ON THE THEORY OF GROUND ANCHORS

Austin Kovacs, Scott Blouin
Bruce McKelvy and Herman Colligan

January 1975
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PREFACE

This report was prepared by Austin Kovacs and Scott Blouin, Research Civil Engineers, and Bruce McKelvey and Herman Colligan, Research Assistants, of the Foundations and Materials Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory.

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<tr>
<td>a</td>
<td>width</td>
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<tr>
<td>$A$</td>
<td>cross-sectional area of anchor base (plate, bell, etc.)</td>
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<tr>
<td>$A_b$</td>
<td>cross-sectional area of chimney or shaft</td>
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<tr>
<td>$A_c$</td>
<td>surface area of chimney above the stress zone</td>
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<td>$A_g$</td>
<td>Wilson and Hilts general load coefficient</td>
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<tr>
<td>$A_1$</td>
<td>circumferential area of pile or earth cylinder formed above an anchor's base</td>
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<td>$A_m$</td>
<td>Wilson and Hilts load coefficient for moment</td>
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<td>$A_s$</td>
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<td>$A_v$</td>
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<td>Wilson and Hilts load coefficient for soil reaction</td>
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<td>$A_y$</td>
<td>Wilson and Hilts load coefficient for deflection</td>
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<td>b</td>
<td>length of rectangular anchor</td>
</tr>
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<td>$b_t$</td>
<td>Tsytovich's temperature dependent parameter of continuous adfreezing strength</td>
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<tr>
<td>$B$</td>
<td>width of stress bulb for belled anchors</td>
</tr>
<tr>
<td>$B_g$</td>
<td>Wilson and Hilts general sublettered moment coefficient</td>
</tr>
<tr>
<td>$B_m$</td>
<td>Wilson and Hilts moment coefficient for moment</td>
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<td>anchor base diameter</td>
</tr>
<tr>
<td>$d_1$</td>
<td>dimension = $h \tan \beta$</td>
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<tr>
<td>$d_2$</td>
<td>side dimension of Universal Ground Anchor (Fig. 27)</td>
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<tr>
<td>$D_d$</td>
<td>relative density</td>
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$D_0$ anchor shaft or rod diameter
$e$ void ratio
$E$ Young's modulus of elasticity
$F_{1-2}$ idealized maximum stress distribution
$f$ coefficient of friction at anchor/soil interface
$f_c$ coefficient of friction between concrete and soil (Table II)
$f'c$ concrete unconfined compressive strength
$F_a$ unit adhesion
$F_f$ unit friction
$F_s$ suction force
$g$ specific gravity in relation to pure water (unitless)
$h$ depth of anchor or stake below soil surface
$h'$ height of stress zone
$h_c$ critical depth of a particular anchor as defined by an established critical depth ratio and the anchor's diameter or $h_c = (d)$ (critical depth ratio)
$h_2$ depth to top of anchor base
$H$ depth of anchor in the active layer
$H_a$ depth of portion of active layer capable of adfreezing
$H_b$ depth of anchor below frozen soil layer (Fig. 62)
$H_f$ depth of anchor into permafrost
$H_r$ horizontal resistance
$I$ moment of inertia of the cross section of a pile or stake
$K_{1-4}$ Matsuo and Tagawa pullout strength factors
$K$ coefficient of earth pressure
$K_a$ coefficient of active earth pressure
$K_d$ Dewberry multiplication factor (Fig. 7)
$K_j$ Jaky's surface area factor for stress bulb influence
$K_k$ Porkhaev's coefficient of anchor pullout force
$K_m$ subgrade modulus
$K_0$ coefficient of earth pressure at rest
$K_p$ coefficient of passive earth pressure
$L$ length
$m$ reduction factor (Biarez and Barraud)
$M$ moment
$M_c$ Biarez and Barraud cohesion coefficient
$M_{c0}$ Biarez and Barraud cohesion term
$M_g$ moment at ground surface

$M_p$ Biarez and Barraud overburden coefficient

$M_t$ plate uplift force factor

$M_{e2}$ Biarez and Barraud anchor plate uplift force factor for rectangular anchors

$M_u$ chimney and pad uplift factor for deep anchors with chimneys when $\phi > 15^\circ$

$M_x$ moment on pile at depth

$M_y$ Biarez and Barraud gravity coefficient (Fig. 20)

$M_{\phi}$ Biarez and Barraud friction coefficient (Fig. 20)

$n$ factor of safety

$n_n$ constant of horizontal subgrade reaction

$N$ exponent of characteristic length for stiffness

$N_c$ bearing-capacity factor for cohesive soils

$N_q$ Terzaghi’s dimensionless bearing-capacity factor of Universal Ground Anchors in cohesionless soils

$p$ rectangular anchors’ horizontal perimeter at any specified height, $2\pi R$ or $2\pi R_o$

$P$ load

$P_a$ constant of horizontal subgrade reaction

$P_b$ uplift resistance of stress zone

$P_c$ total frictional force

$P_g$ long term load

$P_h$ horizontal earth pressure

$P_{\text{max}}$ maximum anchor load

$P_r$ resultant of forces $P_x$ and $P_y$

$P_u$ load per unit length

$P_{\text{ult}}$ ultimate load

$P_x$ lateral or horizontal axial load

$P_y$ perpendicular axial load

$P_1, P_2$ horizontal perimeters around anchor within active and permanently frozen soil, respectively

$q$ surcharge load on soil developed

$Q$ bearing force

$Q_f$ frictional resistance

$Q_s$ shear resistance

$R$ radius

$R_e$ equivalent radius for rectangular anchors
$R_0$ radius of anchor shaft or chimney
$S$ unit shear strength of soil
$S_{\text{max}}$ maximum shear stress of material in which an anchor is placed
$S_{n}$ surface tensile stress perpendicular to shearing stress
$S_x, S_y$ Brom's normal stresses subject to a soil element, $x$ and $y$ direction, respectively
$t$ anchor base thickness
$T$ absolute temperature below freezing, °C
$V$ volume of soil confined within failure planes or shearing boundaries
$V_x$ shear on pile at depth $x$
$V_1$ volume of all soil directly over an anchor
$V_2$ volume of soil within failure boundary less volume of soil directly over the anchor (Fig. 11)
$V_3$ volume of footing shaft
$W_c$ critical lateral soil reaction
$W_x$ soil reaction at depth $x$
$W_a$ weight of anchor or anchor and soil forming fictitious pile
$W_p$ weight of anchor base
$W_s$ weight of soil within failure plane
$W_{s1}$ weight of earth column extending above an anchor plate
$W_{s2}$ weight of soil confined within failure planes less the weight of soil confined directly above the anchor ($W_s - W_{s1}$)
$\Delta W$ anchor weight less weight of soil displaced by the anchor
$X$ expediential constant for the shearing method
$x$ depth below surface
$y$ distance to neutral stress
$y_x$ pile deflection in horizontal direction at depth $x$
$Z$ Wilson and Hilts relative stiffness factor
$Z'$ stiffness characteristic length for stakes and piles
$\alpha$ angle of shear plane
$\beta$ $(45^\circ - \phi/2)$
$\beta'$ assumed angle of failure plane
$\gamma$ unit weight of unfrozen soil
$\gamma_d$ dry unit weight of soil
$\gamma_m$ unit weight of frozen soil
$\delta$ soil deflection due to stake or pile placement
δ'  permissible creep or creep limit of soil for pile and stakes
φ_x  pile slope at depth x due to deflection
λ  anchor depth to diameter ratio or form coefficient
μ  Marinpol'skii dimensionless function
σ  undrained strength
r  Mors' failure plane dimension found by geometry
r_a  adfreeze strength in active layer
r_ad  adfreeze strength between the anchor and frozen soil in lb/ft²
r_p  adfreeze strength developed in the permafrost
r_s  temporary adfreeze strength or ultimate adfreezing strength
ϕ  angle of internal friction
ON THE THEORY OF GROUND ANCHORS

by

Austin Kovacs, Scott Blouin, Bruce McKelvy and Herman Colligan

INTRODUCTION

Foundation design has long presented a problem to engineers. But with the aid of soil mechanics (although this is not an exact science), engineers have in recent years been able to design foundations bearing downward loads with reasonable confidence in the soil’s performance. Furthermore, foundation theory and practice are fairly well documented in textbooks and science journals.

Anchorages are used in the design of many types of structures – power transmission towers, bulkheads, bridges, retaining walls, moorings, pipelines, any type of guyed structure and even temporary buildings and tents. However, the design of anchorages is not as well defined as the design of foundations; and there is no evidence of a general theoretical or scientific method that meets specific engineering needs. Therefore, because soil and anchor parameters vary, there is no single solution for all anchoring situations.

More information is needed on the holding capacity of anchors and on methods for installing them. Soils which possess adequate anchorage capability for one type anchor may, on the other hand, produce a problem in installation, or vice versa.

In short, the design and installation of anchors present complex problems. The objective of this report is to present analytical solutions and test data to enhance the understanding of the limitations of various anchor designs and anchoring techniques. A broad spectrum of theories is presented to make possible analyses of individual anchoring problems. When possible, calculated anchor capacity and field test results are compared. However, these comparisons are few, owing to differences in test techniques, lack of conclusive test results, and vast differences in the types of soils involved.

ANCHOR TYPES

There are such a large number of anchor types that a complete listing will not be attempted here. The type of anchor employed in any specific situation is a function of the load and the soil. Some of the more common anchors used for light loads are the mechanical types such as the screw anchor, expanding or spreading anchor, and various configurations of plates, disks, cones, crosses, etc. (Fig. 1). They are generally used to anchor guy wires against relatively light to moderate loads. For instance, they are extensively used by power companies to brace poles or small towers. Recommended design loads are usually specified by the manufacturer according to anchor type, size and some measure of the soil type and condition. Under ideal conditions the maximum loads recommended
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Figure 1. Typical mechanical anchor configurations (from Chance 1960).
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A. Steel Grillage

B. Formed, Concrete Footing

C. Caisson with Enlarged Base

D. Straight Caisson

E. Piles

F. Malone Anchor

G. Block Anchor

H. Grouted Anchor

Figure 2. Miscellaneous anchor types (after Flucker and Teng 1965).

for the larger mechanical type anchors are generally in the range of 20,000 to 40,000 lb. In practice, power companies rely heavily on past experience in choosing anchors for a particular application. Since these anchors are relatively cheap and easy to install, additional anchors can be utilized at any time, should the initial anchors prove inadequate.

A second class of anchors (Fig. 2) requires considerably more effort to install than the mechanical types, and hence is usually used for loads in excess of the capacities of the cheaper mechanical anchors. Some of these anchor types sometimes serve a dual roll, as combination foundations and anchors, an example being the foundation piers for large power transmission towers. Normally the piers would serve to support the weight of the tower on the underlying soil; however, during periods of high wind the piers may act as anchors in resisting the large negative moments which tend to overturn the towers.

The steel grillage foundation, type A of Figure 2, is commonly used to support power transmission towers. It is installed in open excavations or, where conditions permit, in augered holes. The grillage generally consists of a number of steel beams arranged in a variety of patterns.
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The concrete footing, type B, is costly and therefore is used infrequently. The concrete caisson with an enlarged base, type C, is used in cohesive soil where the cohesion will permit under-reaming the base without cave-in. The enlarged base greatly enhances both bearing capacity and pullout resistance. The straight caisson can be used in cohesionless soils and is formed using the bentonite slurry method.

Piles have a wide range of application as anchors. As combination foundation anchorages they are used where suitable support material is at too great a depth to economically use shallow foundations. Piles are also frequently used to resist lateral loads in such applications as tie-backs for retaining walls and bulkheads and in foundations subjected to wind, explosions, earthquakes and thermally induced lateral forces.

A variation of the pile is the ground stake. This form of anchor usually satisfies simple anchoring requirements and in most cases is temporary. The simplest ground stake is a rod driven into the ground with a sledgehammer. Larger stakes may be driven by some mechanical means and retrieved similarly. For purposes of this paper a ground stake is defined as a rigid body while a pile is treated as non-rigid. Rigidity is described in terms of both the pile and soil properties and is defined in the section on miscellaneous anchors.

