EFFECT OF RATE OF LOADING ON STRENGTH AND YOUNG'S MODULUS OF ELASTICITY OF ROCK

by

R. L. Stowe
D. L. Ainsworth

August 1968

U. S. Army Engineer Waterways Experiment Station
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This paper was prepared for presentation at the Tenth Symposium of Rock Mechanics sponsored by the Intersociety Committee on Rock Mechanics, May 20-22, 1968 at Austin, Tex. The work reported was conducted at the Concrete Division of the U. S. Army Engineer Waterways Experiment Station (WES) under the sponsorship of the Defense Atomic Support Agency (DASA), Nuclear Weapons Effects Research Program A3a, Subtask 13.191A, as authorized by DASA MIPR 503-66. The work was conducted during the period from October 1965 through October 1967, and was under the direction of Messrs. Thomas B. Kennedy, former Chief of the Concrete Division, WES, and Bryant Mather, present Chief of the Concrete Division. The investigation was under the supervision of Messrs. J. M. Polatty, Project Officer, W. O. Tynes, and R. L. Stowe. Mr. Stowe was project leader and prepared this report, assisted by Mr. D. L. Ainsworth.

Directors of WES during the conduct of this study and the preparation of this paper were COL John R. Oswalt, Jr., CE, and COL Levi A. Brown, CE. Mr. J. B. Tiffany was Technical Director.
Abstract

The effect of loading rates ($\dot{\varepsilon}$) on the strength and tangent modulus of elasticity of a basalt, a granite, and a tuff was investigated using static-loading (less than $5 \times 10^3$ psi/sec), rapid-loading ($5 \times 10^3$ to $1 \times 10^7$ psi/sec), and shock-loading (greater than $1 \times 10^7$ psi/sec) techniques. During static loading, specimens of each rock were loaded at a rate of 50 psi/sec. The basalt, found to be the most homogeneous of the three rocks, was then tested at loading rates of 1, 500, and 2250 psi/sec under confined conditions and at loading rates from 1 to $1.6 \times 10^7$ psi/sec under unconfined conditions. Confining pressures of 250, 1000, 4000, and 5000 psi were used during confined testing of all three rocks. The specimens tested using static and rapid techniques were loaded in uniaxial compression, while those specimens tested using the shock-loading method were loaded in one-dimensional strain. Static testing was conducted in a 440,000-lb-capacity universal hydraulic testing machine. Rapid testing at loading rates of approximately $2 \times 10^5$ to $1 \times 10^7$ psi/sec was conducted using a drop tower having a capability of approximately 3800 ft-lb of energy and a 200,000-lb force hydraulic loader. The force loader is a large hydraulic accumulator with a rigid support system containing three pressure chambers. The chambers are actuated by rupture disks serving as rapid-opening valves. Shock loading, at loading rates of about $1 \times 10^{10}$ psi/sec, was carried out using an air-gun apparatus which has the capability of applying pressures
up to approximately 40 kilobars in competent rock. The system continuously measures both shock-wave-propagation velocity and specimen-particle velocities in rock from the passage of a shock wave produced by a flat-plate impact. Stress-strain relations were then deduced from the shock-wave and particle-motion data.

Test data show that both strength and modulus of elasticity increase with an increase in the rate of applied load; strain data from basalt show that total axial strain at failure also increases while total diametral strain at failure decreases with increased loading rates. Ultimate strength was approximately 27, 48, and 70 percent greater for granite, basalt, and tuff, respectively, when rapidly loaded than when statically loaded. A similar comparison is not made for the shock-loading tests since the stress levels induced were not close to the ultimate compressive strength of the materials in one-dimensional strain. Under shock loading, the granite, basalt, and tuff were stressed to $2 \times 10^5$, $6.8 \times 10^4$, and $1.6 \times 10^4$, respectively. However, it should be noted that these materials did not fail at the stress levels indicated above. Results of shock-loading tests indicate that shock response during loading observed for basalt and granite is essentially linear, and stress-strain data for granite were found to agree very favorably with extrapolated data for higher pressure levels. Results of triaxial tests show that as the confining pressure and loading rate increase, the deviator stress and total axial and diametral strain at failure increase.
Data obtained from uniaxial and one-dimensional tests are not comparable due to the specimen geometry under load; however, it does appear that certain physical properties of rock do vary as a function of rates of applied load as do other materials.
Effect of Rate of Loading on Strength and Young's Modulus of Elasticity of Rock

by

Richard L. Stowe* and Donnie L. Ainsworth**

Introduction

The static-, rapid-, and shock-loading response of rock is of interest to many in the field of rock mechanics. For example, the effects of loading rates on strength and stress-strain characteristics of rock are quite important in defining input loading for use in the design of underground protective installations. Response data on rock subjected to shock loading over wide range of pressures are also vital in the field of underground nuclear testing. In addition, material response is needed in design of shock-absorbing liners.

A number of investigators have reported that ultimate compressive strength of rock increases with increased rates of loading.1-6 Wuerker3 stated that modulus of elasticity, obtained from sonic determinations, increased in some rocks by a factor as high as 4.5. It appears that only a limited amount of work has been done on comparing static modulus and modulus obtained from impact testing.6 Phillips4 showed that with increased rates of loading the deformation decreased. More recent work by Atchley and Furr5 showed that when moderate-strength concrete is

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stressed with increased loading rates, the axial strain at failure increased. One would expect this to occur if we think in terms of purely elastic materials. For example, if $E$ is constant and maximum strength, $\sigma_{\text{max}}$, increases, the axial strain would also increase. However, in rock testing we have thus far worked almost exclusively with nonhomogeneous and inelastic rocks. From this fact alone the increase in axial strain with increased load applications is noteworthy.

