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Gordon F.N. Cox and Wilford F. Weeks

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Equations are developed that can be used to determine the amount of gas present in sea ice from measurements of the bulk ice density, salinity and temperature in the temperature range of -2 to -30°C. Conversely these relationships can be used to give the density of sea ice as a function of its temperature and salinity, considering both the presence of gas and of solid salts in the ice. Equations are also given that allow the calculation of the gas and brine volumes in the ice at temperatures other than that at which the bulk density was determined.		

PREFACE

This report was prepared by Dr. Gordon F.N. Cox, Geophysicist, and Dr. Wilford F. Weeks, Geologist, of the Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. The study was sponsored by Shell Development Company, and the Minerals Management Service of the U.S. Department of the Interior with support from Amoco Production Company, Arco Oil and Gas Company, Chevron Oil Field Research Company, Exxon Production Research Company, Gulf Research and Development Company, Mitsui Engineering and Shipbuilding Company, the National Science Foundation, Sohio Petroleum Company, Texaco, the U.S. Department of Energy, and the U.S. Coast Guard.

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EQUATIONS FOR DETERMINING THE GAS AND BRINE VOLUMES IN SEA ICE SAMPLES

Gordon F.N. Cox and Wilford F. Weeks

INTRODUCTION

A knowledge of the total porosity of sea ice is important in interpreting its physical properties. It has been shown, both theoretically and experimentally, that the mechanical, thermal, and electrical properties of sea ice vary with the relative volume of brine existing in the ice (Weeks and Assur 1967, Schwarz and Weeks 1977). However, the air or gas volume in the ice is also important. This is particularly true for low salinity sea ice, such as multiyear ice, where the gas volume may make up a major portion of its total porosity.

The gas volume of sea ice can be calculated given the density, salinity, and temperature of the ice. Generally, Assur's (1958) phase equilibrium table (Table III, p. 124) is used to perform such calculations. This table gives the relative masses of the different components in "standard sea ice" at temperature intervals of two degrees Celsius. These data are primarily based on the experimental determinations of Nelson and Thompson (1954) and Ringer (1928). Here the expression "standard sea ice" is used to refer to sea ice of such a composition that the relative concentrations of ions in its meltwater are the same as in normal sea water. From the density of the various components, the theoretical, or gas-free density of the sea ice can then be calculated and compared to the measured density to determine the actual gas volume present in the ice. However, since the table is based on standard sea ice having a salinity of 34.325 ‰, it is first necessary to reduce the data to the appropriate ice salinity. This is done by multiplying the masses of all the components, except pure ice, by $S_i/34.325$ where S_i is the ice salinity in

parts per thousand. The mass of pure ice is then 1000 g minus the total mass of brine and precipitated salts. Even though these calculations are straightforward, they are both time-consuming and prone to arithmetic errors. In fact they are rarely performed and estimates of the amount of gas present in sea ice and of the total void volume of the ice are rarely obtained.

In the present report, equations are derived from which the gas volume and brine volume can be directly calculated given the ice salinity, temperature, and density. The equations cover the temperature range from -2°C to -30°C and consider the presence of solid salts. Equations are also given to calculate sea ice density and the variation of gas and brine volume of a sea ice sample, if the temperature of the sample is changed from the temperature at which the ice density was originally obtained.

PREVIOUS WORK

Only a few investigators have advanced methods for calculating the density and gas volume in sea ice. Zubov (1945) calculated the density of gas-free sea ice and provided a table of values that could be used at temperatures of -23°C and higher. However, his work was done prior to Nelson and Thompson's (1954) experiments on the chemistry of sea ice brine. Zubov also did not take into account the presence of solid salts. Anderson (1960) published a table giving the density of gas-free sea ice at 14 selected temperatures in the temperature range between 0 and -50°C . This work was based on Assur's (1958) phase relations. As the exact method of calculation is not given, it is not clear whether Anderson considered the presence of

solid salts in the ice. The only relation available that calculates the gas volume of sea ice directly was developed by Schwerdtfeger (1963). However, it is only valid at temperatures warmer than -8.2°C , the $\text{Na}_2\text{SO}_4 \cdot 10 \text{H}_2\text{O}$ crystallization temperature. He also neglected the presence of solid salts and assumed that the volume of brine was equal to that of pure water. For warm sea ice these are reasonable approximations. At the conclusion of this report comparisons are made between the results of these previous authors and the present results.

