WINTER REGIME
OF
RIVERS AND LAKES

Bernard Michel

April 1971
PREFACE

This Monograph was prepared by Dr. Bernard Michel, Professor of Civil Engineering, Laval University, Quebec, Canada. It was written at the U.S. Army Cold Regions Research and Engineering Laboratory (USA CRREL) while the author was on a year’s leave from the University.

The Monograph is published under DA Project 4A062112A894, Engineering in Cold Environments.
## CONTENTS

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surveys of river and lake ice</td>
<td>1</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Ice survey program</td>
<td>2</td>
</tr>
<tr>
<td>Forms of river and lake ice</td>
<td>3</td>
</tr>
<tr>
<td>Ice conditions and mapping</td>
<td>7</td>
</tr>
<tr>
<td>Water temperature measurements</td>
<td>9</td>
</tr>
<tr>
<td>Discharge of moving ice</td>
<td>12</td>
</tr>
<tr>
<td>River stages and discharges in winter</td>
<td>14</td>
</tr>
<tr>
<td>Characteristics of the ice cover</td>
<td>17</td>
</tr>
<tr>
<td>Heat balance on open water in winter</td>
<td>22</td>
</tr>
<tr>
<td>Introduction</td>
<td>22</td>
</tr>
<tr>
<td>Convective and evaporative exchanges</td>
<td>22</td>
</tr>
<tr>
<td>Radiation exchanges</td>
<td>28</td>
</tr>
<tr>
<td>Other heat exchanges</td>
<td>33</td>
</tr>
<tr>
<td>Thermal budget of a free surface flow</td>
<td>33</td>
</tr>
<tr>
<td>Thermal budget of a lake</td>
<td>37</td>
</tr>
<tr>
<td>Frazil</td>
<td>42</td>
</tr>
<tr>
<td>Introduction</td>
<td>42</td>
</tr>
<tr>
<td>Forecasting frazil appearance</td>
<td>43</td>
</tr>
<tr>
<td>Supercooling of water in rivers and lakes</td>
<td>43</td>
</tr>
<tr>
<td>Formation of frazil ice</td>
<td>48</td>
</tr>
<tr>
<td>Forms of frazil</td>
<td>50</td>
</tr>
<tr>
<td>Anchor ice</td>
<td>52</td>
</tr>
<tr>
<td>Deposits of frazil</td>
<td>53</td>
</tr>
<tr>
<td>Types of frazil problems</td>
<td>55</td>
</tr>
<tr>
<td>Remedial action against ice clogging</td>
<td>55</td>
</tr>
<tr>
<td>Ice cover formation</td>
<td>58</td>
</tr>
<tr>
<td>Introduction</td>
<td>58</td>
</tr>
<tr>
<td>Predicting river and lake freeze-up</td>
<td>58</td>
</tr>
<tr>
<td>Shore ice formation</td>
<td>62</td>
</tr>
<tr>
<td>Mechanical progression of an ice cover</td>
<td>65</td>
</tr>
<tr>
<td>Winter regime of rivers</td>
<td>71</td>
</tr>
<tr>
<td>Growth of ice covers</td>
<td>76</td>
</tr>
<tr>
<td>Backwater curves underneath ice covers</td>
<td>80</td>
</tr>
<tr>
<td>Breakup</td>
<td>83</td>
</tr>
<tr>
<td>Introduction</td>
<td>83</td>
</tr>
<tr>
<td>River breakup</td>
<td>83</td>
</tr>
<tr>
<td>Lake breakup</td>
<td>86</td>
</tr>
<tr>
<td>Forecasting</td>
<td>87</td>
</tr>
<tr>
<td>Factors affecting ice jams</td>
<td>88</td>
</tr>
<tr>
<td>Stability of an idealized ice jam</td>
<td>90</td>
</tr>
<tr>
<td>The dry ice jam</td>
<td>98</td>
</tr>
<tr>
<td>Destruction of ice jams</td>
<td>98</td>
</tr>
</tbody>
</table>
CONTENTS (Cont’d)

| Ice control | .......................................................... | Page |
| Control of winter regime | .................................................. | 100  |
| Local melting of ice covers | ........................................... | 108  |
| Control of breakup | .................................................. | 114  |
| Literature cited | .................................................. | 123  |
| Abstract | .................................................. | 131  |

ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Processes of ice formation in rivers and lakes</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Forms of ice</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Ice cover</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>Ice obstructions</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>Graphic symbols for freshwater ice with example</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>Graphic symbols - plan views, and sections and profiles</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>Example of the use of symbols to illustrate river ice formation and breakup</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>Deep-sea reversing thermometer</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>Differential thermometer with telescope</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>Thermograph instrumentation</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>Measurement of ice discharge</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>Set-up for water stage recorder in a cold climate</td>
<td>15</td>
</tr>
<tr>
<td>13</td>
<td>Stream flow gauging</td>
<td>16</td>
</tr>
<tr>
<td>14</td>
<td>Rods for measuring ice thickness</td>
<td>18</td>
</tr>
<tr>
<td>15</td>
<td>Measurement of ice thickness</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
<td>Monocycle radar equipment for measuring ice thickness</td>
<td>19</td>
</tr>
<tr>
<td>17</td>
<td>Forms of ice in an ice cover of a river</td>
<td>20</td>
</tr>
<tr>
<td>18</td>
<td>ACFEL coring auger</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
<td>Universal stage for study of thin sections of ice cores</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>Microtome for thin sectioning of ice</td>
<td>21</td>
</tr>
<tr>
<td>21</td>
<td>Modified shear vane apparatus for slush ice</td>
<td>21</td>
</tr>
<tr>
<td>22</td>
<td>Wind velocity - distribution over water</td>
<td>23</td>
</tr>
<tr>
<td>23</td>
<td>Relation of wind stress to wind speed</td>
<td>24</td>
</tr>
<tr>
<td>24</td>
<td>Evaporation and wind speed</td>
<td>27</td>
</tr>
<tr>
<td>25</td>
<td>Radiation components at an air/water interface</td>
<td>28</td>
</tr>
<tr>
<td>26</td>
<td>Altitude of the sun and diffuse radiation coefficient</td>
<td>31</td>
</tr>
<tr>
<td>27</td>
<td>Net values of longwave radiation emitted from a water body</td>
<td>32</td>
</tr>
<tr>
<td>28</td>
<td>Profile showing flow and thermal conditions</td>
<td>34</td>
</tr>
<tr>
<td>29</td>
<td>Heat losses and air temperature for St. Lawrence River at Montreal</td>
<td>36</td>
</tr>
<tr>
<td>30</td>
<td>Temperature regimes of lakes</td>
<td>38</td>
</tr>
<tr>
<td>31</td>
<td>Diagrammatic representation of the water current patterns and relative velocities in Tub Lake, 27-28 January 1961</td>
<td>40</td>
</tr>
<tr>
<td>32</td>
<td>Strong development of two thermoclines in Great Slave Lake, 7 August 1947</td>
<td>40</td>
</tr>
<tr>
<td>33</td>
<td>Freezing of groups of water droplets of various water qualities</td>
<td>45</td>
</tr>
<tr>
<td>34</td>
<td>Spontaneous freezing point of water droplets as a function of droplet size</td>
<td>46</td>
</tr>
<tr>
<td>Figure</td>
<td>Illustration Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>35.</td>
<td>Air-water temperature distribution before the appearance of frazil in turbulent flow</td>
<td>47</td>
</tr>
<tr>
<td>36.</td>
<td>Thermal boundary layer in water</td>
<td>47</td>
</tr>
<tr>
<td>37.</td>
<td>Growth of ice in supercooled water</td>
<td>48</td>
</tr>
<tr>
<td>38.</td>
<td>Growth of frazil discoids on ice crystals</td>
<td>49</td>
</tr>
<tr>
<td>39.</td>
<td>Water temperature and time during frazil formation</td>
<td>49</td>
</tr>
<tr>
<td>40.</td>
<td>Water depth, distance and velocity during frazil formation</td>
<td>50</td>
</tr>
<tr>
<td>41.</td>
<td>Growth process of frazil into a notched disk-crystal</td>
<td>51</td>
</tr>
<tr>
<td>42.</td>
<td>Development of frazil in a river</td>
<td>51</td>
</tr>
<tr>
<td>43.</td>
<td>Air photos showing growth of ice floes in a river</td>
<td>51</td>
</tr>
<tr>
<td>44.</td>
<td>Deposit of anchor ice growing on weeds</td>
<td>52</td>
</tr>
<tr>
<td>45.</td>
<td>Types of anchor-ice deposits</td>
<td>53</td>
</tr>
<tr>
<td>46.</td>
<td>Channel filled with frazil ice</td>
<td>54</td>
</tr>
<tr>
<td>47.</td>
<td>Typical section of a frazil-ice deposit in the Charny Reservoir on the Chaudiere River</td>
<td>54</td>
</tr>
<tr>
<td>48.</td>
<td>Average dates of river and lake freeze-up in Canada</td>
<td>61</td>
</tr>
<tr>
<td>49.</td>
<td>Probability chart for freeze-up of selected lakes and harbors in Canada</td>
<td>62</td>
</tr>
<tr>
<td>50.</td>
<td>Effect of shore on heat exchange</td>
<td>63</td>
</tr>
<tr>
<td>51.</td>
<td>Critical combination of water temperature and surface velocity for the formation of shore ice</td>
<td>64</td>
</tr>
<tr>
<td>52.</td>
<td>Shore ice in a river</td>
<td>65</td>
</tr>
<tr>
<td>53.</td>
<td>Daily production of slush-ice and air degree-days of frost on open water in various reaches of the Angara River</td>
<td>66</td>
</tr>
<tr>
<td>54.</td>
<td>The development of solid ice covers on two Dutch rivers</td>
<td>67</td>
</tr>
<tr>
<td>55.</td>
<td>Equilibrium of an ice floe</td>
<td>68</td>
</tr>
<tr>
<td>56.</td>
<td>Values of form coefficient for floes</td>
<td>69</td>
</tr>
<tr>
<td>57.</td>
<td>Stability of an ice cover</td>
<td>69</td>
</tr>
<tr>
<td>58.</td>
<td>Equilibrium of the frontal edge of an ice cover</td>
<td>69</td>
</tr>
<tr>
<td>59.</td>
<td>Critical Froude numbers at the head of a pack</td>
<td>70</td>
</tr>
<tr>
<td>60.</td>
<td>Ice cover progression. Angara River, 1950-51</td>
<td>73</td>
</tr>
<tr>
<td>61.</td>
<td>Ice cloggings in a Siberian River</td>
<td>74</td>
</tr>
<tr>
<td>62.</td>
<td>w-s regime in a Siberian River</td>
<td>75</td>
</tr>
<tr>
<td>63.</td>
<td>Polymys in a Siberian River in two successive winters</td>
<td>76</td>
</tr>
<tr>
<td>64.</td>
<td>First snow lying on a fractured ice sheet</td>
<td>78</td>
</tr>
<tr>
<td>65.</td>
<td>Curves of ice thickness and degree-days of frost</td>
<td>79</td>
</tr>
<tr>
<td>66.</td>
<td>Measured river ice sections in Japan</td>
<td>80</td>
</tr>
<tr>
<td>67.</td>
<td>Ice profile on Herriot Creek, Manitoba</td>
<td>80</td>
</tr>
<tr>
<td>68.</td>
<td>Water, shore ice and rapids</td>
<td>84</td>
</tr>
<tr>
<td>69.</td>
<td>Typical breakup on a river stretch</td>
<td>84</td>
</tr>
<tr>
<td>70.</td>
<td>Sheared ice walls</td>
<td>85</td>
</tr>
<tr>
<td>71.</td>
<td>Black ice on a lake</td>
<td>86</td>
</tr>
<tr>
<td>72.</td>
<td>Number of days for ice to disappear and ice thickness at time of last measurement</td>
<td>87</td>
</tr>
</tbody>
</table>
CONTENTS (Cont’d)

ILLUSTRATIONS (Cont’d)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>73. Average dates of breakup in Canada</td>
<td>88</td>
</tr>
<tr>
<td>74. Types of ice jams</td>
<td>89</td>
</tr>
<tr>
<td>75. General stress distribution in an unconsolidated ice cover</td>
<td>91</td>
</tr>
<tr>
<td>76. Hydrodynamic force on frontal edge</td>
<td>93</td>
</tr>
<tr>
<td>77. Forces in the ice cover</td>
<td>93</td>
</tr>
<tr>
<td>78. Forces on a boom caused by a model log jam</td>
<td>94</td>
</tr>
<tr>
<td>79. Thrust measured on a scale model with simulated ice</td>
<td>95</td>
</tr>
<tr>
<td>80. Results of tests with ice floes</td>
<td>95</td>
</tr>
<tr>
<td>81. Section through ice cover normal to an imaginary arch</td>
<td>97</td>
</tr>
<tr>
<td>82. Stability of static ice jams</td>
<td>98</td>
</tr>
<tr>
<td>83. Priming of a dry ice jam</td>
<td>99</td>
</tr>
<tr>
<td>84. Plan of a prismatic channel</td>
<td>101</td>
</tr>
<tr>
<td>85. Water-ice phase diagram for a given prismatic channel with constant inlet water temperature</td>
<td>102</td>
</tr>
<tr>
<td>86. Water intake in drifting ice</td>
<td>103</td>
</tr>
<tr>
<td>87. Johnson-Wahlman intake - interior view</td>
<td>104</td>
</tr>
<tr>
<td>88. Johnson-Wahlman intake in operation</td>
<td>104</td>
</tr>
<tr>
<td>89. Semicircular shaft spillway</td>
<td>106</td>
</tr>
<tr>
<td>90. Effect of a pond on the ice regime of a river</td>
<td>107</td>
</tr>
<tr>
<td>91. Typical ice control structures at a hydropower plant</td>
<td>107</td>
</tr>
<tr>
<td>92. Flow field distant from air bubbler</td>
<td>109</td>
</tr>
<tr>
<td>93. Flow field near air bubbler</td>
<td>109</td>
</tr>
<tr>
<td>94. Typical velocity profile in a vertical jet</td>
<td>109</td>
</tr>
<tr>
<td>95. Influence of depth of nozzle on ice melted</td>
<td>110</td>
</tr>
<tr>
<td>96. Open ferry passage at Babine Lake</td>
<td>111</td>
</tr>
<tr>
<td>97. Measured albedo vs coverage with dust</td>
<td>111</td>
</tr>
<tr>
<td>98. Overflow ice control dams</td>
<td>117</td>
</tr>
<tr>
<td>99. Ice control boom</td>
<td>117</td>
</tr>
<tr>
<td>100. Wooden ice boom</td>
<td>118</td>
</tr>
<tr>
<td>101. Remedial works - correction at the junction of a brook</td>
<td>119</td>
</tr>
<tr>
<td>102. Crater size and explosive charge</td>
<td>120</td>
</tr>
<tr>
<td>103. Depth of placement of explosive charge</td>
<td>120</td>
</tr>
<tr>
<td>104. Plan location of explosive charges at Alaskan sites</td>
<td>121</td>
</tr>
</tbody>
</table>

TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Solar radiation absorbed on a clear day and $C_a$ values</td>
<td>29</td>
</tr>
<tr>
<td>II. Critical velocity for the progression of an ice cover fed by drifting ice</td>
<td>71</td>
</tr>
<tr>
<td>III. Classification of factors affecting winter regime</td>
<td>72</td>
</tr>
<tr>
<td>IV. Values of $n_2$ for ice accumulations at freeze-up time</td>
<td>82</td>
</tr>
<tr>
<td>V. Eutectic temperatures of aqueous solutions of the principal chemical compounds</td>
<td>113</td>
</tr>
<tr>
<td>VI. Volume of melted ice produced by 1 gram of salt</td>
<td>115</td>
</tr>
</tbody>
</table>
EDITOR'S FOREWORD

*Cold Regions Science and Engineering* consists of a series of monographs written by specialists to summarize existing knowledge and provide selected references on the cold regions, defined here as those areas of the earth where operational difficulties due to freezing temperatures may occur.

Sections of the work are being published as they become ready, not necessarily in numerical order but fitting into the following plan, which may be amended as the work proceeds. The monograph series was planned and directed by F.J. Sanger as editor until 1970.

I. Environment

A. General – Characteristics of the cold regions
   1. Selected aspects of geology and physiography of the cold regions
   2. Permafrost (Perennially frozen ground)
   3. Climatology
      a. Climatology of the cold regions: Introduction, Northern Hemisphere I
      b. Climatology of the cold regions: Northern Hemisphere II
      c. Climatology of the cold regions: Southern Hemisphere
      d. Radioactive fallout in northern regions
   4. Vegetation
      a. Patterns of vegetation in cold regions
      b. Regional descriptions of vegetation in cold regions
      c. Utilization of vegetation in cold regions

B. Regional
   1. The Antarctic ice sheet
   2. The Greenland ice sheet

II. Physical Science

A. Geophysics
   1. Heat exchange at the ground surface
   2. Exploration geophysics in cold regions
      a. Seismic exploration in cold regions
      b. Electrical, magnetic and gravimetric exploration in cold regions

B. Physics and mechanics of snow as a material

C. Physics and mechanics of ice
   1. Snow and ice on the earth’s surface
   2. Ice as a material
      a. Physics of ice as a material
      b. Mechanics of ice as a material
   3. The mechanical properties of sea ice
   4. Mechanics of a floating ice sheet

D. Physics and mechanics of frozen ground
   1. The freezing process and mechanics of frozen ground
   2. The physics of water and ice in soil
EDITOR'S FOREWORD (Cont'd)

III. Engineering

A. Snow engineering
   1. Engineering properties of snow
   2. Construction
      a. Methods of building on permanent snowfields
      b. Investigation and exploitation of snowfield sites
      c. Foundations and subsurface structures in snow
      d. Utilities on permanent snowfields
      e. Snow roads and runways
   3. Technology
      a. Explosions and snow
      b. Snow removal and ice control
      c. Blowing snow
      d. Avalanches
   4. Oversnow transport

B. Ice engineering
   1. River-ice engineering
      a. Winter regime of rivers and lakes
      b. Ice pressure on structures
   2. Drilling and excavation in ice
   3. Roads and runways on ice

C. Frozen ground engineering
   1. Site exploration and excavation in frozen ground
   2. Buildings on frozen ground
   3. Roads, railroads and airfields in cold regions
   4. Foundations of structures in cold regions
   5. Sanitary engineering
      a. Water supply in cold regions
      b. Sewerage, and sewage disposal in cold regions
      c. Management of solid wastes in cold regions
   6. Artificial ground-freezing for construction

D. General
   1. Cold-weather construction
   2. Materials at low temperatures
   3. Icings

IV. Remote Sensing

A. Systems of remote sensing
B. Techniques of image analysis in remote sensing
C. Application of remote sensing to cold regions

T.C. JOHNSON
WINTER REGIME OF RIVERS AND LAKES

by

Bernard Michel

SURVEYS OF RIVER AND LAKE ICE

Introduction

The winter regime of rivers and lakes in the cold regions is little known. In northern countries only a few rivers like the St. Lawrence, the Lena, and the Ob have been consistently surveyed during the winter period. One has only to open any report of hydrological data on stages and discharges of Canadian rivers to see a complete blank, for most of them, in the winter months. Books on limnology devote at most a few pages to the winter regime of lakes.

But this winter regime of rivers and lakes is often a determining factor in the control and use of our water resources. The most important force acting on hydraulic structures like bridge piers, dams, and wharfs is usually the ice thrust. The maximum levels attained by rivers in the spring occur normally at breakup time and not at maximum river discharge for ice-free flow, so that flood control depends much on ice movement. The most severe scouring action on banks or river works is produced by moving ice floes. Any water intake built in a river for irrigation, hydroelectric production, industrial or public water supply must be designed specifically with ice problems in mind and occasionally this may become a critical economic factor for a whole project. Because of the shutdown of oxygen supply, the presence of an ice cover in a river or a lake might well be a leading factor in controlling the biomass, the potential for aut purification and, in the end, the biochemical quality of the waters. Finally, the closing to navigation of ice-covered rivers seriously affects the economy of large regions of some countries, especially Canada and the U.S.S.R.

The study of the ice regime of rivers and lakes is not easy. Compared with summer conditions two new variables intervene: water temperature and ice characteristics. Furthermore, even the usual measurements which are easily made during the summer months become very hard to get in winter. Most water-level gauges, manual or automatic, become frozen in ice. The summer stage-discharge relationship does not hold any more. Cold weather and ice impede measurement so much that a winter field team can make very little progress compared to what a summer team can do. It is understandable that very few data are available on the winter regime of rivers and lakes.

Up to now, ice surveys have been mainly conducted on a limited scale for particular engineering problems, such as local flood protection or construction of a hydroelectric dam at a given site. The technique for an overall study of ice conditions in a river or lake is beginning to take form. The introduction of ice charts obtained from aerial photographs has been a major step in this direction. Because of the very high cost of a general ice survey, there is little doubt that it cannot be made for every river that has a hydrological record. These general surveys should be limited to bigger rivers and lakes, which have a good potential for multipurpose water resources development. In the
case of limited ice problems or particular engineering projects, only localized ice surveys may be required. A priority should be given, by government services, to developing instruments and techniques that will enable them to extend the discharge record of rivers they are gauging during the summer months, to the whole year.

Ice survey program

A survey of ice conditions in a river or lake should be prepared according to the future needs for water resources development. It should always take into account not only the immediate projects, but also the long range possible multipurpose developments.

In the broadest possible manner, as for a large water body where many developments are projected, an ice survey should include:

1. Good meteorological data (air temperature, wind velocity and direction, precipitation, insolation and air humidity) for the whole winter period at representative sites along the water body.
2. Good hydrological data, for the period, on water stages, velocities and discharges at critical points along the water body.
3. Complete descriptive survey of ice conditions for the whole water body, from the beginning of frazil production to the disappearance of all ice after breakup.
4. Water temperature measurements at critical areas from the beginning of cold weather to the formation of a complete ice cover.
5. Discharge of ice floes at critical areas and times during ice cover formation.
6. Thickness, overall structure, texture and crystal-axis orientation of the ice cover, at critical locations and times.
7. Oxygen content and chemical analysis of the water under the ice cover, at critical locations and times.

For a complete river development, as for flood protection, navigation or hydroelectric power there is little doubt that most of these measurements will be required as they will have a bearing on the overall development and the design of the particular engineering works.

In the case of a lake or river development for recreation or pollution control the most important general observation might well be the chemical characteristics and oxygen content of the water in conjunction with a general survey of the ice conditions.

For localized engineering works, the ice survey need not be so elaborate. In the case of a water intake a survey of local ice conditions with water temperature measurements before the formation of the solid ice cover (to detect frazil) might be adequate. Of course, a good winter hydrological record is a prerequisite to the complete design of this type of work. Local protection from icy floods may be designed with a local survey of ice conditions at breakup and good meteorological and hydrological data. For the design of bridge piers, all that is required is a knowledge of the movement of ice floes, their structure and characteristics, with the corresponding water stages. Finally the design of a dam might depend to a great extent on the ice pressure and, indirectly, on the meteorological conditions and structure of the ice cover.
Forms of river and lake ice

We present a classification for river and lake ice that is based on its gross macroscopic characteristics and its history, but which is not related to the texture and internal structure of the ice itself.

Any genetic classification of ice must be based on a recognition of the climatic and physical factors which interact to form the ice cover. Since factors which govern ice formation in oceans are similar to those which control river and lake ice formation, we have adopted here a classification scheme very similar to one suggested for sea ice (Monograph II-C1, Snow and Ice on the Earth’s Surface). It is shown in Figure 1.

Figure 1. Processes of ice formation in rivers and lakes.
In most cases the terminology used to describe freshwater ice is similar to sea ice nomenclature. Many of the terms have acquired different meanings in the literature on river and lake ice (as "ice cover" for instance). There are many widely circulated ice glossaries but there has never been any general agreement on the meaning of the terms. We will give a few definitions only to help in understanding the main terms used in this Monograph.

**Frazil:** A group of individual ice crystals having the form of small discoids or spicules which are formed in supercooled turbulent water (Fig. 2a).

**Frazil slush:** An agglomerate of continuous loosely packed frazil floating on the water surface (Fig. 2b).

**Snow slush:** The equivalent of frazil slush for loosely packed snow particles.

**Plate ice:** A thin, transparent and continuous ice layer of limited dimensions in plan (Fig. 2c).

**Ice sheet:** A solid, continuous, smooth, unbroken ice surface, floating on the water.

**Shore ice:** An ice sheet in the form of a longitudinal band of floating ice attached to the shore (Fig. 2d).

**Slush ball:** The result of extreme accretion of slush particles. This is produced either by waves along the shores of lakes or in long stretches of very turbulent and wavy flows in rivers (Fig. 2e).

**Pancake ice or icepan:** An accumulation of slush where a solid ice sheet has formed on top with an overall circular shape. It develops raised rims as a result of repeated collisions with its neighbors and is at most a few feet in diameter (Fig. 2f).

**Ice floe:** A free-floating piece of ice. It may be quite regular and formed by the breakup of an ice cover or it may be a conglomerate of individual slush balls or pancakes frozen together. It may be anywhere from a few feet to many hundreds of feet across. However, in northern lakes and rivers the ice is not thick and as a consequence floes more than 50 ft across are rare.

**Ice cover:** A continuous expanse of ice of any possible form, from shore to shore, in a river or a lake (Fig. 3).

**Hummocked ice:** An ice cover made of floes, fragments of floes and slush, piled up haphazardly by horizontal pressure. The cover may have been modified by melting and snow accumulation (Fig. 3d).

**Underhanging dam:** An accumulation of slush ice, pancakes and plates under a solid ice sheet in a zone of low-velocity flow. It is usually made during the period of ice formation in a river (Fig. 4a).

**Ice jam:** A generic term describing an accumulation of ice floes with some slush in a section of a river, forming a local obstruction raising the water level at its upstream end (Fig. 4b).

**Pool:** Any area of open water of limited extent, surrounded by ice.

**Puddle:** A water-filled depression on the surface of a floe or ice cover.

**Black ice:** New ice of continuous uniform growth appearing dark because of its transparency.

**Snow ice:** Ice formed by the flooding of a snow layer. It has a whitish appearance because of occluded air bubbles.

**Candle ice:** Ice having a columnar crystal structure, which is partly disintegrated by melting. The melting is concentrated at the boundaries of the prisms leaving a weak candle-like structure.
a. Frazil

b. Frazil slush

c. Plate ice

d. Shore ice

e. Slush balls

f. Pancake ice

Figure 2. Forms of ice.
WINTER REGIME OF RIVERS AND LAKES

a. Pure ice cover

b. With snow (partly flooded) on top

c. With frazil accumulation below

d. Unconsolidated ice floes, snow and frazil
   (hummocked ice cover)

Figure 3. Ice cover.

a. Underhanging dam

b. Ice jam

Figure 4. Ice obstructions.
Ice conditions and mapping

Qualitative observations of the formation, evolution and breakup of an ice cover can be made by visual observations along the bank of a river or lake or by aerial observations and photographs. This phase of an ice survey is generally the most important one as it gives an overall view of the winter regime of the water body. In the case of a river, information is collected on all singularities giving rise to special ice phenomena such as jams, frazil accumulations, and hot pools. Because these phenomena have a strong tendency to repeat themselves at the same location, year after year, a few general surveys will readily show the most advantageous overall plan for any development under consideration. In the case of a limited river stretch, a longer period of observation is required in order to form a valid opinion on the probability of occurrence of the worst possible conditions in that particular area.

