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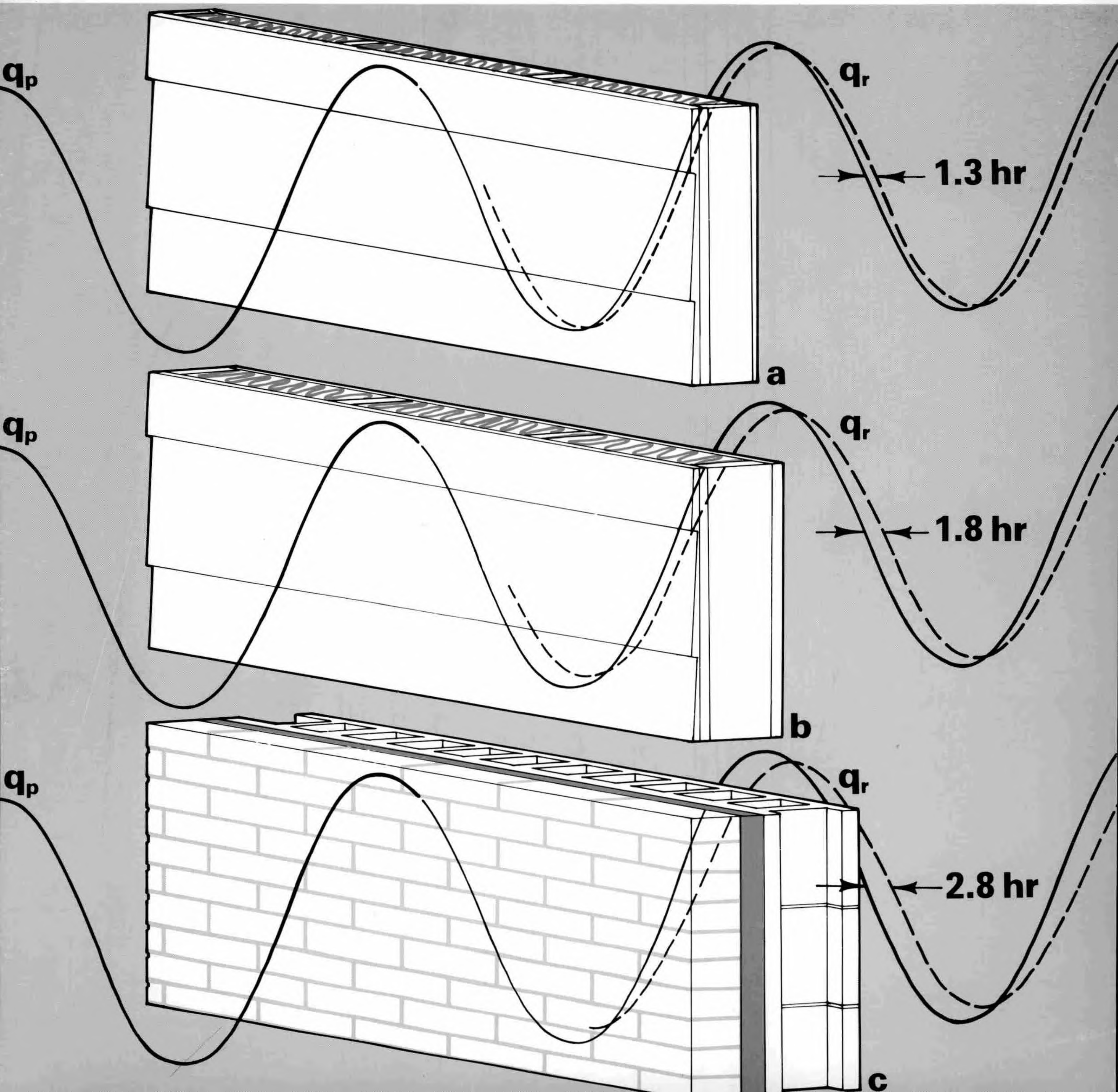
REPORT 80-15

Time constraints on measuring building R-values



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Time constraints on measuring building R-values

Stephen N. Flanders

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This report discusses the time constraints on measuring the thermal resistance (R-value) of building components. Temperature changes on either side of a building component perturb measurement accuracy. Long measurement times and measurement times corresponding to a consistent diurnal cycle can be satisfactory; however, individual temperature changes cause significant error for shorter measurement periods. This report shows how to scale the thermal properties of individual constituent materials in a building element to determine its *characteristic thermal time constant*. The report then demonstrates the size of measurement error resulting from a variety of changes in temperature with representative walls of different time constants.

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PREFACE

This report was prepared by Stephen N. Flanders, Research Civil Engineer, of the Civil Engineering Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding for this research was provided by DA Project 4A762730AT42, *Design, Construction and Operations Technology for Cold Regions,* Task C, *Cold Regions Maintenance and Operations of Facilities,* Work Unit 004, *Maintenance and Rehabilitation of Military Facilities in Cold Regions.*

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The author wishes to thank Ronald Atkins, Kevin O'Neill, and Gary Phetteplace for their assistance in helping to solve the problems treated in this report. Appreciation is also given to Doris VanPelt for her time and technical skills in helping with the laboratory experiments.

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These conversion factors include all the significant digits given in the conversion tables in the ASTM *Metric Practice Guide* (E 380), which has been approved for use by the Department of Defense. Converted values should be rounded to have the same precision as the original (see E 380).

Multiply	*By*	*To obtain*
inch	25.4*	millimeter
foot	0.3048*	meter
hour foot^2 °F/BTU	0.1762280	kelvin meter^2/watt
degrees Fahrenheit	$t_C^\circ = (t_F^\circ - 32)/1.8$	degrees Celsius

*Exact

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Stephen N. Flanders

INTRODUCTION

This report describes the time constraints on the accuracy of measuring thermal resistance (R-values) of building components. It gives a method for estimating the characteristic time constant of building elements and demonstrates the magnitude of error that different weather events may introduce into thermal measurements. This report should help an investigator to determine the appropriate duration of measurement for determining R-values and demonstrate the magnitude of error in the resulting data.

A high R-value helps to slow down heat flow out of a building and thereby conserves space heating energy. Unfortunately, the thermal performance of a building is usually worse than the design thanks to incorrect installation and deterioration of insulation. Therefore, if we want to determine the actual R-value of building components, we must either remove samples to a laboratory or measure the components' thermal properties in place.

A laboratory test can accurately determine building component thermal properties but sample removal damages the building and does not reflect on-site conditions. In the laboratory we can maintain constant temperature and heat flow and determine R-values according to a basic heat flow equation:

$$R = \Delta T/q \tag{1}$$

where

$\Delta T = T_w - T_c$
R = thermal resistance
T_w = temperature on warm side of sample
T_c = temperature on cold side of sample
q = heat flux.

As long as the sample's thermal conditions maintain a steady state, sensors in the test apparatus give meaningful values for the two temperatures and the rate of heat flow.

Field measurement of R-values doesn't damage the building, but it doesn't give us steady-state conditions either. If we measure the R-value of a wall, we apply temperature and heat flow sensors on the indoor surface and a temperature sensor at a corresponding location on the outdoor surface. Figure 1 shows a strip chart recorder monitoring the output of such sensors for several locations on a wall. Fluctuating temperatures on either side of the wall prevent it from passing heat flow at a steady rate, so that at any one time the measured temperature difference across the wall (ΔT) and heat flux (q) do not render accurate R-values in eq 1. Therefore, a major problem of on-site measurement is to *determine the minimum duration of measurement to obtain an accurate R-value.*

This problem occurs because a building component not only *resists* the flow of heat, but also *stores* heat. Each building material has its own resistance to heat flow (R-value), heat storage capacity (specific heat) and density. These factors contribute to the time for a change in temperature across a building component to reach a new steady-state value. If ΔT across the component increases, the component stores heat up to its capacity at that temperature potential until it passes as much through the cool side as enters through the warm side. If the temperature difference decreases, the component gives off heat until it reaches its new diminished capacity and again passes as much as enters. The

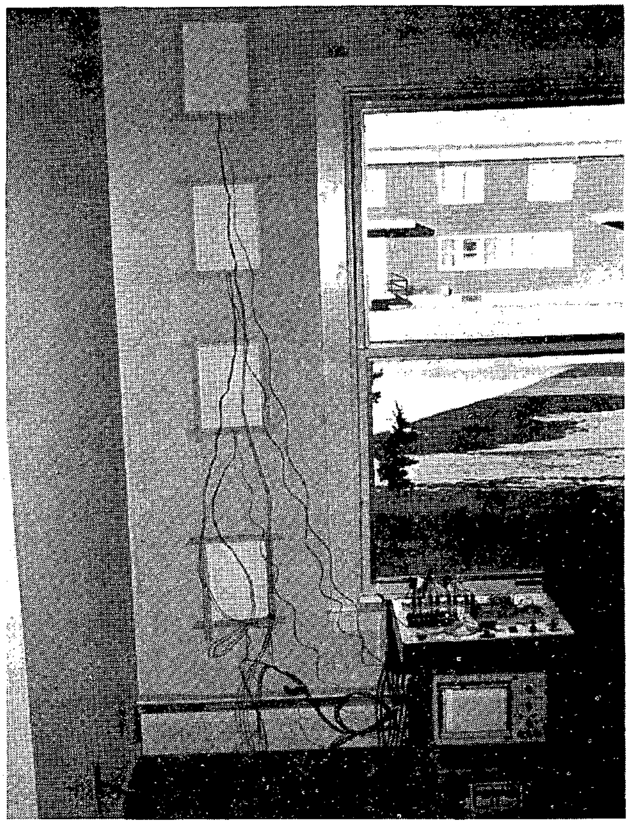

Figure 1. On-site measurement of building R-values. Sensors under the paper on the wall measure heat flow and temperature on the indoor surface while a temperature sensor outdoors completes the needed information which a strip chart records for several locations.

length of time required for the component to reach a steady-state value depends on its characteristic *time constant*.

With these basic principles in mind, we'll see that the accuracy of field measurements of building R-values depends on three main factors:

1. Temperature difference (ΔT) across the component.
2. Variation in ΔT prior to and during the observation period.
3. Characteristic time constant of the component.

Sensor accuracy and the impact of measurement on altering the actual conditions are the subjects of a colleague's unfinished study.

