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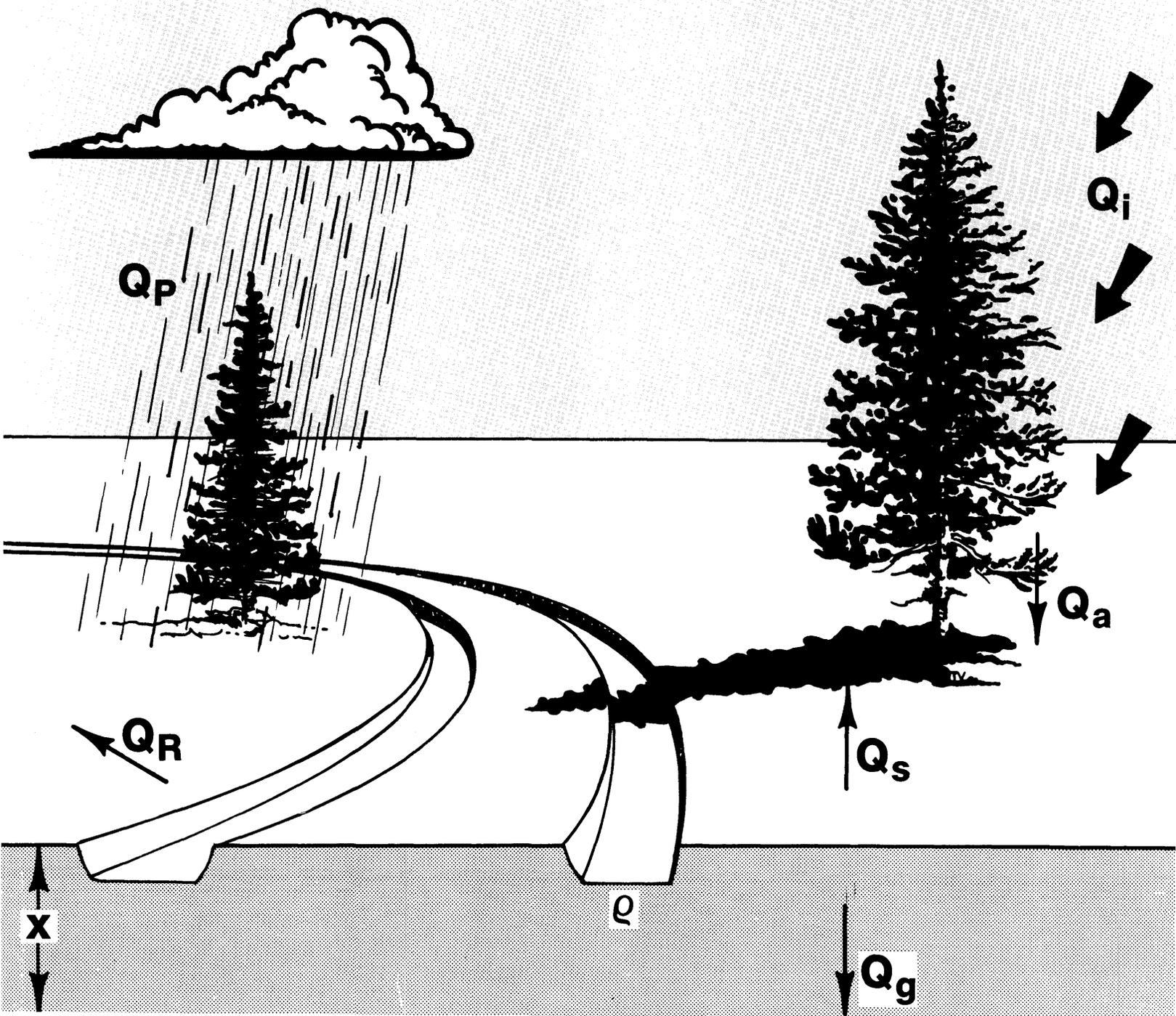
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Developing a model for predicting snowpack parameters affecting vehicle mobility





CRREL Report 83-16

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Roger H. Berger

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The presence of snow on the ground can impose limitations on the mobility of wheeled and tracked vehicles. Snow depth and density are the two most easily measured snow properties that can be related to mobility over snow. Existing models of snowpack accumulation and ablation processes and models of internal snowpack structure were examined to determine if a model of the snowpack can be developed for use in predicting the snow parameters that affect mobility. Simple models, such as temperature index models, do not provide sufficient snowpack details, and the more detailed models require too many measured inputs. Components of the various models were selected from a basis of a snowpack model for predicting snow properties related to mobility over snow. Methods of obtaining the input data for some components are suggested, and areas where more development is needed are described. | | |

PREFACE

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NOMENCLATURE

| | |
|--------------------|--|
| a, b | Empirically determined wind constants |
| C | Specific heat or a constant |
| D | Diffusion coefficient |
| E | Precipitation efficiency |
| e | Vapor pressure |
| g | Acceleration due to gravity |
| H | Hour angle |
| I | Short-wave radiation |
| K | Conductivity |
| L | Latent heat |
| P | Atmospheric pressure |
| p | Probability |
| Q | Energy |
| q | Specific humidity |
| R | Precipitation |
| s | Snowfall depth |
| T | Air temperature |
| t | Time |
| U | Wind speed |
| V | Water vapor transfer |
| W | Liquid water content |
| w | Vertical velocity |
| x | Snow depth |
| z | Height |
| $C(t), K(t), m(t)$ | Weibull distribution parameters |
| $f(U)$ | Surface wind velocity function |
| S, A, L | Slope, altitude and latitude constants, respectively |
| α | Empirically determined constant |
| γ | Psychrometric constant |
| δ | Declination |
| ϵ | Emissivity |
| κ | Conductivity |
| ρ | Density, reflectivity |
| σ | Stefan-Boltzmann constant |
| ω | Albedo |

Subscripts

| | |
|-----|------------------------|
| a | Air, ambient |
| c | Cloud cover |
| d | Empirically determined |
| e | Latent heat, effective |
| f | Freezing |
| g | Soil |
| h | Sensible |
| i | Incident, ice |
| m | Mass |
| n | Net |
| o | Snow surface |

| | |
|----------|---------------------------|
| <i>p</i> | Precipitation |
| <i>r</i> | Reflected, sunrise |
| <i>s</i> | Snow, sublimation, sunset |
| <i>u</i> | Combined effective |
| <i>v</i> | Vegetative cover |
| <i>w</i> | Water |

CONVERSION FACTORS: U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

These conversion factors include all the significant digits given in the conversion tables in the *ASTM Metric Practice Guide* (E 380), which has been approved for use by the Department of Defense. Converted values should be rounded to have the same precision as the original (see E 380).

| <i>Multiply</i> | <i>By</i> | <i>To obtain</i> |
|-------------------|--|------------------|
| foot | 0.3048* | metre |
| mile | 1 609.344* | metre |
| mile per hour | 0.44704* | metre per second |
| degree Fahrenheit | $t_{\text{C}} = (t_{\text{F}} - 32)/1.8$ | degree Celsius |

*Exact.

DEVELOPING A MODEL FOR PREDICTING SNOWPACK PARAMETERS AFFECTING VEHICLE MOBILITY

Roger H. Berger

INTRODUCTION

The primary mission of any vehicle is to transport a payload from one location to another. To assess a vehicle's performance, we must determine the constraints imposed on the vehicle's mobility by the environment in which it operates. Because large areas in the northern hemisphere receive seasonal snowcover and because the snow often severely limits vehicle mobility, it is important to assess this problem. To do this, we must determine how specific snow conditions affect vehicle performance and the geographical and temporal distribution of these conditions. This report addresses part of the latter problem by describing a model that predicts the condition of the snowpack and the snowpack parameters that affect vehicle mobility. The feasibility of this model is then examined in light of the measurements that will probably be available.

For studying mobility the snowpack characteristics may be divided into extensive and intensive properties. The more important extensive snowpack properties are depth and spatial and temporal distribution; the intensive properties are density, grain size distribution, temperature and free water content. A complete snowpack model should predict both classes of properties. To develop a model for predicting these quantities, we must understand the environmental factors that influence the formation and dissipation of the snowpack. In general, the environmental effects with the most influence on the accumulation and the metamorphosis and ablation processes in the snowpack are different, so these two phases can be treated independently.

The accumulation of the snowpack depends initially on the snowfall distribution within a snowstorm and then on the redistribution by wind. Snowstorms are of three types: cyclonic, orographic and convective; each has its own pattern of snowfall.

Cyclonic storms usually extend over 10^5 - 10^7 km². The depth of the snowfall depends on the amount of moisture available, the strength of the low and the distance from the storm center. The distribution of the snowfall is governed mainly by the location of the storm track and the topography.

The distribution of orographic snowfall is governed by the wind speed and direction, the elevation, the slope orientation and the extent of shadowing by upstream barriers. Snowfall correlates well with altitude (Curry and Mann 1965, McPartland et al. 1971, Hendrick and DeAngelis 1976). The snow accumulation patterns on a scale of 10^2 - 10^5 km² can be predicted fairly well if the direction of the winds governing storm motion are known. The accumulation is sensitive to this wind direction and can vary greatly with a change in wind direction.

Convective snowstorms of 1 - 10^2 km² can produce marked variations in snowfall. These storms are generally responsible for the snowbelts associated with bodies of water; cold air passing

over warmer water produces snowfall on the lee side. This effect decreases with decreasing lake size because of the shorter water uptake path across small lakes and because smaller lakes freeze earlier.

The effect of large topographic barriers is well documented, using snowpack records from the windward and leeward sides of mountain ranges (Fiebiger 1976). The macroscale snowpack deficit in the lee of large barriers does not hold true on a microscale, where the opposite frequently occurs. On this scale, wind redeposits snow from the windward side of the crest of a topographical feature to the lee side, where it forms a drift or cornice. Vegetation tends to even out the deposition of snow over a crest by reducing or preventing wind transport of snow. The density, type and maturity of the vegetation determines to what extent the wind field is modified (Reifsnnyder 1955). The vegetative canopy decreases the ground snowcover by intercepting falling snow (Meiman 1968). Some of this intercepted snow evaporates or is redistributed under the vegetation and in adjacent clearings (Ffolliott et al. 1965, Gary 1975). As the vegetative cover increases, the effect of the wind decreases until it plays only a small role in the snow distribution.

In alpine areas above the forest edge and in open areas the distribution of snow is controlled by the wind and the microrelief of the terrain. Fiebiger (1976) found that the reduction of wind in forests produces a snow distribution nearly independent of the microrelief. The snow in forests is generally deeper and less dense than on open land, with the difference correlated with the site, the density and the type of trees.

On prairies and large open expanses the wind, the topography and the vegetation are the dominant factors controlling snow distribution. Snow accumulates in hollows and streambeds and in the lee of hills, ridges and clumps of trees or bushes. Grasses and crop stubble can also trap snow to the height of the vegetative cover (Willis 1979).

