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Thickness and roughness variations of arctic multiyear sea ice

Cover: A multiyear ice ridge and general multiyear ice topography of the type usually found as large floes in the Arctic Ocean. (Photograph by Stephen F. Ackley.)

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Thickness and roughness variations of arctic multiyear sea ice

S.F. Ackley, W.D. Hibler III, F.K. Kugzruk,
A. Kovacs and W.F. Weeks

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20. Abstract (cont'd)

where ρ is the ice density and f' is the ice freeboard plus snow depth in ice equivalent at the point in question. This statistical relationship is consistent with the observed physical properties, which indicate that as the ice freeboard increases, the ice salinity decreases and the higher freeboard or thicker ice therefore decreases in density. Using this variable density with freeboard relationship, a model was constructed to predict the ice thickness, given the ice freeboard and snow depth alone. This prediction is desirable, since the snow depth and freeboard are relatively easy to obtain, whereas the ice thickness can usually be obtained only by drilling through the ice. The model was compared with two other models, one assuming constant ice density (independent of freeboard) and the other using smoothing filters for predicting the ice thickness. It was found that the variable density prediction model gave the best approximation to the observed ice thickness, with a standard error between the measured and predicted value of about 0.4 m, compared with errors from 50 to 100% higher for the other two models. The model was also compared with data on multiyear ice from two other investigations in different regions and was found to give error estimates similar to the error of the data set on which the model was based. It is therefore concluded that the model can be useful to estimate multiyear ice thicknesses from surface elevation information obtained either by ground-based techniques or by aerial methods such as laser profilometry or stereo aerial photogrammetry. The effect of the variable density on estimates of the stress induced in the ice sheet by isostatic imbalance loading was examined and the results are presented in an appendix. Consideration of this property led to the conclusion that stresses from sources other than isostatic imbalance must account for 75% or more of the bending stresses necessary to induce cracking in multi-year ice.

PREFACE

This report was prepared by S.F. Ackley, Research Physicist, Dr. W.D. Hibler III, Research Physicist, F.K. Kugzruk, Physical Science Assistant, and Dr. W.F. Weeks, Glaciologist, of the Snow and Ice Branch, Research Division, and A. Kovacs, Research Civil Engineer, of the Foundations and Materials Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory.

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SUMMARY

The multiyear or perennial ice which accounts for approximately 60 to 75% of the ice cover of the Arctic Ocean is characterized by large variations in thickness, roughness and other physical properties over horizontal distances of the order of a few meters. The physical property variations in the horizontal and vertical make it difficult to obtain the thickness, and surface and depth roughness features of multiyear ice without drilling many holes, a time-consuming and laborious process in ice usually exceeding 2.5 m in thickness. At the same time, this information is critically necessary for a large variety of problems. For example, ice thickness data are needed for predicting and modelling the dynamics of ice drifts, determining routes of icebreakers, estimating the force of ice on offshore structures, and transporting equipment (portable oil rigs) over ice; ice roughness data are needed for modelling the dynamics of vehicles travelling over ice, sound scattering from the underside of the ice, and entrapment of oil spills under the ice cover.

In this study, the feasibility of using one easily obtainable parameter, surface elevation profiles, to predict the ice thickness was investigated. The predicted thicknesses obtained from three models using the surface elevation as input were compared with measured thicknesses obtained on multiyear ice during the 1972 Arctic Ice Dynamics Joint Experiment Pilot Study. One of the models assumed that the thicker ice was less dense and that this density was correlated with high surface elevations. Using a linear regression fit of the density to the surface elevation, the predicted ice thickness at a point was calculated under the assumption of local isostatic equilibrium (i.e., the mass of water displaced by the ice was equal to the mass of the ice at that point). It was found that the model allowed prediction of the ice thickness with a standard error of about 0.4 m and was a considerable improvement over the model which assumed a constant density for the ice irrespective of surface elevation.

Estimates of the error in obtaining ice thickness profiles by airborne instead of ground-based measurements of the surface elevation were also made. It was found that the error increased by only about 10% and that the predicted thickness profiles still accounted for approximately 50% of the ice thickness variation, making them "useful" estimates. Therefore, it is suggested that surface elevations obtained from airborne laser profiles or stereo aerial photogrammetry can be used with the model to provide a remote sensing method for obtaining multiyear ice thickness information.

The information on the density variations on multiyear ice presented in the main part of the report was used to recalculate the possible stresses present in the ice induced by the isostatic imbalance at a point; the results are presented in the appendix. It was found that the stresses are insufficient to produce cracks in multiyear ice when the density variations are taken into account and the small stresses developed are probably relieved by creep of the ice occurring over periods of weeks to months.

THICKNESS AND ROUGHNESS VARIATIONS OF ARCTIC MULTIYEAR SEA ICE

S.F. Ackley, W.D. Hibler III, F.K. Kugzruk,
A. Kovacs and W.F. Weeks

INTRODUCTION

In a study of the ice cover of the Arctic Ocean, two parameters of interest are the variations of ice thickness and the roughness or topographic relief of the ice cover. For example, ice thickness distributions are needed to model correctly the mechanics of ice interaction so that the drift and dynamics of the arctic pack can be predicted (Thorndike and Maykut 1973). Measures of thickness are also needed for accurate calculations of the mass budget of the icepack as an input to climatic models (Koerner 1973). In addition, a wide variety of applied operations, such as those involving icebreaking by ships and transporting heavy equipment over ice, require accurate information on ice thickness variations for route selection and safe load predictions.

