Technical Report 187

TRANSIENT TEMPERATURE DISTRIBUTION WITHIN THERMAL SENSING ELEMENTS

by

John A. Clark

MAY 1967

Conducted for
CORPS OF ENGINEERS, U.S. ARMY

by

U.S. ARMY MATERIEL COMMAND
COLD REGIONS RESEARCH & ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

Contract DA-19-016-ENG-3204
DA Task IV025001A13001

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PREFACE

Authority for the investigation reported herein is contained in FY 195 Instructions and Outline, Military Construction Investigations, Engineering Criteria and Investigations and Studies, Studies of Construction in Areas of Seasonal Frost; Correlation Studies.

The study was conducted for the Engineering Division, Directorate of Military Construction, Office, Chief of Engineers. The program was administered by the Civil Engineering Branch (Mr. T. B. Pringle, Chief). Dr. W. M. Rohsenow and Dr. J. A. Clark, Department of Mechanical Engineering, Massachusetts Institute of Technology, performed this study under Contract DA-19-016-Eng-3204 awarded by the former Arctic Construction and Frost Effects Laboratory (ACFEL)* of the U. S. Army Engineer Division, New England. This report, authored by Dr. Clark, presents one phase of the contractual study. Other contractual phases are reported in: Technical Report 186 (May, 1967), "Application of Method of Predicting Thermal Error in Measurement of Ground Temperatures," W. M. Rohsenow; Technical Report 188 (May, 1967), "The Properties of Thermistors," J. A. Clark and Y. Kobayashi; and Internal Report 5 (Sept, 1966), "The Effect of Temperature Level on Various Thermocouple Circuit Components," J. A. Clark.

Investigations were performed under the general supervision of Mr. K. A. Linell, Chief, Experimental Engineering Division, USA CRREL (Formerly Chief, ACFEL) and the direct supervision of Mr. E. F. Lobacz, Chief, Construction Engineering Branch, USA CRREL (Formerly Coordinator, ACFEL). Mr. W. C. Sayman was the ACFEL project leader and Mr. G. D. Gilman of the Construction Engineering Branch was responsible for the coordination of the final report.

Lt. Colonel John E. Wagner was Commanding Officer/Director of the U. S. Army Cold Regions Research and Engineering Laboratory during the publication of this report, and Mr. W. K. Boyd was Chief Engineer.

USA CRREL is an Army Materiel Command laboratory.

*ACFEL was merged with the former Snow, Ice, and Permafrost Research Establishment (SIPRE) in 1961 to form the Cold Regions Research and Engineering Laboratory (USA CRREL), Hanover, New Hampshire.
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SUMMARY

Errors inherent in temperature measuring elements immersed in fluid streams are discussed for both steady-state and transient temperature conditions. Errors in steady-state conditions are due to calibration error, irregularities in signal transmission, mechanical malfunction of instrumentation, and thermal effects owing to the exchange of heat between the measuring element and its environment. These may generally be reduced to an acceptable minimum by proper design and installation. An additional source of error which must be considered in transient temperature conditions is the "thermal lag" which results from the fact that any physical system immersed in a fluid of changing temperature does not respond with time nor amplitude as does the fluid itself.

This report presents a criterion by which to judge the acceptability of the premise of instantaneous spatially uniform temperature within a temperature sensing element during thermal transients, and the subsequent use of a simplified analysis to determine the dynamic error in measurement.
INTRODUCTION

Temperature measuring elements immersed in fluid streams provide a signal which may be translated, for purposes of control and measurement, into the temperature existing at an instant of time in the fluid. However, owing to many effects such an indicated temperature seldom corresponds to the instantaneous temperature of the fluid. Uncertainties in steady-state measurement arise from many sources, among which are: calibration error, irregularities in the transmission of the pick-up signal, mechanical malfunction of instrumentation, and thermal effects owing to the exchange of heat between the measuring element and its environment. Careful design and installation practice can usually reduce errors from these sources to an acceptable minimum.

In the case of transient temperature conditions, however, design must minimize an additional source of error called the "thermal lag," which results from the fact that any physical system immersed in a fluid of changing temperature does not respond with time or amplitude as does the fluid itself. Hence, a temperature measuring element employed for purposes of control senses changes in temperature which invariably lag behind the true fluid temperature which the element is supposed to control. Thus, the indicated or measured temperature is always in error, even though no error exists from calibration, transmission, instrumentation or from heat exchange with an environment other than the fluid.

