Determining the Effective Modulus of Subgrade Reaction for Design of Rigid Airfield Pavements Having Base Layers

Walter R. Barker and Don R. Alexander

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Abstract

Design and evaluation of rigid pavements are based upon the modulus of subgrade reaction ($K$) as determined by a plate-bearing test. When the pavement support system, which includes the subgrade and/or base beneath the slab, contains a base material, the design or evaluation is based on an effective $K$, which is a function of the subgrade $K$ and the thickness of the base. Technical manual Army TM 5-824-3/Air Force AFM 88-6 contained two charts for evaluating the effective $K$: one for a high-quality base and the other for a low-quality base. A later version of the pavement design manual reduced the two charts to a single chart that eliminated quality of the base as a consideration in the determination of the effective $K$. Because quality of base is not considered, the validity of a single chart has been questioned. Because of the questions raised, a study was conducted that included a review of the history of the plate-bearing test, plate-bearing tests on base materials of different qualities, and an analytical study for determining the effective $K$. The study resulted in a new methodology of determining the effective $K$ and recommendations for modifying the procedure for conducting the plate-bearing tests.

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Preface

The U.S. Army Engineer Research and Development Center (ERDC) was tasked by the U.S. Air Force Civil Engineering Agency (AFCESA) with developing methodology for determining the effective plate-bearing modulus of subgrade reaction ($K$) to be used in the design and evaluation of airfield pavements. The study involved a review of plate-bearing procedure, a review of the development of the current methodology for determining the effective $K$, the construction of test sections for conducting plate-bearing tests, the performance of plate-bearing tests, and an analytical study defining the effective $K$.

Personnel of ERDC’s Geotechnical and Structures Laboratory (GSL), Vicksburg, Mississippi, prepared this publication. The test sections were constructed and the plate-bearing tests were conducted under the supervision of the late Don R. Alexander, Airfields and Pavements Branch (APB). The plate-bearing tests were conducted by Pat McCaffrey and Quint Mason, also of the APB. The computer code for analysis of the plate-bearing data was developed by Alexander and Terry Jobe. Dr. Walter R. Barker, APB, conducted the literature review, performed the analytical analysis, developed methodology for determining the effective $K$, and wrote this report. The research was conducted under the general supervision of Jeb Tingle, Project Manager; Dr. Gary L. Anderton, Chief, APB; Dr. Larry N. Lynch, Chief, Engineering Systems and Materials Division (ESMD); Dr. William P. Grogan, Deputy Director, GSL; and Dr. David W. Pittman, Director, GSL.

At the time of publication, COL Kevin J. Wilson was Commander and Executive Director of ERDC. Dr. Jeffery P. Holland was Director.


1 Introduction

Purpose

The purpose of the study is to develop for the design of rigid airfield pavements an appropriate method for determining the modulus of subgrade reaction while giving credit for base materials.

Background

The technical manual for design of rigid pavements, TM 5-824-3/AFM 88-6, Chapter 3 (1979), provided two sets of curves to determine an effective modulus of subgrade reaction ($k$ or $K$) when given the thickness of the base layer and the subgrade reaction modulus. One set of curves was for well-graded crushed base, whereas the other set was for a natural sand and gravel with a plasticity index (PI) smaller than 8 ($PI<8$). Prior to the 1979 manual, there were no effective $K$ curves in the manual. The guidance in TM 5-824-3/AFM 88-6, Chapter 3 (1970), was to conduct plate-bearing tests on top of the base course and determine the $K$ from the test data. The guidance did include the correction for saturation of the subgrade. The basis for going from the 1970 recommendations to the use of the effective $K$ curves included in the 1979 manual was not found in the literature. In revising the 1979 manual during the early 1980s, the decision was made to combine the two sets of curves (Figures 1 and 2) into a single set of curves (Figure 3) to serve for all granular bases. The logic for combining the two curves was that for typical thicknesses of base material, the differences between the two sets of curves were insignificant, as described below.

The example in the 1979 manual shows that for a base with thickness of approximately 12 in. over a subgrade with $K$ of 100 psi/in., the effective $K$ considering a crushed base would be 210 psi/in., whereas the effective $K$ considering a sand-gravel base would be only slightly less than 210 psi/in. Also in early studies, many researchers, including Dr. A. Casagrande (1945; McLeod 1947), concluded that the quality of the base had little influence on the effective $K$. In particular, the report from the Ohio River Division (1943) concluded: “The presence of a subbase under a concrete pavement will not change the value of the foundation modulus.” The Ohio River Division report contains additional interesting conclusions:
1. “There is no coincidence of size of bearing plate or deformation in a field bearing test which will yield values of foundation modulus which will coincide with the true value for a given slab.”

2. “It will be necessary to correlate, by empirical measurements, the reaction of a particular size of plate, preferably 30-inch diameter, with the actions of loaded slabs, to obtain a correct value of foundation modulus.”

3. “The method of correcting foundation modulus for subbase thickness in accordance with paragraph 2-47b leads to impossible results and should therefore be abandoned.”

![Figure 1. Relationship between CBR and K, from tests at ERDC developed by Gonzalez, compared to the relationship given by Packard, the relationships given in TM5-888-9, and the theoretical relationship between CBR and K.](image-url)
Figure 2. Simulation of composite modulus of subgrade reaction - max. \( K = 500 \).

Figure 3. Simulation of composite modulus of subgrade reaction - max. \( K = 800 \).
Unfortunately, the referenced method for correcting the foundation modulus could not be found; therefore, the method was not evaluated.

Likewise, in a Canadian plate-loading study (McLeod 1947), one of the conclusions was, “Base course support per unit of thickness may be generally independent of the composition of granular base course materials, but appears to be influenced by base course density.”

The review of these past studies and the similarity between the two sets of curves led to the decision to combine the two sets of curves into a single set of curves, independent of the base material, which are the data presented in the 1988 design manual.

The logic of having a single set of curves for determining the effective $K$ has been recently challenged. Logic dictates that the quality of the base should have some influence on the effective $K$. The objective of this study is to develop an appropriate method for determining the effective $K$ when considering base material characteristics.

**Plate-bearing test**

Much of the early development of the plate-bearing test occurred during and immediately following World War II. Such development was needed to support the rigid pavement design procedure and was in conjunction with the development of the $CBR$ test for airfield flexible pavements. In fact, some of the early work with the plate-bearing test was conducted in an effort to develop design criteria for the design of flexible pavements. In the early ’40s a major concern was to determine the proper size test plate. A series of tests conducted by the Ohio River Division Laboratories (1943) and the Waterways Experiment Station (1945) indicated that, as the plate size increases, the measured value of $K$ approaches a constant value.

At an American Society of Civil Engineers (ASCE) symposium on military airfields in 1945, COL James H. Stratton stated, “Plate bearing tests were made to determine the relation of the area of the plate to the bearing value of the subgrade. The results agree closely with those obtained by other investigators and show that, for all practical purposes, the bearing value is the same for all plates with a diameter greater than 30 in. To eliminate the effect of area, a circular plate 30 in. in diameter was selected to determine the modulus of soil reaction for the design of concrete pavement.” Moreover, Stratton made the following observation concerning plate-bearing
tests: “The interpretation of plate bearing test results requires the exercise of judgment based on experience with accelerated traffic tests and upon observation of pavement service performance for extended periods. Such interpretation requires full information on the subgrades, base courses, and concrete which implies extensive laboratory tests of the subgrade, base-course materials, and the concrete; on such factors as the effects of subgrade restraint and of changes in the subgrade induced by change in moisture content; the amount and effect of temperature warping and of frost action and thawing; and the effect of slab dimensions and of stress transfer devices. Field plate bearing tests are so expensive and time consuming, and require the mobilization of such large amounts of equipment for test embankment construction, compaction, load jacking, and base-course construction that such testing often is precluded.” Stratton’s statements were made 65 years ago, and they are just as true today.

Another aspect of the plate-bearing test in the procedure used by the military is that the test is conducted as a static test, allowing for some degree of consolidation under each load increment. This procedure is in contrast to the present-day trend of using a resilient modulus for design and evaluation of pavements. The American Society for Testing and Materials (ASTM) recommends conducting repetitive load plate-bearing tests as given in ASTM D1195-64. Repetitive loading tests also were conducted in the early development of the plate-bearing test (Hittle and Goetz 1947). Even though much work was conducted with the repetitive plate test, the Corps of Engineers adopted the static plate-load test as given in CRD-C 655-95. The procedure is similar to the procedure given in ASTM D1196-64. The reason the military chose the static loading with consolidation was not explicitly given by Stratton (1945). However, the following statement made by Stratton indicates that permanent deformation of the subgrade due to slab warping and load application was a concern: “The adjustment for possible subgrade saturation takes into account neither the attendant remolding of the subgrade, the effects of cyclical swelling and shrinkage of the subgrade, permanent subgrade deformation resulting from slab warping or load application, nor the effects of frost heaving or thawing.” It is apparent that permanent deformation was a concern, and probably this was the reason supporting the static plate test. In his analysis, Stratton also gave credibility to Clifford Older’s observations that “warping of slabs under field conditions will result in permanent subgrade depressions, and that this indicates the fallacy of using stress formulas that are based on the assumption that the subgrade reaction is proportional to slab deflection.”
In addition to conducting the plate-bearing test to consider the influence of soil consolidation, a procedure also was adopted to correct for the difference in the subgrade moisture content at the time of testing and at saturation. The correction for saturation involved consolidation tests of the material at the in-situ moisture content and at saturation. Studies conducted at Waterways Experiment Station (U.S. Army Waterways Experiment Station 1945) recommended the method for correction for saturation as outlined in Chapter XX of the Engineering Manual, Office, Chief of Engineers, which states that if the modulus of soil reaction (ratio of unit load at 0.05-in. settlement to the settlement) is known for the natural field moisture condition, the modulus of soil reaction for a saturated condition can be obtained empirically by the following relation:

\[ K = K_u \cdot \frac{P_s}{P_d} \]  

\( K \) = modulus of soil reaction for the saturated soil  
\( K_u \) = modulus of soil reaction for the soil at natural moisture as found by a field-bearing test  
\( P_s \) = the unit pressure in pounds per square inch used to determine the value of \( K_u \)  
\( P_d \) = the unit pressure in pounds per square inch used in a saturated consolidation test causing the same volumetric strain or settlement as \( P_d \) in a consolidation test made on the same soil at natural moisture.