Many techniques have been devised for studying shock response of rock.\textsuperscript{7,8} Some are based on rigid mechanical considerations and do not consider wave propagation aspects at high loading rates. Plate-impact or explosive-loading techniques which measure the shock-propagation velocity and the free-surface velocity of the sample are successful techniques. These techniques are characterized by the introduction of a known wave form into the material and observing the wave form after propagating through a known distance. These techniques are used primarily in the hydrodynamic region. In low-pressure experiments (< 100 kilobars) the specimen is not in the hydrodynamic region, but rather in a state of uniaxial strain. The shock-loading technique described herein is within the region of uniaxial strain.

**Experimental Techniques**

**Loading rates**

The loading rates used during the static confined (triaxial) and unconfined (tensile and compressive strength) testing were less than
$5 \times 10^3$ psi/sec, specifically, 1, 50, 500, and 2250 psi/sec. Rates between $5 \times 10^3$ and $1 \times 10^7$ psi/sec are defined as rapid-loading rates, while rates greater than $1 \times 10^7$ psi/sec are defined as shock-loading rates.

**Techniques**

Static unconfined and confined tests were conducted using a 440,000-lb universal testing machine and a 10,000-psi-capacity triaxial chamber. The static tests, tensile splitting, compressive strength, and triaxial shear tests were conducted in accordance with the standard procedures, i.e., CRD-C 77-61, CRD-C 19-65, and CRD-C 93-64, respectively, with the exception of loading rates. CRD-C 93-64 was further modified slightly to accommodate wave velocity equipment for purposes of determining the effects of axial and lateral pressures on compressional wave velocities parallel to the core axis. Confining pressures of 250, 1000, 4000, and 5000 psi were utilized during triaxial testing.

The method of triaxial testing as modified with compressional wave velocity equipment involved the use of a cylindrical specimen encased in a flexible membrane and placed in a triaxial chamber, subjected to a constant lateral fluid pressure, then loaded axially to failure.

Fig. 1 is a sketch showing the triaxial chamber and the accessories used inside the chamber for velocity determinations. End plates (housing the transducers) and bearing plates were used to allow for a size reduction to the NX size samples. Aluminum end plates and bearing plates were used because the impedance of aluminum is quite close to that of
TRIAXIAL CHAMBER WITH TRANSDUCERS

Fig. 1
most dense rock. The travel time through the end plates and the bearing plates is measured prior to testing; this travel time is subtracted from the travel time through the plates and rock sample. The equation $V = \frac{d}{t}$ was used to obtain the compressive velocity where $V$ is the velocity, $d$ is the length of the specimen in feet, and $t$ is the pulse travel time in milliseconds through the sample.

The rapid tests were accomplished using both a drop tower and a hydraulically operated 200-kip loader. The drop tower had a capability of 3840 ft-lb of energy with a maximum striking velocity of 22.7 ft/sec produced by a falling mass weighing 384 lb guided by two cylindrical steel columns. The mass was remotely triggered and allowed to fall free from a predetermined height. Friction brakes built into the falling mass prevented any rebound of the mass after impact.

The 200-kip loader consists of a large hydraulic actuator and a rigid support system as shown in fig. 2. The actuator is pressurized with a low-volume, high-pressure multiplier. The actuator has three pressure chambers, above the piston, below the piston, and between the rupture disks. The rupture disks act as a rapid-opening valve. The machine is pressurized by slow buildup of pressure above and below the piston while a slight preload on the specimen is maintained. Concurrently, pressure is built up in the volume between the two rupture disks; the pressure between the rupture disks is maintained at exactly one-half the pressure below the ram, thereby enabling half the total pressure below the piston to be supported by the first rupture disk and the remaining
Fig. 2 200-kip Loader
half of this total pressure to be supported by the second rupture disk. On triggering the machine, the rupture disks burst and move the loading ram onto the specimen which is positioned below the ram.

Shock loading is obtained in specimens using a system involving continuous measurements of both the velocity of propagation of a shock wave and particle velocities in rock and other nonferromagnetic materials from the passage of a shock wave produced by a flat-plate impact. A stress-strain relationship is deduced from the wave and particle motion data. The system has an upper limit of approximately 40 kilobars.

The experimental method is primarily based on a technique used by Frasier and Karpov\textsuperscript{10} for directly measuring the particle velocity in a material. In this technique a fine wire is embedded in the material and placed in a magnetic field. Any subsequent movement of a portion of the wire which cuts the magnetic-flux lines produces an electromotive force ($e$) proportional to the instantaneous velocity as shown in the following equation:

$$\textit{e} = \beta l v$$

where:

$\beta$ = the magnetic field strength,

$l$ = the length of wire cutting the flux lines, and

$v$ = the velocity of the wire (particle velocity).

Several wires are placed in a test specimen and the voltages monitored on oscilloscopes and recorded on photographs. The velocity of propagation can be determined from the initial rises of the voltage signals from the induction wires.
The specimen is mounted at the end of the compressed-air-gun barrel in a magnetic field (fig. 3) and the barrel evacuated to approximately 15 microns. The stress waves will be produced by impacting the specimens with an aluminum projectile (fig. 4). Impact velocities can be obtained up to 1000 ft/sec.

The final state behind the stress wave is determined by the application of the conservation equations. Conservation of mass and momentum across the shock front requires that:

\[ p_0 U_s = p_1 (U_s - U_{pl}) \]  (conservation of mass)

\[ P_o + p_0 U_s^2 = P_1 + p_1 (U_s - U_{pl})^2 \]  (conservation of momentum)

where

- \( p_o \) = initial density,
- \( U_s \) = shock velocity,
- \( p_1 \) = density behind the shock wave,
- \( U_{pl} \) = particle velocity behind the shock wave, and
- \( P_o \) = initial pressure,
- \( P_1 = p_o U_s U_{pl} + P_o \) = the pressure behind the shock wave.

