a. Gow

CRREL

REPORT 78-9

Flexural strength of ice on temperate lakes

Comparative tests of large cantilever and simply supported beams

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A.J. Gow, H.T. Ueda and J.A. Ricard

April 1978

Prepared for
DIRECTORATE OF FACILITIES ENGINEERING
OFFICE, CHIEF OF ENGINEERS
By
CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER CRREL Report 78-9	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) FLEXURAL STRENGTH OF ICE ON TEMPERATE LAKES: COMPARATIVE TESTS OF LARGE CANTILEVER AND SIMPLY SUPPORTED BEAMS		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A.J. Gow, H.T. Ueda and J.A. Ricard		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A161102AT24 Task A1, Work Unit 001
11. CONTROLLING OFFICE NAME AND ADDRESS Directorate of Facilities Engineering Office, Chief of Engineers, Washington, D.C. 20314		12. REPORT DATE April 1978 13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

Cantilever beams
Flexural strength
Ice beams
Ice structure
Lake ice
Simple beams

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

Large, simply supported beams of temperate lake ice were found, generally, to yield significantly higher flexural strengths than the same beams tested in the cantilever mode. Data support the view that a significant stress concentration may exist at the fixed corners of the cantilever beams. Maximum effects are experienced with beams of cold, brittle ice substantially free of structural imperfections; for this kind of ice the strength difference factor, here attributed to the effect of stress concentrations, may exceed 2.0; that is, simply supported beams test a factor of 2 or more stronger than the same beams tested in the cantilever mode. In ice that has undergone extensive thermal degradation, the stress concentration effect may be eliminated entirely. Simply supported beams generally yield higher

20. Abstract (cont'd)

strengths when the top surfaces are placed in tension. This behavior is attributed to differences in ice type; the fine-grained, crack-free top layer of snow-ice which constituted up to 50% of the ice cover in the current series of tests usually reacted more strongly in tension than the coarse-grained crack-prone bottom lake ice.

PREFACE

This report was prepared by Dr. A.J. Gow, Geologist, Snow and Ice Branch, Research Division, H.T. Ueda, Mechanical Engineer, Engineering Services Branch, Technical Services Division, and J.A. Ricard, Civil Engineering Technician, Foundations and Materials Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory.

The study was funded under DA Project 4A161102AT24, *Research in Snow, Ice and Frozen Ground;* Task A1, *Properties of Cold Regions Materials;* Work Unit 001, *Properties of Snow and Ice.*

Technical review of the report was performed by Dr. Donald E. Nevel of CRREL.

The authors wish to thank John Kalafut for his assistance with the instrumentation.

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INTRODUCTION

Any effective use of natural ice covers for constructional or logistical purposes, e.g., drilling platforms, roads and airstrips, depends largely on the surface trafficability conditions and the bearing capacity of the ice. Such ice covers fail whenever the ultimate flexural strength of the ice is exceeded and field experience indicates that the bearing capacity load P can generally be related to the flexural strength σ_f according to the simplified equation

$$P = A\sigma_f h^2 \qquad (1)$$

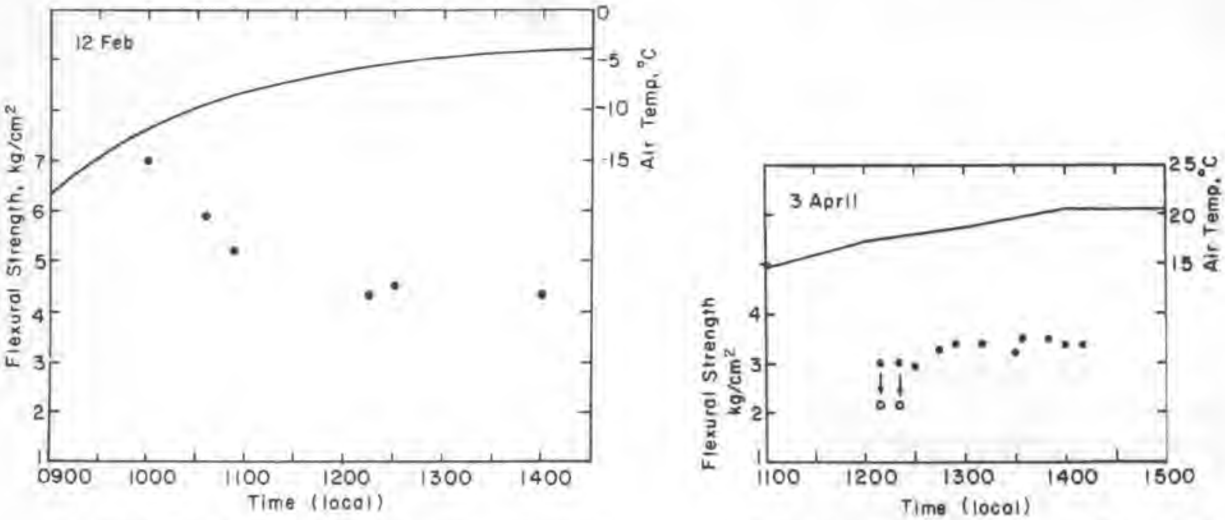
where h is the ice thickness and A is a coefficient that is essentially constant over the range of ice cover thicknesses encountered on lakes and rivers. A very strong dependence of bearing capacity on ice thickness is obvious. However, fully rational utilization of natural ice covers for constructional and logistical purposes has been hampered by inconsistencies in the flexural strength data, due in part to incomplete information on the meteorological and structural factors influencing the intrinsic strength of ice, and in part to difficulties inherent in most test techniques and procedures. For a recent critical survey of the subject, including an extensive bibliography, the interested reader is referred to Kerr (1976).

Of the several types of tests used to measure the flexural strength of natural ice covers, those involving tests on large beams cut from the ice itself probably provide the most realistic values of flexural strength. Such beams are generally tested with one end still attached to the ice cover, the so-called *cantilever beam test,* or with both ends of the beam freely supported. In the series of flexural strength measurements reported here both types of tests were used.