Equation 1 was the basis for the correction for saturation as shown in the 1945 report. In the test conducted at Vicksburg, the corrected modulus of reaction was about 60% of the value determined directly from the test for the clay and about 80% from the test for the silt. The basis for the current correction is given in Equation 2. Equation 2 for correction for saturation is different from Equation 1. Research has not revealed when and why the change for the correction for saturation was made.

\[ K = K_u \left[ \frac{d}{d_s} + \frac{b}{75} \left( 1 - \frac{d}{d_s} \right) \right] \]  

\( K \) = modulus of soil reaction for the saturated soil  
\( K_u \) = modulus of soil reaction for the soil at natural moisture as found by a field-bearing test  
\( d \) = the unit pressure in pounds per square inch used in a saturated consolidation test causing the same volumetric strain or settlement as \( P_d \) in a consolidation test made on the same soil at natural moisture.  
\( d_s \) = the unit pressure in pounds per square inch used to determine the value of \( K_u \).
where

\[ K = \text{corrected modulus of soil reaction, psi/in.} \]
\[ K_u = \text{modulus of soil reaction uncorrected for saturation, psi/in.} \]
\[ d = \text{deformation of consolidometer specimen at in situ moisture content under a unit load of 10 psi} \]
\[ d_s = \text{formation of a saturated consolidometer specimen under a unit load of 10 psi} \]
\[ b = \text{thickness of base course material, in.} \]

In Equation 2, the reference stress for the consolidometer test is 10 psi, whereas in Equation 1 the reference stress is the stress for 0.05 in. of plate deformation. The Equation 1 correction is based on the ratio of pressure for equal deformation; the correction in Equation 2 is based on the ratio of deformation for the same pressure. For the same material, the procedure using the ratio of pressure and the procedure using the ratio of deformation probably would not give significantly different results. Moreover, in Equation 2, the thickness of the base material is a parameter in the equation. When the plate-bearing test is conducted at the surface of a base, the correction for saturation of the subgrade varies from the ratio of deformations for no base to no correction for a base thickness of 75 in. This implies that, for base thickness of 75 in., the expected change in the effective \( K \) due to changes in subgrade modulus would be insignificant. The thickness seems to agree with the plots given in the UFC in which very little change in effective modulus is shown for base thicknesses greater than 50 in. Another aspect to analyze for the plate-bearing test is how the \( K \) value is determined from the load-deflection curve. In the early development of the plate-bearing test, \( K \) was computed at plate deflections of 0.05, 0.1, or 0.2 in. Sometime after 1950, the military started using the deflection at plate pressure of 10 psi as the reference for computing the \( K \) value. The Ohio River Division (1953) recommended computing \( K \) using the deflection for the 10-psi load, reporting that 10 psi was the expected typical vertical stress under a rigid pavement.

The 1953 study by the Ohio River Division Laboratories contained additional considerations related to rigid airfield pavement performance. Airfield evaluations reported that some airfield pavements constructed on granular subgrades deteriorated at a slower rate than expected. After excluding pavement thickness, exceptionally high flexural strength, or low loading frequency, the slow rate of deterioration was explained by the high subgrade
modulus. At the time, the maximum allowable subgrade modulus for design was 300 psi/in. Plate-bearing tests at the better performing airfields indicated subgrade strengths in excess of 300 psi/in. Before revising the criteria to allow higher subgrade modulus for design, a study was conducted to determine whether the plate-bearing test being used was adequately measuring high values of subgrade stiffness. The study consisted of a theoretical analysis, a series of field tests, and a correlation of the field data with the theoretical analysis. Conclusions of the theoretical study (Ohio River Division 1953) include:

1. Bending of the plate occurs regardless of the size of the plate, the load, or the value of the subgrade modulus.
2. The uniform subgrade assumption is justified only for values of $K$ less than 100 psi/in.
3. The value of $K$ should be computed on the basis of the theoretical deflection of the rim.
4. The procedure for the plate-bearing test should be revised.

The field study conducted by the Ohio River Division (1953) considered several aspects of the plate-bearing test, including stacked plates versus a single plate, the thickness of a sand-leveling layer, and the seating load. Some of the most significant conclusions in the report are:

1. Correction of the load-deformation curve for seating conditions is necessary.
2. On high-bearing value subgrades, the 7,070-lb load is not sufficient to develop the subgrade reaction.
3. For loads of 7,070 lb or larger, the load-deformation relationship is a straight line.
4. A sand cushion layer ranging from 3/4 in. to 1 in. thick just below the bearing plate causes an error in determining $K_u$.
5. Bending of the plate occurs regardless of the load, subgrade modulus, or plate arrangement.
6. The correction of the subgrade modulus on the basis of the theoretical rim deflection of a single plate is justified.

With the exception of the third conclusion, the findings of the Ohio River Division study are justified and have influenced the plate-bearing test procedure as it is implemented today. The third conclusion must be considered within the context and extent of the study. In fact, the field study was
for high values of plate-bearing tests; thus, testing was conducted on
granular type soils. A review of data from plate-bearing tests conducted by
Alexander for this report confirms the third conclusion, at least in regard
to plate-bearing tests conducted on granular materials.

In 1945, the Waterways Experiment Station conducted plate-bearing tests
on lean clay, Vicksburg loess, and CH clay (buckshot clay). The load-
deflection curves for these tests were nonlinear, particularly at higher load
levels. On the other hand, a general review of load deflection curves for
granular materials indicated an almost linear relationship between load and
deflection for stress levels above 10 psi. Because of the nonlinearity of the
load-deflection curves and the fact that the concept of a $K$ assumed a linear
relationship between stress and deflection, the procedure used to determine
the value of $K$ from load-deflection curves could have a significant effect on
the value of $K$. As a result of the Ohio River Division study (1953), revisions
were made in the plate-bearing test procedure. Two were:

1. “Loading Reaction: Provide for a sufficient reaction to obtain a complete
load-deflection curve when the test is made on high bearing value sub-
grades. Usually this will be required only for granular subgrades.”
2. “Loading Procedure: The loading procedure will be revised so that a load-
deformation curve may be obtained when the correction for bending will
cause a significant difference in $K$...The total load for the load-deformation
should be increased in regular intervals of 3,535 lb to a maximum load of
21,210 lb. This incremental loading will provide a sufficient number of
points for the curve. Each incremental load should be held for a sufficient
time to allow the major portion of the consolidation to occur.”

The current procedure, as contained in CRD-C 655-95, implements the
Ohio study’s recommended revisions. A comparison of the deflection curves
for granular subgrades and those for clays and silts justifies the increase in
plate loading needed to define the load-deflection curve for granular
material, but not for clays and silts. The deflection curve for clays and silts
at higher water contents is nonlinear over the entire range of stress from
5 to 30 psi. The nonlinearity increases as the stress level increases; there-
fore, carrying the plate test to the higher levels of load does not provide
additional information about the soil behavior. The test procedure in CRD-
C 655-95 recognizes the material nonlinearity and does not require testing
over 10 psi. Stopping the test at 10 psi provides only three data points,
including the 0 loading as a data point, which does not provide sufficient
data to establish error due to seating. The selection of 10 psi as reference for computing $K$ is based on the fact that 10 psi is the expected stress under a properly designed rigid pavement. For this reason, the proper procedure to compute $K$ is to use the tangent to the load-deflection curve at 10 psi. Nevertheless, to establish the tangent at 10 psi requires additional points on both sides of such load level. This suggests that, for low-strength materials, load increments should be smaller with a required maximum load of no greater than 15 psi.

Based on the review of load-deflection curves in the literature, the discussion of the test protocol with technicians conducting the plate load test at ERDC, and the analysis of load curves, the conclusion has been developed that for weak, fine-grained soil ($k < 200, CBR < 8$), the plate-bearing tests should be conducted as discussed in the following section.

The use of the correction for plate bending should be continued. Besides the fact that the Ohio River Division Laboratory study (1953) is the only one addressing plate bending, in the absence of additional data, the correction chart for modulus values between 100 psi/in. and 2,000 psi/in. is represented by the third order polynomial in Equation 3.

$$K_u = 7.6211 \times 10^{-8} \cdot X^3 - 3.2994 \times 10^{-4} \cdot X^2 + 0.85273 \cdot X$$  \ (3)

where

$$X = K'_u - 100$$

$K'_u$ = the uncorrected plate bearing modulus

$K_u$ = the plate-bearing modulus corrected for plate bending but not for saturation

For values of plate-bearing modulus equal to or less than 100 psi/in., no correction is necessary. For values of plate bearing between 100 psi/in. and 200 psi/in., the correction was considered sufficiently small as to be insignificant in the design and/or evaluation of pavements. The procedure is in place for correction; therefore, the correction for plate bending for plate-bearing values between 100 psi/in. and 200 psi/in. may be optional.

Equations 1 and 2 provide for correction of the plate bearing for saturation of the subgrade. The literature review indicated that the correction for
saturation of silts and clays is significant, probably more than the correction for plate bending. For fine-grained subgrades compacted at or near optimum water content, the correction for saturation is strongly recommended. The only exception to the correction requirement is a plate-bearing test performed on an existing subgrade that has achieved constant moisture content and is unlikely to change.

Based on the information found in the literature, the following recommendations are presented:

1. The plate test protocol on fine-grained soils for which the plate-bearing modulus is expected to be lower than 200 psi/in. should be modified to reduce the load increment to 1,767 lb (2.5 psi) and to stop the test at a load level of 10,602 lb (15 psi). The modulus of reaction is to be computed as the slope of the load-deflection curve at 10 psi.

2. A plate-bearing modulus greater than 200 psi should be corrected for plate bending. A plate-bearing modulus equal to or less than 100 psi/in. does not require a correction for plate bending. The correction will be optional for values of plate bearing between 100 psi/in. and 200 psi/in. Perhaps in a criteria document the option of using 100 psi/in. or 200 psi/in. should not be allowed, but the value should be fixed at either 100 psi/in. or 200 psi/in.

3. The correction for saturation is to be required for every plate-bearing test conducted on fine-grained soils. The correction is not needed for free-draining granular base and subbase materials.