The snowpack metamorphosis and ablation processes depend on the net heat transfer between the pack and the environment. The processes may be divided into two classes: radiative and non-radiative heat transfer. Radiative heat transfer depends on the snow surface temperature and albedo, vegetative canopy, cloud cover and insolation (Anderson 1973, 1976, Leaf and Brink 1973). Non-radiative heat transfer is through conduction, convection, latent heat exchange and mass exchange. These processes are functions of the snow, air and ground temperatures, the dew point and the wind velocity (Leaf and Brink 1973, Anderson 1976).

REVIEW OF EXISTING MODELS

Most of the extensive body of literature on snowcover modeling has been developed for hydrological purposes. For the hydrologist the melting or ablation phase of the snowpack is of primary interest, and accumulation processes are only of secondary interest. The general hydrological approach is to develop a model of the snowpack based on energy considerations. This model is calibrated against the meteorological and hydrological records of a specific area or watershed to determine the effects of various unmeasured parameters. The model is then used to predict runoff.

The basic understanding of the relationships between the environment and the behavior of the snowpack that are used in most of the current models can be traced to the pioneering work described in *Snow Hydrology* (U.S. Army Corps of Engineers 1956). This publication describes studies of three western mountain watersheds and the application of the basic sciences to the hydrological cycle. In *Snow Hydrology* each component of the hydrological cycle was determined for the given environmental conditions. To do this, the mathematical relationships between the variables affecting each component were derived and constants of proportionality were determined. With these relationships the behavior of the snowpack at a point was determined; these point values were then used to determine the distribution and behavior of the snowpack with respect to environmental differences.

For hydrological purposes it is impractical and unnecessary to use the kind of point-by-point analysis used in *Snow Hydrology*. A more practical approach is 1) to determine averages for each major topographical and environmental subdivision within a basin and 2) to ignore individual events and deal with time averages. This concept leads to the use of indexes to represent basin averages. To use indexes properly, the physical character of the area and of each measurement point must be considered. Since the use of indexes involves sampling theory and measurement errors, statistical procedures may be used to establish the reliability of estimates and the weightings of individual factors. Indirect methods can then be used to evaluate basin parameters, using observational data commonly available; however, these methods must be used with care, and additional data must be gathered when necessary.

The conclusions reached in *Snow Hydrology* were corroborated by a similar Russian study, *Melting of Snow Cover* (Kuz'min 1961). While the two investigations overlap, the Russian study also includes methods that apply to plains regions, where the snowcover tends to be shallow and discontinuous rather than deep and continuous, as in most mountain regions.

The snowpack modeling that is the direct descendant of *Snow Hydrology* and *Melting of Snow Cover* can be divided into three types: 1) energy and mass balance models, 2) radiation balance and energy exchange models, and 3) temperature index models. These model types are all based on the same heat transfer method or description of the snowpack; the difference is in the detail with which the physical processes within the snowpack and the interactions of the environmental factors and the snowpack are treated. None of these models treat the accumulation process in detail.

The models that treat the accumulation process are more diverse in their underlying principles. They are 1) orographic precipitation models and 2) statistical snowfall models. The statistical models are based on the statistical treatment of either the meteorological processes governing snowfall or historical snowfall records.

Accumulation models

Orographic precipitation models

These models are based on the observation that precipitation increases with increasing elevation in mountainous regions. However, linear regressions between elevation and precipitation (Storr and Golding 1974, Curry and Mann 1965) yield relationships that apply only to the localities for which they were developed. This suggests that elevation is not the only terrain variable affecting precipitation in mountainous regions. Spreen (1947) found that 30% of the variation in precipitation could be accounted for by elevation, but by including terrain slope, exposure and orientation in addition to elevation, 88% of the variation could be explained. Terrain slope is the maximum change in elevation within a five-mile radius, exposure is a measure of the number of barriers within a 20-mile radius that have elevations exceeding that of the point of interest by more than 1000 ft, and orientation is the direction of greatest exposure.

In mountainous terrain the orographic precipitation process is dominant. This process can be expressed by the following formula (Rhea 1978a):

$$R = \frac{E}{\rho_w g} \int_0^t \int_{P_1}^{P_2} \frac{dq_s}{dz} wdPdt \quad (1)$$

where

- R = precipitation
- E = precipitation efficiency
- ρ_w = density of water
- g = acceleration due to gravity

$$\int_0^t dt = \text{time interval}$$

$$\int_{P_1}^{P_2} dP = \text{vertical extent of the saturated air mass producing the precipitation (expressed in terms of pressure).}$$

$$\frac{dq_s}{dz} = \text{vertical gradient of specific humidity}$$

w = vertical velocity of the air parcel.

In orographic precipitation the vertical motion in eq 1 is produced by the slope of the terrain. The upward motion of the moist air causes condensation, some of which reaches the ground as precipitation.

Orographic models couple this precipitation process with slope, exposure, elevation, upper air moisture and wind to predict the distribution of snow accumulation. Rhea (1978a) used a two-dimensional air flow model, in which the topography forces vertical displacements of the air but does not allow the air to deviate horizontally around topographic barriers. The model keeps track of the condensation or evaporation resulting from the vertical motions of the air at the mesh points of a square topographic grid. The model is capable of using a grid as small as 2.5 km over an area of 10^5 km².

Orographic models are designed to use current weather information to predict the water resources stored in the snowpack. They can yield realistic snow accumulations averaged over a season, using as input the atmospheric profiles from a few widely spaced (300–400 km) measurement points. The predictions are often inaccurate for particular storms because the distribution of precipitation is based on the mesoscale wind direction and doesn't allow for the horizontal wind direction changes due to local terrain features. These models also ignore cyclonic storms, which are not significant on a seasonal basis but can be very significant on a short-term basis.

For mobility studies, predictions for individual storms must be more accurate. This may be accomplished by scaling down the size of the region of prediction and by using a three-dimensional flow model with more detailed wind input.

Statistical snowfall models

The first of these models was developed by Sage (1976). Its object is to produce a synthetic hourly snowfall sequence that is statistically indistinguishable from a real snowfall. The model uses partitioning, in which each hour of a storm is designated as a period with or without snowfall. The sequence of snowfall periods is then modeled using a first-order regression equation. Similar equations are developed for sequences of fall periods and snowfall intensity. The three sequences are then combined to represent the hourly sequence of snowfall intensities.

The second kind of model mathematically describes the statistics of individual snowfall events and the meteorological factors affecting the snowfall. The increase in the snowpack due to an individual storm can be expressed as (Isyumov and Davenport 1974, Isyumov and Mikitiuk 1976):

$$\Delta R = g(s, \rho_s, T, U) \quad (2)$$

where

s = snowfall depth

ρ_s = snow density

T = air temperature

U = wind speed.

Since the meteorological variables are random, ΔR is also a random variable.

The probability that a snowfall event will occur depends on the probability that the associated meteorological factors occur. By examining the interdependence of the meteorological factors, the principal dependencies can be identified. The probability that the meteorological conditions exist to produce ΔR can be expressed as

$$p(s, \rho_s, T, U) \approx p(\rho_s/T)p(s/T)p(s/U)p(s) \quad (3)$$

where $p(\rho_s/T)$ is the probability density function for ρ_s , given the occurrence of T ; this expresses the fact that T is the primary meteorological variable on which ρ_s depends. These joint probabilities are determined empirically.

A snowfall of depth s is a random event. The snowfall process is not stationary but varies with time during the snow season. Isyumov and Davenport (1974) showed that the snowfall statistics on a 24-hour basis fit a Weibull distribution, and thus the snowfall probability may be expressed as

$$p(>s, t) = m(t) \exp\left\{- [s/C(t)]^{K(t)}\right\} \quad (4)$$

where

$$\begin{aligned} p(>s, t) &= \text{conditional probability that a snowfall of depth greater than } s \text{ occurred at time } t \\ m(t) &= \text{snowfall expectation at time } t \\ C(t), K(t) &= \text{Weibull distribution parameters.} \end{aligned}$$

The three time functions are obtained by fitting the snowfall distribution for the entire season to a Weibull distribution. These parameters are measures of the length of the snow season and the relative distribution of total snowfall $m(t)$, the probability of large snowfalls $K(t)$, and the mode of the snowfall magnitude distribution $C(t)$.

Expressions similar to eq 4 are developed in an analogous manner for the other meteorological factors and are combined in eq 3. This equation is used in a Monte Carlo calculation to simulate the day-by-day snowfall.

For mobility studies the short-range predictions of statistical snowfall models are not adequate, since they have no provision for including current meteorological data. The utility of these models may lie in Monte Carlo simulations for mobility modeling, rather than in actual snowpack predictions.

Ablation models

Since the temperature index, the radiation balance and energy exchange, and the energy and mass balance snowpack models are all based on the same physical description of the snowpack energy exchange processes, the most detailed will be described first, followed by a description of the simplifications used in the other models.

Energy and mass balance models

These models describe the ablation and metamorphosis of the snowpack in terms of two equations, the energy balance equation and the equation for heat transfer within the snowpack. The snowpack is divided into layers, and implicit finite difference equations are written for each layer. This set of equations is then solved by an iteration technique. Included in the models are the densification of the snow, the change in thickness of each layer, and the retention and transmission of water.

The energy balance equation is derived from the snowpack energy inputs and losses that occur at the air/snow and soil/snow interfaces. Under most conditions the energy exchange at the air/snow interface is much greater than at the soil/snow interface. The energy exchanges across the interfaces are either by the transfer of mass or by radiation. The mass transfer is in the form of

water in any of its three phases. Water is lost by evaporation and sublimation at the air/snow interface and liquid water outflow at the soil/snow interface. Water flows from the bottom of the snow cover when the snow is isothermal at 0°C and saturated with water, when the snow is referred to as “ripe.” The quantity of interest in these instances is the transfer of latent or sensible heat.

The net energy transfer by radiation is the difference between the incoming and outgoing radiation. Both visible and infrared wavelengths have to be considered because a significant amount of energy can be absorbed in the visible and reradiated in the infrared. The incoming radiation comes directly from the sun or is reflected and/or emitted by the atmosphere and terrestrial objects. The outgoing radiation is made up of reflected solar, reflected long-wave, and emitted radiation. For most modeling purposes the emissivity ϵ of snow in the infrared may be taken as one, and the infrared reflectivity and emissivity are related by $\epsilon = 1 - \rho$. The reflected long-wave component may be neglected. The net radiative transfer Q_n can be expressed as (Anderson 1976)

$$Q_n = Q_i - Q_r + (\epsilon \cdot Q_a) - (\Delta t \cdot \epsilon \cdot \sigma \cdot T_0^4) \quad (5)$$

where

Q_i = insolation

Q_r = reflected solar radiation

$\epsilon \cdot Q_a$ = incident atmospheric and terrestrial radiation

$\Delta t \cdot \epsilon \cdot \sigma \cdot T_0^4$ = radiation emitted by the snowpack with surface temperature T_0 during computation interval Δt

σ = Stefan-Boltzman constant.

Several environmental factors affect the net radiative energy transfer. The incident radiation depends on factors such as latitude, atmospheric clarity and cloud cover, topography and vegetation cover. The outgoing radiation is a function of the albedo and the surface temperature of the snowcover; both vary on a small scale, but the surface temperature is also influenced by larger scale variations.

