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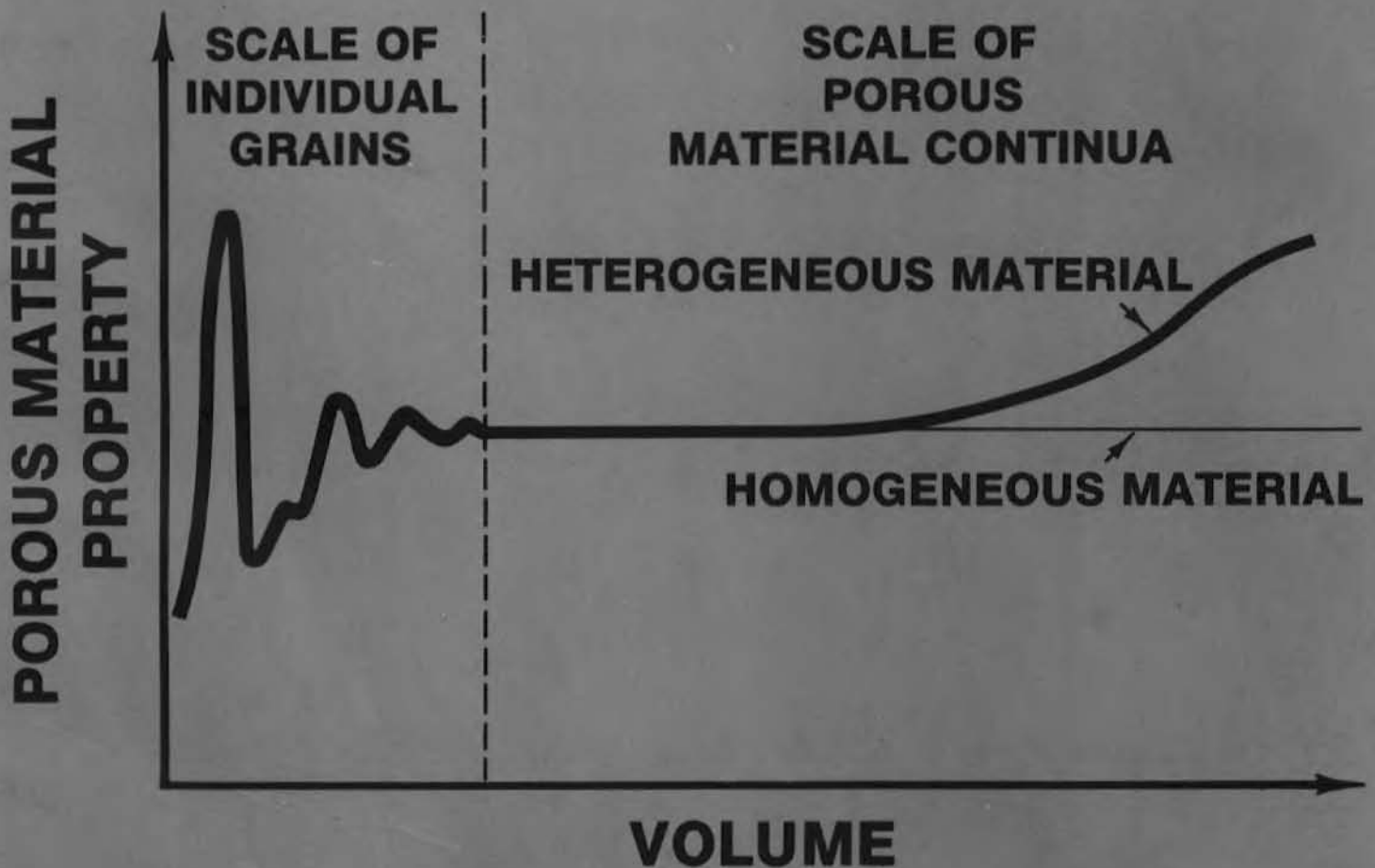
## REPORT 82-41



US Army Corps  
of Engineers

Cold Regions Research &  
Engineering Laboratory

### *Evaluation of procedures for determining selected aquifer parameters*





# CRREL Report 82-41

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Charles J. Daly



20. Abstract (cont'd)

aquifer dispersivity. For each parameter several techniques are described, evaluated and ranked in terms of perceived potential accuracy, simplicity and value to contaminant transport studies. It must be stressed, however, that the evaluations are based principally upon theoretical grounds, and not upon actual conduct of the described procedures.

## PREFACE

This report was prepared by Dr. Charles J. Daly, Hydrologist, Earth Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory.

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# EVALUATION OF PROCEDURES FOR DETERMINING SELECTED AQUIFER PARAMETERS

by

Charles J. Daly

## INTRODUCTION

### Purpose

Effective development, operation or management of a groundwater resource requires reliable quantitative information. This is especially so when serious groundwater contamination problems must be dealt with. The physical parameters characterizing the ability of an aquifer to transmit, store and dilute contaminants must be known, and they must be known in sufficient spatial detail and at sufficient accuracy.

The objective of this report is to describe various aquifer test procedures that are used to estimate the following porous material properties and flow conditions:

1. Piezometric head
2. Hydraulic conductivity-intrinsic permeability
3. Flow direction
4. Specific discharge magnitude
5. Transmissivity
6. Volumetric flow rate
7. Total porosity-effective porosity
8. Average linear velocity
9. Storage coefficient-specific yield
10. Dispersion coefficient-aquifer dispersivity.

The definition of a porous material can be given in terms of three essential characteristics:

1. It is a medium capable of transmitting or storing liquids or gases, or some combination of liquids, gases and dissolved or suspended matter.
2. For a given sample, its ratio of interfacial surface area (solid-liquid, solid-gas) to sample volume is large.
3. On a large scale it can be adequately modeled as a material continuum.

Given these criteria it is clear that clay, loose sand or sandstone all qualify as porous materials. For fractured rock, the situation is not as clear cut. In well-fractured rock the component blocks formed by fracturing may be thought of as "grains" constituting a secondary porous material. These grains may themselves qualify as primary porous materials based upon their own microstructure. Poorly fractured rock's widely spaced fractures may represent sites for significant transmission or storage of fluid; however, such fractures cannot qualify a rock as a porous material.

For this investigation the term "aquifer" refers to all saturated porous materials that may provide avenues for contaminant transport, regardless of how small that potential may be. This word usage is slightly at variance with the usual definition of an aquifer: a saturated, permeable geologic unit capable of economically yielding an appreciable amount of water to wells or springs.

#### Scope

This investigation did not consider those test procedures involving sustained pumping or recharge at a well. The intent of such tests is to estimate parameters by observation of the resultant effects of that pumping or recharge on neighboring wells. There were several reasons for excluding these tests. The high costs of these tests would limit their number. This in turn would necessarily limit knowledge of the spatial variability of the estimated parameters. In addition, pumping tests are well documented elsewhere (Reed 1980).

This study did consider appropriate procedures based on analysis of static water levels, time-water level response of a monitor well to an instantaneous withdrawal or addition of a specified volume of water, physical analysis of borehole material, fate of tracers introduced at a monitor well, and any other information obtained from or during the construction and development of monitor wells.

The literature was searched for materials relating to estimation of the aforementioned aquifer properties. The search resulted in a large collection of articles covering a wide variety of laboratory and field methods. A complete list is provided at the end of the report.



After the collected literature was reviewed, categorized and evaluated, it was clear that there are currently no standardized tests for estimating the selected parameters under all possible conditions. This is understandable, however, when the wide range of porous material types and geologic environments is considered. What has been achieved here is some organization of test methodologies and an evaluation of the conditions under which they may be selected. It must be stressed, however, that such an organization and evaluation cannot be considered authoritative without a program of laboratory and field testing that would also give some means of evaluating the effects of imperfections and variations in monitor well construction.

To further complicate the parameter estimation situation, there appears to be a large gap between many promising theoretical methods and the availability of equipment to make the required measurements. Along with a lab-field test program, it is suggested that instruments be developed and evaluated.

#### Concept of accuracy

Selection of appropriate procedures for identifying aquifer parameters depends upon weighing the accuracy of various procedures against perceived data needs. Unfortunately, it is not a simple matter to judge the accuracy of specific test procedures, nor is it a simple matter to specify reasonable levels of required accuracy for the aquifer parameters. It should also be appreciated that the aquifer parameters are mathematical concepts, not physical quantities that can be measured by comparison with a standard. Before these points are considered in more detail, a rigorous definition of the concept of accuracy is needed.

#### Precision, bias and accuracy

The definition of accuracy, as applied to a parameter estimate or test procedure, is approached in terms of the more basic concepts of precision and bias. Suppose, for example, that a specific test procedure for parameter  $X$  is repeated a number of times. Each test run results in an estimate for  $X$ ; this single value is termed an outcome. Let a final estimate for  $X$  be the mean value of the separate outcomes. The precision of the final

estimate depends on the magnitude of the standard deviation of the test outcomes. If the standard deviation is small then the precision of the final estimate of X is said to be high.

The concept of bias is defined as the difference between an estimate and the "true" value of the parameter. Since the true value of a parameter is unknown, the bias of an estimate cannot be known exactly. However, comparison of the results from several different test procedures performed in the same environment does provide some evidence for evaluating bias. If several different procedures yield final estimates for X that are nearly the same, this is evidence for believing that the bias of any one of the estimates or procedures is small.

An important point should be made here concerning the "true" value of an aquifer parameter. It must be stressed that the true value may exist only in terms of an idealized model of the aquifer being tested. For example, suppose that a single value of hydraulic conductivity K is sought for an aquifer which is in reality heterogeneous. The true value of K then has no physical meaning aside from that given it by the homogeneous aquifer models upon which its estimate may be based. Indeed, none of the aquifer materials may actually have conductivity equal to the "true" value of K!

By use of the terms precision and bias, the accuracy of a test procedure and its results may be defined. A test procedure and its results are called accurate if the outcomes from many runs yield a final estimate having high precision and if the bias of that estimate is believed to be small.

Estimation of the accuracy of a procedure, and the data it yields, thus requires some knowledge of both precision and bias. Unfortunately, the accuracy of a test procedure may not often be easily estimated, since precision and bias are likely to be a function of the specific geologic and hydrologic conditions encountered. For example, a test for hydraulic conductivity may be quite accurate for sandy materials, but it may fail miserably for clays.

#### Spatial variability vs test accuracy

The typically large and unknown spatial variability of aquifer parameters is an important consideration in the selection of aquifer tests.

Since most test procedures are designed to provide point estimates, it is necessary to weigh test accuracy against suspected spatial variation. For instance, even perfectly accurate point estimates of transmissivity are of limited value if transmissivity is known to vary over several orders of magnitude within very short distances.

Point estimates of aquifer parameters are essentially samples drawn from a large population of values. That population is composed of the parameter values associated with each and every location in the aquifer being considered. It is clear that it is not feasible to determine the whole population by sampling so we must be content to infer regional characteristics from a number of geologically and hydrologically appropriate sample points.

It should also be recognized that it is generally not possible to attribute all observed differences between point estimates to spatial variability. Lack of precision and the change of bias under different soil conditions can account for much of the apparent variation.

#### Test selection

The above points demonstrate that test selection must be a matter of judgment that integrates test procedure, anticipated geologic conditions, anticipated precision and bias, and the theory upon which the definitions of the selected parameters are based.

#### DEFINITION OF PARAMETERS

Evaluation and selection of appropriate aquifer test methods must be based upon clear definitions of the parameters to be estimated. In the following paragraphs certain flow and transport parameters are described. In each case the descriptions are presented as part of an organized development of the fundamental principles of groundwater flow and mass transport. An exhaustive treatment of the subject will not be attempted. Instead, emphasis is placed on considerations of the important simplifications, assumptions and theories associated with the aquifer parameters.

In the end analysis it should be remembered that confidence in parameter estimates will be necessarily a matter of considered judgment. Even

the selected test procedures may, quite justifiably, vary from one situation to another, and from time to time, as a result of a variety of factors. In later sections of this report attention will be directed toward discussion of these factors.

### Piezometric head

The concept of piezometric head develops out of applying the classical principles of energy and work to a fluid continuum. To demonstrate this, consider the fluid element, with dimensions  $dx$ ,  $dy$  and  $dz$ , shown in Figure 1. Later, the analysis will consider a limiting process in which the dimensions  $dx$ ,  $dy$  and  $dz$  approach zero.

At some instant of time, suppose the element to be at an arbitrary initial state (location, velocity and ambient pressure) within the fluid moving through a porous material. Using one corner of the element for reference, suppose that initially the element is at location  $\vec{x}_0$ , moving with velocity  $\vec{v}_0$ , under ambient pressure  $p_0$ . It is now useful to calculate the work done on the element as it moves from the initial state to some other state.

From classical mechanics, the work  $W$  done on a particle moving along a curved path from  $\vec{x}_0$  to  $\vec{x}$  is defined as

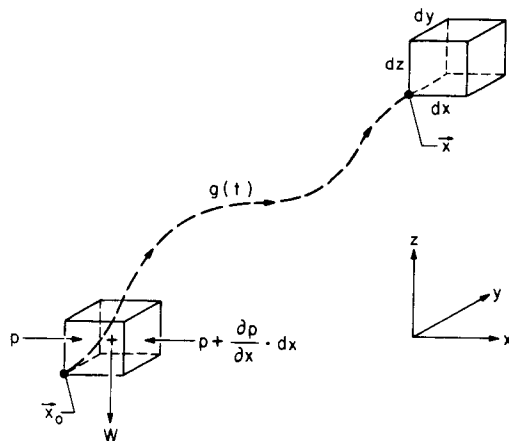


Figure 1. Movement of a fluid element.

$$W(\vec{x}_0, \vec{x}) = \int_{\vec{x}_0}^{\vec{x}} \vec{F} \cdot d\vec{x} \quad (1)$$

where  $\vec{F}$  is a continuous force field that acts on the particle. For flow in saturated materials, this force field is usually assumed to be made up of gravity, pressure and frictional resistance. (For unsaturated flow, surface tension, osmotic and adhesive forces would also be included.)