The roughness characteristics of the surface are important in determining the momentum that the wind imparts to the ice cover (Banke and Smith 1973). This is usually the most important driving force in ice drift. The surface roughness characteristics were necessary in a recent program for developing a large air cushion vehicle to traverse the arctic pack. In this study, terrain roughness information was used in computer models to simulate the dynamic response of the vehicle to the terrain and to predict the detour mileage incurred by vehicles traversing pack ice surfaces (Hibler and Ackley 1973).

Information on the bottom roughness of the ice is needed for evaluating the effects of the water drag force on the ice cover drift (Coachman and Smith 1971, Hunkins 1971), for analyzing operational problems such as sound scattering from the bottom of the ice cover (Berkson et al. 1973), and for determining the confinement of accidental oil spills under the ice cover for cleanup operations (Campbell 1973, Wolfe and Hoult 1972).

The techniques for obtaining this information have had several limitations. The primary factor has been the lack of field portable or airborne equipment to reliably measure ice thickness (Weeks et al. 1974). At present, the only sure method of determining sea ice thickness is to measure it by laboriously drilling a hole. However, because of the rapid variations in thickness, especially in multiyear ice, this procedure for obtaining an accurate representation of the thickness profile is extremely time consuming.

Past observations, however, have indicated some correlation between the surface elevation of the ice cover and its thickness (Bushuev 1966, Hibler et al. 1972, Zubov 1945). In the present report, three profiles of surface elevation and ice thickness of multiyear ice obtained in the 1972 Arctic Ice Dynamics Joint Experiment (AIDJEX) Pilot Study (University of Washington 1972), as well as other data obtained from the literature, are examined to determine the extent of this correlation and the errors produced by different models for predicting the ice thickness from the surface elevation profile.

Profiles of the surface were obtained with standard surveying equipment (level and rod). An ice-thickness profile can be obtained expediently and inexpensively by combining the surface profile with a graph using one of the models tested in this study. The power spectra of the surface and bottom elevations from these profiles were also examined and the variances at high and low frequencies are discussed in terms of the processes changing the ice cover characteristics.

The feasibility of using these predictive methods to determine ice thickness profiles from surface elevation data obtained by airborne laser profilometry such as that used to obtain ridge statistics (Hibler et al. 1974) is examined and estimates of the accuracy of the method are given. This method would allow the surface roughnesses and ice thicknesses of given line profiles over extensive distances to be measured simultaneously. This information would be highly useful for scientific studies concerning ice mass balance and drift and for operational purposes such as picking routes for ice breakers and estimating the forces exerted on offshore structures by moving icepacks of variable thicknesses.

PREVIOUS WORK

The basic conclusion reached in previous investigations (Bushuev 1966, Gakkel' 1959, Hibler et al. 1972, Nazintsev 1971, Yakovlev 1955) was that in all cases the whole floe under study was in overall free-floating isostatic equilibrium. This was verified by calculating the density of the ice from Archimedes' principle

$$\rho_i \sum_j (d_j + f_j) + \rho_s \sum_j s_j = \rho_w \sum_j d_j \quad (1)$$

where ρ_i , ρ_s and ρ_w are the mean densities of the ice, snow cover and sea water, respectively; d_j and f_j are the below sea-level and above sea-level portions of the ice cover at each profile point; and s_j is the snow depth at each point. Using independent measurements of the snow and water density and profiles of the thickness and elevation, it was found that the ice density calculated from eq 1 was in good agreement with mean densities obtained from core samples of the ice. However, when this mean density was then used to predict the thickness of the ice at each point, pronounced deviations from isostatic equilibrium appeared and the predicted thickness profile bore little resemblance to the profile measured (Nazintsev 1971).

From measurements of three profile lines taken on a multiyear floe during the 1972 AIDJEX Pilot Study (University of Washington 1972), we have reexamined this model and have constructed new models for predicting the ice thickness from a surface elevation profile for comparison.

RESULTS

The data were obtained from profile lines surveyed by level and rod and the ice was drilled for thickness. The holes on two profile lines were obtained by using a 3/4-in. electrically-powered drill to drive lightweight auger flights. Steel bits constructed for ice drilling were used as the cutting head (Kovacs et al. 1973, Mellor et al. 1973). Some holes were also obtained by using a steam drill (Hodge 1971). However, problems with operating in the cold (-20° to -30°C) were encountered with this unit, which was developed primarily for work on temperate glaciers. A third profile line was obtained by measuring through a long crack in the ice which formed during the field study. Plots indicating the vertical section along each profile line are shown in Figure 1.

Figure 1. Cross sections of the three profiles studied.