The Malone anchor consists of a rod or angle extending into a ball of grout. It is best suited to cohesive soils where danger of collapse of the cavity is minimized.

The block anchor finds a wide range of application. The block is usually constructed of reinforced concrete and connected to the structure by means of rods and/or cables.

Grouted anchors are used in both rock and soil. Design and installation techniques vary widely.

OVERALL ANCHOR PERFORMANCE

The idealized performance of an anchor under load is shown in Figure 3. Under small loads, movements are elastic and the initial portion of the curve is nearly a straight line. As the load is increased plastic failure zones develop around the anchor and work outward. After a maximum force $P_{\text{max}}$ is reached, the anchor continues to move, even though, in some cases, the load may fall below $P_{\text{max}}$.

Anchor design is governed, in part, by the depth of burial. Generally speaking, if the plastic failure zones around the anchor intersect the ground surface the anchor is considered shallow; if not, it is considered a deep anchor. Anchor depth is normalized with respect to the anchor base width or diameter to give a dimensionless depth ratio, $h/d$. Anchors having depth ratios greater or less than a designated critical depth ratio are considered "deep" and "shallow" respectively. However, there is considerable variation in opinion as to what ratio value is critical.

A critical depth ratio of 6 was arrived at in an investigation performed in sand with circular anchors by Baker and Kondner (1965). This critical depth ratio was based on the shape of a failure surface proposed by Balla (1961). It was observed that for anchors with an $h/d$ ratio less than 6, the pullout test results were very close to those predicted by the Balla equation. At failures with $h/d$ ratios less than 6, a curvilinear failure plane was observed with an accompanying upheaval of the ground surface above the anchor. Anchors having an $h/d$ ratio greater than 6 did not cause an upheaval of the ground surface nor did a curvilinear failure plane appear until the anchor had been drawn upwards such a distance that the $h/d$ ratio became less than 6. At that point, a curvilinear plane was observed as in the case of the shallow anchor. In all cases where the $h/d$ ratio was greater than 6, the Balla analysis gave pullout capacity greater than that actually developed.
Kananyan (1966) made a series of tests on model anchors with base diameters of 15.7, 23.6, 31.5, 39.4 and 47.3 in. A total of 30 experiments were performed: 17 vertical embeddings and 13 oblique embeddings. All models were embedded 39.4 in. The anchors were set on a base of alluvial fine-grained sand. Backfill consisted of densely tamped sand having a unit weight of 101.5 lb/ft$^3$.

These tests revealed that soil deformation was the result of vertical pressure and horizontal thrust and that heaving of the soil was preceded by the formation of radial cracks (Fig. 4), which appeared near the column of the anchor at 70-80% of the ultimate load. The appearance of these cracks coincided with an overall loosening of the soil. As the load increased, the radial cracks propagated to a circular crack where the cracks intersected. After the appearance of the first circular crack, the radial cracks extended further, surface deformations increased markedly, and complete failure of the base quickly followed. Soil movement at failure caused a second circular crack to form (Fig. 4). Consequently, rupturing in radial planes occurred earlier than shearing along the circular planes.

Figure 4 shows that the failure plane is curvilinear. The angle $\beta$ was found to be equal to $(45^\circ - \phi/2)$ where $\phi$ is the angle of internal friction.

Kananyan found that at greater depths less upheaval of the ground surface was exhibited. Therefore, he considered anchors with an $h/d$ ratio greater than or equal to 3 as being deep even though the anchors he tested had $h/d$ ratios of 2.5 or less.

Turner (1962) found from tests on the uplift resistance of transmission tower footings that ground surface movement was relatively small for anchors having an $h/d$ ratio greater than 1.5 (Fig. 5). These results are plotted as the ratio of ground surface movement to footing movement versus the ratio of depth to diameter. Figure 5 shows that at an $h/d$ ratio of 6 the ground surface movement was zero.

From these results, Turner defined anchors with an $h/d$ ratio greater than 1.5 as deep and those with an $h/d$ ratio less than 1.5 as shallow. This ratio is considerably lower than the $h/d$ value of 6 proposed by Baker and Kondner (1965), who gave the value of 0 ground surface movement to mark the division for deep and shallow anchors.
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Figure 4. Sand displacement observed along model anchors tested by Kananyan (1966).

Figure 5. Ratio of ground surface movement to anchor movement vs ratio of anchor depth to anchor diameter (after Turner 1962).

A critical depth ratio of 6 apparently can be used to define a deep anchor in soils exhibiting high viscosity. For ratios greater than 6, failure or displacement of the soil appears to occur in the immediate vicinity of the anchor base with no manifestation of movement at the soil surface. This has been shown by Baker and Kondner for soils and by Kovacs (1967) for polar snow. However, for soil with low viscosity, the critical depth ratio may be lower than 6, owing to continuous flow around the anchor at shallower depths before surface rupture occurs and $P_{ult}$ is reached.

DESIGN OF SHALLOW ANCHORS

There are three basic approaches to the design of shallow anchors (Flucker and Teng 1965): the cone method, the earth pressure method and semiempirical methods. The cone method attempts to estimate the true failure surface surrounding the anchor. In its basic application the uplift resistance is obtained solely from the weight of the anchor plus the weight of the soil within the assumed failure planes. There are many variations of the cone method which include a variety of assumed failure planes. In addition cohesive and friction forces acting along the failure planes are often added to the dead weight resistance.

The earth pressure method disregards the actual shape of the failure planes. The failure plane is assumed to rise vertically from the perimeter of the anchor footing to the ground surface. Pullout
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Figure 6. Soil boundaries for earth cone method (Flucker and Teng 1965).

resistance is obtained by adding the weight of the anchor and soil within the failure surface to the friction developed along the sides of the vertical failure plane.

Cone Method

Perhaps the most common method used in the past for determining the uplift capacity of an anchor with an \( h/d \) ratio < 6 was the earth cone method (Flucker and Teng 1965). This method assumes that failure occurs along a plane inclined at the angle \( \beta' \) (Fig. 6). The uplift capacity as calculated from the earth cone method is:

\[
P_{\text{ult}} = W_s + \Delta W
\]

where

- \( W_s \) = weight of soil within failure plane
- \( \Delta W \) = anchor weight less weight of soil displaced by anchor.

Since only dead weight is considered in this analysis, the ultimate load calculated is equivalent to the ultimate load shown in the idealized performance curve of Figure 3 and may be less than the maximum uplift developed. From Figure 6, \( W_s \) can be derived by geometry:

Rectangular footings:

\[
W_s = h \gamma \left( ab + ad_1 + bd_1 = \frac{\pi}{3} d_1^2 \right),
\]

Circular footings:

\[
W_s = \frac{\pi}{12} h \gamma (3a^2 + 6ad_1 + 4d_1^2)
\]

where

- \( \gamma \) = unit weight of the soil
- \( h \) = depth
- \( a \) = anchor width
- \( b \) = anchor length
- \( d_1 = h \tan \beta' \)
- \( \beta' \) = assumed angle of failure plane.
The uplift capacity as computed by this method obviously varies with the assumed angle of $\beta'$. Different organizations using this method have adopted a value for $\beta'$ dependent upon the underlying soil:

The American Bridge Company

$\beta' = 30^\circ$ for all soils (in the absence of other specifications)

Bureau of Reclamation

$\beta' = 30^\circ$ for footings poured against undisturbed ground with an undercut and incorporating a safety factor of 1.0.

$\beta' = 20^\circ$ for footings backfilled around all sides with a safety factor of 1.5. There is a limitation here to an upward pressure above the anchor not to exceed 1000 lb/ft$^2$ for each foot of embedment below ground.
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Table I. TVA recommended values of $\beta'$ earth cone analysis.

1. Values may be increased 75% for ultimate loads of short duration.
2. Weight of soil per cubic foot = 100 lb for A, 100 lb for B, and 69 lb for C.
3. Vertical pressure intended to cover bearing at bottom of footing and bearing against soil covering footing.

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Vertical pressure (lb/ft$^2$)</th>
<th>Cone angle $\beta'$ (°)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quicksand and alluvial</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Soft clay</td>
<td>1,000</td>
<td>0</td>
</tr>
<tr>
<td>Moderately dry clay, clay and sand</td>
<td>2,000</td>
<td>0</td>
</tr>
<tr>
<td>Dry loam and clay</td>
<td>3,000</td>
<td>0</td>
</tr>
<tr>
<td>Fine firm sand</td>
<td>4,000</td>
<td>0</td>
</tr>
<tr>
<td>Compact coarse sand</td>
<td>5,000</td>
<td>0</td>
</tr>
<tr>
<td>Compact coarse gravel</td>
<td>8,000</td>
<td>0</td>
</tr>
<tr>
<td>Cemented sand and gravel</td>
<td>10,000</td>
<td>0</td>
</tr>
<tr>
<td>Good hardpan and hard shale</td>
<td>12,000</td>
<td>0</td>
</tr>
</tbody>
</table>

Footings against undisturbed natural ground

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Vertical pressure (lb/ft$^2$)</th>
<th>Cone angle $\beta'$ (°)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quicksand and alluvial</td>
<td>1,000</td>
<td>0</td>
</tr>
<tr>
<td>Soft clay</td>
<td>2,000</td>
<td>0</td>
</tr>
<tr>
<td>Moderately dry clay, clay and sand</td>
<td>4,000</td>
<td>0</td>
</tr>
<tr>
<td>Dry loam and clay</td>
<td>6,000</td>
<td>0</td>
</tr>
<tr>
<td>Fine firm sand</td>
<td>6,000</td>
<td>0</td>
</tr>
<tr>
<td>Compact coarse sand</td>
<td>8,000</td>
<td>0</td>
</tr>
<tr>
<td>Compact coarse gravel</td>
<td>12,000</td>
<td>0</td>
</tr>
<tr>
<td>Cemented sand and gravel</td>
<td>16,000</td>
<td>0</td>
</tr>
<tr>
<td>Good hardpan and hard shale</td>
<td>20,000</td>
<td>0</td>
</tr>
</tbody>
</table>

* Condition of soil: A = Naturally well drained.
B = Subject to periodic flooding of short duration.
C = Subject to ground water several months of the year.

Tennessee Valley Authority (TVA): the values for $\beta'$ are listed in Table I.

Dewberry (1962) developed a graphical means for determining the capacity of a circular anchor based upon the earth cone method (Fig. 7). He assumed that soil failure occurs along a conical plane extending up from the anchor base. The volume of earth $V$ inclosed within the failure zone is approximated by:

$$V = Ah + A^{0.5}h^2 + 0.35h^3$$  \hspace{1cm} (4)

where $$A = \text{area of anchor}$$
$$h = \text{depth}.$$  

Thus, anchor holding power is determined by:

$$P = K_d V = K_d (Ah + A^{0.5}h^2 + 0.35h^3)$$  \hspace{1cm} (5)
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Figure 8. Soil boundaries for the Balla cone method (Balla 1961).

Figure 9. Graphs for the Balla coefficient $B_1$ and $B_2$ (for $c = 0$) (after Balla 1961).

where $K_d =$ multiplying factor governing various soil conditions (see Fig. 7).

Balla cone method

A variation of the cone method was proposed by Balla (1961) for circular shallow anchors with $h/d$ ratios < 6. Balla’s method is based on a parabolic failure surface (Fig. 8), the curvature being a function of the soil’s angle of internal friction. In addition to the weight of soil and anchor, Balla’s method takes into account the friction and cohesive forces acting along the failure surfaces. Thus, the uplift capacity calculated by Balla’s method is equivalent to the maximum load shown in the idealized performance curve of Figure 3.
The uplift capacity is given by the following:

\[ P_{\text{max}} = W_s + \Delta W + Q_f \]  

where
- \( W_s \) = weight of soil within failure plane
- \( \Delta W_a \) = weight of anchor less weight of soil displaced by anchor
- \( Q_f \) = frictional and cohesive resistance developed along failure plane.