It should also be noted here that Assur's (1958) brine volume table and the brine volume equations based on this table derived by Frankenstein and Garner (1967) assume a constant ice density of 0.926 Mg/m^3 . Consequently, these reports should be used with caution in gas volume calculations. The brine volume obtained from these sources should be multiplied by $\rho/0.926$, where ρ is the gas-free theoretical density, or better still, the measured ice density in Mg/m^3 at the temperature of interest, to determine the brine volume for a given sea ice density.

DERIVATION OF EQUATIONS

A sample of sea ice is composed of pure ice, brine, solid salts, and gas. In determining the gas volume of sea ice we are interested in the relative masses and volumes of the different components at a given ice salinity, temperature, and bulk density.

In the derivations to follow, m_{ℓ} , ρ_{ℓ} and V_{ℓ} are the mass, density, and volume of component ℓ where the subscripts a , b , i , s , and ss denote the component air, brine, pure ice, salt, and solid salts, respectively. M is the bulk mass, V the bulk volume, and ρ the bulk density. These should not be confused with the gas-free theoretical mass, volume, and density. The terms m_s^b and m_s^{ss} denote the mass of salt in the brine and mass of salt in the solid salts, respectively. The mass of gas in the ice is assumed to be negligible.

Brine volume

The salinity of the ice S_i is defined as

$$S_i = \frac{m_s}{M} = \frac{m_s^b + m_s^{ss}}{m_b + m_{ss} + m_i} \quad (1)$$

The salinity of the brine S_b is defined as

$$S_b = \frac{m_s^b}{m_b} \quad (2)$$

From these two relations an equation for the brine volume of sea ice can be derived. It should be

noted that the brine salinity and the relative amounts of salt in the brine and solid salts are unique functions of ice temperature via phase relations.

From eq 1

$$MS_i = m_s^{ss} + m_s^b$$

and

$$m_s^b = MS_i - m_s^{ss}$$

or

$$m_s^b = MS_i - km_s^b$$

where

$$k = \frac{m_s^{ss}}{m_s^b} \quad (3)$$

Solving for m_s^b from eq 3 and noting from eq 2 that

$$m_s^b = \rho_b V_b S_b$$

we obtain

$$V_b = \frac{\rho S_i}{\rho_b S_b} \left(\frac{1}{1+k} \right)$$

which because $M = \rho V$ can also be written as

$$\frac{V_b}{V} = \frac{\rho S_i}{\rho_b S_b} \left(\frac{1}{1+k} \right) \quad (4)$$

where V_b/V is the relative brine volume. If we neglect the salt present as solid salt, $k = 0$, eq 4 reduces to

$$\frac{V_b}{V} = \frac{\rho S_i}{\rho_b S_b}$$

a relation utilized earlier by Weeks and Lofgren (1967).

Because the brine density can be approximated (Zubov 1945, and Cox and Weeks 1975 by

$$\rho_b (\text{Mg/m}^3) = 1 + 0.0008 S_b (\text{‰})$$

and because S_b and k are unique functions of temperature, the relative brine volume can be expressed as

$$\frac{V_b}{V} = \frac{\rho S_i}{F_1(T)} \quad (5)$$

where S_i is in ‰, ρ is in Mg/m^3 , and

Table 1. Values for S_b , k , C , $F_1(T)$, and $F_2(T)$ at different temperatures.

