Ice Effects on Hydraulics and Fish Habitat

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PREFACE

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Ice Effects on Hydraulics and Fish Habitat

GEORGE D. ASHTON

INTRODUCTION

In the temperate zones of the world the formation of ice in streams and rivers may have significant effects on their water depths and velocities, even when the stream discharge does not change. Velocity and water depth are among the attributes that constitute the habitat of the aquatic species that reside in the streams and rivers. Thus, to evaluate the habitat it is important to be able to characterize the effects of ice on velocity and depth.

In this report I summarize the effects of river ice on hydraulic behavior, with examples meant to provide guidance in evaluating what habitats may be expected in winter in comparison with non-ice conditions of the same stream. While river ice formations may be complex at the small scale of tens of meters or less, on a larger scale they are often sufficiently uniform to enable reasonable comparisons of depth and velocities with and without ice present. The emphasis here is on shallow rivers, such as the Platte River in Nebraska. The Platte is subject to many competing demands for its flow, among which are the demands for habitat. A concern for the required winter flows was the motivation for this study.

THEORETICAL BACKGROUND

Steady flows without ice

For a stream of uniform slope $S$, the mean velocity $V$ (feet per second) is related to the hydraulic radius $R$ (ft) (defined as the cross-sectional area divided by the wetted perimeter) by the Manning equation:

$$ V = \frac{1.49}{n} R^{2/3} S^{1/2} \quad (1) $$

where $n$ is an empirical (Manning’s) coefficient associated with the roughness of the bed surface. By multiplying by the cross-sectional area $A$, this may be put in the form of a discharge relation:

$$ Q = AV = \frac{1.49}{n} AR^{2/3} S^{1/2} \quad (2) $$

where $Q$ is the discharge. Most streams are much wider than they are deep, so the hydraulic radius is nearly equal to the depth ($R = D$), and the cross-sectional area $A = BD$, where $B$ is the width and $D$ is the depth. For non-ice-covered flow, then,

$$ Q_o = AV = \frac{1.49}{n_b} BD_o D_o^{2/3} S_o^{1/2} \quad (3) $$
Table 1. Values of Manning’s n. (After Rouse 1950.)

<table>
<thead>
<tr>
<th>Value of Manning’s n</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.016–0.017</td>
<td>Smooth natural earth channels, free from growths, with straight alignment.</td>
</tr>
<tr>
<td>0.020</td>
<td>Smooth natural earth, free from growths, little curvature. Very large canals in good condition.</td>
</tr>
<tr>
<td>0.022</td>
<td>Average, well-constructed, moderate-sized earth canals in good condition.</td>
</tr>
<tr>
<td>0.025</td>
<td>Very small earth canals or ditches in good condition, or larger canals with some growth on banks or scattered cobbles in bed.</td>
</tr>
<tr>
<td>0.030</td>
<td>Canals with considerable aquatic growth. Rock cuts, based on average actual section. Natural streams with good alignment, fairly constant section. Large floodway channels, well maintained.</td>
</tr>
<tr>
<td>0.035</td>
<td>Canals half choked with moss growth. Cleared but not continuously maintained floodways.</td>
</tr>
<tr>
<td>0.040–0.050</td>
<td>Mountain streams in clean loose cobbles. Rivers with variable section and some vegetation growing in banks. Canals with very heavy aquatic growths.</td>
</tr>
<tr>
<td>0.050–0.150</td>
<td>Natural streams of varying roughness and alignment. The highest values for extremely bad alignment, deep pools and vegetation, for floodways with heavy stand of timber and underbrush.</td>
</tr>
</tbody>
</table>

where the subscript $o$ denotes the open water case and $n_b$ is the roughness of the bottom. The open water case is shown in Figure 1a. The value of the Manning roughness factor is generally determined empirically from measurements of $R$, $S$ and $V$ in the field; however, there is sufficient experience that a good estimate can be made from general descriptions of the boundary surface. Table 1 gives typical values for various surfaces ranging from concrete to earth with weeds. Typical values are on the order of 0.02–0.04 for most natural streams and higher for mountain streams. Note also that $n$ has dimensions of $\text{ft}^{1/6}$ to make eq 1 dimensionally homogeneous, and the coefficient 1.49 has dimensions of $R$ ($g$) where $g$ is gravitational acceleration ($\text{m s}^{-2}$).