The larger the average ΔT across a component, the less a given variation in ΔT will affect the measurement. We can classify each of the many possible causes for variation in ΔT as either *cyclical* or *random*. The passage of seasons and days causes cyclical temperature changes. However, seasons change so gradually that they are essentially constant for the time we would measure an R-value. We can account for the values of ΔT that cause the heat to flow and all the values of q that represent the amount of heat flow in a new formula for R-value:

$$R = \sum_{i=1}^{N} \Delta T_i \bigg/ \sum_{i=1}^{N} q_i \qquad (2)$$

where ΔT_i is the difference in temperature between the inside and outside at the i-th measurement and q_i the heat flow at the same measurement time. This equation works well for cyclical variations in temperature, as long as we measure frequently for one full cycle, but has a drawback for random changes.

If random temperature changes are significant before or during the time we measure, we then must be more careful with our use of eq 2. We should measure for a long enough time period to know that the heat taken into or rejected from storage in the building is not large compared to the amount we saw pass through. The component's thermal time constant determines how much of this heat is unaccounted for.

A component's time constant t_c comes from the thermal properties of resistivity, specific heat, density and thickness for each type of building material in a wall or roof layer. The value t_c helps us to understand how quickly the component responds to changes in ΔT:

$$t_c = \frac{a_k}{\pi^2}\left(\sum_{n=1}^{N} g_n x_n\right)^2 \qquad (3)$$

where

t_c = characteristic time constant of building component

$g_n = (a_n/a_k)^{1/2}$, conversion constant adjusting thickness of layer to make material uniform throughout wall

$a_n = r_n C_n d_n$, reciprocal of diffusivity of n-th layer

r_n = resistivity of n-th layer (published value)

C_n = specific heat of n-th layer (published value)

d_n = density of n-th layer (published value)

x_n = thickness of n-th layer

$a_k = a_n$ at layer k chosen for normalization.

This equation, derived in Appendix G, transforms the thermal properties of each layer into those of one composite material, but adjusts the thickness of each layer to compensate and retain each layer's original time behavior so that we may calculate t_c as if it represented a single material.

With t_c we can predict how rapidly a building element responds to changes in ΔT because t_c reflects both the *resistance to heat flow* and the *heat storage capacity* of the component. If ΔT changes linearly (like a ramp), then q will follow, delayed by a period t_c. If ΔT changes abruptly (like a step), then q follows with time t according to $e^{-(t/t_c)}$ and reaches 63.2% of its new equilibrium value at $t = t_c$. In the next section, we'll see that this value comes from an expression of Carslaw and Jaeger (1959). The time constant also controls the component's response to cyclical changes.

Such *delays* in thermal response occur because the component temporarily gives off or stores more heat than it can over the long term; t_c is the key to the magnitude of error that this transient phenomenon causes. Equation 3 may appear to be academic because the variables are experimental unknowns. Nevertheless, inspection of the component or of construction drawings together with published data about thermal and material properties will give sufficient accuracy for field measurement.

We can get a feel for some typical building wall time constants t_c using eq 3 and data from ASHRAE (1977) in Appendix B. A 2x4 insulated frame wall has a t_c of 1.25 hr, a 2x6 wall 1.78 hr and and an insulated 4-in. brick and 8-in. concrete block masonary wall 2.82 hr. Later we'll use these values in some examples, but first we'll discuss the contributions of other investigators to the problem of determining the time constraints on thermal measurements.

FIELD MEASUREMENT AND ANALYSIS OF TRANSIENT HEAT FLOW

Much literature on the measurement of heat flow under uncontrolled conditions comes from the study of soil temperatures. For example, Lachenbruch (1959) discusses how to analyze the seasonal variation of temperature in stratified soils.

Relatively little is written, however, about how long to use heat flux sensors and thermocouples in measuring the thermal performance of building components, although this is a common investigation. Lorentzen et al. (undated) outline heat flux and temperature sensor application methods and limitations. Peavy et al. (1975) and Burch (1976) of the National Bureau of Standards are two frequent users of this approach to in-situ measurement of building components. Such research often recognizes time constraints by measuring for ample durations or 24-hour periods.

Poppendieck et al. (1976) outline some basic considerations in non-steady-state measurement of heat transfer in buildings. They give the mathematical solutions for temperature as a function of depth and time in an idealized one-material wall for both a sinusoidal and a step change in temperature across the wall. They calculate the phase lag from a sinusoidal variation in temperature across a 2-in.-thick cork board to be 11 hours and then demonstrate that measurement of periodic temperature difference and heat flow for one period should result in a ratio equal to the steady-state R-value. A single laboratory test using this 2-in. cork panel in a guarded hot box corroborated their theoretical expectations for R-value and time constants.

Poppendieck et al. (1976) suggest that measurement over a full 24-hour cycle will give accurate data for the R-value. But they do not answer how to cope with an underlying trend for changing temperature or for a change of amplitude in temperature extremes. They also suggest that a nighttime reading can suffice for walls less than $R = 10$. However, since specific heat, density and wall thickness also influence the length of time for a building component to stabilize its thermal behavior, this recommendation lacks authority.

Further consideration of the time constraints on thermal measurement has come from 1) mathematical analyses, 2) electrical and hydraulic analogs or 3) computer models.

Carslaw and Jaeger (1959) give a mathematical solution for a slab equilibrating to constant temperatures on each side. Their equation demonstrates the basic qualities of a component layer:

$$v = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{l} e^{-(n^2 K \pi^2 t/l^2)} \qquad (4)$$

where

v = temperature at x
a_n = coefficient in a Fourier series representing initial conditions
K = thermal diffusivity
l = thickness of slab
x = distance into slab
t = time.

The sine term pertains to the location in the slab. The exponent is the basis for eq 3 and *describes the time-related behavior of the slab*. Evaluating v to $n = 1$ is usually sufficiently accurate.

Granholm (1971), Trethowen (1972) and Petzold et al. (1974) discuss mathematical techniques for analyzing non-steady-state heat flow through walls. Ullah et al. (1976) refine the multiple harmonic Fourier method to calculate periodic heat flow in building walls. Sonderegger (1977) analyzes the relative advantage of having a massive outside layer with insulation inside the building envelope, or vice-versa, with thermal response factors based on Fourier transforms. Peavy (1978) develops a streamlined method for calculating thermal response factors that includes the effects of convection and radiation where only conduction is usually considered. Unfortunately, such mathematical analyses cannot readily handle typical fluctuations in ΔT across the building walls.

Electrical and hydraulic analogs have also been used to model the thermal properties of building walls. Hawk and Lamb (1963) come closest to representing the problem of describing the transient thermal behavior of building walls with a hydraulic model employing reservoirs and valves to represent heat storage capacity and thermal conductivity of building layers.

Ultimately, computer models are the most versatile of these methods for gaining insight for accommodating unsteady-state conditions in field measurements. The finite difference method (Forsythe 1960) for approximating differential equations describing the heat balance for selected locations in the wall is quite common. Fourier transforms discussed above and finite element techniques (Aziz 1972) are also current.

A CLOSER LOOK AT HANDLING THE CONSTRAINTS

Measurements spanning a long duration will minimize the effect of transient errors on R-values. However, anyone taking a measurement wants to spend the least time possible and yet know that the error is acceptable. The magnitude of error is time-dependent on the nature of the building element and the climatic changes that occur during measurement.

Let's think of the difference between inside and outside temperatures ΔT as corresponding to a *potential heat flux*. Actual heat flux will tend towards the value of this potential until ΔT changes. The true potential heat flux value is $q_a = \Delta T/R_a$, where R_a is the actual R-value of the wall. In practice, we assume a value of R, based on theoretical calculation or prior measurement. This gives heat flux potential as $q_p = \Delta T/R$. This is a convenient way to convert temperature readings into units of heat flow and compare the lagged behavior of the actual heat flow being measured.

Next, we'll compare the difference between the area under curves representing the potential flux q_p and the measured, delayed response in heat flux q_r over the measurement period t_b and determine its significance as an error of measurement. In each case we'll find the error percentage from

$$E_{\%} = \frac{\int_0^{t_b} q_p\,dt - \int_0^{t_b} q_r\,dt}{\int_0^{t_b} q_p\,dt} \tag{5}$$

Cyclical change

Development of eq 2 demonstrates why measurement over one full period (such as 1 day) will render a good calculation of R for true cyclical change. Let's assume that ΔT fluctuates according to a cyclical curve, such as the one at the top of Figure 2, and that q fluctuates according to any other curve of the same period length but out of phase (as the figure shows). Inspection of the areas enclosed by the two curves for any full cycle shows each area to be constant. Equation 2, translated into integrals, is the ratio of these areas and renders R accurately. Any measurement that does not represent a full cycle is in error to the extent that the ratio of the areas is disproportionate.

The diurnal cycle is not strictly sinusoidal. However, an examination of the properties of a sinusoidal signal passing through a building component will give some insight into the nature of the response curve. The delaying effect of a building component may alter the heat flux response in two ways: 1) the measured heat flux q_r may lag behind the potential heat flux q_p by a fixed interval but with the same period, and 2) q_r/q_p may decrease. This is illustrated in Figure 3.

Phase shift and attenuation vary with the time constant of the building component and the period of the signal. One other factor is important: whether the resistance to heat flow within the wall is relatively great compared to the resistance into and out of the wall. The greater the relative internal resistance, the higher the order of delay it represents in Figures 4 and 5. These figures from Forrester (1961) depict the relationship between the time constant of a wall and the attenuation of amplitude of heat flux with the corresponding lag in flux.

Since 24 hours is the most likely cycle period to influence a wall measurement, Table 1 represents for this time period the time ratios, amplitude ratios and response lag for our wall examples as shown on the cover.

The percentage of measurement error during the course of sinusoidal variation in ΔT, given arbitrary starting and ending times for measurement, comes from

$$E_{\%} = \frac{(12/\pi)\,(a\,U - b\,V)}{c(t_2-t_1)-(12a/\pi)\,(U)} \tag{6}$$

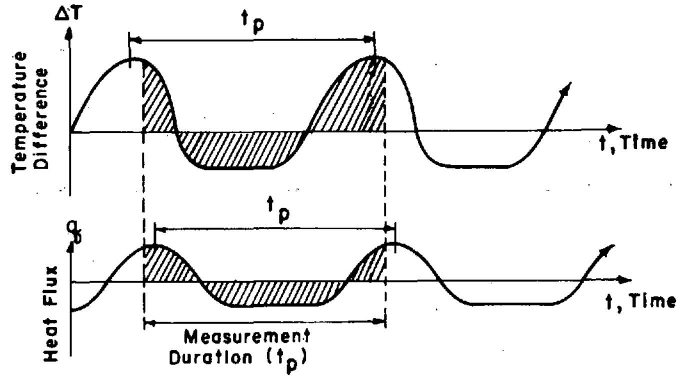

Figure 2. Two arbitrary curves of the same periodicity t_p *but with one lagged behind the other. The ratio of the shaded areas of each curve will always be constant, as long as the measurement duration equals* t_p.