The strain corresponding to \( P_1 \) is

\[ \varepsilon_1 = 1 - \frac{p_o}{p_1} = \frac{U_{pl}}{U_s} \]

These equations are based on the assumption that an equilibrium state of stress is reached behind the shock wave. In this simple form, they are valid only in the region of uniaxial strain. These equations correspond to the Hugoniot or the locus of shocked-end states reached.
Target configuration at muzzle end of gun barrel.

Figure 3    Gas-operated gun.

Figure 4    Schematic of experimental setup.
Rock specimens

The rocks studied all came from the U. S. Atomic Energy Commission's Nevada Test Site (NTS) at Mercury, Nevada. The granodiorite (granite) came from the PILE DRIVER experiment, the basalt came from Buckboard Mesa, and the tuff came from the RED HOT-DEEP WELL experiments. The granite is a light gray, dense, coarse-grained, unweathered rock with phenocrysts of orthoclase feldspar present. The basalt is a light gray, dense, fine-grained, unweathered rock composed of plagioclase feldspar and lesser amounts of pyroxene, olivine, and magnetite. The tuff is a light greenish-yellow to brownish-red, generally poorly indurate rock composed of volcanic ash.

Sample preparation

The rock cores used for static and rapid testing were NX (2-1/8-in.-diameter) in size. The cores were cut to have a length-to-diameter (L/D) ratio of 2 using a diamond saw. After cutting to proper size, the core ends were surface ground and lapped to obtain plane end surfaces. The end surfaces were within 0.001 in. in planeness, were parallel to each other within 0.006 in., and the ends were perpendicular to the sides within 0.5 degrees.

The samples for air-gun tests (fig. 5) were prepared from NX cores sawed into disks and ground plane on a milling machine to a tolerance of 0.01 mm across the 50-mm diameter. These disks were then hand-lapped to a smooth finish, and the thickness measured. The aluminum
Fig. 5 Typical Test Sample for Impact Studies
mounting plate was approximately 1/8-in. thick, flat, and parallel to within 0.001 in., and had a hand-lapped and polished surface.

A 0.0015-in.-diameter wire with an effective length of 0.25 in. (length perpendicular to the field) was placed in the center of each of four disks. The wire was held in this position with a light application of cement (Eastman 910). The disks were then cemented together with epoxy, and a light load was applied until the epoxy cured. The stack of disks was then cemented with epoxy to the aluminum mounting plate with a trigger wire inserted at the aluminum-rock interface. The plexiglas jig shown in fig. 5 is for alignment purposes only and does not separate the rock disks. The assembled specimen was measured to ascertain that the planarity and the glue-layer thickness were within the prescribed limits (i.e., 0.0005 in. and not greater than the diameter of the wire, respectively).

Method of measurement

The rock cores tested statically in unconfined and confined compression had six 13/16-in.-long electrical-resistance strain gages bonded to the core; three gages were placed axially 120 deg apart, and three gages were placed circumferentially 120 deg apart. All gages were located at the midpoint of the core and had a resistance of 120.4 ± 0.2 ohms and a gage factor of 2.01 ± 1 percent. The stress-strain data were recorded on a strip chart oscillograph.

The measurement of compressional wave velocity within the triaxial chamber is accomplished using a through-sample method. A pair of
barium titanate transducers with a lower resonant frequency of 1 megacycle/sec is coupled to either end of the sample with a film of silicone grease. A pressure impulse is imparted to the sample from the expansion of a transducer caused by a step in voltage being applied to the transducer. The incidence of the transmitted pressure impulse on the receiving transducer generates a voltage signal indicating this arrival. These signals are displayed on an oscilloscope and compared with a signal from a crystal-controlled time-mark generator for determining the transit time through the sample. From this measurement of time and the known transmissive path length, the compressional wave velocity can be computed. Fig. 6 shows a typical compressional wave velocity trace. The input signal is on the left and the first arrival is towards the right.

For the drop-tower apparatus a 200,000-lb-capacity SR-4-type load cell, two single-sweep dual-beam oscilloscopes, and two Polaroid cameras were used to record the stress-strain traces. A 1/2-in. piece of celotex was placed on top of the rock sample to mitigate the pulse. The tuff was tested using the drop-tower apparatus. Fig. 7 is a sketch showing the typical traces of the stress-strain pulses recorded during the drop-tower tests.

During testing with the 200-kip loader the load is measured above and below the test specimen by means of strain-gage-type load cells. Accelerations are measured above and below the specimen, using commercial accelerometers. The outputs of all the sensing devices are recorded simultaneously on a multichannel, magnetic tape recorder and later played back using a light-beam galvanometer oscillograph.
Fig. 6 Traces of Compressional Wave Velocity
The top trace is the time mark generator trace with one large division equal to 10 μsec. The bottom trace is the compressional wave velocity signal.
Trace of vertical strain pulse
Strain = 1112 µin./in./cm

Trace of load pulse
Load = 56,380 lb/cm

Time = 1 msec/cm

Trace of horizontal strain pulse
Strain = 1112 µin./in./cm

Trace of load pulse
Load = 56,380 lb/cm

Time = 1 msec/cm

Fig. 7 Traces of Stress and Strain
To observe the particle and shock velocities at various points in a rock specimen, induction wires were mounted at four locations in the specimen, and the specimen was placed in a magnetic field so that the effective length of the wires was perpendicular to the magnetic lines of force (fig. 4).