During the review of the plate-bearing test, a number of issues have evolved concerning the use of the plate-bearing test in pavement design and evaluation. The plate-bearing test is a static test that allows for a degree of consolidation of the subgrade. This method of conducting a plate-bearing test produces a loading scenario that is different from the one of a passing aircraft. Even though, in the early development phases of the plate-bearing test, the repeated load-bearing test received a great deal of attention, the static test was chosen for developing the design criteria. One of the concerns in the early stages of this test was that the subgrade would consolidate under the slab edges due to warping and curling. The thinking was probably that conducting the plate-bearing test as a consolidation test would account for the consolidation at the edges of the slabs due to warping and curling. If the load-deflection curves from static plate tests were compared with those curves from repeated plate tests, where the deflections plotted are the total deflections, then two curves favorably compare. Therefore, static plate-
bearing tests would somehow consider the permanent deformation due to repeated aircraft loads. The fact that the Westergaard rigid pavement design criteria (and evaluation criteria) are based on the static plate-bearing test presents a major issue for pavement evaluation. The aircraft load as it is applied to a pavement at taxi speed is a relatively slow repeated loading. In pavement design, this is not a major issue because the design criteria represent a transfer function from the static plate data to pavement performance under the repeated slow-rolling load. In the layered-elastic design procedure, the resilient modulus is used in a design procedure for which the criteria were developed by relating the resilient modulus to the static plate bearing. The structural evaluation of military pavements almost exclusively is performed using data from nondestructive testing (NDT) employing the falling weight deflectometer (FWD). The FWD loading is a dynamic loading of a very short duration, yet in pavement evaluation the FWD data are being used with the pavement design criteria. The solution is to have a transfer function to go from the FWD data to the static plate modulus of reaction or resilient modulus or to develop evaluation for the FWD data.
2 The Influence of Base Course

Background

The general consensus in the literature is that base courses improve the performance of rigid pavements far beyond the improvement that would be indicated by an increase in $K$. Stratton in his 1945 paper said, “Warping of a concrete pavement tends to deform the subgrade, which in turn affects the magnitude and distribution of the stresses in the pavement. Compaction of subgrades and the use of compacted base courses to improve and to develop uniformity of the subgrade support mitigate these adverse influences.” A little later in the paper, after discussing the loss of support under concrete slabs, he added, “The result of repeated deflections of a slab, particularly at joints where water has ready access, may be a remolding of a plastic subgrade soil, or a pumping action with a consequent marked loss of support at the locations where slab stresses are most critical. Base courses contribute to the alleviation of conditions destructive to the supporting capacity of a subgrade.” Stratton ends his discussion on base courses with this conclusion: “High-quality base materials are less subject to adverse influences which readily deteriorate the susceptible low-quality subgrades. They also serve to insulate and protect the underlying subgrades, and thus to preserve, to a large degree, whatever value and uniformity of support the subgrades offer. Recent plate bearing investigations have disclosed that improvements of pavement support by use of base courses is not always evidenced in the values of $K$ as determined by the bearing tests. Accelerated traffic tests and pavement service performance, however, demonstrate the value of base courses under concrete pavements.”

In the discussions at the ASCE Symposium on Military Airfields (1945), Casagrande makes the following observation concerning compacted bases: “Much confusion still prevails on the value of admixture of fines or binder to a clean coarse-grained material in the construction of base courses. To cite two extremes: (1) excellent concrete pavements have been built in Long Island on hydraulically placed, clean beach sand, without even an attempt at compacting the sand; whereas (2) in a different place, well-graded, fairly clean, glacial gravel (the best base material next to crushed stone) was ‘stabilized’ by adding 15% material passing No. 200 mesh, thereby changing it into a frost-heaving material. Even where there is no frost action, the addition of a plastic binder to gravel has often resulted in
a type of base which, in a condition of capillary saturation (evaporation from its surface prevented by the overlying pavement), was much less resistant to deformation than the coarse material without the binder.”

The value of a high-quality base course long has been recognized as being greater than indicated by a conventional Westergaard analysis. With regard to the definition of high-quality base materials, CPT E.D. Parmer, Commandant of the Army Soils Control School, Graduate School of Engineering at Harvard University, provided the following comments about Stratton’s use of the term “high-quality base materials” (1945):

“Because of a common misunderstanding as to what constitutes a ‘high-quality base material’ and the belief of many engineers that clay should be added to granular materials for use as a base course (simply because the satisfactory performance has been observed of well-graded sand or gravel mixtures containing clay when used as surface courses exposed directly to light wheel loads without the use of substantial pavements), it should be stated that, in general, clean, well-graded granular base materials with a minimum of ‘fines,’ say, not over 4% or 5% passing No. 200 mesh, make the most desirable high-quality bases for both rigid and flexible pavements.” Furthermore, in a report by F.M. Mellinger and J.P. Sale (1956), the following observations were presented:

1. “Non-rigid overlays will perform satisfactorily after considerable breakup or cracking has occurred in the rigid base pavement.”
2. “The rate of breakup of the concrete base slab beneath a non-rigid overlay is very dependent on the subgrade modulus or the amount of subgrade support. This is also true for overloaded plain concrete pavements.”

Mellinger and Sale stated that the base pavement was reduced to pieces 5 to 7 sq ft in area just prior to failure of the overlay and that the rate of failure was much slower for high values of $K$ than for lower ones. The authors indicated as “complete failure” the slab’s breaking into pieces about 7 sq ft in area. From their analysis, Mellinger and Sale went on to propose design criteria utilizing the benefits of high-strength subgrades. For the design of flexible overlays over rigid pavements with high-strength subgrades (subgrade modulus greater than 200), the method proposed by Mellinger and Sale was to redesign the pavement as a flexible pavement, considering the cracked concrete as a high-quality base ($CBR = 100$) and any other base material to be subbase material in the flexible pavement. This approach is part of the current design procedure for non-rigid...
overlays of rigid pavements and clearly indicates the benefits in the use of high-strength bases under concrete pavements.

The extended life of rigid pavements over high-quality bases also was observed in full-scale testing sections at ERDC during the summer of 2010. Three test sections, two of 11-in. concrete and one of 8-in. concrete, were trafficked with a simulated six-tire gear of the C-17 aircraft. Even after the slabs had broken into as many as six pieces, the pavement continued to carry traffic. The sections were constructed over crushed gravel base courses having thicknesses of 7, 14, and 16 in. The uncorrected plate bearings, measured after traffic, were 566, 545, and 625 psi/in., respectively. Because of the very strong base materials, these pavements would have been excellent candidates for rehabilitation using either crack-and-seat or rubblization methodology.

In summary, several benefits of high-quality bases for rigid pavements have been identified by past researchers. They determined that high-quality bases:

1. Increase subgrade support.
2. Prevent loss of subgrade support at joints due to pumping or remolding of plastic subgrade soil.
3. Help provide a uniform subgrade support.
4. Help reduce heave due to frost penetration and swelling of plastic soils.
5. Provide a working platform for construction equipment and support of items such as forms and dowel baskets.
6. Add salvage value to the pavement by qualifying the pavement as a candidate for rehabilitation with a flexible overlay.

**Estimating effective (composite) subgrade modulus**

**Nomenclature**

For clarity, ambiguous terms and names will be redefined. However, most terms agree with the current standard definitions that have gained overall acceptance. The following list contains major terms and definitions used in this report.

1. **Plate-bearing modulus**: The slope of the load-deflection curve from the plate-bearing test conducted, according to CRD-C-655-95 (1995). The plate-bearing modulus corrected for plate bending and saturation is shown
as $K$; the plate-bearing modulus that has not been corrected for plate bending is shown as $K'$; the plate bearing that has not been corrected for saturation is shown as $K_u$.

2. **Modulus of subgrade reaction**: The plate-bearing modulus measured on a very thick homogenous layer beneath the rigid pavement.

3. **Composite modulus of subgrade reaction (composite $K$)**: The plate-bearing modulus measured at the top of a layered subgrade support system (base and/or subbase) that includes the thick homogenous subgrade layer.

4. **Effective modulus of subgrade reaction (effective $K$)**: A pseudo modulus of subgrade reaction that applies to a layered subgrade support system for which the computed tensile stress, using the Westergaard model, is the same as if the subgrade were a single-thick homogenous layer. The composite modulus of subgrade reaction at the top of a base layer is different from the effective modulus of subgrade reaction. This is because the composite modulus of subgrade reaction is a function of only the subgrade plate-bearing modulus, base thickness, and quality of the base material, whereas the effective modulus of subgrade reaction also is a function of the concrete pavement properties and the geometry of the applied load.

5. **Equivalent modulus of elasticity (equivalent modulus of subgrade reaction)**: This defines the relationship between the modulus of elasticity of the subgrade and the modulus of subgrade reaction when the subgrade is considered to be a single homogeneous layer. Ideally, the relationship for equivalency would be based on equal stress in the concrete slab.

**Estimating the modulus of subgrade reaction**

In the early development of the plate-bearing test, correlations of $K$ with other subgrade tests, material types, and basic material properties were evaluated. The Unified Facilities Criteria (2001) presents estimates of typical plate-bearing values based on the soil classification. However, the UFC includes the following statement: “Pavement design should be based on test data or at least historical data of past designs and evaluations at the same facility if at all possible. These default values are suggested for use for preliminary calculations or for small projects or projects where better data simply cannot be obtained. Inadequate testing or evaluation budgets are not an excuse to use these values for final design.” The UFC contains a chart from the Portland Cement Association (PCA) showing the relationships between several soil parameters, such as the unified soil classification, AASHTO classification, dynamic cone penetrometer index, $CBR$, and plate bearing. Nevertheless, the correlations in the PCA chart are to be used only
for small projects or projects without available test data, as specified in the above-mentioned UFC statement.

Another approach to estimating $K$ is by the use of elastic theory. By using the theoretical solution for the deflection of a rigid plate over an elastic foundation, as defined in the University of California publication *Stresses and Deflections in Foundations and Pavements* (Lysmer and Duncan 1969), it is possible to develop a theoretical relationship between $CBR$ and $K$. The equation for the deflection is given in Equation 4.

$$
\Delta_z = q \cdot 2a \left( \frac{1 - \mu^2}{E} \right) \cdot I_{\Delta_z} \tag{4}
$$

where

- $\Delta_z$ = deflection of a rigid plate
- $q$ = pressure on the plate
- $a$ = radius of the plate
- $\mu$ = Poisson’s ratio
- $I_{\Delta_z}$ = an influence factor depending on the thickness of a layer over bedrock; for a half-space the influence factor is 0.785
- $E$ = Young’s modulus

The $CBR$ determined at 0.1-in. penetration is equal to $0.1q$. Combining this relationship and Equation 4, Equation 5 provides the relationship between $CBR$ and $E$.

$$
CBR = \frac{E}{195.44 \cdot (1 - \mu^2) \cdot I_{\Delta_z}} \tag{5}
$$

Likewise, Equation 6 shows the relationship between $K$ and $E$.

$$
K = \frac{E}{30 \cdot (1 - \mu^2) \cdot I_{\Delta_z}} \tag{6}
$$

Combining Equations 5 and 6, the theoretical relationship between $CBR$ and $K$ is

$$
K = 6.5 \cdot CBR \tag{7}
$$
Relationships between $CBR$ and $K$ are given in TM 5-888-9, dated December 1966, as a function of the soil type. The relationships show that, for a given subgrade $CBR$, the corresponding $K$ would be higher for cohesive soils than for less cohesive soils. Carlos Gonzalez of the APB collected historical data for developing a relationship for fine-grained soil between $CBR$ and $K$. From these data and additional data collected from recent testing, he recommended the relationship in Equation 8.

$$K = 20 \cdot CBR$$  \hspace{1cm} (8)

Figure 1 shows the relationship between $CBR$ and $K$ by Gonzalez from tests at ERDC, by Packard (1973), by TM5-888-9, and by the theoretical relationships developed above. All of the relationships, except for a small portion of the relationship in TM 5-888-9 for coarse-grained materials, are above the theoretical relationship of Equation 7. It is believed that the relationship given in TM 5-888-9 for coarse-grained materials was extrapolated, probably without substantiating data, such that for a $CBR$ value of 100, $K$ would be 500 psi/in., which corresponds to the maximum $K$ allowed for design.