The nonradiative heat transfer may be considered as four processes, the first three occurring at the air/snow boundary: 1) sensible heat transfer, 2) latent heat transfer, 3) transfer by mass changes, and 4) one general heat transfer process at the soil/snow boundary. The sensible and latent heat transfer depends on the condition of the atmospheric boundary layer, which is turbulent under most conditions. Within this boundary layer there is a thin (~ 1 mm) viscous sublayer where molecular conduction processes are important. Above this viscous layer is a layer (meters to tens of meters thick) where shear stress and vertical fluxes of heat and water vapor can be considered independent of height. It is this latter layer that is used in estimating latent and sensible heat exchange.

If the hypothesis is correct that the mean vertical fluxes of latent and sensible heat are proportional to their vertical gradients, then the water vapor transfer is predominantly a function of the wind speed and the vapor pressure gradient. The particular relationship depends upon the lapse rate and turbulence, as characterized by the Richardson number.

Since a full set of measurements at two heights above the snow surface is not practical in most cases, the expressions for the latent heat transfer Q_e and the sensible heat transfer Q_h are given in terms of the snow surface quantities and turbulent layer measurements:

$$Q_e = L_s \cdot \frac{\rho_w}{10} \cdot f(U) \cdot (e_a - e_0) \quad (6)$$

$$Q_h = L_s \cdot \frac{\rho_w}{10} \cdot \gamma \cdot f(U) \cdot (T_a - T_0) \quad (7)$$

where

$$\begin{aligned}
L_s &= \text{latent heat of sublimation} \\
\rho_w &= \text{density of water} \\
f(U) &= \text{wind speed function} \\
e_a, e_0 &= \text{vapor pressure in the air and at the snow surface, respectively} \\
\gamma &= \text{psychrometric constant} \\
T_a, T_0 &= \text{temperature of the air and the snow surface, respectively.}
\end{aligned}$$

The snowpack also gains and loses heat by mass changes resulting from precipitation and vapor transfer. If the heat content of the vapor is assumed to be negligible and the water outflow is assumed to be at 0°C , only the heat transferred by precipitation must be considered. The precipitation temperature is assumed to be the wet-bulb temperature, and the heat transfer Q_m is expressed as

$$Q_m = C_w \rho_w R (T_w - 273.16)/10 \quad (8)$$

where

$$\begin{aligned}
C_w &= \text{specific heat of water} \\
\rho_w &= \text{density of water} \\
R &= \text{precipitation (mm)} \\
T_w &= \text{wet-bulb temperature (K)}.
\end{aligned}$$

These four terms represent the energy exchange at the snow/air interface. An additional term Q_g is needed for the exchange at the snow/soil interface. The sum of these five terms is equal to the change in snowpack energy content:

$$\Delta Q = (x \cdot \rho_s)^t \left[(C_i \cdot T_s)^{t+\Delta t} - (C_i \cdot T_s)^t \right] + \frac{L_f \cdot \rho_w}{10} (W^{t+\Delta t} - W^t) \quad (9)$$

where

$$\begin{aligned}
x &= \text{snow depth} \\
C_i &= \text{specific heat of ice} \\
T_s &= \text{temperature of the ice grains inside the snowpack} \\
L_f &= \text{latent heat of fusion} \\
W &= \text{amount of liquid water in the snow, expressed as depth} \\
\Delta t &= \text{time since the last computation.}
\end{aligned}$$

The complete energy balance equation is:

$$Q_n + Q_e + Q_h + Q_m + Q_g - \Delta Q = 0 \quad (10)$$

where Q_g is the heat transfer at the surface of the ground. Combining eqs 5-9 gives

$$\begin{aligned}
&\left[Q_i - Q_r + (\epsilon \cdot Q_a) - (\Delta t \cdot \epsilon \cdot \sigma \cdot T_0^4) \right] + \left[L_s \cdot \frac{\rho_w}{10} \cdot f(U) \cdot (e_a - e_0) \right] \\
&+ \left[\frac{L_s \rho_w}{10} \cdot \gamma \cdot f(U) \cdot (T_a - T_0) \right] + \left[C_w \rho_w R (T_w - 273.16)/10 \right] \\
&+ Q_g - \left\{ (x \cdot \rho_s)^t \left[(C_i \cdot T_s)^{t+\Delta t} - (C_i \cdot T_s)^t \right] - \frac{L_f \cdot \rho_w}{10} (W^{t+\Delta t} - W^t) \right\} = 0 \quad (11)
\end{aligned}$$

This is the first equation on which the model is based; the second expresses the heat transfer within the snowpack. Heat is transferred within the snowpack by conduction, radiation, phase

change and vapor diffusion. The mathematical expressions for these mechanisms include coefficients that are related to the density, grain size and crystal structure of the snowpack. Since the latter two quantities are not generally measured, the density is the only parameter used to characterize the snowpack.

The heat transfer within the snowpack is essentially one-dimensional, with heat either gained or lost through the air/snow and soil/snow interfaces. The one-dimensional Fourier heat conduction equation is used, with modifications to include the different mechanisms of heat transfer:

$$C_i \rho_s \frac{\partial T}{\partial t} - \frac{\partial}{\partial z} \left(\kappa_e \frac{\partial T}{\partial z} \right) + L_f \frac{\partial W}{\partial t} - \frac{\partial I}{\partial z} - L_s \left[\frac{\partial C}{\partial T} \frac{\partial}{\partial z} \left(D_e \frac{\partial T}{\partial z} \right) + D_e \frac{\partial^2 C}{\partial T^2} \left(\frac{\partial T}{\partial z} \right)^2 \right] = 0 \quad (12)$$

where

- C_i = specific heat of ice
- C = concentration of water vapor
- D_e = effective diffusion coefficient
- I = short-wave radiation flux
- κ_e = effective thermal conductivity
- L_s = latent heat of sublimation
- L_f = latent heat of fusion
- T = snow temperature .

The terms in this equation account for conduction, heat sources and sinks, and diffusion.

The one-dimensional terms accounting for the heat conduction within the snowpack are $C_i \rho_s \partial T / \partial t - \partial(\kappa_e \partial T / \partial z) / \partial z$. The effective conductivity is the sum of the conduction by the ice in the snow fabric and by the air in the pores, and the radiant exchange across the pores. The second term also accounts for the fact that this effective conductivity is a function of both the position in the snowpack due to layering and the change in density with depth. Heat sources and sinks are accounted for by $L_f \partial W / \partial t - \partial I / \partial z$. Water within the snowpack acts as either a source or a sink of heat, depending upon whether it is decreasing due to freezing or increasing due to melting. These effects are accounted for by the first term; the second term represents the heat due to insolation. Since the short-wave solar radiation penetrates to significant depths before it is absorbed, it acts as an extended heat source in the upper layers of the pack.

Temperature gradients within the snow are the main driving forces behind the diffusion of water vapor in the snowpack. Associated with the diffusion process are sublimation and condensation, which are the source and sink for the water vapor, respectively. Heat transfer due to the transport of the vapor is negligible, but the heat absorbed and released in sublimation and condensation is significant. These processes decrease the internal temperature gradient. These processes and the metamorphic processes within the snowpack are accounted for in the terms $L_s [\partial C / \partial T \cdot \partial(D_e \partial T / \partial z) / \partial z + D_e \partial^2 C / \partial T^2 (\partial T / \partial z)^2]$.

The effective conductivity is a function of the distribution, size, shape and intergranular bonding of the ice crystals but it can be empirically correlated with density alone. The effective diffusion coefficient can be related to the snowpack temperature, and the absorption coefficient can be related to density if the local relationship between grain size and density is empirically established.

The energy and mass balance model is based on the combined solution of eqs 11 and 12. The information needed to solve the equations is air temperature, air vapor pressure, wind speed, incoming solar radiation, reflected solar radiation, incoming long-wave radiation, and soil temperature. This extensive set of data restricts the use of this model to a few well-instrumented watersheds.

Radiation balance and energy exchange models

These models use a less complete description of the energy exchange and heat transfer processes in the snowpack than that used in the energy and mass balance models. The models compute the energy exchange by one set of equations and the heat deficit of the snowpack by another equation. For example, Leaf and Brink (1973) use the air temperature and the incident solar radiation to estimate the net radiation balance, from which the energy exchange is computed. The one-dimensional Fourier heat-conduction equation is used to compute the temperature at the midpoint of the snowpack, assuming that the snow surface temperature equals the air temperature. The internal and surface temperatures are used to estimate the heat deficit in the snowpack. This simplified treatment reduces the input data requirements while producing a sufficiently accurate snowpack simulation.

Temperature index models

These models use the air temperature as the sole index of snowpack ablation. Maximum and minimum temperatures and degree-days are used most commonly. Anderson (1973) developed a heat-transfer equation that is a function of air temperature, so that the heat exchange at the snowpack surface can be related to the air alone. The simplifications and assumptions involved lead to a loss of precision, and knowledge of the internal metamorphic processes is lost. The big advantage of the procedure is that much less input data is required without losing much prediction accuracy.

Using existing models for studying mobility

The existing models of snowpack accumulation and ablation would have to be modified substantially before they could be used for mobility modeling or prediction. Of the accumulation models, only the orographic model appears to be adaptable for studying mobility. This could be accomplished by reducing the grid size and expanding the two-dimensional wind flow analysis to three dimensions. Unfortunately this model is restricted to areas of relatively large topographical relief, and atmospheric moisture and wind profiles are needed. The other two accumulation models may be of some utility in vehicle systems evaluation modeling but are not suitable for predicting actual snow conditions.

The best ablation model for studying mobility will be one incorporating a compromise between the amount of input data required and the need for snowpack detail. The most reasonable approach is to use a temperature index model and to incorporate as much snowpack detail as possible.

PROPOSED SNOWPACK MODEL FOR MOBILITY STUDIES

The existing snowpack models are capable of predicting the depth, density and temperature of the snowpack. However, they are limited because they cannot predict accumulation events, they can predict snowfall distribution only for limited areas, they require extensive data, and they cannot account for wind effects.

The models that can predict many of the snowpack parameters needed for studying mobility are only of limited use because they require a knowledge of air temperature, dew point, wind speed, incoming and reflected solar radiation, and atmospheric long-wave radiation. To extend the models from point predictors to areal predictors the slope, aspect, vegetative cover, snowpack albedo, and wind field must also be estimated.

Considering this set of input requirements and the errors involved in estimating the areal values and comparing the accuracy of predictions of the conceptual models with an index model, such as a temperature index model, the advantages of an index model are apparent for mobility purposes. In addition, when the probable input data set (which will be available under operational conditions) is considered, the index model becomes more attractive. The operational data set is

assumed to include the data from three sources: 1) a master data bank of geographical and terrain information, 2) reconnaissance of the snow and terrain conditions, and 3) weather forecasts.