\( W_s \) and \( Q_f \) are combined to form

\[ W_s + Q_f = h^3 y B_1 + h^2 c B_2 \]

where
- \( h \) = depth of anchor
- \( y \) = unit weight of soil
- \( c \) = unit cohesion
- \( B_1 \) and \( B_2 \) = values derived as functions of the \( h/d \) ratio, the angle of internal friction \( \phi \), and the unit cohesion of soil \( c \).

Values of \( B_1 \) and \( B_2 \) for cohesionless soils can be obtained from Figure 9.

Balla has shown close correlation between his theoretical load pullout strength values and laboratory tests made on anchors embedded in sand. Photographs of his tests show that the failure plane is convex, with the curve starting out vertically from the upper plane of the foundation slab, curving outwardly from the axis of symmetry of the anchor, and intersecting the ground level at an angle approximately equal to \( (45^\circ - \phi/2) \) (Fig. 8).

Paterson and Urie (1964) made full-scale uplift tests on tower foundations in clay and sandy soils of different shear strengths and compared their findings with load capacities determined by the Balla analysis. The anchors pulled were of the inverted mushroom type (bellbottom) and were constructed of concrete. In most cases the \( h/d \) ratio was less than 6. Excellent agreement was found using Balla's formula for the anchors embedded in sandy soils. However, very poor correlation was found for the same anchor embedded in clay at a comparable depth. In each instance, use of the Balla formula resulted in a gross overestimation of the pullout resistance. For example, the calculated resistance of the mushroom-type foundation in clay with a cohesion of 10 psi was 240,000 lb compared with a test value of 77,500 lb.

Flucker and Teng (1965) caution that the Balla analysis is likely to result in an overestimation of the maximum uplift capacity. This is because failure is progressive and therefore friction and cohesion are fully effective only in a limited zone at any given time. However, the Balla method has the advantage of determining anchor holding capacity by the application of basic soil properties and geometric relations, rather than arbitrary parameters.

**Matsuo and Tagawa cone method**

Matsuo and Tagawa (1968) proposed a modification of the cone method applicable to circular footings evidently in cohesionless soil. They define a failure plane, shown in Figure 10, which is a function of the angle of internal friction and consists of a logarithmic spiral and its tangential
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straight line. The analysis is limited to shallow anchors with \( h_2/R \leq 10 \). The maximum uplift resistance is given by:

\[
P_{\text{max}} = W_a - \gamma V_3 + \gamma R^3 K_1 (h_2/R)^{K_2} + cR^2 K_3 (h_2/R)^{K_4}
\]

where

\[
P_{\text{max}} = \text{ultimate load}
\]

\[
W_a = \text{weight of anchor}
\]

\[
\gamma = \text{unit weight of soil}
\]

\[
V_3 = \text{volume of footing shaft}
\]

\[
R = \text{anchor base radius}
\]

\[
h_2 = \text{depth to top of anchor base}
\]

\[
c = \text{unit cohesion}
\]

\[
\phi = \text{angle of internal friction, in radians}
\]

\[K_1, K_2, K_3, K_4 = \text{pullout strength factors dependent on the scaled depth of burial and angle of internal friction as follows:}\]

<table>
<thead>
<tr>
<th>( (h_2/R) ) limit</th>
<th>( K_1 )</th>
<th>( K_2 )</th>
<th>( K_3 )</th>
<th>( K_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.5 \leq \frac{h_2}{R} \leq 1 )</td>
<td>0.056( \phi ) + 4.0</td>
<td>0.007( \phi ) + 1.0</td>
<td>0.027( \phi ) + 7.653</td>
<td>0.002( \phi ) + 1.052</td>
</tr>
<tr>
<td>( 1 \leq \frac{h_2}{R} \leq 3 )</td>
<td>0.056( \phi ) + 4.0</td>
<td>0.016( \phi ) + 1.1</td>
<td>0.027( \phi ) + 7.653</td>
<td>0.004( \phi ) + 1.103</td>
</tr>
<tr>
<td>( 3 \leq \frac{h_2}{R} \leq 10 )</td>
<td>0.597( \phi ) + 10.4</td>
<td>0.023( \phi ) + 1.3</td>
<td>0.013( \phi ) + 6.110</td>
<td>0.005( \phi ) + 1.334</td>
</tr>
</tbody>
</table>

Although the load capacities determined by Matsuo and Tagawa show good agreement with Balla’s test results, eq 7 may nevertheless be limited by the same factors that seem to affect the universality of Balla’s results.

Marinpol’skii cone method

Marinpol’skii (1965) proposed another variation to the cone method for circular footings having \( h/d < 6 \). According to his analysis (Fig. 11) the maximum load, as defined in Figure 3, is:

\[
P_{\text{max}} = W_a + W_{s1} + \gamma V_2 + Q_s
\]

where

\[
W_a = \text{weight of anchor}
\]

\[
W_{s1} = \text{weight of earth column extending above anchor plate}
\]

\[
\gamma = \text{unit weight of soil}
\]

\[
V_2 = \text{volume of conical soil section (see Fig. 11)}
\]

\[
Q_s = \text{shear resistance developed along failure plane.}\]
Marinpol’skii combined the last three terms of eq 8 into:

\[
W_s + \gamma V_2 + Q_s = \pi (R^2 - R_0^2) \frac{y h [1 - (R_0/R)^2 + K_a \tan \phi (h/R)] + 2Sh/R}{1 - (R_0/R)^2 - \mu h/R}
\]  

(9)

where  
\( R \) and \( R_0 \) = radii (Fig. 11)  
\( K_a = \) coefficient of active earth pressure \([K_a = \tan^2 (45^\circ - \phi/2)]\)  
\( \mu = \) dimensionless function of the angle of internal friction derived from the results of anchor tests made in the laboratory and field (Fig. 12)  
\[ S = \text{unit shear strength of soil} \]

or

\[ S = c + P_h \tan \phi \]  

(10)
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where \( P_h \) = horizontal earth pressure at base of anchor.

Therefore

\[
P_{\text{max}} = W_a + \pi \left( R^2 - R_0^2 \right) \frac{\gamma h \left[ 1 - \left( \frac{R_0}{R} \right)^2 \right] + K_a \tan \phi (h/R) + 2Sh/R}{1 - \left( \frac{R_0}{R} \right)^2 - \mu h/R}.
\] (11)

Marinpol'skii compared the theoretical values for maximum load, calculated from eq 11, to loads obtained by Kananyan (1966), who conducted full-scale tests of anchors in sand and loose and dense silt.

Marinpol'skii's theoretical values were similar to Balla's (1961) results for cohesionless soils and were in reasonable agreement with Kananyan's (1966) test values for shallow anchors (\( h/d \) ratio about 0.83 to 1.67) in cohesionless soil. Again, failure, particularly in a cohesive soil, is progressive and the cohesive force will not be developed over the entire failure surface simultaneously as eq 11 assumes. Flucker and Teng's caution on this would seem to apply to Marinpol'skii's analysis as well as to Balla's.

Earth Pressure Method

The earth pressure method of determining the uplift capacity of an anchor (Flucker and Teng 1965) is also known as the friction cylinder method, the Swiss Formula, or Frohlick Majers' procedure. This method is based upon conditions where the \( h/d \) ratio is less than 6. The method relates anchor pullout force to the friction developed along the sides of a vertical prism (Fig. 13) with a cross section equal to the base of the anchor.

The uplift capacity is given by the following:

\[
P = W_a + W_a + Q_f
\] (12)

\[\begin{align*}
&Q_f \downarrow \\
&P \downarrow \\
&h \\
&h_2
\end{align*}\]

\[\begin{align*}
&Q_f \downarrow \\
&P \downarrow \\
&h \quad h_2
\end{align*}\]

Figure 13. Soil boundaries for earth pressure method (Flucker and Teng 1965). \( Q_f \) = resistance; \( P \) = load; \( d \) = anchor base diameter; \( h \) = depth of anchor below soil surface.

\[\begin{align*}
&Q_f \downarrow \\
&P \downarrow \\
&h \quad h_2
\end{align*}\]

\[\begin{align*}
&Q_f \downarrow \\
&P \downarrow \\
&h \quad h_2
\end{align*}\]

Figure 14. Parameters related to Mueller's version of the earth pressure method (Flucker and Teng 1965).
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Table II. Coefficient of friction $f_c$ for soil against concrete.

<table>
<thead>
<tr>
<th>Smooth surface</th>
<th>Rough surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moist clay and loam</td>
<td>0.2</td>
</tr>
<tr>
<td>Dry sand</td>
<td>0.6</td>
</tr>
<tr>
<td>Wet sand</td>
<td>0.3</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.4</td>
</tr>
</tbody>
</table>

where $W_s$ = weight of soil within failure plane
$W_a$ = weight of anchor
$Q_f$ = frictional resistance developed along failure plane.

Mueller

Mueller (Flucker and Teng 1965) relates the frictional resistance $Q_f$ in the vertical shear plane to the magnitude of the horizontal earth pressure $P_h$ (Fig. 14). The value of $Q_f$ for a concrete anchor is therefore expressed as:

$$Q_f = \left[ \frac{K_h y}{2} 2h(a + b) \tan \phi + 2(h - h_2)(a + b)f_c \right]$$

(13)

where $\phi$ is the coefficient of internal friction for the soil and $f_c$ is the coefficient of friction between soil and concrete (see Table II.)

The parameter $K$ designates the coefficient of earth pressure, where for safety against excessive movement the use of the coefficient for earth pressure at rest ($K_0$) is suggested. This coefficient may be taken as:

$K_0 = 0.35 - 0.60$ for sand and gravel
$K_0 = 0.45 - 0.75$ for normally consolidated clays and silt
$K_0 = 0.80 - 1.36$ for overconsolidated clays.

To allow for safety against pullout, however, it was first decided that the following term for passive earth pressure $K_p$ should be used:

$$K_p = 0.9 \tan^2 \left( 45 + \frac{\phi}{2} \right)$$

(14)

Mors

Mors (Flucker and Teng 1965) later found that the use of $K_p$ in eq 14 generally leads to overestimating the failure load. Consequently, he concluded that the horizontal earth pressure $P_h$ equals the passive earth pressure only at the base of the footing and is distributed along the failure plane according to the function:

$$P_h = \frac{K \gamma}{h^{1-1}}$$

(see Fig. 15)

(15)
where the empirical $j$ values suggested are:

- $j = 13$ for anchor grillage in compacted backfill
- $j = 10$ for formed concrete footings without base in gravel
- $j = 5$ for formed concrete footings with base in gravel
- $j = 1$ for concrete footing poured against stiff clay.

To account for this change in $P_h$, the value of $K_p$ is determined by:

$$K_p = \frac{2}{j + 1} \tan^2 (45^\circ + \phi/2)$$

(16)

which is used in place of $K$ in eq 13.

**Motorcolumbus**

Motorcolumbus (Mors 1964) suggested an empirical modification to the earth pressure method based on numerous full scale tests. Motorcolumbus also found that the magnitude of the shear constant does not vary linearly with depth. If, for instance, the foundation depth were doubled, the shear constant involved would increase only by 20 to 25%; and if the depth were increased three times, the shear constant would increase at a still lower rate. Furthermore, for soils below the water table, it was suggested that normal values of shear constants be reduced by 50%. He derived the following equation:

$$P_{\text{max}} n = W_s + W_a + C_5 h^x$$

(17)

where $P_{\text{max}}$ = maximum anchor load

- $n$ = safety factor
- $W_s$ = weight of soil within failure plane
- $W_a$ = weight of anchor
- $C_5 = 4aK$; $K$ = coefficient of earth pressure
- $h$ = depth
- $a$ = width of square base
- $x = 1.52$. 
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Semiempirical Methods

Baker and Kondner

Baker and Kondner (1965) conducted a thorough study of model circular anchors in sand (Fig. 16) and confirmed that anchor holding capacities are influenced by the \( h/d \) ratio.