The impact creates a shock wave which interacts with the induction wire as it propagates through the specimen, and the wire follows the particle motion. Movement of the wire in the magnetic field generates an electromotive force (emf) by cutting lines of magnetic force. This emf is proportional to the particle velocity in the specimen as shown by the following equation:

\[ E = \beta \ell U_p \]  

(1)

where

\[ E = \text{emf, volts}, \]

\[ \beta = \text{magnetic-field strength, webers/m}^2 \]

\[ \ell = \text{effective length of wire, m, and} \]

\[ U_p = \text{particle velocity, m/sec}. \]

The magnetic-field strength \( \beta \) was \( 775 \times 10^{-4} \) webers/m\(^2\) and the wire length \( \ell \) was \( 6.35 \times 10^{-3} \) m (0.25 in.) for all the tests. The shock-propagation velocity can be determined from the time difference between the particle velocity signals and the known wire spacing.

The impedance mismatch between the wire and rock should not cause a serious perturbation in the shock front since its transit time is
small compared to the rise of the shock front. Reflections within
the wire, which should have a frequency of about 40 MHz, should bring
the wire to equilibrium with the rock very rapidly. Thus, the
assumption that the induction wire follows the particle motion of the
rock seems to be justified.

Each induction wire was connected to the input of a Hewlett-
Packard 450A amplifier. The output signal from the amplifier was
fed to the input of a Type 545 Tektronix oscilloscope with a Type CA
preamplifier through 100 ft of RG-8U cable. The cable was terminated
in its characteristic impedance (51 ohms) at the input to the
oscilloscope. Another induction wire was positioned between the
mounting plate and the rock specimen to trigger the oscilloscopes and
the fiducial generator. The fiducial signal was fed to the B input
of the oscilloscopes and recorded on each signal record. This provided
a time reference which could be used to compare all the signals.

Data reduction

The stress-strain path followed by the material in each shot
was deduced from the measured wave propagation and particle velocities.
Each continuous record of particle velocity was approximated by straight-
line segments, which is equivalent to approximating the wave shape by
a series of successive increments of pressure.

From the four induction wires placed in the material, signals
similar to those shown in fig. 8 will be received.
A particle velocity \( U_{pl} \) of amplitude \( E_1 \) occurs at the time \( t_1 \). The magnitude in meters per second of \( U_{pl} \) at \( E_1 \) is determined from the equation

\[
U_{pl} = E_1 \left( 2.03 \times 10^3 \right)
\]

which is obtained by evaluating equation 1 for \( \beta = 775 \times 10^{-4} \) webers/m² and \( \ell = 6.35 \times 10^{-3} \) m. The propagation velocity \( U_s \) of each increment of particle velocity common to each record is then determined from the time difference measured between the records and the distance \( d \) between the signal wires, i.e.,

\[
U_{s1(2-1)} = \frac{d}{t_1(2) - t_1(1)}
\]

The number of stress-strain points which can be computed from a single shot is dependent on the number of straight-line segments selected to approximate the continuous record of particle velocity.

As has been previously stated, conservation of mass and momentum across the shock front requires that:
\[ \rho_0 U_s = \rho_1 (U_s - U_{pl}) \]  \hspace{1cm} \text{(conservation of mass)} \hspace{1cm} (4)

\[ P_0 + \rho_0 U_s^2 = P_1 + \rho_1 (U_s - U_{pl})^2 \]  \hspace{1cm} \text{(conservation of momentum)} \hspace{1cm} (5)

where:

- \( \rho_0 \) = initial density,
- \( U_s \) = shock velocity,
- \( \rho_1 \) = density behind the shock wave,
- \( U_{pl} \) = particle velocity behind the shock wave, and
- \( P_0 \) = initial pressure.

\[ P_1 = \rho_0 U_s U_{pl} + P_0 \]  \hspace{1cm} \text{the pressure behind the shock wave.} \hspace{1cm} (6)

The strain corresponding to \( P_1 \) is

\[ \varepsilon_1 = 1 - \frac{\rho_0}{\rho_1} = \frac{U_{pl}}{U_s} \]  \hspace{1cm} (7)

These equations are based on the assumption that an equilibrium state of stress is reached behind the shock wave. If the shock structure is stable with pressure (the propagation velocity does not change significantly), the peak steady-state value of particle velocity determined by equation 3 can be used in equations 6 and 7 to compute a point on the stress-strain curve for this material. The points determined from these equations correspond to the Hugoniot or the locus of shocked-end states reached.

For various combinations of material properties and shock-loading pressures, two or more stress waves may develop in the material as the input stress propagates away from the impact surface. Materials with an elastoplastic mode of failure may give rise to a relaxing shock front, i.e., the shock transition region is lengthened with time.
The conservation equations may be successively applied to determine the final state behind the stress wave to yield the following equations for \( n \) approximations to a stress wave:

\[
P_n = P_{n-1} + \rho_0 \left( \frac{U_{sn} - U_{p,n-1}}{1 - \epsilon_{n-1}} \right) \frac{(U_{pn} - U_{p,n-1})}{1 - \epsilon_{n-1}}
\]

\[
1 - \epsilon_n = \frac{i=n}{\Pi} \left( \frac{U_{si} - U_{pi}}{U_{si} - U_{p,i-1}} \right)
\]

These equations are taken from reference 11, which also shows a comparison of a various number of \( n \)-line approximations. This indicates that the calculation process is convergent to the actual Hugoniot, with the accuracy depending on the number of straight-line segments used to approximate the curved record.

