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**WAVE LOADING ON VERTICAL SHEET-PILE GROINS AND JETTIES**

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Groins  Jetties  Mach-stem reflection  Wave loading  Waves

A method is presented for calculating the distribution of force and overturning moment resulting from incident water waves moving along the axis of a groin or jetty with vertical sides. Wave height at the structure is determined from experimental data on Mach-stem reflection. The distribution of force is assumed to be in proportion to the nonlinear shallow-water wave profile given by either the cnoidal or stream-function wave theory. An example problem demonstrates how the cnoidal theory may be used to estimate the wave force and overturning moment distribution along a structure.
PREFACE

This report is an outgrowth of consultations with personnel from the U.S. Army Engineer District, Savannah, concerning wave loading on a concrete sheet-pile groin proposed for Tybee Island, Georgia. The Shore Protection Manual (SPM) provides methods for calculating wave forces due to waves acting normal to vertical-sided structures. The SPM also gives a method for reducing the dynamic component of force when waves approach at an angle to the structure; however, no methodology is presented for estimating the distribution of force and overturning moment along a structure's axis, particularly for waves propagating with crests nearly perpendicular to the structure. This report provides such a methodology. For sheet-pile groins and sheet-pile jetty sections, cost savings can sometimes be realized if wales are assumed to longitudinally distribute some of the wave force. The wave loading methodology presented will provide the information needed to calculate the forces transmitted by the wales. The work was carried out under the coastal structures research and development program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by Dr. J. Richard Weggel, Chief, Evaluation Branch, under the general supervision of N. Parker, Chief, Engineering Development Division.

Comments on this publication are invited.

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TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director
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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

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<th>by</th>
<th>To obtain</th>
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^1To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: \( C = \frac{5}{9} (F - 32) \).

To obtain Kelvin (K) readings, use formula: \( K = \frac{5}{9} (F - 32) + 273.15 \).
WAVE LOADING ON VERTICAL SHEET-PILE GROINS AND JETTIES

by

J. Richard Weggel

I. INTRODUCTION

The Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977) provides guidance for the design of vertical wall structures subjected to either breaking, nonbreaking, or broken waves; however, the design methods presented are for waves approaching perpendicular to a structure. The SPM also presents a rational, though untested, procedure for reducing forces when waves approach a structure at an angle. The SPM force reduction appears to be valid if the direction of wave approach is not too different from a perpendicular, i.e., $\alpha > 70^\circ$, where $\alpha$ is the angle between a wave crest and a perpendicular to the structure. For structures essentially normal to shore such as groins and jetties, waves usually propagate along the axis of the structure rather than normal to it. For this situation, the SPM force reduction factor, which varies as $\sin^2 \alpha$, underpredicts the maximum wave force since $\alpha + 0^\circ$ and $\sin^2 \alpha + 0$. Furthermore, the instantaneous distribution of force along the structure is not uniform when $\alpha$ is small since wave crests act along some parts of the groin or jetty while wave troughs act along other parts. Consequently, the maximum wave force acts over only a small part of the structure at any one time and the point on the structure where the maximum force acts moves landward along the structure as the wave propagates to shore. This report outlines a design procedure for estimating the force acting on a groin or jetty when waves propagate along the axis of the structure, i.e., when $\alpha$, the angle between the incoming wave crests and a perpendicular to the structure, is less than about $45^\circ$.

II. SPM METHODOLOGY FOR COMPUTING FORCES AND MOMENTS

The SPM presents three procedures for calculating wave forces on vertical or near-vertical walls. For nonbreaking waves, the Miche-Rundgren method is given. The SPM design curves for this method were developed from Rundgren's (1958) equations and, for small values of the incident wave steepness $H_i/gT^2$, from Saintlou's (1928) equations where $H_i$ is the incident wave height, $g$ the acceleration of gravity, and $T$ the incident wave period. For breaking waves, the Minikin (1963) method is given in the SPM. When waves break against a structure they exert extremely high impact pressures of very short duration. The impact with the structure of the translatory water mass associated with the moving wave crest causes the high pressures. The Minikin method attempts to describe these pressures but in applying the method the force is assumed to act statically against the structure. The problem is in reality a dynamic one. Wave pressures exceeding those predicted by the Minikin method have been measured; however, their duration is so short that the assumption that the force is a static one makes the method extremely conservative even though the force itself may be underpredicted. Since it is the impact of a translatory mass of water that results in the high pressure, it seems doubtful that waves approaching a vertical wall at an angle could cause them. The Minikin method is thus inappropriate for calculating forces by waves approaching a structure at an angle. For broken waves, the SPM presents a rational method for calculating forces based on computing the stagnation pressure. Again, this method is for normal wave approach.
A modification of the Miche-Rundgren method as presented in the SPM appears to be applicable to breaking, nonbreaking, and broken waves for situations when waves approach at an oblique angle.