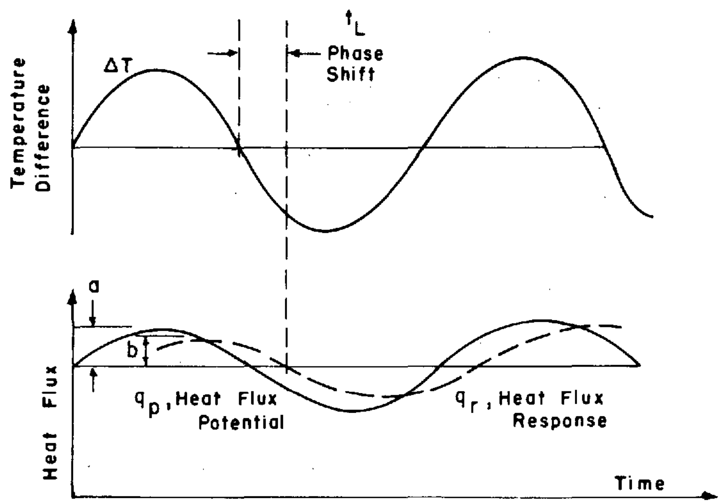

Figure 3. Sinusoidal change in ΔT *and corresponding* q_p *(a-original amplitude of signal, b-attenuated amplitude of response). Measured* q_r *lags behind* q_p *by* t_L *and attenuates by a ratio of* b/a.

where

$U = \cos(\pi t_1/12) - \cos(\pi t_2/12)$
$V = \cos(\pi/12)(t_1 - t_L) - \cos(\pi/12)(t_2 - t_L)$
$E_{\%}$ = error percentage for sinusoidal change
a = amplitude of potential heat flow variation from mean
b = amplitude of heat flow response variation from mean
c = mean value of heat flow
t_1 = time measurement begins
t_2 = time measurement ends.

This equation is derived in Appendix D.

A few applications of eq 6 can quickly convince us to adhere to a measurement duration of approximately one full period. Calculations plotted in Figure 6 demonstrate that large errors in measurement can occur, depending on the phase of the cycle during the course of measurement and when the measurement begins and ends.

The figure shows the thermal response of our sample walls to a sinusoidal variation in ΔT. The percentage error in measurement is plotted as a function of the duration of measurement according to the following conditions:

1. Mean outdoor temperature is either 40°F with a swing of ±30°F or 30°F±10°F.
2. Indoor temperature is 70°F.
3. Measurement begins at various times (t_1 = 0,1,6, 12,18 hr) after sin(0).

A sample calculation demonstrates how to obtain a value on the graph from eq 6. Assume a 2x6 frame wall with R = 20 hr ft^2 °F/BTU, amplitude ratio = 0.93

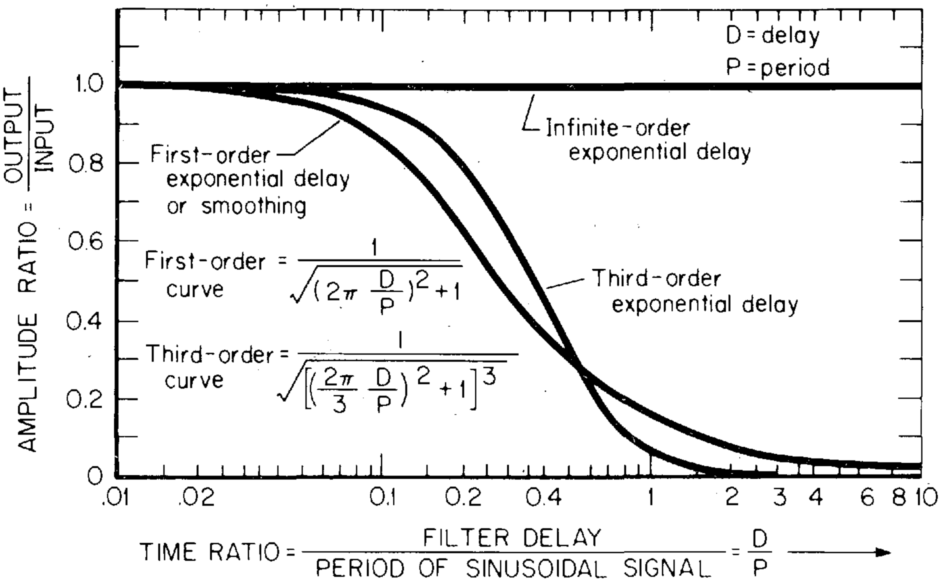

*Figure 4. Ratio of output amplitude to input amplitude vs ratio of delay time to input period.**

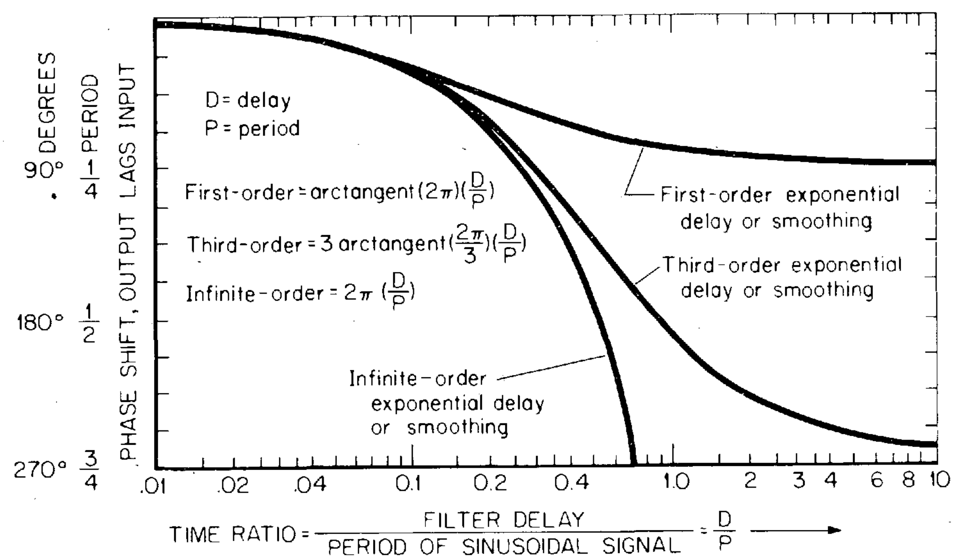

*Figure 5. Phase shift to exponential filter vs ratio of delay to period.**

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*Reproduced with permission from Forrester (1961).

Table 1. **Sinusoidal response properties of insulated wall examples (definitions from Fig. 4 and 5).**

Wall type	*Time ratio*	*Amplitude ratio**	*Response lag** (hr)*
Frame, 2X4	0.052	0.97	1.25
Frame, 2X6	0.074	0.93	1.78
Masonry, brick & block	0.118	0.92	2.77

*Based on a third-order delay.

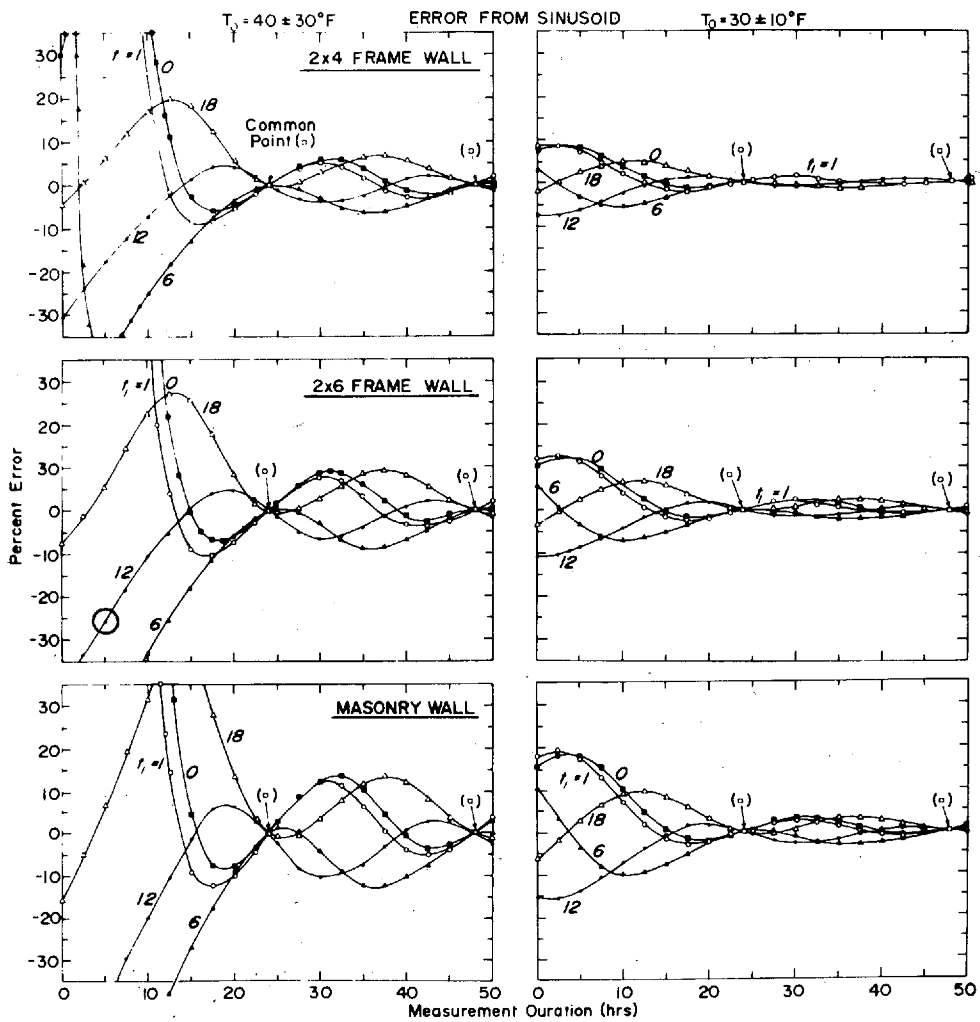


Figure 6. Percentage error from a sinusoidal change in ΔT *as a function of measurement duration for measurement start time* $t_1 = 0, 1, 6, 12, 18$ *hr. The left-hand graphs represent an average temperature of 40°F with a variation of ±30°F. The left-hand graphs represent 30°F±10°F. The top pair of graphs show the behavior of a 2×4 frame wall* (R = 14, *amplitude ratio* (A_r) = 0.96, t_L = 1.25) *the middle pair a 2×6 frame wall* (R = 20, A_r = 0.93, t_L = 1.78) *and the lower an insulated masonry wall* (R = 11, A_r = 0.92, t_L = 2.77).

and response lag = 1.78 hr (from Table 1). What will the error of measurement be after measuring for 5 hours after the 12-hour point in the 40°±30°F sinusoidal cycle? From $q = \Delta T/R$, a = (70–40+30–70+40+30,/2(20), b = 0.93a and c = (70–40)/20. Therefore substituting $U = \cos(\pi/12)(12) - \cos(\pi/12)(12+5)$ and $V = \cos(\pi/12)(12-1.78) - \cos(\pi/12)(12+5-1.78)$ into eq 6 renders $E_{\%} = -25.8\%$ in agreement with the circled data point on the left middle graph. Programs for the Hewlett Packard HP-25 hand calculator that calculate $E_{\%}$ for different changes in ΔT appear in Appendix E.