Moreover, for fine-grained soils with $CBR$ values ranging between 4 and 20, the Gonzalez relationship, the Packard relationship, and the TM 5-888-9 relationship for fine-grained soil with liquid limit (LL) smaller than 50 (LL < 50) are in fairly close agreement. For $CBR$ values less than 4, the Packard relationship yields values of $K$ significantly higher than those of Gonzales and TM 5-888-9. The relationship in TM 5-888-9 for fine-grained soils having a liquid limit greater than 50 yields a value of $K$ higher than that of the field collected data. For dense, coarse-grained soils, the load-deformation curves should be more linear with higher values of $CBR$; thus, the relationship between $CBR$ and $K$ should approach, but never cross, the theoretical relationship. Nevertheless, the TM 5-888-9 relationship for coarse-grained soils crosses the theoretical relationship; similarly, the Packard relationship would cross if extended to higher values of $CBR$.

Consideration of the theoretical relationship suggests that the TM 5-888-9 relationship for the plate bearing as function of $CBR$ for coarse-grained materials should be redrawn to approach, at higher values of $CBR$, the theoretical relationship.
3 Relationship Between Modulus of Subgrade Reaction and Modulus of Elasticity

The Boussinesq solution for the deflection of a rigid plate over an elastic foundation is given by Equation 9.

$$\Delta_z = \frac{\pi \cdot (1 - \mu^2) \cdot q \cdot a}{2 \cdot E}$$

(9)

where

$\Delta_z =$ deflection of a rigid plate

$E =$ modulus of elasticity of the foundation material

$\mu =$ Poisson's ration of the foundation material

$q =$ the load on the plate expressed as a uniform load over the area of the plate

$a =$ radius of the plate.

The deflection of a plate on a half-space foundation as given in Equation 4 for an influence factor of 0.785 is essentially the same as that given by Equation 9. By using the influence factors shown in Table 1, the modulus of subgrade reaction can be computed for cases in which the foundation is underlain with a rigid layer at a specified depth. When either Equation 4 or 9 is applied to the plate-bearing test for a half-space, the modulus of subgrade reaction is equal to the modulus of elasticity divided by a constant that is a function of the Poisson’s ratio. The constant varies from 23.56 for a Poisson’s ratio of 0 to 17.67 for a Poisson’s ratio of 0.5. For a partially saturated subgrade soil and Poisson’s ratio of 0.4, the relationship between the modulus of subgrade reaction and the modulus of elasticity is shown in Equation 10.

$$K = \frac{E}{19.79}$$

(10)
Table 1. Influence factors for Equation 4.

<table>
<thead>
<tr>
<th>Depth Ratio (h/2a)</th>
<th>Poisson's Ratio – (\mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.50</td>
</tr>
<tr>
<td>0.5</td>
<td>0.142</td>
</tr>
<tr>
<td>1.0</td>
<td>0.354</td>
</tr>
<tr>
<td>1.5</td>
<td>0.475</td>
</tr>
<tr>
<td>2.0</td>
<td>0.545</td>
</tr>
<tr>
<td>2.5</td>
<td>0.591</td>
</tr>
<tr>
<td>3.0</td>
<td>0.622</td>
</tr>
<tr>
<td>3.5</td>
<td>0.646</td>
</tr>
<tr>
<td>4.0</td>
<td>0.662</td>
</tr>
<tr>
<td>4.5</td>
<td>0.674</td>
</tr>
<tr>
<td>5.0</td>
<td>0.687</td>
</tr>
<tr>
<td>5.5</td>
<td>0.695</td>
</tr>
<tr>
<td>7.5</td>
<td>0.718</td>
</tr>
<tr>
<td>10.0</td>
<td>0.735</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.785</td>
</tr>
</tbody>
</table>

The case of a foundation containing a rigid layer at some depth beneath the subgrade is not a case of particular interest since the equivalent \(K\) (effective \(K\)) always will be greater than the composite \(K\). For example, if a material having a CBR value of 4 \((k \approx 80)\) has a rigid layer at 30 in., the composite \(K\) for the system will be 156 psi/in. (based on Equation 4). Of greater interest for pavement design and evaluation is the system of a stiff layer over a less stiff layer, as in the case of a base course over a weak subgrade. The analysis of this layer scenario is best accomplished by using an axis-symmetric finite element code to simulate the plate-bearing test. The finite element code used for this study was FE2DAXI2006, developed by Gonzalez. Although the code had the capability for nonlinear analysis, this study was based strictly on linear analysis. First, simulations were run to check the validity of the code by modeling the case of the plate-bearing tests on very thick foundations, which would simulate a half-space as represented by Equations 4 and 9. For the simulations using Poisson’s ratio of 0.4, the constant relating \(E\) to \(K\) was determined to be 19.22, which agreed with the constant that was determined using the Boussinesq solution. Given that the finite element simulation of the plate-bearing test is
adequate, the finite element simulation was used to study the case of the plate-bearing test conducted on a stiff layer of finite thickness over a less stiff layer. The general procedure in the analysis was to assume a $K$ value for the less stiff lower layer representing a half-space. A stiffer material having a finite thickness was assumed for the upper layer. The $K$ value assumed for the stiffer material was the $K$ that would be measured in the plate-bearing test for a material of infinite thickness. The $K$ value for each material is converted to an elastic modulus by the use of the relationship shown in Equation 11.

$$E = 19.22 \cdot K$$

(11)

From the finite element simulation of the plate-bearing test on the layered system, the composite $K$ value was computed. By simulating different combinations of $K$ values and layer thicknesses, the relationships shown in Figures 2 and 3 were developed.

UFC 3-260-02 (2001) includes Equation 12, relating the static modulus of soil reaction to the resilient modulus of elasticity.

$$M_r = 26 \cdot K^{1.284}$$

(12)

For a $K$ value of 100 psi/in., Equation 12 yields a resilient modulus of elasticity of 9,616 psi. For the same $K$ value, Equation 11 produces a theoretical value of 1,922 psi as the static modulus of elasticity. Equation 12 computes a modulus that is 5 times greater than the modulus predicted by Equation 11. Computing the ratio of Equation 12 to Equation 11, the ratio of the resilient modulus of elasticity to the static modulus of elasticity is expressed by Equation 12.

$$\frac{M_r}{E} = 1.35 \cdot K^{0.284}$$

(13)

Equation 5 provides the theoretical relationship between $CBR$ and $E$ which, for an assumed Poisson’s ratio of 0.4, indicates that $E$ is equal to 129 times the $CBR$. For estimating the resilient modulus from the $CBR$, the relationship commonly used shows the resilient modulus of elasticity 1,500 times the $CBR$. The analysis of the above mentioned values indicates the ratio of the resilient modulus to static modulus to be about 12.
In the ASSHTO design guide, the relationship between the $K$ value and the resilient modulus is given by regression Equation 14.

\[
InK_{\infty} = -2.807 + 0.1253(\ln D_{sb})^2 + 1.062(\ln M_R) + 0.1282(\ln D_{sb}) \cdot (\ln E_{sb}) - 0.4114(\ln D_{sb}) - 0.0581(\ln E_{sb}) - 0.1317(\ln D_{sb}) \cdot (\ln M_R)
\]  

(14)

where

- $K_{\infty}$ = modulus of subgrade reaction with no bedrock
- $D_{SB}$ = thickness of the subbase
- $E_{SB}$ = modulus of elasticity of the subbase
- $M_R$ = resilient modulus of the subgrade.

Since Equation 14 is a regression equation, the equation is valid only for a limited range of data. The equation cannot be applied to the case of a plate-bearing test on a subgrade with no base; but as the thickness of the subbase approaches 0, the equation indicates the value of $K$ to be 0.05 times the modulus of elasticity of the subgrade.
4 Relationship Among Composite Modulus, Base Thickness, and Base Quality

As previously defined, the composite modulus is the modulus obtained at the surface of a layered continuum; the effective or equivalent modulus is relative to the stress in a concrete slab on a layered continuum. The composite modulus is independent of the properties of the slab, but the effective modulus is a function of the slab stiffness and thickness. In the 1979 Army/Air Force technical manual *Rigid Airfield Pavements*, two sets of curves (Figure 4) relate the \( K \) on top of the base to the \( K \) of the subgrade and thickness of the base. In the manual’s text, the \( K \) to be used for design of a rigid pavement having a base course is either measured by the plate-bearing test at the top of the base or determined from relationships shown in Figure 4. Even though the manual identifies the \( K \) determined from the relationship as the effective \( K \), the description identifies the \( K \) as the composite \( K \), as defined herein.

In 2004, test sections were constructed at ERDC to evaluate the increase in \( K \) as the thickness of the base increases. Table 2 includes the data from the test section. The materials used for the bases were crushed limestone, clay gravel, sand, and pea gravel. The data in the table clearly indicate an influence due to material type on the measured \( K \) values.

Additional data were collected from the test sections, built to verify the BETA criteria for design of flexible pavements. During the construction of the test sections, plate-bearing tests were conducted on the subgrade and at the top of the different lifts. The data for these tests are in Table 3.

Existing relationships for effective/composite modulus of subgrade reaction versus thickness of non-stabilized granular base

As previously discussed, the military has long used sets of curves to determine the effective modulus of subgrade reaction based on the subgrade plate modulus, the type of base, and the thickness of the base. Figure 4 shows the set of curves as it appears in TM 5-824-3 (1979). Criteria prior to 1979 specified only that a plate-bearing test was to be conducted on the top of the base course for determining the modulus of subgrade reaction. By the definitions within this report, the modulus from a plate test...
Figure 4. Effective $K$ curves from 1979 TM 5-824-3.