III. MACH-STEM REFLECTION

Waves in the vicinity of a vertical wall are comprised of incident waves and superimposed reflected waves. For a perfectly reflecting vertical wall with waves of normal incidence, the reflected wave height equals the incident wave height resulting in wave heights in front of the wall equal to twice the incident height. When waves approach a structure at an angle, reflection is not complete resulting in a wave height less than the sum of the incident and reflected waves. For angles of incidence, $\alpha > 45^\circ$, a phenomenon termed Mach-stem reflection may occur. Instead of reflecting at the angle of incidence, near the structure the incident wave crest turns so that it intersects the structure at a right angle. This gives rise to three separate parts of the wave—an incident wave, a reflected wave, and a Mach-stem wave propagating along the axis of the structure with its crest perpendicular to the structure face (see Fig. 1). The amplitude of the Mach-stem wave, which is the wave causing the force on the wall, was investigated by Perroud (1957) and Chen (1961). The relative amplitude of the Mach-stem wave as a function of angle of incidence and $H_i/2d$ is given in Figure 2 ($H_i$ is the incident wave height and $d$ the water depth). A more general relationship for average reflection data, independent of $H_i/2d$, is given in Figure 3. For $\alpha > 45^\circ$ the relative amplitude of the wave is about 2.0, i.e., the wave height along the wall is the sum of an incident and reflected wave which has a height equal to twice the incident wave height; for $\alpha < 45^\circ$, the relative amplitude of the Mach-stem wave is less than 2.0.

\[
\begin{align*}
H_i &= \text{Height of Incident Wave} \\
H_r &= \text{Height of Reflected Wave} \\
H_m &= \text{Height of Mach-Stem Wave} \\
\alpha &= \text{Angle of Incidence (angle between wave crest and perpendicular to structure)} \\
r &= \text{Angle of Reflection}
\end{align*}
\]

![Diagram of wave crests](image1.png)

Figure 1. Reflection patterns of a solitary wave.
Figure 2. Oblique reflection of a solitary wave, experimental results; water depth, \( d = 0.132 \) foot (from Perroud, 1957).

Figure 3. Oblique reflection of a solitary wave, experimental results (from Perroud, 1957).
IV. CALCULATION OF WAVE FORCE AND OVERTURNING MOMENT

The Miche-Rundgren method in SPM Section 7.32 provides a method for determining wave force and overturning moment when either a wave crest or trough acts against a wall. For oblique incidence some parts of the wall are acted on by wave crests while others are acted on by wave troughs. Thus, there is a variation in force along the wall that can be assumed in proportion to the wave profile along the wall; i.e., the maximum wave force corresponds to the wave crest, the minimum wave force corresponds with the wave trough, and the variation in force between is assumed proportional to the ordinate of the wave profile.

It remains to select an appropriate description of the wave profile. For waves in shallow water the cnoidal theory outlined in Section 2.26 of the SPM provides a satisfactory description. Stream-function wave theory (Dean, 1974) will also provide a satisfactory description of the profile; however, for purposes of illustration and convenience the cnoidal theory will be used here.

V. EXAMPLE PROBLEM

The computation of forces and moments is best illustrated by an example problem.

GIVEN: A concrete sheet-pile groin perpendicular to shore is subjected to waves 6 feet (1.83 meters) high with a period of 8.0 seconds. Waves approach the groin so that the angle between the wave crest and a perpendicular to the groin is 30°. The water depth at the end of the groin is 10 feet (3.05 meters). The beach profile along the windward side of the groin is given in Figure 4.