A series of preliminary tests, conducted during the 1973-74 winter on Post Pond and Mascoma Lake, New Hampshire, was restricted to measurement on in-place cantilever beams. These tests, reported in detail in Gow and Langston (1975), yielded a maximum strength of 7.1 kg/cm^2 (7×10^5 N/m^2). The minimum strength, excluding failure along preexisting cracks in the ice was 2.9 kg/cm^2 (2.8×10^5 N/m^2). The results compare favorably with those obtained by Frankenstein (1959) on lake ice similar to that tested in the present series of measurements. Frankenstein's measurements ranged from a maximum of 9.1 kg/cm^2 (8.9×10^5 N/m^2) to a minimum of 0.8 kg/cm^2 (0.8×10^5 N/m^2).

Data obtained by Gow and Langston (1975) further demonstrated that the intrinsic strength of lake ice decreased significantly as the surface temperature approached 0°C (see Fig. 1a). Ice that had just become isothermal, but had not yet begun to candle, had a strength of about 4 kg/cm^2 (3.9×10^5 N/m^2); ice that had been subjected to prolonged periods of above-freezing air temperatures generally failed at stresses of 3 kg/cm^2 (2.9×10^5 N/m^2) or less (see Fig. 1b). Tests also indicated that cold, unrecrystallized snow-ice was as strong as the underlying lake ice. However, high temperature deterioration of the snow-ice layer could lead to a very significant decrease in the effective thickness of the ice cover as demonstrated in the tests of 3 April 1974 (Fig. 1b).

The cantilever test was chosen in the above series of measurements mainly for its convenience. It is a relatively simple test that has been widely used to measure the flexural strength of river ice and sea ice as well as lake ice. However, the generally low values of flexural strength obtained with cantilever beams in this first series of tests aroused the suspicion that the test results were being unduly influenced by external stress concentrations at the butt ends of the beams.



a. Variation of flexural strength with temperature. *b. Near-constant strength of partially candled, isothermal lake ice.*

Figure 1. Cantilever beam test data. On 3 April 1974, the top 6 cm of snow-ice had been reduced to an essentially strengthless condition, thereby reducing the effective thickness of the ice from 40 to 34 cm. Data denoted by open circles were computed on the basis of the absolute thickness (40 cm) of the ice cover (adapted from Gow and Langston 1975).

Comparative tests of in-place cantilever and simply supported beams by Butiagin (1966) do not seem to show any significant difference in failure strength that can be attributed to external stress concentrations. Frankenstein (1966) also believes that if a stress concentration exists at the butt end of the cantilever beam then the effect must be very small. However, Brown (1963) infers from tests made on plexiglass that the stress concentration factor for sea ice beams could be as high as 2.8.

In an attempt to resolve this problem, a test device was designed that permits the same beams used in cantilever tests to be tested with their ends freely supported, which should eliminate the effect of external stress concentrations.

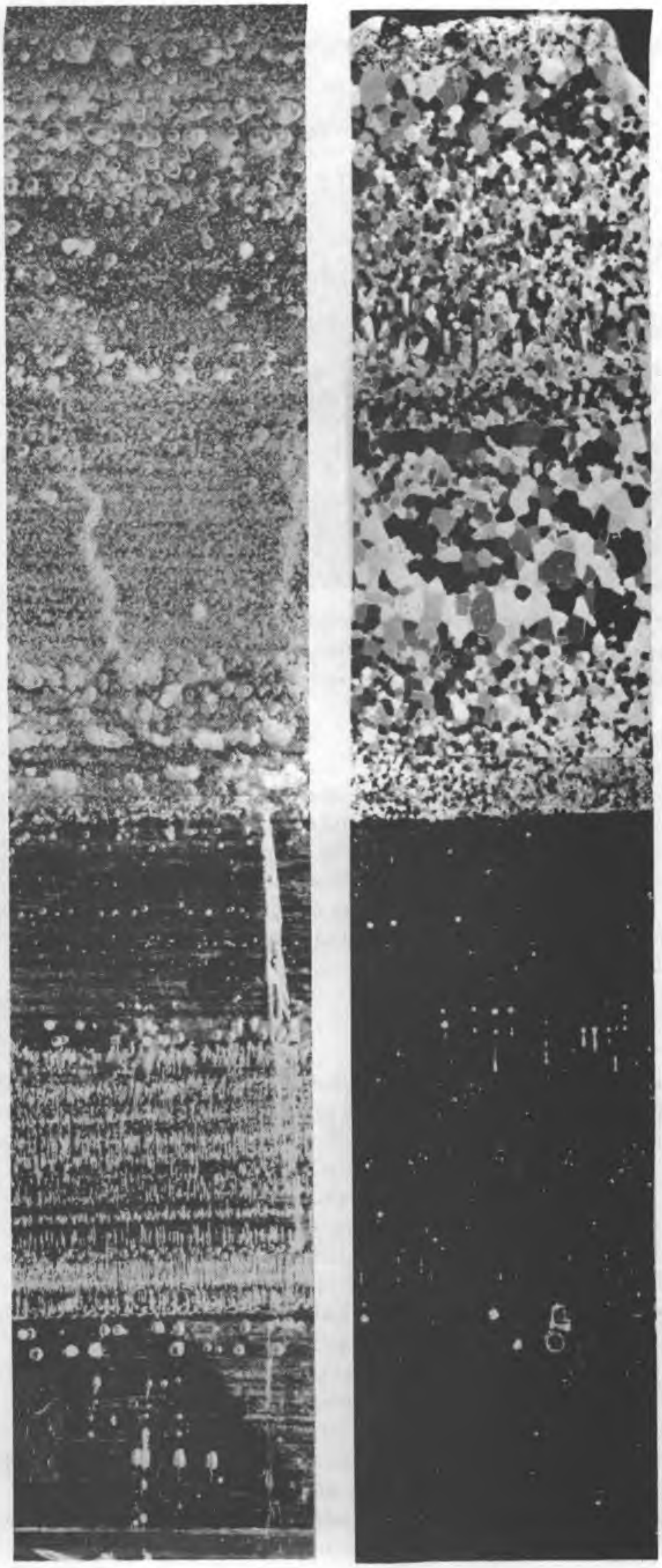
It is the principal purpose of this report to present results of this second series of measurements involving parallel testing of in-place cantilever and simply supported beams. These tests were performed in conjunction with an ongoing program of studies of the growth characteristics, structure and mechanical properties of ice covers on temperate lakes and rivers.