**Steady flow with ice**

A similar analysis of steady flow in a stream of uniform slope with ice may also be done (Fig. 1b). Since the wetted perimeter is now doubled, $R = D_i/2$ and the ice-covered equivalent of eq 3 from eq 2 becomes

$$Q_i = \frac{1.49}{n_c} BD_i \left(\frac{D_i}{2}\right)^{2/3} S_i^{1/2}$$

(4)

where the applicable roughness value $n$ has a subscript $c$ representing a "composite" roughness, that is, one that depends on both the roughness of the bed and the roughness of the underside of the ice. Much work has been done to determine the appropriate value of the composite roughness as a function of the roughness of the bed and the ice undersurface. One widely accepted formula is the Sabaneev expression:

$$n_c = n_b \left[ 1 + \left( \frac{n_i/n_b}{2} \right)^{2/3} \right]$$

(5)
where $n_i$ is the roughness coefficient of the ice undersurface. For a smooth ice undersurface such as often forms when the ice thickens in place during the winter, $n_i$ can be as small as 0.01, while for an ice cover formed from fragments of broken ice, $n_i$ can be as large as 0.06 or more.

The value of $n_i$ for ice covers formed from broken ice seems to depend on the thickness of the accumulation making up the ice cover. Table 2 presents typical data (Nezhikovskiy 1964).

Equation 4 does not directly include the ice cover thickness, only the depth of flow beneath the ice cover. The water level, of course, is higher since the floating ice cover displaces water. The height of the water (above the bed) is denoted by $H$ to distinguish it from the flow depth under the ice cover. Since most ice covers in medium-sized and large streams are floating, the displacement effect is associated with the specific gravity of ice relative to water and

$$H = 0.916 t_i + D_i$$

where $t_i$ is the ice cover thickness, and 0.916 is the ratio of the density of ice to the density of water.

### COMPARISON OF OPEN-WATER AND ICE-COVERED CASES

Equations 3 and 4 provide a basis for comparing the depths and velocities for the open-water case and the ice-covered case. I will assume that in both cases the discharge is the same, since it generally depends on things upstream of the reach of interest. I first assume the

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**Table 2. Manning's $n$ for ice accumulations at freeze-up. (After Nezhikovskiy 1964.)**

<table>
<thead>
<tr>
<th>Thickness of accumulation (m)</th>
<th>Loose slush</th>
<th>Dense slush</th>
<th>Ice floes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>—</td>
<td>—</td>
<td>0.015</td>
</tr>
<tr>
<td>0.3</td>
<td>0.010</td>
<td>0.013</td>
<td>0.04</td>
</tr>
<tr>
<td>0.5</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>0.7</td>
<td>0.02</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>1.0</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
</tr>
<tr>
<td>1.5</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>2.0</td>
<td>0.04</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>3.0</td>
<td>0.05</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>5.0</td>
<td>0.06</td>
<td>0.09</td>
<td>—</td>
</tr>
</tbody>
</table>
uniform flow case for which the slope of the water surface is the same as the slope of the stream bottom and hence the same for both open and ice-covered conditions. From eq 3

\[ D_o = \left[ \frac{n_b Q}{1.49 BS^{1/2}} \right]^{3/5} \]  

(7)

and from eq 4

\[ D_i = \left[ \frac{n_i Q 2^{2/3}}{1.49 BS^{1/2}} \right]^{3/5} \]  

(8)

The ratio of depth under the ice to open-water depth is, under the conditions I have assumed (same \( Q, B \) and \( S \)),

\[ \frac{D_i}{D_o} = \left[ \frac{n_i}{n_b} 2^{2/3} \right]^{3/5} \cdot \frac{n_b}{n_i}^{3/5} \cdot 1.32 \]  

(9)

Neglecting the effect of \( n_i/n_b \) (discussed later), eq 9 shows that for a constant slope the depth under the ice increases 32% over the open-water case. Since the velocity is inversely proportional to the depth, if the discharge is the same \( Q = V_o D_o = V_i D_i \), then the velocity under the ice is reduced by 24% relative to the open-water case. This effect is that due to doubling the wetted perimeter.