Through testing and dimensional analysis, the study provided the formula for the pullout strength capacity when \( h/d < 6 \):

\[
P = C_1 h d^2 \gamma + C_2 h^3 \gamma
\]

where

\( h = \) depth
\( d = \) anchor diameter
\( \gamma = \) unit weight of soil
\( C_1 = 3.0, \) a function of the angle of internal friction \( \phi \), relative density \( D_d \) and void ratio \( e \)
\( C_2 = 0.67, \) also a function of \( \phi, D_d \) and \( e \).

For shapes other than circular, Baker and Kondner suggest that an equivalent diameter can be estimated and approximate holding capacity calculated.

Baker and Kondner ran one full scale verification test on a belled anchor in sand with an \( h/d \) ratio of 5.3. The pullout capacity of the anchor was higher than that predicted by both their recommended analysis and that of Balla.

Turner

Turner (1962) formulated empirical equations based on anchor dimensions and the shear strength of the soil. He concluded after about 50 tests that, for anchors with an \( h/d \) ratio less than 1.5, the uplift capacity is a function of the square of the depth, whereas for anchors with an \( h/d \) ratio of 1.5 or greater, the uplift resistance of the anchor is a function of the base area. From these conclusions, he derived the following equations:

For anchors with an \( h/d \) ratio < 1.5:

\[
P_{\text{ult}} = 2.1 \sigma^{0.5} (h/d)^2 (d^2 - D_0^2).
\]

For anchors with an \( h/d \) ratio \( \geq 1.5 \):

\[
P_{\text{ult}} = 5.8 \sigma (d^2 - D_0^2)
\]

where \( P_{\text{ult}} = \) ultimate load
\( \sigma = \) unit shear strength of soil
\( h = \) anchor embedment depth
\( d = \) anchor base diameter
\( D_0 = \) shaft diameter.
Most of the values computed by Turner’s equation agreed relatively closely with, and all were within 35% of, the actual pullout strength values. Furthermore, most of the computations were conservative.

For the tests performed on 7-ft-diam underreamed (bellbottom) footings, Turner compared some pullout test values with values calculated by the earth cone method (eq 1) and the shearing method of Motorcolumbus (eq 17). He found that load values obtained by the earth cone method were not always conservative. At depths of 13 ft or greater, the capacities computed by the earth cone method were larger than those determined by Turner’s tests and would have resulted in anchors too small to support the design load. Turner also found that except for 9- to 10-ft depths, capacities computed using the shearing method were in excess of test results.

Figure 17 shows that these equations provide holding capacities for shallow footings that are intermediate to those defined by the earth cone method and shear method theories. Turner’s tests also show that his equations yield capacities for deep footings (h/d ratio > 1.5) that are lower and more accurate than those calculated by the other two theories.

**Biarez and Barraud**

Biarez and Barraud (1968) based their design recommendations on an extensive series of model and field tests of various anchorage configurations and soil types and conditions. In addition, their work was supported by an international working group performing a large number of full scale tests in a wide range of soil types. In all tests, care was taken to relate anchorage performance to standard soil parameters. They formulated criteria, based on these parameters, for estimating the uplift capacity of different types of anchors by using different failure planes for various soil conditions as shown in Figure 18. This method can be applied to straight shaft anchors, belled anchors and simple plates.
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Soils of category I
Saturated cohesive soils with low consistency $c \neq 0 \quad 0 < \phi < 10$ to $15^\circ$

Soils of category II
Unsaturated cohesive soils with marked internal friction $c = 0 \quad \phi > 15^\circ$
Powdery soils $c = 0 \quad \phi > 20^\circ$

Figure 18. Illustrations re. shallow anchor design after Biarez and Barraud (1968).

The Biarez and Barraud method is based upon the shear strength of the soil acting along a failure surface described by the angle $\alpha$ as shown in Figure 18. Equation 21 was derived for the holding capacity of shallow anchors where the critical depth ratio $h_2/d = 3$ to $5$ for granular soils and $5$ to $7$ for cohesive soils:

$$ P = A_1 [cM_c + y h_2 (M_\phi + M_y) + qM_q] + W_a $$

(21)

where $A_1 =$ circumferential area of pile or earth cylinder formed above the base, as illustrated in Figure 18 by the dashed lines

$R =$ radius (of pile or pad, whichever is larger)

$h_2 =$ depth to top of anchor pad or plate

$c =$ unit cohesion

$y =$ unit weight of soil

$q =$ surcharge load on soil

$W_a =$ weight of anchor, or weight of anchor and soil forming fictitious pile

$M_c =$ cohesion coefficient $= M_{c0} [1 - 1/2 (\tan \alpha) (h_2/R)]$ (22)
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Figure 19. Biarez and Barraud (1968) cohesion term $M_{c0}$ vs the angle of the shear plane $\alpha$ and the angle of internal friction $\phi$. The factor $\frac{1}{2} \tan \alpha$ (eq 44) is shown where if $h_c < h_2$, $h_c$ is used in place of $h_2$.

Figure 20. Biarez and Barraud (1968) friction plus gravity terms ($M_{\phi 0} + M_{\gamma 0}$) as a function of the angle of the shear plane $\alpha$ and the angle of internal friction $\phi$. The factor $\frac{1}{2} \tan \alpha$ (eq 45) is shown where if $h_c < h_2$, $h_c$ is applied in place of $h_2$.

where

$M_{c0}$ is determined from Figure 19

$\alpha$ = angle of shear plane, either + or - depending on soil conditions (see Fig. 13)

$M_{\phi}, M_{\gamma} = $ friction and gravity coefficients, respectively

$$= (M_{\phi 0} + M_{\gamma 0}) \left[1 - \frac{1}{3} (\tan \alpha) \left(\frac{h_2}{R}\right)\right]$$  \hspace{1cm} \text{(23)}

where

$(M_{\phi 0} + M_{\gamma 0})$ is determined from Figure 20

$M_q =$ overburden coefficient $= M_{c0} (\tan \phi + \tan \alpha) \left[1 - \frac{1}{2} (\tan \alpha) \left(\frac{h_2}{P}\right)\right]$  \hspace{1cm} \text{(24)}

where

$\phi =$ angle of internal friction.
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Figure 21. Theoretical stress zone developed above a bell anchor (after Jaky 1948).

Values of $1/2 \tan \alpha$ and $1/3 \tan \alpha$ are given in Figures 19 and 20.

To determine the capacity of an anchor of rectangular shape, an equivalent radius $R_e$ is calculated, i.e. $R_e = P/2\pi$ where $P$ = periphery of anchor.

DESIGN OF DEEP ANCHORS

Jaky Method

Jaky (1948) developed a relationship for determining the supporting capacity of a pile based upon the support derived from a zone of stressed soil formed around the bottom of an end-bearing pile. A bell anchor is assumed to cause a similar stressed zone when pulled upward through the soil; thus an analysis was made. The theoretical stress zone (Fig. 21) represents the case where the total capacity is the sum of the capacities developed by the stress zone and the friction along the anchor shaft. The soil parameters required for the analysis are unit cohesion $c$, angle of internal friction $\phi$, and the unit friction between anchor and soil $F$.

For analysis, the original Jaky equation for an end-bearing pile is:

$$P_b = cK_f A$$

where $P_b$ = uplift resistance of stress zone

$A$ = cross-sectional area at base of anchor bell
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\[ K_j = \cot \phi \left[ \tan^2 \left( 45 + \frac{\phi}{2} \right) e^{2\pi \tan \phi} - 1 \right] \]  (26)

Equation 26 is predicated upon a complete stress zone being formed, i.e. the anchor must have an embedment depth equal to or greater than the height of the stress zone \( h' \).

To find this height, Jaky derived the equation:

\[ h' = d \tan \left( 45 + \frac{\phi}{2} \right) e^{\pi \tan \phi} \]  (27)

where \( d = \) diameter at base of bell.

Expanding further, if the anchor’s embedment is greater than \( h' \), anchor capacity will be increased as a result of the total frictional force \( P_c \) developed along the shaft above the stress bulb:

\[ P_c = A_c F \]  (28)

where \( A_c = \) surface area of chimney above the stress zone
\( F = \) unit friction (see Table III).

Thus, to calculate anchor capacity by the Jaky method, the stress zone height is determined (eq 27), and \( P_b \) and \( P_c \) are combined, giving the total uplift capacity \( P \) as

\[ P = P_b + P_c = cA \cot \phi \left[ \tan^2 \left( 45 + \frac{\phi}{2} \right) e^{2\pi \tan \phi} - 1 \right] + A_c F. \]  (29)

| Table III. Unit friction between pile and soil (after Chellis 1951). |
|--------------------------|----------------------------------|
| **Material**             | **lb/ft² bounding area of pile*** |
| Fine-grained soils:      |                                  |
| Mud                      | 250 ± 200                        |
| Silt                     | 300 ± 200                        |
| Soft clay                | 400 ± 200                        |
| Silty clay               | 600 ± 200                        |
| Sandy clay               | 600 ± 200                        |
| Medium clay              | 700 ± 200                        |
| Sandy silt               | 800 ± 200                        |
| Firm clay                | 900 ± 200                        |
| Dense silty clay         | 1200 ± 300                       |
| Hard (stiff) clay        | 1500 ± 400                       |
| Coarse-grained soils:    |                                  |
| Silty sand               | 800 ± 200                        |
| Sand                     | 1200 ± 500                       |
| Sand and gravel          | 2000 ± 1000                      |
| Gravel                   | 2500 ± 1000                      |

* The (±) figures indicate a range governed by the character of the soil. Not all soils falling in the same general classification have equal properties.

If not micaceous, muddy, or under hydrostatic pressure or vibration.
A more precise, but involved, method of calculating holding capacity can be used instead of the $A_c f$ term of the above equation. Such an application would be beneficial in "long-chimneyed" anchors where the chimney's anchoring capacity becomes as great as or greater than the bell's capacity. The analysis concludes:

\[ P_c = A_c K y \left( \frac{h - h'}{2} \right) f \]  
(30)

where

\[ K = \text{coefficient of earth pressure} \]
\[ y = \text{unit weight of soil} \]
\[ h - h' = \text{depth of pile above stress zone} \]
\[ f = \text{coefficient of friction at anchor/soil interface}. \]

Although the Jaky method is considered suitable for determining anchor capacity, the calculated capacity is very sensitive to the values of $c$ and $\phi$. Therefore, care should be used when determining $P$.

**Baker and Kondner**

Baker and Kondner developed an equation for deep, round anchors in sand in conjunction with their research described in a previous section (Semiempirical Methods). For depth ratios of $h/d \geq 6$ they suggest:

\[ P = 170 d^3 y + C_3 d^2 t y + C_4 h d t y \]  
(31)

where

\[ d = \text{anchor diameter (see Fig. 16)} \]
\[ y = \text{unit weight of soil} \]
\[ t = \text{anchor plate thickness} \]
\[ C_3 = 2800, \text{ also a function of angle of internal friction $\phi$, relative density $D_d$, and void ratio $e$} \]
\[ C_4 = 470, \text{ also a function of $\phi$, $D_d$ and $e$.} \]

**Barez and Barraud**

Barez and Barraud (1968) also covered deep anchors as shown in Figure 22. Two different situations are defined, where $0^\circ < \phi < 15^\circ$ and $\phi > 15^\circ$. For anchors with $0^\circ < \phi < 15^\circ$ the uplift capacity is the sum of the earth resistance developed within the depth $h_c$ and the resistance developed along the chimney extending above $h_c$ where $h_c$ is equal to the product of the critical depth ratio for the soil times the diameter of the anchor base.

For deep anchors with $\phi > 15^\circ$, two different analyses are used, depending on whether the anchor is a simple plate with cable or tie rod, or a base with a chimney.