TEST SITES AND ICE COVER PROPERTIES

The majority of tests were performed on Post Pond, located near Lyme, New Hampshire. Additional tests were also performed on Canaan Street Lake situated near the town of Canaan, New Hampshire. Combined tests of in-situ cantilever and simply supported beams were initiated in March 1975 and completed the following winter in March 1976.

The ice cover on both lakes was composed typically of two major ice components: true lake ice, formed by the freezing of lake water, and snow-ice that overlies the lake ice and is formed by the freezing of water saturated snow.

Vertical sections of ice sheet stratigraphy and structure from Canaan Street Lake are presented in Figure 2. Structure, as photographed in an ice beam from Canaan Street Lake, is illustrated in Figure 3. In both sections of ice in Figure 2, the lake ice component can be distinguished easily from the overlying snow-ice. Crystals in the structural section (Fig. 2b) are all oriented with their c-axes vertical. Occasionally crystals with subhorizontal c-axes occur in the top or earliest-formed ice but such crystals are soon eliminated in favor of crystals with vertical c-axes. The transition to fine-grained, equigranular snow-ice is especially well marked in the structural section. Both Post Pond and Canaan Street Lake developed vertical c-axis structure during the 1974-75 winter and essentially identical structure was observed in the winter ice cover of 1975-76. The formation of a c-axis vertical structure was invariably associated with the growth of very large crystals. At some test sites, the ice texture approached that of a single crystal and in all cases the sizes of crystals



a. b.

Figure 2. Vertical sections illustrating stratigraphic (a) and structural (b) characteristics of a lake ice sheet, Canaan Street Lake, New Hampshire. Transition from snow-ice to lake ice is especially well marked. In this instance, the lake ice is composed entirely of crystals with vertical c-axes. Ice thickness is 43 cm.

Figure 3. Side view of ice beam showing white snow-ice component overlying distinctively stratified lake ice, Canaan Street Lake, New Hampshire. In this particular example, the snow-ice and lake ice components are present in about equal amounts.

can be best described as massive. It is of interest to note that fluctuations in freezing rate, as clearly reflected in the bubble stratigraphy (Fig. 2a), appear to have exerted little effect on the crystalline texture or fabric of the ice.

Bulk density measurements show that the porosity of even the bubbliest ice rarely exceeded 3-4%. In the case of snow-ice, the porosity seldom exceeded 5%.

Apart from the entrapment of air bubbles, the freezing of water on both Post Pond and Canaan Street Lake resulted in substantial elimination of dissolved materials, the electrolytic conductivity rarely exceeding 2 μmhos/cm compared with 40-80 μmhos/cm for the original lake water.

BEAM TESTING

Actual test sites were carefully examined to ensure that they were as crack-free as possible. The general technique for preparing beams for cantilever tests is demonstrated in Figure 4. The object is to cut slots in the ice so as to isolate a beam with one end still attached to the ice sheet. The cantilever beam is then loaded to failure to determine its flexural strength. All beams were tested in the push-down mode, which places the top surface of the beam in tension. Beam breaking was accomplished by means of a hydraulic jack and the force was measured directly with the aid of a proving ring and dial gage mounted on the unsupported end of the ice beam. With this technique, the beam could be loaded to failure in less than 10 seconds. The flexural strength σ_f of the beam was determined according to the relation:

$$\sigma_f = \frac{6PL}{wh^2} \qquad (2)$$

where P is the applied force, L the length of the beam, w the width of the beam and h the ice thickness. The measurement of failure force depends essentially on the precision with which the dial gage can be read at the instant of tensile failure. It is estimated that flexural strengths can be calculated to an accuracy of better than 7% with this technique.

The test apparatus and technique for measuring the flexural strengths and deflections of simply supported, center-loaded beams are illustrated in Figure 5. The test device (Fig. 5a) consists of an aluminum frame fitted with end supports which provide the reactions for the ice beam. The frame is placed in position over an appropriately cut slot in the ice. The beam from the cantilever test is floated into position as indicated in Figure 5b. The frame is then positioned until the bottom of the free-floating ice beam makes contact with the end supports. Force is applied to the center of the

Figure 4. Preparing the ice for in-place cantilever beam tests. Also note at the left bottom of the figure the free end of the beam with the breaker device set up for a push-down test.

ice beam by means of a manually operated screw jack to which a load cell is attached (Fig. 5c and 5d).

The duration of loading of the beams depended mainly on the intrinsic strength of the ice at the time of testing. Load-time readings generally showed an increasing rate for the first few seconds followed by a constant rate to failure. Beams could be loaded to failure within 20-60 seconds; stress rates computed from the linear portions of recordings varied from 95 $N/m^2 \times 10^2$ to 250 $N/m^2 \times 10^2$ per second. Although higher stress rates would have been desirable, the lower rates resulting from the manual application of the load were tolerated in order to keep the equipment as simple and lightweight as possible. Even at the low rates employed there is still evidence of elastic behavior. According to Butiagin (1966), the flexural strengths of large ice beams do not appear to change appreciably for loading times ranging from several seconds to several minutes duration.