**COMPOSITE ROUGHNESS EFFECTS**

I now examine the effects of composite roughness. Incorporating the Sabaneev formula (eq 5) into eq 9 results in

\[ \frac{D_i}{D_o} = 1.32 \left[ 1 + \left( \frac{n_i}{n_b} \right)^{3/2} \right]^{2/5} \]  

(10)

Figure 2 shows a plot of \( D_i/D_o \) as a function of \( n_i/n_b \). The range plotted for \( n_i/n_b \) is somewhat large but met with occasionally. For example, a very smooth ice cover with \( n_i = 0.01 \) and \( n_b = 0.03 \) causes the depth to increase by 7%, while a rough ice cover with \( n_i = 0.06 \) and \( n_b = 0.03 \) causes the depth to increase by 70%.

Similarly, for the same discharge, the velocity will change from the open-water case to the ice-covered case according to the ratio

\[ \frac{V_i}{V_o} = D_o/D_i = 0.76 \left[ 1 + \left( \frac{n_i}{n_b} \right)^{3/2} \right]^{2/5} \]  

(11)

In summary, if the slope remains constant, the depth will increase and the velocity will decrease as long as \( n_i/n_b \geq 0 \).

In the above analysis I have assumed that the slope is constant for the two cases. For shallower streams this is more likely the case. However, particularly for deeper streams, it is also possible for the energy slope to increase locally with little change in depth. I now examine the change in velocity when the depth is forced to remain constant. From eq 3

\[ S_o = \left[ \frac{Q n_b}{1.49 BD_o^{5/3}} \right]^2 \]  

(12)

and from eq 4
Figure 2. Effect of the ratio of ice cover roughness to bed roughness on the flow depths under the ice cover for a river of constant slope.

Figure 3. Effect of the ratio of ice cover roughness to bed roughness on the slope of a river if the depth is held nearly constant.

\[ S_i = \frac{Q n_c 2^{2/3}}{1.49 B D_i^{5/3}} \]  

\[ \frac{S_i}{S_o} = \left( \frac{n_c}{n_b} 2^{2/3} \right)^2 \left( \frac{n_c}{n_b} \right)^2 2.52. \]

Hence, for the same discharge, width and depth,

After the Sabaneev formula for \( n_c \) is applied, \( S_i/S_o \) is plotted in Figure 3.

Now, what does this slope change mean? Suppose a stream or river has a slope of 0.0001. Increasing the slope by a factor of 2.5 (the case of \( n_i = n_b \)) changes the slope to 0.00025. This means the slope can change over a mile with a depth change of 0.5 feet at the end of the reach. For deeper rivers and canals with controls at the ends of the reach, this is the usual effect, while for shallower streams the depth adjusts.

In actual cases, of course, both the slope and the depth adjust, with the relative amounts of each determined by the end conditions (the "boundary conditions"). While somewhat oversimplified, a generalization is that deep, shorter channels adjust to ice by changing slope, while shallower, longer channels adjust to ice by changing depth.

**EFFECT OF ICE ON STAGE**

The analysis presented above describes the change in the depth of flow, i.e., the distance between the undersurface of the ice and the river bottom. However, what is usually observed is the stage of the river, i.e., the elevation of the top of the water, or how high the water rises in a hole cut in the ice cover. The stage with an ice cover is the depth of the flow plus the
displacement due to the ice cover. This displacement depends on the density of the ice. The stage at any point is thus

$$H = D + \frac{\rho_i}{\rho_w} t_i$$  \hspace{1cm} (15)

where $t_i =$ ice thickness  
$\rho_i =$ density of the ice  
$\rho_w =$ density of the water.

For most ice covers $\rho_i/\rho_w = 0.916$. Thus, the thicker the ice, the higher the stage, even with no change in the depth of flow beneath the ice cover.

**OTHER EFFECTS**

The analysis presented above is somewhat idealistic because it assumes a simple uniformly thick ice cover and straight-sided channel cross sections, and it does not accommodate the boundary conditions. There are many other effects that influence the flow beneath an ice cover. Some of these are discussed here.

**Blockage of the flow cross section by ice deposits**

In many natural rivers, ice is produced as frazil, which consists of small crystals of ice that form in the flow. Frazil is often seen in the form of floating slush. This ice is easily transported by the flow and is deposited in slower portions of the cross section. These deposits may extend over the entire depth, leaving flow passages that occupy only part of the cross section beneath the top, solidly frozen ice cover. Experience suggests that there is a critical velocity below which such frazil is deposited and above which any frazil deposits are eroded. The actual value of this threshold velocity is uncertain, but it seems to be between 2 and 3 feet per second (fps). This suggests that for streams steep enough to have velocities over about 2 fps, a flow area will be maintained to carry the discharge at that velocity and the unneeded portions of the cross section will be filled with frazil deposits. The 2-fps value is also approximately the threshold below which a surface ice cover can form and above which arriving ice is transported beneath the ice cover, thus preventing upstream progression of the ice cover. If the initial open-water velocity is above 2–3 fps, the ice production will accumulate and cause a phenomenon termed “staging,” i.e., the accumulation raises the water level and slows the velocity, and the ice cover then progressively accumulates upstream. The result is a cross section largely blocked by frazil deposits with channels through or beneath with a characteristic mean velocity of 2–3 fps.