**Presentation and Discussion of Results**

**Static**

**Tension.** The averages of three tensile splitting tests for the three rocks are given below. These results are used in conjunction with the triaxial shear results on the plots of Mohr envelopes (fig. 16).

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Average Tensile Splitting Strength, psi</th>
<th>Range in Strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>1700</td>
<td>380</td>
</tr>
<tr>
<td>Basalt</td>
<td>1900</td>
<td>300</td>
</tr>
<tr>
<td>Tuff</td>
<td>170</td>
<td>120</td>
</tr>
</tbody>
</table>
Triaxial compression. Twelve basalt samples were tested under triaxial conditions at confining pressures ($\sigma_3$) of 250, 1000, and 5000 psi. Four specimens at each $\sigma_3$ were tested at loading rates of 1, 50, 500, and 2250 psi/sec. The granite and tuff were tested at a loading rate of 50 psi/sec at $\sigma_3'$'s up to 4000 psi. Compressional wave velocities ($V_p$) were recorded for all specimens at the 50 psi/sec rate only. Table 1 shows the increase in $V_p$, from zero to maximum deviator stress, at each $\sigma_3$. The $V_p$ results for the granite and the basalt indicate that very little consolidation and $V_p$ change took place under relatively low confining pressures to 4000 and 5000 psi. Porosity determinations before triaxial testing confirmed this effect; porosity of the granite and the basalt averaged 0.30 percent and 4.5 percent, respectively.

<table>
<thead>
<tr>
<th>Rock</th>
<th>250</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>4000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>1.09</td>
<td>1.10</td>
<td>1.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basalt</td>
<td>1.03</td>
<td>1.06</td>
<td>1.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tuff</td>
<td>1.23</td>
<td>1.21</td>
<td>1.27</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The $V_p$ for the tuff specimens increased 27 percent under a $\sigma_3$ of 1500 psi. The average porosity of 12 tuff specimens was 20 percent. The increase in $V_p$ under combined stresses is primarily due to the consolidation of the sample resulting in greater density and, hence, larger velocities. The axial deformation at failure was considered in
calculating $V_p$; however, this did not significantly alter the $V_p$ results. Figs. 9-11 show typical plots of $V_p$ versus deviator stress for the three rocks.

Generally, the $V_p$ for the three rocks increased sharply within one-third of the maximum deviator stress and then remained relatively constant to failure.

Figs. 12-14 show the relationship of deviator stress and axial strain. Generally, maximum deviator stress and axial strain at failure increased with increased $\sigma_3$ and with rate of loading as in the case of the basalt.

The tuff was the only rock tested that showed a decrease in deviator stress with increased confining pressures. The rock was tested at natural moisture content of approximately 21 percent and in the undrained state. The pore pressure buildup due to confining pressure and axial loading probably caused the pore pressure to break down some of the rock structure, thereby causing lower strengths at increased confining pressure. Mogi found a similar condition. In additional triaxial testing the effect of pore pressure should be accounted for in terms of effective stresses ($\sigma^1 = \sigma - u$), where $\sigma^1$ = effective stresses, $\sigma$ = total normal stress, and $u$ = pore pressure. Young's modulus also decreased with increased $\sigma_3$'s.

Basalt was tested in triaxial compression at loading rates of 1, 50, 500, and 2250 psi/sec. Fig. 12 shows that at $\sigma_3$'s of 250 and 1000 psi the maximum deviator stress ($\sigma$) increases with increased
Fig. 9  COMPRESSIONAL WAVE VELOCITY versus DEVIATOR STRESS - BASALT
Fig. 10  COMPRESSIONAL WAVE VELOCITY versus DEVIATOR STRESS - GRANITE
Fig. 11  COMPRESSIONAL WAVE VELOCITY versus DEVIATOR STRESS - TUFF
Fig. 12 Increase in Deviator Stress and Axial Strain With Increase in Loading Rate at $\sigma_3$ for Basalt

NOTE: Loading rates are 1, 50, 500, and 2250 psi/sec.
Fig. 13 Deviator Stress Versus Strain Curves for Granite
Fig. 14 Deviator Stress Versus Strain Curves for Tuff
loading rate, with the exception of the specimens loaded at 50 psi/sec. At 5000 psi \( \sigma_3 \) the \( \sigma \) increases through the full range of loading rates used. Total axial strain at failure increases in all cases with an increase in loading rate at each of the \( \sigma_3 \)'s used except at 250 psi \( \sigma_3 \) and at 50 psi/sec loading rate.

Plotting \( \sigma \) at maximum loading rate \( (\dot{\varepsilon}_\text{max}) \) over \( \sigma \) at minimum loading rate \( (\dot{\varepsilon}_\text{min}) \) for each \( \sigma_3 \) used for basalt results in a linear relation, with a positive slope. This is to say that as \( \sigma_3 \) increases the effect of \( \dot{\varepsilon} \) on strength becomes more significant.

![Graph showing the relationship between \( \sigma \) at maximum and minimum loading rates](image)

**Fig. 15**

Generally, the straight line relationship described by Mohr's criterion, \( T = C + \sigma_n \tan \phi \), where \( T \) is the shearing stress, \( C \) is
the cohesion, \( \sigma_n \) is normal stress, and \( \theta \) is the angle of internal friction, fits most of the lower stress circles for the granite and basalt. However, a curvilinear analysis may fit all circles better. There was no envelope drawn for the results of the tuff due to the decrease in deviator stress with increasing \( \sigma_3 \).

Fig. 16 shows that the \( \theta \) for the basalt and granite are very nearly the same at a loading rate of 50 psi/sec. The \( \theta \) for basalt appears to increase between the loading rate of 1 and 2250 psi/sec, which could indicate that shear strength as well as strength and deformation increases under combined stresses and under increasing loading rates. This merits further investigation. Table 2 gives the effect of loading rate on \( \theta \) and \( C \) under combined stresses for basalt.