The effective length of the simply supported beams is determined by the distance between the two end supports. This distance can be adjusted in relation to the ice thickness but in the current series of tests it was maintained at 275 cm. The beam breaker is designed to accommodate beams up to 60 cm thick. With this device, the cantilever beams can be easily rolled over in the water so as to facilitate testing with either the top or bottom surface in tension. The beam widths measured approximately the same as their thicknesses and their lengths were maintained at between 5 and 8 times the ice thickness. Ice beam thickness and width were measured to the nearest 0.5 cm; beam lengths were measured to the nearest 1.0 cm. During testing, frequent measurement of air temperatures were made 10 cm above the ice surface.

The flexural strength σ_f of the simply supported beams was computed according to the relationship

$$\sigma_f = \frac{3PL}{2wh^2} \tag{3}$$

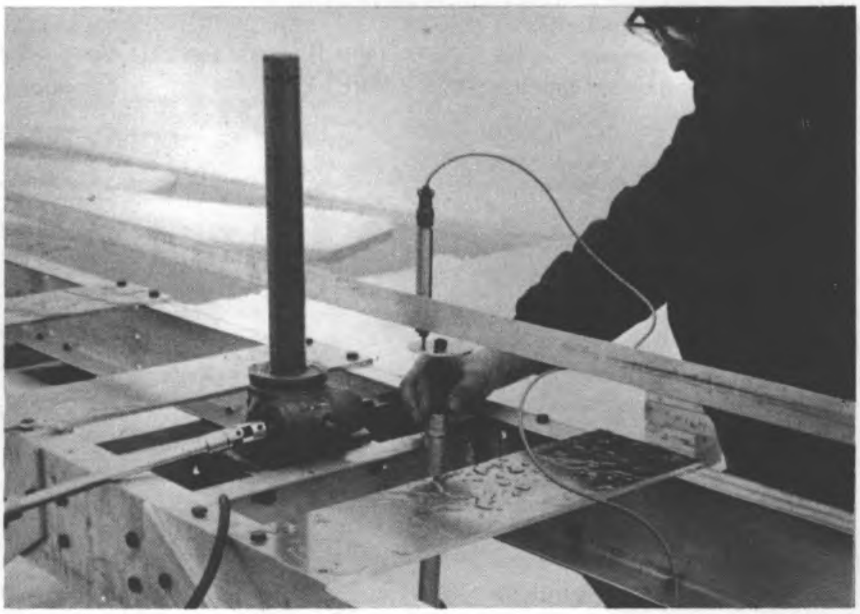
where P is the force at failure, L is the length of the beam and w and h are the width and the thickness of the ice beam respectively. We estimate that the flexural strengths obtained with this technique are accurate to better than 5%.

a. Test frame for supporting beam.

b. Guiding ice beam (previously broken in cantilever mode) into test frame.

Figure 5. Apparatus used for measuring the flexural strengths and deflections of simply supported, center-loaded ice beams.

c. Stabilizing beam. Note brace attached to screw jack in readiness for test.



d. Close-up views of screw jack and the linear variable displacement transformer used for measuring beam deflections.

Figure 5 (cont'd).

Deflection measurements were performed only on simply supported beams. These measurements were made with a linear variable displacement transformer (LVDT) mounted on an aluminum angle as shown in Figure 5c and 5d. The apparatus was independent of the loading frame, ensuring that the measured deflection was solely that of the beam. The actual point of measurement was located about 10 cm from the center of the beam in order to clear the loading bar (attached to the load cell) and to avoid areas of local deformation. This offset resulted in an error of less than 2%.

The elastic modulus E_f of the beams was calculated from

$$E_f = \frac{PL^3}{4wh^3d} \quad (4)$$

where d is the deflection at the center of the beam. All other terms are the same as for eq 2.

It should be emphasized that all values of flexural strength and strain modulus reported here were calculated on the basis of simple elastic beam theory. This application of beam theory may yield only approximate values at best, because of the intrinsically nonhomogeneous and anisotropic nature of lake ice. However, such data permit direct comparison of results from different field locations and are also useful for evaluating measurements obtained on simulated ice in the laboratory.

RESULTS

The current series of tests included some additional measurements on centilever beams, performed mainly on thin lake ice in January and February 1975 during testing and evaluation of the large simply supported beam breaker. Results of these tests, summarized in Table I, agree essentially with those obtained by Gow and Langston (1975) in the initial series of tests of 1973-74.

Fracture surfaces at the butt ends of the cantilever beams were generally planar and vertical. A notch extending inwards about 1-2 cm from the fracture surface and 2-3 cm up from the bottom surface (compression surface) is especially characteristic. Occasional "short" breaks could always be traced to preexisting cracks in the ice. Most of the freely supported beams failed at the center along a fracture surface that was generally vertical and planar. However, in the cases of "off-center" breaks (these generally composed less than 30% of the failures in any set of tests), the fracture

Table I. Summary of cantilever beam tests with top surface in tension, January and February 1975.

Date	Location	Flexural strength ($N/m^2 \times 10^5$) Max	Min	Mean	No. of tests	Ice thickness (cm)
7 Jan	Post Pond	6.82	5.65	6.15	10	16
8 Jan	Post Pond	6.72	3.47	4.74	14	17
21 Jan	Post Pond	6.54	3.94	4.84	8	17
22 Jan	Post Pond	5.43	4.19	4.75	4	19
24 Jan	Canaan Street Lake	6.60	4.26	5.28	8	26
31 Jan	Canaan Street Lake	5.56	4.12	4.75	5	34
21 Feb	Post Pond	5.29	3.86	4.69	6	38

surfaces tended to be curved in the manner depicted by Lavrov (1971, p. 38).

Some typical results obtained with eight separate sets of combined cantilever and simply supported beam tests (two sets from Canaan Street Lake and six sets from Post Pond) are presented in Tables II-IX. Beams are identified according to which surface was placed in tension during the test. Ratios of strengths for the different types of tests were also calculated and are given at the bottom of each table.

Table II. Beam test data, Canaan Street Lake, 6-7 March 1975.

Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	Simply supported	
	4.90 (TT)*	9.40 (TT)	
	6.34 (TT)	10.88 (TT)	
	4.39 (TT)		4.21 (BT)†
	4.09 (TT)		4.79 (BT)
	3.97 (TT)	11.61 (TT)	
	3.95 (TT)		4.72 (BT)
	3.96 (TT)		4.88 (BT)
	5.10 (TT)	8.60 (TT)	
	3.84 (TT)		6.07 (BT)
	3.93 (TT)	10.71 (TT)	
Mean:	4.45 (a)	10.24 (b)	4.93 (c)

b/a = 2.30
c/a = 1.11
b/c = 2.08

The ice cover was 47 cm thick and comprised 23 cm of snow-ice and 24 cm of lake ice. Lake ice was composed entirely of crystals with their c-axes oriented vertically. Air temperatures remained at 0°C throughout the test series on 6 March; however, they varied from +2° to +5°C under sunny conditions on 7 March. Mechanical condition of the ice remained firm throughout the duration of the tests.

* The beam was tested with the top surface in tension.
† The beam was tested with the bottom surface in tension.

Table III. Beam test data, Canaan Street Lake, 12 March 1975.
Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	Simply supported	
	4.58 (TT)		8.19 (BT)
	5.36 (TT)	9.08 (TT)	
	6.41 (TT)		7.04 (BT)
	6.38 (TT)	7.82 (TT)	
	4.85 (TT)		10.80 (BT)
	4.42 (TT)	8.45 (TT)	
Mean:	5.33 (a)	8.45 (b)	8.68 (c)
		b/a = 1.59	
		c/a = 1.63	
		b/c = 0.97	

Total ice cover was 49 cm thick and comprised 23.5 cm of snow-ice and 25.5 cm of lake ice. The lake ice was composed entirely of crystals with vertical c-axes. The surface air temperatures varied from 0° to +2°C during testing. The sky was overcast. The mechanical condition of the ice cover remained firm during the testing.

Table IV. Beam test data, Post Pond, 3 March 1975.
Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	Simply supported	
	7.30 (TT)		7.59 (BT)
	6.42 (TT)		6.03 (BT)
	5.94 (TT)	9.28 (TT)	
	5.97 (TT)	8.45 (TT)	
		8.29 (TT)	
	6.59 (TT)	8.90 (TT)	
	6.65 (TT)	11.24 (TT)	
	3.66 (TT)	8.83 (TT)	
Mean:	6.08 (a)	9.17 (b)	6.81 (c)
		b/a = 1.51	
		c/a = 1.12	
		b/c = 1.35	

The ice cover was 39 cm thick and comprised 8.5 cm of snow-ice and 30.5 cm of lake ice. The lake ice possessed a vertical c-axis structure. Surface temperatures remained at -3°C throughout the test period. The condition of the ice was firm, i.e., not visibly affected by fracturing or candling.

Table V. Beam test data, Post Pond, 22 January 1976.
Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	Simply supported	
	6.11 (TT)	7.40 (TT)	
	5.61 (TT)		5.36 (BT)
	4.85 (TT)	10.73 (TT)	
		6.85 (TT)	
	4.46 (TT)		7.77 (BT)
		7.63 (TT)	
	4.42 (TT)	10.88 (TT)	
		7.50 (TT)	
			7.46 (BT)
		7.63 (TT)	
Mean:	5.09 (a)	8.37 (b)	6.86 (c)
		b/a = 1.64	
		c/a = 1.35	
		b/c = 1.22	

The total ice thickness was 34.5 cm and comprised 15.5 cm of snow-ice and 19 cm of lake ice. The lake ice was composed of crystals with vertically orientated c-axes. It was clear, sunny and windy with surface air temperatures ranging from -12°C to -15°C. There was a tendency for the beams to crack when overturned for the simple support test (TT).

Table VI. Beam test data, Post Pond, 4 February 1976.
Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	Simply supported	
	7.01 (TT)		10.27 (BT)
	8.76 (TT)	13.90 (TT)	
	9.33 (TT)	15.80 (TT)	
	9.88 (TT)	15.70 (TT)	
Mean:	8.75 (a)	15.13 (b)	10.27 (c)
		b/a = 1.73	
		c/a = 1.17	
		b/c = 1.47	

The ice sheet was 48 cm thick and comprised 26 cm of snow-ice and 22 cm of lake ice. The lake ice was composed predominantly of crystals with vertical c-axes. The surface air temperature increased from -7°C to 0°C during the test period. The mechanical condition of the ice was very solid, especially the snow-ice component (see Fig. 6).

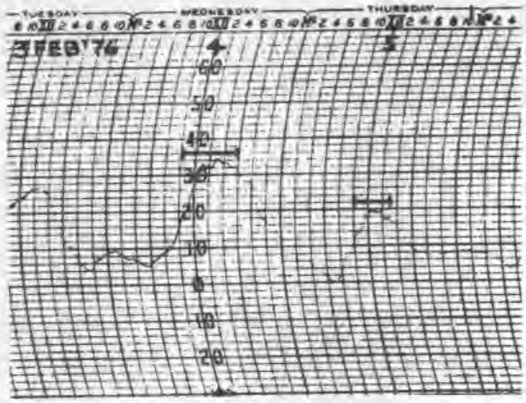


Figure 6. Hygrothermograph air temperature record from Post Pond for 3-5 February 1976. High beam strength measured during this period (see Tables VI and VII) can be attributed essentially to the very cold condition of the ice established prior to testing.

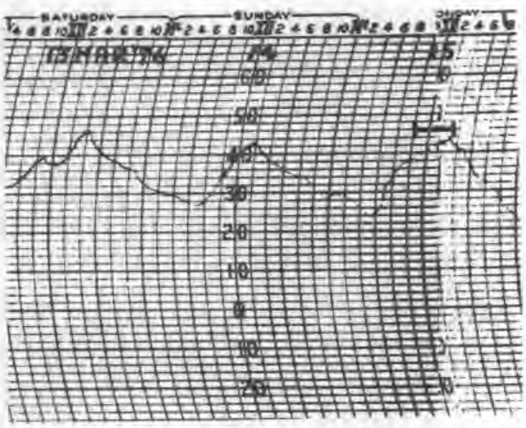


Figure 7. Hygrothermograph air temperature record from Post Pond for 13-15 March 1976. Low beam strength observed on 15 March (see Table IX) can be ascribed to structurally degraded condition of the ice caused by extended period of elevated air temperatures preceding testing.