**Effect of partial ice coverage**

Often the ice cover does not cover the entire channel. The effect is to raise the stage and shift the distribution of flow such that more occurs in the open surface portions of the cross section and less in the ice-covered portions. The stage change is gradual as the coverage goes from zero to complete. This case is analyzed in Appendix A.

**Ice effects on multiple-channel flow distribution**

In rivers with multiple channels, one of the channels may be ice covered while the other remains ice free. The effect is the same as with partial ice coverage of a single channel. More flow is carried by the ice-free channel and less by the ice-covered channel than if both were ice free. The effect on the smaller channel can be significant. This situation is analyzed in Appendix B for the case of two channels. The presence of hydraulic controls other than ice may constrain the ice effects.
This study was motivated by a concern for the habitat associated with the Platte River in Nebraska during the winter. Habitat studies of this river have been ongoing for several years and have included selection of representative study sites, morphologic descriptions and other efforts to evaluate the present habitat and expected effects on that habitat for various flows. This study is preliminary because it relies on existing data and theory rather than on-site measurements during the winter. The study is also limited to an assessment of the distribution of depths and velocities during periods of ice cover. It is expected that these results will provide the basis for data input into simulations of the physical habitat. An example of these simulations is the Physical Habitat Simulation System (Milhous et al. 1984).

The area of concern on the Platte River extends from the confluence of the North Platte and the South Platte near North Platte, Nebraska, to near Grand Island, Nebraska. The morphology of this reach, and subreaches, is more extensively described elsewhere (Bureau of Reclamation 1988). The Platte River over this reach has winter flows typically about 1000–2000 cfs, a slope of about 0.0012 (Fig. 4) and a sand bed. The river typically has a broad dominant channel, often with islands and often with smaller side channels. Depths for flows of 1000–2000 cfs are about 1–2 ft during summer, and mean velocities range from about 1 to 2.5 fps. The dominant channel is typically 300–600 ft wide.

The scope of the present study did not include actual measurements during ice conditions of the study sites in 1989. However, I observed the river on 22 January 1989 from North Platte to Grand Island by driving parallel to it and viewing it at available bridges. I also viewed a videotape taken from an overflight on 12 February 1988. The USGS provided copies of stream gauging notes for water years 1979, 1982 and 1983 for the gauge locations at Overton, Odessa and Grand Island (Fig. 4). These years represent long and short ice seasons and high and low flows. While not strictly representative of the morphology of the river (since gauging

Figure 4. Profile of the Platte River in Nebraska.
a. Overton.

b. Odessa.

c. Grand Island.

Figure 5. Aerial photographs of the gauging stations.
sites are selected for ease in discharge inventory), the data do provide a basis for comparison with the theory presented earlier. In particular it enables comparison of mean depths, mean velocities and stage heights as a function of flow discharge under both ice and non-ice conditions.

Figure 5 shows aerial photographs of the gauging locations at Overton, Odessa and Grand Island. In the analysis that follows, data from measurements with no ice or floating slush only are compared with data from periods of ice conditions. Data for periods of ice conditions were limited to cases where the river was at least 70% ice covered. The effects of partial coverage are discussed in Appendix A.

**Gauge height or stage**

The most easily measured and observed characteristic of rivers is the stage, or the level of the top surface of the water. During periods of ice cover the stage is higher than in summer for the same discharge due to the flow resistance associated with the ice cover and the displacement due to the ice cover thickness. Figure 6 shows the observed gauge heights as a function of discharge for ice and non-ice conditions for the Overton, Odessa and Grand Island gauging locations. The stage–discharge relationship is well defined for non-ice

![Graph](image)

*Figure 6. Stage–discharge observations with and without an ice cover.*
conditions but chaotic for ice conditions, and it is much higher for the latter. From a habitat standpoint these high winter stages may be significant, especially along the banks. However, the irregularity of stage–discharge data during winter obscures a regularity in flow depths below the ice, described next.

