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HEAT DISPERSION IN PHYSICAL ESTUARINE MODELS

Report I

STATE OF THE ART

by

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HEAT DISPERSION IN PHYSICAL ESTUARINE MODELS; Report 1, STATE OF THE ART

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Prototype heat exchange mechanisms and scaling laws for the reproduction of thermal phenomena in hydraulic models are presented for each stage of the dispersion process. These stages are: (a) turbulent or momentum entrainment at the efflux jet, (b) buoyant rise or fall of the heated plume, (c) convective spread of the plume over the surface of the receiving waters, (d) mass transport of the plume by ambient currents, (e) diffusion and dispersion (Continued)
20. ABSTRACT (Continued).

due to turbulence in the receiving waters, and (f) surface heat exchange with the atmosphere. As with any modeling effort, it is impossible to adequately model all phenomena simultaneously with only one model. The area near the efflux jet where turbulent entrainment and buoyancy are the significant dispersion mechanisms is referred to as the "near field." In this area the densimetric Froude number is the scaling criteria for modeling entrainment and the path of the plume. The modeling of turbulent entrainment and buoyancy requires an undistorted-scale model to ensure similitude of turbulent diffusion. The area where convective spread, surface heat exchange, mass transport by ambient currents, and ambient turbulent diffusion are the important dispersion processes is referred to as the "far field." The scaling criterion for convective spread is the densimetric Froude number. The standard Froude number is the scaling criterion for the ambient currents where vertical distortion is usually required to ensure that Reynolds numbers are large enough to ensure fully turbulent flow in the model. If the main dispersion stages in the far field are ambient turbulence and convective spread turbulence, an undistorted-scale model is required since there is no known way of distorting turbulence. Similarity of surface heat exchange coefficients may require a distorted-scale model, but this distortion is to a limited degree compatible with the distortion required to ensure fully turbulent flow.
PREFACE

The investigation reported herein was conducted by personnel of the Hydraulics Laboratory, U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, under the Civil Works Research and Development program.

The study was conducted during the periods 1 June through 31 August 1971 and 1 June through 31 August 1973 under the direction of Mr. H. B. Simmons, Chief of the Hydraulics Laboratory. The investigation was conducted by Dr. Victor L. Zitta, Engineer, and Dr. George W. Douglas, Engineer, with the assistance of Dr. G. H. Keulegan, Resident Consultant, and Mr. C. J. Huval, Research Hydraulic Engineer. Dr.'s Zitta and Douglas were temporary employees affiliated with Mississippi State University and the University of Alabama, respectively.

Directors of WES during the investigation and the preparation and publication of this report were BG E. D. Peixotto, CE, and COL G. H. Hilt, CE. Technical Director was Mr. F. R. Brown.
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HEAT DISPERSION IN PHYSICAL ESTUARINE MODELS

STATE OF THE ART

PART I: INTRODUCTION

Background

1. Distorted-scale hydraulic models have been used to study dispersion of heated discharges into oceans, estuaries, rivers, and lakes at the U. S. Army Engineer Waterways Experiment Station (WES), as well as at other research installations, for a number of years. These models were originally constructed for the study of certain hydrodynamic processes such as tidal heights, velocities, and shoaling effects, in addition to mass transfer evidenced by salinity gradients. The validity of results from such distorted-scale models is established through extensive verification processes whereby the model is made to conform to prototype data. In general, existing models at WES have been neither constructed nor verified for thermal studies and questions have been raised as to how well thermal dispersion in such models correlates with the prototype.

2. Thermal verification has been hampered by the lack of reliable prototype data. Furthermore, dispersion processes based on turbulence which are important in thermal dispersion cannot be adequately scaled in a distorted-scale model. However, distorted-scale physical models can be used for thermal studies, but careful verification procedures need to be applied. This requires prototype data and an understanding of the physical processes involved.