On large watercourses or lakes, the ice survey has to be made from the air and pictures taken to cover the whole water body. The technique of interpreting these photographs has been well developed and much very useful information can be obtained on the ice conditions. The only drawback of this method is that observations cannot be made in cloudy conditions or with mist over the water surface. Unfortunately, this happens most frequently during freeze-up and breakup. Because these two events are decisive for ice surveys, a program of aerial photography should provide for sorties whenever required during these periods. When the ice cover has stabilized over a river or a lake there is no more need for frequent ice reconnaissance surveys.

The results of ice surveys by air can best be represented by ice charts like that shown in Figure 5 for the breakup of Great Slave Lake in 1962.** The descriptive terms used in this representation are taken from seawater nomenclature and the floes are somewhat big for the usual river and lake ice formations. The terminology used is:

- Ice-free: no ice present
- Open water: less than \(\frac{1}{10}\) ice-covered
- Scattered ice: \(\frac{1}{10}\) to \(\frac{5}{10}\) ice-covered
- Broken ice: \(\frac{5}{10}\) to \(\frac{9}{10}\) ice-covered
- Close ice: \(\frac{9}{10}\) to \(\frac{10}{10}\) ice-covered
- Consolidated ice: \(\frac{10}{10}\) ice-covered
- Brash: floes less than 6 ft across
- Block: floes 6 to 30 ft across
- Small floe: from 30 to 600 ft across
- Medium floe: from 600 to 3000 ft across
- Giant floe: from 3000 ft to 5 miles across
- Winter ice: ice of one season's growth which is over 8 in. in thickness
- Young ice: ice that has recently formed in calm water
- Polynya: large area of open water, surrounded by ice, which usually persists in the same position year after year.

The interpretation of Figure 5 is then straightforward. Let us look at some of the symbols:

** Example A: **

\[9+ Pd\] This surface is more than \(\frac{9}{10}\) covered with \(\frac{9}{10}\) slush, brash and blocks,
\[262 3R\] \(\frac{3}{10}\) small and medium floes and \(\frac{3}{10}\) giant floes. There is \(\frac{3}{10}\) coverage of water puddles on the surface and the ice is rotten.
Figure 5. Graphic symbols for freshwater ice with example.
Example B:

\[ \frac{4}{220} \text{ A} \]

This is \( \frac{1}{10} \) covered by winter ice, \( \frac{3}{10} \) slush, brash and blocks and \( \frac{2}{10} \) small and medium floes.

With these symbols we thus see that on that date: "the main ice pack had been pushed against the south shore. With the exception of small polyny at Hay River and Buffalo River, the entire coastline was ice-blocked from Pointe Desmarais to Sulphur Point. A vast area of open water extended from Mission Island and reached north almost to Yellowknife Bay. A shore lead extending from Long Island to Moraine Point, had pushed the ice ten to fifteen miles from the shore. Windy Bay was ice-free. The area located west of a line from Shore Point to Pointe Desmarais was completely open. In the North Arm there was no ice present north of Waite Island. The portion of the ice pack situated southwest of a line from Moraine Point to Pine Point contained consolidated winter ice with thawing holes. Most of the floes were of medium size. There was an area of consolidated winter ice at the entrance of Yellowknife Bay. The remainder of the main body of the lake and the North Arm contained close winter ice with few puddles and thawing holes. In the Eastern Arm of the lake, consolidated winter ice predominated except for few polyny and areas of close ice in the vicinity of the various islands."

The geophysical representation used by the Meteorological Services in Canada is convenient for large water bodies of fresh or sea water. For smaller rivers and lakes the percentage of ice cover at a given time has very little significance and the forms and characteristics of the ice cover itself are the most important features. A geophysical classification proposed by Vasiliskov\textsuperscript{139} appears to be the most logical and representative for these conditions. The symbols are shown in Figure 6. The application of this geophysical representation to the ice formation and breakup in a local river bend is shown in Figure 7. This representation is striking and one does not even need to know the meaning of the symbols to understand it. We believe this type of representation to be particularly useful for observations made from the shore of a small river.

**Water temperature measurements**

A winter survey usually starts with the measurement of water temperatures at the beginning of freezing weather in order to predict the appearance of frazil and give information on ice conditions. In most cases the temperature measurements are used to predict and measure the amount of supercooling of the water at water intakes, which is of the order of a few hundredths of a degree Fahrenheit. Very accurate measurements are required.

Very good measurements can be obtained with the "Deep Sea Reversing Thermometer" (Fig. 8) which is accurate to 0.01°C. This thermometer is rugged, reliable, and takes a reading at any water depth in ice conditions. Its only disadvantages are its bulk and a need for manual operation. If more precision is required, a differential type thermometer can be set in a special telescoping rig and fixed at a certain place in flowing water (Fig. 9). This will read to 0.002°C, but the zero setting is hard to make, and the whole instrument is delicate to handle and easily put out of order by the impact of a small ice floe.

Thermographs with temperature sensors have the obvious advantage of giving a continuous recording so that all intermediate maxima and minima and changes in the rates of decreasing or increasing water temperatures are registered. The usual accuracy is to 0.2°C in the range of cold water measurements. The bridge, amplifier and recorder are delicate elements not designed to be used in adverse conditions, so that they must be housed in a closed cabin kept at an even and adequate temperature. The calibration of the instrument must be verified periodically with a standard thermometer. Figure 10 shows some of this equipment with thermistor and thermocouple probes. The quartz sensor permits a reading to 10\textsuperscript{-4} °C. It is the most accurate in the field but the lag in response is long and much care must be taken in making the measurements.
### WINTER REGIME OF RIVERS AND LAKES

<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Water surface</td>
</tr>
<tr>
<td>2</td>
<td>Uniform ice cover</td>
</tr>
<tr>
<td>3</td>
<td>Border ice at start of freeze-up and at end of break-up</td>
</tr>
<tr>
<td>4</td>
<td>Extension of shore ice</td>
</tr>
<tr>
<td>5</td>
<td>Snow slush</td>
</tr>
<tr>
<td>6</td>
<td>Ice slush</td>
</tr>
<tr>
<td>7</td>
<td>Frazil (number = density or thickness)</td>
</tr>
<tr>
<td>8</td>
<td>Ice cover in motion (number = thickness)</td>
</tr>
<tr>
<td>9</td>
<td>Zone of anchor ice</td>
</tr>
<tr>
<td>10</td>
<td>Ice on isolated boulder</td>
</tr>
<tr>
<td>11</td>
<td>Ice floes</td>
</tr>
<tr>
<td>12</td>
<td>Zone where the freezing extends to the bottom</td>
</tr>
<tr>
<td>13</td>
<td>Ice jam (number = height)</td>
</tr>
<tr>
<td>14</td>
<td>Slush or frazil under ice (number = thickness)</td>
</tr>
<tr>
<td>15</td>
<td>Initial clearing between floes</td>
</tr>
<tr>
<td>16</td>
<td>Secondary clearing between floes</td>
</tr>
<tr>
<td>17</td>
<td>Fractures on ice surface</td>
</tr>
<tr>
<td>18</td>
<td>Superimposed ice layer</td>
</tr>
<tr>
<td>19</td>
<td>Water on ice cover</td>
</tr>
<tr>
<td>20</td>
<td>Water circulating on ice cover</td>
</tr>
<tr>
<td>21</td>
<td>Shores</td>
</tr>
</tbody>
</table>

**Figure 6.** Graphic symbols – plan views, and sections and profiles.
Slush and pans moving on water surface.

An ice bridge of pans and floes is formed in the bend. Shore ice grows.

The ice bridge consolidates.

A hummocked ice cover is formed from the bridge. Some pools remain.

Water appears on the ice surface and floods the banks.

The cover deteriorates and is broken up. An opening is made in high velocity flow.

Ice floes jam and accumulate in the bend.

End of breakup. Some floes are left on the shore.

Figure 7. Example of the use of symbols to illustrate river ice formation and breakup.
If a permanent probe is to be used to record the temperature of a river in winter, it must be set in such a way as not to be affected by the ice movement. The connecting cable should be set in the riverbed, and a special head box, resistant to scour action and impact, should house the temperature sensor. Calibration should be done by comparing the temperature of the water moving around the box to the temperature of the water flowing through it.

**Discharge of moving ice**

On many large rivers where long stretches stay free of ice in winter the amount of floating ice passing a given section close to an ice cover may be one of the important factors affecting the ice conditions.

The discharge of floating ice can be measured from the shore with the help of a transit by a simple method developed on the Volga River. The total discharge of floating ice is given by a formula of the type:
Ia. Sensors

SURVEYS OF RIVER AND LAKE ICE

b. Read-out instrumentation for thermistors

c. Digital read-out for quartz thermometer

Figure 10. Thermograph instrumentation.

\[ I = \sum_{i=0}^{m} n_i v_i h \Delta B \]  \hspace{2cm} (1)

where \( n_i \) is the percentage of ice covering a strip of width \( \Delta B \) on the river surface, \( v_i \) is the velocity of the moving ice in that strip and \( h \) is the total effective thickness of the drifting ice.

The effective thickness of the ice floes can be determined by taking samples of the moving ice along the shore. It is then:

\[ h = h_0 + (1 - e) h_f \]  \hspace{2cm} (2)

where \( h_0 \) is the thickness of the solid part of the floes, \( h_f \) is the thickness of the slush or frazil deposit underneath and \( e \) is the percent of water in the total volume of slush. It can be obtained by weighing part of a given volume of the drained, accumulated slush.
After the transit has been set up on the berm, its elevation above the water level can be obtained with a rod and the moving ice in any strip can be observed (Fig. 11). The position of a point along the axis O - O can be computed with the measured value of the vertical angle α. The percentage of ice covered in a strip can be obtained with the help of a stopwatch by:

\[
n_i = \frac{t}{t_0}
\]

where \( t \) is the time when ice only is passing across the vertical plane of the transit and \( t_0 \) is the total time of the measurement.

The velocity can be obtained directly by moving the instrument with one floe for a few seconds, as shown in Figure 11, and recording the time and angle of movement:

\[
v_i = \frac{Z (\tan \beta_1 + \tan \beta_2)}{t \tan \alpha}
\]

where \( t \) is the interval of time of the observation.

Another more accurate and very quick way of obtaining this discharge is to take a few photographs from the air of the moving ice at preset regular intervals (10 seconds for example). There is then no difficulty in locating particular ice floes and their movements. The main difficulty is the impossibility of doing this at a close enough range at a low speed with an airplane. A helicopter may be necessary.

**River stages and discharges in winter**

One of the most important tasks in winter surveys for rivers is water level measurement and streamflow gauging. It is during the long winter period of low water flows in many rivers that the available water supply must often be determined either for municipal purposes or hydroelectric energy production. Many rivers attain their highest flood levels during the spring breakup.

It is not possible in winter conditions to use staff gauges, either vertical or inclined, because they are most likely to have their datum changed or they may be wrenched loose from their supports by the pressure or movement of the ice cover. At locations where it is feasible (bridge decks, retaining walls, etc.) a chain gauge can, as a rule, be maintained as satisfactorily during the winter months as during the summer. Its datum is no more likely to move and its scale board is not subject to injury by ice. An added problem is to keep a gauge hole open in the ice sheet. Thin ice can usually be broken by the dropping of the gauge weight on it.

Where there is no man-made structure, the water level can be measured either by straightforward leveling from a datum on the shore or by installing a permanent water-stage recorder of the pressure gauge or air-bubbler type. Great care should be taken in installing such a permanent station. The pressure sensor must be well protected at the bottom of the river in a box-type structure and the connecting cable must generally be set in the ground. The recorder itself must be placed in an insulated cabin. It must be designed specifically for cold weather operation (Fig. 12).
One particular phenomenon associated with the breakup of a river is a continuous mound of ice floes on the shores, sheared through and left over just after the ice movement. A survey made afterwards of the highest levels of these floes will give an accurate figure of flood conditions at breakup of any river stretch.

When there is no permanent engineering structure in a river where discharge can be recorded, winter discharge measurements of ice-covered rivers may be made directly at intervals of two to six weeks. The sections to be measured should be carefully chosen so that depth and velocity are fairly uniform, the bottom is smooth and there are no cross currents. The first hole should be made in the middle of the section with an auger, to check for frazil ice. If frazil or floating ice affects more than about 10% of the total cross-sectional area, the measurements should be made at another section where such conditions do not exist.

The first step in measuring under an ice cover is cutting holes through the ice. The holes should be spaced at 5 to 10 ft, the interval depending on the width of the stream. Because the holes have to be large enough to permit the meter to be easily raised and lowered they usually have to be made by hand with an ice chisel or chain saw. The thickness of the ice and the free flow distance between the ice and the riverbed should be recorded at each hole. Usually the water velocity is measured with the flow meter at 0.2, 0.8 and at the midpoint of the free flow depth. It
is sometimes possible to make observations only at mid-depth by using a coefficient to reduce the velocity to the mean. This coefficient may be obtained by observing the velocities at 0.2, 0.8 and mid-depth at a number of sections and comparing the average of the velocities with that observed at mid-depth.11

There are many methods11 available to interpolate the discharge hydrograph for the whole winter period from the actual measurements of discharge under the ice cover. All these methods use, directly or indirectly, the values of the daily gauge height with the open-water stage-discharge curve for the station under consideration and the daily temperature reading. From these data, a guide estimate can be worked out for the hydrograph of the whole winter period. These interpolations are very good for large rivers in frigid zones but their accuracy usually diminishes with a reduction in the size of the river.

Regular discharge measurements in winter may become very costly if extended to a great number of rivers. It is possible in many cases to install a year-round streamflow gauging station that is as accurate in winter as in summer and that does not need regular discharge measurements but only gauge height readings.13 Such a station (Fig. 13) consists of two recording gauges, one set at Sta. 1 at the head of a fall or rapid section in low velocity flow and the other far away upstream (Sta. 2) in a zone of fluvial flow where there is no frazil ice deposit. This station is very similar to the two-gauge stations used in movable bed rivers or at the head of the backwater of a reservoir. For open-water conditions, the stage-discharge rating curve is established at Sta. 1 and 2, and backwater curves are computed to establish the roughness coefficient of the riverbed. For various water levels at Sta. 1, backwater curves are computed for winter conditions with extreme Manning roughness coefficients of 0.011 and 0.020 for the ice cover. A graph is then prepared as shown in Figure 13 which gives the discharge of the river for various values of corrected gauge readings $Y_1$ and $Y_2$.

![Diagram showing stations and stage-discharge curves](image-url)

**Figure 13.** Stream flow gauging.
In winter the gauge readings have to be corrected by subtracting the thickness of the buoyant part of the ice cover. This can be obtained at any time by the formula:

\[ h_0 = a \sqrt{S}, \quad S = \sum (32 - T_a) \, dt \]  

where:

- \( S \) = cumulative sum of degree days of frost at the site since the beginning of ice cover formation,
- \( T_a \) = air temperature,
- \( a \) = experimental coefficient determined previously at the site.

If there is no anchor or shore ice at control section \( 0 \) in Figure 13, there should be a unique stage-discharge relationship at Sta. 1, depending on the roughness. This might be called the normal relationship. Obviously as the winter progresses the actual measurements should always be somewhat on the left side of this normal curve which should verify the proper operation of the gauging station.

### Characteristics of the ice cover

Once an ice cover has started to form, soundings to measure its thickness should start as soon as the cover is thick enough to carry a man safely, which is theoretically on the order of 2 in. However, if the measurements are important at a certain point, light booms designed to hold one or more men could be put across the river, and soundings taken at the very beginning of ice cover formation.

Ice thickness usually varies appreciably in a given section of river or lake, mainly because of ice flow conditions underneath the cover in the first case and snow deposits in the second. In a river, soundings should be made at regular intervals with up to 10 to 20 soundings in a cross section. In a lake soundings may be made in a grid covering total areas of \( 300 \times 300 \) ft to \( 1000 \times 1000 \) ft depending on the size of the lake. The sounding holes should be drilled every 50 ft inside this lattice.

The oldest and at times the only way to make soundings is to use an ice chisel or a chain saw to cut a hole through the ice. Once the solid part of the ice cover has been bored through, a solid graduated and extensible rod, with a fixed or retractable plate attached at the end (Fig. 14), is pushed through the underlying slush accumulation. The thickness of frazil accumulation is measured by feeling the different resistance to rotation of the rod when in water or in frazil ice. As the rod is pulled upward, the plate catches in the bottom of the solid ice cover. The end is extended to the river bed to measure the river depth.

When information is required only on the thickness of the solid part of the ice cover, a very fast method is to use the 1-in.-diam ice drill shown in Figure 15 (designed by ACFEL* and now designated the CRREL ice drill) with tape and weight device to measure the thickness. This drill will cut through ice at the rate of one to two feet per minute.\(^{56}\)

It has been found recently that the thickness of freshwater ice may be measured accurately with monocycle radar equipment (Fig. 16), sending pulses in a band of frequencies of at least 400 MHz.\(^{112}\) The results were usable when the antennae were in a static position a few feet above the ice. The operation is much more delicate from an airplane, because of its vibration, and variation of the ice thickness on larger lake areas. The snow cover or frazil ice accumulations do not seem to interfere with the measurement of the solid part of the cover as long as there is a well delineated interface at each level.

---

*Arctic Construction and Frost Effects Laboratory, now merged in USACRREL, the U.S. Army Cold Regions Research and Engineering Laboratory.
Figure 14. Rods for measuring ice thickness.

a. CRREL 1-inch drill.  
b. Tape and weight device.

Figure 15. Measurement of ice thickness.
In the most general manner the ice cover of a river is made of a variety of ice forms (Fig. 17):

1.) Fresh or compacted snow at the top.

2.) Dense snow ice of granular texture formed when the water rose, flooded the snow and stayed there long enough for a solid sheet to freeze in the snow.

3.) Light snow ice formed when the water level went down and drained the snow underneath the upper solid layer, which refroze afterward.

4.) Solid black ice of the columnar type made from the initial freezing of the water in low velocity conditions.

5.) Solid black ice of congealed frazil formed by the freezing of frazil slush.

6.) Slightly compacted slush, plates and pans accumulated under the solid ice part.

It is useful to obtain information on the structure of an ice cover and the best known coring auger in North America is the CRREL ice coring auger (originally developed by ACFEL) shown in Figure 18. It gives ice cores 3 in. in diameter and about 18 in. long although it might be possible to get a 3-ft length for the first core. It can be operated by hand, or motorized and adapted to a portable motor drill. Core recovery is close to 100% as long as the ice is sufficiently cemented together but it is not designed to take cores of snow or of the slush ice underneath the ice sheet. Another way to obtain ice specimens is to cut out blocks of ice from the ice cover with a portable chain saw of special type.

The texture of the ice is usually examined in thin sections between crossed polaroids. A "universal stage" for doing this is shown in Figure 19. This technique gives the size, shape and orientation of the main optical axes of the crystals. Because the crystals of river and lake ice are big (from a tenth of an inch to a few inches), no magnification is necessary. Thin sections of
Figure 17. Forms of ice in an ice cover of a river.

ice are prepared with a microtome (Fig. 20) and the surface is polished with sandpaper. If the specimens have to be kept some time in a cold room before being analyzed, they are mounted on glass or stored in kerosine or silicone oil to protect their surfaces from evaporation. The birefringence of ice is so low that one has considerable latitude in the thickness of section that can be used. The ice section can be rotated and tilted in the universal stage so that the approximate (±2°) orientation of the c-axis of any crystal can be obtained by finding the position for which it becomes black. A reflex camera is mounted on the polarizing equipment to photograph each specimen.

The study of frazil or snow slush accumulated underneath the solid ice cover can be made with special samplers. The density of the porous ice mass can be determined. In-situ shear strength of the slush mass may also be obtained with a modified shear vane apparatus (Fig. 21).

Figure 18. CRREL ice coring auger (3 inch).

Figure 19. Universal stage for study of thin sections of ice cores.
Figure 20. Microtome for thin sectioning of ice.

Figure 21. Modified shear vane apparatus for slush ice.
HEAT BALANCE ON OPEN WATER IN WINTER

Introduction

The problem of cooling or heating of a free water surface in a river or lake has important applications in northern countries. When natural conditions are unchanged, it is usually possible to predict, from previous surveys, the eventual behavior of a river during the winter months. But when these conditions are unknown, or when engineering works are being considered, it may become necessary to evaluate this probable behavior.

Let us consider some of the questions that may have to be answered:

1. Will an artificial reservoir made on a watercourse freeze in winter? If it does, a completely different approach has to be taken for its design and winter operation, from that if ice does not form.
2. Will there be many open stretches left in a river during the winter months if natural conditions are modified as to discharge and depth? This is indeed a fundamental question insofar as frazil problems are concerned.
3. When and where will ice begin to form on a water body?
4. What is the total quantity of frazil, or of plate ice that can be formed in an open water surface?
5. Can a river be modified to be kept ice-free for winter navigation or other purposes?
6. What climatic changes are brought about by a large, free air/water interface as compared to an ice-covered one?

These are some of the important questions that may be answered by a study of the heat budget on free water surfaces in cold weather. Besides, this question of heat exchange is also of primary importance during the summer months because the temperature of water controls its potential for self-purification, the quality of its biomass and its fish productivity.

The basic modes of heat transfer on an open water surface are convection, evaporation, and radiation, plus heat exchange from precipitation, all of which have been extensively studied. Approximate engineering formulas for application on practical problems are derived in the following paragraphs.

Convective and evaporative exchanges

When a cold wind is blowing over a water surface in nature, waves are formed and there is a heat exchange process cooling the warmer water body by convection and evaporation. If the wind speed is moderate or strong the air is in a state of rough turbulent flow in which the laminar sublayer is small compared to the height of roughness. This is the classical flow problem studied by Prandt199 from whom we can obtain the velocity distribution in the boundary layer and, with the Reynolds analogy, the heat and mass transfer formulas.
Wind velocity distribution over water. Let us consider a fully developed, two-dimensional, turbulent flow of a fluid as shown in Figure 22, where the average velocity is increasing between two flow lines from $\bar{u}$ to $\bar{u} + d\bar{u}$ and where the turbulent velocity fluctuations are $u'$ and $v'$ in the x and z directions. The transfer of momentum between lines 1 and 2 then gives the Reynolds shear stress:

$$\tau = -\rho v' u'$$

By analogy with the kinetic theory of gases, Prandtl postulated that as the masses of fluid migrated laterally, they carried with them the mean velocity (and hence the momentum concentration) of their point of origin. Thus a typical velocity fluctuation $u'$ is of the order of magnitude of $u' \sim (du/dz)$:

$$du = u_0 \frac{dz}{\ell}$$

where $\ell$, the so-called mixing length, is the distance over which a migration takes place. Similarly, a negative $v'$ is of the same order of magnitude as $u'$ and the turbulent shear is then given by:

$$\tau = \rho \ell^2 \left| \frac{d\bar{u}}{dz} \right| \frac{d\bar{u}}{dz}.$$

If, near the boundary, $r$ is taken as equal to $r_0$ and the mixing length is assumed to be proportional to the distance $z$ from a rough boundary, we then get:

$$\bar{u} = \frac{1}{k} \sqrt{\frac{r_0}{\rho}} \ln \left( \frac{z + z_0}{z_0} \right).$$

This formula gives a logarithmic velocity distribution, which has usually been found adequate enough to represent the wind velocity profile in nature. In this equation:

- $z_0$ = equivalent roughness of the surface
- $r_0$ = shear stress at the boundary
- $\rho$ = density of the fluid
- $k$ = Von Karman's universal constant, usually 0.4.

In the case of rough, turbulent air flow over the sea and large lakes, many measurements have been made to obtain the tangential stress over the water surface; some of them are shown in Figure 23. Some of the most widely accepted data, curve a in the figure, show that the equivalent roughness of the wave surface can be taken as $z = 0.6$ cm. Ekman then proposed that the best relation between wind stress (dynes/cm$^2$) on the surface and wind speed (cm/sec) is given by:
\[ \tau_0 = 3.2 \times 10^{-6} v_{50}^2 \]  

(9)

where \( V_{50} \) is the wind speed 50 ft above the water surface.

It is interesting to compare measured values of \( z_0 \) for wind blowing over snow and ice. It appears that \( z_0 \) may vary from 0.04 to 0.25 cm for a smooth ice or snow surface, 0.5 to 0.7 cm for rough, undulated snow and hummocked ice and over 1.1 cm for very rough ridged ice. Suzuki gives 0.5 to 1.8 cm for an ice floe.

Reynolds' analogy of heat and mass transfer. The analogy between the transfer of momentum and the heat and mass transfer in a flowing fluid has been proved to be a very fruitful concept for the analysis of problems dealing with air flow.

Let us consider in Figure 22 the quantity of heat being transported laterally by the velocity fluctuation \( v' \). It must be:

\[ \psi_c = \frac{d\phi_c}{dt} = -\rho v' c_p \frac{dT}{dz} \ell \]  

(10)

where:

- \( \psi_c \) = the convective heat flow per unit time and unit area
- \( \phi_c \) = the convective heat flow per unit area
HEAT BALANCE ON OPEN WATER IN WINTER

\[ c_p = \text{specific heat of the fluid} \]
\[ T = \text{temperature of the fluid.} \]

In the same manner, if we consider a convective mass transfer (water vapor for instance in air) from a level where the concentration is \( W \) to another level, at the mixing distance \( l \), where it is \( W + dW \), we then get:

\[ m = \frac{dM}{dt} = -v' l \frac{dW}{dz} \quad (11) \]

where:

\[ m = \text{mass transfer rate per unit area and time} \]
\[ M = \text{mass transfer rate per unit area} \]
\[ W = \text{mass concentration per unit volume of fluid.} \]

Dividing eq 10 and 11 by eq 6, we obtain:

\[ \psi_c = \frac{r c_p}{\rho u} \frac{dT}{du} \quad (12) \]
\[ m = \frac{r}{\rho} \frac{dw}{du} \quad (13) \]

In a permanent state, \( \psi_c \) and \( m \) are constants throughout the boundary layer because of continuity. If, as before, \( r = r_0 \) we can integrate eq 12 and 13 from \( u = 0 \) to \( u \), to get:

\[ \psi_c = -H_c (T_1 - T_2) \quad (14) \]
\[ H_c = \frac{r_0 c_p}{u} \]

Here, \( H_c \) is the overall heat transfer coefficient.

\[ m = -T_e (W_1 - W_2) \quad (15) \]
\[ T_e = \frac{r_0}{\rho u} \]

Here, \( T_e \) is the overall mass transfer coefficient. It can easily be seen that \( H_c \) and \( T_e \) are uniquely related to the properties of the fluid if the Reynolds analogy holds.

\[ \frac{H_c}{T_e} = \rho c_p \quad (16) \]

The value of \( H_c \) can be obtained from eq 8 in rough turbulent flow:
Formula for heat exchange by convection and evaporation over a cold water surface in nature. The heat transfer coefficient \( H_c \) can be computed for wind blowing over waves in a rough turbulent flow. We then get with eq 14 and 17:

\[
\psi_c = H_c (T_a - T_w)
\]

\[
H_c = 4.4 V_{50}
\]

where \( \psi_c \) is in Btu/ft² day, \( T_w \) is the water surface temperature in °F, \( T_a \) is the air temperature 50 ft above the ground in °F, \( V_{50} \) is the wind speed 50 ft above the ground in ft/sec.

The heat loss by evaporation over a water surface can be related to the mass of evaporated water by:

\[
\psi_e = m L
\]

where \( \psi_e \) is the heat loss in Btu/ft² day, \( L \) is the latent heat of vaporization of water, 1073 Btu/lb.