To examine the correlation between surface elevation and thickness, we define the “effective” freeboard f'_j at each point j as follows:

$$f'_j = f_j + (\bar{\rho}_s)/(\bar{\rho}_i)s_j. \quad (2)$$

Equation 2 simply converts the snow depth to an equivalent ice thickness by utilizing the ratio of the mean snow density to the mean ice density of the floe. This quantity is then added to the existing ice freeboard f_j at each point to create the effective freeboard f'_j .

Models for predicting thickness from ice freeboard

Three models correlating the effective freeboard f'_j with thickness are now described:

Model 1 – Point isostatic. In this predictive model, we assume that the ice floe behaves as a series of free-floating blocks with each profile point allowed to achieve isostatic equilibrium. The floe is assumed to be homogeneous with a density equal to the mean density of the floe calculated from eq 1. Here, we assume the sea water has a density of 1020 kg/m^3 (taken from measurements at the site). The predicted thickness t is then found from isostatic equilibrium at each profile point

$$\rho_i f' + \rho_i d = \rho_w d \text{ or } d = \rho_i f' / (\rho_w - \rho_i) \quad (3)$$

and

$$t = f' + d. \quad (4)$$

Model 2 – Variable density. From core samples taken on multiyear ice, we know that there are large horizontal and vertical variations in the physical properties of the ice (Cox and Weeks 1973), especially in the salinity and brine content. We assume that the higher portions of the ice cover are subjected to more ablation and drainage than the lower parts and therefore contain less brine and are less dense. Figure 2 shows the effective freeboard and density of the ice calculated by solving eq 1 for ρ_i at each profile point, using the measured values of freeboard, ice depth, and snow depth and using the values $\rho_w = 1020 \text{ kg/m}^3$ and $\rho_s = 320 \text{ kg/m}^3$. The solid line is the linear regression fitted to the data, as expressed in the equation

Figure 2. Density of the ice versus effective freeboard calculated from the three cross sections shown in Figure 1. The regression line (solid) is expressed by the eq $\rho = -194f' + 974 \text{ kg/m}^3$. The dashed line indicates the low density cutoff corresponding to an ice sail-height-to-keel-depth ratio of 1 to 3.

Figure 3. Ice thickness plotted as a function of effective freeboard using the variable density prediction (Fig. 2). The hatched area indicates the error (standard deviation) between the measured and predicted values using the density prediction.

$$\rho = -194f' + 974$$

(5)

where ρ is in kg/m^3 and f' is in meters. The dashed portion of the curve indicates a constant ice density (765 kg/m^3 at freeboards greater than 1.05 m). This density gives the ratio of ice above sea level to that below sea level of 1 to 3, which is in agreement with the observed-sail-height-to-keel-depth ratio for multiyear pressure ridges (Kovacs and Mellor 1974, Kovacs et al. 1973).^{*} The model therefore assumes the linear relationship of density to freeboard shown in eq 5 for freeboards below approximately 1.05 m, and a constant density of 765 kg/m^3 for freeboards above this value. Given a measured effective freeboard, a density is calculated from the regression relationship and converted to an ice thickness, using eq 3 and 4. This transformation is done graphically in Figure 3, which plots effective ice freeboard versus ice thickness using this density model. Figure 3 can be used as a field nomogram with reasonable accuracy limits indicated by the hatched areas around the curve.

Model 3 — Distributed isostatic. We make use of the Wittman-Makarov model (Wittman and Schule 1966) to determine the shape of pressure ridges. In this model, which has been supported by field observations (Weeks and Kovacs 1970) and by a kinematic computer model for ridge formation (Parmerter and Coon 1972), the keel or underside of the ridge is wider than the sail or topside of the ridge. In using this model, we assume that for each profile point above sea level an equivalent volume of ice calculated from isostatic equilibrium is distributed over some distance on either side of the profile point in an isosceles triangular shape with the apex of the triangle directly under the profile point. This procedure is equivalent to passing a "sawtooth" linear filter across the freeboard with the gain equal to the average ratio of the depth to the freeboard $= \rho_i / \rho_w - \rho_i$. The output is the depth profile, which is then summed with the freeboard to obtain the thickness at each point.

Other physical arguments also exist for a model of this type. For example, we do not expect high-frequency variations of the surface elevation to be reflected in the bottom profile of the ice, since the plate of ice has some stiffness, causing the deflection from a point load to be distributed over some distance. Also, the processes transforming the surface and bottom of the ice are generally opposite, with shifting snow loads increasing the high-frequency variations on the surface, and the water flow on the bottom generally smoothing out high-frequency variations. We would, therefore, intuitively expect that an operator which smooths the high frequencies of the surface while generally preserving the total isostatic equilibrium might be a reasonable method of approximating the bottom profile. For comparison, we have chosen the base of the triangular filter to be either 10-11 m or 21-22 m.

Comparison between measured and predicted thicknesses

Figures 4-6 give the results of the three models for the measured profiles shown in Figure 1. In each of the figures, the measured ice thickness is shown by the solid line and compared with the appropriately labeled model, which is indicated by the predictor or dotted line. In addition, part e of each figure shows the profile obtained by first using the variable density model (part b) and then filtering it with a unity gain triangular filter as described under model 3. The models used in parts c, d and e of these figures require that endpoints be dropped; so the profile lines are shorter than those in parts a and b, which use all the measured values.