The 40°±30°F case represents an extreme. The mean ΔT could provide adequate measurement accuracy if it were steady-state, but the variation in ΔT which might occur with a southern exposure on a clear spring day in New England makes the duration of measurement crucial for accuracy. The 30°±10°F case offers a more favorable mean ΔT, and together with the diminished variation, makes the error less than 5% for arbitrary measurement periods longer than 15 hours.

The graphs in Figure 6 clearly show how percentage error increases with increasing time constants of building elements.

Since data from diurnal cycles are not sinusoidal, they do not necessarily adhere closely to the guideline for error determination that eq 6 represents. However, the peaks and valleys of the input ΔT values often correspond clearly with extremes in q_r by a fixed lag time. Integrating the data for both variables for corresponding lagged periods should diminish the error of measurement for less than one full cycle's monitoring.

Random change

While the significant cyclical events hinge around a diurnal cycle, random changes in temperature may be sudden or gradual. The most sudden event may be a cloud passing across the sun, or a weather front may change air temperature quite abruptly. Generally, however, air temperature changes gradually for a period after frontal passage. These and many other events can affect the change in ΔT across a building component and be an underlying disturbance to the accuracy of measurement.

Abrupt change in outside temperature might cause temperature and heat flow within a building component to respond as in Figure 7. An immediate reading of ΔT and heat flux would give a misleading calculated R-value from eq 1. Therefore, we would want to measure for an R-value long enough that the amount of heat the component stores or releases when the environment warms or cools is not large compared with the total flow through the wall.

An abrupt (step) change or a linear (ramp) change in ΔT affects the accuracy of eq 1. Since nature produces a combination of such signals, we should appreciate the relative impact of each.

Step change

A cloud passing overhead can cause a rapid change in surface temperature on a wall oriented to the sun. In less than an hour a cold front can cause a rapid change in temperature that persists long afterwards. If t_c of the building component is large enough, either event may approximate an abrupt change in temperature as depicted in Figure 8. This step change creates a potential flux q_p that the actual flux q_r will tend towards. The area between these curves represents the absolute error calculation of R according to eq 2. The following equation as derived in Appendix A represents the percentage of error:

$$E_{\%} = \frac{|Z|\, t_c\, (1 - e^{-U/t_c})}{q_1 t_b + UZ} \tag{7}$$

where

$U = t_b - t_a$
$Z = q_2 - q_1$
$E_{\%}$ = error percentage for a step change
t_a = time when step input occurs
t_b = time when measurement ends
q_1 = initial, steady-state heat flux
q_2 = potential heat flux after step
t_c = characteristic time constant of building component.

Figure 9 illustrates how the example walls would influence the measurement of R-value after a step change for a variety of circumstances:

1. The step in ΔT is up or down 10°F.
2. The indoor temperature is always 70°F.
3. The colder temperature of the step is 50°F or -10°F.
4. There is a variation in the time after measurement began when the step occurs.
5. There is a variation in the time after the step when measurement stops.

The figure caption and key explicitly show all the values used in the equations except that $q_1 = (70° - T_w)/R$ and $q_2 = (70° - T_c)/R$ for a step increase in ΔT; q_1 and q_2 swap these values for a step decrease.

A sample calculation demonstrates the use of eq 7 for a 2x4 frame wall. Assume t_a = 15 hours, t_b = 18 hr, time constant = 1.25 hr, ΔT_1 = (70°F–50°F) and ΔT_2 = (70°F–60°F). Then $q = \Delta T/R$ and R = 14 imply that q_1 = 20/14 and q_2 = 10/14. Therefore, substituting

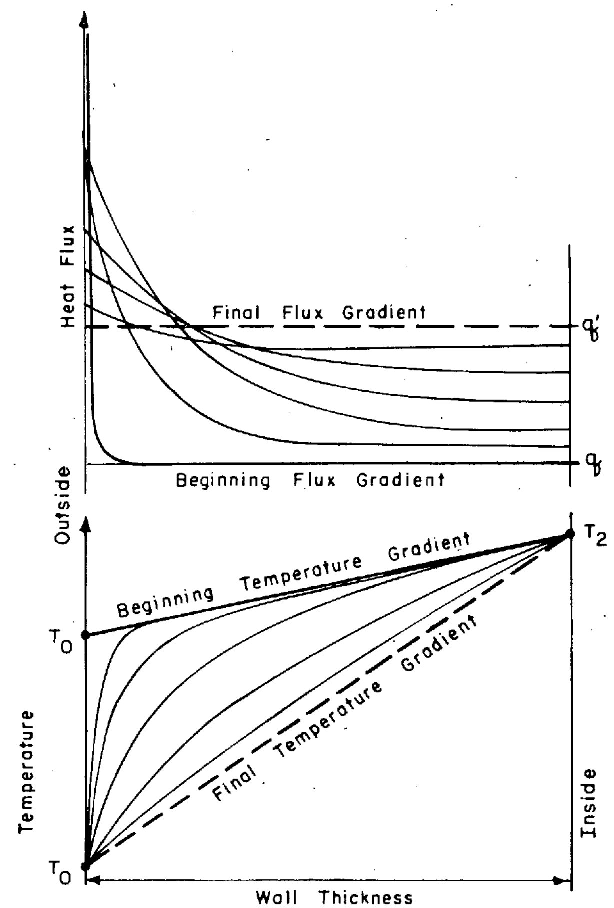


Figure 7. Schematic rendering of corresponding temperature and heat flow changes resulting from a step change in outdoor temperature from T_0 *to* T_0'.

U = 18-15 and Z = 0.71-1.43 into eq 7 renders $E_\%$ = 3.4% in agreement with the circled plot point in the top right graph of Figure 9.

The graphs show that the longer a step input occurs after accumulating measured steady-state values, the smaller the effect on the error. For each wall it is better to stop measuring than to measure a relatively short time longer. However, substantially prolonging the measurement ultimately improves accuracy.

Note that the error increases with larger time constants. The same measurement period results in increasing error when the initial ΔT is smaller.

Ramp change

A slow cooling-off period from dusk to dawn or the passage of a warm front may precipitate a slow change in temperature, which has the approximate effect of a ramp change in ΔT. The q_r will always lag behind a ramp function of q_p as long as it persists. Forrester (1968) demonstrates that the lag shown in Figure 10 will be asymptotic to a line parallel to the ramp displaced one time constant after the heat flux potential line.

The following equation, derived in Appendix C, describes the approximate percentage of error for a ramp change in ΔT:

$$E_\% = \frac{(1/2)\ (U|Z|-SY)}{q_1 t_b + U\,Z/2} \qquad (8)$$

where

$U > t_c$
$U = t_b - t_a$
$Z = q_2 - q_1$
$S = U - t_c$
$Y = S|Z|/U$
$E_\%$ = approximate error percentage for a ramp change
t_a = time when ramp input begins
t_b = time when measurement ends
q_1 = initial, steady-state heat flux
q_2 = potential heat flux when measurement ends
t_c = characteristic time constant of building component.

Figure 11 demonstrates the response to a ramp input of the same example walls as before. It shows the percentage error in measurement according to several assumptions:

1. The rate of change is 5°F/hr up or down.
2. The indoor temperature is always 70°F.
3. The colder temperature of the ramp input is -20°, 0°, 30° or 50°F, depending on starting temperature and duration of measurement.
4. The time between beginning of measurement and ramp commencement varies.
5. The time between ramp commencement and the end of measurement varies.

We calculate q_1 and q_2 as before, using 70°F, T_c and T_w.

A sample calculation demonstrates the use of eq 8 in Figure 11 for a 2×4 frame wall. Assume t_a = 15 hr, t_b = 17 hr, time constant = 1.25 hr, ΔT_1 = (70°-50°F) and ΔT_2 = (70°-60°F). Then $q = \Delta T/R$ and R = 14 imply that q_1 = 20/14 and q_2 = 10/14. Therefore, substituting U = 18-17, Z = 0.71-1.43, S = 2-1.25 and Y = (0.75)(0.17)/2 into eq 8 renders $E_\%$ = 2.6% in agreement with the circled plot point in the top right graph of Figure 11.

The ramp change in ΔT in Figure 11 is a more severe case than a step change of the same magnitude in Figure 9. The top curves in the key of both figures represent a jump in ΔT with a change of outdoor temperatures between 50° and 60°F yet the error magnitude from the ramp change is consistently greater.