T (°C)	S_b (‰)	$k \times 10^3$	$C \times 10^3$	$F_1(T)$ (Mg/m ³)	$F_2(T)$
- 2	37.6	0	0	38.731	0.123
- 4	70.6	0.554	0.148	74.662	0.151
- 6	99.8	1.050	0.387	107.876	0.177
- 8	126.5	1.400	0.660	139.441	0.199
-10	142.8	55.277	18.256	167.865	0.222
-12	157.6	84.141	30.493	192.378	0.240
-14	171.5	97.627	38.421	214.143	0.256
-16	184.4	106.330	44.952	234.033	0.271
-18	197.0	112.570	50.808	253.588	0.284
-20	209.9	118.078	56.851	274.074	0.298
-22	222.6	123.090	63.015	294.496	0.312
-24	230.5	509.787	217.168	412.236	0.394
-26	232.7	1312.694	537.697	638.433	0.556
-28	234.1	2065.827	842.341	852.171	0.708
-30	235.6	2685.708	1098.887	1032.102	0.836

$$F_1(T) = \rho_b S_b (1+k). \quad (6)$$

Values for $F_1(T)$ are given in Table 1 and plotted against temperature in Figure 1. S_b and k are determined from Assur's (1958) phase equilibrium table. The lines through the data in Figure 1 are least-squares curves of the form

$$F_1(T) = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3 \quad (7)$$

where T is the ice temperature (°C). Coefficients for the curves are given in Table 2.

Solid salt volume

Also, via the phase relations, at any given temperature the mass of solid salts (if solid salts are present) is proportional to the mass of brine; that is

$$m_{ss} = C m_b \quad (8)$$

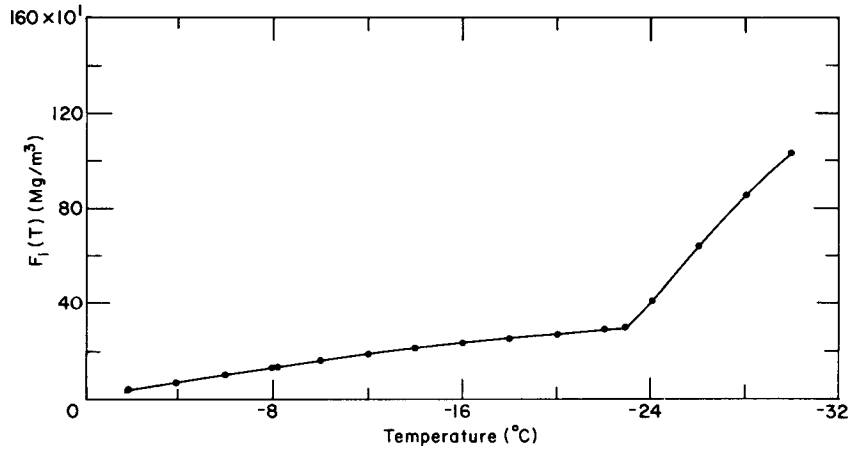


Figure 1. Function $F_1(T)$ versus temperature. Curve determined by method of least-squares.

Table 2. Coefficients for functions $F_1(T)$ and $F_2(T)$ determined by method of least-squares.

T(°C)	α_0	α_1	α_2	α_3	Correlation coefficient
$F_1(T)$					
-2>T>-22.9	-4.732	-2.245×10^1	-6.397×10^{-1}	-1.074×10^{-2}	0.9999
-22.9>T>-30	9.899×10^3	1.309×10^3	5.527×10^1	7.160×10^{-1}	0.9999
$F_2(T)$					
-2>T>-22.9	8.903×10^{-2}	-1.763×10^{-2}	-5.330×10^{-4}	-8.801×10^{-6}	0.9999
-22.9>T>-30	8.547	1.089	4.518×10^{-2}	5.819×10^{-4}	0.9999

where C is a function of T . The relative volume of solid salts, V_{ss}/V , is then equal to

$$\frac{V_{ss}}{V} = C \frac{\rho_b}{\rho_{ss}} \frac{V_b}{V} \quad (9)$$

where ρ_{ss} is the average solid salt density assumed to be constant at 1.5 Mg/m^3 .