Table 2. Composite moduli of materials.

<table>
<thead>
<tr>
<th>Lift Number</th>
<th>Thickness of Base</th>
<th>Limestone</th>
<th>Clay Gravel</th>
<th>Sand</th>
<th>Pea Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>57</td>
<td>60</td>
<td>39</td>
<td>42</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>103</td>
<td>175</td>
<td>42</td>
<td>52</td>
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<td>2</td>
<td>12</td>
<td>220</td>
<td>308</td>
<td>69</td>
<td>80</td>
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<tr>
<td>3</td>
<td>18</td>
<td>238</td>
<td>384</td>
<td>71</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>335</td>
<td>602</td>
<td>145</td>
<td>124</td>
</tr>
</tbody>
</table>
Table 3. Data on composite moduli collected during construction of the BETA test section.

<table>
<thead>
<tr>
<th>Depth</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
<th>Item 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_u'$</td>
<td>$t$ above subgrade</td>
<td>$K_u$</td>
<td>$K_u'$</td>
</tr>
<tr>
<td>Top of Base</td>
<td>404</td>
<td>13</td>
<td>331</td>
<td>302</td>
</tr>
<tr>
<td>Top of Subbase</td>
<td>571</td>
<td>7</td>
<td>436</td>
<td>421</td>
</tr>
<tr>
<td>7&quot; into Subbase</td>
<td>417</td>
<td>7</td>
<td>340</td>
<td>99</td>
</tr>
<tr>
<td>15&quot; into Subbase</td>
<td>0</td>
<td>0</td>
<td>183</td>
<td>232</td>
</tr>
<tr>
<td>Top of Subgrade</td>
<td>201</td>
<td>0</td>
<td>183</td>
<td>232</td>
</tr>
</tbody>
</table>

Note: The correction for plate bending is based on the following reference.
Ref: Figure 3, Determination of High Values of the Subgrade Modulus, $K_u$ from the Plate Bearing Test
Ohio River Division Laboratories; Mariemont, Ohio; October 1953

$K_u'$ as computed for 10 psi
$K_u$ has been corrected for plate bending.

Conducted on the top of the base course is a composite modulus of subgrade reaction and not necessarily the effective modulus of subgrade reaction. The source of the curves appearing in TM 5-824-3 could not be determined, but since the previous criteria called for conducting plate-bearing tests on the base course, the curves probably were developed from plate-bearing tests on different base courses. Thus, these curves would represent the composite modulus, not the effective modulus.

In the 1980s, a single set of curves (Figure 5) was developed to replace the two sets of curves. The elimination of the two sets of curves took the quality of base out of consideration when determining the $K$ for pavement design. The two sets of curves were eliminated because so little difference existed between them and no data were available to assess their reliability. The change also was influenced by the opinion of early experts that, within the limits of acceptable base materials, the base material quality has negligible effect on the effective $K$. Except for some smoothing, the new single set of curves represented an average of the two sets. At higher modulus values, the curves were adjusted such that effective modulus values greater than 500 psi/in. would not be generated. Pittman (1996) approximated the single set of curves by using a regression analysis, given in Equation 15.

$$K_{TB} = 10^{[2.69897 + (\log[K] - 2.69897)10^{-0.01466BT}]}$$

Applying Equation 15 to the example of a 10-in. base over a subgrade with a $K$ value of 100 psi/in., an effective $K$ of 160 psi/in. is obtained. Thus, the $K$ obtained with Equation 15 is somewhat less than the $K$ obtained from the
curves shown in Figures 4 and 5. The conclusion concerning Equation 15 is that the equation gives a close approximation of the combined curves and is adequate for application in a reliability model as developed by Pittman.

In the software for design of airfield pavement (PCASE), each effective $K$ curve is represented by a regression function with an interpolation routine for values of subgrade $K$ that are between curves. In this manner, the effective $K$ can be obtained for a given subgrade $K$ and base thickness. The methodology in PCASE works well and gives an accurate representation of the curves shown in Figure 5.

Figure 5. Effective $K$ curves from 2001 TM 5-824-3.
5 Introducing the Equivalent and Effective Modulus of Subgrade Reaction

Both the equivalent modulus and the effective modulus are defined relative to the tensile stress at the bottom of a concrete slab. With this definition, both $K$ values become a function of the properties of the concrete slab as well as of the subgrade $K$, base thickness, and properties of the base. Vesic and Saxena (1974) presented the concept of the equivalent $K$ based on interior slab stress and used a theoretical approach to develop a relationship between the equivalent $K$ and the properties of a concrete slab on an elastic foundation. The relationship developed by Vesic and Saxena is shown by Equation 16.

\[
K_{\text{equivalent}} = \left( \frac{E_f}{E} \right)^{\frac{1}{3}} \left( \frac{E_f}{1 - \mu^2} \cdot h \right)
\]  
(16)

where

- $E_f$ = modulus of elasticity of the foundation (subgrade)
- $E$ = modulus of elasticity of concrete
- $\mu$ = Poisson’s ratio of the concrete
- $h$ = thickness of the concrete slab.

Similar relationships can be developed by comparing the results from a Westergaard analysis with the results from a layered elastic analysis. The process consists of using the Westergaard model to compute the interior stress for a given pavement then varying the subgrade modulus in the layered elastic analysis to match the stress computed by the Westergaard’s model. Figure 6 shows the comparison between the relationship of Equation 16 and the relationship developed using layered elastic theory. Also shown in Figure 6 are the theoretical relationship between the subgrade $E$ and the composite $K$ and the relationship given in Army TM 5-824-3/Air Force AFM 88-6 (1988) between the subgrade resilient modulus and the composite $K$. The data in Figure 6 indicate excellent agreement between the results obtained using Equation 16 and the results obtained from the Westergaard/layered elastic analysis. The best agreement is obtained when a bonded interface is assumed for the layered elastic modeling.
Based on the comparison, Equation 16 was judged to be an excellent model for computing the equivalent $K$ for a pavement over a subgrade of a given modulus of elasticity.

The data in Figure 6 also illustrate the large difference that can exist in the different relationships between $K$ and the subgrade modulus of elasticity. In the development of the relationships for the effective $K$ for a multi-layered support system, the Westergaard and layered elastic models were utilized similarly to the analysis for a single-layered support system. The procedure for developing the theoretical relationship between base quality, base thickness, and effective $K$ was as follows:

1. The quality of the base and subgrade materials was judged by the equivalent $K$ and the corresponding equivalent modulus of elasticity determined from Figure 6 for a 14-in. bonded-concrete pavement. For example, a subgrade material yielding a $K$ value of 25 psi/in. would be the same quality as a material with a modulus of elasticity of 3,100 psi. Similarly, a subgrade material yielding a $K$ value of 800 psi/in. would be the same quality as a material having a modulus of elasticity of 38,350 psi.
2. Using the layered elastic model, the tensile stress at the bottom of the 14-in. slab was computed for different combinations of subgrade quality, base material quality, and base thickness.

Figure 7 presents one set of data developed from the analysis of a pavement system having different base thicknesses and material quality. Figure 7 suggests that there is a difference in the effective $K$ for different quality base materials when the quality of material is measured by the modulus of elasticity. For thin bases of fewer than 10 in., the difference between the materials, as judged by the increase in effective $K$, is not significant. The effect of the quality (modulus of elasticity) of the base on the effective $K$ increases with increasing thickness and increasing subgrade $K$. It is noted that when the effective $K$ obtained from Figure 7 is compared with the composite $K$ obtained from Figures 3, 4, and 5, the effective is significantly less than the composite.

![Computed Effective K for a 14 Inch Pavement and Dual C-17 Tires](image)

*Figure 7. Computed effective modulus of subgrade reaction.*
Table 4. Values composite $K$ and effective $K$ for different thicknesses of bases with a
subgrade $K$ of 100 psi/in.

<table>
<thead>
<tr>
<th>Thickness of Base (in.)</th>
<th>10</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Plate Simulation (Figure 3)</td>
<td>180</td>
<td>310</td>
<td>420</td>
</tr>
<tr>
<td>From Crushed Stone (Figure 4)</td>
<td>195</td>
<td>360</td>
<td>450</td>
</tr>
<tr>
<td>From Gravel (Figure 4)</td>
<td>190</td>
<td>310</td>
<td>380</td>
</tr>
<tr>
<td>From Combined Curves (Figure 5)</td>
<td>190</td>
<td>360</td>
<td>450</td>
</tr>
<tr>
<td>From Effective $K$ (Figure 7)</td>
<td>130</td>
<td>260</td>
<td>390</td>
</tr>
</tbody>
</table>

The finite element simulation of the plate-bearing test provided values of the composite $K$ that agreed closely with the composite $K$ presented in Figures 4 and 5. The layered elastic analyses of typical airfield pavements indicate the effective $K$ can be significantly less than the composite $K$. As the thickness of the base increases, both the composite modulus and the effective modulus will approach a terminal value of $K$. The terminal $K$ is defined as the value of $K$ that would be measured at the top of the base for a base of infinite thickness.

Figure 8 provides a comparison between the computed composite $K$ and the computed effective $K$. The analysis clearly indicates that, for thinner thicknesses of base, the effective modulus is significantly less than the composite modulus.
6 Development of a Model for the Effective Modulus of Subgrade Reaction

This section explains the development of an analytical model to represent the relationship between effective $K$, subgrade $K$, base quality, and base thickness. All curves included in Figures 2, 3, 4, 5, and 7 are characterized by an “S” shape and approach some terminal value of $K$ as the thickness of the base increases. Equation 17 is a function that has the desirable characteristics to model the effective $K$ curves.

$$y(t) = a \cdot e^{\frac{b}{ct}}$$

(17)

Equation 17 represents an S-shaped curve that begins at a set value when $t = 0$ and approaches another value as $t$ approaches infinite. When Equation 17 is used to model the effective $K$ relationship with base thickness and subgrade $K$, the boundary conditions of the relationship dictate that when $t = 0$, the function $y(t)$ must equal the $K$ value of the subgrade and that when $t$ approaches infinite, the function $y(t)$ must equal the $K$ value of the base material. When these boundary conditions are applied, $a = K_{\text{base}}$ and $b = \ln\left(\frac{K_{\text{subgrade}}}{K_{\text{base}}}\right)$. The values for the constants $a$ and $b$ can be substituted into Equation 16 to arrive at Equation 18.

$$K_{\text{effective}} = K_{\text{base}} \cdot e^{\frac{b}{ct}}$$

(18)

where

$$b = \ln\left(\frac{K_{\text{subgrade}}}{K_{\text{base}}}\right)$$

$$c = \text{a constant that dictates the shape of curve}$$

Equation 18 now can be used to represent the relationship between effective $K$ and the base thickness. In applying Equation 18 for determining the effective $K$, $K_{\text{subgrade}}$ is known from either a plate-loading test or an estimation from subgrade type or other empirical relationships. The value of $K_{\text{base}}$ depends on the stiffness of the base material. By applying the theoretical relationship between the modulus of elasticity and the effective $K$ (Equation 16), values of $K_{\text{base}}$ have been determined for typical values of the
The modulus of elasticity for different base materials. The value of the constant, \( c \), is a function of the material properties, slab properties, and loading. The theoretical effective \( K \) curves, the composite \( K \) curves from the manual, and the finite element simulations allowed a determination of values of \( c \) for each base material type. The following values are suggested for the terminal value of \( K_{base} \) and for the constant, \( c \).