An existing model that can operate from this data base has been selected as the basis for the mobility snowpack model; it is the Snow Accumulation and Ablation Model (Anderson 1973) developed for the National Weather Service River Forecast System. This model indexes each physical process that occurs in the snowpack to the air temperature. Although this results in a significant loss in the accuracy with which the effects of each process is known, this is compensated for by the great reduction in input data and the applicability to a much greater geographical area where only sparse data are available. Anderson (1976) has shown that his model can predict about as accurately as a detailed energy and mass balance model. The agreement of the two models is slightly misleading because a more extensive set of parameters must be determined for the temperature index model before it can be used. The development of a method for determining these parameters from weather station data and extrapolation techniques is one of the areas where further development is needed.

Model components

In the model the snowpack is considered to be a dynamic energy reservoir, the energy being in the form of heat. The capacity of the reservoir depends on the masses of ice and water in the snow and their temperatures. Changes in energy content are measured relative to a dry isothermal snowpack at 0°C. Heat lost by such a snowpack causes an internal temperature drop as the energy is conducted to the surface, where it is lost to the environment. Heat added to the pack will reduce any energy deficit and produce melting in those parts of the pack that reach 0°C. These energy exchanges between the snowpack and the environment are expressed in the energy balance equation, which is the first major component of the model and is described below. The second major component is the heat transfer equation, which deals with the temperature structure and heat transfer within the snowpack.

Heat balance equation

The heat balance equation relates the net change in the energy content of the snowpack to the algebraic sum of the heat gained from or lost to the environment by the various mechanisms of heat transfer:

$$\Delta Q = Q_n + Q_e + Q_h + Q_m + Q_g \quad (13)$$

All of the terms except Q_g represent processes of heat transfer across the snow/air interface.

The radiative heat transfer Q_n is the sum of the incident short-wave solar radiation, the long-wave radiation from the sky and forest cover, and the long-wave radiation emitted by the snow. The short-wave component of the heat balance is affected by the slope, aspect and latitude of the site, the cloud cover, the vegetative cover, and the short-wave reflectivity of the snow. The long-wave components are affected by the cloud cover, the vegetative cover, and the emissivities and temperatures of the various emitting objects. Q_n may be expressed as

$$Q_n = C_c (1-C_v) S \cdot A \cdot L (1-\omega) I + C_c \alpha (1-C_v) \sigma \epsilon_a T_a^4 + C_v \sigma \epsilon_v T_v^4 - C_v \sigma \epsilon_0 T_0^4 - (1-C_v) \sigma \epsilon_0 T_0^4 \quad (14)$$

where

C_c, C_v = cloud and vegetative cover factors, respectively
 S, A, L = slope, aspect and latitude factors, respectively
 ω = snow albedo

I = incident short-wave energy
 α = empirically determined constant (0.757) (U.S. Army Corps of Engineers 1956)
 $\epsilon_a, \epsilon_v, \epsilon_0$ = emissivity of the sky, the foliage and the snow, respectively
 T_a, T_v, T_0 = temperature of the air, the foliage and the snow, respectively.

The first term in eq 14 is the net incident short-wave energy. The next two terms are the incident long-wave radiation from the sky and the foliage, respectively. The remaining two terms are the outgoing long-wave energy to the sky and the foliage, respectively.

Combining the snow-canopy exchange terms and the snow-sky exchange terms yields

$$Q_n = C_c (1-C_v) S \cdot A \cdot L (1-\omega) I + \sigma(T_a^4 C_c \alpha - T_0^4) (1-C_v) + \sigma C_v (T_v^4 - T_0^4) \quad (15)$$

where the emissivities are assumed to be equal to one.

When the energy emitted by the snow is calculated, the snow surface temperature is assumed to be equal to the air temperature for temperatures less than 0°C and equal to 0°C for air temperatures above 0°C. When the ambient temperature is below 0°C the net radiation exchange between the canopy and the snow is assumed to be zero, assuming that $T_0 = T_v$. During periods of precipitation the net long-wave balance is assumed to be zero.

The latent heat transfer Q_e is due to the transfer of water vapor between the snow and the air. This process is governed by the vapor pressure gradient and the wind speed and can be expressed by a relationship derived by Dalton:

$$V = f(U) (e_a - e_0) \quad (16)$$

where

V = water vapor transfer
 $f(U)$ = wind speed function
 e_a, e_0 = vapor pressure of the air and the snow surface, respectively.

The wind function is usually represented as

$$f(U) = a + bU \quad (17)$$

where U is the wind speed and a and b are empirically determined constants.

The values of the constants depend on the lapse rate and the stability of the surface layer. Since stable conditions predominate over snow, the amount of vapor transfer in the absence of wind (represented by the constant a) should be small or zero (U.S. Army Corps of Engineers 1956).

The latent heat transfer is the amount of heat gained by the snow through condensation lost through sublimation. This heat is expressed by

$$Q_e = V \rho_w L_s = bU \rho_w L_s (e_a - e_0) \quad (18)$$

The sensible heat transfer Q_h can be obtained from the ratio of the sensible heat transfer to the latent heat transfer known as Bowen's ratio (Anderson 1973):

$$Q_h/Q_e = \gamma \left(\frac{T_a - T_0}{e_a - e_0} \right) \quad (19)$$

where γ is the psychrometric constant.

On substitution of the latent heat transfer the sensible heat transfer becomes

$$Q_h = bU\rho_w L_s \gamma (T_a - T_0). \quad (20)$$

The precipitation input to the model will be a forecast with both the type and amount of precipitation given. For the heat transfer calculation the temperature of rain is assumed to be the same as the air temperature. The temperature of snow is taken to be 0°C at air temperatures above freezing and the same as the air temperature if it is below freezing. The heat gain from rain is applied against any net heat deficit in the pack, and any excess heat after the deficit is made up goes into melting the pack. In the case of snow, the heat transfer produces no change or increases the net energy deficit. This heat transfer is expressed as

$$Q_m = RC_p (T_p - T_0) \quad (21)$$

where C_p is the specific heat of water or ice, depending on the precipitation form (either C_s or C_i), and T_p and T_0 are the precipitation and snowpack temperatures, respectively.

The heat transfer between the snowpack and the ground Q_g is due mainly to conduction. Since the ground can be considered to be a semi-infinite heat reservoir, its temperature changes only very slowly. Under these conditions a steady-state heat conduction equation can be used to express the heat transfer:

$$Q_g = -K_g \left. \frac{\partial T}{\partial z} \right|_g \quad (22)$$

where K_g is the conductivity of the soil and $\left. \frac{\partial T}{\partial z} \right|_g$ is the temperature gradient in the soil at the soil/snow interface.

The total heat transfer is obtained by substituting eqs 15, 18, 20, 21 and 22 into eq 13:

$$\begin{aligned} \Delta Q = & C_c (1-C_v) S \cdot A \cdot L (1-\omega) I + \sigma (T_a^4 C_c \alpha - T_0^4) (1-C_v) + \sigma C_v (T_v^4 - T_0^4) \\ & + bU\rho_w L_s (e_a - e_0) + bU\rho_w L_s \gamma (T_a - T_0) + RC_p (T_p - T_0) - K_g \left. \frac{\partial T}{\partial z} \right|_g. \end{aligned} \quad (23)$$

The heat transferred by the different processes changes the heat content of the snowpack. A net energy influx will reduce the energy deficit of the pack and if the pack energy deficit is erased and the pack becomes isothermal at 0°C, any additional energy influx will cause melting. On the other hand a net outflow of heat will cause freezing of the free water in the pack and/or an increase in the energy deficit. These phenomena may be expressed by

$$\Delta Q = x\rho_s C_i \Delta T_s + L_f \rho_w \Delta W \quad (24)$$

where x is the snowpack depth, ΔT is the change in snow temperature and ΔW is the change in water in the snowpack.

The water in the snowpack comes either from melting or rain. This water may freeze or remain as liquid within the pack. When the pack becomes isothermal at 0°C and becomes saturated, any additional water from melt or rain becomes outflow at the bottom of the pack.

Substituting eq 24 into eq 23 gives the heat balance in the snowpack:

$$\begin{aligned} x\rho_s C_i \Delta T_s + L_f \rho_w \Delta W = & C_c (1-C_v) S \cdot A \cdot L (1-\omega) I + \sigma (T_a^4 C_c \alpha - T_0^4) (1-C_v) \\ & + \sigma C_v (T_v^4 - T_0^4) + bU\rho_w L_s (e_a - e_0) + bU\rho_w L_s \gamma (T_a - T_0) + RC_p (T_p - T_0) \\ & - K_g \left. \frac{\partial T}{\partial z} \right|_g. \end{aligned} \quad (25)$$

The water equivalent of the snowpack should be held constant during the calculation of the energy balance. When a mass change takes place due to either precipitation or meltwater outflow, the water equivalent of the pack must be adjusted before calculating the heat balance; only the heat content of the mass change is included in the calculation.

In most situations some of the terms in eq 25 may be neglected. The heat transfer at the air/snow interface is usually much larger than that at the soil/snow interface if the ground is frozen, so the latter term may be neglected. When the ground is unfrozen the heat flux is usually small enough to be neglected, except for shallow snowpacks when short-wave radiation penetrates to the ground and causes heating at the interface. The topography, vegetation and time during the snow season will determine which of the other terms become negligible.

Heat transfer equation

The heat transfer within the snowpack must be calculated to determine the change in heat storage. The heat transfer mechanisms within the pack are conduction, radiant energy transfer, and phase change.

The mathematical expressions for these processes are related to the physical characteristics of the snow, such as density, grain size and degree of intergranular bonding. Because the latter two quantities are hard to measure, density is used to characterize the snowpack.

Since heat is gained or lost by a snowpack through the upper and lower boundaries, the temperature gradients within the pack are vertical. The conduction process is therefore a one-dimensional process and may be expressed by the Fourier conduction equation:

$$K_e \partial^2 T_s / \partial z^2 = C_i \rho_s \partial T_s / \partial t \quad (26)$$

where K_e is the effective thermal conductivity and t is time.

The effective conductivity is used to account for the combined effects of heat transfer by conduction and phase changes. Rewriting this equation:

$$\partial^2 T_s / \partial z^2 = K_u \partial T_s / \partial t \quad (27)$$

where $K_u = C_i \rho_s / K_e$.

Schwerdtfeger (1963) proposed a relationship between the diffusivity K_u and density:

$$K_u = 2K_i / (3\rho_i - \rho_s) C_i \quad (28)$$

where K_i is the thermal conductivity of ice and ρ_i and ρ_s are the densities of ice and snow.

In a natural snowpack the density increases with increasing distance from the upper boundary. Incorporating this depth dependence into eq 27 and substituting eq 28 into the revised conduction equation yields

$$\partial^2 T_s / \partial z^2 = K_u \partial T_s / \partial t + \frac{1}{(3\rho_i - \rho_s)} \partial \rho_s / \partial z \partial T_s / \partial z \quad (29)$$

If the heat transfer by the latent heat processes in the snow is accounted for by the use of the effective thermal conductivity, this equation can be used to develop a temperature profile of the snowpack.

Snowpack density

Snow is a dynamic medium. For mobility purposes it is necessary to know the density of the snowpack at any time in its history. The metamorphic process is one of densification, with four processes involved: compaction, equitemperature or destructive metamorphism, temperature gradient metamorphism, and melt metamorphism.