The dot product in eq 1 indicates that small increments of work are equal to the product of displacement and the force component in the direction of displacement (or the product of the force and the component of displacement in the direction of the force). If the path of the fluid element in Figure 1 can be parametrically represented by the vector function  $g(t)$ , where time  $t$  ranges from  $t_0$  to  $t$ , then velocity  $\vec{v}$ , displacement  $d\vec{x}$ , and acceleration  $\vec{a}$  can be given as

$$\vec{v}(t) = \frac{dg}{dt}, \quad d\vec{x} = \vec{v}(t)dt, \quad \vec{a}(t) = \frac{d\vec{v}}{dt} \quad (2)$$

Using Newton's second law ( $\vec{F} = m\vec{a}$ ) and eq 2, we obtain

$$W(\vec{x}_0, \vec{x}) = \int_{t_0}^t m \frac{d\vec{v}}{dt} \cdot \vec{v} dt \quad (3)$$

where  $m$  is the mass of fluid in the element. Noting that

$$\frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{1}{2} \frac{d}{dt} (v^2(t)) \quad (4)$$

we see that eq 3 becomes

$$W(\vec{x}_0, \vec{x}) = \int_{t_0}^t \frac{m}{2} \frac{d}{dt} (v^2(t)) dt = \frac{m}{2} [v^2(t) - v^2(t_0)] \quad (5)$$

Equation 5 is a basic principle of classical mechanics, showing that the work done on a body is equal to the change in its kinetic energy.

To proceed further with this development, consider the forces that act on the fluid element traveling through the saturated porous material shown in Figure 1. Neglecting the possibility of minute chemical, electrical and

temperature effects, we assume that the total resultant force on the fluid element is composed of the body force (weight), pressure related forces, and frictional flow resistance.

The body force has a simple form:  $\vec{F}_B = -mg\hat{z}$ , where  $g$  is the acceleration due to gravity and  $\hat{z}$  is a unit vector directed vertically upward.

The resultant force due to pressure is derived from analysis of the pressures acting on the six faces of the fluid element. Treating the two faces perpendicular to the  $x$  axis in Figure 1, we find that the  $x$  component of the pressure force is

$$F_x^P = p \, dy \, dz - \left( p + \frac{\partial p}{\partial x} dx \right) dy \, dz = - \frac{\partial p}{\partial x} V \quad (6)$$

where  $p$  is pressure and  $V = dx \, dy \, dz$  is the volume of the element. Note that pressure is assumed to be an analytic function, i.e. expandable in a Taylor series; with the limit process where the volume  $V$  will approach zero anticipated, the Taylor series is suitably truncated. Looking at the remaining  $y$  and  $z$  directions, we see that the total pressure force is

$$\vec{F}^P = - \left( \hat{x} \frac{\partial p}{\partial x} + \hat{y} \frac{\partial p}{\partial y} + \hat{z} \frac{\partial p}{\partial z} \right) V = - \nabla p \, V . \quad (7)$$

From eq 7 it is clear that the pressure force acts in the direction of the steepest decline in pressure.

The remaining force to be considered is the frictional resistance. At this point it is sufficient to say that the frictional force depends in a complicated way on the properties of the fluid as well as the properties of the medium through which the fluid travels. Let this force be  $\vec{R}$  and recall that it always acts in a direction opposite to the motion of the particle.

The total resultant force on the fluid element is the sum of body, pressure and frictional forces. Introducing the above expressions for these into eq 1, and using the work-energy principle, we see that eq 5 gives

$$\frac{m}{2} (v^2 - v_0^2) = - \int_{\vec{x}_0}^{\vec{x}} mg\hat{z} \cdot d\vec{x} + \int_{\vec{x}_0}^{\vec{x}} V \nabla p \cdot d\vec{x} + \int_{\vec{x}_0}^{\vec{x}} \vec{R} \cdot d\vec{x} \quad (8)$$

where  $v = v(t)$  and  $v_0 = v(t_0)$ .

Noting that

$$\hat{z} \cdot d\vec{x} = dz ; \quad \gamma = g \frac{m}{V} ; \quad \nabla p \cdot d\vec{x} = dp \quad (9)$$

we find that eq 8 becomes

$$\frac{1}{2g}(v^2 - v_0^2) = z - z_0 + \int_{p_0}^p \frac{dp}{\gamma} + \int_{\vec{x}_0}^{\vec{x}} \frac{|\vec{R}|}{mg} |d\vec{x}| \quad (10)$$

where  $\gamma$  is the specific weight of the fluid. The term in eq 10 involving frictional force is called the head loss and is denoted  $h_L$ . Head loss, as can be seen from eq 10, is strictly a positive quantity.

If the fluid can be assumed to be of constant specific weight, eq 10 can be restated as

$$\frac{v_0^2}{2g} + z_0 + \frac{p_0}{\gamma} = \frac{v^2}{2g} + z + \frac{p}{\gamma} + h_L \quad (11)$$

which is the energy equation of fluid mechanics.

Note that each term in eq 11 has the dimension of length. The first, second and third terms on the left- and right-hand sides of the equation are called the velocity, elevation and pressure heads respectively. Often, elevation and pressure heads are combined; the resulting combination is the piezometric head which was to be defined.

For most groundwater flow problems, velocity head is very small compared to both piezometric head and head loss. Neglecting velocity head, and defining piezometric head as  $h$ , we obtain

$$\frac{p_0 - p}{\gamma} + (z_0 - z) = h_0 - h = h_L \quad (12)$$

that is, the head loss experienced by a fluid element in moving from one state to another is equal to the change of piezometric head (see Fig. 2).

Equation 12 is a basic equation of groundwater flow. As will be shown later, head loss is a function of flow velocity. This fact, plus eq 12, enables the velocity of groundwater flow to be expressed in terms of changes in piezometric head. This expression is known as Darcy's law.

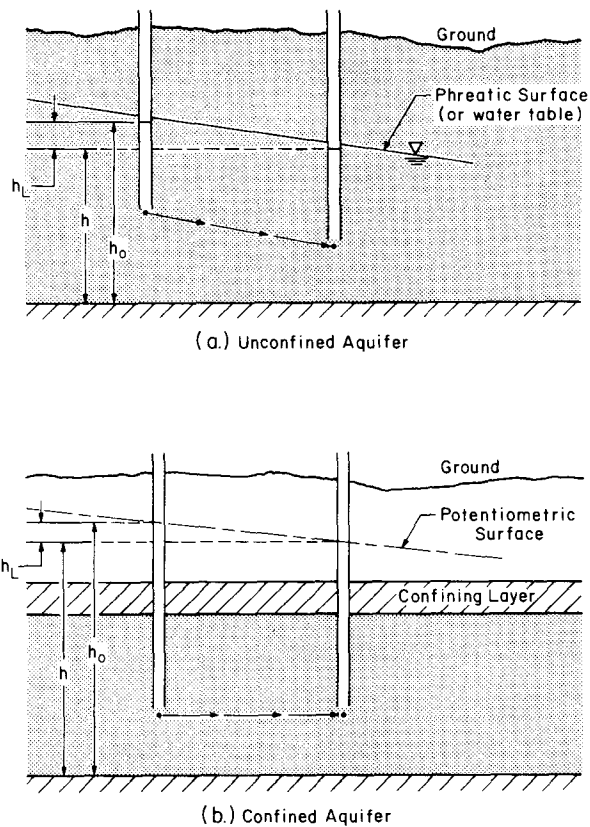


Figure 2. Head loss and flowing groundwater.

### Use of piezometers

Because of its use in determining fluid velocity, piezometric head is a very important quantity for groundwater flow and contaminant advection problems. It can be determined at a known elevation by measuring pressure at that location, or by measuring the elevation of the water/air interface in a tube called a piezometer. In both cases head is usually expressed as a depth of water and the expression  $h = p/\gamma + z$  is applied where  $\gamma$  is the specific weight of water. At this point it is useful to develop the basic fluid statics principles behind the use of piezometers.



The piezometer shown in Figure 3 consists of a simple tube placed in a drilled hole or driven into the ground. The tube has a short screened section or open cavity at a location where the piezometric head is to be measured. Ideally, presence of the tube would not disturb the existing piezometric head distribution, which in general varies in the three coordinate directions. In practice such disturbance can be minimized by making sure there is intimate contact between the tube and undisturbed porous material. Cavities or screens 3 to 12 in. long are recommended (see Reeve 1965).

Drawing a free body diagram for a small cylindrical element within the static fluid in the tube, we write the force balance in the  $s$  direction as

$$p \, dA - \left( p + \frac{\partial p}{\partial s} \, ds \right) dA - W \sin \alpha = 0 \quad (13)$$

where the terms are defined in Figure 3.

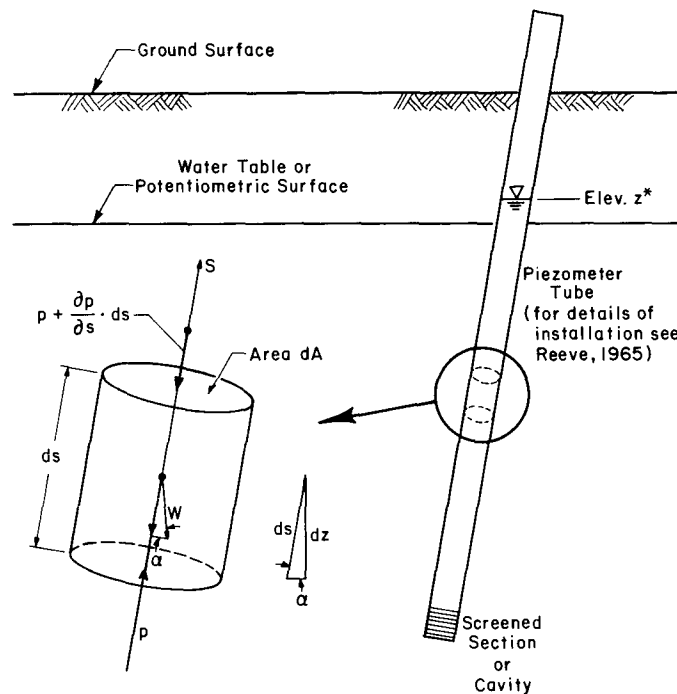


Figure 3. Principles of piezometer operation.

The weight  $W$  of the element is  $\gamma ds dA$  so that eq 13 becomes

$$\frac{\partial p}{\partial s} = - \gamma \sin \alpha. \quad (14)$$

With reference to Figure 3,  $\sin \alpha = dz/ds$  so that the last result becomes

$$\frac{\partial p}{\partial z} = - \gamma \quad (15)$$

which is the equation of hydrostatics used extensively in manometry. If the piezometer is again assumed to be filled with water, eq 15 can be integrated over the length of the piezometer from the screen (where the pressure and elevation are  $p$  and  $z$ ) to the water/air interface where pressure is  $p_A$  (atmospheric) and elevation is  $z^*$ . Thus, after rearranging,

$$p_r = p - p_A = \gamma (z^* - z) \quad (16)$$

where  $p_r$  is gauge pressure measured relative to a constant atmospheric pressure. Using gauge pressure in the definition of head, we are able to write eq 16 as

$$h = \frac{p_r}{\gamma} + z = z^* \quad (17)$$

or the piezometric head in the porous material around the screen is equal to the elevation of the water/air interface in the piezometer.

Note that operation of the piezometer does not require the tube to have constant cross section, nor does it require the tube to be plumb. The latter is a consideration, however, in determining the location and elevation of the screen.

#### Atmospheric pressure, earth tide and entrapped air effects on piezometric head

Records of piezometer water levels often reveal curious variations which cannot be related to any observed recharge, discharge or change of flow pattern. Careful analysis of such records has shown that piezometric head in confined aquifers is influenced by atmospheric pressure (Jacob 1940, Tuinzaad 1954) and earth tides (Robinson 1939, Bredehoeft 1967). For

unconfined aquifers, fluctuations of head have been related to the presence of entrapped air at or near the water table (Tuinzaad 1954, Peck 1960, Turk 1975), and to earth tides (Bredehoeft 1967).

For wells tapping a confined aquifer, a rise in atmospheric pressure  $\Delta p_A$  results in a decrease of head in those wells, i.e. the water level drops slightly. Clark (1967) considers an artesian aquifer at Ocala, Florida, where  $\Delta h = 3$  cm. The analysis of Jacob (1940) gives the relationship:

$$\Delta h = -B \frac{\Delta p_A}{\gamma} = \frac{-\beta n}{\alpha + \beta n} \frac{\Delta p_A}{\gamma} \quad (18)$$

where B is called the barometric efficiency,  $\beta$  and  $\alpha$  are the compressibilities of water and soil matrix, respectively, and n is the porosity as defined in the Porosity section. According to Freeze and Cherry (1979) B usually falls between 0.2 and 0.75; variations in head of as much as 25 cm are clearly possible. Clark (1967) describes a straight-forward technique for determining barometric efficiency from observations of  $\Delta p_A$  and  $\Delta h$ .

For wells in unconfined aquifers the effect of compression of entrapped air by increasing atmospheric pressure is not easily quantifiable. The relationship between atmospheric pressure and water table elevation is again an inverse one; however, the magnitude of the effect depends on the trapped air content at or near the water table. Variations in water table elevation due to this effect have been observed by Turk (1975) where  $\Delta h$  was roughly 6 cm.

When barometric effects are negligible, or removable by knowledge of B, there is often a remaining periodic fluctuation in well water levels. This fluctuation is typically highly correlated with the tidal cycle. For deep confined aquifers fluctuations of as much as 17 cm have been noted. For unconfined aquifers the effect is quite small unless porosity is of the order of  $10^{-4}$ , which would cause  $\Delta h$  to be approximately 1 cm (Bredehoeft 1967).

Barometric and tidal effects need to be considered when values of head taken at different times are compared. The interpretation of regional groundwater flow patterns can be affected by these anomalies.

### Plotting piezometric head distributions

An important point should be mentioned that concerns water table elevation around a piezometer tapping an unconfined aquifer and the water level inside that piezometer. In general the two levels are not equal. Later it will be shown that this condition simply indicates the occurrence of vertical movement of fluid in the porous material.

Freeze and Witherspoon (1966, 1967, 1968) demonstrate that significant local relief or the presence of recharge-discharge areas (springs, ponds, streams, marshes, etc.) may be associated with complex three-dimensional patterns of piezometric head. With such patterns there may be considerable vertical flow and mass transport. Nevertheless, it is still common practice to represent the distribution of piezometric head throughout a water table aquifer by a two-dimensional water table elevation contour map. It will be shown below that this representation is based upon an assumption of horizontal flow (a hydrostatic pressure distribution). For water table aquifers the contour surface is called a phreatic surface. The usefulness of such contour maps depends on the further assumption that water table gradient and piezometric head gradient are equal. Taken together these assumptions imply that both piezometric head and its gradient are invariant with depth throughout the saturated porous material.