1. Crushed Quarry Rock \((CBR = 100)\):
   
   \[
   K_{base} = 1200 \text{pci} \text{ and } c = 0.026.
   \]

2. Crushed Gravels and Shell Base \((CBR = 80)\):
   
   \[
   K_{base} = 1000 \text{pci} \text{ and } c = 0.024.
   \]

3. Gravels \((60 CBR; GW, GP)\)
   
   \[
   K_{base} = 800 \text{pci} \text{ and } c = 0.022.
   \]

4. Sands \((CBR = 40)\)
   
   \[
   K_{base} = 500 \text{pci} \text{ and } c = 0.020.
   \]

Figure 9 provides a comparison between the simulated effective \( K \) and the computed effective \( K \) and indicates that the model for simulation provides a reasonable estimate of the effective \( K \).

Simulations using Equation 18 and the values of the parameters given above now can be used to develop effective \( K \) charts for different base materials. The values of \( CBR \) assigned to the base materials are nominal design values based on material types. Even though the values of \( CBR \) for base materials are assigned based on material classifications, it is interesting to examine the assigned values of the terminal \( K \) and the constant, \( c \), in relationship to the assigned \( CBR \) for the different materials. Figure 10 shows the relationships between the two simulation parameters and the value of the assigned \( CBR \).

The data in Figure 10 indicate the maximum \( K \) value for a granular base to use in Equation 18 for computing the effective \( K \) is approximately 12.4 times the \( CBR \). The 12.4 factor is very close to the relationship between
Figure 9. Comparison of simulation effective $K$ with computed effective $K$.

Figure 10. Relationships between material CBR and the simulation parameters.
CBR and $K$ for granular material given in Figure 1. The shape constant, $c$, in Equation 17 is equal to $0.016 + \frac{CBR}{10,000}$. Figures 11, 12, 13, and 14 include effective $K$ curves for crushed quarry rock, crushed gravel, gravel, and sandy gravel, respectively.

The effective $K$ curves presented in Figures 11 to 14 represent, for the most part, a significant difference from the effective $K$ curves currently in the UFC. For $K$ values less than 300 psi/in. and base thicknesses less than about 30 in., the curves for crushed quarry rock and crushed gravel yield $K$ values that are comparable with the $K$ values in the current UFC. With thicker bases and higher $K$ values, the new effective $K$ curves will give higher values for effective $K$ than those provided in the UFC, particularly with the crushed quarry rock. The new and old curves are different because the development of the old curves limited the effective $K$ to 500 psi/in. and because the curves were sketched to approach 500 psi/in. asymptotically.

The proposed procedure for determining the effective $K$ for a particular base material results in an effective $K$ curve that is limited by the maximum value of $K$.

![Figure 11. Effective $K$ curves for crushed stone.](image-url)
Figure 12. Effective $K$ curves for crushed gravel (80 CBR).

Effective $K$ for Crushed Gravel (80 CBR); $K_{\text{max}} = 1000$ PCI; $c = 0.024$

Figure 13. Effective $K$ curves for gravel (60 CBR).

Effective $K$ for Gravel (60 CBR); $K_{\text{max}} = 800$ PCI; $c = 0.022$
Figure 14. Effective $K$ curves for sand or sandy gravel.
7 Effective $K$ for Stabilized Base

The procedure in the current instructions (UFC 2001) for design of rigid pavements with stabilized bases considers the concrete slab and stabilized layer to be an unbonded composite slab. An earlier version of the design procedure, published as TM5-824-3 (1958), included a chart, shown in Figure 15, for determining a composite $K$, which is used in the design procedure for determining the concrete slab thickness.

Figure 15. Chart in TM 5-824-3 (1958) for determining composite $K$ for stabilized bases.
As shown in the chart (Figure 15), the composite $K$ is a function of the subgrade $K$, the stabilized layer thickness, and the modulus of elasticity of the stabilized material. Although the origin of the chart could not be verified, it is believed the chart was developed under contract from an analysis of the plate-bearing tests.

The development of a new methodology for determining the effective $K$ for stabilized layers might be useful for future pavement design and evaluation. For this purpose, the same approach adopted for developing the effective $K$ criteria for granular layers also can be used to develop the effective $K$ criteria for stabilized layers. Equation 16 by Vesic and Saxena (1974) can be applied to rigid pavements constructed over a stabilized subgrade to determine the equivalent $K$ for the stabilized subgrade. Assuming a modulus of elasticity of 4,000,000 psi for the concrete slab and a Poisson’s ratio of 0.25 for the stabilized base, Equation 19 is obtained.

\[
K_{\text{base}} = \left( \frac{E_f}{4000000} \right)^{\frac{1}{3}} \left( \frac{E_f}{(1 - .25^2) \cdot h} \right) 
\]  

(19)

Equation 19 is reduced to Equation 20, in which the maximum effective $K$ for a stabilized material is a function of the modulus of elasticity of the stabilized layer, $E_f$, and the thickness, $h$ of the concrete slab. Table 5 contains values of $K_{\text{base}}$ for various values of slab thicknesses and stabilized layer moduli of elasticity.

\[
K_{\text{base}} = \frac{E_f^{1.333}}{148.82 \cdot h} 
\]  

(20)

Based on a layer elastic analysis, a value of 0.026 was selected as an appropriate value for the constant, $c$. By applying the parameters for stabilized materials into Equation 16, the model for the effective $K$ for stabilized bases is as shown in Equation 21.

\[
K_{\text{effective}} = K_{\text{base}} \cdot e^{\chi} 
\]  

(21)

where

\[
\chi = \frac{b}{e^{0.026 \cdot t}}
\]
\[
b = \ln \left( \frac{K_{subgrade}}{K_{base}} \right)
\]

\[t = \text{thickness of the stabilized base}\]

\[K_{base} = \text{a value determined from Equation 19 or selected from Table 5.}\]

### Table 5. Values of \(K_{base}\) for various values of modulus of elasticity of stabilized bases and thicknesses of concrete slabs.

<table>
<thead>
<tr>
<th>Maximum Equivalent Modulus of Reaction for Stabilized Material</th>
<th>Thickness of Concrete Slab, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity of Stabilized Base (psi)</td>
<td>10</td>
</tr>
<tr>
<td>50,000</td>
<td>1,238</td>
</tr>
<tr>
<td>100,000</td>
<td>3,119</td>
</tr>
<tr>
<td>200,000</td>
<td>7,859</td>
</tr>
<tr>
<td>300,000</td>
<td>13,495</td>
</tr>
<tr>
<td>400,000</td>
<td>19,804</td>
</tr>
<tr>
<td>500,000</td>
<td>26,667</td>
</tr>
<tr>
<td>600,000</td>
<td>34,005</td>
</tr>
</tbody>
</table>

Notes: Modulus of concrete is 4,000,000 psi. Poisson's ratio of stabilized material is 0.25.

Figures 16, 17, and 18 provide comparisons of the effective \(K\) from the chart in Figure 15 with the effective \(K\) obtained using Equation 21. Generally, the methodology represented by Equations 20 and 21 yields effective \(K\) values that, for most cases, are slightly lower than the \(K\) values derived from the chart shown in Figure 15.

The data given in Figures 16, 17, 18, 19, and 20 were developed using the proposed procedure and compare the proposed procedure with the chart in TM 5-825-3 (1988). Figure 19 indicates that for realistic slab thicknesses, the slab thickness has only a small influence on the effective \(K\). Even though the influence of slab thickness may be minimal, in determining the effective \(K\) a reasonable estimate of the slab thickness should be used.

The influence of the stabilized base modulus of elasticity on the effective \(K\) is more pronounced than the influence of the slab thickness. Figure 20 shows the influence of the base modulus of elasticity on the effective \(K\). As would be expected, the thicker the stabilized base, greater is the influence of the base modulus on the effective \(K\).
Figure 16. Comparison of chart with new procedure (modulus = 100,000).

Figure 17. Comparison of chart with new procedure (modulus = 300,000).
Figure 18. Comparison of chart with new procedure (modulus = 600,000).

Figure 19. Influence of the slab thickness on the effective K.
Figure 20. Influence of base modulus of elasticity on the effective $K$. 

**Effect of Base Modulus of Elasticity on Effective K**

<table>
<thead>
<tr>
<th>Slab Thickness</th>
<th>Concrete Modulus</th>
<th>Poisson's Ratio of Stab. Mat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 IN.</td>
<td>4,000,000 PSI</td>
<td>0.25</td>
</tr>
<tr>
<td>12 IN.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 IN.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8 Determining the Modulus of Elasticity for a Stabilized Base

Because pavements are not linear elastic continuaums, the modulus of elasticity must be defined with reference to the particular material and the manner in which the modulus is being used. The modulus of elasticity of cement- and lime-stabilized materials can be determined in several ways, such as from the compression test, the flexural beam test, and the indirect tensile test or even back-calculated from FWD data. For most applications in pavement analysis, the modulus of elasticity is only a pseudo modulus that is used in a layered elastic model to give pavement response that, in the opinion of the user, matches the observed pavement response.

In the development of design criteria for flexible pavements (Burns 1974), a post-examination by the author of prototype test sections revealed stabilized layers were severely cracked, even in areas that did not sustain test traffic. Load testing of the test pavements also indicated the stabilized bases did not behave as continuous layers. For these reasons, a pseudo modulus based on the compressive strength of stabilized layers was adopted for the design of flexible pavements. Figure 21 shows the relationship between the compressive strength and the pseudo modulus, referred to as the cracked modulus. Strong stabilized layers under concrete slabs can behave as continuous slabs only if the stabilized material has sufficient strength to support construction traffic and cracking is controlled by saw cutting joints to match the concrete slab joints. Considering the difficulty in constructing stabilized layers without cracking, the probability that the stabilized layer will behave as a continuous slab is very small. Therefore, it is appropriate to use the cracked modulus, as determined from Figure 21, for evaluating the stabilized base effective $K$.