<table>
<thead>
<tr>
<th>Loading Rate, psi/sec</th>
<th>( \theta ) at Tangent of 1000 psi ( \sigma_3 ) Circle</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50°</td>
<td>3250 psi</td>
</tr>
<tr>
<td>50</td>
<td>53°</td>
<td>3800 psi</td>
</tr>
<tr>
<td>500</td>
<td>50°</td>
<td>3900 psi</td>
</tr>
<tr>
<td>2250</td>
<td>55°</td>
<td>3900 psi</td>
</tr>
</tbody>
</table>

All but two of the basalt specimens tested in triaxial compression failed in shear; shear angles were measured from the horizontal. The observed angles of failure, table 3, further substantiate the suggestion mentioned above that loading rates may have more effect on the shear properties of basalt than do confining pressures up to 5000 psi.
<table>
<thead>
<tr>
<th>Loading Rate</th>
<th>Basalt</th>
<th></th>
<th>Granite</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\phi)</td>
<td>(C)</td>
<td>Loading Rate</td>
</tr>
<tr>
<td>1 psi/sec</td>
<td>50°</td>
<td>3250 psi</td>
<td>50 psi/sec</td>
</tr>
<tr>
<td>50 psi/sec</td>
<td>53°</td>
<td>3800 psi</td>
<td>500 psi/sec</td>
</tr>
<tr>
<td>500 psi/sec</td>
<td>50°</td>
<td>3900 psi</td>
<td>2250 psi/sec</td>
</tr>
</tbody>
</table>

**Fig. 16** Mohr Envelopes at Various Loading Rates for Basalt and Granite
Table 3

<table>
<thead>
<tr>
<th>Loading Rates, psi/sec</th>
<th>Confining Pressures, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>250</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>500</td>
<td>71</td>
</tr>
<tr>
<td>2250</td>
<td>-</td>
</tr>
</tbody>
</table>

Static and rapid compression

Three unconfined compressive strength tests were conducted for each of the three rocks at a loading rate of 50 psi/sec. The basalt, assumed to be more homogeneous than the granite or tuff, was further tested at higher rates of loading up to $1.60 \times 10^7$ psi/sec. Seven granite and two tuff specimens were tested at approximately $1.00 \times 10^7$ and $4.41 \times 10^5$ psi/sec, respectively. Young's modulus of elasticity ($E$) and Poisson's ratio ($\mu$) were calculated for each specimen. The calculated $E$ is a tangent value taken at one-half the ultimate compressive strength, i.e.,

$$E = \frac{\Delta \sigma}{\Delta e_a},$$

where $\Delta \sigma$ is the change in axial stress and $\Delta e_a$ is the change in axial strain. Poisson's ratio is also calculated at one-half the ultimate strength, i.e.,

$$\mu = \frac{\varepsilon_d}{\varepsilon_a},$$

where $\varepsilon_a$ is the axial strain and $\varepsilon_d$ is the diametral strain. The average unconfined compressive strengths, modulus of elasticity, and Poisson's ratio for the static and rapid loading rates used are given in table 4.
Table 4
Average Static and Rapid Unconfined Test Results

<table>
<thead>
<tr>
<th>Type of Rock</th>
<th>Loading Rate, psi/sec</th>
<th>Ultimate Compressive Strength, psi</th>
<th>Young's Modulus of Elasticity $10^6$, psi</th>
<th>Poisson's Ratio</th>
<th>Number of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basalt</td>
<td>1</td>
<td>20,960</td>
<td>4.28</td>
<td>0.37</td>
<td>3</td>
</tr>
<tr>
<td>Basalt</td>
<td>50</td>
<td>21,570</td>
<td>4.47</td>
<td>0.28</td>
<td>3</td>
</tr>
<tr>
<td>Basalt</td>
<td>500</td>
<td>21,840</td>
<td>4.72</td>
<td>0.29</td>
<td>3</td>
</tr>
<tr>
<td>Basalt</td>
<td>$2.06 \times 10^5$</td>
<td>25,280</td>
<td>3.25</td>
<td>0.34</td>
<td>3</td>
</tr>
<tr>
<td>Basalt</td>
<td>$3.13 \times 10^6$</td>
<td>28,170</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basalt</td>
<td>$1.29 \times 10^7$</td>
<td>32,390</td>
<td>4.68</td>
<td>0.20</td>
<td>1</td>
</tr>
<tr>
<td>Basalt</td>
<td>$1.34 \times 10^7$</td>
<td>34,580</td>
<td>6.66</td>
<td>0.25</td>
<td>1</td>
</tr>
<tr>
<td>Basalt</td>
<td>$1.60 \times 10^7$</td>
<td>32,120</td>
<td>4.73</td>
<td>0.23</td>
<td>1</td>
</tr>
<tr>
<td>Granite</td>
<td>50</td>
<td>24,490</td>
<td>9.26</td>
<td>0.33</td>
<td>3</td>
</tr>
<tr>
<td>Granite</td>
<td>$1.00 \times 10^7$</td>
<td>30,990</td>
<td>11.82</td>
<td>0.22</td>
<td>7</td>
</tr>
<tr>
<td>Tuff</td>
<td>50</td>
<td>1,640</td>
<td>0.54</td>
<td>0.19</td>
<td>3</td>
</tr>
<tr>
<td>Tuff</td>
<td>$1.68 \times 10^5$</td>
<td>1,850</td>
<td>0.33</td>
<td>0.42</td>
<td>1</td>
</tr>
<tr>
<td>Tuff</td>
<td>$3.11 \times 10^5$</td>
<td>2,490</td>
<td>0.41</td>
<td>0.49</td>
<td>1</td>
</tr>
<tr>
<td>Tuff</td>
<td>$8.46 \times 10^5$</td>
<td>4,230</td>
<td>0.91</td>
<td>0.36</td>
<td>1</td>
</tr>
</tbody>
</table>