Pure ice volume

The mass of pure ice is equal to

$$m_i = M - m_b - m_{ss}$$

or from eq 8

$$m_i = M - (1+C) m_b .$$

Since

$$m_i = \rho_i V_i$$

$$M = \rho V$$

$$m_b = \rho_b V_b$$

we have

$$\rho_i V_i = \rho V - (1+C) \rho_b V_b .$$

Solving for the relative pure ice volume, V_i/V , we obtain:

$$\frac{V_i}{V} = \frac{\rho}{\rho_i} - (1+C) \frac{\rho_b}{\rho_i} \frac{V_b}{V} . \quad (10)$$

Air volume

The relative air volume, V_a/V , is equal to

$$\frac{V_a}{V} = 1 - \frac{V_b}{V} - \frac{V_i}{V} - \frac{V_{ss}}{V} \quad (11)$$

Substituting eq 9 and 10 into 11, we obtain

$$\frac{V_a}{V} = 1 - \frac{\rho}{\rho_i} + \frac{V_b}{V} \left[(1+C) \frac{\rho_b}{\rho_i} - C \frac{\rho_b}{\rho_{ss}} - 1 \right] . \quad (12)$$

Equation 12 can be simplified by defining

$$F_2(T) = \left[(1+C) \frac{\rho_b}{\rho_i} - \frac{C\rho_b}{\rho_{ss}} - 1 \right] , \quad (13)$$

substituting for (V_b/V) from eq 4, and recalling the definition of $F_1(T)$ given in eq 6. The resulting relation is

$$\frac{V_a}{V} = 1 - \frac{\rho}{\rho_i} + \frac{\rho S_i}{F_1(T)} F_2(T) . \quad (14)$$

Values for $F_2(T)$ are given in Table 1 and plotted against temperature in Figure 2. C is determined from Assur's (1958) phase equilibrium table. The lines through the data in Figure 2 are also least-squares of the same form as eq 7.

Coefficients for these curves are given in Table 2. In determining the value of $F_2(T)$ at a given temperature, the pure ice density was calculated from (Pounder 1965):

$$\rho_i (\text{Mg/m}^3) = 0.917 - 1.403 \times 10^{-4} T(^{\circ}\text{C}) .$$

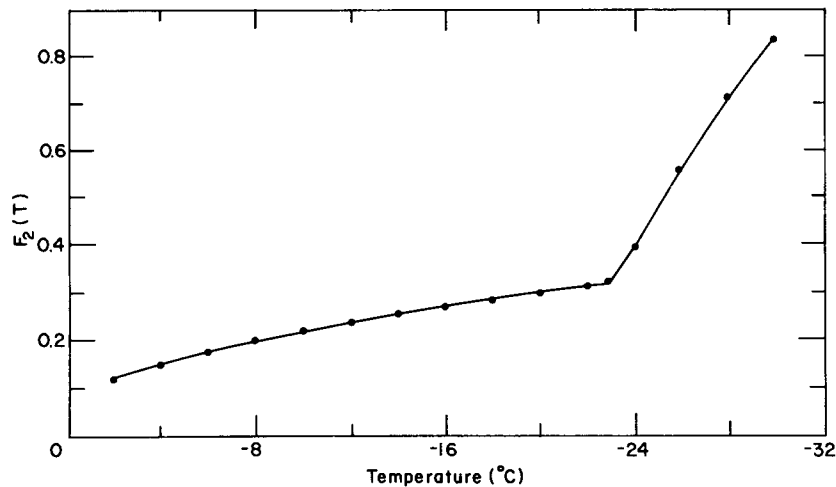


Figure 2. Function $F_2(T)$ vs temperature. Curve determined by method of least-squares.