The equation for the relationship between the unconfined compressive strength ($S_{uc}$) and the cracked modulus ($M_{stab}$) of cement- and lime-stabilized materials is presented as Equation 22.

$$M_{stab} = 2.48 \cdot (S_{uc})^{1.559}$$  (22)
By use of Equation 22 and the methodology presented for computing the effective $K$, it is possible to determine the relationship between compressive strength and the base thickness required to achieve a specified effective $K$. Figure 22 presents the stabilized layer thickness required to obtain a specified effective $K$ as a function of confined compressive strength of the stabilized base when placed over a subgrade with $K$ equal to 100 psi/in.

The modulus for asphalt-stabilized materials might be more complex to evaluate than for cement-stabilized materials. Because asphalt is a viscous material and granular materials have stress-dependent behavior, the asphalt-stabilized base modulus value is greatly influenced by temperature, rate of loading, and level of stress. In the design and evaluation of flexible
pavements, a modulus value of approximately 200,000 psi is commonly used to model asphalt layers. As a base under concrete pavements, an asphalt base is subjected to static loading caused by parked aircraft as well as warping and curling of the concrete slab. Because of the static loading and the possibility of elevated temperatures, the modulus values used for asphalt layers should be in the range of 100,000 psi rather than 200,000 psi.

![Graph showing the relationship between unconfined compressive strength and thickness for subgrade K of 100 psi/in.](image)

**Figure 22.** Relationship between unconfined compressive strength and thickness for subgrade K of 100 psi/in.
9 Equivalency of Base Materials

Figures 11, 12, 13, and 14 are charts for four granular base materials; and Figures 16, 17, and 18 are charts for three stabilized base materials. The charts can be used to compute equivalency factors in terms of required thickness to obtain a given effective $K$. The chart in Figure 11 determines values of the effective $K$ for crushed stone base thicknesses of 10 in., 20 in., and 30 in. over subgrades having $K$ values of 50 psi/in. and 100 psi/in. Table 6 summarizes the effective $K$ values obtained for the different thicknesses of crushed stone. The effective $K$ values in Table 6 then were used to determine the thicknesses from Figures 12, 13, 14, 16, 17, and 18 of the other materials required to match the effective $K$ values in Table 6. An equivalency factor for each material was computed by dividing the thickness of each material by the thickness of crushed stone for which the value of the effective $K$ was matched. Tables 7 and 8 contain thicknesses and equivalency factors for the different materials. Representative equivalency factors for the different materials are 1.2 for crushed gravel; 1.5 for gravel; 2.0 for sandy gravel; 0.8 for a stabilized base having a modulus of 100,000 psi; 0.5 for a stabilized base having a modulus of 300,000 psi; and 0.4 for a stabilized base having a modulus of 600,000 psi.

Equation 18 can be rearranged, as shown in Equation 23, to solve directly for the required thickness of a given material to achieve a specified effective $K$ for a given subgrade $K$.

$$t = \left(\frac{1}{c}\right) \times \ln \left(\frac{\ln \left(\frac{K_s}{K_b}\right)}{\ln \left(\frac{K_e}{K_b}\right)}\right)$$

Table 6. Effective $K$ for different thicknesses of crushed stone.

<table>
<thead>
<tr>
<th>Thickness of Crushed Stone (in.)</th>
<th>Effective $K$ Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>105</td>
</tr>
<tr>
<td>20</td>
<td>180</td>
</tr>
<tr>
<td>30</td>
<td>280</td>
</tr>
</tbody>
</table>

Note: Chart not in range of data.
### Table 7. Data for determining equivalency factors for base on subgrade $K$ of 50 psi/in.

<table>
<thead>
<tr>
<th>Material</th>
<th>$K = 105$ pci</th>
<th>$K = 180$ pci</th>
<th>$K = 280$ pci</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushed Stone</td>
<td>10.0</td>
<td>20.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Crushed Gravel</td>
<td>12.0</td>
<td>24.0</td>
<td>37.0</td>
</tr>
<tr>
<td>Gravel</td>
<td>14.0</td>
<td>29.0</td>
<td>45.0</td>
</tr>
<tr>
<td>Sand/Sandy Gravel</td>
<td>20.0</td>
<td>44.0</td>
<td></td>
</tr>
<tr>
<td>Stab. E = 100,000</td>
<td>8.2</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Stab. E = 300,000</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Stab. E = 600,000</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

Note: * - Data out of chart range.

### Table 8. Data for determining equivalency factors for base on subgrade $K$ of 100 psi/in.

<table>
<thead>
<tr>
<th>Material</th>
<th>$K = 180$ pci</th>
<th>$K = 275$ pci</th>
<th>$K = 380$ pci</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushed Stone</td>
<td>10.0</td>
<td>20.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Crushed Gravel</td>
<td>13.0</td>
<td>24.0</td>
<td>37.0</td>
</tr>
<tr>
<td>Gravel</td>
<td>16.0</td>
<td>30.0</td>
<td>48.0</td>
</tr>
<tr>
<td>Sand/Sandy Gravel</td>
<td>25.0</td>
<td>50.0</td>
<td></td>
</tr>
<tr>
<td>Stab. E = 100,000</td>
<td>8.5</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>Stab. E = 300,000</td>
<td>5.8</td>
<td>9.6</td>
<td>13.5</td>
</tr>
<tr>
<td>Stab. E = 600,000</td>
<td>4.2</td>
<td>8.0</td>
<td>10.9</td>
</tr>
</tbody>
</table>

Note: * - Data out of range of chart.

where

\[ t = \text{ thickness of the base} \]
\[ K_e = \text{ effective modulus of subgrade reaction} \]
\[ K_b = \text{ maximum possible } K \text{ for the particular material} \]
\[ K_s = \text{ subgrade } K \]
\[ c = \text{ shape constant in Equation 18} \]

Equivalency factors, as in Table 10 of the UFC, are used in flexible pavement design and evaluation criteria. These equivalency factors are defined relative to either a granular base or a granular subbase material. Table 10 shows that 1 in. of unbound crushed stone replaces 2 in. of all other unbound granular subbases, but crushed gravel (80 CBR) is
equivalent to other granular basis only on a 1-to-1 basis. The equivalencies determined from Tables 7 and 8 indicate that 1 in. of crushed stone will replace only 1.5 in. of unbound granular subbase, and 1 in. of crushed gravel will replace 1.25 in. of unbound granular subbase.

Table 9 contains data for the thicknesses of the base materials required to develop effective $K$ values of 250 psi/in. and 500 psi/in. for a subgrade with a $K$ of 100 psi/in. The data in Table 9 can be used to develop the equivalency of stabilized bases as a function of the compressive strength of the stabilized material. Figure 23 shows the equivalency of cement-/lime-stabilized materials relative to crushed stone, crushed gravel, and gravel for a subgrade having a modulus of reaction of 100 psi/in. The data indicate that for a stabilized material to be equivalent to crushed stone requires a compressive strength of approximately 630 psi, to be equivalent to crushed gravel requires a compressive strength of approximately 530 psi, and to be equivalent to gravel requires a compressive strength of only 230 psi.

Table 9. Data developed from Equation 22 for subgrade $K$ values of 100 psi/in.

<table>
<thead>
<tr>
<th>Material</th>
<th>UC Strength (psi)</th>
<th>Effective $K$ of 250 psi/in.</th>
<th>Effective $K$ of 500 psi/in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thick (in.)</td>
<td>C.S./Mat.</td>
<td>Mat./C.S.</td>
</tr>
<tr>
<td>Crushed Stone</td>
<td>17.69</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Crushed Gravel</td>
<td>21.14</td>
<td>0.84</td>
<td>1.19</td>
</tr>
<tr>
<td>Gravel</td>
<td>26.41</td>
<td>0.67</td>
<td>1.49</td>
</tr>
<tr>
<td>Sand/Sandy Gravel</td>
<td>42.12</td>
<td>0.42</td>
<td>2.38</td>
</tr>
<tr>
<td>Stab. Base E = 25,000</td>
<td>370</td>
<td>0.35</td>
<td>2.86</td>
</tr>
<tr>
<td>Stab. Base E = 50,000</td>
<td>575</td>
<td>0.84</td>
<td>1.19</td>
</tr>
<tr>
<td>Stab. Base E = 100,000</td>
<td>900</td>
<td>1.31</td>
<td>0.76</td>
</tr>
<tr>
<td>Stab. Base E = 200,000</td>
<td>1,400</td>
<td>1.78</td>
<td>0.56</td>
</tr>
<tr>
<td>Stab. Base E = 300,000</td>
<td>1,825</td>
<td>2.06</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Notes:
* Material is not capable of achieving an effective $K$ of 500 psi/in.
* C.S. stands for crushed stone.
Table 10. Table of equivalency factors from UFC.

### Table 10-2
Equivalency Factors for Army and Air Force Pavements

<table>
<thead>
<tr>
<th>Material</th>
<th>Equivalency Factors</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base</td>
<td>Subbase</td>
</tr>
<tr>
<td>Asphalt-Stabilized</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All- Bituminous Concrete</td>
<td>1.15</td>
<td>2.30</td>
</tr>
<tr>
<td>GW, GP, GM, GC</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>SW, SP, SM, SC</td>
<td>(\text{--}^1)</td>
<td>1.50</td>
</tr>
<tr>
<td>Cement-Stabilized</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW, GP, SW, SP</td>
<td>1.15(^2)</td>
<td>2.30</td>
</tr>
<tr>
<td>GC, GM</td>
<td>1.00(^2)</td>
<td>2.00</td>
</tr>
<tr>
<td>ML, MH, CL, CH</td>
<td>(\text{--}^1)</td>
<td>1.70</td>
</tr>
<tr>
<td>SC, SM</td>
<td>(\text{--}^1)</td>
<td>1.50</td>
</tr>
<tr>
<td>Lime-Stabilized</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ML, MH, CL, CH</td>
<td>(\text{--}^1)</td>
<td>1.00</td>
</tr>
<tr>
<td>SC, SM, GC, GM</td>
<td>(\text{--}^1)</td>
<td>1.10</td>
</tr>
<tr>
<td>Lime, Cement, Fly Ash Stabilized</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ML, MH, CL, CH</td>
<td>(\text{--}^1)</td>
<td>1.30</td>
</tr>
<tr>
<td>SC, SM, GC, GM</td>
<td>(\text{--}^1)</td>
<td>1.40</td>
</tr>
<tr>
<td>Unbound Crushed Stone</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Unbound Aggregate</td>
<td>(\text{--}^1)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\(^1\) Not used as base course.
\(^2\) For Air Force Bases, cement is limited to 4 percent by weight or less.