Flow depths

Figure 7 shows the observed average flow depths as a function of discharge for the Overton, Odessa, and Grand Island sites for both ice and non-ice conditions. The depth for ice conditions is the average depth beneath the ice covers. The depths for ice conditions are somewhat higher for the same discharge than for non-ice conditions, and there is considerable scatter of the data for both ice and non-ice conditions. As a rough estimate, the under-ice flow depths tend to be 10–20% greater than for non-ice conditions for equivalent discharges. From Figure 2 this corresponds to a ratio of $n_i/n_f$ ranging from about 0.5 to 0.7, which, if $n_f=0.03$, corresponds to an ice roughness of about 0.02. This is about the value in Nezhkovskiy’s data (Table 2) for an ice cover 0.5 m thick composed initially of dense slush.

Flow area

Figure 8 shows the cross-sectional flow areas for ice and non-ice conditions, again for the Overton, Odessa and Grand island sites. Since widths changed little in this range of discharge, these plots show the same results as those for depth. In short the flow areas increased 10–20%.

Mean velocities

Figure 9 shows the mean velocities as a function of discharge for ice and non-ice conditions, again for the Overton, Odessa and Grand Island sites. If the depths (and flow areas) increase, then the velocities decrease, and of course this is what is observed. A 10–20% decrease in velocity is estimated in association with ice conditions.
Figure 7. Flow depth–discharge observations with and without an ice cover.
Figure 8. Flow area–discharge observations with and without an ice cover.
Figure 9. Mean velocity–discharge observations with and without an ice cover.
The implications of these changes for habitat simulations for periods of ice cover on the Platte River reach considered are straightforward. During periods of nearly complete ice cover, the flow depths and velocities can be calculated as for non-ice conditions, and the depths then increase by 10–20% and the velocities decrease by 10–20%. This corresponds approximately to a ratio of $n_l/n_b = 0.5$ to $0.7$ in eq 10 and 11. This is a particularly simple result, which is pleasing.

There are some cautions that need to be raised. First, the distributions of velocity across the cross sections as a function of the distribution of depths has not been examined in detail. I expect they will not be much different for ice and non-ice conditions. Second, all the data used here were from gauging locations established by the USGS, and they are at locations with channel confinement by bridges and with single-channel river geometries. Where side channels exist, significant redistributions of flow may occur, particularly when one of the channels is ice free while the other is ice covered. This situation is analyzed in Appendix B for an idealized case of two channels.

Finally the analysis presented here presumes knowledge of whether or not the flow is ice covered. In turn, this depends on air temperatures and, in the case of the Platte River, on thermal inputs and magnitudes of flows in the river. These are significant, and thermal effects were clear in the data. For example, while the data for all three sites covered the same periods, there were significantly fewer observations of ice cover at Overton than at Odessa and Grand Island. The video coverage of February 1988 clearly showed similar effects, with ice conditions increasing in extent in the downstream direction, or conversely thermal effects increasing as one goes upstream.

Application of findings to PHABSIM

The findings of this study are centered on the Platte River but procedures may be applied to other rivers. The procedure is as follows.

Using the hydraulic analyses contained in PHABSIM (Milhous et al. 1984), calculate the velocities $V_o$ and depths $D_o$ for the cross section under consideration for non-ice periods. Then, for the periods when an ice cover is present, the velocities $V_i$ and depths $D_i$ associated with ice conditions are given by eq 11 and 10, respectively, repeated here:

$$V_i = 0.76 \, V_o \left[ \frac{2}{1 + (n_l/n_b)^{3/2}} \right]^{2/5} \quad (11)$$

$$\frac{D_i}{D_o} = 1.32 \left[ \frac{1 + (n_l/n_b)^{3/2}}{2} \right]^{2/5} \quad (10)$$

To use these equations, the Manning's roughness coefficients for the bed $n_b$ and for the ice $n_i$ must be specified. Since $n_b$ must be specified for the open-channel hydraulic analyses, it is presumably already available. The value of $n_i$ should be specified by using Table 2 as an approximate guideline or by direct measurements in the field. Alternatively the combined effect of $n_l/n_b$ can be determined directly by comparing velocity and depth ratios as a function of discharge using detailed cross-section measurements during both open-water and ice-covered conditions, as was done in this report and further summarized in the next section.