For the anchor that is a simple plate with cable or tie rod, Barez and Barraud (1968) found that local soil rotation occurred about the anchor base. They developed the following equation for this condition:

\[ P = AM_4 (y h_2 \tan \phi + c) \]  
(32)
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Figure 22. Illustration re. deep anchor design (after Biarez and Barraud 1968). In the first figure, the anchor shaft above \( h_c \) is treated as in Figure 19 while the equivalent cylinder radius \( R_e \) for rectangular pads is \( R_e = \frac{P}{8} \) where \( P \) is the pad periphery.

\[
M_{11} = \frac{12 \pi}{1 + 6 \tan \phi \left( \frac{\pi}{4} - \frac{2}{\pi} \right)} - 1.6 \text{ for circular anchors}
\]

\[
M_{12} = \frac{4 \pi}{1 + \frac{\pi}{2} \tan \phi} \text{ for infinite-length rectangular anchors}
\]

Figure 23. Biarez and Barraud (1968) plate uplift force factor \( M_i \) for various anchor base plate geometry and as a function of the angle of internal friction \( \phi \). The shape factor \( k \) is equal to \( b/a \) where \( b = \text{length of base} \) and \( a = \text{width of base} \).
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For the limiting value when \( \frac{R_0}{R} = 1 \) then \( M_u = \frac{4 \pi}{1 + \frac{\pi}{2} \tan \phi} \)

\[
M_u \left( \frac{R_0}{R}, \phi \right) = \frac{12 \pi}{1 + 6 \tan \phi \left( \frac{\pi}{4} - \frac{2}{\pi} \right) + 2 \frac{R_0}{R} \left( 1 + \frac{6}{\pi} \tan \phi \right)} - 1.6 \left( 1 - 1.9 \frac{R_0}{R} + 0.9 \frac{R_0^2}{R^2} \right)
\]

Figure 24. Biarez and Barraud (1968) pad and chimney uplift factor \( M_u \) for different values of \( \frac{R_0}{R} \) (where \( R_0 \) is the radius of the anchor shaft and \( R \) is the radius of the anchor pad) vs the angle of internal friction \( \phi \).

where \( A \) = cross-sectional area of anchor base

\( M_t \) = plate uplift force factor (determined from Fig. 23).

Figure 23 gives \( M_t \) values for various shaped anchor bases. However, owing to the lack of adequate evaluation, usage of the "rectangular anchor of infinite length" curve is recommended by Biarez and Barraud for all calculations (making the calculation conservative).

For the anchor that has a base with a chimney, the following equation was developed:

\[ P = (A - A_b)(m)(M_u)\left( \gamma h_2 \tan \phi + c \right) + W_p \]

where \( A \) = cross-sectional area of anchor base

\( A_b \) = cross-sectional area of chimney or shaft

\( W_p \) = weight of anchor base

\( m \) = reduction factor = \( 2\pi(R - R_0) - t/2\pi(R - R_0) \)

\( t \) = anchor base thickness (see Fig. 22)

\( M_u \) = chimney and pad uplift factor (determined from Fig. 24).
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To this equation is added the uplift capacity for the chimney which, as discussed earlier, can be calculated as for a normal friction pile.

MISCELLANEOUS ANCHOR TYPES

Temporary Anchors

Rigid ground stakes

Rigid ground stakes are usually thought of as small, temporary anchors. The most simple ground stake is a rod driven into the ground with a sledgehammer. Larger stakes may be driven by some mechanical means and retrieved similarly. The equations used to describe the load capacities of rigid ground stakes do not limit such stakes to either temporary usage or small size. Hence the lateral load criteria would apply even to a "rigid" pile, but for convenience are presented here.

Extensive use of ground stakes by the Armed Forces has warranted some research on the theory of holding capacity as well as on the development of mechanical devices for driving and extracting these stakes (Gerard 1969, Kovacs and Atkins 1973, Little 1963 and U.S. Army ERDL 1964). Strickland (1964) gave two expressions for determining a stake's holding capacity. One describes the maximum withdrawing force that can be applied axially and the other gives a generalized solution for determining the capacity of a stake loaded perpendicular to its axis. The stake is assumed to be infinite in strength and rigidity and changes in load orientation due to changes in creep are neglected. Furthermore, creep is limited by assuming the soil to be restrained from flowing around the periphery of the stake for an extended radius of \( \frac{1}{4} \) to \( \frac{1}{4} \) the diameter.

The theory is also simplified with empirical coefficients derived from data obtained from a pressure distribution curve observed in an unrestrained soil sample. The result is a rather rapid technique for estimating the holding capacity of a stake.

Driving a stake into the soil causes the soil to be displaced radially a distance (Fig. 25) equal to the radius of the rod. This displacement results in a compressive stress encompassing the embedded portion of the stake. Depending on stake configuration and finish, the axial load \( P_y \) necessary to extract the stake is:

\[
P_y = F_t h \pi D_0
\]

where

\[
F_t = \text{unit friction} = F_a + P_h \tan \phi
\]

\( h \) = depth of stake below soil surface

\( D_0 \) = diameter of stake

\( P_h \) = horizontal earth pressure at stake/soil interface owing to displacement \( \delta \)

\( F_a \) = unit adhesion

\( \phi \) = angle of internal friction.

When a lateral load \( P_x \) is applied, the stake is assumed to rotate at a point of neutral stress, \( y \) distance from the tip (Fig. 26). Rotation is resisted by a force which is proportional to the deflection, resulting in a linear stress distribution \( H_t = P_h \delta \). Here \( P_h \) is measured at \( \delta = R_0 + \delta' \), where \( \delta' \) is the permissible creep:
Figure 25. Parameters related to Strickland's (1964) analysis of an axially loaded stake.

Figure 26. Generalized soil stress distribution related to a laterally loaded rigid stake or pile (Strickland 1964).

where $H_r$ = horizontal resistance
$R_0$ = radius of stake.

By applying the conditions of static equilibrium to Figure 26 and summing forces and moments, the value of $P_x$ is derived:

$$P_x = \frac{dh^2 P_h}{2(3\ell - h)}$$  \hspace{1cm} (35)

where $\ell = $ stake length.

Thus, the idealized maximum holding power of the stake $P_{\text{max}}$ can be found for any loading condition as given by the following equation:

$$P_{\text{max}} = \sqrt{P_x^2 + P_y^2}$$

The equation presented is only an approximate solution and is limited to the assumptions made.

According to Flucker and Teng (1965), the limit to which an assumption of rigidity is permissible is defined by the application of the stiffness characteristic length $Z'$:

$$Z' = \frac{N+4}{K_m} \sqrt{\frac{EI}{K_m}}$$  \hspace{1cm} (36)

where $K_m = $ modulus of soil reaction for soil at bottom of anchor; $K_m = n_h h$ (for $n_h$, see Table IV)
$E = $ Young's modulus of pile or stake
$I = $ moment of inertia of the stake
$N = $ exponent generally set equal to 1.

Up to the value $h/Z' = 3$, the anchor is considered rigid and eq 35 may be used. However, when $h/Z' \geq 3$, the anchor must be analyzed as a flexible pile (see section on Laterally Loaded Piles, p. 30).
Table IV. Constant of horizontal subgrade reaction, $n_h$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$n_h$ in lb/ft$^2$ per ft of depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium-hard caliche</td>
<td>500</td>
</tr>
<tr>
<td>Fine caliche with sand layers</td>
<td>400</td>
</tr>
<tr>
<td>Compact well-graded gravel</td>
<td>400</td>
</tr>
<tr>
<td>Hard dense clay</td>
<td>400</td>
</tr>
<tr>
<td>Compact coarse sand</td>
<td>350</td>
</tr>
<tr>
<td>Compact coarse and fine sand</td>
<td>300</td>
</tr>
<tr>
<td>Medium-stiff clay</td>
<td>300</td>
</tr>
<tr>
<td>Compact fine sand</td>
<td>250</td>
</tr>
<tr>
<td>Ordinary silt</td>
<td>200</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>200</td>
</tr>
<tr>
<td>Adobe</td>
<td>200</td>
</tr>
<tr>
<td>Compact inorganic sand and silt mixtures</td>
<td>200</td>
</tr>
<tr>
<td>Soft clay</td>
<td>100</td>
</tr>
</tbody>
</table>

**Universal Ground Anchors**

Haley and Aldrich (1960) investigated the pullout resistance of Universal Ground Anchors. These are arrowhead-shaped anchors driven into the soil at an angle of 45° with a detachable rod.

Holding strengths were based primarily on the bearing capacity theory suggested by Terzaghi and Peck (1948), i.e. finding the ultimate resistance of the soil to a vertical force imposed on a horizontal bearing surface. It was assumed that the anchor-bearing surface was positioned perpendicular to the direction of the pull (Fig. 27). Also, the soils in which the anchors were embedded were divided into two classifications, noncohesive sands and gravels, and cohesive plastic clays.

The effect of depth and the nature of shearing resistance were both considered. It was assumed that, for anchors driven to relatively shallow depths ($h/d < 5$ or 6) and pulled to the point of failure, the displaced soil would cause an upheaval of the ground. Also, it was assumed that for anchors driven below this limiting depth, ground heave would no longer occur.

The shearing resistance of the soil was assumed due to two sources: 1) sliding friction between soil particles along a failure surface, and 2) cohesion.

In cohesive soils, it was assumed that the cohesive strength was independent of depth in a homogeneous deposit. Therefore, since no increase in ultimate pullout resistance was expected with an anchor embedment greater than the critical depth ($h/d = 6$), the bearing formula was applied. Thus for clays, the ultimate pullout load $P_{ult}$ is given as:

$$P_{ult} = \frac{N_c c d_2^2}{2}$$

where

- $c =$ unit cohesion
- $d_2 =$ side dimension of anchor (Fig. 27)
- $N_c =$ dimensionless bearing-capacity factor assumed equal to 7.
The pullout resistance in eq 38 is directly proportional to the anchor depth, i.e. it is assumed that sliding friction increases with depth in cohesionless soil. Results of seven pullout tests in sand with \( y = 96 \text{ lb/ft}^3 \) are summarized as follows:

<table>
<thead>
<tr>
<th>Size of ground anchor (in.)</th>
<th>Vertical depth (ft)</th>
<th>( h/d_2 )</th>
<th>Failure load (lb)</th>
<th>Total pullout at failure (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.8</td>
<td>7.2</td>
<td>325</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2.2</td>
<td>6.6</td>
<td>800</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2.6</td>
<td>5.2</td>
<td>735</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>2.6</td>
<td>3.9</td>
<td>840</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>4.0</td>
<td>4.8</td>
<td>1680</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>6.1</td>
<td>6.1</td>
<td>5000</td>
<td>25</td>
</tr>
<tr>
<td>17</td>
<td>6.1</td>
<td>4.3</td>
<td>8500</td>
<td>19</td>
</tr>
</tbody>
</table>
Figure 28. Unsymmetrical failure cone developed above a circular anchor loaded at an angle (after Kananyan 1966).

Haley and Aldrich (1960) did not determine the configuration of the failure plane, but they assumed that the soil would be displaced along a surface curved upward and outward from the anchor.

Kananyan (1966) found that for circular anchors pulled at an angle an unsymmetrical yielding cone was formed (see Fig. 28). The outline of the upward yielding zone was an ellipse, and it is assumed that this is the form of the failure plane assumed by Haley and Aldrich (1960). As a consequence of the unsymmetrical yielding conditions, the anchor is displaced not in the direction of the acting force, but rigorously, with an upward deflection.

LaterallyLoaded Piles

According to Broms (1965) the lateral deflection of a pile loaded to less than approximately half its ultimate resistance is usually calculated from either elastic theory or by using a coefficient of subgrade reaction. The use of elastic theory places several limitations on the assumed soil properties. To date, solutions exist only for an idealized isotropic elastic soil mass of constant modulus of elasticity and Poisson's ratio. The soil also is assumed capable of withstanding tensile loads, but since the tensile strength of soil is low, the elastic methods generally underestimate the actual lateral deflections. Broms (1973) outlines several of the elastic methods. These will not be covered here, but detailed descriptions of them can be found in Spillers and Stoll (1964), Douglas and Davis (1964), Poulos (1971) and Oteo (1972).