### Table 10-3
Equivalency Factors for Navy and Marine Corps Pavements

<table>
<thead>
<tr>
<th>Stabilized Material</th>
<th>Equivalency Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm (in.) of lime-stabilized subbase</td>
<td>1.2 mm (in.) of unstabilized subbase course</td>
</tr>
<tr>
<td>1 mm (in.) of cement-stabilized subbase</td>
<td>1.2 mm (in.) of unstabilized subbase course</td>
</tr>
<tr>
<td>1 mm (in.) of cement-stabilized base</td>
<td>1.5 mm (in.) of unstabilized base course</td>
</tr>
<tr>
<td>1 mm (in.) of bituminous base</td>
<td>1.5 mm (in.) of unstabilized base course</td>
</tr>
</tbody>
</table>
Figure 23. Equivalency of stabilized material to different granular materials for subgrade $K = 100$ psi/in.
10 Summary and Conclusions

The history of the development of the plate-bearing test and the current procedure for conducting the test were reviewed. The review indicated significant differences in the ASTM and the CRD-C 655-95 plate test procedures. The ASTM procedure was deemed incomplete as to data analysis for design of military pavements. The procedure contained in CRD-C 655-95 is more complete, in that instructions are given for data analysis.

Based on the experience with plate testing at ERDC, minor modifications to the test procedure are recommended. One modification is that, for modulus values equal to or less than 200 psi/in., the computation for the modulus should be based on the tangent to the load-deformation curve at a plate loading of 10 psi. Also, for weak subgrades (modulus values equal to or less than 200 psi/in.), the load should be applied in increments of 2.5 psi, with the test terminated at a loading of 15 psi. The smaller increment of loading would provide sufficient data points to establish the load-deformation curve for construction of the tangent.

It was noted that the procedure in CRD-C 655-95 requires corrections to the modulus for saturation and plate bending. For some materials, the correction for saturation can be sufficient and, where appropriate, this correction should be made. The correction for plate bending is appropriate only for high-strength material. The correction for bending can be computed using Equation 3, which is a representation of the correction curve given in CRD-C 655-95.

During the development of the plate-bearing test, several variations in plate size, plate thickness, plate arrangement, loading rate, and loading repetition were investigated. Most early procedures for computing the modulus were based on the loading to produce a given plate deflection. At some point in the early development, the military selected the static loading with the 10 psi as the reference for determining the modulus of subgrade reaction. The use of the static test procedure is in conflict with the current trend to use resilient properties to characterize subgrade materials. This is particularly true in pavement evaluation which uses the FWD, in which repeated loading is applied at a very high rate. The effects due to differences in load application are apparent when attempting to correlate (Figure 6) the plate-bearing modulus with the modulus of elasticity of the subgrade materials.
Another aspect of developing a correlation between modulus of elasticity and modulus of subgrade reaction is the stress dependency of soils. Casagrande noted in 1945 that the modulus of subgrade reaction for cohesive materials always will be greater than the modulus predicted from the modulus of elasticity determined from the unconfined compressive test. This behavior is explained by the fact that, in the unconfined compressive test, the entire soil sample is subject to a high level of stress as compared to the soil strength; whereas, in the plate-bearing test, only a small portion of the soil involved is subject to significant stress. The stress dependency also becomes apparent in the correlations between CBR and modulus of reaction. As shown in this report (in the development of Equation 7), the theoretical relationship between CBR and \( K \) is that \( K \) is approximately 6.5 times the CBR. Field test data and existing correlations (Figure 1) indicate that, for fine-grained soils, \( K \) is approximately 20 times the CBR. For granular materials, the observed relationship between CBR and \( K \) is closer to the theoretical relationship. This is particularly true for dense granular materials that have high CBR values and for which the response is more closely represented by elastic behavior. In no case can the conversion factor from CBR to \( K \) be less than 6.5.

Data from plate-bearing tests conducted at ERDC were reviewed. These data indicated quality of the base materials influences the relationship between the thickness of the base and the modulus of reaction measured at the top of the base. Although the variability in the data was very large, the data tended to verify the existing relationship between thickness and modulus of reaction.

In considering the design and evaluation of rigid pavements, the difference in composite \( K \) and effective \( K \) (or equivalent \( K' \)) is recognized. In the past, the composite \( K \) has been used to represent the effective \( K \). In developing the effective \( K \) model of this study, the attempt was to define the effective \( K \) as the \( K \) used for the subgrade in a single-layer stress model that would produce the same tensile stress as computed in a layered stress model. Based on the shape of existing effective \( K \) curves and theoretical considerations, the function represented by Equation 17 was chosen to model the effective \( K \) as a function of the subgrade \( K \) and the thickness of the non-stabilized granular base. Equation 18 was developed from Equation 17 to include the parameters \( K_{\text{subgrade}} \), \( K_{\text{base}} \), and a shape factor, \( c \). By using analytical modeling, values of the parameters \( K_{\text{base}} \) and \( c \) were established for typical base materials. The values for these materials are given in Table 11.
Table 11. Values of parameters to be used in Equation 18.

<table>
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<tr>
<th>Base Material</th>
<th>CBR</th>
<th>Maximum $K$</th>
<th>Constant &quot;c&quot;</th>
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<tr>
<td>Crushed Stone</td>
<td>100</td>
<td>1,200</td>
<td>0.026</td>
</tr>
<tr>
<td>Crushed Gravel</td>
<td>80</td>
<td>1,000</td>
<td>0.024</td>
</tr>
<tr>
<td>Gravel</td>
<td>60</td>
<td>800</td>
<td>0.022</td>
</tr>
<tr>
<td>Sand/Sandy gravel</td>
<td>40</td>
<td>500</td>
<td>0.02</td>
</tr>
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The maximum $K$ and $c$ given in Table 11 can be used when given a particular material, or these parameters can be computed from the relationship that $K_{base}$ is approximately 12.4 times the CBR and $c$ is equal to 0.016 plus the CBR divided by 10,000. Using the relationships developed in this report, charts such as those shown in Figures 11, 12, 13, and 14 can be developed for any non-stabilized granular base material.

Stabilized materials can be modeled using Equation 17 in much the manner that non-stabilized granular materials were modeled. The appropriate value of $K_{base}$ for entering Equation 17 is to be computed using the relationship (Equation 16) developed by Vesic and Saxena. By assuming values for the concrete modulus and Poisson’s ratio, Equation 16 has been simplified to obtain Equation 20. As an additional simplification, the value for $K_{base}$ can be selected from Table 4. For all stabilized bases, the value of the shape factor, $c$, is held a constant 0.026. To determine the modulus of elasticity of a stabilized base, the relationship between unconfined compressive strength and cracked modulus, as defined in the layered elastic procedure for flexible pavements, is used. This relationship, Figure 21, is modeled using Equation 22.

A procedure for developing equivalency factors for different materials has been presented. The procedure is based on Equation 23, which is a rearrangement of Equation 18. Using Equation 23, the thickness of different base materials required to develop a given effective $K$ can be computed. The ratio of the thickness required for the different materials can be considered an equivalency factor for those materials. The data in Table 8 and the data in Table 9 are developed using Equation 23. These equivalency factors are more in agreement with equivalency factors used by the Navy and with those used in highway design.
11 Recommendations

The following recommendations are made concerning the procedure for conducting the plate-bearing test:

1. The basic procedure as given in CRD-C 655-95 (2001) is recommended for use by the military.
2. The loading for weak subgrades should be limited to 15 psi with 2.5 psi load increments.
3. For weak subgrades, the modulus of subgrade reaction should be defined as the tangent to the load-deflection curve at a load of 10 psi.
4. Care is taken to correct the measured $K$ for saturation and plate bending. In this regard, the $K$ value that has not been corrected for saturation should always be $K_u$, and the $K$ that has not been corrected for plate bending should always be identified as $k'$.

The recommendations for estimating the modulus of reaction are:

1. For fine-grained soils such as CHs and CLs, the $K$ value can be estimated as 20 times the $CBR$.
2. For granular soils, the maximum $K$ value for the material can be estimated as 12.0 times the $CBR$.
3. When the modulus of elasticity of the subgrade under a concrete pavement is known, Equation 16 should be used to estimate the $K_{equivalent}$.
4. When the modulus of elasticity of a base material is known, Equation 16 can be used to estimate $K_{base}$.
5. For stabilized bases, the cracked section modulus is estimated using the unconfined compressive strength as given in Figure 21 or by the use of Equation 22.

The recommendations for determining the effective modulus of reaction for layered support systems are:

1. $K_{effective}$ for non-stabilized granular bases over a subgrade should be computed using Equation 18. The parameters to be used in Equation 18 may be selected from Table 10.
2. $K_{\text{effective}}$ for stabilized bases also should be computed using Equation 18, but with the value $K_{\text{base}}$ selected from Table 4 and the value of $c$ being constant at 0.026.

The recommendation concerning equivalency factors is:

1. Typical values for equivalency factors for different materials should be established based on the relationship given in Equation 23.
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# Determining the Effective Modulus of Subgrade Reaction for Design of Rigid Airfield Pavements Having Base Layers

## Abstract

Design and evaluation of rigid pavements are based upon the modulus of subgrade reaction ($K$) as determined by a plate-bearing test. When the pavement support system, which includes the subgrade and/or base beneath the slab, contains a base material, the design or evaluation is based on an effective $K$, which is a function of the subgrade $K$ and the thickness of the base. Technical manual Army TM 5-824-3/Air Force AFM 88-6 contained two charts for evaluating the effective $K$: one for a high-quality base and the other for a low-quality base. A later version of the pavement design manual reduced the two charts to a single chart that eliminated quality of the base as a consideration in the determination of the effective $K$. Because quality of base is not considered, the validity of a single chart has been questioned. Because of the questions raised, a study was conducted that included a review of the history of the plate-bearing test, plate-bearing tests on base materials of different qualities, and an analytical study for determining the effective $K$. The study resulted in a new methodology of determining the effective $K$ and recommendations for modifying the procedure for conducting the plate-bearing tests.

## Subject Terms

- Rigid airfield pavements
- Plate-bearing test
- Subgrade reaction
- Effective $K$
- Subgrade $K$

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