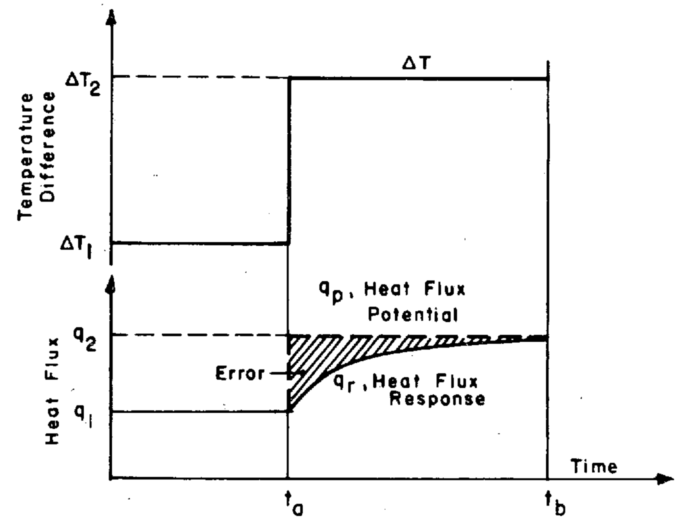
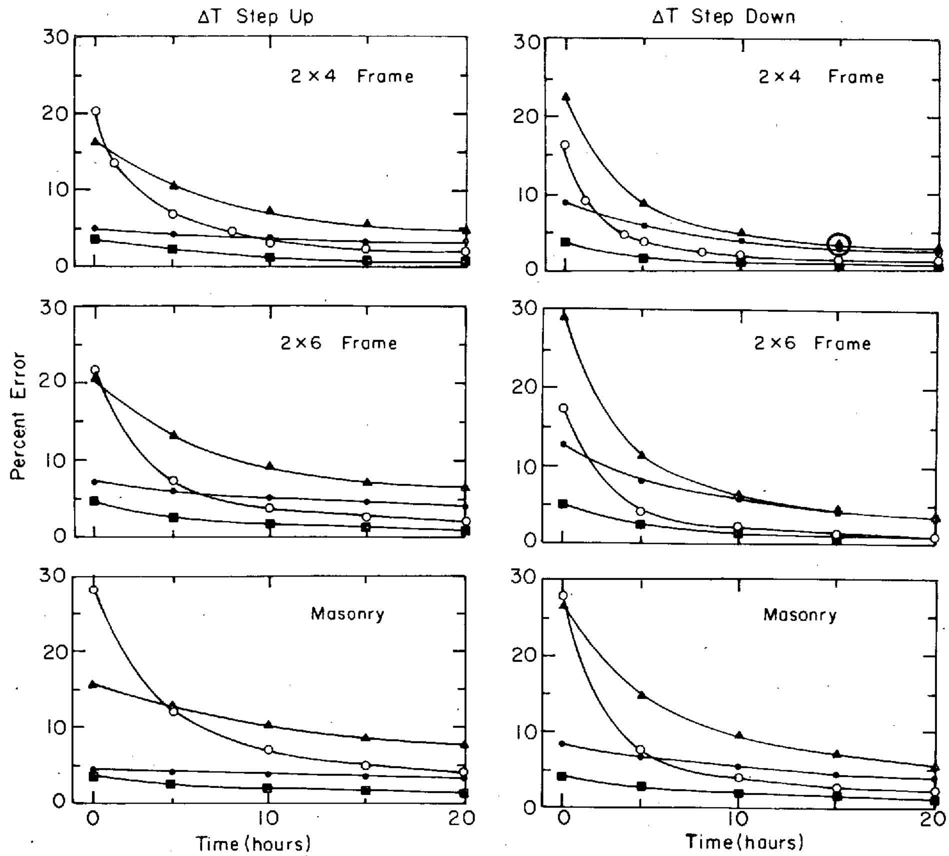


Figure 8. Step change in ΔT *and corresponding* q_p. *The measured* q_r *responds in a delayed manner, creating the error between the curves for* q_p *and* q_r.



	T_c (°F)	T_w (°F)	Duration (hrs) of measurement after step 2x4	2x6	Masonry
○	50	60	0.5	0.5	1.0
▲	50	60	3.0	3.0	8.0
■	-10	0	3.0	3.0	8.0
•	50	60	12	12	32

Figure 9. Percentage error from a step change in ΔT *as a function of time the step occurs after measurement began. The key shows the cold and warm outdoor temperatures,* T_c *and* T_w, *on either side of the step as well as the duration of measurement for each wall after the step. The wall properties are: 2×4 frame (R = 14),* $t_c = 1.25$; *2×6 frame (R = 20),* $t_c = 1.78$; *masonry (R = 11),* $t_c = 2.82$.

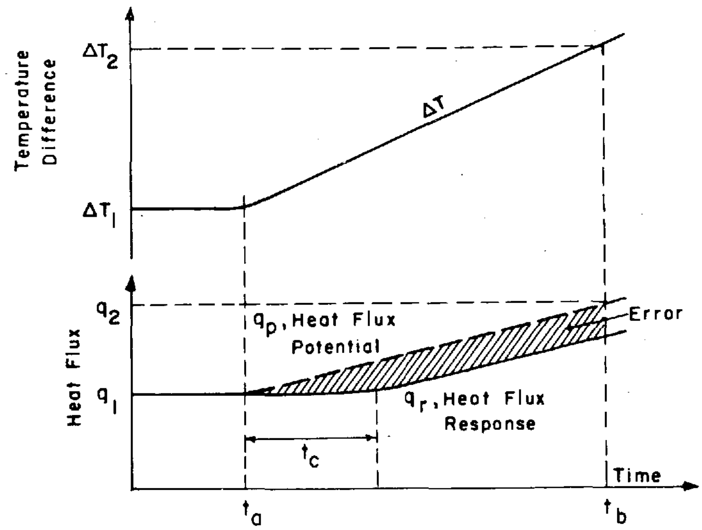


Figure 10. Ramp change in ΔT *and corresponding* q_p. *The measured* q_r *lags behind the* q_p *by one time constant, creating the error between the curves for* q_p *and* q_r.

The graphs show that a ramp change has a smaller impact on measurement accuracy the longer it occurs after accumulating measured steady-state values. For each wall it is better to stop measuring than to measure a relatively short time. However, substantially prolonging the measurement ultimately improves accuracy.

The graphs give us a basic appreciation of the significance of changes in ΔT on measurement accuracy. *Again, the larger* t_c *is, the greater the error.*

APPLICATION OF THEORY

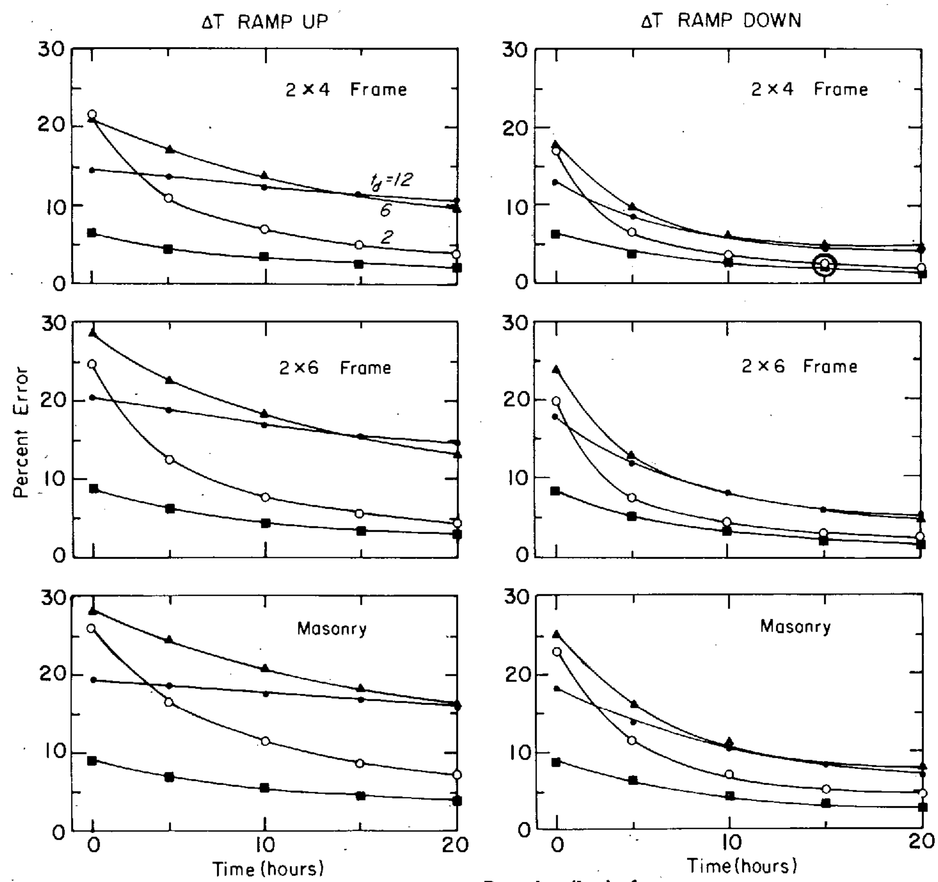
When we prepare to determine the R-value of a building component, we should estimate in advance its characteristic thermal time constant, using eq 3. Duration of measurement depends on the temperature conditions on both sides of the component during measurement. Generally, multiples of 24 hours are appropriate durations for measurement of cyclical changes because the accumulated error is likely to cancel itself out. However, underlying warming or cooling trends may impose themselves upon the diurnal cycle. These warrant analysis as ramp changes according to eq 6 or Figure 11.

Overcast skies and precipitation during an approaching warm front may dampen the effect of the diurnal cycle to where the underlying warming trend is dominant. Such possibilities should prompt us to be aware of the weather conditions during measurement and monitor the data to determine whether enough information is present for satisfactory accuracy.

In analyzing the data, we should try to identify lags between ΔT and heat flux response q_r to corroborate our estimate of the building component's time constant. Then we should base calculations of R-value, using eq 2, on corresponding pairs of ΔT and q_r where the latter value lags the first.

The application of these theoretical considerations to actual field data is imperfect. Figure 12 is a graph of measured temperature changes and the thermal response of a 2x4 frame wall of a building insulated with urea-formaldehyde (UF) foam recorded with the apparatus shown in Figure 1. The measured thermal resistance, calculated according to eq 2 using three days of data, is 22 hr ft^2 °F/BTU. The theoretical R-value from published sources is 19; however, measurement over a 24-hour cycle would have given an R-value of 25.4, 19.9 or 22.5 for each respective day.

There is a three-hour lag between the peak or valley of ΔT over 24 hours and dividing by the corresponding lagged heat flux does not improve the results of the individual daily R-values. The three-hour lag offers another difficulty for analysis. According to Table 1, a



	T_c (°F)	T_w (°F)	Duration (hrs) of measurement after ramp began 2x4	2x6	Masonry
○	50	60	2.0	2.0	4.0
▲	30	60	6.0	6.0	10.0
■	-20	10	6.0	6.0	10.0
•	0	60	12	12	20

Figure 11. Percentage error from a ramp change in ΔT as a function of time the ramp occurs after measurement began. The key shows the cold and warm outdoor temperature, T_c and T_w, at the beginning and end of the ramp as well as the duration of measurement for each wall after the ramp began. The wall properties are 2x4 frame (R = 14), t_c = 1.25; 2x6 frame (R = 20), t_c = 1.78; masonry (R = 11), t_c = 2.82.