These figures show that certain predictors work considerably better than others. In particular, the variable density model (Fig. 4b, 5b and 6b) offers a quite reasonable approximation to the actual

^{*} Later data (Kovacs, personal communication) indicate that this ratio is estimated at 1 to 3.3, making the constant density value 783 kg/m^3 with this correction.

Figure 4. Profile 1: Predicted and measured ice thickness profiles for the various prediction models labeled under the plots.

thickness profile, while perhaps increasing the high-frequency content. Figures 4e, 5e and 6e, which use the density profile and then filter it, seem to offer some improvement over using the density prediction alone. Quantitative measures of the accuracy of the predictions are shown in Table I, which gives the standard deviation (error) between the measured and predicted values for the different profiles for each of the predictions shown in Figures 4-6. From this table and Figures 4a, 5a and 6a, we see that the point isostatic assumption does not give a reasonable prediction,

Figure 5. Profile 2: Predicted and measured ice thickness profiles for the various prediction models labeled under the plots.

against this method, a coefficient of variation has been calculated as follows:

$$r^2 = 1 - (s_{m-p}^2)/(s_m^2) \quad (6)$$

where r^2 is the coefficient of variation, s_{m-p} is the standard deviation between the measured and predicted thicknesses (Table I), and s_m is the measured thickness standard deviation (Table II). When $s_{m-p} > s_m$, $r = 0$. The coefficients for the various predictors are shown in Table III.

since the standard deviation between measured and predicted values is large (0.92 to 1.66 m). The standard deviation for the variable density model is uniformly lower (0.45 to 0.46 m). This is the basic reason we conclude that the variable density model, using Figure 3, can provide useful ice thickness estimates on a field operational basis. In addition, this model is easy to use and offers a considerable improvement over the point isostatic model.

The various filtering techniques, shown as parts c, d and e in Figures 4-6, can also be used with more success than the point isostatic assumption. In particular, the combination of the variable density model with the 10-11-m spread triangular filter (filtered density, part e) best approximates the measured profile, as indicated by the lowest values of the standard deviation for each profile. The variable density profile was combined with the 10-m filter, since this filter affected the frequency content of the data in a less adverse manner than the 22-m filter. Further details of the frequency behavior of the profiles and the predictors are given in the next section.

The distributed isostatic or filter models do have further disadvantages for field situations: 1) they require more extensive computation; 2) the endpoints are lost from the profiles; and 3) the filter gain, calculated from the depth to freeboard ratio, varies from profile to profile. Primarily, however, the predicted profiles from the distributed models (parts c and d) have larger errors than the variable density predictor as indicated by the standard deviations shown in Table I.

Another method of predicting the ice thickness is to guess the mean, if a good approximation to the mean thickness can be obtained. The error in the prediction is then given by the standard deviation of the thickness, as shown in Table II for the three profiles. To test the value of the three prediction models

Figure 6. Profile 3: Predicted and measured ice thickness profiles for the various prediction models labeled under the plots. The filter spreads for profile 3 differ slightly from those used in profiles 1 and 2 because of the different data point spacing (1 m) used for this profile.

Since r^2 is a measure of the “explained” variance, we conclude that a predictor that gives r^2 greater than 0.50 (explains 50% of the variance) represents a valuable prediction. Table III shows that the variable density and filtered density predictions meet these criteria. Also, an 11-point Wiener filter was constructed for profile 2 and gave an r^2 of 0.79. This value compares with $r^2 = 0.76$ obtained for the filtered density prediction for profile 2. Since the Wiener filter represents the optimum linear filter, we can conclude that the prediction schemes are quite good, since they approach the absolute limits in accuracy given by the Wiener filter and can be modeled with some physical justification.

Table I. Standard deviation between measured and predicted ice thicknesses.

<i>Predictor model</i>	<i>Profile 1 101 data pts (m)</i>	<i>Profile 2 56 data pts (m)</i>	<i>Profile 3 77 data pts (m)</i>
Point isostatic	0.92	1.18	1.66
Variable density	0.45	0.45	0.46
Distributed isostatic (10 m-spread)	0.63	0.74	1.07
Distributed isostatic (22 m-spread)	0.48	0.48	0.71
Filtered density (10 m-spread)	0.40	0.35	0.48

Table II. Summary of statistical results.

	<i>Profile 1 (m)</i>	<i>Profile 2 (m)</i>	<i>Profile 3 (m)</i>	<i>Combined profiles (m)</i>
Mean ice depth	2.37	2.57	2.72	2.53
Ice depth standard deviation	0.52	0.57	0.61	0.58
Mean ice thickness	2.65	2.95	3.05	2.85
Thickness standard deviation	0.61	0.72	0.79	0.72
Mean freeboard	0.22	0.32	0.25	0.25
Freeboard standard deviation	0.14	0.22	0.24	0.20
Mean effective freeboard	0.27	0.37	0.33	0.31
Effective freeboard standard deviation	0.13	0.21	0.23	0.19
Mean snow depth	0.13	0.16	0.21	0.17
Snow depth standard deviation	0.07	0.08	0.12	0.10

Table III. Coefficients of variation r^2 .