For confined aquifers it is also common practice to make the horizontal flow assumption. A two-dimensional areal contour plot of piezometric head can then be drawn. The contour surface for confined aquifers is called a potentiometric surface.

### Hydraulic conductivity

Consider the laboratory setup shown in Figure 4 where two large reservoirs are connected by a pipe of diameter  $D$ , cross-sectional area  $A$ , and length  $\Delta x$ . Suppose that the pipe is filled with a porous material and that a discharge  $Q$  (volume per time) is established through it as a result of a difference in fluid levels in the two reservoirs. Assume that fluid specific weight is constant throughout the system. Equation 11 can then be applied to a fluid element that flows from reservoir 1 at point  $x$  through the pipe and into reservoir 2 at point  $x + \Delta x$ . The result is

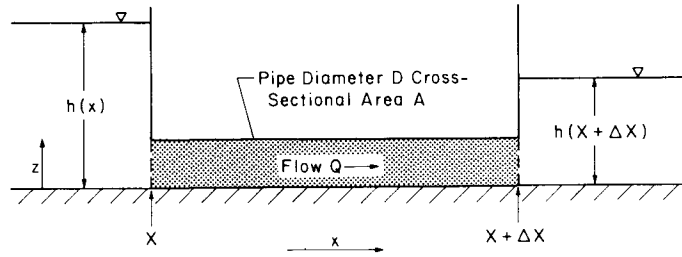


Figure 4. Pipe filled with porous material.

$$\left[ \frac{v^2}{2g} + z + \frac{p}{\gamma} \right]_x = \left[ \frac{v^2}{2g} + z + \frac{p}{\gamma} \right]_{x+\Delta x} + h_L \quad (19)$$

where the subscripts indicate the point where the quantity in brackets is to be evaluated. As previously noted, velocity head is almost always very small compared to piezometric head so that eq 19 becomes

$$h(x) - h(x+\Delta x) = h_L \quad (20)$$

As a result of experiments using a single fluid and a single type of porous material in the equipment of Figure 4, it can be shown, for conditions usually encountered in groundwater flow, that head loss is directly proportional to both the length of pipe and a quantity called the specific discharge v

$$h_L = \frac{\Delta x}{K} v \quad (21)$$

Specific discharge is defined as

$$v = \frac{Q}{A} = \frac{4Q}{\pi D^2} \quad (22)$$

The constant of proportionality K in eq 21 is called the hydraulic conductivity.

Substitution of eq 21 in eq 20 and rearranging gives

$$v = - K \left[ \frac{h(x+\Delta x) - h(x)}{\Delta x} \right] . \quad (23)$$

Equation 23 is called Darcy's law, named after Henri Darcy who proposed this relationship in 1856.

#### Relation to intrinsic permeability

Suppose that further tests are conducted with the equipment of Figure 4 using a variety of fluids and porous materials. In these new experiments, let the change of piezometric head,  $h(x+\Delta x) - h(x)$ , and pipe length  $\Delta x$  be held constant. It requires no great imagination to expect that specific discharge will vary, depending on both the fluid and porous material selected for a test.

It is useful now to quantify the dependence of  $v$  on fluid properties, separate from its dependence on porous material properties. Results show that, as a function of fluid properties,  $v$  is proportional to specific weight  $\gamma$ , and inversely proportional to dynamic viscosity  $\mu$ . Thus Darcy's law can be restated as

$$v = - \frac{k\gamma}{\mu} \left[ \frac{h(x+\Delta x) - h(x)}{\Delta x} \right] \quad (24)$$

where the parameter  $k$ , called intrinsic permeability, is a fundamental property of the porous material alone. Comparison of eq 23 and 24 shows that

$$K = \frac{k\gamma}{\mu} . \quad (25)$$

Equation 25 shows that, while  $k$  is a simple property of the porous material,  $K$  is a function of fluid density and viscosity and thus a function of temperature. By convention,  $K$  is usually given for water at 20°C. When other fluids (or water at another temperature) are considered,  $K$  for that fluid must be calculated from  $k$  and the fluid's specific weight and viscosity. These latter quantities are easily obtained from standard tables.

#### Differential forms of Darcy's law

If, in any of the tests considered so far, the length  $\Delta x$  of the pipe connecting the reservoirs is decreased toward zero in a limiting process,

the differential form of Darcy's law for one-dimensional flow results,

$$v(x) = -K(x) \frac{dh}{dx} = -k(x) \frac{\gamma}{\mu} \frac{dh}{dx} . \quad (26)$$

This differential form of Darcy's law is an expression that is satisfied at all points in a one-dimensional flow where hydraulic conductivity, intrinsic permeability, piezometric head and specific discharge may be functions of coordinate  $x$ .

After generalization of Darcy's law to three-dimensional flows, it at first seems sufficient to write

$$\vec{v} = -K(x,y,z) \left[ \hat{x} \frac{\partial h}{\partial x} + \hat{y} \frac{\partial h}{\partial y} + \hat{z} \frac{\partial h}{\partial z} \right] = -K(x,y,z) \nabla h \quad (27)$$

where  $K$  or  $k\gamma/\mu$  may be used. It turns out, however, that some porous materials exhibit specific discharge components that are not zero in directions in which there is no component of piezometric head gradient. This possibility is clearly not incorporated in eq 27. To explain and describe this often encountered phenomenon requires defining the concepts of porous material homogeneity and isotropy.

#### Homogeneity, isotropy and Darcy's law

Simply stated, a porous material is homogeneous if its intrinsic permeability does not vary as a function of location in the material; otherwise it is nonhomogeneous.

An isotropic material is one in which flows can occur only in the direction of the gradient of piezometric head; otherwise it is called anisotropic. If isotropy is assumed, then the appropriate form of Darcy's law is eq 27.

Classification of a material as homogeneous or isotropic greatly simplifies the mathematical description of groundwater flow. Although no natural material is truly homogeneous or isotropic, many are nearly so. Choice of either simplification becomes a matter of considered judgment of the impact of ignoring nonhomogeneity or anisotropy.

For the most general porous material that is anisotropic and nonhomogeneous, a tensor description of intrinsic permeability (or conductivity) is one that maintains the linearity between specific discharge and head

gradient while accommodating anisotropy. Written out for Cartesian coordinates this is

$$v_x = -K_{xx} \frac{\partial h}{\partial x} - K_{xy} \frac{\partial h}{\partial y} - K_{xz} \frac{\partial h}{\partial z} \quad (28a)$$

$$v_y = -K_{yx} \frac{\partial h}{\partial x} - K_{yy} \frac{\partial h}{\partial y} - K_{yz} \frac{\partial h}{\partial z} \quad (28b)$$

$$v_z = -K_{zx} \frac{\partial h}{\partial x} - K_{zy} \frac{\partial h}{\partial y} - K_{zz} \frac{\partial h}{\partial z} . \quad (28c)$$

According to Bear et al. (1968) the hydraulic conductivity tensor is symmetric. Therefore, an anisotropic, nonhomogeneous porous material is properly defined by specifying six components of the tensor, each component being, in general, a function of  $x$ ,  $y$  and  $z$ . Note that specification of the tensor depends on the orientation of the coordinate system used to define the direction of  $v_x$ ,  $v_y$  and  $v_z$ .

By appropriate rotation of the three orthogonal coordinate axes along which the velocity components are drawn, the tensor description of the porous material can be achieved in terms of only three principal values. The new directions of the rotated coordinate system are then called the principal directions. Often it can be assumed, with some geologic justification, that a horizontal ( $x,y$ ) plane and a vertical  $z$  axis correspond to the principal directions. In that case

$$v_x = -K_{xx} \frac{\partial h}{\partial x} \quad (29a)$$

$$v_y = -K_{yy} \frac{\partial h}{\partial y} \quad (29b)$$

$$v_z = -K_{zz} \frac{\partial h}{\partial z} . \quad (29c)$$

In many groundwater model applications, significant vertical vs horizontal anisotropy has been incorporated. Ratios  $K_{xx}$  and  $K_{yy}$  to  $K_{zz}$  of as much as 1:1000 have been used (Winter 1976). The preferential horizontal deposition of plate-like sedimentary particles is one mechanism suggested to account for this anisotropy.



Further simplification of Darcy's law is possible under a set of fairly restrictive assumptions. If 1) vertical head gradients can be assumed negligible, 2) the x,y principal directions of the hydraulic conductivity tensor are in the horizontal plane, and 3)  $K_{xx}$  equals  $K_{yy}$ , then

$$v_x(x,y) = - K(x,y) \frac{\partial h}{\partial x} \quad (30a)$$

$$v_y(x,y) = - K(x,y) \frac{\partial h}{\partial y} \quad (30b)$$

$$v_z \equiv 0 . \quad (30c)$$

Equations 30a-c correspond to an assumed two-dimensional flow in which vertical vs horizontal anisotropy is ignored. This simple flow description is used extensively in groundwater modeling, often without due regard for the underlying assumptions.

#### Range of validity of Darcy's law

Darcy's law is a linear relationship between specific discharge and piezometric head gradient. Hydraulic conductivity is the constant of proportionality. Over the range of flow conditions usually encountered in groundwater flow problems, the linear relationship proves to be adequate. There is, however, ample evidence to establish an upper limit to the range of applicability of Darcy's law. For moderate flow rates it was pointed out earlier (eq 21) that head loss was proportional to specific discharge; this statement ultimately led to Darcy's law. For high flow rates it turns out that head loss is roughly proportional to the square of specific discharge.

This same dichotomy is observed for flow in pipes where laminar conditions indicate head loss proportional to discharge, and turbulent conditions show head loss proportional to discharge squared. For pipe flows the Reynolds number is the criterion for determining whether a flow is in the laminar or turbulent range. In the case of flow through porous materials an analogy with pipe flow is used to establish a Reynolds number criterion for determining if flow is in the laminar or "Darcy" range.

For pipe flows, the diameter of the pipe becomes a characteristic length required in the definition of the Reynolds number. For flow in porous materials an effective grain size diameter is used. The Reynolds number is then

$$R = \frac{v}{\nu} d_{10} \quad (31)$$

where  $v$  is the specific discharge,  $\nu$  is the fluid kinematic viscosity and  $d_{10}$  is the effective grain diameter that is larger than the effective grain diameters of 10%, by weight, of a porous material sample. It has been shown experimentally that Darcy's law does not apply for  $R$  greater than 10. For  $R$  values between 1 and 10 the flow is transitional, grading from Darcy to turbulent conditions.

There are two noteworthy situations in which the question of the applicability of Darcy's law arises. First is the case of flow in a gravel pack surrounding a well screen. Second is the use of Darcy's law to describe flow in fractured rocks having large fracture widths (or solution channels) or low fracture density. Flow in a gravel pack may exceed the Reynolds number criteria, whereas poorly fractured rocks do not qualify as porous media.

#### Flow direction

Even under the most general conditions of anisotropy and nonhomogeneity, the direction of the specific discharge vector  $\vec{v}$  is always toward some point of lower piezometric head. This statement, however, is not sufficient to determine that direction. In general, there are many lines from a given point along which head decreases.

Determination of the precise direction of flow in a material depends on whether or not the material is anisotropic. For anisotropic materials the direction of movement of a fluid particle in the continuum depends on both gradient and the conductivity tensor (see eq 28 or 29). For isotropic materials, flow direction always follows the piezometric head gradient and is independent of hydraulic conductivity (see eq 27). Thus the assumption of isotropy tremendously simplifies the problem of determining flow direction. Note that this same simplification is possible under the assumptions which led to eq 30.

### Specific discharge magnitude

When the gradient of head and the hydraulic conductivity of a porous material are known, and flow is in the Darcy range, eq 27, 28 or 29 can be used to find the magnitude of  $\vec{v}$ , the specific discharge.

### Transmissivity

If a horizontal flow assumption is made and gradient is assumed constant over the saturated thickness  $b$  of an aquifer, Darcy's law (eq 28a-c) may be integrated over that thickness from  $z=0$  to  $z=b$  to give, after division by  $b$ ,

$$\frac{1}{b} \int_0^b v_x dz = - \frac{\partial h}{\partial x} \left[ \frac{1}{b} \int_0^b K_{xx} dz \right] - \frac{\partial h}{\partial y} \left[ \frac{1}{b} \int_0^b K_{yy} dz \right] \quad (32a)$$

$$\frac{1}{b} \int_0^b v_y dz = - \frac{\partial h}{\partial x} \left[ \frac{1}{b} \int_0^b K_{yx} dz \right] - \frac{\partial h}{\partial y} \left[ \frac{1}{b} \int_0^b K_{yy} dz \right] \quad (32b)$$

Observing that the integration amounts to an averaging over the saturated thickness, we see that eq 32a and b can be rewritten as

$$\bar{v}_x = - \bar{K}_{xx} \frac{\partial h}{\partial x} - \bar{K}_{xy} \frac{\partial h}{\partial y} \quad (33a)$$

$$\bar{v}_y = - \bar{K}_{yx} \frac{\partial h}{\partial x} - \bar{K}_{yy} \frac{\partial h}{\partial y} \quad (33b)$$

where the bars above the variables indicate an average value.

Under the assumption of two-dimensional flow it is useful to define a new quantity called the transmissivity:

$$T = b\bar{K} \quad (34)$$

Thus eq 33a and b can be restated in terms of transmissivity,

$$b \bar{v}_x = - T_{xx} \frac{\partial h}{\partial x} - T_{xy} \frac{\partial h}{\partial y} \quad (35a)$$

$$b \bar{v}_y = - T_{yx} \frac{\partial h}{\partial x} - T_{yy} \frac{\partial h}{\partial y} \quad (35b)$$

It should be stressed that the concept of transmissivity makes sense only for two-dimensional, horizontal flows. For situations where vertical head gradients are substantial, the results of an analysis of an assumed two-dimensional problem may be at variance with field conditions and observations.