The more common method of calculating pile deflections under lateral loads is based on the Winkler foundation shown schematically in Figure 29. The resultant soil reaction on a pile at any given depth is linearly related to the pile deflection at that depth. The method does not assume that the soil mass is a continuum. The soil reaction is given by:

\[ P_h = K_m y \]  

(39)
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Figure 29. Winkler foundation (after Broms 1973).

\[ P_h = \text{horizontal pressure on pile} \]
\[ K_m = \text{subgrade modulus} \]
\[ y_x = \text{lateral deflection.} \]

where \( P_h \) = horizontal pressure on pile
\( K_m \) = subgrade modulus
\( y_x \) = lateral deflection.

Broms (1973) points out that the subgrade modulus \( K_m \) is not a unique soil property, but varies with the dimensions of the pile, the intensity of the applied load and the depth below the ground surface. According to Terzaghi (1955) the subgrade modulus for cohesive soils can be represented as constant with depth and for cohesionless soils as increasing linearly with depth according to the equation:

\[ K_m = \frac{n_h x}{B} \]

(40)

where \( n_h \) = coefficient of horizontal subgrade reaction at a depth of unity for a pile width of unity
\( x \) = depth
\( B \) = width of pile.

Terzaghi called the coefficient \( n_h \) the constant of horizontal subgrade reaction; \( n_h \) is primarily a function of the soil compressibility. Davisson and Gill (1963) illustrated the probable variation of the coefficient of subgrade reaction with depth as shown in Figure 30.

Reese and Matlock (1956) and Matlock (1962) have presented the nondimensional solution for a pile with a coefficient of subgrade reaction increasing linearly with depth. A similar solution for a constant coefficient of subgrade reaction has been presented by Davisson and Gill (1963). A summary of these equations (after Wilson and Hilts 1967) defining deflection, slope, moment, shear and soil reaction is given in Table V. Suggested constants of horizontal subgrade reaction \( n_h \) for different soil types are shown in Table IV. Schematic deflection and loading diagrams are shown in Figure 31 and graphs of the sublettered coefficients \( A \) and \( B \) used in the equations of Table V are shown in Figures 32-41. Wilson and Hilts (1967) recommend that short piles, with maximum depth coefficients of 2 or less, be treated as rigid poles. In this case design should be based on the method outlined in the section on Rigid Ground Stakes.
Table V. Equations describing a laterally loaded pile with constant and increasing subgrade modulus (after Wilson and Hiltz 1967).

<table>
<thead>
<tr>
<th>Term</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Depth coefficient</td>
<td>$C_d = \frac{x}{Z_1}$</td>
</tr>
<tr>
<td>b. Max depth coeff</td>
<td>$C_{d_{max}} = \frac{\ell}{Z_1}$</td>
</tr>
<tr>
<td>c. Relative stiffness factor</td>
<td>$Z_1 = \sqrt{\frac{EI}{n_h}}$</td>
</tr>
<tr>
<td>d. Deflection</td>
<td>$y_x = A_y \frac{P_y Z_1^3}{EI} + B_y \frac{M_g Z_1^2}{EI}$</td>
</tr>
<tr>
<td>e. Slope</td>
<td>$B_x = A_s \frac{P_y Z_1^2}{EI} + B_y \frac{M_g Z_1}{EI}$</td>
</tr>
<tr>
<td>f. Moment</td>
<td>$M_x = A_m P_y Z_1 + B_m M_g$</td>
</tr>
<tr>
<td>g. Shear</td>
<td>$V_x = A_v P_y + \frac{B_v M_g}{Z_1}$</td>
</tr>
<tr>
<td>h. Soil reaction at depth $x$</td>
<td>$W_x = A_w \frac{P_y}{Z_1} + B_w \frac{M_g}{Z_1^2}$</td>
</tr>
</tbody>
</table>

Where:
- $E$ = Young's modulus of pile
- $I$ = moment of inertia of pile
- $n_h$ = constant of horizontal subgrade reaction (Table IV)
- $A_y, A_s, A_m, A_v, A_w$ = load coefficients of deflection, slope, moment, shear and soil reaction (Fig. 32, 34, 36, 38 and 40)
- $A_y', A_m'$ = load coefficients of deflection and moment (see Davisson 1963)
- $B_y, B_s, B_m, B_v, B_w$ = moment coefficients of deflection, slope, moment, shear and soil reaction (Fig. 33, 35, 37, 39 and 41)
- $B_y', B_m'$ = moment coefficients of deflection and moment (see Davisson 1963)
- $M_g$ = moment at ground surface due to load $P_y$
- $P_y$ = horizontal force on pile
- $K_m$ = subgrade modulus
- $x$ = depth below ground surface.
Figure 31. Typical deflection and moment curves for laterally loaded pile (after Wilson and Hilts 1967).

Figure 32. Wilson and Hilts (1967) maximum depth coefficient ($C_d_{\text{max}}$) curves for the $A_y$ deflection coefficient vs the depth coefficient $C_d$. 
**Figure 33.** Wilson and Hilts (1967) maximum depth coefficient ($C_{d_{\text{max}}}$) curves for the $B_y$ deflection coefficient vs the depth coefficient $C_d$.

**Figure 34.** Wilson and Hilts (1967) maximum depth coefficient ($C_{d_{\text{max}}}$) curves for the $A_s$ slope coefficient vs the depth coefficient $C_d$. 
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Figure 35. Wilson and Hilts (1967) maximum depth coefficient $(C_{d_{\text{max}}})$ curves for the $B_s$ slope coefficient vs the depth coefficient $C_d$.

Figure 36. Wilson and Hilts (1967) maximum depth coefficient $(C_{d_{\text{max}}})$ curves for the $A_m$ moment coefficient vs the depth coefficient $C_d$. 
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Figure 37. Wilson and Hilts (1967) maximum depth coefficient ($C_{d_{max}}$) curves for the $B_m$ moment coefficient vs the depth coefficient $C_d$.

Figure 38. Wilson and Hilts (1967) maximum depth coefficient ($C_{d_{max}}$) curves for the $A_v$ moment coefficient vs the depth coefficient $C_d$. 
Figure 39. Wilson and Hilts (1967) maximum depth coefficient (\(C_{d_{\text{max}}}\)) curves for the \(B_v\) moment coefficient vs the depth coefficient \(C_d\).

Figure 40. Wilson and Hilts (1967) maximum depth coefficient (\(C_{d_{\text{max}}}\)) curves for the \(A_w\) soil reaction coefficient vs the depth coefficient \(C_d\).
Grouted anchors are being used in both rock and soil. A Swiss firm, Losinger and Company, SA, has made an extensive study of this anchoring technique and has developed the VSL and Alluvium anchors (Ground Engineering 1968). The VSL anchor is primarily intended for providing anchorage in rock but can also be used in cohesive soils. The Alluvium anchor is designed for use only in soil.

The VSL anchor consists of a high strength cable configured as shown in Figure 42. When used as a tie-back anchorage to shore excavations or to roof bolt tunnels, the cable is tensioned after the grout has cured. Once stressing is complete, final grouting is done to protect the cable passing through the plastic sheath against corrosion.

The load capacity of a VSL anchor in rock is related to the shear strength of the bond at the grout/drill hole interface. The interface strength can be determined by placing a core of the rock in a form and filling the surrounding space with a high strength cement or resin grout. After the grout has cured, the rock core is pressed out and the bond strength between it and the grout determined.

The interface strength between grout and cohesive soils is often determined as being one-half the unconfined compressive strength of the in situ soil. However, Skempton (1959) suggests an adhesive value 0.45 times the undrained strength of clay with a limiting value of 2000 psi, and Littlejohn (1968) suggests an adhesive value 0.3 to 0.35 times the undrained strength when the anchor is embedded in stiff to very stiff clay.

Another variety of rock or cohesive soil-grout anchor is that developed by Universal Anchorage Co., Ltd. (Ground Engineering 1968). This system uses a patented expanding bit to cut an oversized hole or a series of cone sections into the rock or soil at the base of a bored hole (Fig. 43). The base diameter of the cone is two to four times that of the borehole.
Placement consists of drilling a hole to a depth at which the cone of rock projected from the anchorage is capable of providing the retaining force required. It is contended by Ground Engineering (1968) that, “Since no factor need be allowed for skin friction, the depth of the hole is minimized and the elimination of a factor of friction allows design figures to be met without the excessive hole length necessary to allow for unfavorable conditions.” After the hole is reamed, a steel cable (with a ferruled or bushed end) or a tie rod is lowered in place and the hole is filled with injected grout or resin.

Anchors installed in alluvium generally consist of a steel cable separated into strands in the grout zone (Fig. 44). These anchors can be used in any ground capable of carrying a load but the highest resistances are obtained in gravels and coarse sands where the permeability is not less than $4 \times 10^{-3}$ in./sec.

In gravels, the anchors are installed in the sequence shown in Figure 45. The recommended water/cement ratios for installation in gravels and coarse sands are 0.5 and 0.65, respectively; and depending upon ground permeability, injection pressures are from 5 to 10 psi. A high alumina content cement is used where high early strength is required.
Figure 44. Configuration of grouted anchor in alluvium (after Ground Engineering 1968).

Figure 45. Grouting procedure for installation of alluvium anchors (after Littlejohn 1968). a. Installation of lining tube and positioning of anchor cable assembly; b. Grouting procedure.
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Littlejohn (1968) reports that the following empirical rule can be used to determine the ultimate resistance of an anchor grouted in gravel or coarse sands:

$$P_{ult} = \ell N \tan \phi$$

where \( \ell \) = length of grout anchor

\( N = 12-16 \) tons/ft.

Installation of alluvium anchors in fine to medium size sands follows the same procedure as installation in gravels. However, because of the lower permeability of the fine to medium size sands the grout zone tends to be narrower and anchor capacity lower than in gravels. To increase the radius of the grout areas, a larger casing is often used or injection pressures are increased. The capacity of the anchor may also be increased by increasing the embedment depth of the anchor.

In compact, fine grained sands of low permeability \((4 \times 10^{-3} \) to \(4 \times 10^{-5} \) in./sec) chemical grouts with low viscosities \((20 \) cp at \(68^\circ\)F) are used. These grouts do not fill the intergranular voids with an epoxy matrix but do flow throughout, bonding the interparticle contact points together. The cost of chemical grouting is indeed higher than that of cement grouting but the cost of anchorage as a function of load capacity in chemical grouting is sometimes more favorable than that in cement grouting.

Anchorages in clay are occasionally grouted in augered holes but increased capacity is obtained by expanding the base of the borehole by explosive cratering, by belling, or by driving gravel into the clay adjacent to the anchor.

Generally, the strength of the anchor cable or shaft governs the ultimate capacity of anchors grouted in rock or sand. However, considerable uncertainty exists in clay soils. Hanna (1968) believes that the general approach to pile analysis may be used for tentative design of an anchor in clay, provided caution is exercised. His design analysis is predicated upon the anchor configuration shown in Figure 46. Load capacity is the summation of the end bearing force \(Q\) acting over the upper end area of the anchor \(A\) less the area of the shaft \(A_b\), the friction force \(P_c\) developed along the side of the grout cylinder, and the suction force \(F_s\) developed under the base of the anchor. The general equation is:

$$P = Q + P_c + F_s$$

where

$$Q = \sigma N_c (A - A_b)$$

\( \sigma = \) undrained strength of clay at top of anchor

\( N_c = \) bearing capacity factor = 9

$$P_c = \pi d i F$$

\( F = \) average adhesion per unit area between clay and grout

\( d = \) average diameter of grout cylinder

\( i = \) length of grout cylinder.
Although high suction forces have been observed, the value should not be used because the unit suction force developed is dependent upon clay properties as well as on workmanship used in clearing out the base of the borehole. Furthermore, long-term loading causes the force to drop to zero because of swelling of the clay. The existence of a suction force then adds an additional degree of safety to the design in respect to resisting transient loading. The overall uncertainty of this analysis lies in the proper determination of $A$.