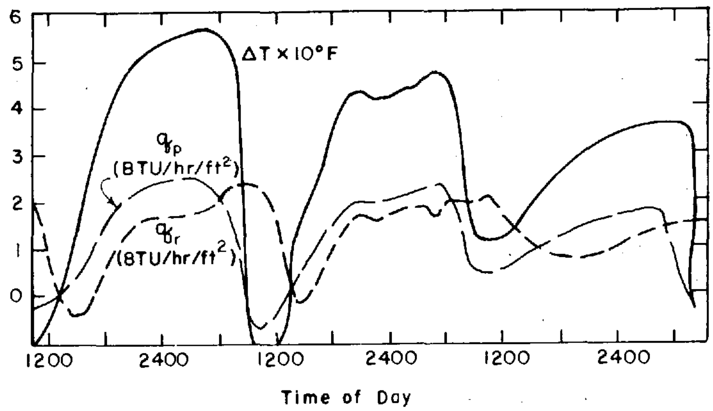


Figure 12. Three-day plot of ΔT *and heat flux on 2x4 frame wall.*

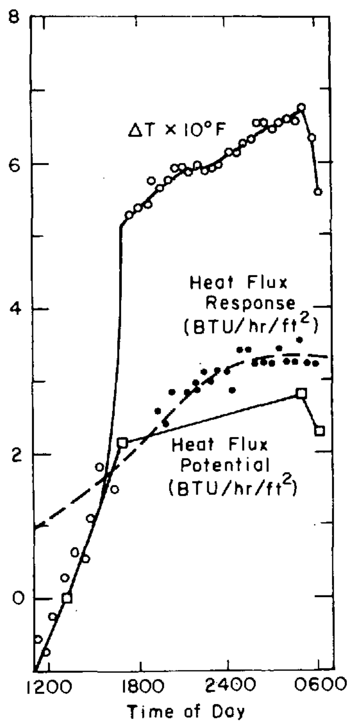


Figure 13. Nineteen-hour plot of ΔT *and heat flux on a 2x6 frame wall.*

fully insulated 2x4 frame wall would have a response lag of half that period. ΔT has been converted to q_p using the measured thermal resistance. The q_r shows slightly more attenuation than the curves in Figure 12 suggest.

Figure 13 depicts the thermal response of a 2x6 frame wall containing UF foam and fiberglass. The expected R-value was 24. However, the measured value, according to eq 2, was only $R = 16$ with 19 hr of data. If we log the last 10.5 hours of data as lagging 1.8 hr to correspond with the estimated time constant of the wall, the R-value is closer to 18.

Laboratory tests employing a Dynatech Rapid-k thermal testing machine also highlighted differences between published values for materials and experimental results. Published values for all parameters except specific heat agreed closely with those measured. Time constants for 1 in. of cork were expected to be 10 min, but 18 min was the measured value. This contradicts the results of Poppendieck (1976). For natural soft rubber 16 min was expected and 40 min was measured. These results are more fully developed in Appendix F.

The time constraints on thermal measurement of building R-values are only part of the problem. These examples of field measurements suggest that measurement techniques need improvement and published data sources may not apply to the material at hand. At least we can now determine the effect of the duration of measurement on the accuracy of our results.

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APPENDIX A. PERCENTAGE ERROR FROM A STEP INPUT

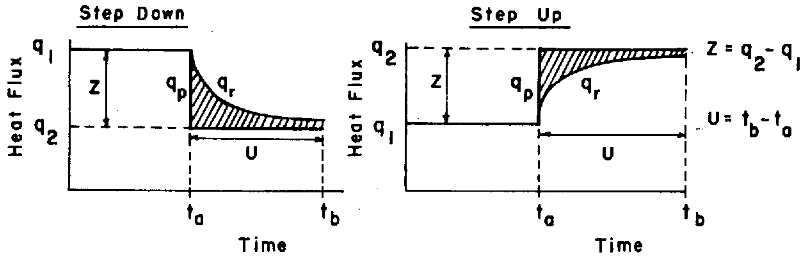

Figure A1. Error from a step input (shaded).

Error = shaded area

$$\int_0^U (q_p - q_r)dt = \int_0^U |Z| e^{-t/t_c} d_t$$

$$= -t_c \; |Z| \int_0^U e^{-t/t_c} \left(-\frac{1}{t_c} dt\right)$$

$$= -t_c \; |Z| e^{-t/t_c} \Big|_0^U$$

$$= t_c \; |Z| \; 1 - e^{-U/t_c}$$

Step down signal area

$$\int_0^{t_b} q_p \, dt = q_1 \; t_b + UZ;$$

$Z = q_2 - q_1$, a negative value

Step up signal area

$$\int_0^{t_b} q_p \, dt = q_1 \; t_b + UZ$$

$Z = q_2 - q_1$, a positive value

% error = error/signal areas

$$E_{\%} = \frac{t_c |Z| \;\; 1 - e^{-U/t_c}}{q_1 t_b + UZ}$$

$Z = q_2 - q_1$ $\qquad$ $U = t_b - t_a$

APPENDIX B. TIME CONSTANTS OF SAMPLE WALLS

Derivation of the technique for determining time constants of building components is in Appendix G.

Definition of variables:

$a_n = r_n d_n c_n$ (1/diffusivity) where
r_n = thermal resistivity (hr ft °F/BTU)
c_n = specific heat (BTU/lb)
d_n = density (lb/ft^3)
$g_n = (a_n/a_k)^{1/2}$ (conversion factor) where
n = layer number
k = layer chosen for normalization
x_n = thickness of layer (ft)

Wood frame wall (2x4 insulated)

(all values from ASHRAE 1977)

n	Material	r_n	d_n	c_n	a_n	x_n
1	Asbestos shingles	3.36	120	0.20	80.6	0.0625
2	Plywood sheathing	14.7	27	0.67	266	0.0625
3	Fiberglass insulation	38.4	2.0	0.22	16.9	0.2917
4	Gypsum wallboard	10.6	50.0	0.26	137.8	0.0625

Normalize to fiberglass: $a_k = a_3 = 16.9$
$g_1 = (80.6/16.9)^{1/2} = 2.18$
$g_2 = (266/16.9)^{1/2} = 3.96$
$g_3 = 1$
$g_4 = (137.8/16.9)^{1/2} = 2.85$
Time constant of wall (from eq 3):

$$t_c = \left(\sum_{n=1}^{4} g_n d_n\right)^2 a_3/\pi^2$$

$$= [(2.18)(0.0625)+(3.96)(0.0625)+(1)(0.2917)$$

$$+(2.85)(0.0625)]^2\ (16.9)/\pi^2$$

$t_c = 1.25$ hr.

Wood frame wall (2x6, insulated)

n	Material	a_n	x_n
1	Asbestos shingles	80.6	0.0625
2	Plywood sheathing	266.0	0.0625
3	Fiberglass insulation	16.9	0.458
4	Gypsum wallboard	137.8	0.0625

Normalization to fiberglass is identical to before.

Time constant:

$t_c = 1.78$ hr.

Masonry wall (insulated cavity, R = 10.8)

n	Material	r_n	d_n	c_n	a_n	x_n
1	Face bricks	1.3	130.0	0.190	32.1	0.333
2	Concrete block webs	0.259	144.0	0.156	5.82	0.417
3	Concrete block faces	1.85	144.0	0.156	41.6	0.167
4	Polystyrene insulation	60.0	3.5	0.29	60.9	0.167

Normalize to block web: $a_k = a_2 = 5.82$
$g_1 = (32.1/5.82)^{1/2} = 2.35$
$g_2 = 1$
$g_3 = (41.6/5.82)^{1/2} = 2.67$
$g_4 = (60.9/5.82)^{1/2} = 3.23$
Time constant:
$t_c = 2.82$ hr.

APPENDIX C. PERCENTAGE ERROR FROM A RAMP INPUT

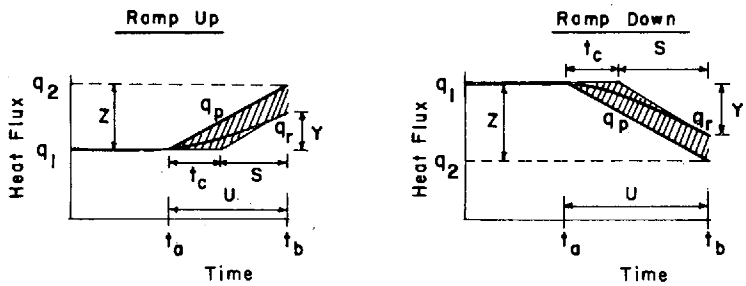

Figure C1. Error from a ramp input (shaded).

$$U = t_b - t_a,\ Z = q_2 - q_1,\ S = U - t_c,\ Y = \frac{S|Z|}{U}$$

Approximate error = shaded area

$$\int_0^U (q_p - q_r)\, dt = 1/2\ U\ |Z| - 1/2\ SY$$

Ramp up signal area

$$\int_0^{t_b} q_p\ dt = q_1\ t_b + 1/2\ UZ;$$

$Z = q_2 - q_1$, a positive value

Ramp down signal area

$$\int_0^{t_b} q_p\ dt = q_1\ t_b + 1/2\ UZ;$$

$Z = q_2 - q_1$, a negative value

% error = error/signal areas

$$E_{\%} = \frac{1/2\ (U|Z| - SY)}{q_1\ t_b + 1/2\ UZ}$$

APPENDIX D. PERCENTAGE ERROR FROM A SINUSOIDAL INPUT

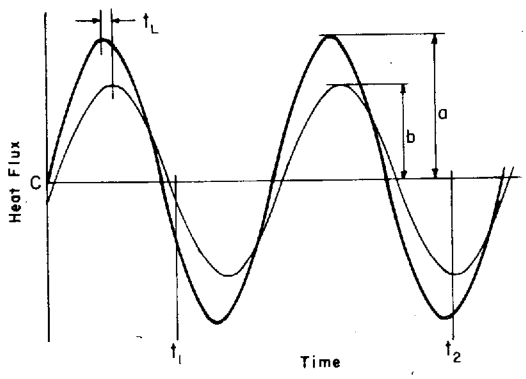

Figure D1. Error from a sinusoidal input (area between curves).