<i>Predictor model</i>	<i>Profile 1 r^2</i>	<i>Profile 2 r^2</i>	<i>Profile 3 r^2</i>	<i>Mean r^2</i>
Point isostatic	0	0	0	0
Variable density	0.45	0.61	0.66	0.56
Distributed isostatic (10 or 11 m)	0	0	0	0
Distributed isostatic (21 or 22 m)	0.39	0.56	0.18	0.36
Filtered density (10 or 11 m)	0.59	0.76	0.67	0.67

Figure 7. Power spectra of the ice depth, freeboard and top snow surface for profile 1 and profile 2.

Figure 8. Comparisons of measured and predicted ice thickness spectra using the variable density and point isostatic prediction models.

Spectral behavior of measured and predicted profiles

Figure 7 shows the power spectra (Hibler and LeShack 1972, Hibler and Mock 1972) of profiles 1 and 2. For profile 3, the data point spacing was 1 m, compared with the 2-m spacing for profiles 1 and 2, and the profile was only 76 m long; this prevented accurate spectra from being calculated for this profile.

The spectra show relatively more variance at low frequencies for the depth than for the top surface of either snow or ice. This indicates that the processes transforming the upper surface increase the high-frequency variations relative to the processes acting on the lower surface which smooth out the high-frequency variations. Random variations in loading on the surface, such as the redistribution of the snow cover, also cause the ice cover to deform. However, these deflections appear as low-frequency variations in the bottom profile, since the rigidity of the ice cover "smooths" the deflection from a point load over distances of the order of tens of meters.

Figure 9. Comparisons of measured and predicted ice thickness spectra using the 10-m and 22-m filter prediction models.

Figure 10. Power transfer functions versus spatial frequency of the 10-m and 22-m filters for profile 2.

Comparisons between the snow elevation and the freeboard spectra in Figure 7 indicate good correspondence for the lower frequencies ($\sim < 0.16 \text{ m}^{-1}$). This finding was observed in a previous study (Hibler et al. 1972) and we may conclude that, for wavelengths exceeding ~ 6 to 8 m , the variations in the ice surface are reflected in the elevation of the snow cover on multiyear ice. The spectra also indicate the low-frequency peak at wavelengths of ~ 25 to 30 m , which appears to be the characteristic spacing for melt pond-hummock features on multiyear ice, as also noted previously. A mechanism causing this periodicity has been postulated and a numerical study to test this hypothesis is currently in progress.

Figure 8 shows the power spectra of the measured and predicted thicknesses for the point isostatic and variable density models. The spectra of the filter outputs (model 3) have been calculated using the power transfer function *PTF* of the filter and are compared with the measured thickness spectra in Figure 9. The frequency response was obtained (for finite digital filters) by (Hibler 1973a):

$$H(f) = 2 \sum_{n=1}^{N-1} H(n) \cos(2\pi f n \Delta x) + H(0) + \cos(2\pi f N \Delta x) H(N) \quad (7)$$

where $2N$ is the number of points used on the filter, f is the spatial frequency, Δx is the data spacing, and $H(n)$ is the filter weight. The power transfer function *PTF* equals $|H(f)|^2$ and is shown in Figure 10 for profile 2.

From Figures 8 and 9, we may infer that the point isostatic spectra bear little resemblance to the measured values for each profile, since the variance is generally an order of magnitude higher across the entire frequency range. On the other hand, the spectra for the variable density predictor are relatively close to the measured spectra for frequencies up to 0.08 m^{-1} . At the higher frequencies (above 0.08 m^{-1}), the variable density spectra are generally higher than the measured values for

profiles 1 and 2. This behavior confirms that seen intuitively in Figures 4b and 5b, which indicated more high-frequency content for the variable density predictor than for the measured values.

Figure 9 shows that the distributed isostatic spectra are generally reproduced at low frequencies by the filter outputs but have essentially no contributions beyond frequencies of $0.12\text{--}0.14\text{ m}^{-1}$, for the 10-m filter. This is also borne out by inspection of the predicted profiles in Figures 4-6. This behavior is due to the sharp cutoff in the power transfer function combined with the general decline in the spectra of the effective freeboard at high frequencies.

Comparisons with previous data

Two other sources of data were examined to determine whether the methods developed were generally applicable or were relevant to only the three profiles measured in 1972.

The first source of data was 31 ice thickness measurements obtained by the authors on a multi-year floe in the eastern Beaufort Sea in 1971 (Hibler et al. 1972). The filter methods for prediction could not be used, since the holes were located randomly on the floe and did not lie along any single line profile. However, the density prediction (Fig. 3) was used on a point-by-point basis and yielded a standard deviation of 0.48 m between the measured and predicted values, which compares favorably with the standard deviations obtained for the three profiles examined here and with the thickness standard deviation of 0.60 m. The coefficient of variation r^2 was 0.41 for this data set.