The compaction of snow layers in seasonal snowpacks has been observed by several investigators (U.S. Army Corps of Engineers 1956, Kojima 1967). Kojima derived an expression relating the change in density of a layer to the weight of the overlying layers. This expression, combined with a relationship among snow temperature, viscosity and density derived by Mellor (1964) yields

$$1/\rho_s \partial\rho_s/\partial t = W C_1 \exp [-0.08 (273.16 - T)] \exp [- C_2 \rho_s] \quad (30)$$

where

- W = weight of snow compressing the layer of interest, expressed as water equivalent
- C_1 = empirically determined constant for the fractional increase in density at 0°C and $\rho_s = 0$
- C_2 = empirically determined constant.

The constants depend on the snow type. Equation 30 may be used for all conditions except low-density new snow, depth hoar or windpacked snow.

Destructive metamorphism is a process in which the water molecules rearrange themselves on the snow crystals so that the free surface energy is reduced. It is by this process that the various snowflakes, with their relatively complicated shapes and large surface-to-volume ratios, are metamorphosed into rounded, smooth ice grains. This is how a newly fallen snow layer is incorporated into the existing snowpack, with a marked settling of the layer and an increase in density. Anderson (1976) has developed an expression for densification by this process:

$$1/\rho_s \partial\rho_s/\partial t = C_3 \exp [-C_4 (273.16 - T_s)] \exp [-46 (\rho_s - \rho_d)] \quad (31)$$

where

- C_3 = fractional settling rate at 0°C for densities less than ρ_d (hr^{-1})
- C_4 = empirically determined constant
- 46 = constant (cm^3/gr) necessary to reduce the settling rate when $\rho_d < \rho_s$
- ρ_d = empirically determined density.

This equation incorporates the increasing rate of metamorphism with increasing temperature and the preference for destructive metamorphism in the new snow layers, which tend to be less dense.

Constructive metamorphism takes place when there is a temperature gradient in the snowpack. Water vapor moves from the warmer parts of the snow to the cooler parts, producing a density gradient. The increase in temperature also makes the pore air unsaturated. Some of the vapor may not be transferred but may be used to keep the pore air saturated. If the effective diffusion coefficient D_e for water vapor is a function of the local temperature in the pack, the rate of change of density with temperature is (Anderson 1976)

$$\frac{\partial\rho_s}{\partial t} = D_e \frac{\partial C}{\partial T} \frac{\partial^2 T}{\partial z^2} + T \left(\frac{\partial T}{\partial z} \right)^2 \frac{\partial}{\partial T} \left(D_e \frac{\partial C}{\partial T} \right) \quad (32)$$

where C is the water vapor concentration.

Melt metamorphism is important during the spring when the pack becomes isothermal, during rains, and when the air or ground temperatures become greater than 0°C . In a low-density fresh snow layer, water increases destructive metamorphism and the settling rate. Freeze-thaw cycles in general cause compaction and thus a density increase in the affected layer. Anderson (1976) introduced a multiplicative parameter into the density change equation for destructive metamorphism. This factor is greater than one and has to be determined empirically whenever water is present.

Implementation of the model

Calculation of the energy balance (eq 25) and the conduction (eq 29) requires data describing both the terrain and the meteorology. A calculation is performed to determine the snow conditions at a particular location. These snow conditions are then assumed to be the same over the surrounding area or terrain unit. The extent of a terrain unit is determined by the homogeneity of the slope aspect, elevation and vegetation of the area. The meteorological data needed are cloud cover, temperature, wind speed, dewpoint or relative humidity, precipitation, and snow and ground temperatures. The following section is an explanation of the terms in the energy balance and conduction equations with comments on the constants and the method of calculation where further explanation is needed.

The short-wave radiation reaching the snow surface consists of two components, direct and diffuse. The direct radiation is that fraction of the solar radiation transmitted through the atmosphere without being scattered or absorbed. The diffuse radiation reaches the snow surface after being scattered in the atmosphere and/or reflected by the surrounding terrain. Diffuse radiation reaches the snow surface from all directions but is most intense in the direction of the sun. Although diffuse radiation may be a significant fraction of the total short-wave energy flux, it is not easy to express mathematically and is often ignored in energy balance calculations. In this model only the direct beam is calculated (in the first term on the right side of eq 25), although some of the forward scattered energy may be accounted for by increasing the cloud cover factor C_c .

The amount of solar radiation transmitted through the atmosphere to the snow surface depends on the latitude, time of year, time of day, slope and orientation of the surface, atmospheric transparency, cloud cover and vegetative cover. The seasonal, latitude and surface orientation effects can be calculated using a method formulated by Garnier and Ohmura (1968). If the meridian of solar noon is used as a reference, then the motion of the sun is north and south along this meridian, and the unit vector \vec{S} , representing the sun's position, may be specified by the declination δ (Fig. 1):

$$\vec{S} = (0, \cos \delta, \sin \delta) . \quad (33)$$

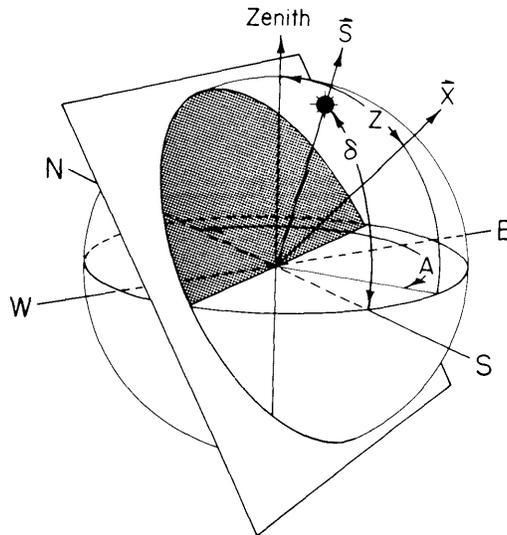


Figure 1. Geometry used to calculate the insolation on a surface (shaded). \vec{X} = unit normal to the surface, \vec{S} = position vector of the sun, δ = declination, A = azimuth and Z = zenith angle.

The orientation of the snow surface is described with respect to an altitude-azimuth coordinate system, with the axes in the local horizon plane and going through the zenith. The unit normal to the surface is expressed in terms of the slope angle and azimuth:

$$\vec{X} = (-\cos A \sin Z, \sin A \sin Z, \cos Z) \quad (34)$$

where A is the azimuth angle (north = 0° , east = 90° , etc.) and Z is the zenith angle.

This vector is then transformed to the global coordinate system:

$$\begin{aligned} \vec{X}' = [& (-\sin \phi \sin H \cos A \sin Z + \cos H \sin A \sin Z \\ & + \cos \phi \sin H \cos Z), (-\sin \phi \cos H \cos A \sin Z \\ & + \sin H \sin A \sin Z + \cos \phi \cos H \cos Z), (\cos \phi \cos A \sin Z \\ & + \sin \phi \cos Z)] \end{aligned} \quad (35)$$

where ϕ is the latitude and H is the hour angle measured from noon (H is negative in the forenoon and positive in the afternoon).

The reduction in insolation due to the angle at which the radiation strikes the slope is equal to cosine of the angle between the sun's direction and the line normal to the slope:

$$\begin{aligned} \vec{X}' \cdot \vec{S} &= \cos H \\ &= (-\sin \phi \cos H \cos A \sin Z + \sin H \sin A \sin Z \\ &+ \cos \phi \cos H \cos Z) \cos \delta \\ &+ (\cos \phi \cos A \sin Z + \sin \phi \cos Z) \sin \delta . \end{aligned} \quad (36)$$

In addition to the effects of geometry, the energy reaching the snowpack is reduced by atmospheric absorption and scattering. Garnier and Ohmura (1968) presented a simple relationship due to Haltner and Martin (1957), by which the energy I incident on a horizontal surface may be calculated from the atmospheric transmissivity and the solar constant:

$$I = I_0 P^m \quad (37)$$

where

$$\begin{aligned} I_0 &= \text{solar constant } (1.35 \times 10^3 \text{ J/m}^2 \text{ s}) \\ P &= \text{mean zenith path transmissivity} \\ m &= \text{optical air mass.} \end{aligned}$$

The optical air mass is the ratio of the atmospheric path length traveled by the sun's rays to the atmospheric depth at the zenith. This may be obtained from tables (List 1963, Table 137). It may also be calculated using the solar elevation angle α (Egleson 1970):

$$\sin \alpha = \sin \delta \sin \phi + \cos \delta \cos \phi = \cos \left(\frac{\pi H}{12} \right) \quad (38)$$

and an approximation given by Choudhury (1979a):

$$m = \frac{1}{\sin \alpha} \quad \text{for } \alpha \geq 30^\circ$$

and

$$m = \frac{1}{\sin \alpha} - 2.8/\alpha^2 \text{ for } \alpha < 30^\circ . \quad (39)$$

For a clear sky the factor in the first term representing incident short-wave radiation in the energy balance equation can now be written as

$$S \cdot A \cdot L \cdot I = I_0 \int_{H_1}^{H_2} P^m \cos H dH \quad (40)$$

where the limits of integration are the sun's hour angle at the beginning and end of the energy balance calculation interval. The limiting values of H_1 and H_2 are the local sunrise and sunset, respectively. For flat terrain the local sunrise and sunset may be obtained from *The American Ephemeris and Nautical Almanac** or calculated by solving eq 38 with $\alpha = 0$. For areas where the topography changes the local sunrise and sunset times, the times found for flat terrain may be substituted into

$$\sin Z_s = -\cos \delta \sin H / \cos \alpha \quad (41)$$

which is solved for Z_s , the azimuth of the sun. With the azimuths of sunrise and sunset, the altitudes of the topographical features at these azimuths is then substituted into eq 38 to calculate the local sunrise or sunset times.

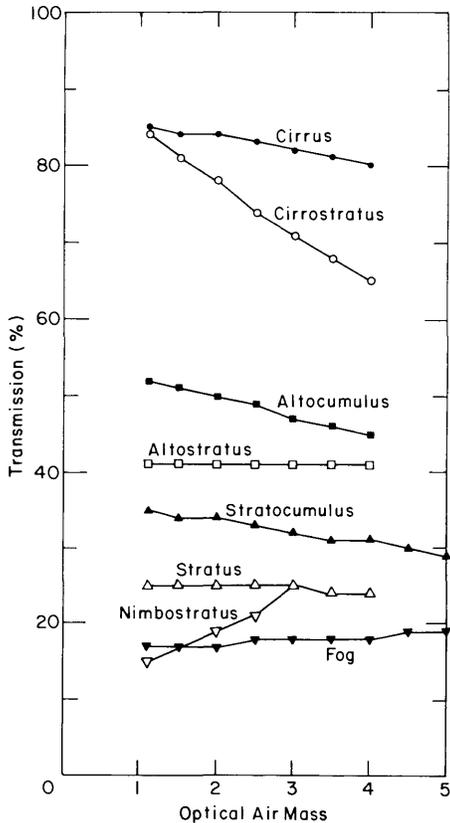


Figure 2. Generalized transmission through clouds as a function of cloud type and air mass. (From List 1963.)