All times are from $q_p\ (0) = c$. Values for b and T_1 come from Figures 11 and 12.

$$q_p = a \sin (\pi/12)\ t{+}c \quad q_r = b \sin(\pi/12)\ (t-t_L){+}c$$

Error = area between curves

$$\int_{t_1}^{t_2} (q_p - q_r)\ dt = a \int_{t_1}^{t_2} \sin (\pi/12)\ t dt$$

$$+c \int_{t_1}^{t_2} dt - b \int_{t_1}^{t_2} \sin (\pi/12)\ (t+t_L) dt - c \int_{t_1}^{t_2} dt$$

$$= \frac{12}{\pi} \Big[-a \cos (\pi/12)\ t + b \cos(\pi/12)\ (t-t_L) \Big]_{t_1}^{t_2}$$

$$= \frac{12}{\pi} \Big\{ a\big[\cos(\pi/12)\ t_1 - \cos(\pi/12)\ t_2\big]$$

$$-b\big[\cos(\pi/12)\ (t_1 - t_L) - \cos(\pi/12)(t_2 - t_L)\big]\Big\} = E$$

Signal area

$$B = \int_{t_1}^{t_2} q_p\ dt = c\ (t_2 - t_1) - \frac{12a}{\pi} \times \ [\cos(\pi/12)\ t_1 - \cos(\pi/12)\ t_2]$$

% error = error/signal areas

$$E_{\%} = E/B$$

APPENDIX E. PERCENTAGE ERROR PROGRAMS FOR HEWLETT-PACKARD HP-25 CALCULATOR

Title: Percentage error: step, ramp & sinusoid — Page 1 of 4

Programmer: S. Flanders

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
	Step Input	p.2					
1	Store warm flux	q_w	STO	1			
2	Store cold flux	q_c	STO	2			
3	Store time constant	t_1	STO	3			
4	Beginning time	t_a	ENTER				
5	End time	t_b	R/S				$E_{\%}$ Down
6	Percent error		R/S				$E_{\%}$ Up
	Repeat 1-6 or 4-6						
	Ramp Input	p. 3					
	Repeat 1-6 as above						$E_{\%}$ Up
	Up & down are reversed						$E_{\%}$ Down
	Sinusoidal Input	p. 4					
1	Convert to radians		g	RAD			
2	Store $\pi/12$	12	g_0	π	⊗	STO	
3	Store a	a	STO	1			
4	Store b	b	STO	2			
5	Store t_1	t_1	STO	3			
6	Store c	c	STO	5			
7	End time	t_2	ENTER				
8	Beginning time	t_1	R/S				$E_{\%}$

Title: Percentage error from a step input

Switch to PRGM mode, press [f] [PRGM], then key in the program.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	X	Y	Z	T	COMMENTS
00			t_b	t_a			Step up of ΔT
01	73 07	STO 7	t_b	t_a			
02	41	⊖	$-U$				$U = t_b - t_a$
03	23 00	STO 0	$-U$				
04	24 03	RCL 3	t_c	$-U$			
05	71	⊕	$-U/t_c$				
06	15 07	e^x	e_1				$e_1 = e^{-U/t_c}$
07	01	1	1	e_1			
08	41	⊖	e_1-1				
09	24 01	RCL 1	q_1	e_1-1			
10	24 02	RCL 2	q_2	q_1	e_1-1		
11	41	⊖	$-Z$	e_1-1			$Z = q_2 - q_1$
12	23 06	STO 6	$-Z$	e_1-1			
13	61	⊗	$Z(1-e_1)$				
14	24 03	RCL 3	t_c	$Z(1-e_1)$			
15	61	⊗	E				$E = Z\,t_c(1-e^{-U/t_c})$
16	23 05	STO 5	E				
17	24 01	RCL 1	q_1	E			
18	24 07	RCL 7	t_b	q_1	E		
19	61	⊗	$q_1\,t_b$	E			
20	24 00	RCL 0	$-U$	$q_1\,t_b$	E		
21	24 06	RCL 6	$-Z$	$-U$	$q_1\,t_b$	E	
22	61	⊗	UZ	$q_1\,t_b$	E		
23	23 04	STO 4	UZ	$q_1\,t_b$	E		
24	51	⊕	$q_1\,t_b$ & UZ	E			
25	71	⊕	$E_\%$				$E_\% = E/q_1\,t_b+UZ$
26	74	R/S	$E_\%$				Percent error
27	24 05	RCL 5	E				Step down of ΔT
28	24 02	RCL 2	q_1'	E			$q_1' = q_2$
29	24 07	RCL 7	t_b	q_1'	E		
30	61	⊗	$q_1'\,t_b$	E			
31	24 04	RCL 4	$-UZ'$	$q_1'\,t_b$	E		$Z' = -2$
32	41	⊖	$q_1'\,t_b+UZ'$	E			
33	71	⊕	$E_\%$				$E_\% = E/q_1'\,t_p+UZ'$
34							Percent error
35							
36							
37							
38							
39							
40							
41							
42							
43							
44							
45							
46							
47							
48							
49							

REGISTERS	
R_0	$-U$
R_1	q_1 warm temp. heat flux
R_2	q_2 cold temp. heat flux
R_3	t_c time constant
R_4	UZ
R_5	E error
R_6	$-Z$, $Z = q_2-q_1$
R_7	t_b end time

Title Percentage error from a ramp input Page 3 of 4

Switch to PRGM mode, press [f] [PRGM], then key in the program.

DISPLAY LINE	CODE	KEY ENTRY	X	Y	Z	T	COMMENTS
00			t_b	t_a			Input times
01	23 07	STO 7	t_b	t_a			
02	47	⊖	$-U$				$U = t_b - t_a$
03	23 00	STO 0	$-U$				
04	24 01	RCL 1	q_w	$-U$			q_w: warm flux
05	24 02	RCL 2	q_c	q_w	$-U$		q_c: cold flux
06	41	⊖	$-Z$	$-U$			$Z = q_c - q_w$
07	23 06	STO 6	$-Z$	$-U$			
08	61	⊗	UZ				
09	23 05	STO 5	UZ				
10	24 03	RCL 3	t_c	UZ			t_c: time const.
11	24 00	RCL 0	$-U$	t_c	UZ		
12	51	⊕	$-S$	UZ			$S = U - t_c$
13	15 02	X^2	S^2	UZ			
14	24 06	RCL 6	$-Z$	S^2	UZ		
15	61	⊗	$-S^2Z$	UZ			
16	24 00	RCL 0	$-U$	$-S^2Z$	UZ		
17	71	⨸	SY	UZ			
18	41	⊖	$UZ-SY$				
19	02	2	2	$UZ-SY$			
20	71	⨸	E				E: error
21	23 04	STO 4	E				
22	24 01	RCL 1	q_w	E			
23	24 07	RCL 7	t_b	q_w	E		
24	61	⊗	$q_w t_b$	E			
25	24 05	RCL 5	UZ	$q_w t_b$	E		
26	02	2	2	UZ	$q_w t_b$	E	
27	71	⨸	$\frac{1}{2} UZ$	$q_w t_b$	E		
28	23 05	STO 5	$\frac{1}{2} UZ$	$q_w t_b$	E		
29	51	⊕	D_u	E			$D_u = \frac{1}{2}UZ + q_w t_b$
30	71	⨸	$E_{\%}$				Ramp up
31	74	R/S	$E_{\%}$				% error
32	24 04	RCL 4	E				
33	24 02	RCL 2	q_c	E			
34	24 07	RCL 7	t_b	q_c	E		
35	61	⊗	$t_b q_c$	E			
36	24 05	RCL 5	$\frac{1}{2} UZ$	$t_b q_c$	E		
37	41	⊖	D_D	E			$D_D = t_b q_c - \frac{1}{2} UZ$
38	71	⨸	$E_{\%}$				Ramp down
39							% error
40							
41							
42							
43							
44							
45							
46							
47							
48							
49							

REGISTERS
R_0 $-U$
R_1 q_w warm heat flux
R_2 q_c cold heat flux
R_3 t_c time constant
R_4 E error
R_5 UZ $\frac{1}{2} UZ$
R_6 $-Z$
R_7 t_b end time

Title Percentage error from a sinusoid Page 4 of 4

Switch to PRGM mode, press [f] [PRGM] , then key in the prog ·m.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	X	Y	Z	T	COMMENTS
00			t_1	t_2			Input times
01	23 06	STO 6	t_1	t_2			
02	24 00	RCL 0	$\pi/12$	t_1	t_2		
03	61	$\otimes$	α_1	t_2			$\alpha_1 = (\pi/12)\ t_1$
04	14 05	COS	COS α_1	t_2			
05	21	$X \leftrightharpoons Y$	t_2	COS X_1			
06	23 07	STO 7	t_2	COS X_1			
07	24 00	RCL 0	$\pi/12$	t_2	COS α_1		
08	61	$\otimes$	α_2	COS α_1			$X_2 = (\pi/12)\ t_2$
09	14 05	COS	COS α_2	COS α_1			
10	41	$\ominus$	$C–C$				$C–C$ = COS α_2-COS α_1
11	24 00	RCL 0	$\pi/12$	$C–C$			
12	71	$\oplus$	C_1				$C_1 = (12/\pi)(C–C)$
13	24 01	RCL 1	a	C_1			
14	61	$\otimes$	$a\ C_1$				
15	23 04	STO 4	$a\ C_1$				
16	24 03	RCL 3	t_L	$a\ C_1$			t_L : lag time
17	24 06	RCL 6	t_1	t_L	$a\ C_1$		t_1: begin meas.
18	41 51	$\oplus$ $\ominus$	t_1+t_L	$a\ C_1$			
19	24 00	RCL 0	$\pi/12$	t_1+t_L	$a\ C_1$		
20	61	$\otimes$	α_3	$a\ C_1$			$X_3 = (\pi/12)\ (t_1+t_L)$
21	14 05	COS	COS α_3	$a\ C_1$			
22	24 03	RCL 3	t_L	COS α_3	$a\ C_1$		
23	24 07	RCL 7	t_2	t_L	COS α_3	$a\ C_1$	
24	41 51	$\ominus$	t_2+t_L	COS α_3	$a\ C_1$		
25	24 00	RCL 0	$\pi/12$	t_2+t_L	COS α_3	$a\ C_1$	
26	61	$\otimes$	α_4	COS α_2	$a\ C_1$		$\alpha_4 = (\pi/12)(t_2+t_L)$
27	14 05	COS	COS α_4	COS α_3	$a\ C_1$		
28	41	$\ominus$	$C–C$	$a\ C_1$			$C–C$ = COS α_3-COS α_4
29	24 00	RCL 0	$\pi/12$	$C–C$	$a\ C_1$		
30	71	$\oplus$	C_2	$a\ C_1$			$C_2 = (12/\pi)(C–C)$
31	24 02	RCL 2	b	C_2	$a\ C_1$		
32	61	$\otimes$	$b\ C_2$	$a\ C_1$			
33	41	$\ominus$	E				$E = a\ C_1 - b\ C_2$
34	24 07	RCL 7	t_2	E			
35	24 06	RCL 6	t_1	t_2	E		
36	41	$\ominus$	t_2-t_1	E			
37	24 05	RCL 5	C	t_2-t_1	E		
38	61	$\otimes$	$c(t_1-t_1)$	E			
39	24 04	RCL 4	$a\ C_1$	$c\ (t_2-t_1)$	E		
40	41	$\ominus$	S	E			$S = C(t_2-t_1)-\cancel{a}\ C_1$
41	71	$\ominus$	$E_{\%}$				$E_{\%} = E/S$
42							
43							
44							
45							
46							
47							
48							
49							

REGISTERS
R_0 $\pi/12$
R_1 a q_p ampl.
R_2 q q_r ampl.
R_3 t_L Lag time
R_4 $a\ C_1$
R_5 C mean a
R_6 t_1 begins meas.
R_7 t_2 end meas.