Anchors being installed in rock having a temperature below 32°C must be specially considered. According to Myska Ltd (1967) the recommended average ultimate bond strengths for grout anchors installed in freezing conditions, and subsequently thawed and cured, are based upon the American Concrete Institute (ACI) code values. The bond strengths recommended for the surface between the anchor rod and the grout are:
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Figure 48. Influence of salt on the compressive strength of cement (after Shideler 1952).

\[
\text{Bond strength} = \frac{9.5 \sqrt{f'_c}}{D_0}
\]  

(45)

where \( D_0 \) = rod diameter

\( f'_c \) = concrete unconfined compressive strength.

Using a concrete strength \( f'_c \) of 2000 psi, the formula gives rod/grout interface strengths of 425, 380 and 340 psi for reinforcing bar numbers 8, 9 and 10, respectively.

The suggested bond strength at the grout/rock interface is 200 psi. However, grout freezing before curing can be a serious problem. This concern stems from observations that, although the uncured grout is capable of resisting load while frozen, upon thawing it returns to a physical state similar to its state when poured (Fig. 47). Upon thawing, the curing process commences again but with permanent damage to the grout, i.e. reduction of strength. The overall consequence of uncured grout thawing around an anchor rod is obvious.

To avoid grout freezing before curing, the mixture should be poured at a temperature \( \geq 70^\circ \text{F} \). Also, the anchor rod should be heated to a similar temperature before being inserted into the grout-filled hole.

Another precaution against grout freezing is the addition of salt. Salt not only lowers the freezing and curing temperatures of the grout, but increases the compressive strength under most conditions (see Fig. 48 and 49). The ACI code states that "no more than 2% (by weight of cement) calcium chloride" should be used. However, Stormer (1970) shows that 2, 3 or more times as much salt can be used without damaging effects if the temperature is low enough.

By preheating and adding salt, many pre-curing-freezing problems may be overcome. However, the science of combining the two procedures must be explored further. Also, a study of soil
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Figure 49. Strength behavior of concrete cured under different temperatures as influenced by salt additives (after Stormer 1970). (Mixture 50/50 is an equal molar concentration of NaCl and CaCl₂).

temperatures versus required salt percentages and preheat temperatures required to ensure adequate curing and soil-grout adhesion would be of considerable value when anchors are being designed for frozen ground installation.

Some quantitative testing conducted by Dorman et al. (1969) confirms that slurry backfill could be an answer to some permafrost anchoring problems. Their anchors consisted of small diameter rods of different configuration slurried in place with a silt-water or quick setting cement backfill. Although the anchors could sustain high transient loads, long-term loading resulted in appreciable creep. More tests must be undertaken, however, to define the load-creep behavior versus temperature for anchors embedded in a frozen backfill. A beginning in this direction has been made by Johnston and Ladanyi (1971).
Block Anchors

Tschebotarioff (Leonards 1962) presents a summary of design criteria for block anchors, sometimes termed deadmen, in both cohesive and non-cohesive soils. Hansen (1953) presented an equation, based on the Coulomb and Rankine-Résal criteria, for an anchor block of rectangular cross section pulled through a plastic clay, as shown in Figure 50. It was concluded that the total resistance $P_u$ per unit of anchor block width was:

$$P_u = 11.4ct$$ (46)

where $c =$ unit cohesion

$\tau =$ anchor thickness (see Fig. 50).

MacKenzie (1955) performed model tests on rectangular anchor blocks in two plastic clays and concluded that:

$$P = 8.5ct$$ (47)

for scaled depths, $h/t$, greater than 12. MacKenzie's experimental curve for $P$ as a function of depth is represented in Figure 50. Tschebotarioff (Leonards 1962) recommends that MacKenzie's experimental curve be used for design in conjunction with appropriate factors of safety.

For anchor blocks in cohesionless soils Tschebotarioff (Leonards 1962) presents equations derived by Buchholz (1930/31) and Streck (1950) based on experiments in medium density sand with a friction angle $\phi = 32.5^\circ$. Equations are given for a continuous block and for a block whose length equals its height, as shown in Figure 51. The resistance of a continuous block per unit width of block is given by:

$$P_u = \frac{yh^2}{2} K_p$$ (48)

where $K_p$ values are obtained from Figure 51. Note that the scaled depth has little influence on $K_p$.

The resistance of a square block is given by:

$$P = \frac{yh^2}{2} K'_p b$$ (49)

where $b$ is the length of the block and just equals the height $t$, and $K'_p$ is obtained from Figure 51.
FIELD EVALUATIONS

Reinart

A study was made by Reinart and Adalan (1969) and Reinart et al. (1968) to develop design criteria for anchoring transmission line towers in permafrost. They concluded that permafrost would most likely degrade beyond the economical anchorage depth within the lifetime of the transmission line and therefore decided that the anchors should be designed on the basis of the thawed soil properties. However, it was necessary to install the test anchors in frozen ground to account for installation problems associated with permafrost. Therefore, once the anchors were installed, the ground was artificially thawed.

Cast-in-place concrete bell anchors were selected for the tests. These anchors had shaft diameters of 18 to 24 in. and bell diameters of 36 to 48 in. Three 18-in.-diam 12-ft-long straight-shaft anchors were also tested.

For both straight-shaft and bell anchors, the test load was applied in increments of 5 kips and anchor deflection was measured at 5-min intervals. The load level was not increased until creep had virtually stopped or the rate of creep had maintained a constant value for a period of at least 2½ hours, the anticipated duration of the maximum design load. The creep behavior observed was similar to the generalized curve shown in Figure 52. Additional load increments were applied until the creep rate increased with time (the tertiary point) or until the total deformation made further testing impractical. For the straight-shaft anchors, failure was defined as the load that caused the onset of tertiary creep within the 2½ hour time limit.

Because of jack travel limitations, the failure load of the bell anchors was considered to have been reached when the creep rate was no longer "appreciably" decreasing with time, i.e. failure was selected as some arbitrary creep rate along the primary stage of the creep curve (see Fig. 52). This occurred when the creep rate still exceeded 3 in./hr.
Maximum capacities of the bell anchors were developed after a relatively large anchor movement of approximately 6 in.

Straight-shaft anchors first exhibited capacities between 15 and 18 kips. However, when they were again tested, approximately three weeks later, their holding capacity had increased approximately 50%. Reinart and Adalan (1969) stated that this increase was related to increased consolidation of the soil.

Excavation and visual examination of 9 of the 20 test anchors showed that the 18-in.-diam shafts were formed as designed, but that the 24-in. shafts were oversized by 2 to 3 in. Also, the inclined bell anchors had larger bell diameters than specified and were slightly oval in shape; this was considered to be a contributing factor to their higher capacities. The 36-in.-diam bells were undersized, measuring only about 29 in. This undersizing was thought to be caused by the fact that the belling tools were designed for cutting 48-in. bells and had functioned improperly in cutting the smaller size. The 48-in.-diam bells of the vertically installed anchors were of the proper size.

Of the four inclined bell anchors tested, two exhibited capacities of about 60 kips while the other two sustained and probably could have surpassed 70 kips, the capacity of the test equipment. The apparent higher capacities of the inclined anchors were believed to have resulted from larger bell sizes and a greater degree of consolidation of the soil owing to a longer period of time between thawing and testing.

The results of the anchor tests were compared to values calculated from theoretical methods developed by Jaky (1948) and Biarez and Barraud (1968). Reinart and Adalan found that the calculated values were in good agreement with the test results when the results were extrapolated to account for consolidated soil properties assumed to have existed during the tests.

**American Electric Power Service Anchor Tests**

A report on load versus anchorage requirements based on economical considerations was prepared by the American Electric Power Corporation (Zobel 1965). In their study, the following types of anchorages were evaluated: Malone anchors, steel grillage anchors, concrete bell anchors, steel grillage and screw anchor combinations, and Never-Creep anchors.
All anchors were tested in uplift. Loads were applied in 10,000-lb increments and anchor movement was measured to the nearest \( \frac{1}{8} \) in. All of the tests were performed in what appeared to be dry clay.

The Malone anchor is a concrete anchor formed in a cavity made by drilling and blasting. Placement procedures are as follows: 1) a hole is augered to the desired depth, 2) a charge is detonated at the bottom of the hole to form a spherical cavity, 3) the cavity and shaft are then filled with concrete, and 4) a steel tie rod is inserted into the concrete. An idealized sketch of the resulting anchor configuration is shown in Figure 53.

Two blasting methods are used to form the cavity. The first method uses several charges: an initial charge to start the cavity, and then additional charges to expand the cavity to the desired size. After each blast, concrete is measured and poured into the cavity through a steel casing. The addition of concrete helps to concentrate the explosion and to prevent caving in of the hole.

The second method consists of forming the cavity with a single charge; concrete is placed in the augered hole prior to and after detonation. Although this method is more expedient than the first, the load test results of the anchors so formed were found to be erratic. This installation method is therefore considered unsatisfactory.

Anchors installed by the first method, at depths ranging from 6 to 12 ft, in dry, compact soil, were satisfactory. An average of four tests indicated a pullout capacity of 80,000 lb, but the size of the concrete "bells" was not determined. For additional test results of Malone anchor installations, see Abels (1967).

Two types of steel grillage anchors were used: 1) the standard grillage shown in Figure 54a, with earth or rock backfill, and 2) the pyramid grillage shown in Figure 54b, also with earth or rock backfill. The pyramid grillage anchor was designed with a steel plate bolted to its bottom and had numerous connections. (This anchor required about twice as many man-hours to assemble as the standard grillage anchor.)
Test results from the steel grillage were consistent. Anchors installed 10 and 13 ft deep with earth backfill had average total movements of 2 in. for the pyramid grillages and 4 3/4 in. for the standard grillages under uplift loads of 90,000 lb. Anchors installed 7 ft deep failed at 70,000 and 80,000 lb for the pyramid and standard grillage anchors, respectively.

Pyramid grillage anchors installed 10 ft deep with 3 ft of rock backfill moved 7/8 in. and standard grillage anchors moved 1/4 in. under an uplift load of 90,000 lb. The pyramid grillage anchor with the steel plated bottom is more costly than the standard grillage anchor. However, certain features of the design are effective in resisting shear loads. The holding power of the grillage anchors was greatly increased when crushed rock backfill was used. AEPS recommended that crushed rock backfill be specified at locations where "wet soil" conditions exist, since no test information was obtained for such soil.

Three types of concrete bell anchors were tested (Fig. 55) and all except one performed well. The anchors accepted loads up to 90,000 lb and moved a total of 1/4 in. or less. At these loads there was no indication that any one type was superior to another. The bell anchor was considered superior to the grillage anchor in compact soil because its movement was resisted by undisturbed soil. Further testing was recommended to determine whether bell anchors were economical for general use and to gain experience with the installation of the anchors in very wet soils using bentonite to prevent hole collapse during augering.

Six installations were tested to investigate the performance of a steel grillage – screw anchor combination (Fig. 56). The steel grillages used were 5-ft square with a screw anchor attached to each cover. Eight-in.-diam power-installed screw anchors were used in three tests, and 11-in.-diam power-installed screw anchors were used in the remaining tests. The screw anchors were installed below the bottom of the 3-ft-deep open excavation for the steel grillage. The grillage was then set in place and connected to the anchor rods (Fig. 56).

Several complications were experienced in installing the screw anchors. In one test an 8-in. anchor was fractured during installation when it struck a rock. In four of the six test installations, rock interference prevented the screw anchors from being installed to the depth desired.
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Figure 55. Configuration of bell anchors tested by the American Electric Power Service Corporation (after Zobel 1965).

Figure 56. Configuration of steel grillage - screw anchor combination (after Zobel 1965).
Three tests with the 11-in.-diam screw anchors and one test with 8-in.-diam screw anchors showed that the grillages with screw anchors performed better than those without. The two remaining tests with 8-in. screw anchors failed at the lightest test loads. AEPS felt that the screw anchor-grillage combination would perform satisfactorily provided screw anchor installation problems could be overcome.