Sea ice density

The sea ice density can be found from eq 14 by solving for ρ :

$$\rho = \left(1 - \frac{V_a}{V}\right) \frac{\rho_i F_1(T)}{F_1(T) - \rho_i S_i F_2(T)}. \quad (15)$$

Unlike the calculated densities given in Schwerdtfeger (1963) and Anderson (1960), this equation considers the presence of both air and solid salts in the ice. A comparison between the different density estimates is given later in this report.

Gas volume at another temperature

The gas volume and brine volume of sea ice are determined by measuring the volume and mass of the ice sample at a given temperature and the salinity of the melted sample. If tests are conducted at a temperature different than the temperature at which the bulk density was determined, the air and brine volumes must be recalculated.

Equations are now derived to calculate the air and brine volumes at a temperature that differs from the temperature at which the ice density was determined. Primed variables denote the initial value of the different variables.

It is initially assumed that the brine and air pockets are interconnected. From eq 5 we know that

$$V_b = \frac{M S_i}{F_1(T)}.$$

Because the mass and bulk salinity of the ice do not change with temperature

$$V_b = V'_b \frac{F'_1(T)}{F_1(T)}. \quad (16)$$

By rearranging eq 14, substituting for ρ based on eq 5 and defining

$$F_3(S_i, T) = \frac{\rho_i S_i}{F_1(T) - \rho_i S_i F_2(T)} \quad (17)$$

it can be shown that

$$V_b = (V - V_a) F_3(S_i, T). \quad (18)$$

Also dividing both sides of eq 16 by V and noting that

$$V = \frac{\rho' V'}{\rho}$$

we obtain for the new relative brine volume

$$\frac{V_b}{V} = \frac{\rho}{\rho'} \frac{V'_b}{V'} \frac{F'_1(T)}{F_1(T)}. \quad (19)$$

From eq 18 we can also conclude that

$$\frac{V_b}{V'} = \frac{(V - V_a) F_3(S_i, T)}{(V' - V'_a) F'_3(S_i, T)}. \quad (20)$$

Substituting eq 19 into eq 20 and solving for V_a/V we obtain for the new relative air volume

$$\frac{V_a}{V} = 1 - \left(1 - \frac{V'_a}{V'}\right) \left(\frac{\rho}{\rho'}\right) \left(\frac{F'_3(S_i, T) F'_1(T)}{F_3(S_i, T) F_1(T)}\right). \quad (21)$$

If the volume of the sample is not known at the new temperature, one can assume that the change in the external dimensions of the sample is due to the volumetric change in the ice matrix; that is

$$\frac{\rho}{\rho'} \approx \frac{\rho_i}{\rho'_i}.$$

In the temperature range where many investigations are carried out

$$\frac{\rho_i}{\rho'_i} \approx 1.$$

If the brine and gas pockets are not connected, a change in gas volume may also take place if the ice is warmed, that is, as less dense ice is melted to dilute the brine. In this case the new relative air volume is equal to

$$\frac{V_a}{V} = \frac{V'_a}{V} + 1 - \frac{\rho}{\rho'} \frac{F'_3(S_i, T) F'_1(T)}{F_3(S_i, T) F_1(T)} \quad (22)$$

and in many applications

$$\frac{\rho}{\rho'} \approx \frac{\rho_i}{\rho'_i} \approx 1.$$

If the brine and air pockets are not connected and the ice is cooled, the relative air volume will not change. However, due to phase changes, some brine will be lost from the sample as a result of brine expulsion. The new relative brine volume can be calculated from eq 19.

DISCUSSION

As methods for calculating the gas content of sea ice invariably contain a calculation of the density of gas-free sea ice, it is interesting to compare the density estimates developed in this report with previously

Table 3. The calculated densities (Mg/m^3) of gas-free sea ice at different temperatures and salinities as given by different authors.