The concentration \( W \) of water vapor in air can be related to the water vapor pressure by the universal gas law:

\[
p_e = W R T_a
\]

where \( p_e \) is the water vapor pressure in lb/ft², \( R \) is the universal gas constant, 85.75 ft°/°F, \( T_a \) is the absolute temperature (492°F at the freezing point). Using formula 15 with the convenient modifications and units and computing Bowen's number from eq 16 we finally get:

\[
\psi_e = H_e (p_a - p_w)
\]

\[
H_e = \frac{H_c}{\beta}
\]

where \( \psi_e \) is the heat loss by evaporation in Btu/ft² day, \( p_a \) and \( p_w \) are the water vapor pressure in inches of mercury, \( p_a \) 50 ft above the water surface. \( \beta \) is the Bowen’s number, which is computed to be equal to 0.011. The value of \( H_e \) is then:

\[
H_e = 400 V_{50}
\]

There are many empirical formulas obtained from measurements in the field that give a value of the heat transfer coefficient by evaporation \( H_e \). These values were usually obtained with pan evaporimeters which, because of their limited areas and edge effects, may not be fully representative of physical conditions for a larger water body in nature. We have compared formula 21 with these well-known formulas in Figure 24. It can be seen that straightforward Reynolds similitude with appropriate boundary roughness gives results which are certainly comparable with the data from
HEAT BALANCE ON OPEN WATER IN WINTER

this main group of observations. The proposed formulas are not, however, valid for low wind velocity nor for natural convection. These states are not usually significant however in the average heat budget for a free water surface in nature.

Simplification of formulas. The direct relation between the heat exchange formulas by convection and evaporation, and the fact that water vapor pressure is related only to temperature and relative air humidity, make it possible to combine them. Equation 20 can be written:

$$\psi_e = \frac{H_c}{\beta} p_w \left( e \frac{p_{sa}}{p_w} - 1 \right)$$  \hspace{1cm} (22)

where $p_w$ is 0.18 in. Hg for water close to 32°F, $e$ is the relative humidity of the air and $p_{sa}$ is the saturated water vapor pressure of the air at temperature $T_a$. When the air is colder than 0°F, the absolute air humidity is very small and can be neglected. As a first approximation it can be assumed in engineering applications that:

$$\frac{p_{sa}}{p_w} = \frac{T_a}{32} \quad \text{for } T_a > 0$$  \hspace{1cm} (23)
and \( \frac{P_{sa}}{P_w} = 0 \) for \( T_a \leq 0 \).

With eq 20 and the value of \( \beta \), this gives:

\[
\begin{align*}
\psi_e &= -H_c (16 - 0.5 e T_a) & T_a > 0 \\
\psi_e &= -H_c (16) & T_a \leq 0
\end{align*}
\]

(24)

This equation of heat transfer by evaporation could thus be combined on a daily basis with eq 18 for heat transfer by convection. If we assume that the water temperature \( T_w \) is close to 32°F, we get:

\[
\psi_{ce} = \psi_c + \psi_e = -6.6 V_{50} [32 - (0.86 + 0.33 e \xi) T_a]
\]

(25)

where \( \xi = 1 \) for \( T_a > 0 \), \( \xi = 0 \) for \( T_a \leq 0 \). \( \psi_{ce} \) is in Btu/ft\(^2\) day, \( T_a \) is in °F, \( e \) is the relative humidity and \( V_{50} \) is the wind velocity in ft/sec, both 50 ft above water.

**Radiation exchanges**

Heat exchange between a water surface and the surroundings includes radiation exchanges which are of two forms: shortwave and longwave radiations.

Figure 25 shows the various processes in radiation at the surface of a water body. Shortwave radiation from the sun is partially reflected on the earth's atmosphere. In its passage through the atmosphere part of the solar radiation is absorbed by the various molecules in it and another part is scattered by them or reflected by the larger particles. The scattered portion of the radiation is not entirely lost, however, because a chain process of multiple scattering and reflection results in a fraction of this radiation eventually reaching the earth's surface in diffuse form. Some is finally absorbed by the water body; the fraction of solar radiation reaching the water which is reflected at the water surface is the albedo of the surface.

The earth (its land, water and atmosphere) also emits substantial quantities of heat in the form of longwave radiation. Almost all of the radiated energy of the earth consists of wavelengths greater than 4\( \mu \) and this is related to the surface temperature of the earth itself. Thus a water surface will emit heat and receive some from its surroundings in the form of longwave radiation.

**Direct radiation gained from the sun.** The basic equation of heat gain per unit surface and time, from direct solar radiation, is:

\[
\psi_{r1} = \beta_1 S C \sin \alpha.
\]

(26)
HEAT BALANCE ON OPEN WATER IN WINTER

S is the solar constant, which is the intensity of solar radiation outside the atmosphere at normal incidence. Its value varies slightly with seasons but is close to 442 Btu/ft² hr.

C is a reduction factor taking into account the dispersion of radiation by the atmosphere, water vapor, and dust in suspension in air, as well as absorption by water vapor and ozone. The determination of each of these components has been the object of intensive studies. One of them gives the value of the product \( S \times C \) by taking the most probable value of the concentration of ozone, dust and water vapor in the atmosphere. Table I gives these integrated values of \( SC \sin \alpha \) for latitudes of 40°, 50°, 60° and 70° from September 1st to May 15th, as well as values of \( C_s \) (an insolation factor - see below).

\( \beta_1 \) is the factor of absorptivity of water. The radiation from the sun is partially reflected at the water surface. The factor \( \beta_1 \) depends on the incidence of the radiation and its wavelength. From Fresnel’s law, the reflectivity \( \rho \) of natural light is given by:

\[
\rho = \frac{1}{2} \left[ \frac{\sin^2 (i - r) + \tan^2 (i - r)}{\sin^2 (i + r) + \tan^2 (i + r)} \right].
\]

The angle of incidence \( i \) is the complement of the altitude \( \alpha \) of the sun, known for all latitudes and hours of the day. The refraction angle \( r \) is related to the incidence angle \( i \) by Snell’s law:

\[
\sin \alpha = \frac{\sin i}{\sin r}.
\]

Table I. Solar radiation absorbed on a clear day and \( C_s \) values.

<table>
<thead>
<tr>
<th>Latitude</th>
<th>40°N</th>
<th>50°N</th>
<th>60°N</th>
<th>70°N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Btu/ft² day</td>
<td>( C_s )</td>
<td>Btu/ft² day</td>
<td>( C_s )</td>
</tr>
<tr>
<td>1 Sept</td>
<td>2239</td>
<td>3.498</td>
<td>1750</td>
<td>2.734</td>
</tr>
<tr>
<td>15 Sept</td>
<td>2034</td>
<td>3.178</td>
<td>1540</td>
<td>2.406</td>
</tr>
<tr>
<td>1 Oct</td>
<td>1794</td>
<td>2.803</td>
<td>1270</td>
<td>1.984</td>
</tr>
<tr>
<td>15 Oct</td>
<td>1509</td>
<td>2.358</td>
<td>1027</td>
<td>1.605</td>
</tr>
<tr>
<td>1 Nov</td>
<td>1270</td>
<td>1.984</td>
<td>766</td>
<td>1.197</td>
</tr>
<tr>
<td>15 Nov</td>
<td>1088</td>
<td>1.700</td>
<td>563</td>
<td>0.880</td>
</tr>
<tr>
<td>1 Dec</td>
<td>878</td>
<td>1.372</td>
<td>437</td>
<td>0.683</td>
</tr>
<tr>
<td>15 Dec</td>
<td>831</td>
<td>1.298</td>
<td>388</td>
<td>0.608</td>
</tr>
<tr>
<td>1 Jan</td>
<td>837</td>
<td>1.308</td>
<td>391</td>
<td>0.611</td>
</tr>
<tr>
<td>15 Jan</td>
<td>922</td>
<td>1.440</td>
<td>465</td>
<td>0.727</td>
</tr>
<tr>
<td>1 Feb</td>
<td>1211</td>
<td>1.892</td>
<td>625</td>
<td>0.977</td>
</tr>
<tr>
<td>15 Feb</td>
<td>1305</td>
<td>2.039</td>
<td>873</td>
<td>1.364</td>
</tr>
<tr>
<td>1 Mar</td>
<td>1636</td>
<td>2.556</td>
<td>1128</td>
<td>1.762</td>
</tr>
<tr>
<td>15 Mar</td>
<td>1971</td>
<td>3.079</td>
<td>1394</td>
<td>2.178</td>
</tr>
<tr>
<td>1 Apr</td>
<td>2240</td>
<td>3.500</td>
<td>1718</td>
<td>2.684</td>
</tr>
<tr>
<td>15 Apr</td>
<td>2405</td>
<td>3.758</td>
<td>1925</td>
<td>3.008</td>
</tr>
<tr>
<td>1 May</td>
<td>2523</td>
<td>3.942</td>
<td>2102</td>
<td>3.284</td>
</tr>
<tr>
<td>15 May</td>
<td>2592</td>
<td>4.050</td>
<td>2214</td>
<td>3.459</td>
</tr>
</tbody>
</table>
The refractive index $\sigma$ does not vary much with the wavelength so it can be taken as a constant equal to 1.33 for an air/water interface. It must, however, be noticed that this is true for a calm interface but that nothing is known of this index for a natural wavy water surface. Supposing it has the same value, the factor of absorptivity $\beta_1$ can then be obtained with:

$$\beta_1 = 1 - \frac{1}{2} \left[ \frac{\sin^2 (i - r)}{\sin^2 (i + r)} + \frac{\tan^2 (i - r)}{\tan^2 (i + r)} \right]$$

in which $i = 90^\circ - \alpha$ and $\sin i = 1.33 \sin r$.

When it is cloudy, observations have shown that only 25% of the sun's radiation reaches the earth surface. A widely used relation is then:

$$\psi_{t1} = \beta_1 SC \sin \alpha \left( 0.25 + 0.75 \frac{m}{N} \right)$$

where $m/N$ is the fraction of hours of clear sky during daylight.

**Diffuse radiation from the sun.** For solar radiation diffused and transmitted indirectly through the atmosphere, a basic relation is:

$$\psi_{t2} = f \beta_2 SC \sin \alpha \left( 0.25 + 0.75 \frac{m}{N} \right)$$

where $f$ is the ratio of the intensity of indirect to direct radiation. This parameter varies with the altitude of the sun and its value may be taken from experimental data. These values are shown in Figure 26.

$\beta_2$ is the absorptivity of water for indirect radiation in clear weather. A theoretical and experimental study has shown that its value is close to 0.83. These values are only a rough approximation of the diffuse radiation component, which is very complex. The exact influence of clouds is particularly difficult to determine and an extensive treatment of this subject is beyond the scope of this work.

**Longwave radiation.** A water body emits longwave radiation and receives some from the surrounding bodies. The quantity of heat emitted in a half space per unit area and time is given by the classical Stefan-Boltzmann equation:

$$\psi_{t3} = - \epsilon_1 \sigma T_{*w}^4$$

The emissivity factor for water $\epsilon_1$ is 0.95 and it does not vary much with the temperature. The Stefan-Boltzmann constant $\sigma$ is equal to $0.173 \times 10^{-8}$ Btu/ft$^4$ hr $^\circ$F. $T_{*w}$ is the absolute water temperature on the Rankine scale.

The quantity of radiant heat received by a horizontal water surface from surrounding bodies, principally water vapor and $CO_2$, has the form:

$$\psi_{r3} = \epsilon_2 \sigma \beta_2 T_{*a}^4$$

The factor of absorptivity $\beta_2$ for water has a value of 0.83, as seen previously. The emissivity factor $\epsilon_2$ depends on the cloudiness. For a cloudy sky it can be taken as 0.96. For a clear sky many formulas have been proposed and this one is widely used:
HEAT BALANCE ON OPEN WATER IN WINTER

\[ \epsilon_2 = 0.55 + 0.33 \sqrt{p_a} \quad \text{(31)} \]

where \( p_a \) is the water vapor pressure in inches of mercury.

The net quantity of longwave radiant heat gained by a water body is then given by:

\[ \psi_{r3} = - \epsilon_1 \sigma T_{*w}^4 + \epsilon_2 \beta_2 \sigma T_{*a}^4. \quad \text{(32)} \]

The values of the actual vapor pressure obtained from eq 31 play a small role in the total value of \( \psi_{r3} \). For a relative humidity of 75% the values of \( \psi_{r3} \) are shown in Figure 27.

Simplification of the expression for radiation exchanges. The total radiation received from the sun can be obtained from eq 28 and 29:

\[ \psi_{r1} + \psi_{r2} = (\beta_1 + 0.83 I) SC \sin a \left( 0.25 + 0.75 \frac{m}{N} \right). \quad \text{(33)} \]

The \( \beta_1 \) and \( I \) factors depend on the sun elevation \( a \). Their values in eq 33 were computed for all daylight hours during the cold months at latitudes of 40°, 50°, 60° and 70°. The overall integration had the form:

\[ 640 C_g = \int_{\text{daylight hours}} (\beta_1 + 0.83 I) SC \sin a. \quad \text{(34)} \]
The \( C_s \) values are of the order of unity in this formula and they are given in Table I for the period under consideration. The total solar radiation absorbed by water is then:

\[
\psi_{t1} + \psi_{t2} = 640 \ C_s \left(0.25 + 0.75 \frac{m}{N}\right)
\]

where \( \psi_{t1} \) and \( \psi_{t2} \) are in Btu \( \cdot \) ft\(^2\) \cdot hr; \( C_s \) has the same unit and \( m/N \) is the insolation written as a fraction during daylight hours.

The longwave radiation exchange can be computed by supposing, as before, that the water temperature is 32°F and by developing the air temperature terms in eq 32 around \( T_{*a} = 460°F \) and keeping only the first linear term. The second term has no significant effect over a wide range of air temperatures. Supposing furthermore that the cloud effects are directly proportional to coverage we get in simplified form:

\[
\psi_{r3} = -12.7 \ (65 - T_a) - 480 \frac{m}{N}
\]

(35)

The total heat gain by radiation is then:

\[
\psi_r = 640 \ C_s \left(0.25 + 0.75 \frac{m}{N}\right) - 12.7 \ (65 - T_a) - 480 \frac{m}{N}
\]

(36)

where \( \psi_r \) is in Btu/ft\(^2\) \cdot day, \( m/N \) is the clear sky ratio, \( T_a \) the air temperature in °F and \( C_s \) is a solar gain coefficient obtained from Table I. This formula can be applied on an hourly basis at night by putting \( C_s = 0 \).
Other heat exchanges

Precipitation and ground water. The heat needed to supply the latent heat of fusion of snow falling into water can be represented by the following equation:

$$\psi_{sp} = -750 i_s$$  \hfill (37)

where $\psi_{sp}$ is in Btu/ft$^2$ day; $i_s$ is the snow precipitation in equivalent inches of water during the day. The heat required to raise the temperature of snowfall to the water temperature is usually considered negligible.

In the same manner the heat gain from water precipitation is:

$$\psi_{wp} = 5.2 i_w (T_a - T_w)$$  \hfill (38)

$i_w$ is the precipitation in inches of water during the day at the air temperature $T_a$.

For groundwater this formula becomes:

$$\psi_{wg} = \frac{62.4 q_g \Delta T_g}{B}$$  \hfill (39)

where $q_g$ is the groundwater flow in ft$^3$/day per foot of river length having a temperature difference $\Delta T_g$ over the river water temperature and $B$ is the width of the river.

Generally, the effect of precipitation on a river or a lake is not sensible on the average heat budget but groundwater flow may explain many local phenomena during ice formation and remains an important factor in a river once the ice cover has formed.

Heat conduction through the earth. There are few measurements in the literature of geothermal flow. For some Swedish rivers it was found that the terrestrial heat flow in December was 1.3 Btu/ft$^2$ day. For a few Russian rivers, 2.7 to 5.4 Btu/ft$^2$ day have been measured and a value of 0.3 Btu/ft$^2$ day is cited for locations in Canada. On this basis it can be concluded that the effect of conduction through the earth is negligible in the overall budget of an open water area.

Heat gain from friction losses. The heat added because of flow friction losses in a river can be easily computed. It is given by:

$$\psi_f = 6840 q s_e$$  \hfill (40)

where $\psi_f$ is again in Btu/ft$^2$ day, $q$ is the river discharge per unit width in ft$^3$/sec ft and $s_e$ is the slope of the energy line of the flow at the section considered.

Thermal budget of a free surface flow

Consider in Figure 28 an element of a river flow whose geometry is constant in space and time. Let us assume that the meteorological conditions remain constant above the water surface for an interval of time $dt$. The length of the element is $dL$, its width $b$ and the water depth is $Y$. The average velocity of this uniform flow is $V$ and the water discharge is $Q$.

At section 1-1, the water temperature is $T_w$ and the quantity of heat $\phi$ advected to this volume element by unit time is:
$$\phi = \gamma_w V Y b c_p T_w - \epsilon Y b \frac{dT_w}{d\ell}.$$  \hspace{1cm} (41)

The first term represents the actual heat transported into the section by the flow and the second term represents the convective turbulent heat diffusion at the boundary. The specific weight of water is $\gamma_w$, $\epsilon$ is the coefficient of eddy diffusivity and $c_p$ is the specific heat of water. The heat content of water is referred to $32^\circ F$.

The heat taken out of section 2-2 is:

$$\phi + d\phi = \gamma_w V Y b c_p \left( T_w + \frac{dT_w}{d\ell} d\ell \right) - \epsilon Y b \left( \frac{dT_w}{d\ell} + \frac{d^2 T_w}{d\ell^2} d\ell \right).$$  \hspace{1cm} (42)

Taking into account the total heat gain at the water surface $\psi_s$, the internal heat gains $\psi_f$ and $\psi_{wg}$ from friction losses and groundwater and further supposing an isotropic external heat gain $\psi_e$ per unit area and unit time, we obtain the equilibrium equation:

$$\Sigma \psi = \psi_s + \psi_f + \psi_{wg} + \psi_e = \gamma_w c_p V Y \frac{dT_w}{d\ell} - \epsilon Y \frac{d^2 T_w}{d\ell^2}.$$  \hspace{1cm} (43)

Neglecting the second order term $\psi_s$ (turbulent diffusion) finally gives:

$$T_{w,\ell} - T_{w0} = \frac{1}{\gamma_w c_p} \int_{\ell_0}^{\ell} \frac{\Sigma \psi d\ell}{V Y}$$ \hspace{1cm} (44)

or

$$\ell - \ell_0 = \gamma_w c_p \int_{T_{w0}}^{T_{w,\ell}} \frac{V Y dT_w}{\Sigma \psi}.$$ \hspace{1cm} (45)
Using the values of the heat components previously determined for $\Sigma \psi$, we finally get an approximate engineering formula for the variation of the temperature of water, when it is close to freezing point, in a river on a daily basis:

$$\Sigma \psi = -6.6 V_{50} \left[ 32 - (0.66 + 0.33 e \xi) T_a \right] +$$

$$+ 640 C_s \left( 0.25 + 0.75 \frac{m}{N} \right) - 12.7 (65 - T_a) - 480 \frac{m}{N}$$

$$- 750 i_s + 6940 q_s e + \frac{62.4 q_g \Delta T_g}{B} + \psi_e$$

with $\xi = 1$, $T_a > 0$ and $\xi = 0$, $T_a < 0$.

And:

$$T_w - T_{w0} = 1.85 \times 10^{-7} \int_{L-L_0}^L \frac{\Sigma \psi \, dL}{q}$$

$$L-L_0 = 5.4 \times 10^6 \int_{T_w=T_{w0}}^{T_w} \frac{q \, dT_w}{\Sigma \psi}$$

where:

- $\Sigma \psi$ = total heat transfer per unit surface area, Btu/ft$^2$ day
- $\psi_e$ = external artificial addition of heat, Btu/ft$^2$ day
- $L$, $L_0$ = river length along axis, ft
- $B$ = river width, ft
- $q$ = river discharge per unit width, ft$^3$/sec ft
- $q_g$ = ground water discharge per unit of river length, ft$^3$/day ft
- $T_w$, $T_{w0}$, $T_{wL}$ = average water temperature at river sections, °F
- $T_a$ = air temperature 50 ft above surface, °F
- $\Delta T_g$ = groundwater temperature difference from water, °F
- $V_{50}$ = wind velocity 50 ft above surface, ft/sec
- $e$ = relative humidity of the air
- $m/N$ = fraction of clear sky
- $i_s$ = snow precipitation, in. of water/day
- $s_e$ = slope of energy gradient for water flow
- $C_s$ = insolation coefficient (Table I).
There are some overall formulas\textsuperscript{121} \textsuperscript{141} \textsuperscript{144} in the literature that give an average monthly value of the exchange coefficients for the winter months. For water close to 32°F, which is the case in most applications, these formulas have the form:

$$\Sigma \psi = K_1 - K_2 T_a .$$

Two of these\textsuperscript{121} \textsuperscript{144} were derived for average winter conditions at Montreal for the St. Lawrence River. Further measurements were taken from a 10-ft-diam water tank in the Ottawa region.\textsuperscript{145} If we use average winter conditions at Montreal:

- average $V_{50} = 8$ mph $= 11.7$ ft/sec
- mean humidity $e = 0.8$
- hours of sunshine $m/N = 0.3$. 

\begin{figure}[h!]
\centering
\includegraphics[width=\textwidth]{figure29}
\caption{Heat losses and air temperature for St. Lawrence River at Montreal.}
\end{figure}
Using these values in eq. 46 with \( s_e = 0 \), \( C_s = 1.5 \), \( s_e = 0 \), \( q_g = 0 \) and \( \psi_e = 0 \) we get the average formula:

\[
\begin{align*}
\Sigma \psi &= 85 \, T_a - 3000 & \text{for } T_a > 0 \\
\Sigma \psi &= 64 \, T_a - 3000 & \text{for } T_a \leq 0
\end{align*}
\]

This formula is compared with previously cited ones in Figure 29 and it can be seen that it gives a good representation of these local conditions.

**Thermal budget of a lake**

In the case of a lake the equations giving the basic components of the heat exchange at the water surface \( \psi_s \) are essentially the same as the ones derived for a river. One difference is that the surface temperature cannot usually be taken to be 32°F. Another difference is that the friction-loss term does not exist.

The fundamental difference between a river and a lake is the effect of the temperature distribution inside the water body itself. In the case of a river it is essentially uniform in a section because of turbulent exchanges. In a lake it is not normally uniform. The heat content of the water mass is important and its thermal inertia has to be taken into account.

**Temperature distribution in a lake.** From a widely used nomenclature describing the thermal regime of lakes in temperate and northern regions there are three orders of lakes (Fig. 30):

a. Lakes of the first order where the bottom temperature is so near to 39°F, the temperature at which water has its highest density, that extrapolation of the summer temperature curve would involve negligible heat transfer in this lower zone. Very deep lakes belong to this order.

b. Second-order lakes, thermally stratified but with bottom temperature significantly above 39°F. Most deep and fairly deep lakes (the Great Lakes in North America for instance) are of this order.

c. Third-order lakes not thermally stratified (shallow lakes).

The temperature distribution in lakes is shown schematically in Figure 30 for critical periods of the year. The temperature distribution follows identical processes for the three orders of lakes. During the summer months, the upper region of a lake contains a layer of more or less uniformly warm, circulating and fairly turbulent water termed the epilimnion. The turbulent exchange in this zone is caused by the wind which makes currents and waves leading to the mixing and downward transport of momentum and heat.

The deep, cold, and relatively undisturbed region at the bottom of lakes of order 1 and 2 is called the hypolimnion. In this stable area there appears to be much more a process of molecular heat diffusion than turbulent convection.

Between the epilimnion and the hypolimnion there is a region of rapid decrease in temperature, all of which is called the metalimnion. The plane of maximum rate of decrease in temperature is defined as the thermocline, where, formally:

\[
\frac{d^2 T_w}{dy^2} = 0.
\]
Figure 30. Temperature regimes of lakes.

Yearly thermal regime of a lake. We will describe the principles of the various heat transfer processes that occur around the year for temperate lakes showing ideal behavior.

It is convenient to start when the ice melts in the spring and a small amount of surface heating theoretically produces, for any kind of lake, an isothermal condition where all the water is at 39°F.

From this time on we have summer conditions with the heat transfer process 1-2 shown in Figure 30. This may be called a density-stable heating process because the water heated at the surface is always less dense than the water underneath. In the epilimnion, because of the wind, there is turbulent heat mixing, which produces a fairly uniform temperature distribution. In shallow-water lakes, where this exchange goes down to the bottom, there is thus only one layer of practically isothermal water. In lakes of the second order the heat taken at the surface is transmitted to the hypolimnion and affects its whole thickness so that the temperature at the bottom rises above 39°F during the summer. In lakes of the first order the heat transmitted through the thermocline affects the upper part of the hypolimnion but is not sufficient to raise the temperature at the bottom which stays stable at 39°F the whole year round.
When a lake has attained its maximum heat content, its temperature is usually stable for a few weeks at the end of the summer, and it then starts to cool according to process 2-1 indicated in Figure 30. This is essentially a density-unstable cooling process because the cooled water on top is heavier than the water underneath. This heavier water has a tendency to sink to the level where it finds its own temperature. Because this process is added to the turbulent mixing in the epilimnion, a much more uniform temperature distribution is attained during cooling than during warming-up periods. Furthermore, the temperature gradient is much steeper in the metalimnion where this water meets the cooler stable water underneath. In the case of a shallow lake the temperature is quite uniform practically from the start of the cooling phase (Section A-A). In the case of a second order lake, the temperature becomes uniform in the whole section somewhat later when the surface temperature becomes equal to the bottom temperature. From that time on, there is also complete mixing of the whole lake water from top to bottom when heavier water is cooled on the top of this unstable state. For a first order lake, the temperature becomes uniform only in the epilimnion and part of the hypolimnion. There is no density mixing in the lower, denser water which stays at 39°F and is stable the whole year round. Process 2-1 is completed, in all cases, when all the water in a lake is again, theoretically, at 39°F.

From this isothermal state, further cooling follows process 1-3 which is a density-stable cooling process because the cooler water on top is lighter than that underneath. For all types of lakes the total heat taken out during this process is relatively small. The cooling is limited to the epilimnion up to the time the freezing temperature is attained at the surface and the ice starts to form.

After an ice cover has formed on the surface of a lake, there is no more turbulent mixing in the epilimnion by the wind and also there is no heat transfer (except some radiation) from the atmosphere, because the lower ice cover boundary acts as a sink at constant freezing temperature for any heat conducted through the ice sheet. But some heat is gained from the ground and groundwater, and more may come from solar radiation through clearer ice and from biochemical sources. Thus, the water temperature increases again under the ice cover and tends to become isothermal at 39°F except for a small boundary layer of cooler water on top. This process, 3-1 on Figure 30, is essentially an unstable-density heating one. Warmer water on the bottom is lighter than that at 39°F on top and tends to go up. Heated water on top is heavier than that underneath and tends to sink so that a quite uniform temperature state is attained.

Mechanism of density convection. The water of a lake in winter under an ice cover is not stagnant. Some recent measurements have shown that there is a measurable velocity field, induced by the convection of water that is in an unstable density state, as shown in Figure 31. This helps to explain the tendency toward uniformity of temperature of the water, near 39°F, at the end of a winter in smaller lakes. The total heat gain might not, however, be adequate to attain this state in a deeper lake.

Inherent instability of the thermocline. Because the metalimnion is essentially formed by convective currents caused by the wind it has to be an unstable temperature boundary.

The most frequent divergence from the classical temperature curve is caused by secondary thermoclines as shown in Figure 32. They can be made in a number of ways but let us consider one example. A summer storm on a lake usually has the effect of lowering the thermocline (high winds) and steepening the temperature gradient at point A (unstable cooler water on the top). If it is followed by relatively calm and warm weather, a second thermocline at B, much closer to the surface, will be produced (Fig. 32). It is often said that the overall temperature of a lake is an integrator of past climate. It may also be said that the form of the thermocline is the signature of past storms.
There are many other reasons for the instability of the thermocline. In a lake there is always a general circulation of the water in the epilimnion depending on its form and that of its surroundings, on the influents and effluents and, mainly, on the characteristics of the wind. One of these effects is the piling up and thickening of the epilimnion downwind and the bringing up of the colder metalimnion upwind. This is, indeed, continuously in a changing state, as is the cause itself. Even in calm water a seiche may start in the metalimnion of the whole water body because of the previous inclination of this layer when the wind was blowing.