In the case of unsteady flow, the saturated thickness may change with time, resulting in changes in transmissivity. Occasionally in these cases it is assumed that transmissivity is strictly a linear function of saturated thickness. Note from the above analysis that this assumption is truly correct only when conductivity does not vary with depth (the average  $K$  is constant over any portion of the saturated thickness). More often, changes in transmissivity are assumed small and ignored.

All of the preceding considerations become much more serious for groundwater contamination problems since the precise description of flow patterns is crucial for determining contaminant transport.

Vertical averaging is not so serious a problem for predicting the propagation of piezometric head (the effect of a well, for example), but it is a crucial consideration for predicting mass transport. A correct prediction of the rate of movement of contaminants depends on recognition and appreciation of the large variance in travel times for particles in adjacent layers of very different porous materials. This fact has lead many to question the usefulness of the concept of transmissivity for mass transport prediction.

#### Volumetric flow rate

The formulas for the calculation of volumetric flow rate, or discharge, through a certain surface area  $S$  are based on the definition of specific discharge. Consider first an arbitrary small planar patch of the surface  $S$  as shown in Figure 5a. Suppose the patch has area  $dA$ . Next, express the spatial orientation of the patch by the direction of a vector, normal to its surface. Let this vector have length  $dA$ . Thus the vector  $\vec{dA}$  represents both the orientation of the patch and the magnitude of its surface area.

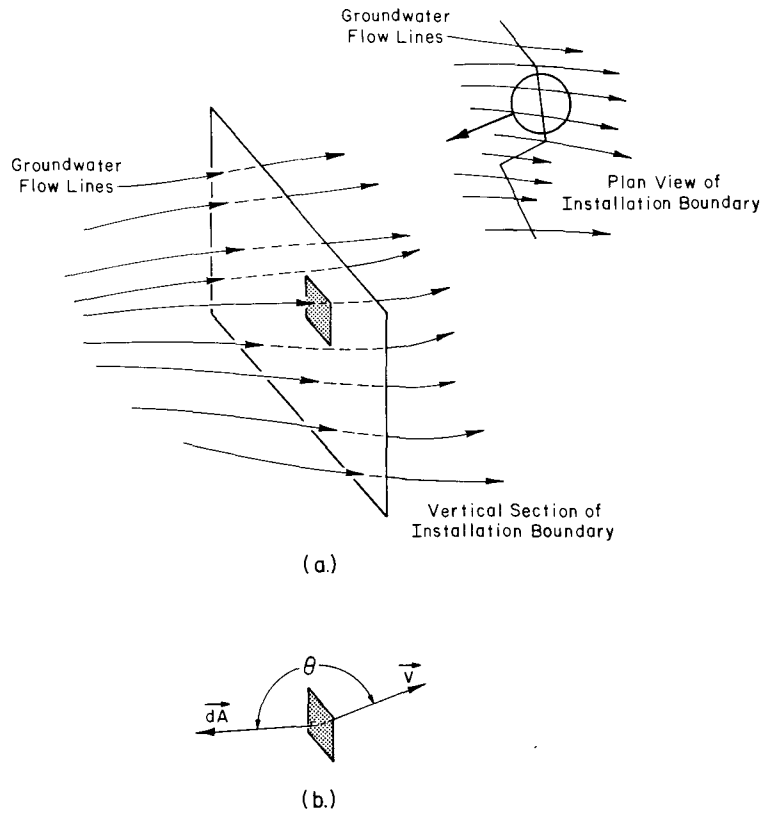


Figure 5. Calculation of volumetric flow rate (after Loeltz 1968).

Now suppose that a discharge of fluid is moving through the surface  $S$ , and that over the patch being considered the specific discharge is  $\vec{v}$ , expressing both its magnitude and direction.

From Figure 5b it is clear that the small portion of the discharge passing through  $S$  via the patch is

$$dQ = |\vec{v}| |d\vec{A}| \cos \theta = \vec{v} \cdot d\vec{A} . \quad (36)$$

To find the total volumetric flow rate through  $S$ , the individual discharges through all the patches covering  $S$  are summed,

$$Q = \int_S \vec{v} \cdot d\vec{A} . \quad (37)$$

In practice, an approximation of the integral may be used,

$$Q \approx \sum_{i=1}^{NP} \vec{v}_i \cdot \vec{A}_i \quad (38)$$

where the surface  $S$  is divided into NP patches.

For two-dimensional flows, the volumetric flow rate through any vertical "curtain" can be approximated by applying eq 38 to a series of vertical strips defining the extent of the curtain.

### Porosity

Total porosity  $n$  is the ratio of the volume of void space  $V_v$  to the total volume  $V_T$  of a porous material sample,

$$n = \frac{V_v}{V_T} . \quad (39)$$

Defining  $\rho_T$  and  $\rho_s$  as the mass density of a dried sample of material, and the mass density of the grains respectively, we find that the following is easily obtained:

$$n = 1 - \frac{\rho_T}{\rho_s} . \quad (40)$$

Total porosity is an important parameter because it indicates the amount of fluid that may be held in a saturated sample. Of equal or perhaps greater importance, however, is a related parameter called the effective porosity  $\phi$ . Effective porosity is a measure of the amount of space available for fluid flow through a sample. The difference between  $n$  and  $\phi$  can be explained in terms of two porous material phenomena: 1) dead-end pores and 2) adhesive films around grains.

Dead-end pores are void spaces within a porous matrix that have no outlet communicating with the bulk of the remaining void spaces. Thus they provide no avenue for fluid flow. Adhesive films around the grains of a porous material result from strong, short-ranged electrical forces. These films are held very tightly to grain surfaces and may result in a signifi-

cant blockage of the pores in a fine-grained material. Clays are good examples of this phenomenon, having a total porosity of 0.45 to 0.55, while their effective porosity is around 0.01 to 0.10.

Effective porosity is clearly, at most, equal to the total porosity. For coarse unconsolidated materials and some consolidated rocks, the difference between  $n$  and  $\phi$  is not dramatic. In general, however, a clear distinction should be made between them, and they should not be presumed equal.

#### Average linear velocity

Earlier, in a description of flow through a pipe filled with porous material, specific discharge  $v$  was defined as the ratio of discharge  $Q$  to the total cross-sectional area normal to the flow. Consider now a cross section of area  $A$  taken through the pipe of Figure 4. The cross section would show that the actual area available for flow is not  $A$ , but  $\phi A$ , due to the presence of the grains, surface films and dead-end pores. An average linear velocity of fluid particles in the pores of the material can be defined as

$$v^* = \frac{Q}{\phi A} = - \frac{K}{\phi} \frac{dh}{dx} . \quad (41)$$

Generalizing to three dimensions, we obtain

$$\vec{v}^* = \frac{\vec{v}}{\phi} . \quad (42)$$

From the definition, it is clear that  $\vec{v}^*$  is a representative average velocity at which fluid particles (and advected contaminants) are carried along through the pores by the flow. It should be stressed that  $\vec{v}^*$  is only an average. The actual speed of fluid particles moving through the void spaces ranges from near zero at the grain-film boundaries to many times  $|\vec{v}^*|$  in the larger spaces between grains. More will be said concerning this point when mechanical dispersion is discussed in the Aquifer Dispersivity section.

#### Storage coefficient-specific yield

Darcy's law is only one of the two essential parts of a complete mathematical description of flow through porous materials. The other part

of the description is based upon the principle of mass conservation.

In presenting Darcy's law, piezometric head and hydraulic conductivity (intrinsic permeability) were the important quantities introduced. Consideration here of the principle of mass conservation leads to the definition of parameters associated with the ability of a porous material to store and release fluid mass.

The concept of mass conservation may be stated as: the net rate of mass flow across the boundaries of a volume must equal the rate of change of mass stored within that volume. Thus the conservation principle is nothing more than a bookkeeping for fluid mass. This statement of mass conservation is the most general, applying to both unsteady (time variable) and steady (time invariant) flows. In the case of steady flows, however, the conservation principle can be reduced to a simpler form.

For steady flow the mass stored in a volume of porous material cannot change; thus the flow rates into and out of the volume must be equal, and no consideration need be given to storage characteristics. For unsteady flow, no simplification of the conservation principle is possible and the storage characteristics of the porous material and fluid are important. The remainder of this section considers only unsteady flow.

There is a fundamental difference in the description of storage characteristics for saturated vs partly saturated porous material volumes. For a saturated volume, storage can vary as a result of both the compressibility of the porous material and the compressibility of the fluid itself. For partly saturated volumes the mass storage is a function of the level of saturation.

For saturated or partly saturated volumes it will be shown that changes in mass storage can be related to piezometric head, just as the flow of mass was related to head through Darcy's law. This common dependence on head can be used to unite the mass conservation principle and Darcy's law, resulting in a single governing equation for flow in a porous medium.



## Mass storage in saturated materials

Compressibility of the porous matrix. Consider a small volume of porous material that lies at some depth below the ground surface. According to basic principles of soil mechanics, the weight of overburden acting on the small volume is balanced by 1) grain-to-grain transfer of forces within the solid structure of the porous matrix, and 2) fluid pressure forces acting on the walls of pores within the matrix. That part of the overburden stress  $\sigma_B$  (force per cross-sectional area) being borne by the grain structure is called the effective stress  $\sigma_e$ , while the remaining stress borne by the fluid is simply the pressure  $p$ , or

$$\sigma_B = \sigma_e + p \quad . \quad (43)$$

If small changes of  $\sigma_B$ ,  $\sigma_e$  and  $p$  occur,

$$d\sigma_B = d\sigma_e + dp \quad . \quad (44)$$

To proceed with this analysis, assume that pressure changes within the small volume are not the result or cause of significant changes in the fluid content of overburden. If the weight of overburden (both fluid and soil) remains constant, eq 44 gives

$$d\sigma_e = - dp \quad (45)$$

or the change in effective stress is equal and opposite to the change of pore water pressure.

Intuitively, it is reasonable to expect that an increase of effective stress  $\sigma_e$  results in compression of the soil matrix, and a decrease of  $\sigma_e$  results in matrix expansion. If the matrix compresses, there will necessarily be a corresponding release of fluid mass from the volume. If it expands, fluid will be drawn into storage (a sponge serves as a useful analogy). Over a limited range of effective stress the relationship between  $d\sigma_e$  and changes in soil volume  $V_T$  may be expressed in terms of a compressibility constant:

$$dV_T = - \alpha V_T d\sigma_e \quad . \quad (46)$$

A compression of the aforementioned soil volume produces a volume of fluid equal to the amount of compression, or

$$dV_w = - dV_T . \quad (47)$$

Using the definition of piezometric head for a constant elevation  $z$ , we find

$$dp = \gamma(dh - dz) = \gamma dh . \quad (48)$$

Combining eq 45 through 48, we get the result,

$$dV_w = - \alpha \gamma V_T dh . \quad (49)$$

Compressibility of the fluid. Changes in the amount of fluid mass contained in a soil volume also result from fluid compressibility. For the fluid volume  $V_w$  within the soil, its relation with ambient pressure is given in terms of a fluid compressibility constant:

$$dV_w = - \beta V_w dp . \quad (50)$$

The volume of fluid in the soil is related to the total volume through porosity  $n$ , or

$$V_w = n V_T . \quad (51)$$

Combining eq 50, 51 and 48 we obtain the result:

$$dV_w = - \beta n \gamma V_T dh . \quad (52)$$

Specific storage and storage coefficient. With both the effects of matrix and fluid compressibility under consideration, eq 49 and 52 can be combined to give

$$\frac{dV_w}{dh} = - \gamma(\alpha + \beta n)V_T = - S_s V_T \quad (53)$$

where  $S_s$  is defined as the porous material specific storage. Equation 53 is a general purpose relationship relating the change of fluid storage to changes in head for any small, fully saturated volume  $V_T$ .

Consider the special case of assumed two-dimensional flow in a confined aquifer that is fully saturated over its thickness  $b$ . If a vertical

parallelepiped of horizontal area  $A$  and length  $b$  is divided along its length, then each segment of length  $dz$  eq 53 can be written as

$$d(dV_w) = - S_s A dz dh . \quad (54)$$

Integration of this expression from  $z=0$  to  $z=b$  gives

$$dV_w = - A dh \int_0^b S_s dz = - S A dh \quad (55)$$

where  $S$  is defined as the storage coefficient. Equation 55 may be explained by noting that  $dV_w$  is the volume of fluid produced from the parallelepiped given a decline in head of  $dh$ . Written out, the storage coefficient is

$$S = \int_0^b S_s dz = \gamma \int_0^b (\alpha + \beta n) dz . \quad (56)$$

Specific yield. By definition, a partly saturated soil volume is one that contains a phreatic surface separating a fully saturated zone below from a fully drained zone above. Changes in mass storage in a partly saturated volume are dominated by movement of the phreatic surface, i.e. by actual emptying or filling of pore spaces. The effects of compressibility are typically very small by comparison.

Consider again a narrow vertical parallelepiped of horizontal cross-sectional area  $A$ . This time suppose the parallelepiped to contain a phreatic surface. For a given decline,  $dh$ , in the level of the phreatic surface, a volume of fluid  $dV_w$  is yielded. The obvious relation follows:

$$dV_w = S^* A dh \quad (57)$$

where  $S^*$  is called the specific yield.

In applying eq 57 it is usually assumed that drainage is immediate, that all drainable fluid is yielded simultaneously with a decline of phreatic surface. Often, however, the portion of a soil column above the phreatic surface will continue to yield fluid for some length of time after the decline. This phenomenon is called delayed drainage. The usual practice

of ignoring delayed drainage may represent a source of error in some flow analyses.

### Aquifer dispersivity

The parameters considered thus far have been related to the hydraulics of fluid flow in porous materials. At this point consideration will be given to the processes of mass, or contaminant, transport.

The relations describing mass transport in porous materials are generally thought to include both advective and dispersive processes. Advective flux refers to the movement of contaminants that can be described in terms of the energy and momentum of fluid particles. The equation governing advection is based on the mass conservation principle. Dispersive flux is a process dependent on the concentration gradient and is postulated to account for the disagreement between observed mass transport and that predicted by pure advection. The equation for dispersion is essentially a gradient-discharge law analogous to Fick's law for molecular diffusion. This analogy can be demonstrated by the following heuristic arguments.