Table VII. Beam test data, Post Pond, 5 February 1976.
Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	*Simply supported*
	6.23 (TT)	13.21 (TT)
	7.54 (TT)	10.31 (TT)
	5.15 (TT)	12.19 (TT)
	5.84 (TT)	
Mean:	6.19 (a)	11.90 (b)

b/a = 1.92

The ice conditions were very similar to those at the 4 February test site. The surface air temperature increased from -11°C at the start of the tests to -7°C midway through the testing, and then decreased to -9°C by the end of testing (see Fig. 6).

Table VIII. Beam test data, Post Pond, 26 February 1976.
Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	*Simply supported*	
	5.02 (TT)		3.64 (BT)
	5.96 (TT)	4.88 (TT)	
	3.91 (TT)	5.23 (TT)	
	4.38 (TT)		5.39 (BT)
	3.84 (TT)	5.88 (TT)	
	3.63 (TT)	4.28 (TT)	
	3.96 (TT)	4.97 (TT)	
	3.98 (TT)		5.09 (BT)
	3.82 (TT)	5.56 (TT)	
	4.09 (TT)		5.53 (BT)
	4.48 (TT)		5.79 (BT)
Mean:	4.28 (a)	5.13 (b)	5.09 (c)

b/a = 1.20
c/a = 1.19
b/c = 1.01

The ice sheet was 46.5 cm thick and comprised 28 cm of snow-ice and 18.5 cm of lake ice. The c-axes of the lake ice crystals were predominantly vertical. The surface air temperatures increased from +5°C at the beginning of the tests to +14°C by the end of the tests. The mechanical condition of the ice visibly deteriorated during testing. By the end of the tests, 3 cm of ice had been lost from the top surface and candling had begun to affect the underlying lake ice. Also, puddles of water began to form on the surface.

Table IX. Beam test data, Post Pond, 15 March 1976.
Flexural strength ($\times 10^5$ N/m^2).

	Cantilever	*Simply supported*	
	6.67 (TT)	5.12 (TT)	
	4.30 (TT)		2.61 (BT)
	3.24 (TT)		1.99 (BT)
	3.39 (TT)	4.82 (TT)	
	3.29 (TT)	4.42 (TT)	
	3.22 (TT)	4.43 (TT)	
	2.83 (TT)	2.75 (TT)	
	3.14 (TT)	2.66 (TT)	
	3.48 (TT)	2.16 (TT)	
		2.95 (TT)	
Mean:	3.73 (a)	3.66 (b)	2.30 (c)

b/a = 0.98
c/a = 0.62
b/c = 1.59

At the outset of the tests, the thickness of the ice cover was 40.5 cm, including 21 cm of snow-ice and 19.5 cm of lake ice. By the end of the tests, the ice sheet thickness had decreased to 38 cm. That loss of ice was accompanied by severe granulation of the snow-ice layer (which ultimately lost all cohesion with the underlying lake ice) and by extensive candling of the lake ice component. The surface air temperature varied from +4 to +6°C during testing. There was much puddling of the surface (see Fig. 7).

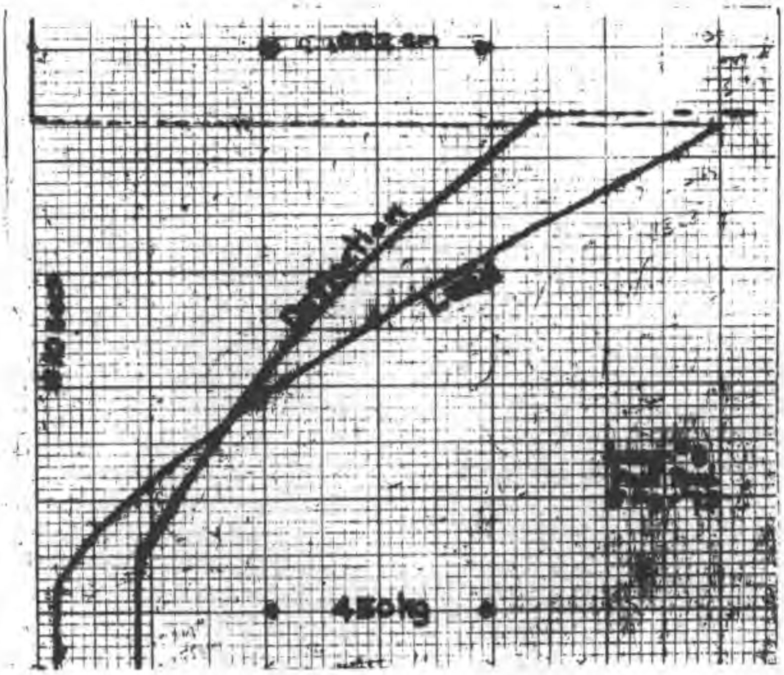


Figure 8. Typical chart record showing load-deflection traces for a large, simply supported beam of lake ice.

Table X. Elastic modulus data for Post Pond and Canaan Street Lake ice beams.