APPENDIX F. EXPERIMENTAL DETERMINATION OF TIME CONSTANTS

Laboratory tests of the thermal properties of cork and natural soft rubber were to:

1. Corroborate the results of Poppendieck et al. (1976).
2. Demonstrate the time constant for multiple layers of different materials.
3. Demonstrate the thermal response to a cyclical temperature input.

In summary, the time constants (in minutes) were:

Material	*Expected Constant*	*Observed Constant*
1 in. cork	9.88	17.8
1 in. rubber	15.5	40.0
1 in. cork & 1 in. rubber	–	98.0

These results contradict those of Poppendieck et al. (1976) who expected an 11.6-hr time constant and obtained a 10-hr figure to "the ten percent value" with a step input. In this report one time constant occurs when a thermal response has gone 63% of the way to a step input and 2.3 such constants bring the response within 10%. Poppendieck et al.'s cork samples were 2 in. thick, and therefore would respond one-fourth as rapidly. Based on the above results, the sample of Poppendieck et al. (1976) should have only a 2.73-hr time constant, according to their definition.

The observed results were in reasonable agreement with the method for calculating the time constant of multiple layers of different materials developed in Appendix G.

The cyclical test produced poor agreement between expected and observed values of amplitude attenuation of the input potential heat flux as an actual heat flux response, as shown in Figure 11. There was excellent agreement between expected and observed phase shift, using Figure 12. The discrepancy may be due to the fact that the cyclical input was not a sine curve as treated in the figures.

The measurement apparatus was a Dynatech Corp. Rapid k thermal testing instrument with electrically heated and water cooled plates controlling the ΔT across the sample. Temperature and heat flux sensors registered on a strip chart recorder.

The test materials were cork and soft rubber. Both were 1 ft square and 1 in. thick.

The procedure for measuring the time constant of a sample was:

1. Allow the heat flux on the cool side of the sample to reach equilibrium at a low ΔT.
2. Set the warm side to a substantially higher temperature, 109.6°F.
3. Continuously record the heat flux response to the new equilibrium temperature value, ΔT = 37.8°F.
4. Determine time for the response to reach 63% of its final value.

Samples of cork, rubber and a cork-rubber composite underwent this procedure.

The procedure for measuring the frequency response of the cork and rubber sample was:

1. Allow the heat flux on the cool side of the sample to reach equilibrium at a high ΔT.
2. Change the setting hourly to the low or high alternative temperature on the warm plate.
3. Continuously record the heat flux response to temperature.
4. Determine the attenuation and phase shift of the heat flux response to the heat flux potential, according to Figures 11 and 12.

The temperature controllers on the machine took 2.2 min to achieve the criterion temperature in the step temperature increase. This would cause a longer response time than a true step. However, the ΔT overshot the set level by 14%, which would decrease the response time. Figure E1 shows a plot from sample data.

Measurements of the density and thermal resistivity of each material agreed well with published data. However, there was no independent check of the specific heat of each material. Therefore the descrepancy between the expected and the observed time constants

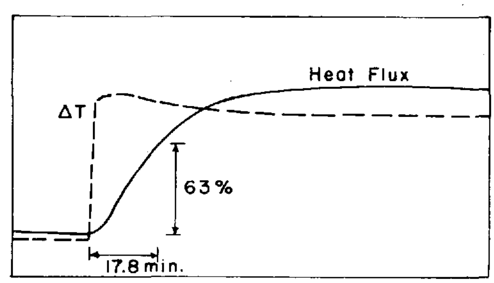

Figure F1. Plot of the heat flux response of a 1-in. cork sample to a step increase in ΔT.

Material	Density (lb/ft^3)	Thickness (ft)	Resistivity ($ft\,°F\,hr/BTU$)	Specific Heat (BTU/lb)	Time Constant (min)
Cork	14.8*/15+	0.083/NA	32.6/35.7	U/0.485	17.8/9.9e
Rubber	62.4/68.6	0.038/NA	12.4/12.5	U/0.48	40/15.5e

e = expected, NA = not applicable, U = no independent check, * = Measured
+ = Published

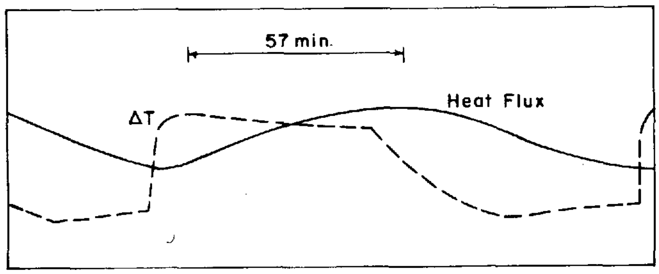

Figure F2. Plot of the heat flux response of combined 1-in. cork and rubber samples to a cyclical input of ΔT with a 2-hr period.

remains unexplained. The above summarizes the measured and published data for the step inputs.

The step change determination of the time constant of rubber and cork together yielded $t_c = 1.6$ hr.

The temperature controller took a fairly long time to cool down the face plate with tapwater during the cyclical test. Consequently, the cyclical inputs resulted in the rapid warming and slow cooling shown in Figure F2.

The period, $t_p = 2$ hr, of the cyclical input to the samples implies a time ratio of $t_c/t_p = 0.817$. Using Figures 11 and 12, the expectations compare with observations as follows:

	Expected Value	Observed Value
Attenuation (output/input)	$[(2\pi t_c/3t_p)^2+1]^{-3/2} = 0.129$	11/24 = 0.458
Phase lag	$(3 \text{ arc tan } (2\pi t_c e t_p)(2 \text{ hr}/2\pi) = 1.0$	1.02 hr

These tests represent good indications of the behavior sought. Independent testing for specific heat and further cyclical testing with different time ratio will help explain unanswered questions.

APPENDIX G. DERIVATION OF TIME CONSTANT FORMULA FOR MULTIPLE LAYERS

The exponential time-dependent term in equation 4 from Carslaw and Jaeger (1959) suggest that the time constant, $t_c = ax^2/\pi^2$, increases with the square of the distance for a uniform slab. If we define a = 1/diffusivity and x = thickness of slab, then a 6-in. slab would have a time constant 36 times longer than a 1-in. slab.

Consider three different slabs of the same material whose individual time constants are known and compare their individual time constants to the time constant of the three combined in one slab (Fig. G1).

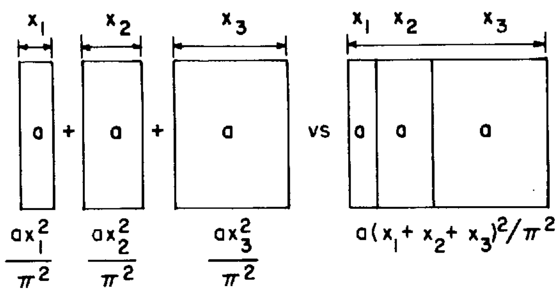

Figure G1. Time constants for individual slab thicknesses at the same material vs the time constant when combined into one slab.

Since the chosen thicknesses can be arbitrary and add up to the same total thickness, one constant D will convert the individual time constants to the appropriate single-slab figure:

$$t_c = D(a/\pi^2)(x_1^2+x_1^2+x_3^2) = (a/\pi^2)(x_1+x_2+x_3)^2.$$

Therefore,

$$D = \frac{\text{time constant of single slab}}{\text{sum of time constants of individual slabs}}$$

$$= \frac{(a/\pi^2)\ (x_1+x_2+x_3)^2}{(a/\pi^2)\ (x_1^2+x_2^2+x_3^2)}$$

$$= \frac{(x_1+x_2+x_3)^2}{(x_1^2+x_2^2+x_3^2)}. \qquad \text{(G1)}$$

Suppose that each segment is a different material. Equation G1 can represent this if the thickness of each segment is adjusted to convert each separate material into the same equivalent material (Fig. G2).

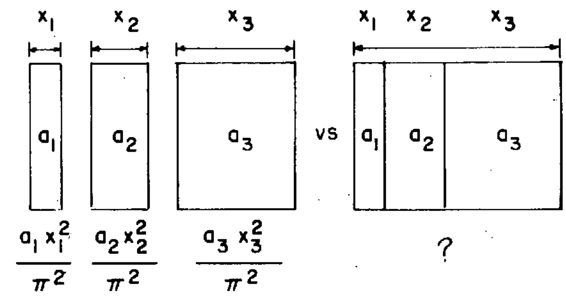

Figure G2. Time constants for individual slab thickness of different materials vs the time constant when combined into one slab.

Choose a material, say a_3, to normalize the other properties to according to a constant g_n for each. Then,

$$a_1 = g_1{}^2 a_3 \text{ and } t_1 = a_3(g_1x_1)^2/\pi^2$$

$$a_2 = g_2{}^2 a_3 \text{ and } t_2 = a_3(g_2x_2)^2/\pi^2$$

$$a_3 = g_3{}^2 a_3 \text{ and } t_3 = a_3(g_3x_3)^2/\pi^2,$$ where g_3 = 1, since this material is the basis for normalization.

Therefore eq G1 becomes

$$D = \frac{(g_1x_1+g_2x_2+g_3x_e)^2}{[(g_1x_1)^2+(g_2x_2)^2+(g_3x_3)^2]}. \qquad \text{G2}$$

In the case of the rubber and cork composite sample, $x_c = x_r$ = 1/12 ft. From Appendix F:

$$a_r = (0.67\ \text{hr})\pi^2/x_r^2 \text{ and } a_c = (0.30\ \text{hr})\pi^2/x_c^2$$

$g_r{}^2$ = 0.67/0.30 and g_r = 1.49. Therefore D for the two layers is

$$D = \frac{[(1.49)(1/12)+(1)(1/12)]^2}{(1.49/12)^2+(1/12)^2}$$

$$= 1.92.$$

The measured time constant of the composite sample should be 1.92 times the sum of the two constituent time constants. Therefore, from Appendix F

$$t_{expected} = (1.92)(0.67+0.30)$$

$$= 1.86 \text{ hr.}$$

This is 16% over the observed value of 1.6 hours, indicating that the analysis is in reasonable agreement with the observed value.

In conclusion, the time constant of a composite material is:

$$t_c = (a_k/\pi^2)\left[\sum_{n=1}^{N} g_n x_n\right]^2 \text{ where}$$

$$g_n = (a_n/a_k)^{1/2} \text{ and}$$

a_k is the reciprocal of diffusivity for a chosen material. This is, of course, the numerator of D.