This report presents a criterion by which to judge the acceptability of the premise of instantaneous spatially uniform temperature within a temperature sensing element during thermal transients, and the subsequent use of simplified analysis for the determination of the transient heat conduction in solids with specific application here to cylinders and spheres which are subjected to sudden changes in temperature. Results for these two geometries should have broad application to a great many types of temperature sensing elements. It is recognized that other types of transients occur, such as oscillations in temperature, but it is believed that the nature of the transient is of secondary importance to the major conclusions presented herein.

NOTATION AND CONVERSION TABLE

<table>
<thead>
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<th>Symbol</th>
<th>Definition</th>
<th>Multiply by</th>
<th>To obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Thermal diffusivity, sq ft/hr</td>
<td>0.092903</td>
<td>m²/hr</td>
</tr>
<tr>
<td>c_p</td>
<td>Specific heat, Btu/lb °F</td>
<td>4.1868</td>
<td>J/g°C</td>
</tr>
<tr>
<td>h</td>
<td>Heat transfer coefficient, Btu/sq ft hr °F</td>
<td>4.882</td>
<td>kg cal/hr m² °C</td>
</tr>
<tr>
<td>J₀</td>
<td>Bessel function of zero order</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J₁</td>
<td>Bessel function of first order</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity of solid, Btu/sq ft hr °F/ft</td>
<td>1.488</td>
<td>kg cal/hr m °C</td>
</tr>
<tr>
<td>k_a</td>
<td>Thermal conductivity of air, Btu/sq ft hr °F/ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>Coordinate normal to surface at radius r₀</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TEMPERATURE DISTRIBUTION WITHIN THERMAL SENSING ELEMENTS

NOTATION AND CONVERSION TABLE (Cont'd)

\( r \) Radius, ft 30.48 cm
\( r_o \) Outside radius, ft
\( t \) Temperature of measuring element, °F 5/9 (°F - 32) °C or °K
\( t_f \) Ambient fluid temperature, °F
\( t_c \) Temperature at center of solid, °F
\( t_s \) Temperature at surface of solid, °F
\( t_i \) Initial temperature of solid and ambient fluid, °F
\( z \) Axial coordinate
\( \theta \) \( t - t_f, \) °F
\( \theta_i \) \( t_i - t_f, \) °F
\( \theta_s \) \( t_s - t_f, \) °F
\( \theta_c \) \( t_c - t_f, \) °F
\( p \) Density, lb/cu ft 16.0185 kg/m³
\( \tau \) Time, hr
\( \psi \) Angular coordinate
\( \beta_n \) Boundary value
\( \beta_i \) Boundary value
\( \beta_1 \) First boundary value

TEMPERATURE DISTRIBUTION DURING A THERMAL TRANSIENT

Evaluation of thermal error

Two separate considerations are required in the estimation and minimization of time and amplitude of transient temperature errors. First, a transient thermal condition originating from the fluid causes a spatial variation of temperature within the temperature sensing element itself, so that both the mean temperature and the centerline temperature of the measuring element differ from the temperature of the element's surface in contact with the fluid. Second, the surface temperature differs from the temperature of the main body of fluid because of a thermal resistance between the surface of the measuring element and the fluid, originating from what is sometimes called the thermal boundary layer. These errors are shown graphically on Figure 1 at some instant of time for a solid cylindrical measuring element. On Figure 1, the fluid temperature is shown to be lower than the surface temperature; however, the following discussion is also applicable if the fluid temperature is greater than the surface temperature.

As seen from Figure 1, during a transient a difference may be expected between the temperatures at the centerline and at the surface of a temperature sensing element, both of which differ from the element mean temperature. The mean temperature is registered only when the sensing element occupies the entire
TEMPERATURE DISTRIBUTION WITHIN THERMAL SENSING ELEMENTS

Figure 1. Instantaneous temperature distribution in a solid cylindrical temperature sensing element.

cross-section of the solid, as in the case of a bare thermocouple, resistance element or liquid thermometer. However, because of other considerations, such as protection against contamination and corrosion, a sheath or protection tube may be placed around the sensing element, in which case it may measure a temperature nearer that of the centerline. Both of these conditions present difficulties in the evaluation of thermal error since it is the surface temperature that is most conveniently related to the instantaneous fluid temperature (the desired quantity), through the concept of the heat transfer coefficient.