	3 March 1975	*6 March 1975*	*12 March 1975*
No. of tests	4	11	6
Average L/h	7.05	5.97	5.73
Average elastic modulus from initial tangent ($\times 10^9$ N/m^2)	5.44 (TT)	7.87 (TT) 6.75 (BT)	7.78 (TT) 7.57 (BT)
Average elastic modulus from tangent at 50% of ultimate load ($\times 10^9$ N/m^2)	5.55 (TT)	4.50 (TT) 4.28 (BT)	5.49 (TT) 4.55 (BT)

Load-deflection curves were obtained for 21 separate, simply supported beam tests. A typical example of data recorded from the beginning to the end of a test is shown in Figure 8. Using eq 4 with its underlying assumptions from elastic beam theory, two values for the elastic modulus were obtained for each test. One values was determined from a tangent drawn to the initial part of the load-deflection curve and the second value was determined from a tangent drawn to the curve at 50% of the ultimate load. Values of the initial tangent modulus varied from 4.33×10^9 to 9.86×10^9 N/m^2. Values determined from the tangent at 50% of the ultimate load varied from 3.6×10^9 to 5.68×10^9 N/m^2. A summary of the averaged results is presented in Table X.

DISCUSSION

In all tests of the current series of measurements, the ice cover was of composite structure and included a more substantial proportion of snow-ice than is usually the case for lakes in this part of New Hampshire. Stratigraphic and crystalline characteristics of the underlying lake ice were essentially identical at both sites during 1975 and these conditions did not change significantly in 1976. The highest flexural strengths were obtained on cold lake ice sheets (Tables VI and VII) that were unaffected by cracking or thermal degradation, i.e., candling. These observations imply that the strength on any particular day is sensitive to both the ambient air temperature and to the prior history of weather conditions, which largely determine the mechanical state of the ice at the time of testing (see Fig. 6 and 7 for hygrothermograph air temperature records).

In cold, structurally unmodified ice, center-loaded, simply supported beams generally yielded much higher flexural strengths than the same beams tested in the in-place cantilever mode. This was especially true of simply supported beams tested with tension induced in the top (snow-ice) surface. Only after the ice cover had been subjected to prolonged periods of above-freezing air temperatures did this difference in flexural strength between simply supported and cantilever beams begin to diminish. (See, for example, Tables VIII and IX.)

The ratios of strengths of simply supported and cantilever beams tested with the top surfaces in tension (expressed as b/a in Tables II-IX) frequently exceeded 2.0 in individual tests but a mean value of between 1.2 and 1.7 appears to be more typical of the tests as a whole. These test data deviate appreciably from those of Butiagin (1966), who detected no significant differences between the flexural strength of cantilever and simply supported beams. However, the present results agree generally with those obtained by Frankenstein (1961). In three separate series of tests involving cantilever and simply supported beams with tensions induced in the top surface, Frankenstein's data are found to yield b/a values of 1.71, 3.40 and 1.26 respectively.

The present series of tests also shows that the flexural strengths of simply supported beams generally are higher when the top surfaces are in tension. These differences (expressed as b/c ratios in Tables II-IX) also tend to diminish in ice that has undergone extensive structural modification, especially thermal degradation leading to granulation of the snow-ice layer. In three similar series of tests by Frankenstein (1961), a reevaluation of his data in terms of b/c ratios has yielded values of 1.44, 1.10 and 1.49. These ratios agree very closely with results from the current series of tests. Our b/c ratios occasionally exceed 2.0 but the mean values are generally located in the range of 1.2-1.6.

The top layer of all beams tested in the current series of measurements was composed of snow-ice. The observation that tests showed higher flexural strengths for simply supported beams when the top of this layer was placed in tension is entirely compatible with the mechanical properties of snow-ice: it is characteristically fine-grained and firmly bonded at subfreezing temperatures and is generally crack-free. The bottom ice, by contrast, is usually composed of very large crystals, is always at temperatures close to its melting point and is usually intersected by cracks – factors that collectively could be expected to cause beams to fail more rapidly when tension is applied to the bottom surface.

It is perhaps significant that beams containing snow-ice as the major component usually yielded the highest flexural strengths – both in the simply supported and cantilever modes. Flexural strengths in excess of 15×10^5 N/m^2 (see Table VI) obtained with cold ice of this kind are among the highest ever reported with simply supported beams. The common practice of using just half the true thickness of the snow-ice layer to calculate its effective strength may be valid for thermally degraded snow-ice, but it certainly does not apply in the case of cold, unrecrystallized snow-ice which the present observations show can possess flexural strengths that frequently exceed those of the underlying lake ice.

The current method of testing ice beams differs from the procedures of others in that the same beam is used for both tests. For most of the beams the two tests were performed within 10 minutes of each other. These procedures ensure that the cross-sectional dimensions and structure of the beam are kept constant and that the temperature distribution in the beam will not change substantially during testing.* Accordingly, the generally lower strengths observed with the cantilever tests must be due either to the existence of an appreciable stress concentration at the corners of these beams or to fundamental differences between the failure mechanics of simply supported and cantilever beams.

Both Butiagin (1966) and Lavrov (1971) argue against the existence of significant external stress concentrations: Butiagin on the basis of comparative tests that failed to demonstrate any apparent difference between strengths of simply supported and cantilever beams, and Lavrov on the basis of tests performed on cantilever beams with their corners filleted to reduce external stress concentration. However, Lavrov, unlike Butiagin, acknowledges that the flexural strengths of simply supported beams generally exceed those of cantilever beams. Lavrov attributes these differences in strength to fundamental differences between the failure mechanics of cantilever and simply supported

* Because of thermal disturbances induced in the ice during the preparation of beams, the temperature distributions in beams at time of testing could differ appreciably from those of the undisturbed ice cover. However, the important point here is that the two tests were generally performed fast enough to prevent any significant changes in the thermal condition of the beam *during actual testing*.