Fick's law for molecular diffusion states that the mass transport of, say, a gas A, into a gas B follows the relationship

$$F_x = - D_{AB} \frac{\partial C_A}{\partial x} \quad (58)$$

where  $F_x$  is the flux (mass flow rate) of gas A in the x direction,  $D_{AB}$  is the molecular diffusion coefficient, and  $C_A$  is the local concentration of gas A. Generally,  $D_{AB}$  depends on the types of gases and their concentration.

Considering a molecular scale of the diffusion process, we can see that gas A will mix with gas B due to the random motion of molecules. The motion of any individual molecule in a region of mixing is governed by energy and momentum principles, much the same as one of a collection of billiard balls. This implies that the mixing of gases A and B could be described by a strictly advective process that keeps track of the trajectory of every molecule of gas. This proposal is of course absurd, considering the immense number of molecules in even a small volume and the hopelessness

of determining their initial energy and momentum. The solution of this problem is to view the collection of molecules as a fluid continuum. This idealization and the use of principles from statistical mechanics are sufficient to justify the use of Fick's law.

Consider now the motion of fluid particles moving through an aggregate of porous material grains. Let the scale of motion about the grains be called the microscale. Observation of elemental fluid particles at the microscale would establish a pattern of particle trajectories. These trajectories would demonstrate that fluid particles, which are initially close together, tend to become separated as they travel along tortuous paths through the material. In principle, this separation could be described by a strictly advective process. This would require detailed knowledge of the pore channel geometry, as well as the initial energy and momentum of all fluid particles. Again, however, this method is untenable.

To overcome the difficulties of describing particle movement at the microscale, the continuum approach is again used. Porous materials are idealized and represented by spatial distributions of conductivity and porosity. The values of these properties at any point are, in some sense, local averages. The averages are taken over volumes that are small but still large enough to contain very many grains of the material.

When the actual paths of fluid particles are not important, the idealization of porous materials as continua is sufficient to account for the gross fluid flow by advection. A strictly advective mechanism, however, using the local average values of porosity and conductivity, will not properly account for the separation of particles around individual grains.

The mechanism of dispersion is introduced to reconcile the observed transport of fluid particles and the transport predicted by advection alone. Typically, the dispersion is given by

$$\vec{F}_A = - D_{AB} \nabla C_A \quad (59)$$

where  $\vec{F}_A$  is a vector quantity giving the magnitude of the dispersion flux of material A in each coordinate direction,  $C_A$  is the local concentration of material A, and  $D_{AB}$  is a dispersion coefficient tensor. The disper-

sion coefficient is written as a tensor since there may be dispersion flux across the direction of the concentration gradient.

A third and final scale of mass transport problems is that one called the regional, or macroscale. In this case the movement of fluid particles over tens or thousands of meters is considered. At this scale the transport of fluid particles depends, of course, on the regional distributions of porous material properties. Unfortunately, the detailed and precise distribution of material properties is generally unavailable. This leads to the possibility of discrepancies between field-determined mass transport and that which could be predicted by advection alone.

On the molecular scale, the principle of diffusion was introduced to overcome the lack of information on the kinetics of every gas molecule. On the microscale, the dispersion was introduced to overcome the lack of data on pore channel geometry. On the macroscale, it is a common practice to compensate for imprecise knowledge of regional porous material properties by use of a similar dispersive mechanism.

Use of a dispersive mechanism for macroscale transport problems has at least one serious drawback. Since dispersion is introduced to make up for the lack of detail in the spatial definition of parameters, the magnitude of the dispersion, and hence the dispersion coefficient, must depend on the degree of detail of the available data. This is not a very desirable situation. It would be much better to base the dispersion coefficient on some intrinsic porous material property.

In the case of microscale dispersion, dispersion coefficients are related to an intrinsic material property called dispersivity. This property has not yet been extended to macroscale problems. The values of calibrated "aquifer dispersivity" are typically orders of magnitude greater than values of dispersivity derived from lab analyses of small porous material samples.

Solute dispersion in porous materials was first observed during experiments to determine the rate and direction of regional groundwater flow. Using a combination of source wells and several nearby observation

wells, Slichter (1905) noted that an injected electrolytic tracer did not arrive at down gradient observation wells as a concentrated slug. Instead, the electrical conductivity of fluid in the observation wells increased gradually to a maximum and then decreased as the tracer continued to move downgradient.

After a series of laboratory flume experiments, Slichter discounted his earlier belief that molecular diffusion was responsible. He proposed that the phenomenon was probably due to the faster motion of fluid along the central thread in an interstitial pore than that near the walls of the pore. This difference would account for the stretching out or dispersion of the injected tracer mass. Analyzing the broadening of a tracer cloud in directions transverse to the general direction of flow, Slichter attributed this effect to branching or subdividing of capillary pores around porous material grains.

Dispersion associated with fluid flowing slowly through straight capillary tubes was attributed by Taylor (1953) to two mechanisms. He proposed that the spreading was due to the combined action of molecular diffusion and velocity variation over the cross section of a tube. Considering a plane, perpendicular to the longitudinal axis of a capillary tube and moving with the mean flow velocity, Taylor concluded that, in the frame of reference of this plane, dispersion of solute mass could be given by a Fickian "diffusion" process. He determined that the appropriate or "effective" diffusion coefficient was a function of both mean velocity and the molecular diffusion coefficient.

Representation of a porous material by a bundle of capillaries was held by Scheidegger (1954) to be an unsatisfactory explanation for some well-known phenomena observed in laboratory experiments. In place of a very ordered capillary model, he proposed a completely disordered model. Paralleling the theory of Brownian motion, Scheidegger applied the statistics of random processes to flow of fluids in porous materials. He introduced the porous material quantity called dispersivity.

Conducting experiments of longitudinal mixing (in the direction of flow), Brigham et al. (1961) related the longitudinal dispersion coefficient to flow velocity and an effective diffusion coefficient. Grane and

Gardner (1961) concluded that at sufficiently slow flow rates the transverse and longitudinal dispersion coefficients were equal. At sufficiently high flow rates they found that both coefficients were proportional to flow velocities and independent of fluid properties.

A tensor form of the dispersion coefficient was suggested by Bear (1961). He reasoned that the tensor would in general contain 36 independent values in the case of an anisotropic material. For isotropic conditions the tensor is adequately described by only two independent values.

Scheidegger (1961) proposed a relationship for obtaining the elements of the dispersion tensor. His formula divides porous material and flow velocity effects. The material is described by the dispersivity coefficient. Scheidegger's formulation is widely used and may be written as

$$D_{ij} = \alpha_{ijmn} \frac{v_m^* v_n^*}{|\vec{v}^*|} \quad (60)$$

where  $v_m^*$  and  $v_n^*$  are components of the average linear velocity in directions  $m$  and  $n$ ,  $|\vec{v}^*|$  is the magnitude of  $\vec{v}^*$ , and  $\alpha_{ijmn}$  is the dispersivity tensor. For isotropic conditions eq 60 reduces to two dispersion coefficients, one for a direction longitudinal to and one for a direction transverse to the piezometric head gradient:

$$D_L = \alpha_1 |\vec{v}^*| \quad (61a)$$

$$D_T = \alpha_2 |\vec{v}^*| \quad (61b)$$

The coefficients  $\alpha_1$  and  $\alpha_2$  are defined as the dispersivities of the porous material.

#### PARAMETER ESTIMATION TECHNIQUES

In this section a number of procedures for estimating aquifer parameters are reviewed. In each subsection, where a particular parameter is considered, an attempt is made to rank the procedures. Ranking is based upon perceived potential accuracy and utility under the most commonly occurring test conditions. The most promising methods, which should receive



consideration as a standard method, are indicated by an asterisk after their title.

#### Piezometric head

Piezometric head is probably the most useful and fundamental aquifer parameter. It largely determines flow patterns and mass transport, and it is involved in a number of tests for the other aquifer parameters. Techniques for accurate measurement of head, and sometimes for accurate measurement of rapid changes in head, are essential parts of many parameter estimation methods.

As explained earlier (see eq 17) head can be measured by finding the water surface elevation in a piezometer, or by measuring relative pressure at a known elevation. The first approach is the most common, while the latter may be better suited for detection of rapid changes in head. Installation of piezometers is described by Reeve (1965).

#### Radio frequency sounders\*

Boulanger et al. (1970) describe three sounding devices that employ radio (or microwave) frequency electromagnetic radiation. Advantages of each of the devices include their potential use in narrow piezometers (as small as 1.25 cm), high precision ( $\pm 0.5$  cm), and relatively low unit costs. Continuous measurement, sensitivity, ease of installation, durability and adaptability to recording-transmission modules are suggested as significant features.

Each radio frequency (RF) sounder includes a circular conducting wave guide pipe that is lowered into a piezometer until part of the pipe's lower end is filled with water. RF signals sent down the conducting pipe are reflected at the water/air interface and return to their source. The principle of operation of the device is to measure the travel time of reflected signals, and thus to determine the unfilled pipe length. According to Boulanger et al. (1970), the device consisting of a high frequency oscillator source (whose output is modulated by the return signal) is the alternative with the lowest cost and the best accuracy.

\* A promising method that has the potential to become a standard.

Possible undesirable features of the RF sounder are related to the effects of temperature variations and bending or distortion of the wave guide pipe; each device must be calibrated when it is installed. Neither of these considerations should prohibit adapting the RF sounder to piezometers or monitoring wells, although the device should be tested before it is used in the field.

#### Electric sounding probes\*

Water, with even a small amount of a dissolved ionic species, is an appreciable conductor of electric current. A sounding device can be constructed that uses this feature to detect the level of water within a piezometer or well casing.

The essential component of electric sounding devices is a probe that usually consists of two simple electrodes. When the probe is lowered into a well and held above the water surface, the electrodes form an open circuit. When the electrodes are lowered into the water, the circuit is completed and current can flow and be detected. At the point where the circuit is just completed, a measurement of the length of cable from the well head to the probe tips can be used to determine piezometric head.

Use of the basic electric sounding device just described requires an operator who lowers the probe and reads cable lengths. If used carefully, highly accurate water level measurements can be made in this way. Actual error will depend upon depth, characteristics of cable and probe, and operation of the sounder. To collect a continuous record of water level, Lovell et al. (1978) designed and built a motorized electric sounder with connected data logger. Their device will now be described.

In order to continuously monitor a well with an electric probe, provision must be made for automatically lowering or raising the probe in response to water level changes. This can be done by giving a probe three electrodes, two "long" electrodes of equal length and one "short" electrode. The long electrodes extend 0.48 cm further from the probe than their short neighbor.

A reversible electric motor is used to raise and lower the probe so as to keep the water level between the long and short electrodes. When the water level rises to the height of the short electrode, a circuit is com-

pleted, and the motor slowly raises the probe until the short electrode is withdrawn from the water. When the water level falls below the two long electrodes, the motor slowly lowers the probe until they again contact the water surface. The distance between the long and short electrodes thus establishes the sensitivity limit of the device. Water surface tension is an important factor limiting sensitivity. The inertia of the motor, probe and cable also must be considered.

The cable length is measured by passing the cable over a pulley connected to the shaft of a potentiometer. As the cable moves up and down, the electrical resistance of the potentiometer is changed. The resistance is measured and recorded by a data logger. Calibration of the device enables translation of the resistance into well water level. Lovell et al. (1978) report a  $\pm 0.5$  cm accuracy over probe cable lengths of 15 cm to 3 m. They describe the motor activation circuitry and give a schematic diagram. It is possible that improvements can be made to their design.

#### Cable and float devices \*

A counterweighted cable connected to a float that rides up and down with well water level can also be used to measure piezometric head. Joined to a continuous recorder, these devices are similar in design to electric sounding-data logging devices. If carefully constructed, sensitivity and accuracy are likely to be comparable. A number of reliable paper and drum recording devices have been developed, primarily for measuring river stage in stilling wells.

#### Pressure transducers \*

In deep wells where the casing is often not plumb, and in cases where rapid changes in head are to be measured, it may be more convenient to measure piezometric head in terms of its components, namely pressure and elevation head (see eq 17). A pressure transducer located somewhere below the water level at a known distance from the well head can do this.

Various transducers with different operating ranges, sensitivities and accuracies are available; calibration is required. A pressure transducer, being an electrical device, has great advantages over mechanical devices in terms of compatibility with data logging equipment (see Norton 1969).

### Bubbler tube-manometers \*

Consider a rigid tube, one end of which is submerged below the water level in a well. The basic principle behind bubbler tube-manometers is to measure the gas pressure required to drive all of the water out of the rigid tube. In practice this pressure is maintained such that an occasional bubble of gas is released from the submerged end of the tube.

Pressure differentials in static columns of gas are very small. Thus the gas pressure measured anywhere in the rigid tube equals the water pressure at the submerged end of the tube. The pressure and elevation at the submerged end of the tube enables head to be calculated.

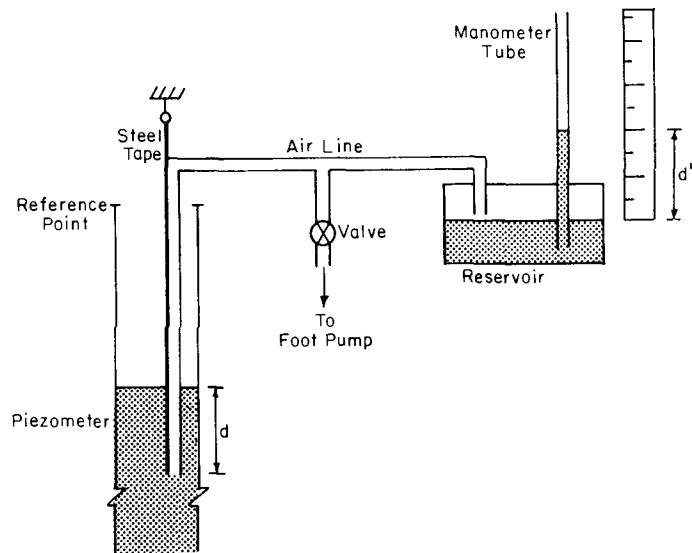
Various bubbler tube-manometer designs are described in the literature. The only substantial differences between them are their gas supply systems and pressure measurement methods. The basic and common principle of these devices makes them attractive for measurement of head in narrow wells or piezometers that are not plumb nor of constant cross section.

A relatively simple device described by Loeltz (1968) uses a foot pump to supply air under pressure to a flexible plastic tube. The flexible tube is attached to a steel measuring tape which is lowered into a well or piezometer. The steel tape allows the elevation of the submerged end of the plastic tube to be determined. This obviates the need for a cumbersome rigid tube, and makes transportation of the device much easier.