![Figure 4. Beach and groin profile.](image)

FIND: Ignoring changes in wave direction as the wave propagates toward shore (refraction) and changes in depth near shore (shoaling), determine the maximum wave force and overturning moment acting on the groin when the wave crest passes a point 400 feet (121.9 meters) from shore (100 feet or 30.5 meters from the seaward end of the groin) and estimate the distribution of force and moment along the groin at that instant.
SOLUTION: An estimate of the reflection coefficient can be obtained from Figure 2 or 3. Calculate

\[
\frac{H_i}{2d} = \frac{6}{2(10)} = 0.30
\]

From Figure 2, with \( H_i/2d = 0.3 \) and \( \alpha = 30^\circ \), read \( H_{ms}/2d = 0.47 \). Therefore,

\[
H_{ms} = 0.47(2d) = 9.4 \text{ feet (2.87 meters)}
\]

Alternatively, from Figure 3 for \( \alpha = 30^\circ \), \( H_{ms}/H_i = 1.61 \) or

\[
H_{ms} = 1.61(6) = 9.7 \text{ feet (2.96 meters)}
\]

This estimated Mach-stem wave height is subject to limitations imposed by the maximum breaker height that can exist in the given water depth. This breaking wave height is given approximately by \( H_b/d = 0.78 \) or, for the example

\[
H_b = 0.78d = 0.78(10) = 7.8 \text{ feet (2.38 meters)}
\]

The wave at the structure is thus limited to a height of 7.8 feet. The remainder of the calculations are based on this maximum wave height.

The maximum wave force (crest at the wall) can be estimated using SPM Figure 7-70 by assuming that the maximum wave height of 7.8 feet is the sum of an incident wave and a reflected wave, each 3.9 feet (1.19 meters) high. Calculating

\[
\frac{H_i}{d} = \frac{3.9}{10} = 0.39
\]

and

\[
\frac{H_i}{gT^2} = \frac{3.9}{(32.2)(8)^2} = 0.00189
\]

From SPM Figure 7-70, read from the top part of the figure,

\[
\frac{f_C}{wd^2} = 1.14
\]

and from the bottom part of the figure,

\[
\frac{f_t}{wd^2} = 0.38
\]
where \( F_c \) and \( F_t \) are the wave force when the crest and trough are at the structure respectively, and \( w \) is the specific weight of the water (64 pounds per cubic foot (10,000 newtons per cubic meter) for seawater). Therefore,

\[
F_c = 1.14(wd^2) = 1.14(64)(10)^2
= 7,300 \text{ pounds per foot (106,500 newtons per meter)}
\]

and

\[
F_t = 0.38(wd^2) = 0.38(64)(10)^2
= 2,400 \text{ pounds per foot (35,000 newtons per meter)}
\]

Similarly, from SPM Figure 7-71, the maximum and minimum overturning moments are

\[
M_c = 0.64(wd^3) = 0.64(64)(10)^3
= 41,000 \text{ foot-pounds per foot (182,400 newton meters per meter)}
\]

\[
M_t = 0.11(wd^3) = 0.11(64)(10)^3
= 7,000 \text{ foot-pounds per foot (31,100 newton meters per meter)}
\]

where \( M_c \) and \( M_t \) are the overturning moments caused by the waves when the crest and trough are at the structure.

To determine the distribution of force along the groin, the appropriate cnoidal wave profile must be found. Calculate

\[
T = \sqrt{\frac{\pi^2}{\alpha}} = 8 \sqrt{\frac{32.2}{10}} = 14.36
\]

and

\[
\frac{H}{d} = 0.78
\]

which is the limiting value of \( H/d \). From SPM Figure 2.11 for the calculated values of \( T \sqrt{\frac{g}{d}} \) and \( H/d \), find the value of \( k^2 = 1 - 10^{-4.5} \). From SPM Figure 2-12 with \( k^2 = 1 - 10^{-4.5} \), find

\[
\frac{L^2 H}{d^3} = 240
\]

The wavelength, \( L \), is therefore

\[
L^2 = 240 \frac{d^3}{H} = \frac{240(10)^3}{7.8} = 30,770
\]

\[
L = 175 \text{ feet (53.3 meters)}
\]
This wavelength is measured perpendicular to the wave crests. To obtain the wavelength along the structure, this value must be divided by \( \cos \alpha \). Thus,

\[
L' = \frac{L}{\cos \alpha} = \frac{175}{\cos 30^\circ} = 203 \text{ feet (61.9 meters)}
\]

where \( L' \) is the wavelength along the structure. The appropriate wave profile is obtained from SPM Figure 2.9 by interpolating between the profiles for \( k^2 = 1 \times 10^{-4} \) and \( k^2 = 1 \times 10^{-5} \). This profile is plotted in Figure 5 in dimensionless form.