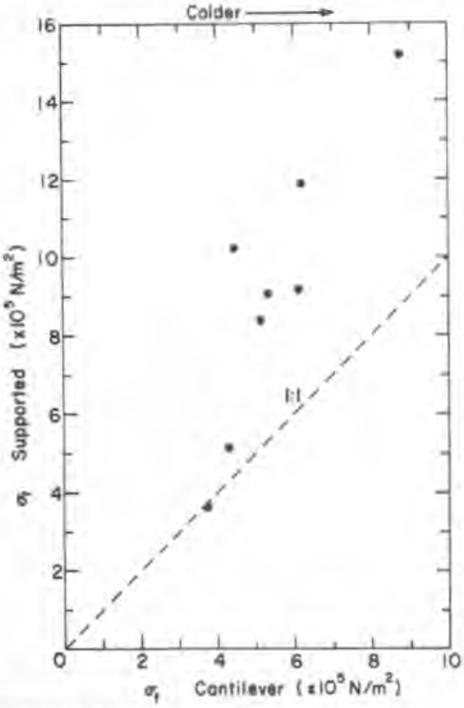


Figure 9. Average values of flexural strength of lake ice as determined from parallel testing of in-place cantilever and simply supported beams. Data from Tables II-IX. Qualitative dependence of data on temperature is also indicated.

ice beams. He suggests that the bending strength of the ice cover can be obtained from cantilever beam tests by multiplying the measured strengths by a correction factor that varies with the length-to-thickness ratio of the beams. For beams with a length-to-thickness ratio of 7 to 8, this factor is about 1.5, which agrees rather well with the average strength difference factor we observed in our tests. A similar strength difference factor was also observed in the several series of tests conducted by Frankenstein (1961).

In order to explain the current series of tests it is suggested that a significant stress concentration may exist at the corners of cantilever beams, but that the magnitude of the stress concentration depends critically on the thermal and structural condition of the ice. In the case of cold, brittle ice, this factor may exceed 2.0 if the strength difference ratio (b/a) can be attributed entirely to stress concentration effects; i.e., cantilever tests may underestimate flexural strengths by a factor of 2 or more. In ice that has undergone extensive thermal degradation, any appreciable loss of cohesion between grains and crystals in the ice sheet is probably sufficient to override stress concentration effects. This condition apparently occurs when flexural strengths fall below about 3×10^5 N/m^2. In ice of intermediate strength, the strength difference ratio tends to be in the range of 1.2-1.7. These differences between the flexural strengths σ_f of beams in the two test modes, and their general dependence on temperature, are further illustrated in Figure 9.

In view of the convenience and simplicity of the cantilever beam test, it would be very useful to know if the kind of empirical relationship observed in Figure 9 is of general application, that is, can be used for adjusting flexural strengths of ice covers obtained solely on the basis of cantilever beam tests. The relationship expressed in Figure 9 is probably valid only for the type of ice cover tested, which in this case was composed of approximately equal proportions of snow-ice and frozen lake water. Different types of ice cover could be expected to react differently, depending on the amount of snow-ice present and on the structural state (texture and c-axis orientation) of the lake ice component. It would seem important, in light of the present observations, to determine just how strength difference ratios for cantilever and simply supported beams vary with changes in composition of the ice cover.

Observations by Gold (1971) of the failure and successful use of fresh water ice covers for vehicular traffic indicate that good quality ice can support loads of up to $P = 250\,h^2$, where P is the total load in pounds and h is the ice thickness in inches. Failures were reported, however, for loadings as low as $P = 50\,h^2$. Gold is quick to emphasize the fact that the stress at failure may not be directly related to P/h^2 as he tacitly assumed in presenting his results. However, if his numerical constants 50 and 250 are transformed from British (psi) units to SI units, then we obtain values of 3.45×10^5 N/m^2 and 17.25×10^5 N/m^2, which correspond very closely with the lower and upper limits of flexural strength observed in the current series of tests. This suggests that the numerical coefficient A in the relationship

$$P = A\sigma_f h^2$$

approximates unity for the type of ice cover we tested and that the bearing capacity of the ice can be adequately expressed in the form $P = \sigma_f h^2$ where σ_f and h are the measured flexural strength and ice thickness at the time of testing.

Very few data on the static modulus in flexure from in-situ large scale beam tests on lake ice are available. Values obtained in the current series of tests (Table X) are generally higher than the few values reported previously, such as those published by Butiagin (1966); in some cases our values approach those obtained for the dynamic modulus of ice. The present results indicate no systematic change in strain modulus with increasing flexural strength such as Lavrov (1971) observed in tests on small beams of laboratory-prepared ice. Scale factors may be important here because the dimensions of Lavrov's beams were approximately an order of magnitude smaller than the beams used in the present series of field tests.

CONCLUSIONS

Parallel tests of large in-place cantilever and simply supported beams of lake ice have yielded the following data on flexural strengths:

1. Center-load, simply supported ice beams generally yield much higher flexural strengths than the same beams tested in the cantilever mode. Strength difference ratios frequently exceeded 2.0. Ice of intermediate flexural strengths generally gave values of 1.2-1.7. Strength difference ratios reduced to about 1.0 when the average flexural strength fell below about 3×10^5 N/m^2. A maximum strength of 15.80×10^5 N/m^2 was obtained with a simply supported beam with tension induced in the top layer; the same beam tested in the cantilever mode yielded a strength of 9.33×10^5 N/m^2. These data support the contention that sizable stress concentrations can exist at the fixed corners of cantilever beams. The maximum effect is experienced with cold, brittle ice substantially free of structural imperfections; in structurally degraded ice, the stress concentration factor may be eliminated entirely.

2. Center-loaded, simply supported beams generally are much stronger when the top surfaces are placed in tension. This behavior is attributed largely to differences in ice types; the fine-grained, crack-free top layer of snow-ice that characterized all beams in the current series of tests generally reacted more strongly in tension than the coarse-grained, crack-prone, bottom ice. The ratio of strength for the top layer in tension to that for the bottom ice in tension may occasionally have exceeded 2.0, but average values generally occurred in the range of 1.2-1.6.

3. Examination of the differences between the strength of in-place cantilever beams and the same beams tested in the simply supported mode indicate that the relationship between the two could be used to obtain "corrected" flexural strengths from tests based solely on measurements of cantilever beams, which are much simpler to test than simply supported beams. The relationship of the two strengths could be expected to vary with the percentage of snow-ice and the structural state of the frozen lake water component. Such relationships should be evaluated for the several different types of ice cover known to form on temperate lakes.

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