The second source of data was a 75-m profile reported by Nazintsev (1971). Nazintsev's data were obtained from a line drawing in his article; so some error in transcribing the data may have occurred. Using the methods described in "*Models for Predicting Thickness from Ice Freeboard*" (p. 3), standard deviations in thickness for the variable density and filtered density models of 0.51 m and 0.46 m were obtained compared with the measured values. Although these data cannot be considered exact, the standard deviations can be considered to be similar to those observed for the profile and random hole data obtained here.

From these other data (Hibler et al. 1972, Nazintsev 1971) on multiyear ice, we conclude that the *variable* density prediction, and for equally spaced profile data, the *filtered* variable density prediction, are generally applicable to profiles on multiyear ice and are not limited to the three profiles on which they were based.

ESTIMATION OF ICE THICKNESS USING AIRBORNE LASER PROFILOMETRY

Laser profiles of the pack ice surface which have been obtained over the past few years have been used to characterize the terrain for vehicle crossing purposes (Hibler and Ackley 1973, Hibler et al. 1974). These data have been used primarily to define the ridging intensity as a function of season and location. However, general surface elevation profiles are also obtainable using data of this type, since the data acquisition rate and relative elevation accuracy between points are comparable to data obtained on the ground (Mock et al. 1974).

In order to use the profile data to predict ice thickness, several other acquisition and analysis problems need to be solved. One of these is the removal of aircraft altitude variation to determine an absolute sea level reference. This problem has been solved in a manner adequate for estimating ridge heights by using a three-step filtering process (Hibler 1973b). However, to obtain the accuracy required for ice thickness estimation, various fiducial water level marks are needed. It is probable that indicators of open water (loss of reflection) or very thin ice (a low freeboard) can be correctly identified on the record and used to reference the profile to sea level. Another problem is the

identification of ice type, since the predictions given here apply only to multiyear ice. Preliminary examination of the spectra indicated differences between first and multiyear ice (Hibler and LeShack 1972); so this problem may be tractable. Further work in this area to obtain the different spectral behavior is planned.

Another major problem is determination of the effect of the snow cover on the accuracy of the prediction. If laser profilometry data are to be used, it appears that the snow cover will have to be accounted for by computational procedures, since the data will not differentiate between bare ice and snow covered surfaces. To estimate how this factor affects the prediction accuracy, we recalculated the predicted thickness, using the three surface profiles, assuming (a) the top surface elevation represents the top of the ice (the snow cover is assumed to be solid ice), and (b) the surface elevation consists of an ice surface plus a snow depth at each point equal to the mean snow depth, i.e. constant snow depth at each point. The predicted ice thicknesses for each profile were then calculated for the variable density and filtered density predictors using these two estimates of the ice freeboard. The standard deviations between the measured and predicted thicknesses, obtained by using these two assumptions, are shown in Table IV.

Table IV. Standard deviations between measured and predicted thicknesses under assumptions necessary for airborne laser profilometry data.

<i>Assumptions</i>	<i>Profile 1</i>		<i>Profile 2</i>		<i>Profile 3</i>	
	<i>Variable density (m)</i>	<i>Filtered density (m)</i>	<i>Variable density (m)</i>	<i>Filtered density (m)</i>	<i>Variable density (m)</i>	<i>Filtered density (m)</i>
a) Surface elevation all ice	0.64	0.53	0.62	0.55	0.56	0.57
b) Mean snow thickness at every point (see Table II)	0.54	0.44	0.49	0.38	0.55	0.49

Comparison of Table IV and Table I (in which the predicted thicknesses are based on the profiles using the actual rather than assumed snow depths) shows that the filtered density predictions, using the assumption of the mean snow cover at each profile point (b), increase the standard error on the predicted thickness by only about 0.04 m (10%) for each of the three profiles. Testing the best prediction (filtered variable density) against the mean value as a prediction by using the coefficient of variation described earlier gives r^2 values as follows: profile 1: $r^2 = 0.48$, profile 2: $r^2 = 0.72$, profile 3: $r^2 = 0.61$. Since these values account for close to or more than 50% of the variance using this prediction, we conclude that the prediction method with surface profiles obtained by airborne techniques can give a useful estimate of the real thickness profile.

We, therefore, conclude that estimating ice thickness profiles on multiyear ice by using airborne laser profilometry could be quite successful, with an accuracy close to that obtained from ground surveys, if the procedures described here were used. The success of this method, however, is dependent on 1) obtaining frequent updates of true sea level (probably on the order of every few hundred meters), 2) correctly identifying the ice type as multiyear, and 3) obtaining information on representative snow depths on the ice.

CONCLUSIONS

It is concluded that predictive methods for obtaining multiyear ice thicknesses from profiles of surface elevation offer considerable improvement over methods that assume either the mean thickness or point-by-point isostatic equilibrium of the ice with constant density. In particular, the method which combines the natural density variation in the ice with the variation in the freeboard, and a 10-m-wide linear filter to smooth the high-frequency roughness, gives estimates of the thickness to within 0.4 m and accounts for over 50% of the variance in thickness. This method can also be applied to remotely obtained profiles of the top surface (which might be estimated by airborne laser profilometers), with only a 10% increase in the thickness error over that obtained with ground-based surveys.