Cooling of a lake. The period when a lake is cooling down before the ice sheet starts to form is certainly the one which lends itself best to computations.
Let us consider an origin of time such that after a certain amount of cooling has taken place, we get a temperature distribution shown as section A-A for the three orders of lakes in Figure 30. It can be seen that the temperature is then theoretically uniform from top to bottom, in second and third order lakes, which are the most usual kinds. The temperature of the surface of these lakes from this time to when the temperature becomes uniform at 39°F can then be obtained from:

$$\int \Sigma \psi A \, dt = \gamma_w \, c_p \, dT_w \int y \, dA$$

(50)

where:

- $\Sigma \psi$ = the total external heat transfer at the lake surface, Btu/ft² day
- $A$ = surface of the lake, ft²
- $t$ = time, days
- $\gamma_w, c_p$ = specific weight and specific heat of water
- $T_w$ = temperature of the water surface, °F
- $y$ = water depth in the lake, ft.

In a first-order lake the temperature will be uniform from the top to the depth of the stable hypolimnion; from there it will then be stratified to the bottom where the temperature is 39°F. The maximum temperature distribution in the hypolimnion may change little from year to year. If we take an origin of time such that the zone of uniform temperature gets down to the hypolimnion, the surface temperature can be computed from:

$$\int \Sigma \psi A \, dt = \gamma_w \, c_p \int y \, dA \, dT_w$$

(51)

In this case $y$ is the equivalent lake depth equal to the depth of the water at temperature $T_w$ in the hypolimnion, during the cooling period.
FRAZIL

Introduction

One of the most remarkable phenomena of ice formation in rivers and lakes is the sudden appearance of frazil particles in a mass of turbulent water. This type of ice formation is not usual in lakes but occurs regularly in northern rivers. It is usually the main ice formation process in larger rivers and sometimes, as for the Angara and Chirchik rivers, the prevailing one during the whole winter period.\(^5\)

Frazil is born on the open water surface of a turbulent stream and from the moment the particles are nucleated they are continuously changing form.\(^7\) They may take the form of small circular plates, porous flocs inside water, slush on the surface, ice pans and, finally, parts of an ice cover. Because frazil ice is always changing in an evolutive process, definition has always been difficult. We will use here the usual interpretation which limits this type of ice to the forms originating from the frazil particles, excluding the continuous solid ice pieces. This includes frazil particles, frazil flocs and slush, frazil lumps and balls, and frazil deposits under ice covers.

Frazil, in all its forms, is one of the most dangerous types of ice affecting river works. It is essentially a cause of obstruction of the flow passages, and this leads to four types of engineering problems:

- Partial or total obstruction of flow openings for various types of water intakes.
- Freezing of movable mechanical parts in flowing water.
- Rise of water levels causing floods.
- Decrease in usable capacity of reservoirs.

Not a winter passes, in northern countries, without reports of water intakes being shut off by frazil. Schaefer\(^24\) reports: "Nothing is more dramatic than to witness a 30,000 kw hydroplant removed from the operating electrical network in less than an hour by the accumulation of these tiny frazil particles on the intake racks." In 1914 the Leningrad water supply lines were blocked, which left the whole town without water. The bottom of the Neva River was covered with a thick crust of porous anchor ice originating from frazil particles.\(^2\)

One important aspect of the frazil ice problem is the tremendous quantity of ice which is produced in an open water area as compared to the quantity which is formed, under identical atmospheric conditions, by the static growth of an ice cover. That is why a river rapid producing frazil for a long period of time is called an ice-making machine or an ice factory. Furthermore, frazil accumulates into masses of very porous form where the actual volume of ice is small compared to the volume of the deposit.

Under those conditions frazil travels, deposits under a cover, and reduces the flow passage in a most effective way. Many cases are known where reservoirs of power stations have been completely filled with frazil except for a very small water passage.\(^7\) Some power plants have thus been rendered inoperative for the whole winter period and others have had to be abandoned altogether. When a natural frazil factory is in operation, the frazil flocs and slush fill the low-velocity areas underneath the downstream ice cover, reducing the flow section and raising the water level.
Some flooding may occur but it may not be serious because of the reduced water discharge during the winter months. However, these deposits consolidate somewhat and when breakup commences, the increase in discharge is not compensated by an increase in free flow area so that frazil contributes in no small way to spring floods.

**Forecasting frazil appearance**

For most water intakes where frazil is known to clog the racks, special measures are taken to fight this menace just before it appears. The success of these emergency techniques depends mainly on the ability of the operator of the works to predict, at least half an hour in advance, the onset of frazil.

This can usually be done fairly well by measuring the water temperature. Knowing the rate of water cooling and the fact that frazil occurs in supercooled water at 0.02°F to 0.05°F of supercooling, it is not difficult, by extrapolation, to predict the time it should make its appearance. The critical temperature from which the measurement should be started is normally 33°F.

Frazil of consequence occurs at night and disappears with sunlight because the heat added from solar energy usually stops the cooling of the water. This can be put to use. If, for example, in the early morning hours after a cold night the rate of cooling of the river is, say, 0.2°F/hour, but the water temperature is still 33°F, there is no need to do anything because it will be five hours before the freezing point is reached and the sun will be coming up in two hours. The fact that some supercooling of the water is required before the appearance of frazil gives the operator a last chance when the temperature of the water attains normal freezing point at 32°F. From one operational experience the trash screens become choked with frazil approximately one hour after this point is reached.

To make any kind of prediction of the appearance of frazil within a temperature range of 1°F it is necessary to have a method of precise water temperature measurement with an accuracy of at least ±0.02°F. A continuous recording of temperatures in a heated room is usually the best solution. One of the difficulties involved in measuring the temperature of supercooled water is that ice forms on the temperature sensor and then the only temperature recorded is the normal freezing point.

Frazil may be detected in flowing water by putting a frayed rope with a weight, or a steel chain, in the water. As soon as frazil particles are nucleated they stick strongly to these objects. Its concentration may also be obtained by measuring the electrical conductivity of the water filled with frazil particles.

**Supercooling of water in rivers and lakes**

It is a well known fact that the phenomenon of supercooling of liquids is apparently universal for the beginning of the solid phase. Water is no exception. There is considerable supercooling of water droplets in clouds before the appearance of snow or hail. The temperature of freezing of quiet water masses in the laboratory is generally many degrees below 32°F. Thus, it is not surprising to observe that crystals of surface ice or frazil, in flowing water, also appear at temperatures below 32°F.

But there is quite a difference in the amount of supercooling for different cases. Water droplets might freeze at minus 35°F, but frazil particles form in rivers with only a few hundredths of a degree F of supercooling. There is only one coherent explanation in the literature for these discrepancies, which we will give here.

The temperature of freezing of water droplets in clouds is usually very low, of the order of -35°F to 0°F. Research by physicists has shown that this temperature depends mainly on the presence of foreign particles in the droplets. Mason says that we can imagine, in the following...
way, the freezing of a droplet: "As the temperature is lowered the molecular arrangement of the supercooled water particle is getting closer and closer to that of ice. If there is no foreign particle, nucleation will occur by a lucky orientation of water particles in space and time to form the crystalline structure of ice. However, the presence of a foreign substance of favorable crystallographic boundary will give a better orientation to the molecules under the interfacial force field and will maintain this configuration for a longer time in the Brownian motion of the fluid. The probability that the water molecules in contact with this particle will attain the critical configuration to start the solid phase will be much increased. Thus, it is clear that the presence of favorable foreign particles will decrease the amount of supercooling before the freezing of the droplet." In the presence of foreign particles the freezing of water is called heterogeneous nucleation and with very pure water, which is nonexistent in nature, homogeneous nucleation.

An important extension to the theory of heterogeneous nucleation for the freezing of water samples of finite sizes has been proposed by Dorsey. His results on specimens many cubic centimeters in size can be summarized as follows:

- The temperature of nucleation is much closer to 32°F than for droplets.
- It is consistent for the same sample.
- It is independent of the cooling rate.
- It is influenced by the state of agitation of the sample. Most forms of agitation decrease the amount of supercooling necessary to cause nucleation.

Dorsey proposes an extension to the theory of heterogeneous nucleation to explain the effect of agitation and rubbing. He says: "It is generally agreed that the molecules of a liquid that are immediately adjacent to a foreign body are packed quite closely together, are bound strongly to that body and have a preferred orientation with reference to the interface between the body and the liquid. In successively more distant layers the packing becomes less close, the binding weaker, and the orientation less complete, until presently the status characteristic of the liquid in bulk and far from foreign bodies is attained. For simplicity, this entire group of layers is called the adsorbed layer of liquid.

"If the adsorbed layer can be torn (impact of free molecules of the melt, gross mechanical impact or rubbing) so that a sufficient number of loosened molecules remain in one another's field of force for such a time and with such a degree of freedom that they can reorient themselves and become bound together in the manner characteristic of an embryo that can persist and grow at the existing temperature, they will do so...."

"An embryo of a crystal is any structural aggregation of the molecules of the melt that maintains its identity as an individual, distinct from the ambient melt, for an interval that is long as compared with the mean time between consecutive molecular collisions.... [The embryo may be simple (one composed of molecules of the melt only) or complex (one with a foreign particle as a center and molecules of the melt adhering to it).].... The retention of identity as an individual does not preclude a limited mutual transfer of molecules between it and the ambient melt.... The stability of an embryo varies with its size; so that an embryo of a given kind can at a given temperature be in equilibrium with its melt only if it is of a particular size. That will be called its critical size.... Embryos larger than the critical will mature into macroscopic crystals; those smaller than the critical will wane (decay) to evanescence. Hence, at any given temperature the initiation of a viable embryo involves the organization per saltum of an embryo of at least the critical size."

In short, Dorsey's extension of the heterogeneous theory contains a description of some possible intermediate mechanisms in the formation of ice crystals originating from foreign nuclei. This theory is particularly fruitful for explaining the effects of agitation, rubbing and supersonic radiation, which can nucleate a sample of water of finite size at a higher temperature than that of quiescent water.
The theory of heterogeneous nucleation can be developed from the empirical relation:

\[ n = n_c e^{\beta (|T_s| - |T_c|)} \]  

(52)

where:

- \( n \) = concentration of active particles at supercooled temperature \( T_s \)
- \( n_c \) = concentration of active particles at critical temperature \( T_c \)
- \( T_c \) = critical temperature below which the foreign particles become active.

Taking into account the probability that a droplet of volume \( V \) contains at least one active nucleus at supercooled temperature \( T_s \), it is found that:

\[ |T_s| - |T_c| = \frac{1}{\alpha} \left\{ \ln \left( \frac{\ln 2}{n_c} \right) - \ln V \right\} \]  

(53)

in which for a volume \( V \) and a temperature \( T_s \), 50\% of the droplets are frozen. This relation applies well to experimental data\(^{11,51}\) as shown in Figures 33 and 34. In Figure 33 each curve corresponds to a different concentration of foreign particles, the lowest curve on the graph corresponding to the highest concentration \( n_c \). The slope of these curves depends on the affinity of nucleation of the particles \( \alpha \). Figure 34 shows without any doubt that above a critical temperature \( T_c \), no particle in the droplets is active as a nucleus in quiescent water.

![Figure 33. Freezing of groups of water droplets of various water qualities.\(^{11}\)](image)
Much difficulty has been experienced in the past in trying to apply the theory of heterogeneous nucleation to frazil formation. This was so because in turbulent flow the amount of supercooling barely exceeds a few hundredths of a degree Fahrenheit. At such temperatures no substance in suspension in water is able to nucleate ice\(^{10}\) and, in fact, no nucleation of sizable volumes of water normally occurs\(^{34}\) above 26°F. We will see here that the Dorsey concept of the embryo applies equally well in frazil formation, if use is made of the temperature distribution in the thermal boundary layer at the air/water interface.\(^{90}\)

Let us consider first, in a river, the temperature distribution along a vertical section of an idealized turbulent flow (Fig. 35). There is a steep temperature gradient at the water surface. The water temperature at the interface must be lower than the bulk water temperature because of the thin boundary layer at the top of the water flow. Below the very thin thermal layer on top, there is an almost uniform temperature distribution in the remainder of the flow. In cold weather viable heterogeneous embryos are formed on the surface. As they start to give up the heat of fusion of ice, they are carried underneath by the turbulence; they decay and disappear in the warm water. The whole body of water cools down to the freezing point. Because there is no heat sink to allow the viable embryos to turn into ice at that time, some undercooling of the mass is required to nucleate into finite ice elements the viable embryos of critical size that have been carried down by turbulence. Depending on the number and size of the embryos that have been nucleated and the amount of heat taken out at the surface at the time, the amount of supercooling of the water can then be computed.
Figure 35. Air-water temperature distribution before the appearance of frazil in turbulent flow.

Figure 36. Thermal boundary layer in water.²

Consider, finally, the case of the cooling of the surface of a calm lake. The temperature distribution will then be similar to that shown in Figure 36. The surface layer will start to supercool and only a prism of supercooled water will be formed on top. When the temperature at the interface attains the critical value required to form viable embryos, nucleation will start and propagate quickly at the surface, giving up the latent heat of fusion, which is taken away in this supercooled layer.

Figure 36 shows some actual measurements of the temperature in the top layer of turbulent and laminar flows.² The strong temperature variation at the very top surface is striking. These measurements show the existence of a very thin boundary layer at the top with a strong temperature gradient where supercooling at the very surface could not be measured with existing instrumentation. The only possible temperature of the air-water interface to start nucleation is the critical temperature that forms viable embryos. Figure 37 shows the measured temperature of an ice crystal growing in a prism of supercooled quiescent water.³¹
Formation of frazil ice

Two basic mechanisms may account for the appearance of the first frazil particles in a turbulent flow. The main one is the entrainment and nucleation in the mass of water of the viable embryo formed in the highly supercooled water of the top layer of the flow; the second mechanism might well be the mechanical action or multiplication of crystals by splitting.

Disk-shaped ice crystals have been observed to grow on the surface of bulk ice when it was placed in slightly undercooled water. These disks only touch the surface of the bulk ice at one place on their periphery (Fig. 38). The disks, which remain circular during their growth, have been grown to a diameter of 1 cm. If a disk is broken off by an external force, the point of attachment becomes the site where a new disk grows. In this manner a single site can create two or three crystals per second. Shore ice is the first type of ice to appear in rivers in highly supercooled calm or laminar flowing water. Crystals grown on the edge can thus be entrained in the turbulent river flow, multiply and form frazil. It must, however, be remembered that lateral diffusion and mass transport is a slow process in a turbulent river flow and it is doubtful that this type of frazil production is important in larger rivers. Connected to this process is also the growth and splitting of ice crystals from snow falling in supercooled water.

Frazil forms in turbulent flows and the maximum supercooling of the bulk water observed in nature has been 0.1°F; the normal supercooling is from 0.02°F to 0.05°F. There are two ways of considering the evolution of the water temperature when frazil is formed. One consists of studying an element of water as it flows along its path, the other of studying temperature distribution along the flow. Tests made in laboratory water tanks or flumes represent the first case. A typical curve of water temperature changes with time is shown in Figure 39. The mass of water initially above 32°F is cooled at a constant rate. Water temperatures reach 32°F and keep on cooling at the same rate for a few hundredths of a degree. The rate of cooling then starts to decrease until it rapidly becomes zero. Water temperature has then reached its minimum point. As the rate of cooling starts to decrease, small ice particles start to appear uniformly in the turbulent flow. Those particles are then too small to be seen by the naked eye, but their presence is revealed by the fact that they reflect light well. They rapidly grow and form small ice disks. After the water temperature has reached its minimum it returns to 32°F, first rapidly and then more and more slowly, giving to the curve a more or less asymptotic appearance as the temperature tends toward 32°F. During that period the individual particles agglomerate to form porous flocs whose dimensions depend on the turbulence of the flow. Frazil production lasts for a few minutes only, and
then no more particles of that type are produced. During the frazil formation period a considerable reserve of "cold" exists in the supercooled water. The potential of particle growth is high and that is why the particles form rapidly and agglomerate into flocs. Frazil formed in that period may be called active frazil\textsuperscript{33} to differentiate it from frazil that has already formed and evolved in the water whose temperature has returned to the normal freezing point.

For very high cooling rates of water (over 2\(^\circ\)F/hr) the temperature curve does not return to the normal freezing point and residual supercooling is left in the body of water to account for the heat given by the growing ice particles.\textsuperscript{23} These conditions are exceptional in nature, however.

Let us now examine the temperature changes along the axis of a river with the help of Figure 40, depicting uniform flow with depth \(y_0\) and mean velocity \(V_0\). If we suppose that the temperature at the chosen point of origin along the distance axis is constant and that the rate of heat transfer towards the atmosphere per unit area is also constant, the temperature curve is then quite similar to the one represented in Figure 39 except that it differs by a change in abscissa \(x = V_0 t\). But this curve is much more representative of frazil appearance in rivers. In this ideal case, frazil is produced indefinitely in the same zone and the point of origin is fixed. The zone of active frazil extends only over a few hundred feet. In nature, the principal difference comes from the continuous variation in the heat transfer process because of atmospheric changes; one most important aspect of this is the abrupt and unexpected changes in wind conditions. A continuous lowering of air temperature provokes an upstream displacement of the frazil production front. If it is warm enough during the day to increase the water temperature and cold enough at night to decrease it again, the same section of a river will produce a certain amount of active frazil every
night. If energy losses due to friction at the foot of a rapid maintain the water temperature above 32°F, the zone of active frazil formation may oscillate slightly around a point downstream of that rapid, and the supercooling of the water may sometimes last many days, in the same river stretch.  

**Forms of frazil**

If a very small spherical crystal of ice is placed in slightly supercooled water it grows into a circular disk (Figure 41). It was found that the transition from sphere to disk occurred at a diameter of about 10 microns and that the ratio of diameter to thickness is between 5 and 100 for crystals whose diameters range between 500 and 3000 microns. As growth proceeds, however, flat dendrites grow out from the edges of the flat disks and may produce needle-like fragments if supercooling lasts a long enough time. Many observations in nature have shown disk thicknesses in the range of 25 to 100 microns for particles 1000 to 5000 microns in diameter.

During the period of active frazil production the ice crystals have a great potential for growth and they agglomerate to form frazil flocs of a very low ice content. Once the water has come back to 32°F the ice in the flocs has no further potential for spontaneous growth and it may be called **inactive** frazil. The flocs oscillate in the turbulent flow and those reaching the surface serve as nuclei for the regular growth of ice from heat exchanges with the atmosphere.

Inactive frazil evolves in a river as shown in Figure 42. As soon as the flocs are formed they have a tendency to concentrate at the surface of the flow. Their ice content is small and the total density of a floc is not much different from the density of water. The turbulence of the flow brings them successively to the surface for a while. New crystals are then formed on the side exposed to the air, without changing their external dimensions much. Interstices inside the flocs are closed up and they become lighter and concentrate more and more at the surface to form frazil slush. If the flow velocity is low, some stay at the surface long enough for a continuous layer of ice to form at their upper part, and finally ice pancakes are made, overtopping porous masses of frazil flocs. They take a saucer-like form because of collisions with neighbors. In the case of less turbulent flow, flocs concentrate more rapidly at the surface and ice floes form more rapidly.
Figure 41. Growth process of frazil into a notched disk-crystal.

Figure 42. Development of frazil in a river.

Figure 43. Air photos showing growth of ice floes in a river.
If the flow is very fast, the flocs become denser and denser, forming frazil balls without the solid ice layer on top. If frazil can travel over long distances large ice floes develop, mainly by the sintering of individual pancake floes together. Figure 43 shows air photographs of this process.

In lakes frazil formation and evolution is essentially the same except that the driving force causing the turbulent flow is the wind instead of gravity, and frazil formation is limited to the epilimnion.

Anchor ice

One of the remarkable and much discussed forms in the evolutive process of frazil formation is anchor ice. When frazil is being formed and the water is supercooled, there is a strong potential for growth, and frazil particles stick and grow on certain materials of favorable crystalline structure. They also deposit in separated no-flow areas in front of and behind obstacles in flowing water. The frazil particles deposit and adhere to projections like stones and weeds on the river bottom. These projecting bodies being in the supercooled water itself, the adhesion is much stronger than on a flat muddy bottom where the heat conducted from the ground prevents supercooling of the surface.

In river stretches with low flow velocity, these initial deposits grow in the supercooled water and form transparent ice plates growing at various angles (Fig. 44). In high velocity flow, growth is practically impossible and frazil is shingled in a snow-like crust. Various types of anchor ice deposits are shown in Figure 45.

It must be remembered that anchor ice can be produced only in supercooled water, which happens frequently only at selected river reaches. When the sun comes up during the morning following a night of frazil formation, some anchor ice may float up. It looks like any other frazil slush on the surface except that it contains dirt or debris.

Figure 44. Deposit of anchor ice growing on weeds.
Deposits of frazil

Because of its eventual evolution into ice floes, frazil is essentially at the origin of ice covers in larger rivers as we will see in the next chapter. Let us limit ourselves here to the more usual form of mushy ice and the formation of an underhanging dam as shown in Figure 4. In this case an ice cover normally progresses in a stretch of low velocity flow and the ice floes accumulate at the foot of a rapid. Velocities (more precisely Froude number) become too high for the ice cover to progress any farther upstream. That often happens when a relatively short section of flow is kept open in a rapid. Sections upstream of the rapid are also ice-covered. Frazil is then produced over a short length of river and consequently the ice cover downstream is mainly fed with frazil slush and small ice cakes.

Since the ice cover cannot progress upstream, frazil is carried downstream under the cover until it reaches a point of sufficiently low velocity where it is deposited under the bottom of the cover. It accumulates there and blocks part of the flow section until the velocity reaches a value high enough to carry newly arrived floes and small floes farther downstream. Thus an underhanging dam develops, occupying considerable volume in zones of low-velocity flow. Head losses increase, the upstream level rises and the ice cover may progress a little more upstream.
Frazil produced in a rapid will, in a general way, progressively fill up part of any channel section downstream until it is frozen up or until the production is stopped because of milder weather (Fig. 46). The limiting velocity for frazil deposit under an existing cover depends essentially on its physical aspect and varies greatly from slush made up principally of small disks to slush containing sizable plates. Measurements show a critical velocity of about 3 ft/sec. Other indications are that this velocity may be between 3 and 5 ft/sec.

Frazil deposited under an ice cover is essentially unstable. Figure 47 shows a section through such a deposit in a reservoir where, between successive soundings, the clear water channel had changed place. Furthermore, the ice itself was progressively changing form in the deposit. Samples taken out of this deposit showed small ice plates and particles of granular form more or less welded together, with a mean diameter of about 0.1 in. (2½ mm). There was no doubt that there had been metamorphosis of the deposit where crystals sintered together and took a more spherical shape. This could probably be explained by thermodynamic instability at the ice/water boundary producing a migration of ice molecules from the angles to the convex parts.
**Types of frazil problems**

There are not many water intakes completely immune to frazil problems in northern countries except, of course, those where the water never gets down to the freezing point. Even in deep lakes frazil occasionally clogs water intakes, and a case is known of frazil clogging an intake 60 ft deep in a lake 25 miles long in the tenth year of operation. In extreme cases the reservoir may fill with frazil every winter, stopping all flow to the intake. One hydroelectric plant with this problem is known to have been abandoned. Because of the physical behavior of frazil there are essentially two completely different types of frazil problems:

The first and usual problem is the clogging of water intakes during the period of frazil formation. In a lake frazil is formed under the influence of a cold wind that mixes the upper layers of water in the epilimnion and produces frazil particles in its mass. In a river it is formed practically any place where there is a turbulent flow, at different times. One major aspect of this problem is the fact that dangerous frazil formation is generally of very short duration (of the order of a few minutes), at a given site, so that the frazil-clogging problem occurs for at most a few hours at the beginning of the winter. But there are a few sites in a river, particularly in strong rapid stretches, where the water stays supercooled for an appreciable length of time and active frazil is produced. These sites are obviously more subject to intake clogging during all winter months.

The second and major problem caused by frazil is its accumulation somewhere on the riverbed, if it is produced in sizable quantity. It may pack a river section in front of an intake from top to bottom. There is no solution to this problem, once it has occurred, so it has to be foreseen.

Frazil formation is a basic consideration in the design of river structures in northern countries. Because this problem is related to the complete winter regime of a river it will be fully developed in the next chapter. One solution is to create a reservoir of adequate size to accumulate all frazil slush well before the deposit gets close to the intake and another solution is to divert all ice past the intake with a particular design to allow only the clearer water to enter.

**Remedial action against ice clogging**

It is useful before reviewing the means of fighting the clogging of trash racks and screens of water intakes to consider the basic behavior of frazil in sticking and clogging.

When frazil is being produced (in active form) in supercooled water there is a great potential for growth of the ice particles. Particles coming in contact with others or with foreign materials of favorable crystalline structure group together to form continuous structures. Frazil flocs are produced and frazil particles adhere strongly to materials such as steel, cast iron and copper. During that stage, frazil sticks to the metallic racks of water intakes, various pieces of metal it touches and even objects on the riverbed. Because the duration of the active phase is only a few minutes for the normal condition when the front of frazil production is progressing upstream, water intake racks are subjected to sticky frazil only for a short period of time and growth is limited. Power plant operators have observed that the first particles of frazil are much stickier than succeeding ones.

The second mode of clogging of intake racks follows the first, although it is physically much different. When the front of active frazil has progressed upstream, there comes inactive frazil made up of porous flocs, followed by slush with thin ice plates, thin pans, etc. (Fig. 42). This type of frazil does not adhere to the objects it meets. The mushier forms deposit in separated, low velocity flow areas. The stronger forms block passages by simple mechanical action. The first effect is not really dangerous because it improves the hydrodynamic circuit. The second is more dangerous and its effects are particularly felt on the upper part of the flow where the larger pieces of ice concentrate.
Active frazil may be fought by using certain materials or protective coatings to prevent adhesion. If a frazil run is of short duration and the rack openings are large, there will be no sensible head loss, because of the small amount of deposit. Frazil becomes inactive in a short distance so that the covering of the water surface before an intake ensures no active frazil passage at that point. If a frazil run is of short duration and the rack openings are large, there will be no sensible head loss, because of the small amount of deposit. Frazil becomes inactive in a short distance so that the covering of the water surface before an intake ensures no active frazil passage at that point. Finally, active frazil does not stick to slightly heated surfaces.

Ice pans and plates coming from frazil may be bypassed at an intake by reducing the turbulence and decanting them on the top. Otherwise, frazil slush may easily pass through intakes where openings are big enough. If necessary, the upper racks may be taken out to let it through the intake. Heat is of no use for inactive frazil but mechanical cleaning is often useful for both active and inactive types.

A standard and effective practice at many power stations is the raising of the intake racks or their upper section when frazil comes. As long as the water passages are large there is no problem with the passage of debris that enters the turbine scroll case except in log driving rivers where large logs may travel submerged in the water. There is little trouble with frazil adhering to the runner or wicket gate if it is not a spot of prolonged active frazil formation.

Mechanical rack cleaning can be carried on with success when the frazil run is not too heavy. The scraper usually consists of a frame structure to which is fastened suitably-spaced horizontal beams provided with teeth that mesh with the openings between the screen bars. The frame is driven by a motor to slide up and down the screen. When the frazil formation is too large, mechanical scrapers have been known to seize up entirely.

A cheap way to get rid of frazil problems in areas where they are not too severe is to use screens with large openings (4 in. or more) and to coat the steel bars with a nonadhesive coating. Recent research has shown that plastic resins, polyethylene coatings and silicone grease are effective in preventing the adhesion of active frazil.