It can be seen from table 4 that the strength and modulus of the basalt are not greatly affected by small increases in loading rates. The ratios of strength and modulus of specimens tested at 500 psi/sec and 1 psi/sec are 1.04 and 1.10, respectively. However, the ratios of
rapid to static ($F_r$) strength and modulus ($E_r$) are somewhat different; the strength ratio is 1.48, and the modulus ratio is 1.17. In calculating $E_r$, the moduli obtained from specimens tested at $2.06 \times 10^5$ psi/sec were omitted for the following reasons. These specimens were tested at the end of the investigation in order to fill in 3-1/2 decades of loading rates. At this time only foil-backed strain gages were available. The foil gages were bonded to the basalt with epoxy, while the paper-backed gages used on all other specimens were bonded with Eastman 910 adhesive; the gage length of the foil gages was 50 percent less than the paper gages. It appears that the different gages, different adhesives, and gage lengths affected the strain in relationship to stress for these three specimens, and, hence, the modulus of elasticity appears lower than expected.

The results in table 4 show an $F_r$ for granite and tuff of 1.27 and 1.70, respectively; $E_r$ is 1.28 for the granite and 1.02 for tuff. The very slight increase in modulus with increased rates of loading for the tuff suggests that this parameter is not as sensitive to increased loading in soft rock as it is in brittle rock. Watstein found the opposite to be the case between weak and strong concrete.

Figs. 17-19 give average static and rapid stress-strain curves for the basalt, granite, and tuff. The stress and strain data points are averages of from three to seven values each. In fig. 20 the rates of loading are plotted to a logarithmic scale against the ultimate strength.
Fig. 17 Static and Rapid Stress-Strain Curves for Basalt

Average Rate of Loading

- Static: 180 psi/sec
- Rapid: $1.41 \times 10^7$ psi/sec
Fig. 18 Static and Rapid Stress-Strain Curves for Granite
Fig. 19 Static and Rapid Stress-Strain Curves for Tuff
As shown in fig. 20, the strength increases with an increase in the logarithmic rate of loading. The general trend of the results in this paper compare quite well with other investigators. The results of Kumar, et al, are only different from the results in this paper in that their rates of loading are higher. Fig. 20 indicates that the rocks they used were inherently stronger by about 6000 psi. This could account for the higher ultimate strength. Also, this difference could be due to test equipment and the size (L/D) of specimens used.

Additional testing of the weaker materials, tuff and concrete, in unconfined compression at loading rates greater than $10^7$ psi/sec would clarify a suggestion put forth by Atchley and Furr that "The trend of the curves indicates that as the rate of stress increases, the dynamic strength approaches a constant value." This statement refers to concrete with nominal strengths up to 5000 psi. The results presented for the tuff appear to be in accord with this statement. However, under shock loading the tuff was stressed to $1.6 \times 10^4$ psi without failing at an average loading rate of $1.09 \times 10^9$ psi/sec. If the test geometry is neglected, it can be stated that the weaker materials would increase in strength with loading rates greater than $10^7$ psi/sec instead of remaining constant as Atchley and Furr suggest. The maximum stress level for tuff is plotted in fig. 20. A stress level for basalt of 4.7 kilobars is
Fig. 20  Effect of Loading Rate on Ultimate Compressive Strength

- ▲ Basalt 23 C After Kumar, et al
- ○ Granite 23 C
- △ Basalt This Report
- ○ Granite
- □ Tuff
- ♦ Concrete 5000 psi After Atchley and Furr
also plotted in fig. 20, which appears to agree with the basalt curve presented by Kumar, et al,\(^1\) if it were projected. These two stress points are plotted to indicate that compressive strength does increase with increased loading regardless of test geometry.

Total axial strain (fig. 21) observed at failure in the rapid tests was greater than the corresponding strain in the static tests. Diametral strain decreased slightly with increased rates of loading for the basalt (fig. 22). As mentioned previously, the Young's modulus of elasticity for the basalt tested at \(2.06 \times 10^5\) psi/sec was suspected to be low. For this reason, these values are not included in the plot of Young's modulus versus loading rate (fig. 23). The method of least squares was used to obtain a straight line equation of best fit for the data in figs. 21-23.

Shock loading

The shock-loading work was accomplished as part of a different research program from that in which the static and rapid work was done. Therefore, a few of the figures in this portion of this paper include data for quartzite which will not be discussed.

Fig. 24a is a plot of pressure versus particle velocity computed from a single impact test on granite. Three curves were computed from the four records obtained for the particle velocity wires, which were separated by \(1/4\) in. The scatter in the data is probably caused by local inhomogeneities within the specimen. Fig. 24b shows a comparison of stress-strain data for granite obtained from two separate tests. The
Fig. 21 $\dot{\varepsilon}$ VERSUS TOTAL AXIAL STRAIN AT FAILURE FOR BASALT

$Y = 3.9528 + (0.5290) (x)$
\[ Y = 3.6790 + (-0.1642) \] (x)

**Fig. 22** \( \dot{\varepsilon} \) versus TOTAL DIAMETRAL STRAIN AT FAILURE FOR BASALT
Fig. 23 Loading Rate Versus Modulus of Elasticity for Basalt
Figure 24  Test results for granite
maximum stress reached in Shot 1 was 3.2 kilobars, and in Shot 2 it was 13.8 kilobars. As can be seen from this figure, the computed stress-strain curves from these two tests are identical within experimental error. The modulus, which can be computed from these data points, is a constrained modulus.