Investigation of the spectra confirms earlier data, indicating consistent peaks in the top and bottom roughnesses at 25 to 30-m wavelengths for multiyear ice. The predictive methods for obtaining ice thicknesses also give good approximations to the true spectra for wavelengths longer than 6 to 8 m. Little correlation is seen between surface and bottom roughnesses for wavelengths shorter than 6 to 8 m.

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APPENDIX A: MISGIVINGS ON ISOSTATIC IMBALANCE AS A MECHANISM FOR SEA ICE CRACKING

S.F. Ackley, W.D. Hibler III and F.K. Kuzruk

INTRODUCTION

Two assumptions of the AIDJEX model are that the pack ice can be modeled as an elastic-plastic material and that, on the scale considered by this model, the ice is densely fractured (Coon et al. 1974). It is assumed that enough cracks are formed by other processes so that the model can ignore how cracks are formed. The model then seeks to describe the ice motion by deformation and movement at the preexisting cracks.

The mechanism of how cracks are formed is not an easy problem for materials as complex as sea ice. Our purpose here is to examine one of the invoked mechanisms, isostatic imbalance, as presented by Schwaegler (1974), since we feel that this particular mechanism needs closer examination based on a more realistic picture of the physical properties of the ice. This mechanism has already been referred to in various articles as a cracking mechanism (Coon et al. 1974, Rothrock 1975), even though several assumptions in the analysis require rethinking.

PREVIOUS WORK

Briefly, Schwaegler (1974) found, by using a model containing thickness variations similar to those observed on multiyear ice (Hibler et al. 1972, Ackley et al. 1974), that the isostatic imbalance from the thickness variations leads to sufficiently large bending stresses to cause cracking. In this analysis, the ice floe was assumed to be a homogeneous, isotropic, linear elastic plate of variable thickness. By various formulations of beams on elastic foundations, it was found that the stress induced within the floe exceeded the cracking stress over a range of roughness element sizes that are typically found in multiyear ice.

PHYSICAL PROPERTIES OF MULTIYEAR ICE

Along with the documented thickness variations of multiyear ice, calculations of this type should also consider the evidence that multiyear ice is an inhomogeneous medium with measurable density variations. Examples of the variation are given in Figure A1, which shows vertical profiles of a multiyear floe obtained during the 1971 AIDJEX Pilot Study (Ackley et al. 1973). Further evidence for these variations is given in Cox and Weeks (1974) from observations obtained during the 1972 Pilot Study.

In the main part of this report, a model for predicting multiyear ice thickness, assuming constant density of the ice [equivalent to the homogeneous, isotropic assumption used by Schwaegler (1974)],

Figure A1. Brine volume and density vertical profiles of multiyear sea ice.

Figure A2. Estimated density of multiyear ice (assuming local isostasy) versus effective freeboard. The regression line (solid) has the form $\rho = -194f' + 974 \text{ kg/m}^3$. The dashed line indicates the low density cutoff corresponding to an ice-sail-height-to-keel-depth ratio of 1 to 3.

was compared with one using a variable density. The variable density used in the predictive model was based on the data shown in Figure A2. In this figure, the effective freeboard at each measured point (ice freeboard plus the snow depth in ice equivalent) is plotted against the density calculated from assuming isostatic equilibrium at each point; i.e., solving the following equation for the ice density:

$$\rho_i T + \rho_s s_d = \rho_w (T - f) \quad (\text{A1})$$

where ρ_i , ρ_s , ρ_w are the ice, snow and water densities, respectively, and T , f and s_d are the ice thickness, freeboard, and snow depth, respectively.

The density-freeboard relation was then obtained by fitting the least-squares line to the data over the range shown in the figure. The equation for this line is:

$$\rho_i = -194f' + 974 \quad (\text{A2})$$

where ρ_i is in kg/m^3 and f' is the effective freeboard defined by

$$f' = f + (\langle \rho_s \rangle) / (\langle \rho_i \rangle) s_d. \quad (\text{A3})$$

In this equation $\langle \rho_s \rangle$ and $\langle \rho_i \rangle$ are the mean ice and snow densities of the floe. A relation of this type compares favorably with the measurements reported in Cox and Weeks (1974), who observed a drop in the mean salinity (and hence less brine volume and a lower density) of the top meter of multiyear ice as the freeboard increased. Using eq 2, a predicted thickness curve was

Figure A3. Comparison of predicted thickness (using freeboard) of multiyear ice, assuming a) a constant density and b) a variable density linearly dependent upon ice freeboard.

constructed and compared with the measured values of thickness obtained by drilling. Figure A3 compares the predicted values with the measured values for (a) the constant density assumption and (b) the variable density assumption. This figure clearly indicates that a better estimate of the measured thickness is obtained by considering a systematic variation in the density. The standard errors between the measured and predicted values were 0.92 m for the constant density assumption and 0.45 m for the variable density assumption.