The effects of cloud cover on the radiation budget are expressed as averages for each interval of integration:

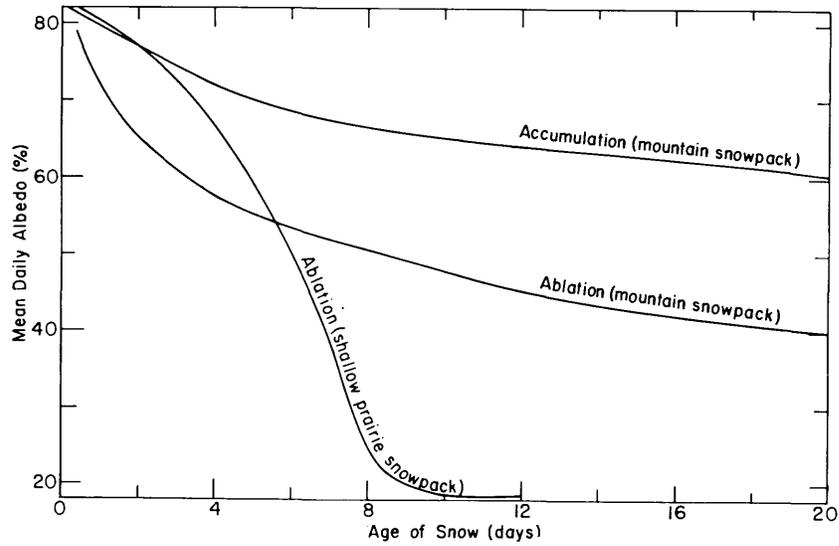
$$C_c = 1 - N + NT \quad (42)$$

where N is the fractional cloud cover in tenths and T is a cloud transmission coefficient.

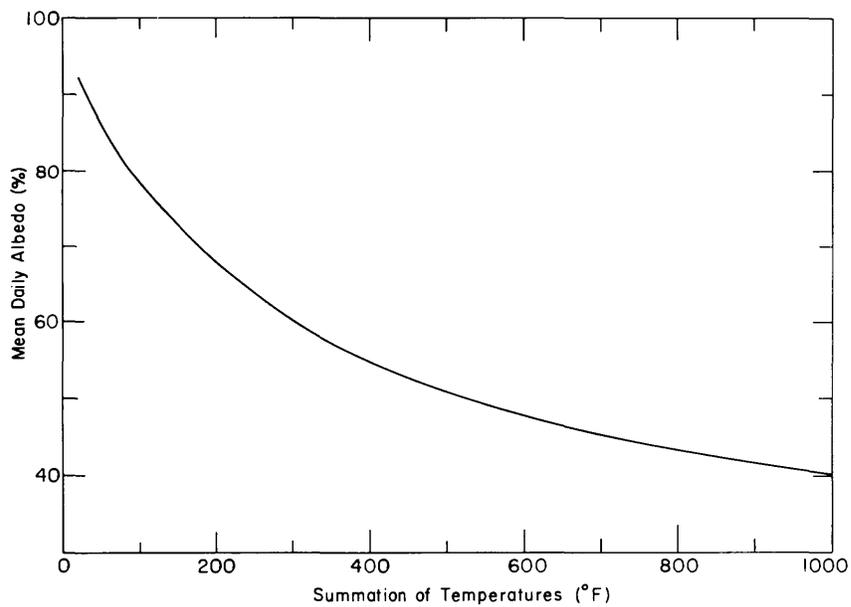
The cloud transmission coefficient depends on the cloud type, thickness and height; no general relationship exists between these quantities. Figure 2 shows transmission as a function of cloud type and optical air mass (List 1963). These measurements do not separate direct and diffuse radiation, partially compensating for the lack of a separate diffuse radiation term.

The albedo ω depends on many factors, including size, shape and structural arrangements of the snow crystals, snowpack water content, snowpack age, sun angle, cloud cover, and amount of foreign material on the surface. Newly fallen snow generally has a high albedo, 0.8 or higher. This decreases as the snow ages and can attain values as low as 0.4 for pure snow. The albedo decrease is correlated with grain size increase, densification, temperature and time. Temperature is the independent variable that affects albedo the most, because the rate of

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a. The albedo as a function of the number of days since the last snowfall. The prairie albedo shows effects of the underlying surface (Male and Gray 1975). The mountain snowpack curves are from U.S. Army Corps of Engineers (1956).



b. The albedo as a function of the sum of the maximum daily temperatures since the last snowfall (U.S. Army Corps of Engineers 1956).

Figure 3. Decay of snowpack albedo.

snow metamorphism depends on it. Figure 3 illustrates the decrease in albedo as functions of temperature and time. The difference in albedo between the accumulation and ablation seasons illustrates how the higher temperatures during the late winter and spring increase the rate of snow metamorphism from that during the accumulation season. The curve for the shallow prairie snowpack illustrates the effects of the underlying surface on the albedo as bare patches appear and the remaining snow becomes thin enough for the underlying surface to affect the albedo.

Indexing the albedo to temperature provides a general guide for clean, deep snowcover (Fig. 3b). Where the snow is contaminated with other material (soot or dust, for example), the initial albedo is lower than that of pure snow. As the contaminated snow melts, more low-reflectivity material is exposed, so that the albedo decays faster during the ablation season than during the accumulation season. The relationship for the albedo of clean snow as a function of temperature is

$$\omega = 140.15 - 14.4 \ln(T) \quad (43)$$

where T is the sum of the daily maximum temperatures since the last snowfall.

The two long-wave radiation exchange terms are $\sigma(1-C_v)[aC_c T_a^4 - T_0^4]$ for the exchange between the snowpack and the sky and $\sigma C_v [T_v^4 - T_0^4]$ for the exchange between the snowpack and the vegetation canopy. The vegetative cover factor C_v ranges from 0 for no canopy to 1 for a complete cover, such as a dense evergreen canopy.

Figure 4 illustrates the effect of the canopy on the radiation budget of the snowpack on a clear day. The canopy has the greatest effect on the incoming short-wave radiation. As the canopy

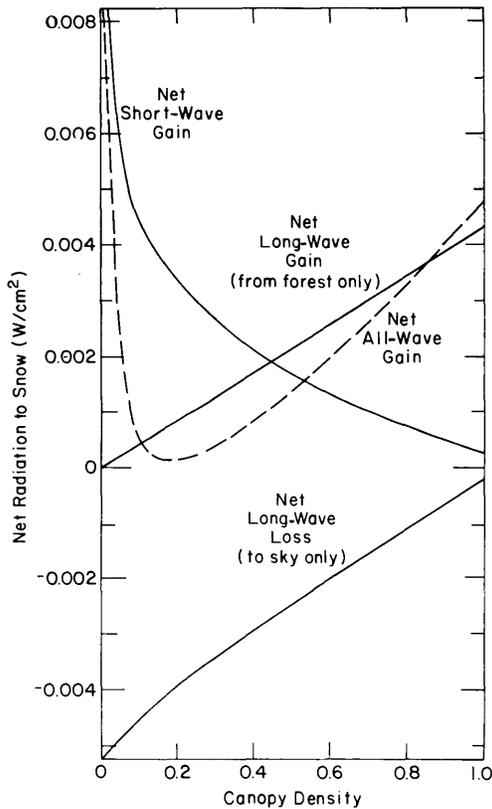


Figure 4. Effect of canopy density on radiant energy exchanges at the snow surface (Jeffrey 1968).

density increases, the loss in incoming short-wave energy is partially compensated for by the decrease in long-wave energy loss to the sky and the increase in long-wave energy received from the canopy. The net radiational energy received by the snowpack is the algebraic sum of three components. The net energy received by the snowpack has a minimum at about 20% canopy density and then increases with increasing canopy density. The amount of net radiation received under a canopy of density 1 is about half that for the same conditions without a canopy.

Figure 4 represents the radiation balance during the day when the canopy is relatively warm. The canopy temperature varies diurnally, being influenced most by the air temperature and insolation. The insolation may raise the temperature of the unshaded parts of the canopy several degrees above the ambient air temperature, but the effect of this on the radiation to the snow is small, so the canopy temperature T_v may be approximated by the air temperature.

The temperatures and temperature gradients within the snowpack vary on a daily basis due to the diurnal cycle and on a longer time scale associated with weather patterns. The incident solar radiation warms the snow indirectly through higher daytime air temperatures and directly through radiation

absorption in the top 50 cm of the snowpack. The diurnal variation of air temperature is also exhibited by the snowpack. The amplitude of the variations in the snowpack decreases with increasing depth and the phase lag increases with depth, with most of the effects damped out at a depth of 15–20 cm (Quick 1967, LaChapelle and Armstrong 1977). The variations in internal snowpack temperatures due to weather patterns also exhibit damping and phase lag effects but are irregular.

Although the snowpack is relatively transparent to short-wave radiation, it is highly absorbing in the infrared, so that the temperature of the top centimeter governs the infrared radiation characteristics. The temperature of this surface layer can be considered to be the same as the air temperature for calculating the long-wave radiation balance except when the air temperature is above 0°C. For air temperatures greater than this the snow temperature is set at 0°C.

The terms for the latent and sensible heat exchange, $bU\rho_w L_s (e_a - e_0)$ and $bU\rho_w L_s \gamma (T_a - T_0)$, depend on the conditions in the turbulent boundary layer, the first two or three meters above the snow surface. The turbulence in this layer is determined by the wind, the insolation, and local conditions such as surface roughness, topography and vegetation. These terms are evaluated by measuring the air temperature and humidity gradients and the surface wind at the point of interest.

DEVELOPING INPUT DATA

The model presented here has been developed from components that require extensive input data. The feasibility of using this model for predicting the snowpack characteristics that affect mobility depends on whether the input data can be extrapolated from measurements made at locations other than where the model is to be applied.

Several techniques are used for extrapolating measurements from one location to another. Anderson (1973) developed a technique for extrapolating the air temperature at a point from measurements at several locations bracketing that point. Slightly different methods are adapted to mountainous areas, where elevation changes have a significant effect on the temperature, and to low relief areas, where elevation does not affect the temperature significantly. These methods, possibly refined by using estimates of insolation and surface conditions, can be used to estimate the air and snow temperatures. The snow albedo may also be estimated using Figure 3b.

Kuz'min (1961) developed a relationship for estimating the humidity as a function of altitude. This may apply to mountainous regions, but the influence of local environment and surface conditions is also very important; a general relationship describing these influences has yet to be developed.

Topographic and reconnaissance maps are readily available, so that the insolation and exposure can be found easily. The reconnaissance maps will probably contain the necessary information on the vegetative canopy. A quantitative estimate of the canopy is therefore available, but its relationship to the snowpack is difficult to quantify.

Meiman (1968) tabulated the results of 24 investigations of forest cover and accumulation. These investigations covered a wide variety of conditions and had widely varying results. Some of the studies using multiple-factor regression analysis showed that accumulation is inversely correlated to canopy density. This agrees with the results shown in Figure 5. Canopy density nullifies the effects of wind (Fig. 6) and small-scale topography, producing a more uniform snowfall distribution under the canopy.

