Predicting lake ice decay

George D. Ashton
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**Author**: George D. Ashton

**Performing Organization**: U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire 03755

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**Abstract**: A nine-year record of the lake ice decay pattern of Post Pond in Lyme, New Hampshire, is analyzed using a simple algorithm. Quite good correlations between decay rates and thawing degree-days are obtained using heat transfer coefficients on the order of 15-20 W m\(^{-2}\) °C\(^{-1}\).
PREFACE

This report was prepared by Dr. George D. Ashton, Chief, Geophysical Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. Sandra J. Smith processed much of the data. Anthony Gow and John Govoni provided ready access to the original, detailed thickness data. The report was technically reviewed by Michael Bilello and Darryl Calkins. The work was funded by project CWIS 31750, *Prediction of Ice Formation*.

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PREDICTING LAKE ICE DECAY

George D. Ashton

BACKGROUND

In dealing with the effects of ice on water bodies or in assessing ice problems, the most important characteristic of the ice cover is its thickness. Over the years much attention has been devoted to predicting the thickening of the ice cover under the influence of subfreezing air temperatures, but relatively little attention has been devoted to the decay of thickness during the spring. In part the greater attention to the thickening stage is due to the availability of a simple heat-conduction model by which to predict the growth rates (the “Stefan” problem); a simple analysis yields the well-known “square-root-of-freezing-degree-days” method. During the decay period both the top and the bottom of a floating ice cover are at 0°C; hence, the heat-conduction framework of analysis is not applicable, and the problem of predicting thinning rates is conceptually unclear. This report presents a simple algorithm for predicting decay of lake ice and tests it on data from a nine-year study of the lake ice cover on Post Pond in Lyme, New Hampshire.

There is little published work on the decay period of lake ice covers. Bilello (1980) presented extensive data from a variety of locations and examined several empirical approaches to predict decay. He found that an accumulated thawing degree-day (ATDD) index provided the best overall correlations, particularly when the ice cover was not subjected to water currents or other action that would mechanically break up the ice cover. The relationship used by Bilello (1980) is

$$ I = I_m - S(\text{ATDD}) $$

(1)

where

- $I$ = thickness of the decaying ice sheet (cm)
- $I_m$ = maximum ice thickness at the start of ice decay (cm)
- $S$ = “slope” of the ice thickness vs ATDD index plot (cm °C⁻¹)
- ATDD = accumulated thawing degree-days (°C with 0°C base).

This report uses a similar approach but puts it in a more rigorous form by performing a simple heat balance to obtain the same result.

ANALYSIS

This analysis consists of equating the energy flux associated with a melting rate of $d\eta/dt$ to a sensible heat flux relationship based on the difference between 0°C and the above-freezing air temperature; that is,

$$ \rho_i \lambda \frac{d\eta}{dt} = -H_{ai} (T_m - T_a) $$

(2)

where

- $\rho_i$ = density of the ice (916 kg m⁻³)
- $\lambda$ = heat of fusion (3.34 × 10⁵ J kg⁻¹)
- $d\eta/dt$ = thinning rate (m s⁻¹) of an ice cover of thickness $\eta$
- $t$ = time
- $H_{ai}$ = transfer coefficient (W m⁻² °C⁻¹)
- $T_m$ = melting temperature (0°C)
- $T_a$ = air temperature.
Integration of eq 2 yields

\[ \eta = \frac{-H_{ai} (T_m - T_a)}{\rho_i \lambda} + C \] (3)

where \( C \) is the constant of integration. If \( \eta_m \) is the ice thickness at the beginning of melt, then \( C = \eta_m \), the term \((T_m - T_a) t\) is the ATDD (with appropriate units), and eq 3 is equivalent to eq 1.