A rational design of a measuring element, for minimum uncertainty in temperature measurement, must therefore take into consideration the spatial variation of temperature within the element itself. It may be possible by appropriate design to reduce spatial variation to essentially zero in some cases but certainly not in all cases. In any event, it is important to estimate its magnitude in order to evaluate the performance of a given application.

The most ideal condition would be one in which $t_c$ and $t_s$ (Fig. 1) were always identical. Such a condition would exist if the thermal conductivity, $k$, of the solid were assumed to be infinitely large in the radial direction. This assumption removes any further consideration of the spatial variation of temperature within the solid along this as well as other directions of infinite thermal conductivity. The temperature may vary with time within the solid, but the result of the assumption of infinite thermal conductivity is that this variation is spatially uniform; i.e., the
temperature is uniform along any axis coincident with an axis of infinite thermal conductivity at any instant of time. Should the thermal conductivity be assumed infinitely large in all spatial directions, at any instant of time the entire solid is at a uniform temperature. The analysis of the time response and dynamic error of such systems is then greatly simplified and can be practically employed. Analysis of the response of these systems is presented in Bailey (1931) and Clark (1955). However, in many cases it is not always readily evident whether the above assumption is practical and, of equal importance, what the consequences of it may be.

As a measure of the magnitude of the spatial variation of temperature within an element, this analysis is concerned primarily with the instantaneous difference in temperature, \( t_c - t_s \), between the centerline and the surface of a measuring element. For generalization this temperature difference is put into dimensionless form as \( (t_c - t_s)/\theta_i \), where \( \theta_i \) is the sudden change in temperature to which the fluid surrounding the element is subjected. Hence, if at all times, or for certain times following the introduction of the transient, the quantity \( (t_c - t_s) \) is small in relation to an acceptable uncertainty in temperature specified for a given design and application, the spatial variations in temperature within the element may be ignored.\(^*\) Other steps must be taken, including changes in design and/or installation, if this quantity exceeds an allowable maximum.

**Generalized theory**

The instantaneous temperature distribution in a homogeneous isotropic solid not containing heat sources may be determined from the following expression of the First Law of Thermodynamics and Fourier's Law of Conduction in cylindrical coordinates:

\[
\frac{\partial \theta}{\partial \tau} = \frac{k}{\rho c_p} \left[ \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \psi^2} + \frac{\partial^2 \theta}{\partial z^2} \right].
\]

The solution to this equation for various boundary conditions has been the subject of much effort by mathematicians and engineers. A great many of these solutions for particular cases have been published in mathematical forms (Boelter et al., 1948; Carslaw and Jaeger, 1948; Ingersoll et al., 1948; Jakob, 1949) and are available also in the form of graphs and charts (Groeber, 1925; Heisler, 1947; McAdams, 1954).

Generalized but unwieldy solutions of eq 1 have been obtained (Carslaw and Jaeger, 1948, Chapter VIII) for the finite, semi-finite, and infinite cylinders. For many applications considerable mathematical simplification may be achieved without significant loss in generality by assuming both \( \partial^2 \theta / \partial \psi^2 \) and \( \partial^2 \theta / \partial z^2 \) equal to zero. The case of \( \partial^2 \theta / \partial z^2 \) not equal to zero is treated in Clark (1955) for a stepped temperature probe. The case of \( \partial^2 \theta / \partial \psi^2 \) not equal to zero is unimportant for probes whose thermal element occupies the entire cylindrical cross section placed in a fluid stream, where radial symmetry may be reasonably assumed, and is probably not of first importance for solid cylinders made of materials with large values of the property \( k/\rho c_p \). Equation 1 will be so modified for this analysis.

