1 = 23.03 \times 10^6 \text{ lb-min/ft}^2 \) were determined by a least-square fit of the data to eq 27. For this calculation, the data were eliminated for \( t < 1.2 \text{ min} \), since eq 28 is not applicable for small \( \xi \), and for \( t > 7.5 \text{ min} \), since the time is approaching the failure time. Thus in our example \( l = 13.77 \text{ ft} \) and \( a = 0.0726 \). Equation 13 is also shown in Figure 5 for the above values of \( E_1 \), \( \eta_1 \), and \( a \).
The ratio of the constant load moment in eq 11 to the elastic moment in eq 16 is shown in Figure 6. The ratio of the constant load rate moment in eq 14 to the elastic moment in eq 16 is shown in Figure 7. It can be seen that the moments for any time are reduced very little from the elastic moments. In our example, failure occurred at \( t = 9.7 \) min. Thus the stress at failure was

\[
\sigma = 6 \text{ m}\text{h}^2 = 306.9 \text{ lb/in.}^2.
\]

![Figure 3. Constant load deflections.](image)

![Figure 4. Constant load rate deflections.](image)
Figure 5. Observed deflections.

Figure 6. Ratio of constant load viscoelastic moment to the elastic moment.
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Figure 7. Ratio of constant load rate viscoelastic moment to the elastic moment

LITERATURE CITED


Frankenstein, G. (In prep.) Strength data of an Arctic lake, U. S. Army Cold Regions Research and Engineering Laboratory (USA CRREL).


## APPENDIX A: INVERSE BILATERAL LAPLACE TRANSFORMS

<table>
<thead>
<tr>
<th>$g(s)$</th>
<th>$L^{-1}[g(s)]=f(t)$</th>
<th>Erde'lyi, 1954</th>
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<td>$s^{-1}$</td>
<td>$H(t)$</td>
<td></td>
</tr>
<tr>
<td>$s^{-2}$</td>
<td>$H(t) t$</td>
<td></td>
</tr>
<tr>
<td>$s^{-3}$</td>
<td>$H(t) t^2/2$</td>
<td></td>
</tr>
<tr>
<td>$s^{-3/2}$</td>
<td>$H(t) (2t/\pi)^{1/2}$</td>
<td>p 235, eq 29</td>
</tr>
<tr>
<td>$s^{-5/2}$</td>
<td>$H(t) (4/3) (t^3/\pi)^{1/2}$</td>
<td>p 235, eq 29</td>
</tr>
<tr>
<td>$(s+c)s^{-2}$</td>
<td>$H(t) (1+ct)$</td>
<td>p 229, eq 2</td>
</tr>
<tr>
<td>$(s+c)s^{-3}$</td>
<td>$H(t) t(1+ct/2)$</td>
<td>p 229, eq 6</td>
</tr>
<tr>
<td>$s^{-1} \ln s$</td>
<td>$H(t) (-\ln \gamma t)$</td>
<td>p 250, eq 1</td>
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<td>$s^{-2} \ln s$</td>
<td>$H(t) \gamma (1-\ln \gamma t)$</td>
<td>p 250, eq 2</td>
</tr>
<tr>
<td>$s^{-3} \ln s$</td>
<td>$H(t) (3-2 \ln \gamma t)(t^2/4)$</td>
<td>p 250, eq 2</td>
</tr>
<tr>
<td>$\ln [(s+c)/s]$</td>
<td>$H(t) (1-e^{-ct})t^{-1}$</td>
<td>p 251, eq 12</td>
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<td>$s^{-1} \ln[(s+c)/s]$</td>
<td>$H(t) [E_1(\gamma ct)+\ln \gamma ct]$</td>
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</tr>
<tr>
<td>$s^{-2} \ln [(s+c)/s]$</td>
<td>$H(t) t[(1-e^{-ct})(ct)^{-1}-1+E_1(\gamma ct)+\ln \gamma ct]$</td>
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</tr>
<tr>
<td>$(s+c)s^{-2} \ln[(s+c)/s]$</td>
<td>$H(t) [1-e^{-ct}-ct(1+ct)(E_1(\gamma ct)+\ln \gamma ct)]$</td>
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</tr>
<tr>
<td>$(s+c)s^{-3} \ln[(s+c)/s]$</td>
<td>$H(t) (t/2)[(1-e^{-ct})(ct)^{-1}-e^{-ct}+3ct/2+E_1(\gamma ct)+\ln \gamma ct)]$</td>
<td></td>
</tr>
<tr>
<td>$(s+c)^{1/2} s^{-3/2}$</td>
<td>$H(t) e^{-ct/2}[(1+ct)I_0(ct/2)+ctI_1(ct/2)]$</td>
<td>p 235, eq 25</td>
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<tr>
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<td>$H(t)(t/3)e^{-ct/2}[(3+2ct)I_0(ct/2)+(1+2ct)I_1(ct/2)]$</td>
<td></td>
</tr>
</tbody>
</table>

*Convolution:* E$_1(\gamma ct)+\ln \gamma ct$
A solution for a viscoelastic plate on an elastic foundation is presented for an infinite bulk modulus and a shear modulus which obeys Maxwell's model. Observed deflections of a floating ice sheet agree with this solution. A solution for a viscous shear modulus is also presented.
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