Loeltz's device uses a fluid manometer to measure air pressure in the submerged tube. A schematic diagram of the device is given in Figure 6. Using the basic fluid statics presented in the Definition of Parameters section, we can easily show that

$$d = \frac{\gamma_m}{\gamma_w} d' \quad (62)$$

where  $\gamma_m$  and  $\gamma_w$  are the specific weights of the fluid in the reservoir and water in the well respectively. Equation 62 thus enables quick determination of the water surface elevation in the well (especially when  $\gamma_m = \gamma_w$ ). Detailed drawings and photographs as well as a good practical description of the use of the device are given by Loeltz (1968). Note that



no provision is made for continuous recording, although careful operation of the device may enable gradual changes in head to be determined. Loeltz makes no specific remarks about accuracy.

A continuous record of head is afforded by two bubbler tube-manometer devices described by Beck and Goodwin (1970). Closely resembling the device in Figure 6, both use a tank of nitrogen gas to supply the submerged tube and both have a mercury-filled reservoir and manometer. The devices are operated as follows.

Note that as the head in the well changes, the fluid level (in this case mercury) in the manometer changes. Both devices of Beck and Goodwin contain sensors for detecting the mercury level in the manometer. They also have motors for raising or lowering the reservoir and manometer. The up-down motion required to return the manometer level opposite a specific point on the measuring scale is recorded. The motion  $\Delta d'$  of any one restoration event translates into a change in well water level  $\Delta d$  by the equation

$$\Delta d = \frac{\gamma_m}{\gamma_w} \Delta d' . \quad (63)$$

One of the devices detects the manometer fluid level by a float on the surface of the mercury. The position of the float may or may not contact an electrical switch that causes the manometer to rise or fall. The second design has a magnetic float that is detected by a coil surrounding the manometer. Any change in the electrical characteristics of the coil causes relative movement and reestablishment of a null condition.

Morrison and Meneley (1971) describe a nitrogen gas bubbler tube-manometer device that uses a large water reservoir and a cable and float device (see Cable and Float Devices section) to measure manometer fluid level. Advantages of their design include 30 days of unattended operation, a 100-day gas supply and a claimed  $\pm 0.3$ -cm precision.

For each of the bubbler tube-manometer devices described so far, a fluid level is observed in a manometer tube. Typically, this results in a rather slow response. If a pressure transducer is used to measure the gas pressure, a faster response is possible. This alternative also makes data logging considerably easier. Pressure transducers are described in an earlier section. The accuracy of such a device has been reported by Brockett (pers. comm.) at  $\pm 1.15$  cm.

Use of carbon dioxide gas in place of air or nitrogen has been suggested by Robbins and Hughes (1968) as a means to prevent precipitation of calcium carbonate about the tip of the submerged tube in the well or piezometer. Under fairly common water conditions, clogging of the tube may otherwise be a problem.

#### Sonic echo sounders \*

The water surface in a piezometer or well provides a boundary from which sonic echos can be reflected. Measurement of the travel time of sonic signals thus can provide a means of determining the depth to water. Although no device currently is usable in 10-cm or smaller casings, manufacturers contacted did claim the ability to build such a device. Accuracy of  $\pm 1.0$  cm is expected. One disadvantage of the sonic device is that the well casing must remain unobstructed.

### Steel tape \*

A steel measuring tape is an accurate, reliable means of determining the depth to water. The lower few meters of the tape are coated with chalk, the tape is let down the well so that the lower end is slightly submerged, a measurement of the tape length is made, the tape is hauled up out of the well and the place where the chalk is wet is noted. A simple subtraction gives an accurate depth to water. The principal disadvantage of the steel tape method is in the large amount of time required for a single measurement.

### Steel tape with popper \*

If a lead weight with a shallow depression in its base is attached to a steel tape, measurements of depth to water can be made without withdrawing the tape. The base of the weight, when dropped against the water surface, should produce an audible popping sound. The length of the tape is then noted. Ferris and Knowles (1954) describe the use of the popper and tape; they claim that accuracy is the same as for the chalked-tape method.

### Rock technique

Stewart (1970) describes a technique where the fall of a marble or BB dropped into a well is timed by listening for the splash. A simple calculation then gives the depth to water. An accuracy of  $\pm 1$  m is claimed if care is taken in making measurements with a stop watch. Wells that are not plumb pose obvious problems with ensuring free fall.

### Bong technique

This technique depends upon timing a number of echos and reechos resulting from an initial sonic impulse (a "bong"). From a knowledge of the speed of sound, the depth to water may be calculated. Stewart (1970) recommends this approach only for wells deeper than 30 m. Under ideal conditions an accuracy of  $\pm 5\%$  is claimed.

### Seismic methods

Burwell (1940) suggests that seismic signals can be used to locate groundwater table surfaces since the velocity of the disturbance is related

to water content. He claims "close" agreement between such estimates and observed water table elevation in some test borings. Due to the noise that is characteristic of seismic records, however, great accuracy cannot be expected.

### Hydraulic conductivity

The methods for determining hydraulic conductivity considered by this report may be divided into two groups: laboratory tests and field methods. Laboratory tests involve the analysis of typically small samples that may or may not be representative of undisturbed aquifer material. Field methods are in situ tests that usually consider appreciable volumes of material around the test site.

The close relationship between hydraulic conductivity, intrinsic permeability and transmissivity should be kept in mind. It can often be assumed that saturated thickness and fluid kinematic viscosity enable any two of these parameters to be estimated from the third. Accordingly, the Transmissivity section should be viewed as complementary to this section.

### Piezometer, tube and auger hole methods\*

Piezometer, tube and auger hole methods include a number of relatively simple field techniques for estimating aquifer hydraulic conductivity. Piezometer methods involve the installation of a casing that ends in saturated material above a typically short section of screen or unlined hole (15 cm is recommended). Tube methods are those in which the end of a casing is flush with or driven into aquifer material. Auger hole methods are those that use an uncased cylindrical cavity bored to a point below the piezometric surface. Diagrams of these three types of installations are shown in Figure 7 --  $d$  is the inside diameter of the tube and  $D$  is the cavity diameter.

Piezometer and tube casings can be installed using a hollow stem auger. If the auger is run through a section of casing, both auger and casing can be advanced into the soil together. For piezometers without screening, an unlined cavity remains below the casing (see Kirkham 1954).

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\* A promising method that has the potential to become a standard.



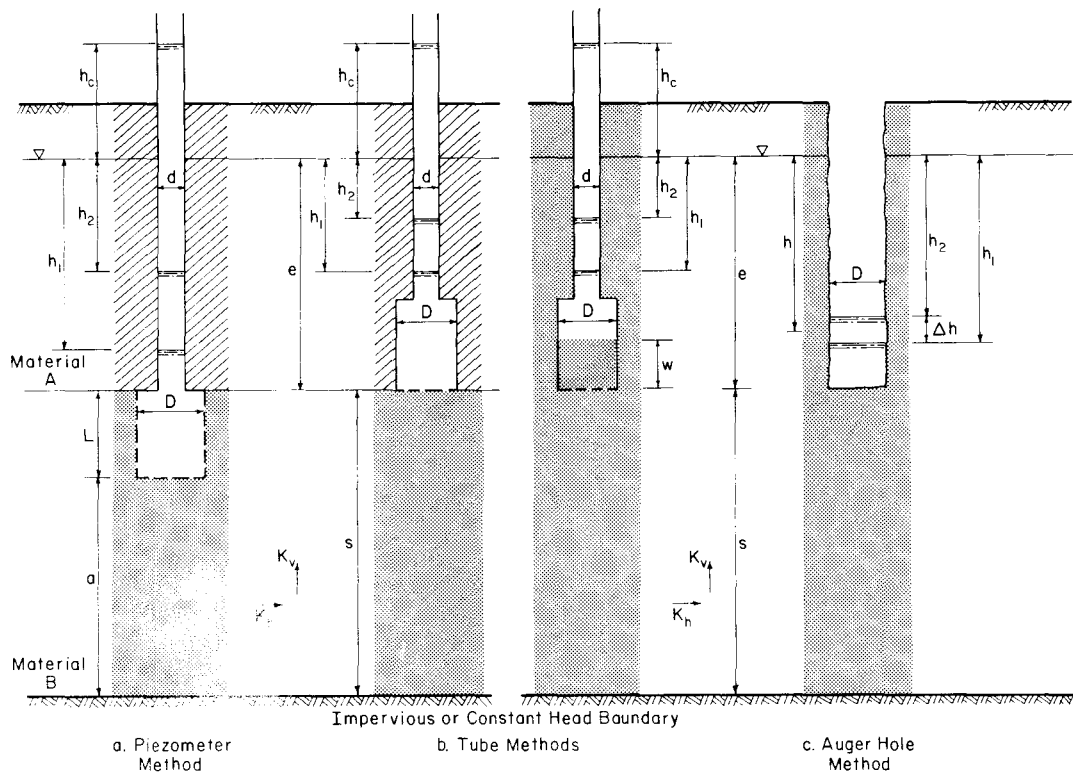


Figure 7. Piezometer, tube and auger hole tests.

Hollow stem augers are recommended so that a suction can be avoided as soil material, or the auger itself, is withdrawn from the hole. Appreciable suction in an uncased hole or cavity could produce undesirable caving. Smearing of the walls of an uncased hole or cavity could render the walls somewhat impermeable. Kirkham (1954) recommends use of a small pump to bail the hole and remove the puddled soil.

The tube methods described below are best suited for the most unconsolidated materials. Piezometer and auger hole methods must use a screen in such circumstances. Tube methods tend to measure vertical components of the hydraulic conductivity tensor, especially if the tube is driven into the porous material filling some portion of its lower end. Piezometer, and even more so, the auger hole methods tend to measure the horizontal components of  $K$ . If isotropy can be assumed, all three methods should yield the same scalar value of  $K$ .

Piezometer, tube and auger hole methods differ in the amount of material around them that they "sample." Tube methods and piezometers with short screened sections tend to measure localized point values of  $K$ . The auger hole approach tends to measure a value of  $K$  that in some sense is an average over the depth of the unlined hole.

Hydraulic conductivity tests using piezometers, tubes or auger holes can be performed under constant head or variable head conditions. For constant head, water is added (or withdrawn) at a constant rate  $q$  until a stable water level elevation  $h_c$  is obtained. For variable head a given volume of water is suddenly added (or withdrawn) and the return of the water surface to its initial static elevation is monitored.

Variable head tests are easier to do than constant head tests. "Pulsing" of the hole is possible using a weight of known volume to displace an equivalent volume of water. Lowering the weight into the hole is equivalent to adding water and it raises the water surface elevation. If equilibrium is then attained, withdrawal of the weight appears as a loss of an equivalent volume of water. Of course, bailing and recharge accomplish the same end.

Theoretically, the direction of flow, either to or from a piezometer, tube or auger hole, is immaterial. Flow from the hole into the surrounding material, however, does present the possibility of the water carrying suspended fines into the pores of the aquifer material, clogging them and preventing an accurate estimate of  $K$ . Therefore, it is preferable to perform tests by causing flow out of the porous material toward the hole.

The researcher conducting piezometer, tube or auger hole tests should take into account the type of material being encountered. Materials with very high conductivity may respond so rapidly to a pulse that no accurate record of the response (water level record) can be obtained. Materials with very low conductivity may respond so slowly that a very long time would be required to note any appreciable changes. Proper choice of piezometer, tube or auger hole dimensions can alleviate some of these problems. Reeve and Kirkham (1951) recommend 1-in. (2.5-cm) piezometers with cavities 6 in. (15.2 cm) long, tubes 8 in. (20.3 cm) in diameter, and 4-in. (10.2-cm) diameter auger holes.

The following is a list of piezometer, tube and auger hole tests as well as the appropriate mathematical equations for determining K. The derivation of each equation assumes that over the course of a test there is a negligible change of piezometric surface about the casing or unlined hole.

Piezometer method. 1. Conditions for this case are: uniform porous material, aquifer of effectively infinite thickness, constant head procedure, and materials A and B are identical. For this case the equation for conductivity is

$$K_h = \frac{q}{2\pi L h_c} \ln \left[ \frac{mL}{D} + \sqrt{1 + \left(\frac{mL}{D}\right)^2} \right] \quad (64)$$

where  $m \equiv \sqrt{K_h/K_v}$  (65)

and the remaining variables are as shown in Figure 7. Lambe and Whitman (1969) and Cedergren (1977) cite eq 64 as a result of Hvorslev (1951). If the quantity mL/D is greater than 4.0, eq 64 can be approximated as

$$K_h \approx \frac{q}{2\pi L h_c} \ln \left( \frac{2mL}{D} \right) \quad (66)$$

for isotropic materials  $K_h = K_v$  and  $m=1$ . Cedergren (1977) cites eq 64 and 66 from standardized tests compiled by the NFEC (1974).

2. Conditions for this case are: uniform porous material, aquifer of effectively infinite thickness, variable head procedure, and materials A and B are identical. Suppose that values of head  $h_1$  and  $h_2$  are measured at times  $t_1$  and  $t_2$  after a sudden pulse of water is withdrawn (or added) to the piezometer. The following result (see also Lambe and Whitman 1969, Cedergren 1977) from Hvorslev (1951) is

$$K_h = \frac{d^2}{8L(t_2 - t_1)} \ln \left[ \frac{mL}{D} + \sqrt{1 + \left(\frac{mL}{D}\right)^2} \right] \ln \left( \frac{h_1}{h_2} \right) \quad (67)$$

If mL/D is greater than 4.0, eq 67 becomes

$$K_h \approx \frac{d^2}{8L(t_2-t_1)} \ln \left( \frac{2mL}{D} \right) \ln \left( \frac{h_1}{h_2} \right) \quad (68)$$

(see NFEC 1974, Cedergren 1977).

3. Conditions for this case are: screen or cavity located directly below an impermeable material, aquifer of effectively infinite thickness, constant head procedure, and material A is impermeable.