3. Historically, three methods have been used to establish scaling laws for physical models: the dynamical, the dimensional, and the equational methods. The dynamical method was first developed. However, inadequacies in the dynamical method led to the development of the dimensional method. Both these methods were based on geometrically similar models, and, hence, did not allow distortion. For this reason, the advent of the equational method expanded physical modeling.
applications, since it is not based on geometric similarity and allows model distortion in the reproduction of hydraulic phenomena. The equa­tional method has been used most often in developing scaling laws for heated effluents. The most complete development is presented by Stolzenbach and Harleman; however, Ackers and Silberman and Stefan have presented similar developments which concur with those presented by Stolzenbach and Harleman.

**Objective**

The objective of this study is to determine the capability of distorted-scale physical models for reproducing the far field dispersion characteristics of thermal waste in estuaries. This report is a review and evaluation of present scaling laws necessary for the reproduction of thermal phenomena in hydraulic models.
5. The atmosphere will be the ultimate sink for excess heat discharged into a water body. The dispersal of the heat throughout the receiving water is, however, of concern since certain areas may experience a temperature rise. Interest in the magnitude and location of isotherms stems from the fact that areas of excess heat may adversely affect the biological components of the system, as well as its ability to supply cooling water to industry. Quantitative data may be needed to locate specific isotherms; qualitative results may be needed only to determine the general location of a plume. In general, several of the exchange mechanisms to be discussed herein will affect the heat dispersal simultaneously; usually, however, one mechanism will dominate.

Classification of Heat Dispersion

6. The stages or modes of heat dispersion in nature are listed below with a brief description of each dispersion process.

Turbulent entrainment of the efflux jet

7. In cases where the heated water is discharged into the receiving water by a jet, turbulent entrainment causes rapid mixing of the two streams (Figure 1). This condition results from turbulence generated at the borders of the submerged jet due to friction. As the receiving water is entrained, there is a momentum transfer and the jet

![Figure 1. Turbulent entrainment of heated plume](image-url)
decelerates. The entrainment rate is a function of the initial jet inertia and the availability of entraining fluid, not density differences.

Buoyant rise or fall of the thermal plume

8. As a jet enters a body of receiving water, its trajectory is influenced by several factors. The initial inertia of the jet affects the distance to which it will penetrate the receiving body before appreciable deceleration occurs due to entrainment. The presence of ambient stream currents will alter the path of the plume and, at the same time, possibly increase the turbulent structure at the plume boundary. Any density difference between the plume and receiving water will, of course, result in a rise or fall of the plume. Since power plant cooling water intakes located in estuaries are sometimes several miles from the cooling water outfalls, a variety of plume buoyancy conditions can occur (see Figures 2 and 3). Since the salinity of the heated effluent may be significantly greater than that of the receiving water, the plume could sink rather than rise. This condition arises because the effluent, even though heated, is denser than the receiving water because of its higher salinity. Also, when the receiving water is well stratified with regards to salinity, a neutrally buoyant plume may result, i.e., the heated plume is less dense than the bottom receiving waters but denser than the surface receiving waters. If the plume has a positive buoyancy, it will tend to rise to the surface with a two-fold effect: the three-dimensional character of the entrainment mechanism will be altered as the plume nears the surface and the heated plume may begin to dissipate heat to the atmosphere. If the plume does not rise to the surface, the transfer of heat from the plume to the receiving waters is due primarily to entrainment by turbulence at the plume boundary.

Convective spread mixing

9. Outside the initial dilution zone, the plume may still exhibit a density difference sufficient to cause mixing at the interface. This mixing is due not to large-scale turbulence (e.g., the stages described in paragraphs 7 and 8 above), but to the tendency of small interfacial
Figure 2. Discharge of effluent into nonstratified water body

Figure 3. Discharge of effluent into stratified water body
disturbances to be amplified. If these small disturbances are dampened instead of amplified, the interfacial structure is stable, thereby eliminating mixing from convective spreading.

Mass transport by ambient currents

10. Ambient currents will tend to disperse the thermal plume as well as other exchange mechanisms (see Figure 1a).

Diffusion and dispersion due to turbulence in the receiving waters

11. The turbulent structure of the receiving water is characterized by the instantaneous velocity fluctuations and by eddies formed by velocity gradients. The steady-state form of the equation governing conservation of heat in the advective turbulent flow field is given below:* 

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{\partial}{\partial x} \left( \frac{D_x}{\partial x} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{D_y}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{D_z}{\partial z} \frac{\partial T}{\partial z} \right) \]  

(1)

Dispersion by ambient turbulence is considered to be superimposed upon the other dispersion processes.