One very effective and widely used method of protecting trash racks or any other object against adhering frazil particles, and also against other types of icing, is the use of heat. The important parameter is the amount of heat needed to keep a screen free of ice. This can be computed by standard methods of heat transfer theory. The power required to keep a round bar at a temperature difference $\Delta T$ in flowing water is:

$$E' = 33 \Delta T (Vd)^{0.52}$$  \hspace{1cm} (54)

where:

$E'$ = amount of power required per foot length of bar, watts
$V$ = water velocity past the bar, ft/sec
$\Delta T$ = difference between bar temperature and $32^\circ$F.
$d$ = diameter of the bar, in.

If $s$ is the spacing between bars in inches and the total temperature differential $\Delta T$ to prevent freezing with supercooled water is $0.2^\circ$F, we then find that the power required to prevent freezing is:

$$E = \frac{79 (Vd)^{0.52}}{s} \text{ w/ft}^2 \hspace{1cm} (55)$$
For an approach velocity of 3 ft/sec and a 4-in. spacing between 1-in.-diam rods this gives an energy of 34 w/ft². This compares with 200 w/ft² which has been successfully used. Whatever the form of the structure or object to be protected against icing, the computation can be made similarly.

There are many electrical systems available for heating trash racks. It can be done by connecting metallic bars in series or parallel arrangement to a three-phase low-voltage supply. It can also be done by putting electric wire elements in hollow rack bars.

Other heating methods use hot air, hot water or steam. The warm fluid may be carried inside the piece or it may be directed from the outside to the piece. In one water intake warm water from wells is distributed around the rack structure, through many outlets, when frazil formation is imminent. In a deep reservoir warm water could be brought up from below by use of an air-bubbling system.

All these heating systems are effective as long as heating is started before the ice forms on the intake racks. It is practically impossible to get rid of the ice once there is a strong deposit. That is why a good thermometric detector is essential for predicting the appearance of frazil and starting the heating system in time. Because frazil is formed at night (during off-peak hours) the use of heating power is not usually critical.

The two main problems connected with frazil have not been discussed here. One is the evident inability of an intake to pass large ice floes and the other is the plugging of the intake by a large frazil deposit under an ice cover in front of it. There is no remedy to these problems once they occur so the location and design of an intake must be made with them in mind. This will be dealt with later.
ICE COVER FORMATION

Introduction

Interest in the field of ice formation in rivers and lakes has increased slowly. It was first aroused by the construction of hydraulic works in rivers, mainly hydroelectric power plants where ice problems were encountered and had to be dealt with. The question also arose in transportation, either for winter navigation or for road crossings and aircraft landings on ice. Most of the problems were dealt with directly in the field and some sound engineering rules were derived for the formation of ice covers in rivers, their growth and the characteristics of ice jams.

New interest has been developing in the last two decades, principally because of cities expanding into low areas along rivers, greater use of river and lake water, heavy bridge construction and increased use of ship channels in winter. New approaches have been made to an understanding of the physical phenomena involved in these processes and more particularly of the mechanics of ice cover progression and jam formation. Studies of this type have led to the use of artificial materials to model ice in order to reproduce these phenomena in the laboratory. Unfortunately, the thermal aspects, which are so dominant in ice formation, have not received similar attention so that a universal foundation for the complete understanding of processes involved in ice cover formation is only just beginning to take form.

Predicting river and lake freeze-up

The appearance of first ice on a river or lake, whether it be frazil, ice plates or the first ice skim, is essentially dependent on the thermal exchange between the water surface and the atmosphere, or more simply, on weather conditions.

In the heat budget of a water surface heat is lost through convection, evaporation, radiation and other processes already discussed. It has been shown that during fall the rate of heat loss by evaporation is approximately equal to the heat gain by solar radiation, so that the net heat loss from the water surface can be related directly to the air-water temperature difference by a relation of the type:

\[ \Sigma \psi = H_0 (T_a - T_w) \]  \hspace{1cm} (56)

where

\[ \Sigma \psi = \text{total external heat transfer at the interface, Btu/ft}^2 \text{day} \]
\[ T_a, T_w = \text{air and water temperature, } ^\circ\text{F} \]
\[ H_0 = \text{coefficient of heat transfer from the surface, Btu/ft}^2 \text{day } ^\circ\text{F}. \]

In the cooling process of a lake, discussed earlier, there are essentially two phases. The first is an isothermal process when the water cools down to the uniform temperature of maximum density at 39\(^\circ\)F. The second occurs when the top layer only is cooled down to the freezing point.
In the first phase the temperature is uniform from the surface to the depth $y$ of the thermocline and its evolution can be computed using eq 51:

$$A \int \Sigma \psi \, dt = y_w \, c_p \, \int y \, dA \, dT_w \quad \text{(57)}$$

Defining an equivalent depth of water in a lake $Y_e$ such that:

$$AY_e \int dT_w = \int y \, dA \, dT_w$$

we get:

$$\int \Sigma \psi \, dt = y_w \, c_p \, Y_e \, dT_w \quad \text{(58)}$$

In differential form with eq 56, this gives:

$$H_0 \, (T_a - T_w) \, dt = y_w \, c_p \, Y_e \, dT_w \quad \text{(59)}$$

where:

- $y_w \, c_p$ = specific weight and specific heat of water
- $Y_e$ = equivalent depth of convective mixing, ft.

This is the basic relation for water temperature forecasting in a lake. It is valid only for the first cooling stage in the fall, up to the time when the water temperature drops to 39°F.

The equivalent equation for predicting the appearance of ice in a river is relation 44. The temperature of water is given for a unit volume of water that travels over a distance $(L - L_0)$ along the river axis. It is:

$$dT_w = \frac{1}{y_w c_p} \int_{L_0}^{L} \frac{\Sigma \psi \, dt}{y} \quad \text{(60)}$$

With eq 56, in similar form as eq 59, it gives:

$$H_0 \, (T_a - T_w) \, dt = y_w \, c_p \, Y^* \, dT_w \quad \text{(61)}$$

where $Y^*$ is the average depth of water over the area traveled by the unit volume of water whose temperature has change, $dT_w$. In the case of river flow, where there is good turbulent mixing, this equation is valid down to the freezing point or the appearance of frazil.
The basic equations 59 and 61 can be reduced to a simpler form:

\[ dT_w = K (T_a - T_w) \, dt. \]  

\[ K = \frac{H_0}{\gamma_w c_p Y_e}. \]  

The \( K \) value, in \( \text{sec}^{-1} \), depends essentially on average meteorological data other than air temperature; mainly wind velocity. The value of the depth of convective mixing \( Y_e \) may be quite constant for a given lake as discussed above.

For a lake:

\[ K = \frac{H_0}{\gamma_w c_p Y_e}. \]

For a river:

\[ K = \frac{H_0}{\gamma_w c_p Y_e}. \]

The \( K \) value depends here not only on average weather data but also on the river stages at time of freeze-up.

The investigation of eq 62 can be made by dividing the period between \( t = t_0 \) and \( t = t_n \) into \( n \) equal intervals of time \( \Delta t \). Letting \( T_{a_i} \) represent the mean during the interval from \( t_{i-1} \) to \( t_i \), we then get:

\[ T_{wn} = T_{w0} e^{-K(t_n - t_0)} + \]

\[ + (1 - e^{-K\Delta t}) \sum_{i=1}^{n} T_{a_i} e^{-K(t_n - t_i)}. \]

The water temperatures can be predicted with this relation from the air temperature forecast for the coming \( n \) intervals of time. Various numerical methods and computer programs have been devised to solve this equation.

For a lake, the preceding method is valid up to the time when the water is isothermal at \( 39^\circ \text{F} \). After that stage there is no more isothermal mixing caused by density difference and the surface water only is cooled down to the freezing point. The sensitivity of surface ice formation to variable weather conditions during the final stage of cooling is one reason why freeze-up fluctuates within large limits. A very small layer of water, on the top, may be cooled to its freezing point in calm weather. On the contrary, most of the water of a shallow lake must be cooled to freezing in a windy storm. It was found that from 70 to 100 accumulated degree-days of frost were required for a small freshwater lake to cool from \( 39^\circ \text{F} \) to \( 32^\circ \text{F} \) and have 4 to 6 in. of ice form on it.

The appearance of frazil in rivers can be predicted theoretically with the above method. But the process of ice cover formation is complex and depends much on hydrodynamic conditions. This makes it difficult to predict the time of freeze-up except in stretches of low-velocity flow where the ice cover is formed practically at the time of frazil formation.
There is a completely different method for predicting the time of freeze-up — analyzing the records of the dates of ice appearance. In these records, two dates are usually reported: the date of appearance of the first skim ice on the water surface and the date when the lake is first completely frozen over.

Statistical analysis of these records can be made. Distribution curves are prepared from which the probability of freeze-up occurring between certain dates can be estimated. Maps like the one in Figure 48 can also be prepared showing the "isopleths" of average dates of freeze-up in Canada. A graph showing the chance of freeze-up (first ice) of selected lakes and harbors before a given date is shown in Figure 49.

**Figure 48. Average dates of river and lake freeze-up in Canada.**

<table>
<thead>
<tr>
<th>Lake or bay</th>
<th>Closes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Bear Lake</td>
<td>15 Oct-1 Nov</td>
</tr>
<tr>
<td>Great Slave Lake</td>
<td>15 Oct</td>
</tr>
<tr>
<td>Lake Athabasca</td>
<td>20 Oct</td>
</tr>
<tr>
<td>Hudson Bay</td>
<td>Mid Oct*</td>
</tr>
</tbody>
</table>

*Covered with ice floes by Jan.*
Shore ice formation

Two basic processes contribute to the formation of a continuous ice cover in a river. The first might be called the frazil evolution process which starts with the supercooling of water and the production of frazil crystals, which eventually develop ice pans and floes, and finally form the solid ice cover. The second might be called the shore ice growth process. Although it does not account for the production of large quantities of ice in big rivers it is important because it provides the means for covering up zones of rapid flow. This process is the dominant one in small rivers and brooks. Because this latter process is slow compared to the frazil evolution process it takes a long time to cover the surface of strong rapids unless the winter weather is severe.

Shore ice is the first type of ice to appear in a river in areas of laminar flow along the banks. Because in laminar flow there is no intermixing of the top layer with the bottom layers, the temperature differences are important both in the vertical direction and horizontally away from the banks. The top layer adjacent to the bank goes through considerable undercooling while the average temperature of water in the middle of the river is still far above the freezing point. Ice is nucleated, starting in contact with the colder (because more conductive) material of the banks. This nucleation propagates on the surface toward the middle of the flow, forming a clear and solid ice sheet.

The edge of this ice sheet finally comes in contact with the turbulent water and its further progress depends only on the thermal atmospheric exchange as compared with the temperature and turbulence of the water. By no means does the growth process stop only because the water is above the freezing point.

Let us consider the case where the edge of the shore ice is parallel to the direction of flow (Fig. 50). There is a velocity gradient in a direction x normal to the edge along the top surface layer. The velocity is zero at the boundary and attains a reference value outside the boundary layer. Very close to the edge is a laminar sublayer followed by the turbulent one. Because of the Reynolds similitude of heat transfer to velocity distribution, if the water temperature is above the freezing point there is also a temperature gradient of similar shape from the warm water temperature at the reference point to the freezing point temperature at the ice boundary. The quantity of heat given by the warm water to the boundary is:

\[ \psi_w = K_w \left| \frac{dT_w}{dx} \right|_{x=0} \] (66)
where $K_w$ is the thermal conductivity of water.

Consider now (Fig. 50) the wind velocity profile in air in a direction normal to the ice sheet at the ice boundary. Neglecting the heat extraction from water by radiation or evaporation, the quantity of heat taken out by convection from a thin surface film of water at the boundary is, for similar reasons:

$$
\psi_A = K_A \left. \frac{dT_A}{dy} \right|_{y=0}
$$

(67)

where $K_A$ is the coefficient of thermal conductivity of air.

The ice boundary grows if $\psi_A$ is greater than $\psi_w$. Because of the values of $K_w$ and $K_A$, the temperature gradient at the boundary has to be about 25 times higher in air than in water. But in winter wind velocities and air temperature differences below freezing are often so much greater than water velocities and water temperature above freezing that the border ice will progress in water if the weather is cold enough.
Relation 66 can be represented in another manner by using in the boundary layer an overall heat transfer formula similar to eq 18:

\[ \dot{\psi}_w = C_w V_w \Delta T_w \tag{68} \]

where \( C_w \) is a constant, \( V_w \) and \( \Delta T_w \) are the water velocity and temperature difference above freezing, outside the boundary layer.

It can be understood from this last relation that for a given heat exchange with the atmosphere there is a limiting combination of the water velocity and temperature (of hyperbolic form) that will permit the growth of border ice. From measurements made on Norwegian rivers during periods of intense cold this type of relation takes the form shown in Figure 51.

It can be seen from these results that an ice cover may grow by this process at any water velocity if the temperature of the water is close enough to the freezing point. It is, however, a very slow process than can be somewhat accelerated when the water is carrying frazil flocs. They cling in parallel rows to the growing dendrites at the boundary and form successive clearly marked layers in the solid ice sheet.

Shore ice grows not only from the shore. It forms around emerging boulders or from anchor ice that has grown on a submerged obstacle in high velocity flow. The ice island will take a hydrodynamic shape corresponding probably to an isothermal surface in the flow. Figure 52 shows some aspects of shore ice formation in a river.

![Figure 51. Typical combination of water temperature and surface velocity for the formation of shore ice during period of intense cold.]

---

*Figure 51. Typical combination of water temperature and surface velocity for the formation of shore ice during period of intense cold.*
Figure 52. Shore ice in a river.

The shore ice growth process is dominant in smaller rivers and brooks and accounts for the formation of the ice cover, even in areas of high velocity flow. It is also noticeable on lakes, particularly in calm weather. But because of its slow development in flowing waters, it cannot explain the rapid progression of ice covers in the larger rivers of northern countries.

### Mechanical progression of an ice cover

The frazil evolution process has been described above. It starts with the supercooling of water and the production of frazil crystals. These crystals agglomerate to form frazil flocs which then evolve into frazil slush or ice pans drifting on the river.

The quantity of frazil slush that can be produced in a river stretch at the freezing point can be computed from the heat extracted at the water surface $\psi$ as given by eq 46. The volume of solid ice produced will then be:

$$Q_i = \frac{\sum \psi_i b_i \Delta L_i}{\gamma_i L}$$  \(69\)

where

$Q_i$ = discharge of solid ice, ft³/day

$\psi_i$ = average heat extracted from stretch $i$, Btu/ft² day

$b_i$ = equivalent average width of open water on stretch $i$, ft

$\Delta L_i$ = length of stretch, ft

$L$ = latent heat of fusion of ice, Btu/lb.

$\gamma_i$ = specific weight of ice, lb/ft³.
The correlation between the total quantity of slush measured in three different stretches of the Angara River and the total heat exchange as represented by the main term of the heat budget
\[ S = \sum (32 - T_A) dt \] (cumulative degree-days of frost) is shown in Figure 53. It can be seen that there is a good linear relationship of the form of eq 49 where the constants depend on the regional meteorological conditions and the morphology of the river.

If the ice is drifting over long river stretches, a sizable area of the water surface becomes covered with floating slush or ice pans. In deep sections with low surface velocity, or between bridge piers, the coverage may reach 100% of the surface, and the ice floes will press together and freeze to form a continuous ice bridge. The sectors susceptible to formation of an ice cover from drifting ice can be determined with the continuity equation:

\[ q_i = n_i V_s B = \text{constant} \] (70)

where

- \( q_i \) = surface ice discharge, ft\(^2\)/sec
- \( n_i \) = ice concentration on the surface, tenths (may exceed 1.0 when floes slide over and under each other)
- \( V_s \) = average surface velocity in the section, ft/sec
- \( B \) = effective width of the section, ft.

Figure 54 shows the ice concentration \( n_i \) as a function of time at critical sections on two Dutch rivers. It was shown that the following conditions were favorable for the bridging of a solid ice cover:

1. Ice cover concentration \( n_i \) greater than 1.0 for a period of 5 to 8 hours.
2. Water velocity less than 1.6 ft/sec during the 5 to 8 hours.
3. Average air temperature below 16°F.

Figure 53. Daily production of slush-ice, \( N \), and air degree-days of frost \( S \), on open water in various reaches of the Angara River.
Once an ice cover has started to form it may progress very quickly by simple juxtaposition of incoming frazil slush and ice pans. In larger rivers, as in low-velocity stretches of the St. Lawrence river, the ice pack may advance as much as 25 miles a day.

The equilibrium of an ice floe coming in front of an ice cover can be computed as shown in Figure 55. The downward hydrodynamic force tending to drag the floe under water is given by the classical expression:

\[ F^- = \frac{C_L \rho A V^2}{2} \]  

(71)

where:

- \( C_L \) = coefficient of downward force depending on the form of the floe
- \( A \) = projected horizontal area of the floe
- \( V \) = surface velocity of approach to the floe.

The maximum resisting force to submersion is readily computed from hydrostatics:

\[ F^+ = g (\rho - \rho') (1 - e) A h' \]  

(72)

where:

- \( \rho, \rho' \) = the densities of water and solid ice respectively
- \( g \) = the acceleration of gravity
- \( e \) = the porosity of the ice pan, i.e., the ratio of the volume of voids filled with water to the total volume of the ice pan
- \( h' \) = the average thickness of the floe taking the maximum horizontal section \( A \) as a reference.

From eq 71 and 72, or a similar analysis taking the equilibrium of moments instead of forces, the ice pan will be entrained under the solid ice sheet when:
where $V_s$ is the critical surface velocity and $K_3$ a form coefficient that has been measured in the laboratory and the field.\(^{24,52,100}\) Its value is given in Figure 56.

This formula shows that a frazil floc 6 in. in diameter surmounted by a small ice plate, with $e = 0.8$, will be entrained under an ice cover at a surface velocity close to 0.5 ft/sec and that a big drifting ice pan at freeze-up, 1 ft thick, of solid ice, will become unstable at a velocity close to 3 ft/sec.

It has been shown that this condition of progression of an ice cover is not generally a limiting one.\(^{100}\) The incoming frazil slush and pans may underturn and pile up under the cover until it is able to progress upstream at a determined thickness. This condition of stability can be studied\(^{44}\) with the help of Figure 57. The Bernoulli equation between section 1 and 2 gives:

\[
Y + \frac{V^2}{2g} = y + \frac{V_u^2}{2g} + e x + (1-e) \frac{\rho \cdot h}{\rho}.
\]

The condition of non-submersion of the frontal edge of the cover at $A$ is:

\[
Y + \frac{V^2}{2g} = y + h.
\]

And the continuity equation:

\[
VY = V_u y.
\]

These three relations give the equation of equilibrium of the ice cover:

\[
N_{FR} = \frac{V}{\sqrt{Y}} = \sqrt{2g \frac{(\rho - \rho')}{\rho} (1-e) \frac{h}{Y} [1 - h/Y]},
\]

where $N_{FR}$ is the Froude number in front of the cover and $h/Y$ its relative equilibrium thickness. This equation has been verified in the laboratory\(^{17}\) with polyethylene blocks, as shown in Figure 58.
ICE COVER FORMATION

Figure 56. Values of form coefficient, $K_3^*$, for floes.100

Figure 57. Stability of an ice cover.68

Figure 58. Equilibrium of the frontal edge of an ice cover.77
This equation shows that there is a limiting value of the Froude number (or the flow velocity) over which an ice cover cannot progress. It occurs when the thickness of the ice accumulation is \( h/Y = 0.33 \) and the critical Froude number is obtained from eq 77:

\[
N_{FR} = 0.154 \sqrt{1 - e}
\]  

(78)

It can be seen that the porosity of the ice accumulation plays a major role in the progression of ice covers. If frazil travels over a short distance before reaching the upstream edge of the cover, the floes will have a very high porosity. If the ice pans have traveled and thickened over long distances the porosity will be much smaller. Even at the same river section the drifting ice will have variable characteristics depending upon time and meteorological conditions.

Figure 59 gives this limiting value of the Froude number for many rivers of different types. In spite of the understandable dispersion of the above data because of the ice porosity, Kivisild has proposed a critical Froude number:

\[
N_{FR_c} = 0.08
\]  

(79)

It corresponds to slush ice and pan accumulations of porosity 0.73 for average conditions of ice cover progression. We believe that in difficult conditions of progression with newly formed frazil floes the porosity might be as high as 0.9. The corresponding Froude number is then \( N_{FR_c} = 0.05 \). Table II gives the critical velocity corresponding to these two cases.

![Figure 59. Critical Froude numbers at the head of a pack.](image-url)
Table II. Critical velocity for the progression of an ice cover fed by drifting ice.

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>Velocity (ft/sec) for progression: 6-in. ice pans with slush</th>
<th>Velocity (ft/sec) for newly formed frazil or snow slush</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>average conditions</td>
<td>unfavorable conditions</td>
</tr>
<tr>
<td>40</td>
<td>2.9</td>
<td>1.8</td>
</tr>
<tr>
<td>30</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>20</td>
<td>2.0*</td>
<td>1.3</td>
</tr>
<tr>
<td>10</td>
<td>2.0*</td>
<td>0.9</td>
</tr>
</tbody>
</table>

*Limited by velocity of juxtaposition of floes

When an ice cover is progressing with drifting slush and pans a solid boundary crust develops at the surface between the ice pieces. This solid part of the cover takes most of the hydrodynamic thrust developed by the friction of the water flowing under the cover. Depending on the weather this crust is more or less thick and resistant. In areas of high velocity flow and thick covers, the thrust may, at times, become higher than the resistance of the crust and the cover suddenly breaks at a point of weakness with a loud rustling and forms ice hummocks which locally may rise several feet above the water level. These shoves are much facilitated by periods of thaw alternating with cold spells and by high winds, whose influence is very great, as observed on the St. Lawrence River.

The upstream edge of an ice cover may also progress and have time to consolidate in place at velocities higher than the critical one if the discharge of ice feeding the cover is greater than the discharge which is being pulled under the progressing upstream edge.

When the upstream edge of an ice cover reaches a section of river, such as the foot of rapids, where the velocity is such that it cannot progress further upstream, slush and ice pans are forced to pass underneath the cover and are carried downstream until the velocity of the flow becomes low enough for them to deposit on the underside of the cover. The ice accumulates and forms an underhanging dam, reducing the flow section until the velocity gets high enough to carry it farther downstream. The limiting velocity for the ice pieces to deposit varies widely with the form and type of the ice. For frazil floes it might be very low and for bigger ice pans up to 6 or 7 ft/sec.

Large accumulations are thus formed over great distances in zones of low velocity flow below long rapids. As the underhanging dam lengthens, hydraulic losses get more and more important, the water level is raised upstream, the velocity gets lower and the ice cover can progress a little upstream.

It is interesting to note that nature floods out a rapid or a large river in winter by the long process of frazil accumulation under the ice cover and that it attains the same purpose in a swift brook by forming sills of deposited anchor ice on the bed, of the same origin, damming up the stream.

Winter regime of rivers

The most important factors affecting the ice regime of a river are, in order of importance: the thermal effects, the hydraulic effects, the wind and the morphology of the riverbed.
The effects of the first two factors have been classified by Estifeev according to the simplified scheme shown in Table III. There are four possible combinations of river regimes according to this scheme: v-s, v-S, V-s and V-S.

Table III. Classification of factors affecting winter regime (Estifeev)

<table>
<thead>
<tr>
<th>Thermal factors</th>
<th>HydRAUlic factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Cooling alternates with thawing — mild climate.</td>
<td>Velocity of water lower than critical value to form an ice cover — calm winter flow.</td>
</tr>
<tr>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Continuous cooling of the river — cold climate.</td>
<td>Velocity of water higher than critical — fast winter flow.</td>
</tr>
</tbody>
</table>

Regime v-S. The v-S regime corresponds to low water velocity with continuous cold weather. Estifeev states that the velocity must be less than 1.6 ft/sec and that the air temperature should be less than 14°F with wind on the order of 5 mph. This regime produces a quickly progressing stable ice cover in the river stretch, as shown in Figure 60, where a progression of as much as 15 km (10 miles) per day was observed.

Regime V-S. The V-S combination corresponds to high velocity with continuous cold weather. Because an ice cover cannot form normally on such a river stretch it continuously produces an immense quantity of frazil, and also induces important anchor ice deposits.

Because the presence of a continuous rapid is somewhat theoretical, there are always pools of calm water in such a river stretch, where an ice cover can be primed. This permits the deposition of large quantities of frazil under the cover in the pools and progression of the cover in the rapids. Such an occurrence is shown in Figure 61 where the frazil deposits are considerable, leading to rises in water level of from 14 to 17 ft.

Regime v-s. The v-s regime combines low-velocity flow with alternating freezing and thawing weather. Thin ice covers form at different locations in the river reaches where bridging can start. Frazil and anchor ice are formed during each cold spell, and because the frazil frequently forms in short open areas it is mainly in the form of individual particles in supercooled water, dangerous for clogging. Each thaw brings with it a small-scale breakup where the deposited masses of frazil get loose and the solid ice breaks to form limited jams. A typical regime like this is shown in Figure 62 where these various phenomena can be observed, particularly the water level rises following each thaw.

Regime V-s. With a high velocity and changing weather conditions no ice cover can ever be formed in the reach. This situation produces much less frazil and anchor ice than regime V-S. One main difference might be that frazil forms at precise locations in a permanent way with regime V-S, but the formation front travels all over the reach with the cold weather in regime V-s.

Effect of wind. The primary effect of wind in winter is, of course, to increase heat exchange at the water surface by convection and evaporation. This influence is indeed a major one, as can be verified from formula 46.
In open rivers wind has two mechanical effects: it produces waves and slows down or accelerates the surface velocity of water, depending on its direction relative to the flow. Wind against the direction of the flow decreases the discharge of ice feeding the cover but facilitates its progression, at least as far as reduced surface velocities are concerned. In the other direction, the waves and increased surface velocities prevent the formation of, or destroy, the solid ice edge, thus hampering the formation of a cover.

A strong wind blowing on a river ice sheet develops an important thrust in the cover that may produce shoves and destroy the cover as demonstrated later (p. 118).

The wind effect is particularly important in the formation of an ice sheet on a lake because in the mechanics of the phenomenon it acts as the equivalent of the discharge of a river. The wind sets the depth of the thermocline and the amount of water that has to be cooled to initiate
Figure 61. Ice cloggings in a Siberian River (V-S regime).
ICE COVER FORMATION

Figure 62. v-s regime in a Siberian River.**

ice formation. Currents set up in a lake by the wind control the progression of the ice sheet when drifting ice goes under the shore ice. The same mechanisms are found, and eq 73 and 77, used for river ice formation, can be applied if an equivalent depth of flowing water can be determined.

Morphology of Rivers. The main features of a river have an evident bearing on its winter regime. The effect of the slope on velocity has just been discussed. What is even more important is the relative combination and importance of the regimes v and V along the river course. A v regime followed by a V one is not a source of trouble, but the opposite is. If a stretch with a V regime is long enough, a sizable underhanging dam can be formed. All types of slope and thermal combinations in successive reaches can be discussed with the principles just outlined.

The ice regime of a river depends very much on the early ice bridging points** that develop along its course, at natural or artificial obstacles and river restrictions with deep flow sections. The ice cover forms more readily at higher velocities in the lower part of a reach between two ice bridges because of bigger drifting floes in higher quantity. It is much harder to fill the remaining upper gap, except by shore ice growth, when only small frazil particles are produced.
Features of the river channel, except the particular ones starting the ice cover, do not have the same importance for ice formation as for breakup. In the cases of islands and secondary channels, shore ice quickly covers the low-velocity areas, leaving only the main channel to carry the drifting ice. Bends and tributary junctions are favorable spots for polynyas. In a bend, the spiral flow of water brings the warmer water at the bottom up to the top of the convex side of the curve and the polynya is normally found at the same place year after year (Fig. 63).