If, as shown in Figure 7, a short screened or unlined cavity is made below an impermeable layer, the following equations of Hvorslev (1951) apply (see also Lambe and Whitman 1969, Cedergren 1977):

$$K_h = \frac{q}{2\pi L h_c} \ln \left[ \frac{2mL}{D} + \sqrt{1 + \left( \frac{2mL}{D} \right)^2} \right]. \quad (69)$$

If  $mL/D$  is greater than 2.0, eq 69 becomes

$$K_h \approx \frac{q}{2\pi L h_c} \ln \left( \frac{4mL}{D} \right). \quad (70)$$

4. Conditions for this case are: screen or cavity located directly below an impermeable material, aquifer of effectively infinite thickness, variable head procedure, and material A is impermeable. The equation for  $K$ , from Hvorslev (1951), is again cited by Lambe and Whitman (1969), and Cedergren (1977):

$$K_h = \frac{d^2}{8L(t_2-t_1)} \ln \left[ \frac{2mL}{D} + \sqrt{1 + \left( \frac{2mL}{D} \right)^2} \right] \ln \left( \frac{h_1}{h_2} \right). \quad (71)$$

If  $mL/D$  is greater than 2.0,

$$K_h \approx \frac{d^2}{8L(t_2-t_1)} \ln \left( \frac{4mL}{D} \right) \ln \left( \frac{h_1}{h_2} \right). \quad (72)$$

5. Conditions for this case are: uniform porous material, aquifer of finite thickness that is bounded below by impermeable material, variable head procedure, and materials A and B are identical. Reeve and Kirkham (1951) give the following relationship for this case:

$$K = \frac{\pi d^2}{4A(t_2-t_1)} \ln \left( \frac{h_1}{h_2} \right). \quad (73)$$

The coefficient A, having dimensions of a length, depends on the geometry of the piezometer installation. Where, with reference to Figure 7,  $e \gg L$  and  $a \gg L$ , A is found from Figures 2 and 3 of Luthin and Kirkham (1949). Boersma (1965) states that A can be assumed constant and equal to 43.2 cm for a cavity 4.9 cm in diameter and 10.2 cm in length, as long as  $e \geq L$  and  $2a \geq L$ ; error is less than  $\pm 8\%$  under these conditions. Bouwer and Rice (1976) develop a general equation for determining K for wells partially penetrating an aquifer. Their equation is

$$K = \frac{d^2}{8L(t_2 - t_1)} \ln \left( \frac{2R_o}{D} \right) \ln \left( \frac{h_1}{h_2} \right) . \quad (74)$$

Using an electric resistance analog model, we can find  $R_o$ , or equivalently the natural log of  $(2R_o/D)$ , as a function of D, L, e and s. Bouwer and Rice provide the equations and graphs necessary for determining  $R_o$ . Their approach is especially useful as L approaches  $(e+s)$ , that is as the length of screen or cavity approaches the saturated thickness. (See also case 1 in the Auger Hole section, which may apply.)

6. Conditions for this case are: uniform porous material, aquifer of finite thickness that is bounded above and below by impermeable material, constant head procedure, and material A is impermeable. Cedergren (1977) presents the results of the NFEC (1974) for piezometers penetrating a layer of finite thickness bounded above and below by impermeable material. For piezometer cavities beginning at the base of the upper impermeable layer and extending various degrees into the underlying aquifer, the results are

$$K = \frac{2q}{h_c DC_s} , \quad L \leq 0.2s \quad (75)$$

$$K = \frac{q}{2\pi L h_c} \ln \left( \frac{2L}{D} \right) , \quad 0.2s < L < 0.85s \text{ and } L > 4D \quad (76)$$

$$K = \frac{q}{2\pi L h_c} \ln \left( \frac{2R_o}{D} \right) , \quad L \approx s . \quad (77)$$

In eq 75  $C_s$  is a dimensionless function of  $L/D$ . The references for the above equations provide graphs for obtaining  $C_s$ . A comparison of eq 69 and 75 shows that

$$C_s \approx \frac{4\pi L}{D} \ln \frac{2L}{D} + \left[ \sqrt{1 + \left(\frac{2L}{D}\right)^2} \right]^{-1} . \quad (78)$$

In eq 77  $R_0$  is an effective radius, the distance to a fictitious source at constant head. NFEC (1974) recommends using  $R_0 = 100 D$ .

7. Conditions for this case are: uniform porous material, aquifer of finite thickness that is bounded above and below by impermeable material, variable head procedure, and material A is impermeable. Piezometers, described in case 6, may be tested by the variable head procedure.

Cedergren (1977) gives the following results from NFEC (1974):

$$K = \frac{\pi D}{2C_s (t_2 - t_1)} \ln \left( \frac{h_1}{h_2} \right) , L \leq 0.2s \quad (79)$$

$$K = \frac{D^2}{8L (t_2 - t_1)} \ln \left( \frac{2L}{D} \right) \ln \left( \frac{h_1}{h_2} \right) ,$$

$$0.2s < L \leq 0.85s \text{ and } L > 4D \quad (80)$$

$$K = \frac{D^2}{8L (t_2 - t_1)} \ln \left( \frac{2R_0}{D} \right) \ln \left( \frac{h_1}{h_2} \right) , L \approx s \quad (81)$$

The constants  $C_s$  and  $R_0$  are the same as described in case 6. Nguyen and Pinder (1981) describe a slug test procedure for determining hydraulic conductivity under the conditions stated above. Their formula for  $K$  is

$$K = \frac{Gd^2}{16PL} \quad (82)$$

where  $G$  is the negative slope of a straight line fit to a log-log plot of  $h(t)$  vs  $t$ , i.e.

$$G = \frac{-\ln (h_1/h_2)}{\ln (t_1/t_2)} = \frac{-\log_{10} (h_1/h_2)}{\log_{10} (t_1/t_2)} \quad (83)$$

and  $P$  is the slope of a straight line fit to a semi-log plot of  $-(\Delta h / \Delta t)$  vs the reciprocal of  $t$ ,

$$P = \frac{t_1 t_2}{t_2 - t_1} \log_{10} \left[ -\left(\frac{\Delta h}{\Delta t}\right)_1 \right] - \log_{10} \left[ -\left(\frac{\Delta h}{\Delta t}\right)_2 \right] . \quad (84)$$

In using Nguyen and Pinder's results it is necessary to estimate  $(\Delta h / \Delta t)$  as a function of time. This can be accomplished by assuming

$$\left(\frac{\Delta h}{\Delta t}\right)_i \approx \frac{h(t_{i+1}) - h(t_i)}{t_{i+1} - t_i} . \quad (85)$$

As they point out, when fitting straight lines to the two graphs mentioned above, we should avoid early values of  $h$ ,  $t$  and  $\Delta h / \Delta t$ . Note that values of  $h$ ,  $t$  and  $\Delta h / \Delta t$  used in eq 83 and 84 correspond to points lying on the fitted straight lines.

Tube method. 1. Conditions for this case are: uniform porous material, aquifer of effectively infinite thickness, porous material flush with bottom of tube, constant head procedure, and materials A and B are identical. The following formula, apparently due to Hvorslev (1951), can be found in many references (Lambe and Whitman 1969, U.S. Bureau of Reclamation 1973, 1974, NFEC 1974, Cedergren 1977):

$$K = \frac{q}{2.75 Dh_c} . \quad (86)$$

2. Conditions for this case are: uniform porous material, aquifer of effectively infinite thickness, porous material flush with bottom of tube, variable head procedure, and materials A and B are identical. Hvorslev's (1951) result for this case is

$$K = \frac{\pi d^2}{11D(t_2 - t_1)} \ln \left(\frac{h_1}{h_2}\right) \quad (87)$$

(Lambe and Whitman 1969, NFEC 1974, Cedergren 1977). Kirkham (1954) developed the following equation for  $K$  for the case  $d=D$ :

$$K = \frac{\pi D^2}{4A(t_2 - t_1)} \ln \left(\frac{h_1}{h_2}\right) . \quad (88)$$

His shape factor  $A$  depends on the depth and diameter of the tube and can be found from a table by Spangler (1951).

3. Conditions for this case are: uniform porous material, aquifer of effectively infinite thickness, bottom of tube driven into a depth of

porous material, and constant head procedure. For this case K can be found from

$$K_v = \frac{4q}{\pi D^2 h_c} \left( \frac{\pi D}{11m} + w \right) \quad (89)$$

where w, as shown in Figure 7, is the depth of material in the tube, and  $m=K_h/K_v$  (Hvorslev 1951, Lambe and Whitman 1969, NFEC 1974, Cedergren 1977).

4. Conditions for this case are: uniform porous material, aquifer of effectively infinite thickness, bottom of tube driven into a depth of porous material, and variable head procedure. Under these conditions  $K_v$  is found from

$$K_v = \frac{d^2}{8D(t_2-t_1)} \left( \frac{\pi D}{11m} + w \right) \ln \left( \frac{h_1}{h_2} \right) \quad (90)$$

(Hvorslev 1951, Lambe and Whitman 1969, NFEC 1974, Cedergren 1977).

5. Conditions for this case are: base of tube located at the boundary between an impermeable layer and an underlying aquifer of effectively infinite thickness, constant head procedure, and material A is impermeable. Hvorslev's result (1951) (see also Lambe and Whitman 1969, Cedergren 1977) is

$$K = \frac{q}{2 D h_c} \quad (91)$$

6. Conditions for this case are: base of tube located at the boundary between an impermeable layer and an underlying aquifer of effectively infinite thickness, variable head procedure, and material A is impermeable. Hvorslev's result (1951) (see also Lambe and Whitman 1969, Cedergren 1977) is

$$K = \frac{\pi d^2}{8D(t_2-t_1)} \ln \left( \frac{h_1}{h_2} \right) \quad (92)$$

7. Conditions for this case are: uniform porous material, aquifer bounded below by an impermeable layer, variable head procedure, and



materials A and B are identical. Reeve and Kirkham (1951) present the formula for the case  $d=D$ ,

$$K = \frac{\pi D^2}{4A (t_2 - t_1)} \ln \left( \frac{h_1}{h_2} \right) . \quad (93)$$

The shape factor A depends upon the geometry of the tube installation. It can be found from Figure 3 given by Frevert and Kirkham (1948).

Auger hole method. 1. Conditions for this case are: uniform porous material, hole fully penetrates to an impermeable boundary, and variable head procedure. Reeve and Kirkham (1951) and Kirkham (1954) present the following equation for K:

$$K = \frac{-\pi^2 D}{32 e S} \frac{dh}{dt} \approx \frac{-\pi^2 D}{32 e S} \frac{(h_2 - h_1)}{(t_2 - t_1)} . \quad (94)$$

The factor S is called a shape factor, and is a function of e, D and h (see Fig. 7 for definition of these variables). Kirkham and van Bavel (1948), Spangler (1951) and Kirkham (1954) give S in both algebraic and graphical form. Written out it is

$$S = \sum_{n=1,3,\dots}^{\infty} (-1)^{\frac{n-1}{2}} \frac{\cos [n\pi(e-h)/2e]}{n^2} \frac{K_1 (n\pi D/4e)}{K_0 (n\pi D/4e)} \quad (95)$$

where  $K_0$  and  $K_1$  are modified Bessel functions of the second kind (e.g. Dwight 1971, eq 804, 815 and 816).

Boast and Kirkham (1971) and Boersma (1965) reformulate eq 94 and 95 as follows

$$K = \frac{-\pi D^2}{4Ah} \frac{dh}{dt} \approx \frac{-\pi D^2}{4Ah} \frac{(h_2 - h_1)}{(t_2 - t_1)} . \quad (96)$$

The factor A, having dimensions of a length, is calculated from

$$A = \frac{8De}{\pi h} \sum_{n=1,3,\dots}^{\infty} \frac{\sin (n\pi h/2e)}{n^2} \frac{K_1 (n\pi D/4e)}{K_0 (n\pi D/4e)} . \quad (97)$$

Derivation of this latter formulation is presented by Kirkham (1965). Note that the two sets of formulae are equivalent since

$$S = \frac{A h \pi}{8 D e} . \quad (98)$$

Boersma (1965) gives a graph from which A can be estimated.

Ernst (1950) presented an approximate formula for this case,

$$K \approx \frac{-2.08 D^2 e}{h (2e + 10D) (2e - h)} \frac{(h_2 - h_1)}{(t_2 - t_1)} . \quad (99)$$

Ernst's formula gives results that are fairly close to the exact expressions, eq 94 or 96.

Kirkham (1954) points out that the derivations of eq 94-99 are based on the neglect of the effect of a cone of depression that develops about the auger hole after a sudden withdrawal of water. Using a flow net analysis he shows that this assumption is valid so long as h is greater than  $3e/4$ .

In eq 94, 96 and 99, the derivative  $dh/dt$  is replaced by its finite difference approximation  $(h_2 - h_1)/(t_2 - t_1)$ . Typically, the accuracy of this substitution increases as  $(t_2 - t_1)$  approaches zero. In practice, readings of  $h_1$  and  $h_2$  should be made at times  $t_1$  and  $t_2 = t_1 + \Delta t$ , where  $\Delta t$  is small. The value of h required in the above equations should be the average of  $h_1$  and  $h_2$ .

If a continuous recording of  $h(t)$  vs  $t$  is made (using perhaps a sensitive pressure transducer), K can be estimated from the data taken at various points during the test. This provides some feeling for the precision of the final estimate of K.

2. Conditions for this case are: uniform porous material, hole partially penetrates an aquifer bounded below by an impermeable or constant head boundary, and variable head procedure. For the case where the aquifer is underlain by an impermeable boundary, Reeve and Kirkham (1951) and Kirkham (1954) use eq 94 for K, with the substitution of  $S'$  for S. The new shape factor  $S'$  is a function of e, D, h and s, where s is the distance from the bottom of the auger hole to the impermeable boundary. Van Bavel and Kirkham (1948) and Johnson et al. (1952) provide values of  $S'$  for a limited number of values of  $D/2e$ ,  $(e-h)/e$  and  $S/e$ . Their results are based on electrical analog models.

Kirkham (1958) analyzed the auger hole problem for partially penetrating holes. He provides an exact technique for obtaining  $S'$ .