	1	3	5	10	20 ‰
$T = -2^\circ\text{C}$					
Zubov (1945)	0.922	0.926	0.930	0.939	
Anderson (1960)	0.9198	0.9252	0.9307	0.9447	0.9739
Schwerdtfeger (1963)	0.9191	0.9233	0.9275	0.9379	0.9588
This report	0.9200	0.9254	0.9308	0.9448	0.9740
$T = -8^\circ\text{C}$					
Zubov (1945)	0.920	0.923	0.925	0.932	0.944
Anderson (1960)	0.9192	0.9216	0.9240	0.9301	0.9426
Schwerdtfeger (1963)	0.9175	0.9186	0.9196	0.9222	0.9275
This report	0.9193	0.9218	0.9242	0.9303	0.9429
$T = -10^\circ\text{C}$					
Zubov (1945)	0.920	0.922	0.925	0.929	
Anderson (1960)	0.9194	0.9216	0.9239	0.9296	0.9412
This report	0.9195	0.9218	0.9240	0.9297	0.9413
$T = -30^\circ\text{C}$					
Anderson (1960)	0.9216	0.9230	0.9245	0.9281	0.9354
This report	0.9219	0.9233	0.9247	0.9281	0.9352

published estimates. Table 3 gives such a comparison. As can be seen, the agreement between the present results and those of Anderson (1960) is very good (the present values are at most $+0.0003 \text{ Mg/m}^3$ higher with a modal difference of only $+0.0001 \text{ Mg/m}^3$). This agreement is, however, not surprising in that both Anderson's and our calculations are based on Assur's phase equilibrium tables. The agreement essentially verifies both sets of calculations. When comparisons are made with Schwerdtfeger's (1963) results for sea ice warmer than -8.2°C , much larger differences are found. Within this temperature range these differences vary from $+0.0009$ at warm temperatures (-2°C) and low salinities (1 ‰) to $+0.0154 \text{ Mg/m}^3$ at -8°C and 20 ‰. The agreement with the earlier values of Zubov (1945) is better than with Schwerdtfeger, with the largest difference being $+0.0058 \text{ Mg/m}^3$.

It is also interesting to use the relations that have been developed to examine the importance of considering the presence of solid salts in making gas volume calculations. For sea ice warmer than -8.2°C , Schwerdtfeger (1963) calculated the relative air volume from

$$\frac{V_a}{V} = 1 - \frac{\rho}{1000} \left(\frac{1000 - S_i}{0.917} + \frac{4.98 S_i}{T} \right) \quad (23)$$

where ρ is in Mg/m^3 , S_i in ‰, and T in $^\circ\text{C}$. As mentioned earlier he neglected the presence of solid salts and assumes that the volume of brine in the ice is equal to the volume of pure water. If we neglect the presence of solid salts in our equations, eq 12 simplifies to

$$\frac{V_a}{V} = \left(1 - \frac{\rho}{\rho_i} \right) + \frac{\rho S_i}{S_b} \left(\frac{1}{\rho_i} - \frac{1}{\rho_b} \right) \quad (24)$$

The results from eq 23 and 24 are compared to the results from eq 12 in Table 4. The air volume of sea ice of different assumed salinities, densities, and temperatures are calculated using these three equations. Schwerdtfeger's results are given in column 1, our results neglecting solid salts in column 2, and our results considering solid salts in column 3.

All the calculated values at a given salinity, density, and temperature are in close agreement. This indicates that, even for sea ice below the $\text{NaCl} \cdot 2\text{H}_2\text{O}$ eutectic temperature, neglecting the presence of solid

Table 4. Air volume (‰) calculations from three methods for sea ice having difference densities, salinities and temperatures (see text for explanation).

T (°C)	$\rho(\text{Mg/m}^3)$								
	0.890			0.910			0.930		
	1	2	3	1	2	3	1	2	3
$S_i = 1 \text{ ‰}$									
- 6	31.1	31.8	31.8	9.4	10.0	10.0	-	-	-
-10	-	32.0	32.1	-	10.2	10.4	-	-	-
-20	-	33.4	33.4	-	11.7	11.7	-	-	-
-30	-	34.8	34.6	-	13.1	12.9	-	-	-
$S_i = 10 \text{ ‰}$									
- 6	46.5	44.9	44.8	25.1	23.5	23.3	3.7	2.0	1.9
-10	-	42.7	42.7	-	21.2	21.2	-	-	-
-20	-	42.2	42.1	-	20.7	20.6	-	-	-
-30	-	43.1	41.0	-	21.6	19.4	-	-	-

salts in air volume calculations is a reasonable assumption. For warm, high density, high salinity sea ice (0.930 Mg/m³), Schwerdtfeger's value differs by 1.8 ‰, or nearly 100%. However, the brine volume of this ice is relatively much greater (85.1 ‰) and there is little difference in the total ice porosity.