The boundary conditions appropriate to the case of transient response are shown in Figures 2a, b according to the following:

\[
\theta = \theta_i = t - t_f \quad (2a)
\]

when \( \tau = 0 \);

\(^*\)An exception, which is not evaluated herein, is the effect of axial heat conduction during a transient. This effect is examined in Clark (1955), and in many applications it may be essentially eliminated by installation techniques.
TEMPERATURE DISTRIBUTION WITHIN THERMAL SENSING ELEMENTS

\[ \frac{\theta}{\theta_i} = \frac{t - t_f}{t_i - t_f} = 1 \] (a)

\[ \frac{\theta}{\theta_i} = \frac{t - t_f}{t_i - t_f} < 1 \] (b)

Figure 2. Temperature distribution before and following a sudden change in fluid temperature.

\[ \frac{\partial \theta}{\partial r} = -\frac{h}{k} \theta_s \] (2b)

when \( r = r_0, \tau > 0; \)

\[ h = \text{constant, } t_f = \text{constant} \] (2c)

when \( \tau > 0. \)

For these boundary conditions Jakob (1949) gives the solution to eq 1 for solid cylindrical shapes for the temperature excess, \( \theta, \) of both the surface and the center as:

\[ \frac{\theta_c}{\theta_i} = \sum_{n=1}^{\infty} \frac{2}{\beta_n} \frac{J_1(\beta_n)}{J_0(\beta_n) + J_1(\beta_n)} \exp \left( -\beta_n^2 \frac{ar}{r_0^2} \right), \] (3)

\[ \frac{\theta_s}{\theta_i} = \sum_{n=1}^{\infty} \frac{2}{\beta_n} \frac{J_1(\beta_n)}{J_0(\beta_n) + J_1(\beta_n)} J_0(\beta_n) \exp \left( -\beta_n^2 \frac{ar}{r_0^2} \right), \] (4)

where the \( \beta_n \) are the roots of the equation \( m/\left(hr_0/k\right) = J_0(m)/J_1(m), \) demanded by the boundary condition, eq 2a. The quantity of interest \( (t_c - t_s), \) which is being used as the criterion for non-uniformity of temperature within the solid, may be found directly from eq 3 and 4, noting that

\[ \frac{t_c - t_s}{\theta_i} = \frac{\theta_c - \theta_s}{\theta_i} = \frac{\theta_c}{\theta_i} - \frac{\theta_s}{\theta_i} \]
hence,

\[
\frac{t_c - t_s}{\theta_i} = \sum_{n=1}^{\infty} \frac{2}{\beta_n} \frac{J_1(\beta_n) [1 - J_0(\beta_n)]}{J_0^2(\beta_n) + J_1^2(\beta_n)} \exp \left( -\frac{\beta_n^2 \alpha r}{r_o^2} \right). \tag{5}
\]

Since the series in eq 3 and 4 are convergent, eq 5 is also convergent. This equation is the principal result for solid cylindrical temperature sensing elements and will be discussed below.

Should the geometry of a temperature measuring element more closely resemble a sphere, the solutions for the transient temperatures at both the center and the surface, subject to the boundary conditions of eq 2, are also given by Jakob (1949) as:

\[
\frac{\theta_c}{\theta_i} = \sum_{n=1}^{\infty} 2 \frac{\sin \beta_n - \beta_n \cos \beta_n}{\beta_n - \sin \beta_n \cos \beta_n} \frac{\exp \left( -\beta_n^2 \frac{\alpha r}{r_o^2} \right)}{\exp \left( f_3 n r \right)} \tag{6}
\]

\[
\frac{\theta_s}{\theta_i} = \sum_{n=1}^{\infty} 2 \frac{\sin \beta_n - \beta_n \cos \beta_n}{\beta_n - \sin \beta_n \cos \beta_n} \left( \frac{\sin \beta_n}{\beta_n} \right) \exp \left( -\frac{\beta_n^2 \alpha r}{r_o^2} \right). \tag{7}
\]

The various values of \( \beta_n \) are the roots of the equation \( \lambda r / k = 1 - \beta_1 \cot \beta_1 \), as demanded by the boundary conditions, eq 2a.

The difference between the temperature at the center and that at the surface is found from eq 6 and 7 as:

\[
\frac{\theta_c - \theta_s}{\theta_i} = \frac{t_c - t_s}{\theta_i} = \sum_{n=1}^{\infty} 2 \left[ \frac{\sin \beta_n - \beta_n \cos \beta_n}{\beta_n - \sin \beta_n \cos \beta_n} \right] \left[ 1 - \frac{\sin \beta_n}{\beta_n} \right] \exp \left( -\frac{\beta_n^2 \alpha r}{r_o^2} \right). \tag{8}
\]

The series in eq 8 is convergent.