A lake is a heat accumulator and would normally freeze later if it is on a river system. Where an ice cover can form on it, a lake is also an ice accumulator, as it can store considerable quantities of frazil that has been produced upstream in the river system. But before a continuous ice cover forms on a lake it may contribute much ice to the river system. The wind may produce frazil in open areas and the waves will break newly formed shore ice that will then drift with the lake currents. The way ice can move between a lake and its river system is a major factor in modes of ice formation.

Figure 63. Polynyas in a Siberian River in two successive winters.

Growth of ice covers

The growth of solid continuous ice in a river or lake, from heat exchange with the atmosphere, is not a simple, straight-forward process.
ICE COVER FORMATION

Let us consider first the vertical static growth of ice in a river, or of an ice sheet in a lake formed from an initial skim, in very calm weather. Transparent black ice then grows in the water. If we further take into account the effect of a snow layer on top of the ice sheet and assuming that the temperature at the snow surface is the temperature of the air, the basic heat transfer equation of ice growth is:

\[ K_i \frac{(32 - T_s)}{h} \, dt = K_s \frac{(T_s - T_a)}{h_s} \, dt = \gamma_i L \, dh \]  

in consistent units, where:
- \( T_a \) is the temperature of the air, °F
- \( T_s \) is the temperature at the snow-ice interface, °F
- \( \gamma_i \) is the specific weight of ice
- \( K_i \) is the coefficient of thermal conductivity of ice
- \( K_s \) is the coefficient of thermal conductivity of snow
- \( L \) is the latent heat of fusion of ice
- \( h \) is the thickness of the ice sheet at time \( t \)
- \( h_s \) is the depth of snow.

Equation 80 gives:

\[ \frac{(32 - T_a)}{(h/K_i) + (h_s/K_s)} \, dt = \gamma_i L \, dh \]

If we define the number of degree-days of frost \( S \) as:

\[ S = \int_{t_0}^{t} (32 - T_a) \, dt \]  

and integrate (80) between limits of \( h_0 \) to \( h \) for a constant snow thickness \( h_s \), we get:

\[ h = \sqrt[3]{h_0 + \frac{K_i}{K_s} h_s} \, h_s}^2 \, \frac{2K_i}{\gamma_i L} \, S - \frac{K_i}{K_s} \, h_s \]  

where \( h_0 \) is the initial thickness of black ice.

Equation 80 has been solved for different cases.\(^*\) Usually when the snow accumulated by successive snowfalls it is solved by finite differences methods.

One particular case is when \( h_s = 0 \); we then get the Stefan\(^{130}\) formula for ideal conditions of black ice growth:

\[ h = \sqrt{\frac{2K_i}{\gamma_i L}} \, S. \]  

With appropriate units in the English system where \( h \) is in inches and \( S \) in °F-days

\[ h = \sqrt[3]{S}. \]
It is obvious that these formulas 82 and 83 will give ice thickness values in excess of what will be found in nature for many reasons. Air bubbles trapped in the ice reduce its thermal conductivity. Solar radiations will be absorbed in the ice sheet and they are not included in the heat budget. Finally a boundary layer at the ice surface makes the temperature of the surface ice one or two degrees Fahrenheit higher than the air temperatures. 35

Let us now consider the growth processes of snow ice on an ice cover (Fig. 64). It is very much different from black ice growth. If the weight of a snowfall is sufficient it will drown the solid ice and water will move through cracks (thermal or other) to flood the ice up to a condition of equilibrium given by:

$$ h_w = \frac{(y - y_s) h}{y_s} + \frac{(y - y_i)(1 - e)}{y_s} (h_s - h_w) \tag{84} $$

where $y$ and $y_s$ are the specific weights of water and snow, respectively.

The second term is negligible before the first one and with the usual values 0.9 for the density of ice and 0.2 for newly wind driven deposited snow, this shows that the water level will adjust itself to a distance from the top of very nearly half the ice thickness. It also means that a very small amount of deposited snow is sufficient to stop the growth of black ice underneath; snow about half the thickness of the black ice sheet suffices. A first snowstorm usually floods the ice of a river or a small lake and a new type of ice called snow ice begins to form in the water-saturated snow layer.

Equation 82 can also be used for different boundary conditions. For snow ice growth in a saturated snow layer as shown in Fig. 64, it gives:

$$ h^* = \sqrt{\left(\frac{K_i}{K_s} h_w\right)^2 + \frac{2K_i}{y_i L} h_s - \left(\frac{K_i}{K_s} h_w\right)} \tag{85} $$

where $h^*$ is the thickness of snow ice only, formed under constant snow submergence $h_w$.

One other interesting limiting condition is when only snow ice forms. This would give the lower limiting thickness of ice that may occur in nature when water is completely free to move through a fractured ice cover. This happens when:

$$ h_0 \to 0. $$

It was shown above that we may reasonably let:

$$ h_s = \frac{h}{2} $$

and we get:

$$ h_{\text{min}}^* = \sqrt{\frac{1}{1 + \frac{K_i}{2K_s}}} \cdot h \tag{86} $$

where $h_{\text{min}}^*$ is the minimum thickness of snow ice and $h$ is the thickness of black ice that would have been produced under the same meteorological conditions without a snow cover according to the Stefan formula.
Figure 65. Curves of ice thickness and degree-days of frost.

Figure 65 shows different cases of ice cover growth. The curve showing the maximum thickness is the Stefan law for black-ice formation. The curve of minimum thickness is that of only snow-ice formation when the fractured cover can be immediately flooded. Curve A shows the growth of snow-ice after a 12-in. snowfall on a fractured 6-in. cover of black ice. Curve B shows the growth by accretion of black ice under the same conditions but when the original 6-in. cover cannot be flooded because of restraint on the sides.

In the case of an ice cover formed on a river by the accretion of frazil slush and pans the continuous ice will grow through these before the first snowstorm, and form an ice agglomerate. Snow will lie more easily on top of the rough surface of the cover. In a river the water discharge decreases considerably during the winter and the cover goes down with it and usually breaks along the shore. When there is an important frazil deposit underneath, the cover may stay on top of it and the frazil accumulation will refreeze together as very porous ice. Typical sections through ice covers in rivers are shown in Figure 66. In northerly rivers that become practically dry in winter an ice bridge, empty underneath, may be formed. Figure 67 shows an ice cover formation in a northern country.

Because of the practically impossible task of predicting when snow-ice will begin to form in a river or lake or when to use eq 82, 83 or 85, the Stefan formula has been used with a numerical coefficient (which must theoretically always be smaller than one) to take into account the other meteorological conditions. The final engineering formula is then:

$$ Z = a \sqrt{S} $$  \hspace{1cm} (87)

where $Z$ is the thickness of the total solid part of ice in inches, $S$ is the degree-days of frost in °F-days. The values of the coefficient $a$ which are derived from experience are:

<table>
<thead>
<tr>
<th>Description</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Windy lakes with no snow</td>
<td>0.8</td>
</tr>
<tr>
<td>Average lake with snow</td>
<td>0.5-0.7</td>
</tr>
<tr>
<td>Average river with snow</td>
<td>0.4-0.5</td>
</tr>
<tr>
<td>Sheltered small river with rapid flow</td>
<td>0.2-0.4</td>
</tr>
</tbody>
</table>
The prediction of probable ice thicknesses in rivers and lakes for various intervals of recurrence can be made from statistical study of the cumulative degree-days of frost at a specific area and formula 87.

**Backwater curves underneath ice covers**

Backwater curves underneath ice covers could be computed by dividing the river into short stretches and applying to each a formula for uniform flow. Working from section to section a backwater curve can be traced in the same way as with the standard step method in free surface flow.

It is difficult to predict the stages of a river in winter, because it is practically impossible to determine beforehand the thickness of the cover, taking into account the accumulation of frazil and loose ice under the solid part of the cover, and because it is difficult to assess the value of the roughness coefficient at the boundary of the accumulation.

Many formulas have been advanced to compute the head losses under an ice cover. They all start with the basic Chezy-Manning equation:

$$V_0 = \frac{1.49}{n_0} \frac{r^{2/3}}{s^{1/2}}$$

(88)
where:

\[ V_0 = \text{average velocity of the flow under the ice cover, ft/sec} \]
\[ r = \text{hydraulic radius, ft} \]
\[ s = \text{slope of the energy gradient} \]
\[ n_0 = \text{Manning's roughness coefficient of overall flow}. \]

Difficulty in using this formula arises when it comes to determining the value of \( n_0 \) and \( r \), taking into account the different roughnesses of the river bottom and the ice cover. Many formulas have been proposed to do that.\(^{127}\) The one with the soundest theoretical basis\(^{61}\) was obtained by Torok-Sabaneev,\(^{129}\) which gives for a river:

\[
\begin{align*}
  n_0 &= \left( \frac{n_1^{3/2} + n_2^{3/2}}{2} \right)^{2/3} \\
  r &= \frac{Y_0}{2}
\end{align*}
\]

where:

\[ n_1 = \text{Manning's roughness coefficient of the ice cover alone} \]
\[ n_2 = \text{Manning's roughness coefficient of the bed with free surface flow} \]
\[ Y_0 = \text{water depth up to line of buoyancy in the ice cover, ft} \]

Recently a complete compilation of hydrologic data was made in the U.S.S.R.\(^{98}\) for the period 1936-1959 to determine the value of the roughness coefficient \( n_1 \) under an ice cover.

About 500 measurements were made on smooth ice covers, formed by thermal exchange with the atmosphere, with no loose ice accumulation underneath. The values of \( n_1 \) were very consistent; they were:

\[ 0.010 < n_1 < 0.012 \text{ (beginning of freeze-up)} \]
\[ 0.008 < n_1 < 0.010 \text{ (middle of winter)}. \]

The roughness of a smooth ice cover varies during the winter. At the beginning of the winter there is usually some small frazil deposit underneath that is gradually smoothed out so the roughness coefficient decreases. Then later on ripples and dunes are formed on the underside of the solid ice sheet that may increase the coefficient to 0.014 in certain cases.

In the above-mentioned compilation there were 436 measurements for ice covers with loose ice accumulations underneath. They were classified into three types according to the type and depth of the accumulation:

1. Frazil slush
2. Frazil slush with ice pans; compact slush
3. Ice floes.
It was found that just after the formation of the ice accumulations the values of $n_1$ were, on the average (standard deviation 14%), those given in Table IV.

### Table IV. Values of $n_1$ for ice accumulations at freeze-up time.

<table>
<thead>
<tr>
<th>Initial thickness of the accumulation of the ice (ft)</th>
<th>Type of accumulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.01 0.013 0.015</td>
</tr>
<tr>
<td>1.0</td>
<td>0.01 0.02 0.04</td>
</tr>
<tr>
<td>1.6</td>
<td>0.02 0.03 0.06</td>
</tr>
<tr>
<td>2.2</td>
<td>0.03 0.04 0.07</td>
</tr>
<tr>
<td>3.2</td>
<td>0.04 0.06 0.08</td>
</tr>
<tr>
<td>4.8</td>
<td>0.04 0.07 0.09</td>
</tr>
<tr>
<td>6.4</td>
<td>0.05 0.08 0.10</td>
</tr>
<tr>
<td>9.7</td>
<td>0.06 0.09</td>
</tr>
<tr>
<td>16.0</td>
<td></td>
</tr>
</tbody>
</table>

It was further found that the coefficient of roughness of the bottom surface of the slush-ice cover gradually diminished during the winter period in nearly all cases. It is significant that on some rivers the diminution happened rapidly and on others slowly. Moreover on the same river, the coefficient dropped steeply in one year and gently in another. Thus, for example, on the Dniester River in the winters of 1950-51 and 1957-58 it dropped from an initial value of 0.07 to 0.08 to a final value of 0.008 to 0.012 in only 15 to 25 days. The decreases of $n_1$ are determined, to some extent, by the properties of the ice material from which the accumulation was formed. When the cover is formed mainly from ice floes, it decreases more slowly than when it is formed from loose frazil floes. However, the meteorological conditions play the principal role. Thus, let us assume that for some reason there are many open stretches on the river. If the winter is mild, with many thaws, then the water has time to warm up somewhat during the thaw, frazil ice melts, the roughness is smoothed out, and as a result the coefficient $n_1$ is rapidly reduced. On the contrary if the winter is severe, then in the presence of open water stretches the water is cooled, more frazil ice is produced, or the existing cover is not melted so quickly; as a result the coefficient falls slowly. If there are no open stretches the meteorological conditions have a smaller effect on the changes of the coefficient with time.

It is interesting to note that there were two measurements available for ice jams in spring, when it was found that the coefficient $n_1$, taking into account the thickness of the accumulations, was smaller than for the freeze-up accumulations formed with ice floes. Apparently this might be because in the spring the ice floes are smoother than in the fall; their edges are not as sharp and there is snow slush in the interstices. In the first approximation a value of $n_1$ given for ice accumulation of type 2 was recommended.
**BREAKUP**

**Introduction**

Ice breakup on rivers usually happens very quickly, mainly at night, and it is practically impossible to make measurements on the fast-moving water and ice. Because of this it is among the least known natural ice phenomena.

Breakup is characterized by the formation of ice jams, and numerous icy floods have been recorded. Some of these were severe; for example, the Yukon River rose 65 ft in the spring of 1930 to flood the village of Ruby. Water-level rises of more than 20 ft are common when an ice jam is formed.

What is more difficult to visualize, in a river breakup, is that two processes are happening at completely different rates. One is the slow process of weakening and fracturing of the ice cover along the river course and the other is the general movement of the ice cover breaking up into ice floes in a short period of time from the source to the mouth of the river.

The equivalent processes in a lake are the gradual melting and weakening of the ice cover and the sudden breaking up, movement and accumulation of the ice remnants on the shore, caused by the wind.

**River breakup**

The breakup of a river has been well described. It is divided into three phases, although one or two of them may not fully occur in particular cases. These phases are the pre-breakup period, the drive, and the wash.

**Pre-breakup period.** This period begins with the start of runoff on the watershed when solar radiation begins to melt the snow cover, even before the average daily air temperature has exceeded 32°F. The discharge of the river starts to increase, putting the ice cover under uplift pressure. With increasing discharge, the ice cover fractures at various points. In the case of a reach of low-velocity flow, this break occurs along both shores. The central part of the cover floats freely but the water floods the remaining bands of shore ice (Fig. 68). In areas of higher flow velocity, the ice cover, or parts of it, is usually attached to numerous boulders in the riverbed. The water goes up and floods most of the ice cover through numerous checker-patterned uplift cracks. Water then starts to flow on the ice itself, the snow cover quickly melts and a few pieces of fractured ice may succeed in detaching themselves and moving downstream on the remaining ice cover.

As the discharge increases, and principally because of repeated small daily flood waves, the ice pieces detach themselves in the sections of rapids and accumulate at the upstream end of the stronger floating ice sheet of the low-velocity reaches. In tributaries, the daily flood wave is more important than in the main river, and the ice moves down earlier to form accumulations in front of the solid parts of the ice cover of the main river course. These first smaller accumulations, with their supporting ice covers, may be called ice-reaches. A typical ice condition at the end of the pre-breakup period is shown in Figure 69a.
Figure 68. Water, shore ice and rapids.

Figure 69. Typical breakup on a river stretch.

The drive. The causative mechanism of these first ice-reaches is essentially a mechanical process of destruction by the action of hydrodynamic forces acting on the ice cover. At the end of this first phase, the ice cover in zones of low-velocity flow is still amply strong and well preserved from melting by solar radiations because of the snow cover or the snow-ice surfaces on top of it.

The next movement of the ice cover depends on the possible combinations of river discharge and strength of the ice sheet. Let us first suppose, as an extreme case, that there is little snow
on the watershed, that it melts slowly and that there is no precipitation during the melt period. The river discharge will not increase sensibly, so it will not develop strong forces on the solid ice sheet of an ice-reach and the ice will slowly disintegrate in place until it becomes weak enough to be gently pushed through by the water. No appreciable drive will occur.

But let us now look at the opposite case, much closer to what usually occurs at breakup. There is rain or intensive melt on the watershed, the discharge increases considerably and the tangential friction forces of water under the ice-reach become high enough for it to become unstable. The frontal accumulation breaks through the ice sheet and one complete reach is destroyed and moves downstream (Fig. 69 b).

This moving ice pack then gets to the following ice-reach downstream, which may not have moved at the same time. The ice accumulates in front of this stable reach and a jam of unconsolidated ice floes forms at its upstream end. Depending on the site and the discharge conditions, this jam may be stable enough to stay in place. Ice jams may form at various sites along the river with increasing discharge, each jam being stopped by a more resistant jam priming site, in a lower reach.

Finally, with ever-increasing discharge and the ice cover weakening by thermal effects, one of the bigger jams gives way and its impact carries all others along its course, freeing the river of ice in a matter of hours (Fig. 69 c). This general ice movement makes a grinding noise that can be heard well in advance of its coming. The ice accumulation moving down pushes on the parts of the ice cover that are still solid and breaks them into very large floes, which are rammed on the banks with a tremendous force to form heaps of ice. Some of these floes are projected into the air, then fall back and break. As they move downstream the floes are continually rotated, overridden, underridden, turned over, and broken up by impact with adjacent ones. Between the floes are smaller blocks and accumulations of remaining snow or frazil slush.

Because most river breakups occur when there is still a strong ice cover, the determining factor is usually the river discharge. The advent of rain while the runoff is rapid because of frozen ground is a major factor affecting a general ice breakup.

The wash. After the breakup, vertical walls of ice of smooth, masonry-like form are left along the shore (Fig. 70). They attest to the shearing action of the ice pack and the maximum levels attained by the ice. Experience shows that, after the drive, the spring flood comes and cleans the ice floes left over on the banks and bordering lowlands.

Figure 70. Sheared ice walls.
Lake breakup

The melting and breakup of lake ice has been well studied and we will repeat here the accurate description of them by G.P. Williams.\textsuperscript{153}

The ice first starts to melt on the shore because it is thinner there and because more heat comes from the adjoining ground surface. A free water surface appears along the shoreline leaving the main body of ice floating free. At that time the ice cover still has considerable strength.

Williams\textsuperscript{153} states: "During the second stage of melting there is melt of the snow and of the snow ice on the floating ice. The melt water flows along drainage patterns on the surface and it drains to the open water at the shore-line or to holes that appear to develop preferentially along old thermal cracks. When the melt water drains away, the surface has a porous, white and crumbly structure which reflects solar radiation and retards internal melting. As the melt season progresses some melt water accumulates beneath the ice surface. A typical ice-cover profile will then consist of a shallow, porous surface layer 2 to 3 inches thick, a layer of water-logged ice several inches thick and then solid unmelted ice extending to the water.

"In the final stages, the underlying entrapped water layers result in darkened surface ice and most of the incoming solar radiation is absorbed."

Figure 71 shows large patches of this darkened ice. "When the ice cover reaches this advanced stage of melt it is ripe for breaking by wind and currents and is unsafe for over-ice transportation. The currents created by strong winds will break up the ice cover and induce circulation that brings the warmer subsurface water to the ice. This can cause rapid weakening and melting and indeed, the final disappearance of ice covers has occurred so quickly at times that early observers believed the ice actually sank."
Forecasting

From the description of the breakup that has just been given it is clear that no general method can predict these phenomena because, contrary to freeze-up, they depend only partly on the heat exchange with the atmosphere.

To illustrate the fact that the mechanical breaking of ice in rivers and lakes by wind and currents is sometimes more important than its melting, the breakup records for several lakes, rivers and salt water harbors in Canada were analyzed by Williams\textsuperscript{150} (Fig. 72): "The number of days from the time the ice thickness was last measured, in the spring, to the time it was completely cleared was related to the total ice thickness at the time of the last measurement. The comparison shows the great variation in the rate at which ice disappears from different bodies of water. The lower limit, where the ice melt is due entirely to the heat from the atmosphere, is about 1 in./day. This is less than one tenth the upper limit where the ice is cleared from a river or harbour largely by wind and current effects."

Except for sheltered lakes where atmospheric heat exchanges are dominant\textsuperscript{150} the prediction of breakup in rivers and lakes may be attempted only with a multiple correlation analysis of precipitation, wind and heat exchange.

As for freeze-up, however, it is also possible to determine the average date of breakup for various regions and prepare a map of isopleths of breakup dates. This was done for Canada\textsuperscript{150} (Fig. 72). Because there is a considerable difference in time, when the comparison is made between rivers and lakes in the same area, it was found more convenient to take only the date of a break in ice along the shore in the case of a lake. Indeed this depends very much on heat exchange conditions only.

![Figure 72. Number of days for ice to disappear and ice thickness at time of last measurement.\textsuperscript{150}](image-url)
Factors affecting ice jams

There are essentially two types of ice jam\textsuperscript{29} that can form on a river stretch: the simple ice jam and the dry ice jam (Fig. 74). The simple ice jam is caused by the regular accumulation of ice floes in front of a solid ice cover. It is of uniform shape and the water flows freely underneath the accumulation. It is stable in a static manner, and produces a regular increase in water level along its length. It is destroyed by an increase in river discharge or by the impact of further oncoming ice floes. This type of ice jam occurs frequently. It is accessible to computation and prediction.
A dry ice jam is formed by the jamming of ice floes at an obstacle which may be an existing ice accumulation or bed irregularity. The ice completely blocks the whole flow section down to the river bottom. The water has to flow by infiltration through the ice plug and its level increases rapidly upstream. The jam is essentially unstable and it will go out when the upstream water level increases sufficiently. It is practically unpredictable.

The main factors affecting the formation of ice jams are the previous winter conditions, the water discharge at breakup and the relative position of rapids, fluvial transitions or other hydraulic features.

Winter ice conditions. Winter conditions have an important influence on breakup. Ice formation may happen in such a way that an important quantity of frazil will be formed and deposited under the cover, hindering the passage of ice at breakup. Even worse is the case of a winter breakup when partial ice jams are formed and refrozen in place before the real breakup occurs. In a long, very cold winter with little snowfall the ice is thick and dangerous at breakup time.

Precocity, intensity and duration of spring flood. An early, large spring flood will cause the highest jams in a river because the ice is still very strong. A late flood will more easily move the decayed ice sheet.

One factor that causes large spring floods is early rainfall when there is a quick runoff on frozen ground. The operation of dams during that period may also affect ice jams.

The orientation of a large river is important. In a river flowing from south to north, the flood coming from melting in the southern part of the watershed will get to the northern area where the ice cover is still well preserved. This favors extensive ice jamming.

The torrential-fluvial transition. The hydraulic feature most likely to give rise to an ice jam is the upstream end of a fluvial flow reach at the foot of a rapid. At such a section the downstream ice cover is usually strong, with shore leads only, and resistant to ice thrust. Furthermore, there is often an underwater frazil accumulation increasing the resistance of the cover. The broken-up ice floes can accumulate to considerable length and depth in the low velocity area until the head created by the water flow is adequate to shear through and move the pack ice. The importance of
a jam that can be formed at such a site depends on the discharge and the total quantity of ice that can be fed to that point. Such a location is much more resistant to a jam if the channel is also crooked.

Hydraulic singularities. Ice jams are also caused by islands, shoals, narrows, sharp bends, bridge piers or abutments. One very likely place that usually forces the ice pack to become grounded and form a dry ice jam is a shallow widening section forming a rocky ridge at the end of a small river pool.

Stability of an idealized ice jam

Let us consider an unconsolidated ice accumulation of constant thickness \( h \) in a rectangular channel of width \( B \) with uniform flow (Fig. 75).

The flow and the wind exert a hydrodynamic force on the frontal edge of the cover of \( p_0 \) per unit width, at a short distance from this edge. They also produce a tangential force \( r \) per unit area in the direction of the flow.

The problem of the distribution of the stresses in such an ice accumulation is then similar to the one of a two-dimensional grain elevator with a top load. Both materials are granular where the unit force \( r \) takes the place of the weight of the grain. The stress distribution in a grain elevator has been studied for a long time. A theory that verifies well the measurements is that of Caquot, which assumes that part of the total thrust is transmitted to the edges by an arching action of the material. Because of the constant loading, this system consists of parabolic arches that correspond to the directions of the principal stresses in the material.

If we consider an element of this system limited by axial planes (Fig. 75), one of the principal stresses is \( \sigma_{n1} \), which is a constant all along the arch, and the other is the normal stress \( \sigma_y \) in the \( y \) direction. From considerations of the conjugate stress relationships at equilibrium in granular media we have for "filling" conditions:

\[
\sigma_{n2} = K_\alpha \sigma_x
\]

where \( K_\alpha = \tan^2 \left( 45^\circ - \alpha/2 \right) \) is the coefficient of active thrust for an element of obliquity \( \alpha \). The limiting condition of equilibrium will be attained for the highest value of \( \alpha \) at the wall, where \( \alpha = \psi \), \( \psi \) being the angle of friction of the material with the wall. We then have:

\[
\sigma_t = \sigma_{n1} \cos \psi K_\psi.
\]

It is now possible to write the equation of equilibrium along the \( x \) axis for an arch element of the system:

\[
\tau B \, dx - d\sigma_{n1} B h - 2 \sigma_t \sin \psi \, h dx = 0.
\]

With eq 2, this gives

\[
\frac{d\sigma_{n1}}{dx} + \frac{\sin 2\psi \, K_\psi}{B} \sigma_{n1} - \frac{\tau}{h} = 0.
\]
Figure 75. General stress distribution in an unconsolidated ice cover.

whose solution is, with $\sigma_{n1} h = p_0$ for $x = 0$:

$$T' = \sigma_{n1} B h = p_0 B e^{-\xi \frac{x}{B}} + \frac{\tau B^2}{\xi} \left[ 1 - e^{-\xi \frac{x}{B}} \right]$$  \hspace{1cm} (94)

where $T'$ is the total thrust exerted on an arch at a distance $x$ from the edge, and:

$$\xi = \sin 2\psi \tan^2 \left( 45^\circ - \frac{\psi}{2} \right)$$

There is an important practical advantage in the application of this method. If the angle of friction of ice on the shores varies from $15^\circ$ to $30^\circ$ the coefficient $\xi$ varies very little. By taking a value:

$$\xi = 0.3$$

the error is less than 4% in that range. This is very useful in the case of ice floes where this angle has not been measured but it most probably lies in that range, as for many other granular materials.

With these numerical values we can now write the final formulas:

$$T = T_\infty \left[ 1 - a e^{-0.3x/B} \right]$$

$$T_\infty = 3.3 \frac{\tau B^2}{\rho_0}$$

$$a = 1 - \frac{p_0}{3.3 \frac{\tau B}{\rho_0}}$$  \hspace{1cm} (95)
Computation of the hydrodynamic force on the frontal edge, $p_0$. Let us consider the frontal edge of an ice cover where the ice pieces have taken a hydrodynamic configuration and where we neglect the friction forces, which are taken into account in other terms of the formula. We then have the conditions shown in Figure 76. At a certain distance from the edge, normal buoyancy of the cover is attained. Considering hydrostatic pressure distribution in sections (1) and (2):

From energy conservation:

$$ Y + \frac{V^2}{2g} = Y_1 + \frac{V_0^2}{2g} \quad (96) $$

Because of continuity:

$$ V Y = V_0 Y_0 \quad (97) $$

The momentum theorem gives, between sections (1) and (2):

$$ p_0 = \frac{\gamma Y^2}{2} - \frac{\gamma Y_1^2}{2} + \rho Y V^2 - \rho Y V V_0 \quad (98) $$

And finally, neglecting terms of second order of smallness:

$$ p_0 = \gamma Y \left(1 - \frac{Y_0}{Y}\right)^2 \frac{V_0^2}{2g} \quad (99) $$

Computation of the tangential force $\tau$ on the cover. The total tangential force on the cover is made up of the component of the weight of the cover $\tau_g$ and of the water friction force $\tau_w$ under the cover.