Surface heat exchange with the atmosphere

12. The atmosphere is the ultimate sink for all excess heat discharged into a water body. The mechanisms of heat transfer across the water surface have been presented in a comprehensive study by Edinger and Geyer \(^5\) (1965). The mechanisms shown in Figure 4 with the range of values for northern latitudes are incoming short-wave solar radiation and long-wave atmospheric radiation; outgoing long-wave back radiation, heat loss due to evaporation, and reflected solar and atmospheric radiation; and loss or gain by conduction. Obviously the net rate of heat transfer is not constant but varies considerably within a diurnal period. The equation describing heat loss from the free surface of a well mixed body of water with an artificial input is

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* For convenience, symbols and unusual abbreviations are listed and defined in the Notation (Appendix A).
Figure 4. Heat transfer mechanisms across a water surface

\[ \rho \cdot c_p \cdot V \cdot \frac{\partial T}{\partial t} = -K\alpha(T - T_e) + H \]  

(2)

K must be recognized as a net heat exchange coefficient that includes all of the heat exchange mechanisms shown in Figure 4. If \( T_e \) can be considered constant, then an ambient temperature \( T_a \) can be defined by the equation

\[ \rho \cdot c_p \cdot V \cdot \frac{\partial T_a}{\partial t} = -K\alpha(T_a - T_e) \]  

(3)

Equilibrium conditions exist when no net surface heat exchange occurs following an artificial heat addition into a body of water.

Near Field-Far Field

13. The dispersion stages described above are more generally classified "near field" and "far field." The near field consists basically of the first two stages, i.e., the zone where dispersion is due mainly to momentum entrainment and buoyancy effects. The far field consists of the next four stages, i.e., the region where convective turbulence, ambient currents, ambient turbulence, and/or surface heat exchange dominate the heat dispersion process.
PART III: MODELING LAWS GOVERNING HEAT DISPERSION

14. The primary variables that must be scaled from prototype to model in heat dispersion modeling are:

\[ L_z = \text{vertical length} \]
\[ L_y = \text{lateral length} \]
\[ L_x = \text{longitudinal length} \]
\[ t = \text{time} \]
\[ u = \text{velocity} \]
\[ \Delta T = \text{temperature difference} \]
\[ \Delta \rho = \text{density difference} \]
\[ Q = \text{discharge} \]
\[ K = \text{net surface heat exchange coefficient} \]

15. The model should duplicate, at a reduced scale, prototype values \( u \), \( \Delta T \), \( \Delta \rho \), \( Q \), and \( K \) at some location \( L_z \), \( L_y \), \( L_x \), and at some time \( t \). It should be noted here that for all "distorted-scale" models discussed in this report, \( L_x = L_y \), i.e., distorted-scale physical models are distorted vertically, not longitudinally or laterally.

Receiving Waters Modeling

16. Flow in the receiving waters is governed by gravitational forces represented by the Frouadian relationship known as the Froude number which can be stated

\[
\frac{u_m}{u_p} = \left( \frac{L_z}{L_z} \right)^{1/2}
\]  (4)

Equation 4 ensures similitude of velocity magnitudes. Use of this Frouadian relationship and the law of continuity results in the following discharge description

\[
\frac{Q_m}{Q_p} = \frac{u_m (L_x) (L_z) m}{u_p (L_x) (L_z) p}
\]  (5)
Also, the time relationship can be stated as follows:

\[ \frac{t_m}{t_p} = \frac{\left( \frac{L_x}{m} \right)}{\left( \frac{L_x}{p} \right)} \]

(6)

The Froudian relationship is the basis for the scaling laws of existing estuary models at WES.

Heated Effluent Modeling

17. As in any modeling effort, it is not usually possible to adequately model all phenomena simultaneously. Ackers\(^3\) notes that in the modeling of heated effluents when motion already exists prior to heated effluent discharge, as in the case of a tidal estuary, the similarity of flow patterns in the receiving fluid must first be obtained. As previously mentioned, this modeling is ordinarily achieved by equating model and prototype Froude numbers. The second step involves recognition of the dominant mode of dispersion. The prototype dispersion or heat exchange mechanisms have been presented in Part II. The heat dispersion modeling requirements for each mechanism are as follows.