**Thermal error for dimensionless time**

The thermal "error" may be easily computed for both a cylinder and sphere for values of the dimensionless time, \( \alpha r / r_o^2 \), greater than that given in Figures 3 and 4. For values of \( \alpha r / r_o^2 \) greater than about 1/2, only the first term in the series of both eq 5 and 8 is important. Therefore, the time variation in \( (t_c - t_s) \) for large times may be written:

**Cylinder:**

\[
\frac{t_c - t_s}{\theta_i} = \frac{2}{\beta_1} \frac{J_1(\beta_1) [1 - J_0(\beta_1)]}{J_0^2(\beta_1) + J_1^2(\beta_1)} \exp \left( -\beta_1^2 \frac{\alpha r}{r_o^2} \right). \tag{9}
\]

**Sphere:**

\[
\frac{t_c - t_s}{\theta_i} = \frac{2}{\beta_1} \left[ \frac{\sin \beta_1 - \beta_1 \cos \beta_1}{\beta_1 - \sin \beta_1 \cos \beta_1} \right] \left[ 1 - \frac{\sin \beta_1}{\beta_1} \right] \exp \left( -\beta_1^2 \frac{\alpha r}{r_o^2} \right). \tag{10}
\]
Figure 3. Temperature distribution within a solid cylinder during a thermal transit.

Figure 4. Temperature distribution within a solid sphere during a thermal transit.
TEMPERATURE DISTRIBUTION WITHIN THERMAL SENSING ELEMENTS

It will be noted that both eq 9 and 10 approach zero at large values of \( a \tau / r_0^2 \). The values of \( \beta_1 \) depend upon the magnitude of \( h r_0 / k \). These have been computed by Jakob (1949), and are given in Figure 5 for values of \( h r_0 / k \) from 0 to infinity. The Bessel functions \( J_0(\beta_1) \) and \( J_1(\beta_1) \) are found in mathematical handbooks, such as Jahnke and Emde (1945).

![Figure 5. Boundary value as a function of \( h r_0 / k \).](image)

DISCUSSION

The principal results of this analysis are embodied in eq 5 and 8. Inspection of these equations and the boundary condition relating to heat exchange with the fluid, eq 2a, indicates that the difference in temperature between the center of the probe and its surface, \( t_C - t_S \), is a function of \( \theta_1 \), \( h r_0 / k \) and \( (k/\rho c_p) \tau / r_0^2 \). These results are plotted in dimensionless form in Figures 3 and 4 as \( (t_C - t_S)/\theta_1 \) vs \( (k/\rho c_p) \tau / r_0^2 \) with \( h r_0 / k \) as a parameter.

The quantity \( (k/\rho c_p) \tau / r_0^2 \) is a dimensionless time, relating the probe geometry, characterized by \( r_0 \), and the probe material, characterized by the thermodynamic property \( k/\rho c_p \) called the thermal diffusivity, with the time, \( \tau \), subsequent to the transient step change in fluid temperature. The parameter \( h r_0 / k \) is of the form of a "Nusselt modulus" and may be regarded as a measure of the ratio of the thermal resistance of the solid to that of the thermal boundary layer between the surface and the fluid.

A very broad range of conditions is represented by the range of variables shown in Figures 3 and 4. The parameter \( h r_0 / k \) is given for a range of zero to infinity, which includes all possible combinations of the quantities \( h \), \( r_0 \), and \( k \). The term \( (t_C - t_S)/\theta_1 \) can never be less than zero nor greater than 1.0; hence the range 0 to 1.0 represents all possible values. The dimensionless time, \( (k/\rho c_p) \tau / r_0^2 \), has zero as its lower limit and infinity as its upper limit, since time may be extended to very large values. The range of greatest practical interest is from 0 to about 0.4, as shown, since the largest values of \( (t_C - t_S)/\theta_1 \) occur in this interval of dimensionless time for all possible values of the parameter \( h r_0 / k \).
These results may be easily extended to any value of \((k/\rho c_p)r/r_0^2\) to infinity, as is discussed below.