![Dimensionless cnoidal wave profile](image)

**Figure 5.** Dimensionless cnoidal wave profile, \( k^2 = 1 \times 10^{-4.5} \).

The distribution of force along the structure is then obtained by letting the maximum correspond to 7,300 pounds per foot and the minimum to 2,400 pounds per foot. Then \( F_c - F_t = 7,300 - 2,400 = 4,900 \) pounds per foot or 71,500 newtons per meter and

\[
F(x) = 2,400 + 4,900 \eta(x)
\]

where \( F(x) \) is the variation of force with distance along the structure, and \( \eta(x) \) is the variation of the dimensionless surface profile with \( x \). Similarly, the distribution of the overturning moment along the structure can be determined. \( M_c - M_t = 41,000 - 7,000 = 34,000 \) foot-pounds per foot or 151,200 newton meters per meter and therefore,

\[
M(x) = 7,000 + 34,000 \eta(x)
\]
Values of $F(x)$ $M(x)$ are tabulated for the example in the Table and plotted in Figure 6. The force distribution along the groin is shown in Figure 7.

<table>
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<tr>
<th>$x/L'$</th>
<th>$x$ (Ft)</th>
<th>$n(x)$</th>
<th>$F(x)$ (lb/ft)</th>
<th>$M(x)$ (ft-lb/ft)</th>
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<td>1.0</td>
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<td>101.50</td>
<td>0.0</td>
<td>2,400</td>
<td>7,000</td>
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</table>

1 Computed from $x = 203 x/L'$
2 Computed from $F(x) = 2,400 + 4,900 n(x)$
3 Computed from $M(x) = 7,000 + 34,000 n(x)$

Figure 6. Variation of force and moment along groin.
As a check on the wave force when the trough is at the structure, the hydrostatic force can be computed using the minimum water surface elevation. From SPM Figure 2-13, the value of \( \frac{y_t - d}{H} + 1 \) is found to be 0.850 where \( y_t \) is the height of the wave trough above the bottom. Therefore,

\[
\frac{y_t - d}{H} + 1 = 0.850
\]

rearranging and solving for \( y_t \),

\[
y_t = (0.850 - 1.0) H + d
\]

or

\[
y_t = -0.150 (7.8) + 10 = 8.83 \text{ feet (2.69 meters)}
\]

Computing the hydrostatic pressure

\[
F_t = \frac{w y_t^2}{2} = \frac{64(8.83)^2}{2} = 2,495 \text{ pounds per foot or 36,400 newtons per meter}
\]

compared with \( F_t = 2,400 \) pounds per foot for the value computed from the Miche-Rundgren figures in the SPM.
When actually computing forces on sheet-pile groins and jetties, the restoring force caused by water and wave action in the structure's leeward side must also be considered. The critical design situation occurs when the water surface on the leeward side is a minimum; i.e., when a wave trough acts there. For the example, the worst case for overturning moment exists when the water level on the leeward side is equal to $y_t = 8.83$ feet. This corresponds to a minimum restoring force of 2,400 pounds per foot.

A critical factor not included in the example that must be considered in any real design problem is the force arising because of the differential sand elevation on each side of the structure. These forces must be based on estimates of the maximum deposition and scour that will be experienced during the lifetime of the structure. It is also possible that this critical condition will occur during construction unless a scour blanket is placed adjacent to the structure.

VI. SUMMARY

The proposed method for computing the distribution of wave force and overturning moment along a vertical sheet-pile groin or jetty is approximate. It assumes that the force and moment are in proportion to the nonlinear wave profile as given by the cnoidal wave theory. Alternatively, any other appropriate wave theory to describe the profile could be used. The assumption that a wave crest or trough acts uniformly along structures oriented nearly perpendicular to shore grossly overpredicts the total force since only a small part of the structure is acted on by a wave crest at any instant. The use of wales on sheet-pile groins and jetties can distribute these forces longitudinally, allowing the safe use of smaller structural sections.
LITERATURE CITED


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