Turbulent entrainment and buoyant rise or fall of the plume

18. These two stages must be modeled at a natural or undistorted scale (horizontal scale equals vertical scale). Vertical distortion of the model would cause loss of similarity with respect to vertical turbulent mixing. In other words, a distorted-scale model cannot be used to adequately predict prototype performance where these two stages are the dominant means of dispersion, since there are no means available for distorting the turbulent structure of a jet. In general, the scaling law required for modeling these stages in an undistorted-scale model is termed the densimetric Froude number, \( F_\Delta \), which can be stated
The densimetric and regular Froude numbers give identical velocity ratios if

\[ \left( \frac{\Delta \rho}{\rho} \right)_m = \left( \frac{\Delta \rho}{\rho} \right)_p \]  

(8)

Since the regular Froude number is normally used as the scaling criteria for receiving waters, it is convenient to use an undistorted density ratio, represented by Equation 8, when modeling the two stages.

**Convective spread mixing**

19. For studies of convective spread, Barr has developed a densimetric Froude-Reynolds number for a geometrically similar or undistorted-scale model to specify equality in model and prototype. Keulegan calls this term the densimetric Reynolds number \( F_{\Delta R} \), which can be stated

\[ \left[ \left( \frac{\Delta \rho}{\rho} g \right)^{1/2} L_x^{3/2} \right]_m = \left[ \left( \frac{\Delta \rho}{\rho} g \right)^{1/2} L_x^{3/2} \right]_p \]  

(9)

Barr notes, however, that if turbulent flow is assured in both model and prototype, strict equality of this parameter is not needed. In fact, convective spread models are usually based on the densimetric Froude law with a densimetric Reynolds number high enough to provide turbulent conditions; this factor may actually require vertical distortion of the model. The functional relation describing the two-layered flow conditions during convective spreading is

\[ \frac{dh_2}{dx} = \phi(f_b, f_i, \eta, F_{\Delta}, q) \]  

(10)

Assuming the model has similarity of \( F_{\Delta} \) and \( q \), Equation 10 becomes

\[ \frac{dh_2}{dx} = \phi(f_b, f_i, \eta) \]  

(11)
If \( f_b \), \( f_i \), and \( \eta \) are properly scaled in the model, the scaling law becomes
\[
\phi_r = \frac{(L_z)_r}{(L_x)_r} \tag{12}
\]

In a distorted-scale model where \( (L_z)_r > (L_x)_r \), the model friction, \( f_b \), must be adjusted so that \( \phi_r(f_b, f_i, \eta) \) is also > 1.

**Mass transport by ambient currents**

20. The Froude law is used when the ambient currents are superimposed on the jet discharge. These ambient currents can be treated by the standard laws for homogeneous fluids, including the Froude number, with a Reynolds number sufficient to ensure turbulent flow. Therefore, vertical distortion is allowed in the modeling of mass transport by ambient currents.

**Dispersion due to turbulence in the receiving water**

21. A natural or undistorted-scale model is required for this stage to maintain similarity of horizontal and vertical diffusion. As stated previously, the steady-state form of the equation governing conservation of heat in an advective turbulent flow field is given in Equation 1. Assuming that the velocities \( u \), \( v \), and \( w \) are correctly modeled according to Equations 4 and 7, and correctly represented by a velocity scale \( u_m / u_p \), then, from Equation 1, the diffusion scales are as follows:
\[
\frac{(D_x)_m}{(D_x)_p} = \frac{u_m}{u_p} \left(\frac{L_x}_m\right)^\frac{2}{\left(\frac{L_x}_p\right)} \tag{13}
\]
\[
\frac{(D_y)_m}{(D_y)_p} = \frac{u_m}{u_p} \left[\left(\frac{L_y}_m\right)^\frac{2}{\left(\frac{L_y}_p\right)}\right] \left/(\frac{L_x}_m\right) \frac{L_x)_p}{(L_x}_p \tag{14}
\]
The magnitude of the diffusion coefficients is relative to the turbulent structure of the ambient velocity. At present, there is no way of relating the effect of model distortion to the turbulent structure and, thus, to the dispersion coefficients.