Procedures for determining $n_l/n_b$

Using values of $n_l$ and $n_b$ from other studies is clearly less preferable than directly determining values at the site of interest. The value of $n_b$ is ordinarily determined from open-
channel measurements of discharge, hydraulic radius (or depth), flow area and slope using the Manning’s equation (eq 2). It is desirable to obtain \( n_b \) from periods when the water is cold, since temperature can affect the bed roughness. It is also assumed that \( n_b \) stays constant when the ice cover is added although this assumption has not been adequately validated. With \( n_b \) specified and with measurements during ice cover periods of discharge, flow areas, depths and slopes, the composite roughness \( n_c \) can be determined from eq 4 and \( n_i \) can then be determined from eq 5. The value of \( n_i \) does not stay constant throughout the winter. Newly formed covers tend to have higher roughness values, which attenuate somewhat during the winter, only to increase somewhat just prior to break-up. The major effect of the ice cover is the addition of the second boundary, and the roughness, while of significance in extreme cases, is generally less influential than the effect of this second boundary. For example, a 50% error in the estimate of the value of \( n_i \) results in only about a 20% error in the estimate of depth or velocity.

LITERATURE CITED

**Bureau of Reclamation** (1988) Biological assessment of the Prairie Bend Unit on the whooping crane, interior least tern, piping plover, bald eagle, peregrine falcon, eskimo curlew and black-footed ferret. Prairie Bend Unit, Bureau of Reclamation, Great Plains Region, Billings, Montana.


APPENDIX A: EFFECT OF PARTIAL ICE COVERAGE

INTRODUCTION

It is relatively common for stationary ice to cover only part of the width of a river. This occurs sometimes during formation when ice grows out from the shore. It also often occurs when there is a warm tributary flow that results in a narrow band of open water downstream of the entry point. For these cases the equations developed in the main report may be applied to the open and ice-covered portions and the flows partitioned between the covered and open-flow areas since the energy slope is the same for both. The following analysis of partially covered flows is essentially the same as published by Calkins et al. (1982), but it also includes an explicit result for the limiting case of a very thin ice cover. This explicit result allows the effect of roughness of the ice cover on the increased stage to be evaluated.

ANALYSIS

The analysis starts with the Manning’s equation:

\[ V = \frac{1.49}{n} R^{2/3} S^{1/2} \]  \hspace{1cm} (A1)

For the open area, denoted by the subscript \( o \),

\[ V_o = \frac{1.49}{n_b} D_o^{2/3} S^{1/2} \]  \hspace{1cm} (A2)

where \( n_b \) = bed roughness
\( D_o \) = flow depth
\( S \) = slope.

For the ice-covered area, denoted by the subscript \( i \),

\[ V_i = \frac{1.49}{n_c} \left( \frac{D_i}{2} \right)^{2/3} S^{1/2} \]  \hspace{1cm} (A3)

where \( n_c \) is the composite roughness, which depends on both the bed roughness \( n_b \) and the ice roughness \( n_i \), and \( D_i \) is the depth of flow under the ice. For ice with a density of \( \rho_i \) in water with a density of \( \rho_w \) and for an ice thickness \( t_i \), the stage \( H \) is

\[ H = D_o = D_i + \frac{\rho_i}{\rho_w} t_i \]  \hspace{1cm} (A4)

The coverage \( C \) has a value between 0 (no ice) and 1 (complete ice coverage). The total discharge \( Q \) is the sum of the discharges in the open and covered portions, so that, for a total width \( B \),

\[ Q = CBV_i D_i + (1 - C)BV_o D_o \]  \hspace{1cm} (A5)

Substituting for \( V_i \) and \( V_o \) results in

\[ Q = CB \frac{1.49}{n_i} \left( \frac{D_i}{2} \right)^{2/3} D_i S^{1/2} + (1 - C)B \frac{1.49}{n_b} D_o^{2/3} D_o S^{1/2} \]  \hspace{1cm} (A6)

Using eq A4 this may be expressed in terms of the stage \( H \):
\[ Q = CB \frac{1.49}{n_c} \left( \frac{H - \frac{P_l}{\rho_w} t_i}{2} \right) \left( H - \frac{P_l}{\rho_w} t_i \right) S^{1/2} \]

\[ + \ (1 - C) B \frac{1.49}{n_b} H^{5/3} S^{1/2}. \]  