Fig. 25a shows a plot of pressure versus particle velocity, or the shock impedance, for all materials tested. Each curve was computed from a single experiment. Fig. 25b shows the stress-strain relation for all the materials tested. The maximum stress only represents the stress reached in the respective tests and does not indicate elastic failure. Under conditions of one-dimensional strain these materials, with the exception of tuff, would not be expected to fail in compression at these pressure levels. For comparative purposes the static, rapid, and shock-loading stress-strain data are plotted in fig. 26 to indicate the increase in stress, strain, and modulus for the full range of loading rates presented in this paper.

The shock response during loading observed for basalt and granite is essentially linear. The structure of the shock wave is not significantly altered over a propagation distance of 1 in. except for random scatter between sets of particle velocity transducers. This scatter is believed to be caused by local inhomogeneities in the rock samples.

The tuff specimen did not exhibit a stable shock structure. The width of the shock transition increases continuously with distance propagated. This could be a result of local yielding, shear effects,
Fig. 25  Test results for all materials tested
Fig. 26 Static, Rapid, and Shock-Loading Stress-Strain Curves
or viscosity effects; however, the data are too limited to determine the cause. Since there is a much greater relaxation of the shock front in tuff, the approximation method of analyzing the data is less accurate than that for the other materials. This method of analyzing the data can, however, be adjusted to make the error comparable with that associated with the data from the other materials.

Plexiglas was used as a reference material to verify the test method. As shown in fig. 27a, these data correlate closely with the data points reported by Wagner, et al\textsuperscript{13}; however, this comparison requires extrapolation of their data from approximately 6 kilobars to the pressure (approximately 2 kilobars) achieved in the present tests. The plexiglas data also seem to correlate well with low-pressure stress-strain data from compressive strain-rate tests on lucite presented by Maiden and Green\textsuperscript{14} (fig. 27b). The shock response of these two materials is approximately the same; therefore, a comparison of the data would seem justified although the data from Maiden were determined under conditions of uniaxial stress and our data correspond to a uniaxial strain geometry.

Very little low-pressure data are available on granite; however, some stress-strain data are presented by Dennen\textsuperscript{15} on a log-log plot for pressures both above and below our data points. As shown in fig. 27d, our data compare favorably with those from Dennen's work.

Data that would be directly comparable were not available on basalt and tuff below 50 kilobars. A rough comparison of granite
Fig. 27  Comparison with Results of Previous Studies
and tuff data with data from unconfined static and dynamic tests shows the constrained modulus determined in these tests to be somewhat higher in each case.

**Conclusions**

On the basis of results obtained from the static-, rapid-, and shock-loading tests of the basalt, granite, and tuff, the following conclusions are drawn:

a. Both maximum deviator stress and axial strain at failure increase with increasing loading rates up to 2250 psi/sec at confining pressures of 250, 1000, and 5000 psi. Shear stress also increases at loading rates of 1, 50, 500, and 2250 psi/sec up to 5000 psi confining pressure. For the loading rates and confining pressures investigated, confining pressure has considerably more influence on deviator stress than does loading rate. Loading rates up to 2250 psi/sec affect the observed angle of shear more than do confining pressures up to 5000 psi.

b. The influence of loading rates on the deviator strength of basalt becomes more significant at higher confining pressures.

c. The straight line relationship described by Mohr's criterion describes the triaxial failure of basalt and granite only at the lower confining pressures. Von Mise-Hencky's yield criterion does not describe the failure of basalt and granite as well as does Mohr's criterion.
d. The compressional wave velocity of the three rocks is affected by increases in both the applied axial stress and confining pressure. Velocities recorded in the direction of maximum applied stress increase sharply within about one-third of the maximum deviator stress and then generally level off until failure.

e. The unconfined compressive strength of the three rocks studied increases with an increase in the logarithmic rates of loading. The ratio of rapid to static strength $\frac{F_r}{F_s}$ for the basalt was 1.48, for granite 1.27, and for tuff 1.70. The $\frac{F_r}{F_s}$ of tuff is close to the $\frac{F_r}{F_s}$ of weak concrete reported by several other investigators.6,7

f. For basalt the axial strain at failure increases with an increase in the logarithmic rate of loading, while diametral strain at failure decreases.

g. Young's modulus of elasticity increases with an increase in the logarithmic rate of loading up to about $2 \times 10^{11}$ psi/sec. This is based on a comparison of the constrained modulus calculated from the shock-loading data and unconstrained modulus of all three rocks.

h. Uniaxial stress-strain or Hugoniot equation of state data can be determined by the method described in this paper for solid materials in the pressure range from 0 to 40 kilobars. The velocity of propagation can be determined, and the final state behind the stress wave is determined, by the application of the conservation of mass and momentum equations. The data obtained from these equations correspond to the Hugoniot or the locus of shocked-end states reached. This method of
obtaining continuous records of propagation velocity and particle
velocity does not depend on a prior knowledge of the unloading
characteristics of a material as does the free-surface technique.
The shock response of the basalt and granite was found to be linear.
The tuff specimen exhibited an unstable shock structure.

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References

1. Kumar, A., Hauser, F. E., and Dorr, J. E., The Effect of Stress Rate and Temperature on the Strength of Rocks, Publisher Unknown.


16. Stowe, R. L., "Static and Dynamic Strength and Stress-Strain Properties of Rocks of Four Types," U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.; In Preparation, Unclassified.