For lake ice in Canada and Alaska, Bilello found values of \( S \) in eq 1 (or \( H_{ai}/\rho_i \lambda \) in eq 3) to vary between \( 0.43 \times 10^{-7} \) m s\(^{-1}\) °C\(^{-1}\) (0.37 cm day\(^{-1}\) °C\(^{-1}\)) and \( 1.49 \times 10^{-7} \) m s\(^{-1}\) °C\(^{-1}\) (1.29 cm day\(^{-1}\) °C\(^{-1}\)), with an average value for 15 lakes of \( 0.79 \times 10^{-7} \) m s\(^{-1}\) °C\(^{-1}\) (0.68 cm day\(^{-1}\) °C\(^{-1}\)). Not surprisingly the equivalent values of \( H_{ai} \) are very similar to typical heat-transfer coefficients for sensible heat transfer from air to water surfaces, which are on the order of 15-25 W m\(^{-2}\) °C\(^{-1}\).

APPLICATION TO POST POND DATA

Post Pond is a small eutrophic lake of glacial origin located in Lyme, New Hampshire, at a latitude of 44°. Its area is 0.46 km\(^2\) and its maximum depth is 12 m. The ice thickness measurements used here were obtained during the winters from 1973-74 to 1981-82 and are reported more fully by Gow and Govoni (1983). Ice thickness was measured about once a week.

The present method begins with the maximum ice thickness of each season and calculates the expected daily thickness change using eq 2 written in the form

\[ \Delta \eta = \frac{-H_{ai}}{\rho_i \lambda} (T_m - T_a) \Delta t \] (4)

where \( T_a \) was taken as the average of the daily maximum and minimum air temperatures measured at Hanover, New Hampshire, some 20 km south of Post Pond. Figure 1 provides a comparison of predictions with observations for \( H_{ai} \) values of 10, 15 and 20 W m\(^{-2}\) °C\(^{-1}\). When the ice was thickening (i.e. when \( T_a < 0^\circ \text{C} \)), the algorithm used was (Ashton 1980)

\[ \Delta \eta = \left( \frac{1}{\rho_i \lambda} + \frac{1}{H_{ai}} \right) \frac{(T_m - T_a)}{\eta_i} \Delta t . \] (5)

DISCUSSION OF RESULTS

Examination of all nine cases leads to the conclusion that a reasonable value for \( H_{ai} \) for this lake is between 15 and 20 W m\(^{-2}\) °C\(^{-1}\). The use of equation 5 for cold spells (< 0°C) seems to adequately treat delays of thawing or slight increases in thickness. Overall, the agreement between calculated and observed thicknesses is adequate, particularly in light of the extreme simplicity of the method. To include the processes and detailed calculations necessary to improve the method would require that the calculations be an order of magnitude more complex and that the input data, such as wind speed, radiation components and cloudiness be of much greater detail. For many applications the simple method presented here should prove adequate and (for short-term predictions) much better than historical averages.

LIMITATIONS

The extreme simplicity of this method of predicting lake ice decay necessarily has limitations. One limitation is that the appropriate value for the transfer coefficient has been determined for one site only and may not be appropriate for sites, times, or locations where other components of the energy budget are of different relative values. The method clearly does not apply near the shorelines or where stream inlet flows cause enhanced melting.

Implicit in the method is the assumption that the ice surface is reasonably well drained of meltwater. This is usually true, either as a consequence of the porosity that develops inside the ice cover during the deterioration process or as a result of natural cracks or holes in the ice cover. Wake and Rumer (1979) discussed some of the difficulties that occur when the ice is not well drained.

This method does not consider the snow on top of the ice cover. During the initial melt period and after subsequent snowfalls this may lead to some inaccuracy, although generally during spring the snow (but not the snow ice) rapidly melts to the top of the ice.

The method also should not be applied for periods less than a day or so because of the highly variable effects of radiation exchange that occur on a diurnal basis. Finally predictions based on this method are only as good as the forecasts of air temperature.
LITERATURE CITED


*Figure 1.* Predicted ice thicknesses during the decay period starting from maximum observed ice cover thickness and using different H_{al} values. The plotted points are observed thicknesses.
Figure 1 (cont'd). Predicted ice thicknesses during the decay period starting from maximum observed ice cover thickness and using different $H_{ai}$ values. The plotted points are observed thicknesses.