Surface heat exchange with the atmosphere

This stage could significantly affect surface plume characteristics. Therefore, similarity of surface heat exchange may be sought, although practicality often prevents strict similarity. If test durations are sufficiently short, for example, effects of cooling may be slight. Driver and Elder note that for a model operated in a laboratory without a controlled environment, humidity is likely to be greater and wind movement less than for the prototype, both factors reducing evaporative heat loss. Unless the model is outdoors and in the same geographical locale as the prototype, incoming radiation will be different. In addition, depth of penetration might be only a small portion of prototype depth but greater than the entire model depth. To evaluate the influence of all these factors, tests should be conducted concurrently with the operation of the thermal model to determine the surface heat exchange coefficient appropriate to the model. After comparison with the prototype, model results can be adjusted accordingly.

Stolzenbach and Harleman have shown that if the densimetric Froude similarity is adhered to and \( \frac{\Delta \rho / \rho}{\Delta \rho / \rho} \) equals 1.0, the model law for surface heat exchange similarity is

\[
\frac{K_m}{K_p} = \left( \frac{L_z}{L_z} \right)^{3/2}
\]

The application of Equation 16 assumes a knowledge of the surface heat
transfer coefficient for both the model and prototype. When the model already exists at a given scale, the model surface heat transfer rate must be controllable to satisfy this similarity law; however, such a situation is highly impractical for two reasons. First, an accurate estimate of \( K_m \) is difficult to obtain. Usually, \( K_p \) is determined from empirical relations which are largely a function of wind velocity and solar radiation. Since neither of these factors are present in an enclosed model, the empirical relations do not apply. Second, there is reason to believe that, in some instances, appreciable heat transfer can occur between the model base (usually concrete) and the water, in addition to the heat exchange with the atmosphere. In this case, \( K_m \) will be an overall coefficient and will, in general, not be a function of the parameters as associated with \( K_p \). At present, the best way to determine \( K_m \) is to determine the rate of change of the temperature, \( \frac{dT}{dt} \), of a test pool at the model conditions; Equation 2 can then be applied. The equilibrium temperature, \( T_e \), must first be determined, however, and for the model this is also a problem. By definition, \( T_e \) is that temperature reached by the water body when the heat transfer in and out are equal. If ambient conditions (air temperature, humidity, concrete temperature) could be controlled, a test pool could be allowed to come to equilibrium, thereby yielding \( T_e \). This is not practical however, and some relationship between easily measured parameters is needed.
PART IV: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

23. Based on the model scaling laws discussed in Part III, the densimetric Froude number, $F_r$, must be satisfied in all thermal models. Also, $(\Delta \rho / \rho)_r$ should equal 1.0 (Equation 8) in which case the densimetric and the standard Froude number scales will be identical. It should be mentioned here that satisfying the equation $\Delta T_r = 1.0$ does not necessarily satisfy Equation 8, since the temperature-density curve for water is nonlinear. Strict similarity would require also that $T_r = 1.0$; however, satisfying this equation is impractical, and except for extreme cases, the error introduced by ignoring the absolute temperature criteria is minimal.

24. In addition to the magnitude of velocities being correctly scaled, the large-scale ambient currents in the receiving water must be correctly reproduced in the model. In the deeper areas of an estuary, the ambient currents are most likely to be dominated by tides and, thus, correctly scaled by Froude number criteria. However, in shallow bays, the dominant currents may be dominated not by tides but by wind stress, which is not accounted for in a physical model.

25. If the dispersion process is caused by turbulent mixing or turbulent diffusion, then an undistorted model is required ($L_x = L_y = L_z$), and thus, based on Equations 13, 14, and 15, $D_x = D_y = D_z$.

26. For convective spread of a stratified flow, Equation 12 must be satisfied. If a distorted model is used, the model friction must be adjusted to satisfy Equation 12.