CONCLUSIONS

Equations have been derived to quickly calculate the air volume of sea ice considering the presence and absence of solid salts. The results show that neglecting the presence of solid salts is a reasonable assumption. However, in this era of computers and sophisticated hand calculators, we recommend considering the presence of solid salts and the use of eq 12 to calculate the air volume of sea ice. A program for making such calculations on a Hewlett-Packard 41C calculator is given in Appendix A.

Earlier we mentioned that very few direct determinations of the gas content of sea ice have been undertaken. This might lead the reader to question the usefulness of the calculations outlined here because of a lack of available comparisons between calculated gas porosities and measured gas porosities. Fortunately such a comparison has now been completed for first-year sea ice (Nakawo 1983) indicating a very good agreement between the calculated and measured gas porosities. Inasmuch as the calculation procedure we have developed is very rapid and the measurements required to utilize it are simple, it is

hoped that gas volume will, in the future, join brine volume and specimen temperature as part of the sample description for all physical property tests on sea ice.

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APPENDIX A. HP-41C AIR VOLUME PROGRAM

The following HP-41C program was written to calculate the gas volume (VA), brine volume (VB), and total porosity (N) of sea ice samples. The program first prompts the user for the ice density (g/cm^3), salinity (‰), and temperature ($^{\circ}\text{C}$) at which the ice density was determined. The gas volume, brine volume, and total porosity are then calculated for this temperature. Next the program prompts the user for the test temperature and calculates the test gas volume, brine volume, total porosity, and new density. It is assumed that the brine and air pockets are interconnected.

To initialize the program, the contents of storage registers 32 to 50 are entered. The program is then executed and after each real time input (PROMPT), RUN is entered. During execution, storage registers 1 through 15 are used. A sample output is also given below.

MAIN PROGRAM

PRP "AIRVOL"	38 *	80+LBL 03
	39 RCL 04	81 XEQ 13
01+LBL "AIRVOL"	40 /	82 STO 09
02 "DEN G/CC?"	41 +	83 XEQ 14
03 PROMPT	42 STO 07	84 STO 10
04 STO 01	43 RCL 50	
05 "SAL PPT ?"	44 *	85+LBL 04
06 PROMPT	45 FIX 1	86 RCL 06
07 STO 02	46 "VA PPT = "	87 RCL 02
08 "TEMP C ?"	47 ARCL X	88 *
09 PROMPT	48 AVIEW	89 RCL 04
10 STO 03	49 RCL 01	90 ENTER↑
11 -22.9	50 RCL 02	91 RCL 06
12 X<>Y	51 *	92 RCL 02
13 X<=Y?	52 RCL 04	93 *
14 GTO 01	53 /	94 RCL 05
15 XEQ 11	54 STO 08	95 *
16 STO 04	55 RCL 50	96 -
17 XEQ 12	56 *	97 /
18 STO 05	57 "VB PPT = "	98 STO 11
19 GTO 02	58 ARCL X	99 XEQ 15
	59 AVIEW	100 STO 12
20+LBL 01	60 RCL 07	101 RCL 02
21 XEQ 13	61 RCL 08	102 *
22 STO 04	62 +	103 RCL 09
23 XEQ 14	63 RCL 50	104 ENTER↑
24 STO 05	64 *	105 RCL 12
	65 "N PPT = "	106 RCL 02
25+LBL 02	66 ARCL X	107 *
26 XEQ 15	67 AVIEW	108 RCL 10
27 STO 06	68 "TEST TEMP C ?"	109 *
28 1	69 PROMPT	110 -
29 ENTER↑	70 STO 03	111 /
30 RCL 01	71 -22.9	112 STO 13
31 RCL 06	72 X<>Y	113 1
32 /	73 X<=Y?	114 ENTER↑
33 -	74 GTO 03	115 1
34 RCL 01	75 XEQ 11	116 ENTER↑
35 RCL 02	76 STO 09	117 RCL 07
36 *	77 XEQ 12	118 -
37 RCL 05	78 STO 10	119 RCL 11
	79 GTO 04	120 *