(A7)

For the stage \( H_0 \) associated with no ice \((C=0)\), eq A7 becomes

\[ Q = \frac{1.49}{n_b} B H_0^{5/3} S^{1/2} \]  

(A8)

or

\[ H_0 = \left( \frac{Q n_b}{1.49 B S^{1/2}} \right)^{3/5}. \]  

(A9)

Equation A7, after some algebra, can also be written in the form

\[ H^{5/3} + \left( \frac{H - \frac{P_l}{\rho} t_i}{2} \right)^{5/3} C n_b \frac{Q n_b}{2^{2/3} (1 - C) n_c} = \frac{Q n_b}{1.49 (1 - C) B S^{1/2}}. \]  

(A10)

This is the result of Calkins et al. (1982) and can be solved for \( H \) by iterative procedures.

For the special case of \( t \to 0 \) (a rigid but infinitely thin ice cover), eq A10, again after some algebra, can be reduced to the form

\[ H^{5/3} = \frac{Q n_b}{1.49 B S^{1/2}} \left[ \frac{2^{2/3} (1 - C) n_c + C n_b}{2^{2/3} (1 - C) n_c} \right]. \]  

(A11)

The stage relative to the ice-free conditions, that is, \( H/H_0 \), is

\[ \frac{H}{H_0} = \left[ \frac{1}{(1 - C) + \frac{C n_b}{2^{2/3} n_c}} \right]^{3/5}. \]  

(A12)

\[ \text{Figure A1. Effect of partial coverage by a stationary ice cover on the flow depth as a function of the roughness ratio.} \]
for the limiting case of a very thin ice cover. Equation A12 can be put in a form containing
\( n_t/n_b \) explicitly using the Sabaneev formula, which is

\[
\frac{n_c}{n_b} = \left[ \frac{1}{2} \left( \frac{n_t}{n_b} \right)^{3/2} \right]^{2/3}.
\]  
(A13)

This results in an expression for \( H/H_o \) explicitly in terms of \( n_t, n_b \) and coverage \( C \) in the form

\[
\frac{H}{H_0} = \left( \frac{1}{(1 - C) + C \left[ \frac{1}{1 + \left( \frac{n_t}{n_b} \right)^{3/2}} \right]^{2/3}} \right)^{3/5}.
\]  
(A14)

Figure A1 shows eq A14 for various values of \( n_t/n_b \) as a function of \( C \). The value of \( H/H_0 \) is the “hydraulic effect” of the top boundary and roughness. Actual \( H/H_0 \) values should be larger due to the displacement effects of finite-thickness ice covers. The case \( n_t/n_b = 0 \) is, of course, physically impossible since it represents an ice cover with no roughness or friction.

The usefulness of Figure A1 is that it shows that the stage rise associated with imposition of an ice cover does not abruptly change as final closure occurs, but rather increases steadily as the coverage goes from 0 to 1.
APPENDIX B: ICE EFFECTS ON MULTIPLE-CHANNEL FLOW DISTRIBUTION

INTRODUCTION

It is not uncommon for rivers to have multiple channels, and the distribution of the total flow to the individual channels depends on their relative hydraulic characteristics of depth, width and surface roughness. During winter the formation of ice covers on these channels may change the distribution of the total flow. Here I examine the extent of some of these ice effects on multiple channel flow distribution for the case of a two-channel situation.

ANALYSIS

I start by imposing the end conditions, and I denote the channels with subscripts 1 and 2, and characterize their widths \( B_1 \) and \( B_2 \), their mean depths \( D_1 \) and \( D_2 \), and their discharges \( Q_1 \) and \( Q_2 \). The total discharge is \( Q \) and the total width is \( B \). I assume that the channels emerge from a single channel and join again at distance \( L \) downstream, with both channels having the same bed slope \( S \) and equal open-channel roughnesses characterized by a Manning's \( n_b \) value, where the \( b \) subscript denotes the roughness associated with the bottom under open-surface conditions.