Weight of the cover: The component of the weight of the cover in the direction of the flow is (Fig. 77):

$$ \tau_g = \gamma (1 - e) \rho_0 s_0 \quad (100) $$

where:

$$ s_0 = \sin i $$

$$ e = \text{porosity of the accumulation} $$

Water friction force: The boundary stress distribution can be obtained by the Torok-Sabaneev hypothesis (Fig. 77):

$$ \tau_w = \gamma Y_0 s_0 $$

$$ \tau_{w1} = \gamma Y_1 s_0 $$

$$ \tau_{w2} = \gamma Y_2 s_0 $$

$$ \tau_w = \tau_{w1} + \tau_{w2} \quad (101) $$
where \( r_w \) is the total friction force of the flow, \( r_{w1} \) under the cover and \( r_{w2} \) on the bottom.

With the use of the Chezy-Manning equation:

\[
\begin{align*}
T_w &= \frac{V_0^2 n_0^2}{2.22 (Y_0/2)^{4/3}} = \frac{V_0^2 n_1^{4/3}}{2.22 Y_1^{4/3}} = \frac{V_0^2 n_2^{4/3}}{2.22 Y_2^{4/3}} \\
&= \frac{V_0^2 n_0^2}{2.22 (Y_0/2)^{4/3}} \\
&= \frac{V_0^2 n_1^{4/3}}{2.22 Y_1^{4/3}} = \frac{V_0^2 n_2^{4/3}}{2.22 Y_2^{4/3}}
\end{align*}
\]

we then have:

\[
2 (n_0)^3 = n_1^{3/2} + n_2^{3/2}
\]

\[
Y_1 = \frac{Y_0 n_1^{3/2}}{2 n_0^{3/2}}
\]

\[
Y_2 = \frac{Y_0 n_2^{3/2}}{2 n_0^{3/2}}
\]

The friction force caused by the flow under the cover is then:

\[
T_{w1} = \frac{\gamma Y_0 s_0 n_1^{3/2}}{2 n_0^{3/2}} = \frac{\gamma V_0^2 n_1^{3/2} n_0^{1/2}}{1.76 Y_0^{1/2}}
\]

Some experimental verifications. Extensive investigations were carried out by Kennedy\textsuperscript{44} to determine the force acting on a boom by a log jam. Starting with the Janssen theory developed for grain elevators he obtained the final formula:

\[
T = T_\infty (1 - e^{-0.4 L/B})
\]
where $T\infty = 2.5 rB^2$.

This formula is exactly similar to the one derived here except for the coefficient 0.4 instead of 0.3 and the fact that the initial thrust is not taken into account. Tests carried out in flumes with model logs showed that for the damping coefficient $\alpha$, 0.3 is better than 0.4, as shown in Figure 78. Because of model-length limitations Kennedy was not able to arrive at $T\infty$ and this value could not be checked.

Delagrave made experimental studies in a 6-ft-wide flume with polyethylene pieces simulating ice floes. These tests were made for thick covers ($h/Y$ in the range of 0.2 to 0.3). It was found that the value of $T_0$ was important here and that eq 95 represented the phenomenon fairly well in all cases (Fig. 79).

Some investigations were made in a small channel, 1.6 m wide, in the U.S.S.R. in 1946. The ice accumulation consisted of a single layer of ice floes and it was found that the force attained a maximum when $L \approx (2.5$ to $3.0) B$. For practical purposes the maximum force was given by the empirical relation:

$$T\infty = 1.72 rB^2 + p_0B \quad (106)$$

It can be suspected that the value of $p_0 B$ was also very high in those tests as the ratio $h/Y$ varied from 0.15 to 0.22. Moreover, the hydrodynamic thrust on the edge is higher for a single floe than for an accumulation of ice pieces of the same thickness. Equation 106 checks with eq 95 if $p_0 B$ is taken equal to $1.88 rB^2$. In that case the results of the testing and the theory give a very good fit as shown in Figure 80, if we take a limiting value of 0.8 $T\infty$ to measure the very slowly increasing force starting from a length of $3L/B$.

![Figure 78](image-url)
Figure 79. Thrust measured on a scale model with simulated ice.28

Figure 80. Results of tests with ice floes.44
Stability of a jam. There is a physical limit to the thickening of an ice cover under the effect of the hydraulic forces. If the discharge is slowly raised under an ice accumulation the cover thickens by shoves until the increase in the hydraulic thrust because of the reduction of the flow area gets higher than the resistance of the cover to this thrust.

Let us consider a section through the cover normal to an imaginary arch (Fig. 81). The resistance of the cover developed under the acting principal stress $\sigma_{n1}$ is:

$$p_1 = \frac{1}{K\phi} (h - h_0) \gamma' (1 - e) =$$

$$= \frac{(y - \gamma') (1 - e) h_0}{K\phi}$$

$$\sigma_{n1} h = \frac{p_1 h_0 \gamma'}{2 \gamma'}$$

The reaction of the shore is given by eq 91:

$$R = \sigma_r h = \sigma_{n1} \cos \psi \quad K_\psi h = \frac{1}{2} \cos \psi \frac{K_\psi}{K_\phi} \frac{(y - \gamma') \gamma' (1 - e) h_0^2}{\gamma'}$$

$K_\psi$ and $K_\phi$ being respectively the coefficient of active resistance of the accumulation to the thrust at the shore and inside the accumulation.

Because of the stress distribution in the cover (Fig. 75) the reaction $R$ at infinity is given by:

$$\tau B = 2R \sin \psi$$

In the case of hydraulic thrust only, the value of $\tau$ is given by eq 103 and 107:

$$\tau = 1.14 \gamma \left[ (1 - e) h_0 + \frac{1}{2} \left( \frac{n_1}{n_0} \right)^{\gamma_2} Y_0 \right] \frac{Q^2 n_0^2}{B^2 Y_0^{\gamma_2}}.$$ 

If we write:

$$\zeta = \sqrt{\frac{2.28 \gamma' n_0^2 K_\phi}{\sin 2\psi K_\psi (1 - e) (y - \gamma') Y_0^{\gamma_2}}}$$

the conditions of equilibrium of the cover with eq 107, 108, 109 and 110 become:

$$\left[ \frac{(1 - e) h_0 + \frac{1}{2} (n_1/n_0)^{\gamma_2} Y_0}{h_0^2 Y_0^{3/2}} \right] = \frac{B}{\zeta^2 Q^2}$$
This can be simplified with:

\[ Y' = Y_0 + \frac{h_0}{\beta} \]

\[ \beta = \frac{1}{2} \left( \frac{n_1/n_0}{\frac{1}{2}} \right) \]

\[ h' = \frac{h_0}{Y'} \]

\[ y' = \frac{(Y')^{2}B}{e^2Q^2} \]

and we get:

\[ y' = \frac{\beta + (1 - e - \beta)h'}{(1 - h')^3(h')^2} \]

This function has a minimum that can be obtained by differentiation. Because \((1 - e - \beta)h'\) is always smaller than \(\beta\), this minimum is given very closely by:

\[ h' = \frac{h_0}{Y'} \approx 0.4, \quad y' = 31 \beta \]

These expressions represent the limit of stability of an ice jam. In the case where most of the roughness comes from underneath the ice jam, \(\beta = 1\), and the condition of limit equilibrium of the jam is:

\[ \frac{Y'\sqrt{B}}{\sqrt{e}Q} = 2.36 \]

This equation of equilibrium of an ice accumulation (Eq 113) is shown in Figure 82 and the results of tests carried on with polyethylene blocks of density 0.92, porosity 0.4 and \(\phi = 30^\circ\) are shown on the same graph. It can be seen that the general criterion for the stability of a jam is then given by eq 115.

The limiting tangential stress for this condition of stability is then:

\[ T_{\infty} \approx (Y')^2B \]

And the maximum discharge before shoving is:

\[ Q_{\text{max}} = \frac{0.037}{n} (Y')^{2.16} B^{0.5} \]

where:

\[ n = \sqrt{n_1^{\frac{1}{2}} n_0^{\frac{1}{2}}} \]
The analysis of the stability is valid only for an idealized jam in a straight channel. There is a considerable difference in conditions for stability in nonprismatic channels. In a complex natural situation only a scale model would show these critical conditions.

**The dry ice jam**

When the hydraulic conditions are such that a simple ice jam cannot be maintained, or when the discharge becomes higher than that given by eq 117, a static ice jam will crush. With favorable riverbed irregularities, the flow section might become completely blocked, forming a dry jam.

Observations have shown that the form of the waterline in a dry ice jam differs considerably from that in a simple jam. It has a parabolic form along the whole length of ice grounding and the head losses are considerable compared to those of a simple jam. It has been impossible to determine these head losses in a general manner because of the seemingly fortuitous length of grounding in each case and the variable solidity of the accumulation of floes.

The conditions favorable to the priming of a dry ice jam have been studied and the results are shown in Figure 83. It was found that the critical factor was the ratio of the vertical depth \( Y_B \) underneath the rigid ice cover below the dam to the largest dimension of the blocks. As can be expected, the ratio \( Y_B/A = 1.0 \) always provoked a dry jam, but the ratio \( Y_B/A \geq 1.3 \) never did for conditions of these tests.

**Destruction of ice jams**

An ice jam can be destroyed by disgorgement, diversion, or melt. The mechanism of disgorgement is essentially that of shearing of the ice floe accumulation. In the case of a simple jam, the floes are pushed under the retaining section and resume their descent downstream. In the case of a dry jam the plug is suddenly released when the water level becomes high enough upstream. In both cases sheared vertical ice walls are left on the banks.
If the plug or the jam itself is sufficiently resistant, water often rises upstream and finds a new route on the major bed of the river, bypassing the jam. In many cases with dry jams, most of the discharge can thus be deflected, possibly even leading to the formation of a new river channel.

Ice jams rarely melt in place, except where there are permanent ice obstacles.
ICE CONTROL

Control of winter regime

Although there has been no systematic study of the effects of engineering works on the winter regime of rivers it has become apparent to ice observers that these effects are considerable. Korzhavin says: “The construction of large hydroelectric stations on Siberian rivers, which created new reservoirs, led to considerable changes in the rivers’ thermal regime. This has to be taken into account when designing and building new hydroelectric stations. These changes are the consequences of the new hydraulic and thermal conditions in the deep and slow flowing artificial reservoirs. The observations that have been conducted for many years on the Angara, the Yenisey and the Ob Rivers show that a large reservoir changes the time of freeze-up and breakup and even the properties of the ice sheet. The winter fluctuating flow breaks up the ice in the tailwater, leading to the formation of open pools; the spring ice run becomes smoother, underhanging ice dams and ice jams occur less frequently and are less dangerous to the structures.”

Many artificial means are known to completely prevent or accelerate the formation of an ice cover. Hence, dangerous ice jams at breakup can be completely eliminated or controlled by adequate engineering works.

On p. 55 ways of preventing active frazil from clogging intakes were discussed. We will now discuss, in a more general way, the control of the river regime at the beginning of winter, in order to prevent, beforehand, any problem that might originate from frazil or a solid ice cover. The more dangerous conditions that occur at that time normally come from partial and unstable ice cover formation. A river stretch, under these conditions, is the site of intensive frazil formation of the active form, of accumulation of unstable underhanging dams and of ice jamming. There are two extreme solutions to such problems: either a completely free-flowing channel or a continuous ice cover.

Free-surface channel. Let us consider a prismatic channel (Fig. 84) where the water temperature at the inlet is \( T_{w0} \) and at the outlet \( T_w \). This temperature will be given by eq 47:

\[
T_{w0} - T_w = 1.85 \times 10^{-7} \left( K_1 - K_2 \frac{T_a}{Q} \right) L B
\]

where:

- \( L, B \) = canal length and width, ft
- \( Q, V \) = canal discharge, ft\(^3\)/sec and average velocity, ft/sec
- \( T_a \) = air temperature, °F, assumed constant during the interval of time necessary for the water to travel from one end of the channel to the other.
- \( K_1, K_2 \) = average coefficients related to heat exchanges between water and air, during the same interval of time.
With this formula it is easy to discuss the conditions necessary to maintain a free-surface flow in this channel during the winter months. The only other condition required is the critical flow velocity that will not permit the hydrodynamical progression of an ice cover. This is given in Table II (p. 71) where the higher values of this critical velocity, $V_{c2}$, vary from 2.0 to 2.9 ft/sec.

**Effect of velocity:** An ice cover will never form if:

$$V > V_{c2}$$ \hspace{1cm} (119)

It will always form when $V \leq V_{c2}$ (if ice is being produced in the channel), $V_{c2}$ being the lowest possible limit of progression of an ice cover fed by frazil slush, as given in Table II.

**Effect of inlet temperature and heat addition:** From eq 118 it can be seen that ice will not form in the channel if:

$$T_w > 32^\circ F$$

This gives:

$$(T_{w0} - 32) > 1.85 \times 10^{-7} \left( K_1 - K_2 T_a \right) \frac{LB}{Q}$$ \hspace{1cm} (120)

If $T_w$ is less than $32^\circ F$, a frazil production front will appear in the channel at a distance $L_0$ from its edge as shown in Figure 84. As discussed previously this frazil will not form an ice cover if the velocity is high enough.

It is also possible to keep a channel free from ice by the addition of external heat at the inlet. If at the inlet the whole stream is thoroughly mixed the quantity of heat required is:

$$\phi_e = \left( K_1 - K_2 T_a \right) LB$$ \hspace{1cm} (121)

where $\phi$ is in Btu/day.

It was shown recently that the 20-mile-long ship channel between Montreal Harbour and Lake St. Louis could be kept free from ice all winter long by the cooling requirements (thermal pollution output) of a 500-MW nuclear plant.
Effect of channel geometry: In the design of the St. Lawrence Seaway some attention was given to a geometrical form of the channel that would prevent ice formation during the winter months. Because the channel width and velocity were fixed by navigation requirements, the remaining parameter that could be varied was the depth. The depth required to prevent freezing is then given by:

\[
y > \frac{1.85 \times 10^{-7} \left[ K_1 - K_2 T_a \right]}{T_{w_0} - 32} L^n
\]

(122)

With an inlet temperature of 36°F (water coming from Lake Ontario) it was found that a 60-ft deep channel would remain unfrozen under the most severe conditions whereas with even moderately severe weather, ice formation would occur in a 35-ft-deep channel before reaching Montreal.

Representation of results: The water-ice conditions in the prismatic channel that has been taken as an example are represented in Figure 85. This corresponds to a channel of given length, depth and width where the water temperature is also fixed at the inlet.

For conditions corresponding to zone A on this graph, the air is too warm and the velocity too fast for water to have time to drop to the freezing point.

In zone B the water gets to the freezing point at a location which moves closer to the inlet as the air temperature gets lower. Because of the low velocity there is no difficulty in forming an ice cover in the ice-forming area.

Zone C has the same conditions as zone B except that because of higher velocity the pack cannot progress so easily upstream. It forms partially and the frazil flocs and slush are entrained underneath.

In zone D, frazil is produced but cannot pack to form an ice cover because of too-high velocities. The quantity of frazil increases as the water velocity and air temperature get lower.

Figure 85. Water-ice phase diagram for a given prismatic channel with constant inlet water temperature.
Water intakes in drifting ice. In a free-flowing channel, water intakes must be designed so as to take in only the water without the floating ice. Some frazil may be entrained but it will be in the form of small, suspended particles that can be dealt with by means given earlier.

The principles of design of such an intake were established long ago. It was suggested that surface water be used for transportation and bypassing of ice in the direction of the streamflow, while intake water would be drawn from the middle or bottom strata, preferably at right angles to the stream direction. Guide vanes in the bottom or gathering tubes might facilitate the cross flow in the lower strata. The water might pass under a curtain wall built parallel to the direction of the stream. The velocity of the surface water should then be low enough to avoid pulling the ice underneath by suction.

Such a principle has been applied in Europe on smaller rivers (Fig. 86). Normally, in a smaller river, the intake should be placed on the convex side of a bend so that most ice would make for the surface in a helicoidal flow.

This type of intake has been built successfully on a large scale in the Niagara River where water is directed to many hydroelectric stations totaling more than 2GW capacity. The Niagara River has been called a giant refrigerator, with more than 40 square miles of open surface between Lakes Erie and Ontario, producing an estimated 3 million cubic yards a day of frazil and slush, in 0°F weather. One of the more spectacular intake designs, called the Johnson-Wahlman type, was built at the Sir Adam Beck Station No. 2 (Fig. 87). It consists of a series of port-vanes on the side of a tunnel for a length of 500 ft. These ports are submerged at least 10 ft at all times. They are grouped at varying angles to the flow to ensure equal draft distribution. This intake can be seen in operation in Figure 88 with heavy ice passing freely along its length without being entrained in the intake. No difficulty has been experienced with these intakes even with very heavy runs of ice.

![Diagram of water intake in drifting ice](image-url)

*Figure 86. Water intake in drifting ice.*
Figure 87. Johnson-Wahlman intake - interior view.

Figure 88. Johnson - Wahlman intake in operation.
*Ice spillways.* Ordinary spillways seem to be adequate to pass drifting ice and even with a large ice run at breakup, only a fraction of the river width is needed to pass the ice without jamming. During the construction of the Bratsk hydroelectric station, a third of the river width, divided by 40-ft openings between piers, was adequate to evacuate the ice.

It is, however, obvious that the most advantageous spillway to pass ice is the one with the longest uninterrupted crest length and enough depth over the crest to accommodate the thickest pieces of ice. The roller gate, 60 ft long or more, has many advantages over the relatively short-span Tainter, or Stoney gates. Better yet are wicket gates which leave a clear crest line with a minimum amount of water when they are lowered.

In smaller and older power plants, where frazil problems were met at the beginning of the winter, a standard sluice was provided to clear the floating ice in case of trouble. These sluices were rectangular, 12 to 15 ft wide, with a minimum of 2 ft of water at the lowest operating level.

A more recent ice spillway is the "Control Structure" on the Niagara River which has 18 sluices with downward-opening gates 100 ft wide. The maximum depth over the sills is about 10 ft. These gates can be operated with the minimum depth of water to pass ice, in order to lose the minimum amount of water.

A spillway can be designed specifically for ice passage and its hydraulic requirements should be:

1. To pass without delay all floating ice that can be brought to the structure because, if some slowdown occurs in the movement of ice floes, a solid bridge might form in front of the spillway, preventing further evacuation of ice and providing a start for the formation of a jam.
2. To break the thickest ice sheets that can be expected whose dimensions in plan are bigger than those of the entrance to the downstream channel.
3. To have an operating discharge as small as possible compared to the total available one.

In many cases the maximum amount of ice that might be expected will cover 100% of the surface at the approaches to the structure. In order that no slowing down will occur in the passage of ice toward the spillway, it is necessary that:

\[ I = V_s B = \text{constant} \]  

where:

- \( I \) = discharge of floating ice, \( \text{ft}^2/\text{sec} \)
- \( V_s \) = average surface velocity of approach, \( \text{ft/sec} \)
- \( B \) = approach channel, width, \( \text{ft} \).

The discharge of water in an ice spillway will be a minimum if the ice capacity of the spillway is equal to \( I \). To maintain a constant capacity toward a spillway control section, it may be necessary to extend a slab in front of the structure in order to clear the zone of no-velocity flow. Finally, the depth of water on the spillway slab should also be a minimum. This is attained if the depth is a little higher than the thickest floes than are expected so that no solid friction will retard the flow.
One such spillway, the semicircular shaft spillway (Fig. 89) was tested and found to have an ice capacity of 100% of that of the approaches for the conditions of the tests. The thickest expected blocks would be less than 3 ft thick and could be broken down by a shaft 30 ft in diameter. For a normal depth of 3 ft on the slab the loss of discharge was 8% of that of the adjoining turbine.

**Effects of a reservoir on river ice.** One of the best ways to deal with ice problems at an intake is to create a pond of adequate size on which an ice cover will form and stabilize during the winter period. Such a cover will form readily and be very stable at the lowest critical velocities given in Table II. It will form with more difficulty but be stable in the range of velocities up to the highest given in that table.

Before the formation of the ice cover some frazil problems may still be expected on the racks of an intake, but they can be dealt with by the methods given previously.

It is of primary importance that the ice cover should form on a pond of adequate size as shown in Figure 90. Usually, frazil produced in rapids of higher reaches will deposit and progress downstream in the reservoir. If the pond is too small, this deposit may reach the intake structure, creating worse ice problems than would occur without the ice cover. The frazil deposit would also raise by $\Delta H$ the upstream level of the pond, much above the backwater levels of an open surface flow. Many dams have thus raised the upstream water level in winter to heights and distances far above the upstream limit of their expected summer conditions.

Another effect of a dam is the fact that the water taken from the lower levels in a reservoir is warmer than freezing point and keeps a stretch of river downstream of the dam ice-free for a length $L$ (Fig. 90). This in turn produces more frazil.

A pond in which the cover forms only partially, leaving polynyas on its surface, might be worse than no pond at all because frazil and snow slush contribute to important ice accumulations in the water body. Polynyas can be frozen very quickly, in an artificial manner, by covering them with brushwood wired together.

The ideal ice pond would be designed for low water velocity. It would flood rapids upstream so that no frazil slush could be deposited underneath. The water would be taken at the surface of the dam and discharged in a calm stilling basin so that the ice cover would form downstream immediately.
Examples of ice control in power plants. We will give three examples of measures taken to solve ice problems in typical hydroelectric stations, as given by Estifeev.37

Hydropower A fed by lake water: When the air temperature is low, the headrace of this plant is partially covered by an ice cover except for a few high velocity polynyas within 6 miles of the plant. To prevent frazil from forming in the polynyas and clogging the trash racks, the original reservoir level was raised by means of a special superstructure. The speeds were reduced in the range of 0.7 to 1 ft/sec, the polynyas froze, and now a continuous ice cover forms from the lake to the plant.

Hydropower B: The river flows from north to south and there is a strong rapid in the upper reaches that is flooded at higher elevations but which appears when the storage reservoir is drawn down. This causes clogging of the intake. The velocity is very low in the reservoir and a control of the winter level is required during very cold winters.

Hydropower C: This station is situated in a region of periodic cooling at the downstream end of a high velocity stream (more than 6 ft/sec). After many trials the final solution consisted of letting the frazil through the ice spillway in the main channel (Fig. 91), and forming an ice cover at velocities lower than 1.5 to 2.0 ft/sec in the diversion channel to the station. Because of frazil deposits under the ice cover of the diversion channel, a frazil accumulation pit was excavated right at the channel inlet. This pit could be flushed with the aid of a special outlet.
Local melting of ice covers

The best known way to induce local melting of ice, other than by the direct use of heat in any form, is with air bubbling systems or by dusting or putting chemicals on the ice.

Air bubbling. In 1917 an air-bubbling system was installed at the Keokuk dam. It was successful and opened the way for one of the most ingenious devices that can be used for local melting of ice.

Air-bubbling systems depend on the fact that subsurface warm water can be brought to the surface by rising air bubbles and used to prevent surface ice formation at specific locations. In rivers the water mass is kept at nearly the same temperature by mixing, even if the velocity is quite low. In seawater this temperature gradient does not normally exist and only small benefits can be derived from a positive salinity gradient. So for all practical purposes, the air system is efficient and has been successfully used for lakes and reservoirs where there is a vertical temperature gradient.

Experimental study has shown that the general circulation of water around a rising column of air bubbles has the form shown in Figures 92 and 93. For usual air discharges, the bubbles issue from the orifice in a heterogeneous mixture, the sizes varying from about 1/16 to 1 in. in diameter. The bubbles rise at varying rates, depending on their size, and oscillate rapidly from side to side. The large bubbles induce turbulence which moves the smaller bubbles laterally. Thus, the size of the cloud of bubbles increases with vertical distance and it appears that they are contained in a cone with a total included angle of about 12°.

The effect of the bubbles extends beyond the cone containing them. The water around the bubble cone is accelerated and moves upward with a velocity distribution shown in Figure 94. The force exerted by the air bubbles on the surrounding water is roughly constant during the period of ascent. If the installation is in very deep water the effect of compressibility is such that the bubble expands as it rises. Measurements show that the peak velocity in the water jet is constant and that there is an increase of momentum accomplished by a steadily widening jet. It is thus evident that more water is transported to the surface for deeper bubbler systems of identical air capacity.

Since the vertical jet is increasing in volume with distance above the source, there must be a lateral flow to supply the fluid. This entrained flow, as shown in Figure 93, greatly increases the area influenced by the air bubbles.

Upon reaching the water surface the air bubbles stop but the momentum of the water jet is converted into a radially spreading surface jet. This is a relatively thin, high velocity jet (6 in. thick or less). The radial spread of the surface jet indicates that the velocity should decrease. Measurements confirm this trend and show thickening of the jet with entrainment of water underneath. Consequently the jet disappears quickly and its energy is absorbed into the general pattern of circulation.

Because of the entrainment of water in the vertical jet and its dissipation in the surface jet, the large scale circulation induced by the air bubbles is a giant ring vortex as shown in Figure 93. The flow velocities are very small everywhere except in the jets and decrease with distance away from the bubbler.

Not much is known of the optimum conditions for the operation of a bubbler system. The orifice sizes are usually 1/32, 1/16, and 1/8 in. in diameter. Special care should be taken to machine the holes in the pipe wall so they will be perfectly smooth. The temperature and pressure of the air inside the pipe should be such that the decompression of air in the water will not freeze the water at the outlet and block the orifice.
The optimum spacing and discharge of the orifices has not been determined on a systematic basis. Actual working practice is to space the orifices from one-eighth to one time the water depth, the latter condition applying for operation in mild weather. The air discharge usually varies between 0.75 and 2 ft³/min per hole. Figure 95 shows results of measurements giving the volume of ice melted as a function of the depth of the orifice and the discharge of air.
Air-bubbling systems have been used successfully to prevent ice formation at the face of dams, gates and docks. They have been used in log ponds, fish ponds, marinas, lakes, and around bridge piers. A unique and most recent adaptation of the system was made to keep a 2-mile-long ferry passage open through winter ice at Babine Lake, together with landing areas large enough to allow the ferry to be turned around. The main line consists of galvanized 1-1/2-in. and 2-in. steel pipe laid on the bottom with air holes at 20-ft centers and 2-1/2-in. polyethylene pipe moored at a depth of 134 to 161 ft with holes also at 20-ft centers. The air is provided by a 625 ft³/min compressor at a pressure of 115 psi.

Figure 96 shows the open channel formed with this system. At temperatures of -35°F the width of the channel is reduced to about 35 ft. At higher temperatures, the width is correspondingly greater. For example, at -30°F, 0°F, and +30°F it is about 40 ft, 60 ft and 150 ft wide respectively.

Dusting. A layer of suitable dust on a snow or ice surface increases the amount of solar radiation absorbed and the amount of heat available for melting. The percentage of the total shortwave radiation reflected from a surface is termed the albedo. For new snow the albedo may be as high as 80 to 90%. For melting snow and ice it varies from 40 to 60%. The albedo of an ice cover darkened by a suitable dust ranges from 10 to 20%. That of a water surface is also low. The increase in solar radiation absorbed because of dusting will depend on how much the albedo of the natural surface can be reduced by dusting.

The amount of incoming shortwave radiation available at the site during favorable dusting periods controls the amount of additional ice that can be melted. In the south of Canada, this ranges from 1100 to 1300 Btu/ft² day (in March) and in the Far North from 2200 to 2600 Btu/ft² day (in June). Assuming a reduction in albedo from 50% to 80% this corresponds to a theoretical increase in the melting rate of about 1/2 to 1 in. of ice per day.