Forest cover affects the ablation phase of the snowcover by reducing the net radiation at the snow surface, reducing the melt rate. Hendrie and Price (1979) found that the turbulent energy exchange for a deciduous forest was reduced to the point where it can be neglected, and although the net solar radiation is reduced, it can be used to calculate the energy balance. Ffolliott et al. (1965) studied the effects of ponderosa pine stands on the accumulation and ablation of snow in

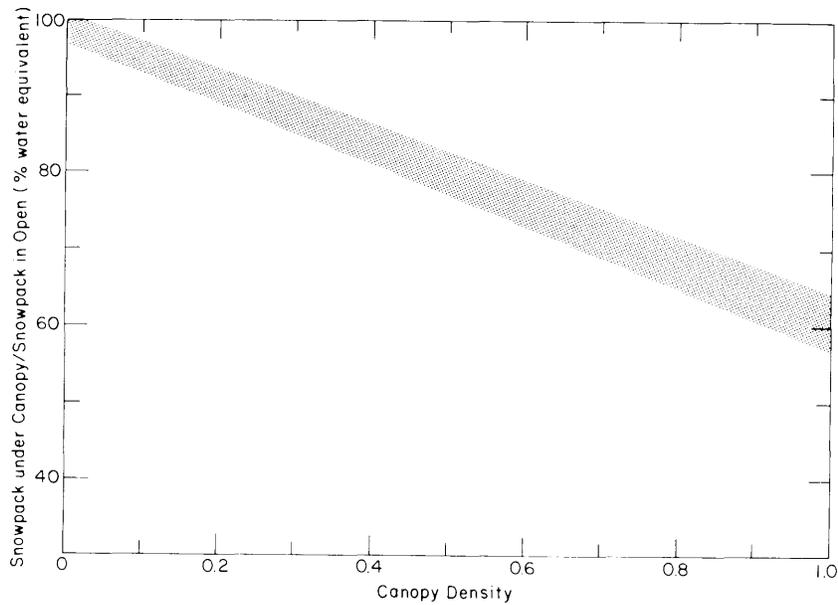


Figure 5. Effect of vegetative canopy on the accumulation of snow under the canopy (U.S. Army Corps of Engineers 1956).

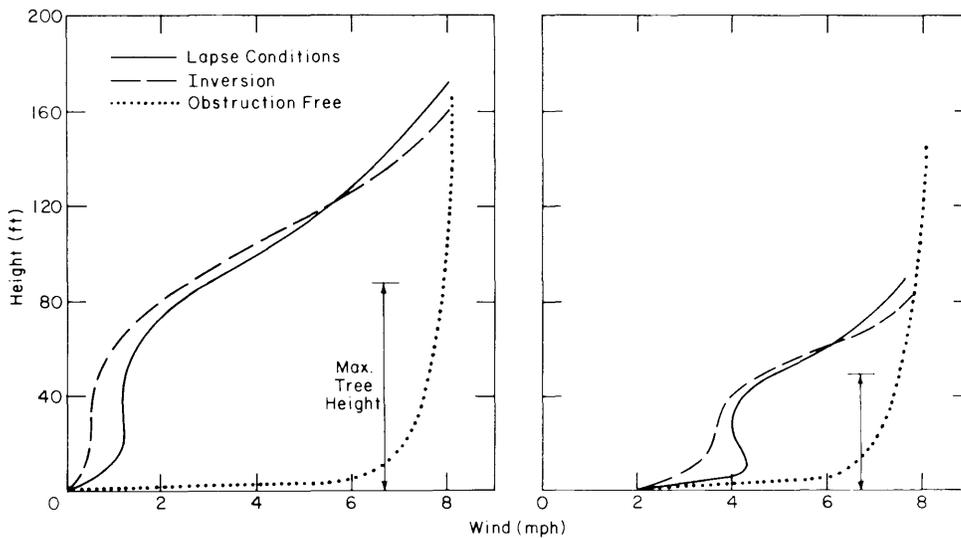


Figure 6. Effect of vegetation on the wind profile for two tree heights. (After Jeffrey 1968 and Izumi 1968.)

adjacent clearings. Figure 7 illustrates the effect of the stand on the distribution of snowcover as a function of distance from the forest edge. For the prevailing winds the clearing was in the lee of the forested area so that the accumulation pattern was due to wind transport of snow from the forest to the forest edge.

During the ablation phase the melt-rate distribution in Figure 7 was governed by radiative exchange. The forest was to the southwest of the clearing, reducing the short-wave radiation incident on the snowpack. This produces a situation analogous to that in Figure 4 (to the left of the

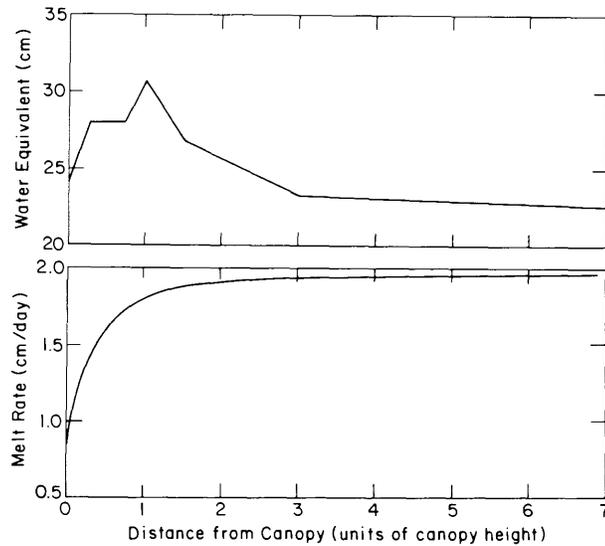


Figure 7. Modification of snowpack distribution near the edge of a stand of trees. The top graph shows the typical snowpack distribution during the accumulation phase, while the bottom graph shows the distribution in melt rate due to the canopy. (After Ffolliott et al. 1965.)

minimum in the all-wave gain curve), where increasing distance from the canopy corresponds to decreasing canopy density. These figures illustrate how the forest cover and wind combine to affect the snowpack accumulation in Figure 7, and forest cover and the aspect of the forest edge affect the solar radiation and the resulting melt.

Vegetation on prairies and steppes affects snow accumulation in an entirely different manner. In these regions, where the ground is relatively flat, the vegetation (generally either grass or crop stubble) affects the surface roughness. The variation in surface roughness, combined with the prevalent winds on the prairies, produces a wide variation in snow depths. Moderate winds and a relatively uniform vegetation cover produces a snowpack with a depth equal to the vegetation height, because additional snow is blown away. High winds can strip away the accumulation or prevent the pack from forming. Variations in vegetation height can produce an even snowcover or a cover of drifts and bare ground.

The effects of the wind are probably the most difficult to quantify. As shown in the above examples, the effects of each of the environmental factors such as vegetation are modified by wind. The wind also plays an important role in the heat exchange processes at the air/snow interface. The transfer of sensible and latent heat are governed by turbulent exchange in the boundary layer. This turbulence is controlled in part by the wind speed and wind speed profile. Male and Granger (1979) showed the relationships among the stability parameter and the gradients of wind and temperature, from which the turbulent energy terms may be derived. Scott (1964) presented a relationship between the large-scale "gradient wind" and surface roughness, from which the wind profile in the region near the surface layer may be obtained. If these relationships can be refined through further research, it may be possible to predict the turbulent energy exchange terms.

The wind also has an effect on the snowcover density. In the absence of wind the density of newfallen snow depends on the crystal structure of the snowflakes and varies between 10 and 190 kg/m³ (Diamond and Lowry 1953). The lighter snows are formed of plate and dendritic snowflakes. When these flakes are fractured by wind-induced saltation, they can pack into a denser snowcover,

with densities approaching 210 kg/m^3 , depending upon the wind speed. Bilello (1969) showed the combined effects of wind and temperature on density to be

$$\rho = 0.152 - 0.0031 T + 0.019 V \quad (44)$$

where

- ρ = average seasonal snowcover density
- T = average seasonal air temperature
- V = average seasonal wind speed.

Although this relationship applies only to seasonal averages, it illustrates the importance of wind speed. There have been many observations of snowpack metamorphosis with time and temperature, but there do not appear to be any general relationships that can be used to predict the model input parameters. The components of the model that provide for the increase in density due to metamorphosis and compaction require a number of empirical constants. Methods of predicting these constants and other relationships of metamorphism have yet to be developed.

In addition to these limitations the model is limited because the various model components have been developed for relatively deep snowpacks. For snowpacks less than about 20 cm the model results don't apply; this would not affect the predictions for most vehicles, because snow that shallow would be completely compacted under the weight of the vehicle. Then the snow acts as a low adhesion layer, and its other properties are negligible.

CONCLUSIONS

The modeling elements put forth in this study can provide a basis for a model to predict snowpack parameters that effect mobility. In general the extensive snowpack characteristics can be predicted with more accuracy and from fewer measurements than the intensive characteristics.

Two gross snow parameters, depth and density, are probably the most significant for mobility use. Predicting the depth requires the development of a suitable accumulation model. This model could be based on the weather forecast, with elements to adjust the accumulations for specific terrain and vegetation features. Once the pack depth is determined, the density can be determined if a method can be developed to remotely determine the empirical model parameters.

The model elements that have been presented are a starting point from which to build the model. To develop the model further will require effort in two areas: 1) refinement and definition of terrain elements over which the snowpack can be considered uniform, and 2) development of either simple techniques to measure parameters such as the vapor pressure and snow densification constants, or models to eliminate the need to measure these parameters.

Ultimately the feasibility of the model depends on the accuracy with which the desired parameters can be predicted. Since the model inputs are a weather forecast, topographical data and parameters extrapolated from one or more sets of remote measurements, the uncertainty of the inputs can be large. The utility of the model will depend on the reduction of these uncertainties to an acceptable level.

LITERATURE CITED

Adams, W.P. (1976) Areal differentiation of snow cover in east central Ontario. *Water Resources Research*, 12(6): 1226-1234.