27. When surface heat exchange is important, a distorted model is required to satisfy Equation 17,

$$K_r = \frac{(L_z)^{3/2}}{(L_x)^{1/2}} \left(\frac{\Delta \rho}{\rho}\right)_r^{1/2}$$
or, if Equation 8 is satisfied:

\[ K_r = \frac{(L_z)_r^{3/2}}{(L_x)_r} \]  

(18)

It should be noted that if \((L_x)_r = 1/1000\) and \((L_z)_r = 1/100\) then \(K_r = 1.0\) or the heat exchange coefficient should be the same in model and prototype. With the distortion ratio held constant at 10, \(K_r\) increases as the size of the model is increased.

**Recommendations**

28. From this study the following general recommendations seem justified:

a. In a modeling effort where turbulent entrainment and buoyancy effects are not significant modes of dispersion, the model scales can be distorted, thereby achieving adequate reproduction of far-field thermal phenomena.

b. In a modeling effort where turbulent entrainment and/or buoyancy effects are significant modes of dispersion, two models are required. First, an undistorted-scale model must be used to study the near-field dispersion processes of turbulent entrainment and buoyancy effects, which possibly could be accomplished in a steady-state model without ambient currents. The results of the undistorted-scale model can then be used to duplicate the boundary conditions of the near field in a distorted-scale model of the far field. However, if ambient turbulence in the far field is a significant dispersion process, then it is necessary to treat the results obtained from a distorted-scale model qualitatively only.

29. The collection of prototype data for comparison with model results is a research requirement, since model-prototype comparison is the best way to evaluate the effect of model-scale distortion. Occasionally, model tests are conducted involving problems where the heated discharge already exists in nature. In such cases, model observations can be compared with field observations of the existing heated discharge to verify the model results. More frequently, however, model tests are conducted for a proposed heated discharge in nature. In other words, no prototype observations of the heated discharge are available for
verification of model results since the prototype heated discharge does not exist. In this case, two different approaches can be used to verify the model with regards to far-field thermal dispersion.

30. The first approach involves a continuous dye release at the prototype site of the proposed heated discharge for a period of several weeks to several months. The dye dispersion is monitored periodically during the release. This dye release can then be simulated in the model and the model dye dispersion results compared with the field observations. Additionally, the field dye dispersion observations can be analytically converted to the equivalent heat dispersion patterns, if certain assumptions are made.

31. The second approach to model verification is possible if a heated discharge already exists in the vicinity of the proposed heated discharge. The existing heated discharge is simulated in the model and results are compared with field observations.

32. However, for either approach, several years may elapse after model testing before comparative prototype data of the discharge are available. In numerous instances, no provision is made for even securing such follow-up data. Hopefully, future field data will strengthen physical modeling as a valuable tool for studying heated discharges. Although a multitude of analytical techniques will undoubtedly be proposed for treatment of geometry effects and other constraints, models will continue to play an important role in the future, especially in regions subject to tidal action and irregular geometry.
REFERENCES


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APPENDIX A: NOTATION

A  
Surface area

$\text{cp}$  
Specific heat

$D$  
Turbulent diffusion coefficient

$f_b$  
Bed friction factor

$f_i$  
Interfacial friction factor

$F_A$  
Densimetric Froude number

$F_{AR}$  
Densimetric Reynolds number

$g$  
Gravitational acceleration

$h_1$  
Depth of upper layer

$h_2$  
Depth of lower layer

$H$  
Artificial heat input

$K$  
Net surface heat exchange coefficient

$L$  
Length

$m$  
Denotes model

$p$  
Denotes prototype

$q$  
$Q_1/Q_2$ where $Q_1$ and $Q_2$ are upper and lower flows, respectively

$Q$  
Discharge

$r$  
Denotes ratio, model to prototype

$t$  
Time

$T$  
Temperature

$T_a$  
Ambient temperature

$T_e$  
Equilibrium temperature

$u$  
Velocity in the longitudinal (x) direction

$v$  
Velocity in the lateral (y) direction

$V$  
Volume

$w$  
Velocity in the vertical (z) direction

$x, y, z$  
Denote Cartesian coordinate system

$\Delta$  
Denotes a difference

$\eta$  
Ratio of upper depth $h$, to total depth, $h_1 + h_2$

$\rho$  
Density

$\nu$  
Kinematic viscosity

$\phi$  
Denotes a function of