121 RCL 04	166 AVIEW	209 RCL 41
122 *	167 GTO "AIRVOL"	210 RCL 03
123 RCL 13	168*LBL 11	211 *
124 /	169 RCL 32	212 +
125 RCL 09	170 ENTER↑	213 RCL 03
126 /	171 RCL 03	214 X↑2
127 RCL 12	172 RCL 33	215 RCL 42
128 *	173 *	216 *
129 RCL 06	174 +	217 +
130 /	175 RCL 03	218 RCL 03
131 -	176 X↑2	219 3
132 STO 14	177 RCL 34	220 Y↑X
133 RCL 50	178 *	221 RCL 43
134 *	179 +	222 *
135 *TEST VA PPT = "	180 RCL 03	223 +
136 ARCL X	181 3	224 RTN
137 AVIEW	182 Y↑X	
138 1	183 RCL 35	225*LBL 14
139 ENTER↑	184 *	226 RCL 44
140 RCL 14	185 +	227 ENTER↑
141 -	186 RTN	228 RCL 45
142 RCL 13		229 RCL 03
143 *	187*LBL 12	230 *
144 STO 15	188 RCL 36	231 +
145 RCL 50	189 ENTER↑	232 RCL 03
146 *	190 RCL 37	233 X↑2
147 *TEST VB PPT = "	191 RCL 03	234 RCL 46
148 ARCL X	192 *	235 *
149 AVIEW	193 +	236 +
150 RCL 14	194 RCL 03	237 RCL 03
151 RCL 15	195 X↑2	238 3
152 +	196 RCL 38	239 Y↑X
153 RCL 50	197 *	240 RCL 47
154 *	198 +	241 *
155 *TEST M PPT = "	199 RCL 03	242 +
156 ARCL X	200 3	243 RTN
157 AVIEW	201 Y↑X	
158 RCL 15	202 RCL 39	244*LBL 15
159 RCL 09	203 *	245 RCL 48
160 *	204 +	246 ENTER↑
161 RCL 02	205 RTN	247 RCL 49
162 /		248 RCL 03
163 FIX 3	206*LBL 13	249 *
164 *TEST DEN G/CC ="	207 RCL 40	250 +
165 ARCL X	208 ENTER↑	251 RTN
		252 END

INITIALIZATION OF THE PROGRAM

032.050 ENTER†
PRREGX

R32= -4.732+00
R33= -2.245+01
R34= -6.397-01
R35= -1.074-02
R36= 8.903-02
R37= -1.763-02
R38= -5.330-04
R39= -8.801-06
R40= 9.899+03
R41= 1.309+03
R42= 5.527+01
R43= 7.160-01
R44= 8.547+00
R45= 1.089+00
R46= 4.518-02
R47= 5.819-04
R48= 9.170-01
R49= -1.403-04
R50= 1.000+03

SAMPLE RUN

XEQ "AIRVOL"
DEN G/CC? .910 RUN
SAL PPT ? 4.500 RUN
TEMP C ? -15.000 RUN
VA PPT = 14.7
VB PPT = 18.3
N PPT = 33.0
TEST TEMP C ? -5.0 RUN
TEST VA PPT = 17.2
TEST VB PPT = 44.0
TEST N PPT = 61.2
TEST DEN G/CC =0.909
DEN G/CC?