- Adams, W.P. and D.R. Barr (1976) The evolution of snowcover in east central Ontario with special reference to the International Field Year for the Great Lakes (IFYGL) and to stratigraphy. *Proceedings of the 33rd Annual Eastern Snow Conference*, pp. 1-21.
- Anderson, E.A. (1973) National Weather Service River Forecast System—Snow accumulation and ablation model. NOAA Technical Memorandum NWS HYDRO-17, National Weather Service, Silver Spring, Maryland.
- Anderson, E.A. (1976) A point energy and mass balance model of a snow cover. NOAA Technical Report NWS 19, Office of Hydrology, National Weather Service, Silver Spring, Maryland.
- Anderson, H.W. and A.J. West (1965) Snow accumulation and melt in relation to terrain in wet and dry years. *Proceedings of the 33rd Annual Western Snow Conference*, pp. 73-82.
- Baker, M.B., Jr. and D.R. Carder (1977) Comparative evaluations of four snow models. *Proceedings of the 45th Annual Western Snow Conference*, pp. 58-62.
- Banon, H. (1978) A Markov model of snow depth. Massachusetts Institute of Technology, Department of Civil Engineering, Research Report R78-1.
- Bilello, M.A. (1957) A survey of arctic snow-cover properties as related to climatic conditions. SIPRE Research Report 39. AD-158 191.
- Bilello, M.A. (1969) Relationships between climate and regional variations in snow-cover density in North America. CRREL Research Report 267. AD-700 990.
- Bilello, M.A. (1971) Frozen precipitation: Its frequency and associated temperatures. *Proceedings of the 28th Annual Eastern Snow Conference*, pp. 68-80.
- Choudhury, B.J. (1979a) Thermal conditions in a snowpack. I. The energy balance components. Report prepared by Computer Sciences Corp. for National Aeronautics and Space Administration under Contract NAS5-24350.
- Choudhury, B.J. (1979b) Thermal conditions in a snowpack. II. Temperature distribution due to solar heating. Report prepared by Computer Sciences Corp. for NASA under contract NAS5-24350.
- Choudhury, B.J. (1979c) Snow albedo variation with solar altitude and fractional cloud cover. Report prepared by Computer Sciences Corp. for NASA under contract NAS5-24350.
- Curry, G.E. and A.S. Mann (1965) Estimating precipitation on a remote headwater area of western Alberta. *Proceedings of the 33rd Annual Western Snow Conference*, pp. 58-66.
- Diamond, M. and W.P. Lowry (1953) Correlation of the density of new snow with 700 mb temperature. SIPRE Research Paper 1.
- Farnes, P.E. (1971) Mountain precipitation and hydrology for snow surveys. *Proceedings of the 39th Annual Western Snow Conference*, pp. 44-49.
- Eagleson, P.S. (1970) *Dynamic Hydrology*. New York: McGraw-Hill.
- Ffolliott, P.F., E.A. Hanson and A.D. Zander (1965) Snow in natural openings and adjacent ponderosa pine stands on the Beaver Creek watersheds. U.S. Forest Service Research Note RM-53.
- Fiebiger, G. (1976) The influence of relief and vegetation on snow deposition in the Alps with special regard to the effects on skiing areas. *Proceedings of the 33rd Annual Eastern Snow Conference*, pp. 156-162.
- Galway, J.G. (1952) The effect of altitude and latitude on New England snowfall estimates. *Proceedings of the 9th Annual Eastern Snow Conference*, pp. 57-61.
- Garnier, B.J. and A. Ohmura (1968) A method of calculating the direct shortwave radiation income of slopes. *Journal of Applied Meteorology*, 7(5): 796-800.
- Gary, H.L. (1975) Airflow patterns and snow accumulation in a forest clearing. *Proceedings of the 43rd Annual Western Snow Conference*, pp. 106-113.
- Gray, D.M. (1979) Snow accumulation and distribution. *Proceedings of a Meeting on Modeling of Snow Cover Runoff, 26-28 September 1978, Hanover, New Hampshire*. CRREL, pp. 3-33.
- Haltner, G.J. and R.F.L. Martin (1957) *Dynamical and Physical Meteorology*. New York: McGraw-Hill.

- Hendrick, R.L. and R.J. DeAngelis** (1976) Seasonal snow accumulation, melt and water input—A New England model. *Journal of Applied Meteorology*, **15**(7): 717-727.
- Hendrie, L.K. and A.G. Price** (1979) Energy balance and snow melt in a deciduous forest. *Proceedings of a Meeting on Modeling of Snow Cover Runoff, 26-28 September 1978, Hanover, New Hampshire*. CRREL, pp. 211-221.
- Isumov, N. and A.G. Davenport** (1974) A probabilistic approach to the prediction of snow loads. *Canadian Journal of Civil Engineering*, **1**: 28-49.
- Isumov, N. and M. Mikitiuk** (1976) Climatology of snowfall and related meteorological variables with application to roof load specifications. *Proceedings of the 33rd Annual Eastern Snow Conference*, pp. 41-69.
- Izumi, Y.** (1968) Kansas 1968 field data program. Air Force Cambridge Laboratories, Hanscom Air Force Base, Bedford, Massachusetts.
- Jeffrey, W.W.** (1968) Snow hydrology in the forest environment. *Snow Hydrology, Proceedings of the Workshop Seminar*. Canadian National Committee, International Hydrological Decade, pp. 1-19.
- Kind, R.J.** (1976) A critical examination of the requirements for model simulation of wind-induced erosion/deposition phenomena such as snow drifting. *Atmospheric Environment*, **10**(3): 219-227.
- Kojima, K.** (1967) Densification of seasonal snow cover. *Physics of Snow and Ice—Proceedings of the International Conference on Low Temperature Science—Sapporo*, Vol. 1, Part 2. The Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan, pp. 929-952.
- Kuz'min, P.P.** (1961) Protsess tayaniya skezhnogo prokrova. *Gidrometeorologicheskoe Izdatel'stvo*, Leningrad (Melting of Snow Cover. Israel Program for Scientific Translations, TT 71-50095, 1972).
- LaChapelle, E.R. and R.L. Armstrong** (1977) Temperature patterns in an alpine snow cover and their influence on snow metamorphism. Institute of Arctic and Alpine Research, University of Colorado, Boulder, Colorado.
- Leaf, C.F. and G.E. Brink** (1973) Computer simulation of snowmelt within a Colorado subalpine watershed, USDA Forest Service Research Paper R11-99, Rocky Mountain Forest and Range Experiment Station, U.S. Forest Service, Ft. Collins, Colorado.
- List, R.J.** (1963) *Smithsonian Meteorological Tables*. Smithsonian Institution, Washington, D.C.
- Male, D.H. and D.M. Gray** (1975) Problems in developing a physically based snowmelt model. *Canadian Journal of Civil Engineering*, **2**: 474-488.
- Male, D.H. and R.J. Granger** (1979) Energy and mass fluxes at the snow surface in a prairie environment. *Proceedings of a Meeting on Modeling of Snow Cover Runoff, 26-28 September 1978, Hanover, New Hampshire*. CRREL, pp. 101-124.
- McKay, G.A.** (1968) Problems of measuring and evolution of snowcover. In *Snow Hydrology, Proceedings of the Workshop Seminar*. Canadian National Committee, International Hydrological Decade, pp. 49-62.
- McKay, G.A. and B.F. Findlay** (1971) Variation of snow resources with climate and vegetation in Canada. *Proceedings of the 39th Annual Western Snow Conference*, pp. 17-25.
- McKay, D.C. and G.W. Thurtell** (1978) Measurement of the energy fluxes involved in the energy budget of a snow cover. *Journal of Applied Meteorology*, **17**(3): 339-349.
- McPartland, J.T., A.B. Super and V.L. Mitchell** (1971) Snowpack accumulation in relation to terrain and meteorological factors in southwest Montana. Report prepared for Army Research Office—Durham, Durham, North Carolina, by Montana State University.
- Meiman, J.R.** (1968) Snow accumulation related to elevation, aspect and forest canopy. *Snow Hydrology, Proceedings of the Workshop Seminar*. Canadian National Committee, International Hydrological Decade, CRREL, pp. 35-47.
- Mellor, M.** (1964) Properties of snow. CRREL Monograph III-A1. AD-611 023.
- Myers, V.A.** (1962) Airflow on the windward side of a large ridge. *Journal of Geophysical Research*, **67**(1): 4267-4291.

- Nakaya, U.** (1954) *Snow Crystals: Natural and Artificial*. Cambridge, Massachusetts: Harvard University Press.
- Nielsen, B.C.** (1966) A technique for forecasting the rate of snowfall at Alta, Utah. Alta Avalanche Study Center, Project D, Progress Report #1, U.S. Dept. of Agriculture, Forest Service, Wasatch National Forest.
- Obled, C. and H. Harder** (1979) A review of snow melt in the mountain environment. *Proceedings of a Meeting on Modeling of Snow Cover Runoff, 26-28 September, 1978, Hanover, New Hampshire*. CRREL, pp. 179-204.
- **Quick, M.C.** (1967) A comparison of measured and theoretical snowpack temperatures. *Journal of Hydrology*, 5: 1-20.
- Rhea, J.O. and L.O. Grant** (1974) Topographic influences on snowfall patterns in mountainous terrain. In *Advanced Concepts and Techniques in the Study of Snow and Ice Resources*. Washington, D.C.: National Academy of Sciences, pp. 182-192.
- Rhea, J.O.** (1978a) Orographic precipitation model for hydrometeorological use. Department of Atmospheric Sciences, Colorado State University, Atmospheric Science Paper No. 287.
- Rhea, J.O.** (1978b) Performance of a simple orographic precipitation model for hydrological and climatological studies. Presented at ASCE Spring Convention, Pittsburgh, Pennsylvania.
- Reifsnyder, W.E.** (1955) Wind profiles in a small isolated forest stand. *Forest Science*, 1: 289-297
- Sage, J.D.** (1976) An operational model for hourly snowfall. *Proceedings of the 33rd Annual Eastern Snow Conference*, pp. 98-128.
- **Scott, R.F.** (1964) Heat exchange at the ground surface. CRREL Cold Regions Science and Engineering Monograph IIA1. AD-449 934.
- Schwerdtfeger, P.** (1963) Theoretical derivation of the thermal conductivity and diffusivity of snow. International Association of Scientific Hydrology, Publication No. 61, pp. 75-81.
- Spreen, W.C.** (1947) A determination of the effect of topography upon precipitation. *Transactions of the American Geophysical Union*, 28(2): 285-290.
- Stephuhn, H. and G.E. Dyck** (1974) Estimating true basin snowcover. In *Advanced Concepts and Techniques in the Study of Snow and Ice Resources*. Washington, D.C.: National Academy of Sciences, pp. 314-328.
- Storr, D. and D.L. Golding** (1974) A preliminary water balance evaluation of an intensive snow survey in a mountainous watershed. In *Advanced Concepts and Techniques in the Study of Snow and Ice Resources*. Washington, D.C.: National Academy of Sciences, pp. 294-303.
- Tabler, R.D.** (1975) Predicting profiles of snowdrifts in topographic catchments. *Proceedings of the 43rd Annual Western Snow Conference*, pp. 87-97.
- U.S. Army Corps of Engineers** (1956) *Snow Hydrology: Summary Report of Snow Investigations*. North Pacific Division, Portland, Oregon.
- Vance, H.M. and B.L. Waley** (1971) Snow frequency analysis for Oregon and Utah. *Proceedings of the 39th Annual Western Snow Conference*, pp. 34-38.
- Willen, D.W., C.A. Shumway and J.E. Reid** (1971) Simulation of daily snow water equivalent and melt. *Proceedings of the 39th Annual Western Snow Conference*, pp. 1-8.
- Willis, N.O.** (1979) Snow on the Great Plains. *Proceedings of a Meeting on Modeling of Snow Cover Runoff, 26-28 September 1978, Hanover, New Hampshire*. CRREL, pp. 56-62.