\[
D_2 = D_1 - \frac{\rho_i}{\rho_w} \frac{\rho_w - \rho_i}{t} \quad \text{(B1)}
\]

where \( \rho_i \) and \( \rho_w \) are the densities of ice and water, and \( t \) is the ice thickness. The velocity in channel 1 is

\[
V_1 = \frac{1.49}{n_b} D_1^{2/3} S^{1/2} \quad \text{(B2)}
\]

and in the ice-covered channel 2,

\[
V_2 = \frac{1.49}{n_c} \left( \frac{D_2}{2} \right)^{2/3} S^{1/2} \quad \text{(B3)}
\]
where \( n_c \) is the composite roughness of the ice-covered channel. The Sabaneev formula for composite roughness gives \( n_c \) in terms of the open-channel (bottom) roughness \( n_b \) and the ice-cover roughness \( n_i \):

\[
n_c = n_b \left[ 1 + \left( \frac{n_i}{n_b} \right)^{3/2} \right]^{2/3}.
\]

As \( n_i/n_b \) ranges from 0.5 to 2, \( n_i/n_b \) ranges correspondingly from 0.77 to 1.54 (Fig. B2).

The total discharge is composed of the discharges in the two channels:

\[
Q = Q_1 + Q_2 = B_1 V_1 D_1 + B_2 V_2 D_2
\]

\[
= B_1 \frac{1.49}{n_b} D_1^{5/3} S^{1/2} + B_2 \frac{1.49}{n_c} D_2^{5/3} S^{1/2}.
\]

The fraction of the discharge carried in channel 1 is

\[
\frac{Q_1}{Q} = \frac{B_1 \frac{1.49}{n_b} D_1^{5/3} S^{1/2}}{B_1 \frac{1.49}{n_b} D_1^{5/3} S^{1/2} + B_2 \frac{1.49}{n_c} D_2^{5/3} S^{1/2}}.
\]

which can be written in the form

\[
\frac{Q_1}{Q} = \frac{1}{1 + \frac{B_2}{B_1} \frac{n_b}{n_c} \left( 1 - \frac{\rho_i}{\rho_w} \right) \frac{D_1^{5/3}}{2^{2/3}}}.
\]

Clearly, if both channels are ice free, the flow is distributed proportional to the widths, that is.
The limiting case of a thin ice cover corresponds to a skim of ice on channel 2 such that \( t/D \to 0 \) and

\[
\frac{Q_1}{Q} = \frac{1}{1 + \frac{B_2}{B_1}} \quad \text{(B8)}
\]

\[
\frac{Q_1}{Q} = \frac{1}{1 + \frac{B_2}{B_1} \frac{n_b}{n_c} \frac{1}{2^{2/3}}} \quad \text{(B9)}
\]

This relationship is shown in Figure B3 as a function of the alternate terms \( B_2/B \) and \( n_b/n_c \). Figure B4 shows the discharge carried by the ice-covered channel 2 relative to the discharge carried by the same channel with no ice is shown (in both cases with the larger channel ice free). This ratio is denoted by \( Q^* \).

\[\text{Figure B3. Effect of flow in channel 1 of a skim of ice on channel 2 as a function of roughness ratio.}\]

In Figure B3 the effect of a thin skim of ice on one channel is to increase the proportion of flow in the ice-free channel relative to the "both channels ice free" case (equivalent to the \( n_b/n_c = 0 \) line in the figure). The effect is not much for the larger channels, ice free or ice covered. In Figure B4 the effect on discharge of the smaller channel is shown. Here the effect is significant, especially for a small side channel covered with ice. As an example for a side channel with a width that is 20% of the total width of both channels and with a ratio of \( n_b/n_c = 1 \), the discharge in the smaller channel is 68% of what it would be under non-ice conditions; adding finite thickness to the ice cover will reduce the side-channel discharge even further.

Similar results hold in the opposite case, that is, a small channel maintained ice free while the wider main channel is ice covered. In this case flow is diverted to the smaller channel.
These results are shown in Figure B5. In this case covering the main channel with ice while maintaining the smaller channel ice free results in a significant increase in the discharge in the smaller channel.

These results are not meant to be definitive for actual field cases, which should be analyzed using conventional hydraulic analyses with ice-cover effects. In some cases the distribution of discharge is determined not by ice-cover effects as analyzed here but by other hydraulic controls. However, it is clear that ice can have significant effects on the flow distribution among channels.
### Ice Effects on Hydraulics and Fish Habitat

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The effects of an ice cover on the flow depths and velocities beneath the ice are analyzed. Data from gauging records for the Platte River in Nebraska are analyzed using this context. A procedure to use the results for habitat simulations during winter periods is suggested. The effects of partial coverage by a stationary ice cover and the effects of coverage of ice on multiple channel flow distributions are analyzed.