EFFECTS OF VARIABLE DENSITY ON THE CALCULATED BENDING STRESSES

We may now compare the bending stresses induced by assuming a homogeneous or constant density model to those present assuming a variable density model. The bending stresses are due to forces of the type $\rho_i \Delta T g$ (per unit area) where ΔT is the deviation in the thickness between what *should be* there, based on isostasy, and what *is* there (the measured thickness). Since the standard deviation σ_T between the measured and predicted thickness values is approximately halved (from 0.92 m to 0.45 m) by using a statistical variable density model consistent with the physical properties of the ice, we would expect that the predicted bending stresses, in the mean, would also be approximately halved, since the variable thickness ΔT values would be reduced.

For a more quantitative assessment of the effect of density variations on bending stresses, the individual stress values at each point should be examined, since the ice may be "crackable" at only a few locations where the bending stresses are at the extremes. To make such a comparison, using the profile in Figure A3, we calculated the deflections using an infinite beam approximation with the loading at each point x given by $g < \rho_i > \Delta T(x)$ for the constant density case and $g\rho_i(x) \Delta T(x)$ for the variable density case. A Green's function approach (Greenberg 1971, p. 30) was used for the calculation, with a finite symmetric digital filter to approximate the infinite Green's function. The convolution operation necessitated the loss of a number of endpoints at both ends of the profile, equal to one-half the length of the filter. The comparison between the two models is shown in Figure A4. This figure compares the second derivative ($-d^2w/dx^2$) of the deflection w at each point. In the analysis given by Hetenyi (1946), this value is proportional to the bending moment from which the maximum stress (on the outside surfaces of the beam) is calculated according to the following equations:

$$M = -EI(d^2w/dx^2) \quad (A4a)$$

$$\sigma = (MT/2)/I \quad (A4b)$$

where M is the induced bending moment, E the Young's modulus, I the moment of inertia, σ the stress at the extremities, and T the thickness of the beam.

To account, to some extent, for the thickness variations, the stresses were recalculated with a variable moment of inertia based on the thickness variations after using a constant I to calculate the bending moment. For this case the stress is proportional to $(-1/T^2)(d^2w/dx^2)$, which is plotted in Figure A5.

In these figures the appropriate values of (d^2w/dx^2) and $(-1/T^2)(d^2w/dx^2)$, corresponding to the failure stress (3 bars) used by Schwaegler (1974) are indicated by the light lines labeled "critical cracking values" (note the break in scale for Fig. A5). The Young's modulus used in these calculations is also the same as that given by Schwaegler (1974). The figure indicates that the extreme values of the calculated stresses are of the order of 50 to 60% of the cracking value for the constant

a. Point isostatic.

b. Variable density.

Figure A4. Second derivative of calculated deflection using profile in Figure A3 assuming a) a constant density and b) a variable density linearly dependent upon ice freeboard.

density model but are reduced to 25% or less of the cracking value, by using the variable density. We conclude that the mean and extreme values of stress in multiyear sea ice due to thickness variations cannot alone account for the cracking of the ice. When the density variations are taken into account, stresses from other sources must account for 75% or more of the cracking stress.

DISCUSSION

Our main purpose was to compare the effects of density variations in the elastic bending model for sea ice beams. Two other points should also be mentioned. First, the Young's modulus used here and by Schwaegler (1974) is not very representative of sea ice. Weeks and Assur (1967)

a. Point isostatic.

b. Variable density.

Figure A5. Second derivative of calculated deflection divided by thickness squared using profile in Figure 3 assuming a) a constant density and b) a variable density linearly dependent upon ice free-board.

indicate that this value is 10 to 50% higher than the data from dynamic measurements (acoustic and seismic) would indicate for sea ice and is an order of magnitude higher than the extreme values seen for beam tests on sea ice (Tabata 1966). Equations A4a and A4b show that the stresses generated are directly proportional to the modulus used and that a proportional decrease in the modulus will lead to similar stress reductions, i.e., 10 to 50% less than those shown in Figure A4.

The second point is related to the reduction in modulus for static tests, such as the beam experiments, as compared with the dynamic measurements. This effect has been attributed to creep

processes occurring during the loading period which are not present to the same degree during the time of an acoustic or seismic test (Tabata 1966). Tabata's experiments indicate that the relaxation time for this process (qualitatively the time to onset of steady-state creep) is of the order of 10 minutes. Given this observation, we must question the validity of a purely elastic analysis, especially when the physical process that causes the thickness variation loading to occur is examined. This process is probably a combination of small changes in ablation and accretion, rafting, brine drainage, and melt pond formation, that is, incremental loading changes occurring over periods of days, weeks, or months. If large stresses do occur within the ice, the properties are such that creep should occur within the ice and thereby continuously reduce the stress level, especially over the periods, probably weeks or months, during which the stresses are present, and the incremental way in which they are probably applied. If the only factor we assume is that the modulus is consistent with the beam experiments on sea ice (an order of magnitude lower than that used for Figures A4 and A5), even if the loads are applied instantaneously, then the resulting stresses are an order of magnitude lower than those shown in Figures A4 and A5 and only of the order of 2 to 5% of the value necessary to crack the ice.

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