The results for a cylinder are similar to those for a sphere. For fixed values of \(hr_0/k\) and \((k/\rho c_p)r/r_0^2\) the quantity \((t_c - t_s)\theta_1\) is generally less for a sphere than for a cylinder because of the greater area-to-volume ratio associated with a sphere. As is observed from an inspection of both Figures 3 and 4, curves of a given value of \(hr_0/k\) cross and fall below the curves of lesser value of this parameter at some unique value of the dimensionless time, \((k/\rho c_p)r/r_0^2\). This cross-over point depends upon any two values of \(hr_0/k\) considered, and divides the dimensionless time into two domains, the first corresponding to small values of time in which systems having the greater magnitude of \(hr_0/k\) have the greater \((t_c - t_s)/\theta_1\) and the second corresponding to larger values of time in which systems with the greater \(hr_0/k\) have the smaller "error" or value of \((t_c - t_s)/\theta_1\). At the cross-over point, the value of \((t_c - t_s)/\theta_1\) is the same for each of the two systems having different values of \(hr_0/k\). Thus, it is not possible to generalize on the effect of the parameter \(hr_0/k\) on the quantity \((t_c - t_s)/\theta_1\) which is being taken as a measure of the uniformity of temperature within the solid. At values of the dimensionless time less than that corresponding to the cross-over point, larger values of \(hr_0/k\) produce greater non-uniformity of temperature within a solid temperature measuring element and hence greater uncertainty in measurement. The reverse is true for values of the dimensionless time greater than that corresponding to the cross-over point.

The effect of the magnitude of \(hr_0/k\) on the difference \(\theta_m\) between the fluid temperature and the mean temperature of a solid cylinder at the cross-over point is shown schematically in Figure 6. In this instance \((t_c - t_s)/\theta_1\) is the same for each case represented by a different value of \(hr_0/k\). However, the magnitude of \(\theta_m\), the dynamic error in the mean temperature, is significantly less for the case having the larger value of \(hr_0/k\). This is very important to the discussion of thermal errors in the measurement of temperature during thermal transients. This subject is taken up elsewhere (Bailey, 1931; Clark, 1955) and is not appropriate to the present discussion, which is limited to an evaluation of the degree of uniformity of temperature within a solid temperature measuring element. It is generally true that the larger the heat transfer coefficient \(h\), the more rapid is the rate of change of temperature at all points within a solid of given shape and geometry.

Both Figures 3 and 4 show that for a given value of \(hr_0/k\) the greatest non-uniformity in temperature within the solid occurs at relatively small values of the dimensionless time \((k/\rho c_p)r/r_0^2\). At larger values of this quantity the temperature distribution rapidly approaches one of uniformity, i.e., \((t_c - t_s)/\theta_1\) approaches zero. The magnitude of the physical time \(\tau\) (hours, seconds, etc.) for a given value of dimensionless time depends upon the quantity \((k/\rho c_p)/r_0^2\). The physical time \(\tau\) is small or large accordingly as the quantity \(r_0^2/(k/\rho c_p)\) is small or large for a given magnitude of the dimensionless time. The physical time corresponding to maximum nonuniformity of temperature is the least for temperature sensing elements having the smallest practical diameter and made from a material with the largest possible magnitude of thermal diffusivity, \(k/\rho c_p\). The value of this property for several common materials is shown in Figure 7. As might be expected, magnesium, aluminum, copper and silver have the largest values. Some of the usual construction materials such as nickel, iron, carbon steel and stainless steel have the lowest values of \(k/\rho c_p\). For comparison purposes the thermal diffusivity of atmospheric air is shown. Its magnitude is in the same range as the steels, nickel and titanium, which means that air would have about the same thermal equalization rate for a shape of the same geometry.
Figure 6. Effect of heat transfer coefficient on the dynamic error.

Figure 7. Thermal diffusivity of common materials (Eckert, 1950).

LITERATURE CITED


TRANSPORT TEMPERATURE DISTRIBUTION WITHIN THERMAL SENSING ELEMENTS

This report presents a criterion by which to judge the acceptability of the premise of instantaneous spatially uniform temperature within a temperature sensing element during thermal transients, and the subsequent use of a simplified analysis to determine the dynamic error in measurement. Errors inherent in temperature measuring elements immersed in fluid streams are discussed for both steady-state and transient temperature conditions. Errors in steady-state conditions are due to calibration error, irregularities in signal transmission, mechanical malfunction of instrumentation, and thermal effects owing to the exchange of heat between the measuring element and its environment. These may generally be reduced to an acceptable minimum by proper design and installation. An additional source of error which must be considered in transient temperature conditions is the "thermal lag" which results from the fact that any physical system immersed in a fluid of changing temperature does not respond with time nor amplitude as does the fluid itself.
Thermometry
Thermostats (Low temperature)
Probes (Thermal conductivity)