The increase in albedo obtained by dusting does not depend much on the type of material used but it depends very much on the amount of the surface that is covered by the dust.

The materials most frequently used are coal slag, coal dust, cinders, soot, ashes and different types of soils, sands and crushed stones.
Figure 96. Open ferry passage at Babine Lake.\textsuperscript{126}

Optimum density of application for:

- \(d = 1.5 \text{ mm} \) (150 tons/mile\textsuperscript{2})
- \(d = 0.5 \text{ mm} \) (475 tons/mile\textsuperscript{2})

Figure 97. Measured albedo vs coverage with dust.\textsuperscript{151}

Williams\textsuperscript{154} has shown that the albedo changes almost linearly with the area covered by the dust from that of the original surface to the albedo of the dust (Fig. 97). The coverage obtained from an application is given by:

\[
A_N = 1.5 \frac{w}{yd}
\]  

(124)
where:

\[
A_N = \text{area covered by dust per unit area}
\]
\[
w = \text{density of application, lb/ft}^2
\]
\[
\gamma = \text{unit weight of the particles, lb/ft}^3
\]
\[
d = \text{diameter of particles, ft.}
\]

It can be seen from this formula that the efficiency per unit weight of a given dust is inversely proportional to the average diameter and density of the material. The practical range for average particle diameter of dusts that are used is probably about 0.1 to 2.5 mm so that it will take about 25 times as much material by weight with the bigger particles than with the smaller ones. As an example, it takes about 1000 tons/square mile for a dust of average particle diameter 1 mm and specific weight 2.4 lb/ft³.

There is little information available on the technique for applying dust. It seems that the techniques used to apply fertilizers either by airplane or with a tractor are also suitable for the purpose.

Extensive experimentation on dusting has been carried on by Williams¹⁵¹ and his results are most revealing:

"Dusting snow and ice covers to increase the rate of melting of ice is not very effective if the minimum daily air temperature falls much below 32°F because the dusting period is usually followed by periods of freezing which will refreeze the melt water produced.

"Small amounts of new snow, which might (in weather records) be classified as a trace, can effectively cover dusted areas and prevent increased melting by dusting."

Dusting increases the melting rate significantly when the surface has a relatively high albedo but it is no longer effective as soon as meltwater accumulates on the surface.

"Dyes and powders applied in the form of sprays to ice covers that are porous are not as effective as dusts because they penetrate the ice leaving a surface with a high albedo. Pumping a thin layer of bottom mud onto the ice cover of shallow lakes or rivers is an effective means of increasing the melt rate of ice."

These conclusions show that dusting can be carried on successfully when atmospheric conditions can be closely predicted and are favorable or when extensive applications can be made repeatedly. In most cases where dusting cannot be done on a large scale, it is not a reliable ice melting process.

Chemicals. As is known from physical chemistry, the effect of dissolving a salt in water may be to lower the melting point to what is called the eutectic temperature.

Interaction of salt with ice is possible at eutectic temperatures of the saturated solution. At temperatures below the eutectic point, there is no melting of the ice; at temperatures above it there are several stages in the ice-salt reaction.

In the first stage of salt application, the surrounding ice is cooled to the eutectic temperature of the solution. In the second stage the heat given up by the cooling ice melts the ice in contact with the salt to form a brine. This melting continues until the limiting concentration of the salt in the brine is attained for the corresponding temperature. A large number of substances which interact with ice have been listed in Table V, with their eutectic temperatures."
<table>
<thead>
<tr>
<th>Formula and name of anhydrous substance</th>
<th>Eutectic temp, °C</th>
<th>Formula and name of anhydrous substance</th>
<th>Eutectic temp, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaO</td>
<td>-0.15</td>
<td>NH₄NO₃</td>
<td>-16.9</td>
</tr>
<tr>
<td>NH₄Al(SO₄)₂</td>
<td>-0.20</td>
<td>NH₄Br</td>
<td>-17.0</td>
</tr>
<tr>
<td>SrO</td>
<td>-0.30</td>
<td>LiNO₃</td>
<td>-17.8</td>
</tr>
<tr>
<td>Na₂B₄O₇</td>
<td>-0.45</td>
<td>NaNO₃</td>
<td>-18.1</td>
</tr>
<tr>
<td>BaO</td>
<td>-0.50</td>
<td>(NH₄)₂SO₄</td>
<td>-18.5</td>
</tr>
<tr>
<td>Ba(NO₃)₂</td>
<td>-0.55</td>
<td>SrCl₂</td>
<td>-18.7</td>
</tr>
<tr>
<td>KMnO₄</td>
<td>-0.58</td>
<td>NaNO₂</td>
<td>-19.5</td>
</tr>
<tr>
<td>K₂Cr₂O₇</td>
<td>-0.63</td>
<td>SrI₂</td>
<td>-20.0</td>
</tr>
<tr>
<td>K₂SO₄</td>
<td>-1.52</td>
<td>NaCl</td>
<td>-21.2</td>
</tr>
<tr>
<td>CuSO₄</td>
<td>-1.60</td>
<td>CoCl₂</td>
<td>-22.5</td>
</tr>
<tr>
<td>FeSO₄</td>
<td>-1.80</td>
<td>KI</td>
<td>-23.0</td>
</tr>
<tr>
<td>NaHCO₃</td>
<td>-2.33</td>
<td>Co(NO₃)₂</td>
<td>-24.0</td>
</tr>
<tr>
<td>CoSO₄</td>
<td>-2.70</td>
<td>NH₄CNS</td>
<td>-25.2</td>
</tr>
<tr>
<td>Pb(NO₃)₂</td>
<td>-2.70</td>
<td>Cu(NO₃)₂</td>
<td>-26.4</td>
</tr>
<tr>
<td>KH₂PO₄</td>
<td>-2.75</td>
<td>NH₄I</td>
<td>-27.5</td>
</tr>
<tr>
<td>KNO₂</td>
<td>-2.85</td>
<td>Ni(NO₃)₂</td>
<td>-27.8</td>
</tr>
<tr>
<td>Na₂SO₃</td>
<td>-3.35</td>
<td>NaOH</td>
<td>-28.0</td>
</tr>
<tr>
<td>NiSO₄</td>
<td>-3.40</td>
<td>NaBr</td>
<td>-28.0</td>
</tr>
<tr>
<td>NH₄HCO₃</td>
<td>-3.90</td>
<td>Ca(NO₃)₂</td>
<td>-28.0</td>
</tr>
<tr>
<td>MgSO₄</td>
<td>-3.90</td>
<td>KCN</td>
<td>-29.6</td>
</tr>
<tr>
<td>Al₂(SO₄)₃</td>
<td>-4.0</td>
<td>KCNS</td>
<td>-31.2</td>
</tr>
<tr>
<td>Na₂CrO₄</td>
<td>-4.9</td>
<td>Mg(NO₃)₂</td>
<td>-31.6</td>
</tr>
<tr>
<td>KH₂CO₃</td>
<td>-5.43</td>
<td>NaClO₄</td>
<td>-32.0</td>
</tr>
<tr>
<td>K₂S₂O₇</td>
<td>-5.5</td>
<td>Zn(NO₃)₂</td>
<td>-32.0</td>
</tr>
<tr>
<td>NaF</td>
<td>-5.6</td>
<td>MgCl₂</td>
<td>-33.6</td>
</tr>
<tr>
<td>Sr(NO₃)₂</td>
<td>-5.75</td>
<td>FeCl₂</td>
<td>-36.6</td>
</tr>
<tr>
<td>ZnSO₄</td>
<td>-6.56</td>
<td>K₂CO₃</td>
<td>-36.5</td>
</tr>
<tr>
<td>AgNO₃</td>
<td>-7.3</td>
<td>CuCl₂</td>
<td>-40.0</td>
</tr>
<tr>
<td>BaCl</td>
<td>-7.8</td>
<td>HNO₃</td>
<td>-42.3</td>
</tr>
<tr>
<td>Na₂SO₄</td>
<td>-9.05</td>
<td>K₂SO₄</td>
<td>-46.5</td>
</tr>
<tr>
<td>Na₂S</td>
<td>-10.0</td>
<td>Th(NO₃)₄</td>
<td>-46.8</td>
</tr>
<tr>
<td>KCl</td>
<td>-10.7</td>
<td>AlCl₃</td>
<td>-55.0</td>
</tr>
<tr>
<td>Na₂S₂O₃</td>
<td>-11.0</td>
<td>FeCl₃</td>
<td>-55.0</td>
</tr>
<tr>
<td>K₂CrO₇</td>
<td>-11.55</td>
<td>CaCl₂</td>
<td>-58.0</td>
</tr>
<tr>
<td>MgSO₄</td>
<td>-11.40</td>
<td>Be(NO₃)₂</td>
<td>-59.0</td>
</tr>
<tr>
<td>KBr</td>
<td>-13.0</td>
<td>ZnCl₂</td>
<td>-62.0</td>
</tr>
<tr>
<td>NaI</td>
<td>-13.5</td>
<td>KOH</td>
<td>-65.2</td>
</tr>
<tr>
<td>NH₄Cl</td>
<td>-16.0</td>
<td>H₂SO₄</td>
<td>-74.5</td>
</tr>
<tr>
<td>Cd(HO₃)₂</td>
<td>-16.0</td>
<td>HCl</td>
<td>-96.0</td>
</tr>
</tbody>
</table>
Table VI gives the theoretical value of the volume of ice that 1 gram of a particular salt can melt. From this table it can be seen that certain substances are more effective than others in melting ice. For example, at a temperature of \(-5^\circ\text{C}\) potassium bicarbonate can melt 59 times its volume of ice, sodium fluoride 33 times, sodium sulfide 21 times, and calcium chloride 10 times. With a decrease of temperature the quantity of ice which can be dissolved decreases. At temperatures down to \(-21^\circ\text{C}\) it is more rational to use chloride salts of ammonium, sodium or potassium. They are stable, inexpensive and do not require special precautions in their use. Sodium chloride causes melting of ice at a temperature above \(-21^\circ\text{C}\), ammonium chloride at \(-16^\circ\text{C}\) and potassium chloride at \(-10.7^\circ\text{C}\). Many other substances are less suitable because they are toxic, hygroscopic or otherwise undesirable.

In melting ice it is possible to use chemicals that react with sea or fresh water to release heat instead of forming eutectic mixtures. For example, a mixture of sodium hydroxide and powdered aluminum was applied to the ice cover.\(^{103}\) A reaction occurred that released hydrogen and heat. It began in 3 to 4 minutes and was violent at the start. In two days the ice thawed to a depth of 42 to 43 inches.

Interesting results were obtained\(^{103}\) in an experiment using magnesium (99%) in a mixture with nickel (1%). This mixture reacted well with seawater and melted the ice in a proportion of 1 to 60. It does not work, however, with freshwater ice unless salt is added to the mixture.

The use of chemicals to melt river and lake ice must be limited to very special applications as in the example cited by Peschanski of melting the ice packed at the intake and scroll case of a turbine. It may be of interest for taking out the plug of a stubborn ice jam where water pollution is not feared. Calcium chloride is particularly effective for that purpose.\(^{5}\)

**Mushy ice formed with foreign substances.** It might be interesting, for navigation or other purposes, to form slush ice on a water body instead of solid ice by the application of a foreign substance on the surface. This was studied\(^9\) and it was found that sawdust, camphor and oils were not effective for that purpose but that very mushy, spongelike and weak ice would be produced by a saponated substance derived from fatty acids.

**Aquaterm.** The Aquaterm is a motorized propeller unit, which directs a jet stream of water toward the surface to be kept free from ice.\(^{104}\) It can either be a permanent installation or a temporary one like an outboard motor. It was found, in \(-20^\circ\text{F}\) weather, that a 10-hp unit could maintain an open area about 5 ft wide and 100 ft long in front of a spillway and a 3/4-hp unit placed 40 ft upstream of a 32-ft floodgate kept it free of ice.\(^{104}\)

Such effective control of ice without thermal stratification is possible because this equipment causes erosion of ice rather than melting, and frazil might form instead of a solid ice sheet.

**Control of breakup**

Ice jams and the dangerous floods they produce can be permanently controlled by the construction of adequate engineering works. Local preventive action can also be taken when needed, by dusting, using chemicals, explosives or mechanical means like icebreakers.

**Ice dams and booms.** The principle of operation of ice dams and booms is the accumulation of drifting ice floes upstream in order to prevent jam formation downstream. A dam essentially induces a jam at the upper end of its reservoir, so its final effect is to displace the site of jam formation to one where the jam can be controlled.
Table VI. Volume (cm$^3$) of melted ice produced by 1 gram of salt.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>KHC0$_3$</td>
<td>59.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaF</td>
<td>32.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sr(NO$_3$)$_2$</td>
<td>3.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZnSO$_4$</td>
<td>3.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AgNO$_3$</td>
<td>2.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BaCl$_2$</td>
<td>5.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K$_2$S$_3$O$_3$</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na$_2$S</td>
<td>20.9</td>
<td>10.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>9.6</td>
<td>6.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KCl</td>
<td>10.3</td>
<td>4.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na$_2$S$_3$O$_4$</td>
<td>5.0</td>
<td>2.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K$_2$CrO$_4$</td>
<td>8.6</td>
<td>2.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MgSO$_4$</td>
<td>3.6</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KBr</td>
<td>5.4</td>
<td>2.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NH$_4$Cl</td>
<td>14.0</td>
<td>7.1</td>
<td>4.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cd(NO$_3$)$_2$</td>
<td>4.3</td>
<td>2.6</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SrCl$_2$</td>
<td>3.4</td>
<td>4.6</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaNO$_3$</td>
<td>9.4</td>
<td>4.9</td>
<td>3.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaNO$_4$</td>
<td>7.5</td>
<td>3.4</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaCl</td>
<td>12.2</td>
<td>6.7</td>
<td>4.7</td>
<td>3.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CaCl$_2$</td>
<td>10.1</td>
<td>6.0</td>
<td>4.5</td>
<td>3.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KI</td>
<td>3.7</td>
<td>4.5</td>
<td>2.8</td>
<td>2.0</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NH$_4$CNS</td>
<td>9.5</td>
<td>3.9</td>
<td>2.9</td>
<td>2.4</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cu(NO$_3$)$_2$</td>
<td>6.5</td>
<td>3.7</td>
<td>2.9</td>
<td>2.4</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ni(NO$_3$)$_2$</td>
<td>5.9</td>
<td>3.7</td>
<td>2.9</td>
<td>2.4</td>
<td>2.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaOH</td>
<td>18.1</td>
<td>9.9</td>
<td>7.0</td>
<td>5.7</td>
<td>4.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaBr</td>
<td>6.6</td>
<td>3.9</td>
<td>2.9</td>
<td>2.3</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ca(NO$_3$)$_2$</td>
<td>6.8</td>
<td>3.3</td>
<td>2.3</td>
<td>1.8</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KCN</td>
<td>10.6</td>
<td>5.5</td>
<td>3.8</td>
<td>2.9</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KCNS</td>
<td>6.9</td>
<td>3.5</td>
<td>2.3</td>
<td>1.7</td>
<td>1.4</td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg(NO$_3$)$_2$</td>
<td>5.8</td>
<td>4.7</td>
<td>3.6</td>
<td>3.0</td>
<td>2.6</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaClO$_4$</td>
<td>6.2</td>
<td>3.7</td>
<td>2.7</td>
<td>2.2</td>
<td>1.9</td>
<td>1.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zn(NO$_3$)$_2$</td>
<td>8.7</td>
<td>8.7</td>
<td>6.2</td>
<td>5.3</td>
<td>4.6</td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>14.5</td>
<td>5.7</td>
<td>6.2</td>
<td>5.3</td>
<td>4.6</td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FeCl$_3$</td>
<td>9.6</td>
<td>6.1</td>
<td>4.7</td>
<td>3.9</td>
<td>3.3</td>
<td>2.9</td>
<td>2.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mn(NO$_3$)$_2$</td>
<td>7.8</td>
<td>4.0</td>
<td>2.8</td>
<td>2.2</td>
<td>1.9</td>
<td>1.7</td>
<td>1.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K$_2$CO$_3$</td>
<td>7.2</td>
<td>3.9</td>
<td>3.0</td>
<td>2.4</td>
<td>2.1</td>
<td>1.9</td>
<td>1.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CuCl$_2$</td>
<td>8.7</td>
<td>5.2</td>
<td>3.7</td>
<td>3.0</td>
<td>2.6</td>
<td>2.3</td>
<td>2.0</td>
<td>1.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HNO$_3$</td>
<td>12.0</td>
<td>6.8</td>
<td>4.7</td>
<td>3.8</td>
<td>3.2</td>
<td>2.8</td>
<td>2.5</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K$_2$SO$_4$</td>
<td>6.2</td>
<td>3.3</td>
<td>2.3</td>
<td>1.9</td>
<td>1.6</td>
<td>1.4</td>
<td>1.3</td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Th(NO$_3$)$_4$</td>
<td>3.0</td>
<td>1.5</td>
<td>1.2</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AlCl$_3$</td>
<td>15.9</td>
<td>8.1</td>
<td>6.6</td>
<td>5.7</td>
<td>5.1</td>
<td>4.6</td>
<td>4.3</td>
<td>4.1</td>
<td>3.8</td>
<td>3.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FeCl$_3$</td>
<td>10.0</td>
<td>6.0</td>
<td>4.5</td>
<td>3.7</td>
<td>3.3</td>
<td>3.0</td>
<td>2.7</td>
<td>2.5</td>
<td>2.3</td>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Be(NO$_3$)$_2$</td>
<td>8.2</td>
<td>6.3</td>
<td>3.5</td>
<td>2.6</td>
<td>3.1</td>
<td>2.7</td>
<td>2.5</td>
<td>2.3</td>
<td>2.0</td>
<td>1.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZnCl$_2$</td>
<td>7.8</td>
<td>4.3</td>
<td>3.0</td>
<td>2.3</td>
<td>1.9</td>
<td>1.6</td>
<td>1.5</td>
<td>1.3</td>
<td>1.1</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KOH</td>
<td>12.8</td>
<td>8.0</td>
<td>6.0</td>
<td>4.9</td>
<td>4.3</td>
<td>3.7</td>
<td>3.4</td>
<td>3.1</td>
<td>2.7</td>
<td>2.6</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>H$_2$SO$_4$</td>
<td>11.2</td>
<td>8.7</td>
<td>5.1</td>
<td>4.2</td>
<td>3.6</td>
<td>3.2</td>
<td>2.8</td>
<td>2.6</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>HCl</td>
<td>21.7</td>
<td>13.1</td>
<td>9.7</td>
<td>8.0</td>
<td>6.9</td>
<td>6.2</td>
<td>5.8</td>
<td>5.2</td>
<td>4.5</td>
<td>4.3</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td>CrO$_3$</td>
<td>6.2</td>
<td>3.7</td>
<td>2.6</td>
<td>2.1</td>
<td>1.7</td>
<td>1.5</td>
<td>1.3</td>
<td>1.2</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>
An ice dam should obstruct the ice passage so that the whole quantity of ice coming from upstream will eventually melt in place. Because the ice accumulation will be subjected to the full spring flood, adequate freeboard should be left upstream of the reservoir for the backed up water. Some backwater effect may also occur during the winter, because of frazil accumulation.

An ice dam not only stops all ice movement from upstream but it may also be beneficial in indirectly stabilizing the ice cover downstream. When there is a stretch of low velocity fluvial flow below the dam, there is no moving ice accumulation left to destroy the ice cover, so the ice will stay in place longer, melt, and be weaker when the water above finally moves it downstream.

An overflow dam of the type shown in Figure 98 a and b is well adapted for retaining the ice. A special rack must be built for this purpose in front of the spillway section. A stilling basin with a submerged jet is appropriate to dissipate the energy because an ice cover can form in it without frazil production.

An ice boom can usually do part of the work of an ice dam. The main difference is that the boom does not usually retain all the ice, but only delays its course downstream. This delay may be beneficial when the downstream ice cover is rotten or on its way to disappearing. An ice boom is very effective if combined with an overflow weir as shown in Figure 99. If the weir is high enough, this combination can even keep all the drifting ice in place. Another advantage is that it is always possible to release the boom and let the ice go, if the water level gets too high upstream. A simple wooden ice boom is shown in Figure 100.

Ice booms have also been extensively used to assist in the formation of an ice cover, particularly on the St. Lawrence and Niagara Rivers. It was found, in those cases, that they should be designed in accordance with the condition of hydrodynamical progression of an ice cover given by eq 79.

In the case of an ice dam it is preferable that only a static jam be formed in the reservoir so that the ice is not crushed to the bottom to form a dry jam. This would cause unexpected water level rises. The limiting condition of stability of a static jam is given in a prismatic channel by eq 117:

$$ Y \geq \frac{4.6 \ (a \ Q_{\text{max}})^{0.48}}{B^{0.23}} $$

where:

- $Y$ = average water depth, ft
- $n$ = equivalent Manning's roughness coefficient
- $B$ = width of prismatic channel, ft
- $Q_{\text{max}}$ = maximum channel discharge, ft$^3$/sec.

The maximum static pressure that an ice accumulation transmits to a hydraulic structure is not caused in most cases by the hydraulic thrust, but by the effects of wind.

By assuming an equivalent roughness of the hummocked surface of an ice jam of $z_0 = 1$ cm, the tangential stress caused by the wind is given by eq 8:

$$ r_A = 1.6 \times 10^{-5} \ V_{50}^2 $$
a. Pictorial sketch

b. On Chaudiere River

Figure 98. Overflow ice control dams.

Figure 99. Ice control boom.
where $V_{50}$ is the wind velocity (mph) 50 ft above the ground. Considering an extreme 80-mph wind, the tangential stress on a jam is:

$$
\tau_A = 0.1 \text{ lb/ft}^2
$$

The resulting thrust per unit width that the static ice accumulation is giving to the structure is then obtained from eq 95:

$$
T_{\infty} = 0.33 \ B^2
$$

But there is also the limitation that the cover cannot thicken more than $T_0 = Y$. The thrust is then obtained from eq 107 and 108:

$$
T_{\infty} = 6 \ Y^2 \ B
$$

Either eq 126 or 127 gives the maximum thrust that an ice jam can exert on a structure in a prismatic channel. Experiments show that there might be a significant increase in the value of that thrust if the channel is not prismatic.51 52

In the case of a flexible boom the impact of an isolated ice floe is not a determining ice force. However, for a rigid structure like a dam, some failures show that this effect has to be taken into account, and might lead to the worst loading conditions on the structure.

Channel improvements. In some rivers ice jams are caused by riverbed irregularities that can be readily corrected. Rocky ridges can be blown off with beneficial effects. It is, however, very difficult to make drastic corrections to the movable bed of a river because the sediments transported by the river flow may act adversely on any work which is done. In fluvial hydraulics, it is axiomatic
that any correction should be made with a view to preserving the general character of the riverbed. For instance, a sharp river bend should not be cut off but only smoothed out so as to attain the average curvature of other river bends. In a shallow reach it is preferable to concentrate the flow with a spur dike rather than make excavations that a later flood may fill up.

The experience, up to now, has been that it is extremely difficult to correct a torrential-fluvial transition that is an ideal site for the formation of an ice jam. If there is no way to control the ice itself, the only remedy might be to build protective walls and levees to prevent flooding. Figure 101 shows remedial works against ice jamming in a river.

**Explosives.** The explosives used for blasting ice jams are dynamite, TNT, ANFO (a mixture of ammonium nitrate and fuel oil) and Thermit. This technique is certainly one of the most widely used for the removal of ice jams. Its success depends on the experience of the people applying it. The main limitation of the use of explosives is that adequate water depth is needed to obtain an efficient blast. Explosives are not very effective on smaller streams.

Experiments conducted by the U.S. Army Corps of Engineers show that ANFO is one of the most desirable explosives because of the following characteristics. It has a high energy release at a lower rate than dynamite, which results in more lift with less danger. It is readily available in either the prepackaged or premixed form, or it can be mixed at the site if the proper fertilizers are available. In storage, ANFO is less sensitive to shock and flame than other explosives. In addition it is cheap.

The size of the crater formed by a charge of ANFO of given weight is given by Frankenstein in Figure 102. These charges should be placed as shown in Figure 103.

The effectiveness of hand-placed charges depends mainly on their arrangement in respect to the ice jam. By changing the size of the charges to suit the conditions and by properly placing them, ice work can be carried on in congested areas with no apparent danger of damage to buildings and personnel. There is then no shock wave and little debris above the ice surface for optimum conditions. Typical locations of explosive charges are shown in Figure 104.

In uninhabited areas one good way to break an ice jam is by airplane bombing. As with hand-placed explosives, if the bombs are properly located the bombing effort is very successful. The jam will immediately start to move and clear the area in a matter of minutes.

![Figure 101. Remedial works – correction at the junction of a brook. Chaudiere River.](image)
Figure 102. Crater size and explosive charge.  \[\text{CUBE ROOT OF CHARGE WEIGHT, } \text{lb}^{\frac{1}{3}}\]

<table>
<thead>
<tr>
<th>Point</th>
<th>Weight</th>
<th>Location</th>
<th>Type of explosive</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>940</td>
<td>Alaska</td>
<td>U.S. Army C-4</td>
</tr>
<tr>
<td>b</td>
<td>132</td>
<td>Alaska</td>
<td>U.S. Army C-4</td>
</tr>
<tr>
<td>c</td>
<td>50</td>
<td>Minnesota Elk River</td>
<td>ANFO and TNT</td>
</tr>
<tr>
<td>d</td>
<td>33.0</td>
<td>Minnesota Elk River</td>
<td>ANFO</td>
</tr>
<tr>
<td>e</td>
<td>6</td>
<td>Sartell Minnesota</td>
<td>ANFO</td>
</tr>
</tbody>
</table>

Location of explosive charge dependent on water depth and charge weight. Weight size dependent on current.

Figure 103. Depth of placement of explosive charge.
Icebreakers. Powerful vessels have been used in numerous instances to break up ice jams and they are certainly one of the most effective means for that purpose. It must be remembered that frazil accumulation may block condenser water intakos and adhere to the hull of the ship. An icebreaker with a fore propeller is well suited to maneuvering under those circumstances because the propeller produces lubrication between the vessel's hull and the slush. An icebreaker's bow
propellers can be used as drills to dislodge the ice pieces from the slush accumulation. Canadian icebreakers are fitted to resist the impact of heavy ice and their propellers can effectively drill slush ice accumulations as high as 20 to 30 ft.

Icebreakers work better in pairs to dismantle an ice jam in a river. They move upstream in the central part of the jam, cutting out masses of ice that can drift downstream with the current.
LITERATURE CITED


45. Gorunov, I.V. (1958) *Controlling anchor ice and slush by accelerating the formation of the ice cover*. Translation, Geophysics Research Directorate, American Meteorological Society, 1 p.


WINTER REGIME OF RIVERS AND LAKES


100. Meteorological Branch, Department of Transport, Canada (1963) Aerial ice observations and reconnaissance, Athabaska River, Lake Athabaska, Slave River and Great Slave Lake 1962, CIR-3820, TEC-461. 33 p.


102. Parmelee and Aubele (1951) Radiant energy emission of atmosphere and ground. Heating, Piping and Air Conditioning, no. 11, p. 120-129.


134. Thréilkel and Jordan (1957) Direct solar radiation available on clear days. *Heating, Piping and Air Conditioning*, vol. 29, no. 12, p. 135-145.


WINTER REGIME OF RIVERS AND LAKES


The monograph summarizes existing knowledge of river and lake ice surveys, heat balance on open water in winter, frazil, ice cover formation, ice breakup and ice control.

14. KEY WORDS

- Anchor ice
- Frazil ice
- Ice
- Ice breakup
- Ice control
- Ice forecasting
- Ice formation
- Lakes
- Rivers