For the Never-Creep anchor tests, a special 24- x 60-in. nonproduction anchor was built by the A. B. Chance Company (Fig. 57). These anchors were geometrically similar to the familiar Never-Creep anchors on the market today. However, in an effort to improve holding power and restrain vertical movement, rectangular plates were welded at each end of these anchors to serve as creep guards. Ten anchors were tested. Five of these were installed in vertical holes and five in holes augered at an angle (Fig. 57). The anchors were forced flush against the side of the hole in which they were installed and then backfilled with earth with standard tamping (i.e. hand tamping around the anchor, and machine tamping in 12-in. levels thereafter).

The types of failures experienced in testing the Never-Creep anchor were noteworthy. A structural failure was experienced with the anchors installed in vertical holes; here the rod guides, which were welded to the anchor, sheared off under loads approaching 50,000 to 60,000 lb. This structural failure did not cause complete failure of the anchor, but did allow additional deformation to occur.

A common failure was experienced in the tests in which the anchors were installed in sloping holes. Here failure was complete because the anchor rods pulled completely out of the anchor plate. Failure occurred at loads of 60,000 to 80,000 lb for anchors installed 7 to 10 ft deep. AEPS suggested that the Never-Creep anchor would perform satisfactorily provided the connection between anchor plate and rod was redesigned.

As in other AEPS tests, information was not obtained on the performance of this anchorage in "wet" soil.

**Expandable Land Anchor**

In a study of an expandable land anchor, Dantz (1966) found that it could effectively resist a 30,000- to 40,000-lb load. Installation of this anchor requires that a hole be augered, the anchor inserted, the hole backfilled, and a seating load applied to the anchor (Fig. 58). Tests were conducted on a 6-in.-wide by 18-in. expanded-width anchor in sand, clay and sandy loam soils.

Figure 59 presents the results.
ANCHORAGES IN FROZEN GROUND

There are both advantages and disadvantages to installing anchorages in frozen soil. All things being equal, an anchorage in frozen soil will have a greater load bearing capacity than the same anchorage in the same soil in the thawed state. Crory et al. (1969) ran a series of tests on universal ground anchors in thawed and frozen soil (summer and winter conditions). As shown in Table VI, they found the ultimate load capacity to be much higher in the frozen soil than in the thawed soil. Similar load capacity results were reported by Kovacs (1973) for hook anchors tested in frozen and unfrozen ground. Clevett and Barry (1955) found that 9-in.-long aluminum pins held up to several hundred pounds in frozen muskeg.
Table VI. Results of Universal Ground Anchor tests in frozen and unfrozen soil.

<table>
<thead>
<tr>
<th>Anchor size (in.)</th>
<th>Depth (in.)</th>
<th>Failure load winter series (lb)</th>
<th>Summer series (lb)</th>
<th>Winter series tested in summer (lb)</th>
<th>Driven and tested 1 hr later (lb)</th>
<th>Average of all summer tests (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>4000 (3)</td>
<td>250 (1)</td>
<td>250 (1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>4000 (3)</td>
<td>800 (4)</td>
<td>800 (4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>4000 (1)</td>
<td>1400 (5)</td>
<td>1400 (5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>3000 (1)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>3500 (2)</td>
<td>600 (5)</td>
<td></td>
<td>600 (5)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>3800 (1)</td>
<td>1460 (5)</td>
<td>1460 (5)</td>
<td>1360 (7)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>4000 (2)</td>
<td>2860 (5)</td>
<td>2860 (5)</td>
<td>2590 (7)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>5000 (1)</td>
<td>580 (5)</td>
<td>580 (5)</td>
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<td></td>
</tr>
<tr>
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<td>5000 (1)</td>
<td>1340 (5)</td>
<td>1340 (5)</td>
<td>1410 (1)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>5500 (1)</td>
<td>2760 (5)</td>
<td>2760 (5)</td>
<td>2760 (7)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>5000 (1)</td>
<td>800 (2)</td>
<td>800 (2)</td>
<td>800 (2)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>5700 (1)</td>
<td>1060 (2)</td>
<td>1060 (2)</td>
<td>1350 (4)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>5000 (1)</td>
<td>2700 (2)</td>
<td>2700 (2)</td>
<td>2800 (4)</td>
<td></td>
</tr>
</tbody>
</table>

* This anchor was the only anchor of the winter group that was successfully pulled out of the ground. However, the test was made by pulling on both the holding and retrieving cables simultaneously.

NOTES: 1. All the anchors except one of the winter group failed either by the anchor or cable breaking.
2. All the anchors of the summer group were successfully pulled out of the ground.
3. All loads are average.
4. Numbers in parentheses refer to number of tests performed.

However, frozen soil may present problems in installing conventional anchors. Bernadin (1961) found that it was impossible to drive universal ground anchors far enough into frozen ground to hold desired loads. Likewise, Fletcher (1966) found that in -65°F soil aluminum stakes could not be driven deep enough to hold desired loads. Also, removal of the stakes was reported to be exceedingly difficult. It is evident that such anchors have high potential in frozen ground if placement difficulties can be overcome.

One of the greatest difficulties in using anchors in frozen ground is maintaining the area around the anchorage in the frozen state. Thawing of permafrost is greatly enhanced once the insulating organic cover is even slightly disturbed and the subsequent thawing results in a tremendous loss in strength, especially with fine-grained soils. Sanger (1969) states that anchors in permafrost have given considerable problems in the past and as a result are often designed on the basis of dead weight alone. In critical cases it has been necessary to employ some means of refrigerating anchors to maintain the surrounding soil in the frozen condition.

Another problem involving anchors in cold climates is the upward forces which develop as a result of freezing of the active layer during the winter. These can be large enough to pull an anchor from the ground independently of the live load which the anchor was designed to resist. The Russian permafrost code suggests an average value of about 1700 lb/ft² to be used as an adfreeze strength for the entire thickness of the active soil layer. The uplift force resulting from frost heaving is estimated to be about 560 and 950 lb/linear inch for the peripheral surfaces of foundations embedded in soils having active layer thicknesses of 4.3 and 8.6 ft, respectively.
For anchor design in a permafrost region, the Russian permafrost code recommends that the anchor capacity be greater than the sum of the frost force and the long-term guy load. This may be computed from the following equation:

$$0.9P \geq (1.4r_{ad}A_a) + (0.9P_g)$$

where 

- $P =$ design load
- $r_{ad} =$ adfreeze strength between the anchor and frozen soil
- $A_a =$ area of anchor embedded in an active layer
- $P_g =$ long-term load.

Backed by much study and testing, Tsytovich (1958) established that the adfreeze strength is most directly related to the moisture content and grain size (Fig. 60). Both higher moisture content and generally smaller grain size resulted in higher adfreeze strengths.

Figure 60. Relationship between ultimate adfreeze strength and moisture content, and ultimate adfreeze strength and grain size (after Tsytovich 1958).
Tsytovich then determined the basic principles of adfreezing. He defined $r_a$ as "the value of continuous force of adfreezing" within the adfreeze portion of the active layer ($H$ in Fig. 61) and $r_p$ as "the value of continuous force of adfreezing" at the depth of the anchor into a permanently frozen layer $H_f$. He found that where the active layer is underlain by frozen soils, frost heaving stresses do not appear through the entire depth of the active layer. Thus, adfreezing occurs only in the portion of the active layer capable of adfreezing ($H_a$, Fig. 61).

This phenomenon is explained mainly by the fact that the lower part of the active layer is dried up in the process of moisture migration toward the freezing front. As an approximation, $H_a$ was two-thirds $H_{max}$, the maximum depth of frost penetration. By summing all forces in the $y$ direction (Fig. 61) the following equation is presented.

$$-P - r_a H_a P_1 + r_p H_f P_2 = 0$$

where $P_1$ and $P_2$ = mean perimeters of the foundation within the active and permanently frozen soil layers, respectively

which rearranged gives the required depth of anchor embedment in permafrost $H_f$ as

$$H_f = \frac{r_a H_a P_1 + P}{r_p P_2}$$

where $P$ = foundation or anchor load with downward being +

$$r_a = c_t + b_t (T) = \text{values of continuous adfreeze force within the active layer, where } c_t \text{ and } b_t \text{ are parameters dependent upon the absolute value of temperature below freezing, }$$

$$c_t = 4.26 \text{ to } 5.68 \text{ psi and } b_t = 1.42 \text{ to } 2.13 \text{ psi }^\circ C$$

$$T = "\text{mathematical absolute value}" \text{ of the temperature below } 0^\circ C, \text{ i.e. } -15^\circ C = 15^\circ C.$$ 

$$r_p = \frac{1}{10} r_s$$

where $r_s$ = temporary soil adfreeze strength (found from field tests).

Equation 52 is accurate only for temperatures higher than $-15^\circ C$ ($6^\circ F$).

Through an understanding of adfreeze forces, proper steps can be taken to reduce their effects. One solution is described by McKinley (1952) and an overall discussion of foundations in frozen ground is given by Sanger (1969).
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FOUNDATION ANCHORING IN THAWED GROUND

Porkhaev (1958) published an analysis, Foundation Anchoring in Thawed Ground, describing the holding strength of an anchor exposed to frozen soil conditions. In his analysis, based upon the soil weight directly over the anchor (Fig. 62), Porkhaev concluded that the pullout resistance is:

\[ P = \left( \frac{y_m H_a}{K_k} + \frac{y H_b}{K_k} \right) (A - A_b) \]  

(53)

where

- \( P \) = anchor resistance to pullout
- \( K_k \) = coefficient of anchor pullout force determined from Table VII
- \( y_m \) = unit weight of frozen soil in the seasonal frost layer
- \( y \) = unit weight of unfrozen soil
- \( A \) = area of anchor bottom
- \( A_b \) = cross-sectional area of anchor column
- \( H_a \) = depth of portion of active soil layer capable of freezing (see Fig. 62)
- \( H_b \) = depth of anchor below frozen soil layer (see Fig. 62).

In this analysis Porkhaev made conservative assumptions. This implies that eq 53 has an adequate built-in safety factor. However, it should be noted that Porkhaev did not consider heaving forces.

CONCLUSIONS

The reports cited herein and the information presented demonstrate the wide spectrum of anchor types, applications and design methods found today. This report is far from all-inclusive, and touches only briefly on the more common anchor types and design techniques; but it does point to the fact that the design of any anchorage system is dependent on the individual situation. There is no single anchor type or design method suitable for all applications. The most reliable design method is, of course, previous experience in identical or similar situations. Where this is lacking, the techniques outlined here can be used to suggest the more practical anchor types and to give some idea of the loads they will hold. Under these circumstances, however, a specific design still requires testing to assure that it will satisfy design requirements. The need for such testing can be reduced as more conclusive data are accumulated from field and laboratory tests.

Collection of such data is in itself a complex and time-consuming undertaking. Over the past few years a tremendous amount of research has been done on anchorages. Broms (1973) claims that over 150 articles have been published since 1960 on laterally loaded piles alone. Clearly, there is a great need to collect, analyze and disseminate this wealth of data. Likewise, some organization of on-going and future effort is badly needed. Perhaps the international effort of CIGRE Study Committee No. 7 (Biarez and Barraud 1968) could serve as a model. Here an evidently successful
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international effort was made by interested companies to test various configurations of power transmission tower anchors in different soil types throughout the world. The effort was centrally coordinated and all data were submitted in standardized form. As more and more such data are assimilated the science of anchorage design and construction will become more refined and economical.

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The findings of a literature review of anchor design are presented to give a synopsis of the numerous theoretical and empirical techniques available for predicting anchor capacity. The review revealed that anchor capacity is related to anchor configuration, soil characteristics and depth of anchor embedment and that the mode of soil failure as a result of anchor loading is dependent upon soil type and state as well as on the ratio of the depth of anchor embedment to anchor diameter. As a result it was found that no single equation can be used to predict anchor capacity under all soil conditions or anchor embedment depths.

14. Key Words

    Anchors (structures)
    Bridge anchorages
    Cold weather construction
    Foundations