Boast and Kirkham (1971) present the most comprehensive treatment of the auger hole problem. They treat the case of partial penetration for the aquifer underlain by an impermeable (no flow) or gravel (constant head) boundary. Their simplified result is

$$K = \frac{-C}{864} \frac{dh}{dt} \approx \frac{-C}{864} \frac{(h_2 - h_1)}{(t_2 - t_1)} \quad (100)$$

The constant  $C$  is a function of  $e$ ,  $D$ ,  $h$  and  $S$ . The values of  $C$ , presented in tabular form in the reference, are such that if  $h$  and  $t$  are given in centimeters and seconds, respectively, then  $K$  is measured in meters per day. The results of Boast and Kirkham are not subject to the errors involved with the previous use of electric analog models.

3. Conditions for this case are: uniform porous material, hole penetrates an aquifer which may be assumed infinitely thick, and variable head procedure. Reeve and Kirkham (1951) point out that the effect of soil beneath an auger hole may be neglected if the depth of the hole is large compared to its diameter. Thus if  $e \gg D$ , we may assume  $s=0$  and use the results of case 1 (see also NFEC 1974).

When  $e$  is not much greater than  $D$ , the above assumption cannot be made. Johnson et al. (1952) extrapolated electric analog model results for partially penetrating holes. Their analysis yields values of  $C$  for the equation

$$K = \frac{-C}{864} \frac{dh}{dt} \approx \frac{-C}{864} \frac{(h_2 - h_1)}{(t_2 - t_1)} \quad (101)$$

where  $dh/dt$  is in cm/s and  $K$  is in m/day. More accurate results using this extrapolation principle are given in tabular form by Boast and Kirkham (1971).

Ernst (1950) provides an approximate solution for  $C$  in this case. His formula for the coefficient  $C$  of eq 101 is

$$C \approx \frac{2000 D^2 e}{h (2e + 20D)(2e - h)} \quad (102)$$

Estimates based on identification with representative porous materials of known hydraulic conductivity \*

The hydraulic conductivity of a material can be estimated from a table of geologic or soils classifications and corresponding values of K. Admittedly, the estimates obtained in this manner are crude. The uncertainty of the value of K may easily be more than two or three orders of magnitude. There are, however, several reasons for appreciating the usefulness of a table of hydraulic conductivities.

The principal advantage of a table is that no special procedure or equipment is required. Classification of aquifer material can be based on samples obtained in the course of installing wells, piezometers, tubes or auger holes.

A crude estimate of K can be of great value when specifying the particular conditions under which a more accurate test is to be performed. It has already been noted, for instance, that the rate of recovery of a depressed or elevated water level in an auger hole is proportional to K. If the hydraulic conductivity of material about a hole is very high, the recovery may be too rapid to be monitored accurately. Using an estimate of K, we can anticipate this sort of problem and suitably modify or replace the test procedure.

The relationship between hydraulic conductivity and porous material classifications enables the spatial variability of K to be inferred from knowledge of geology. A large part of the information needed to determine the path of contaminated groundwater is available from a knowledge of geology and the associated spatial variability of K.

Table 1 is a compilation of hydraulic conductivity data from several sources (Bedinger 1961, Norris 1963, Rasmussen 1963, Morris and Johnson 1967, Davis 1969, Lambe and Whitman 1969, Walton 1970, Freeze and Cherry 1979, Todd 1980).

Laboratory permeameters \*

Permeameters are laboratory devices for determining the hydraulic conductivity of small volumes of unconsolidated porous materials. Although many variations of the basic design are found in the literature, the fundamental principles involved in their operation are the same.

Table 1. Hydraulic conductivity of various materials. Single values indicate an average; question marks indicate that upper or lower limits were not determined.

<u>Material</u>	<u>K (cm/day)</u>
gravel	4,100 - 8,600,000
gravel, coarse	15,000
gravel, medium	27,000
gravel, fine	45,000
sand and gravel	810 - 20,000
sand, very coarse, and very fine gravel	24,000 - 61,000
sand	41 - 86,000
sand, very coarse	12,000 - 37,000
sand, coarse and very coarse	6,100 - 16,000
sand, coarse	3,300 - 8,100
sand, medium and coarse	1,600 - 4,100
sand, medium	810 - 2,000
sand, fine and medium	410 - 1,000
sand, fine	200 - 570
sand, very fine and fine	81 - 240
sand, very fine	41 - 120
dune sand	2,000 - 26,000
beach sand	7,000 - 17,000
sand, very fine and uniform, lacustrine and marine offshore	8.6 - 550
silty sand	0.60 - 6,000
sand, Scituate	350 - 820
sand, Plum Island	1,600 - 2,300
sand, Fort Peck	150 - 250
sand, Ottawa sand	480 - 730
sand, Union Falls	3,600 - 8,600
sand, Franklin Falls	78 - 130
sand, dam filters	130 - 8,600
silt	8
silt, loess	0.0086 - 150
sandy silt	0.0006 - 0.0026
silt, Boston	0.00086 - 0.17
silt, North Carolina	0.048 - 11
glacial till	0.0000086 - 10
glacial till, predominantly sand	49
glacial till, predominantly gravel	3,000
glacial till, northeast Ohio	0.0033 - 3.7
glacial till, surficial, Montgomery County, Ohio	0.041 - 2.0
glacial till, buried, Rohrsers Island, Ohio	0.12 - 0.55
glacial till, Southern Illinois	0.33 - 2.5
glacial till, South Dakota	0.0012 - 2.0
glacial deposit, outwash plains	4,300 - ?
glacial deposit, esker, Westfield, Massachusetts	860 - 11,000
glacial deposit, delta, Chicopee, Massachusetts	8.6 - 1,300
clay	? - 0.0086

Table 1 (cont'd).

unweathered marine clay	0.0000043 - 0.011
silty clay	0.0041 - 8.1
sandy clay	0.000022 - 22.
lean clay	0.00017 - 0.0024
sodium Boston blue clay	0.000014 - 0.0086
Vicksburg buckshot clay	0.000026 - 0.000095
compacted Boston blue clay	0.00031 - 0.0026
clay/loess	0.00035 - 0.00043
compacted caliche	0.00043 - 0.0086
calcium kaolinite	0.18 - 1.0
sodium montmorillonite	0.0016
peat	570
sandstone	0.41 - ?
sandstone, fine-grained	20
sandstone, medium grained	310
shale	0.0000010 - 0.41
slate	0.008
limestone and dolomite	0.005 - 41
limestone	94
dolomite	0.10
karst limestone	20 - 410,000
tuff	20
basalt	1.0
permeable basalt	4.1 - 410,000
schist	20
gabbro, weathered	20
granite, weathered	140
unfractured metamorphic and igneous rocks	0.00000020 - 0.0012
fractured metamorphic and igneous rocks	0.0081 - 4,100
fractured gneiss	0.0041 - 230
fractured arkosic sandstone, siltstone and shale	0.0041 - 610
fractured shale	0.20 - 490
fractured quartzite	0.016 - 220
fractured coarse-grained igneous (granite, diorite, gabbro)	0.041 - 370
fractured sandstone	0.029 - 470
fractured greenstone	0.49 - 860
fractured tight, fine-grained igneous (rhyolite, trachyte, basalt)	5.3 - 1,100
fractured schist	0.041 - 1,000
fractured carbonate rocks, augmented by tubes, tunnels, cavities	0.0041 - 9,800
fractured, fine-grained, cavernous igneous rocks	10,000 - 700,000

The central component of a permeameter is a sample tube, some portion of which is filled with the material to be tested. When a permeameter is in operation, a fluid (usually water) is permitted to pass through the sample tube. Conductivity is found from measurements of discharge and head loss accompanying the flow through the porous material in the tube.

Permeameters may be divided into two classifications: constant head or variable head. The difference depends on whether or not the hydraulic gradient established along the axis of the sample tube is permitted to change during the test. The adjective "falling head" is often used to refer to those variable head permeameters designed to be operated under a decreasing hydraulic gradient. Diagrams illustrating the two kinds of permeameters are given in Figure 8. Klute (1965) recommends the constant head device for materials with  $K$  greater than 15.0 cm/day and the falling head device for those materials having smaller  $K$  values. This does not imply that suitably modified designs cannot be operated outside these limits, however. More detailed drawings are given in the references cited below.

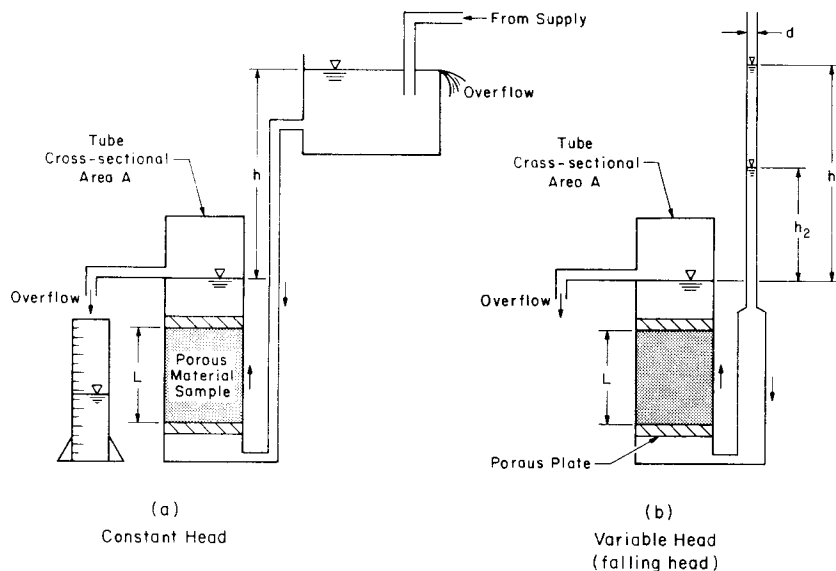


Figure 8. Permeameters.

The essential features of constant and variable head permeameters, the basic experimental procedures, and the equations for determining K are presented in a number of references (Wenzel 1942, Lambe and Whitman 1969, Cedergren 1977, Freeze and Cherry 1979, Todd 1980). Detailed laboratory procedures are given by Stearns (1927), Lambe (1951) and Klute (1965).

Modifications of permeameter design and experimental procedure have been described by many researchers. Lambe (1954) presents a constant head design that measures K for fine-grained materials. The device uses a large hydraulic gradient to obtain a significant, measureable rate of discharge through the sample. The required gradient is produced by pressurizing the compartment that supplies fluid to the sample tube. One possible disadvantage of Lambe's procedure is the potential compaction of the sample resulting from high operating pressures.

Yemington (1954) describes a modified falling head permeameter that can accurately measure hydraulic conductivity for very coarse aggregates (K up to  $3 \times 10^6$  cm/day). By comparison, he reports that the standard device can be used for materials with K values of  $18 \times 10^4$  to  $21 \times 10^4$  cm/day at most.

Compaction of porous materials can significantly alter their hydraulic conductivity. In the laboratory it may be desirable to reproduce the in situ conditions of buried sediments by loading a sample prior to determining K. A permeameter that attempts to do this is described by Jones (1954). Applying a constant head difference across a loaded sample, he was able to measure the hydraulic conductivity of various sands and gravels having K values between  $15 \times 10^4$  and  $6 \times 10^6$  cm/day.

So far in the discussion of permeameters, it has been tacitly assumed that the operating fluid is water. Recall that K is a fluid-dependent parameter and therefore is a function of interstitial fluid density and viscosity. Consequently, a value of K is not independent of fluid type or temperature. For groundwater hydrology, K is usually defined for water at 20°C.

Earlier, in the Relation to Intrinsic Permeability section, the fluid-independent conductivity parameter called intrinsic permeability k was introduced (see eq 25). From the relationship between K and k it is



clear that if the density and viscosity of various fluids are known as functions of temperature, then the "hydraulic" conductivity of a material (to any one of the fluids) can be found from its value for any other fluid. Thus  $K$  for water at a standard temperature can be calculated from a value of "hydraulic" conductivity obtained from a permeameter test using an arbitrary fluid at a temperature  $T$ .

Many other liquids have been used in permeability tests. Various gases have also been used both in constant head and variable head permeameters. The use of gas, notably air, was proposed very early in the development of measuring techniques for  $K$  (e.g. Stearns 1927). A constant head gas permeameter is described by Klinkenberg (1941); Weaver (1954) discusses a falling head design. Klinkenberg (1941) points out that at low pressures, gas permeameters produce unreliable results due to a peculiar phenomenon called "slip." He demonstrates that the use of large pressures and an extrapolation to the case of "infinite" gas pressure gives a more meaningful estimate of true conductivity. It should be noted, however, that large gas pressures may introduce undesirable compaction of the sample, and also that the necessary drying of the sample may significantly affect its internal structure. Gas permeameters are recommended only for loose sands and gravels.

The correspondence between a value of  $K$  obtained from a permeameter and that  $K$  characterizing an in situ volume of porous material depends on the degree to which the permeameter column represents the in situ conditions. Sampling of material and preparation of the laboratory column are the most important considerations when evaluating the reliability of results.

Samples used in permeameter tests may be "undisturbed" or "disturbed." Undisturbed samples are those that are carefully removed from the parent material so as to retain the existing internal porous material structure. Disturbed samples are the rule for loose, unconsolidated sands and gravels; they are often reconstituted from homogeneous mixtures of parent material. Undisturbed samples can be obtained from cohesive materials that are neither too wet nor too dry. A cylindrical push sampler similar to that of Smith and Stallman (1954) or de Ritter and Wit (1965) may be used.

For packed columns of disturbed material, density provides some means of correlating the sample with in situ material. Values of K determined at different sample densities provide valuable information that can be used to infer the range of the spatial variability of K in the field.

Care must be taken when packing disturbed samples in the permeameter tube to avoid segregation of fines. Chu et al. (1954) describe a useful technique for avoiding segregation of particles when filling the sample tube. They first fill the bottom fourth of the sample tube by pouring the sample in at the top and allowing it to fall through the length of the tube. The tube is then inverted and the sample is allowed to fall again over the length of the tube, collecting again at the lower end. The permeameter test is performed on the sample in this position. Chu et al. report far more consistent results after employing this procedure.

Preparation of a laboratory column for determination of K requires saturating the material and driving out all air trapped in the pores. The sample must be saturated with minimal disturbance of the packed column. This is to avoid roiling or settlement due to seepage forces. Gradual saturation of a sample (from the base of the sample tube upwards) may take anywhere from several minutes to many hours (Stearns 1927).

Chu et al. (1954) recommend a carbon dioxide ( $\text{CO}_2$ ) treatment of the packed column to displace air. The treatment is followed by saturation with degassed water to dissolve the highly soluble  $\text{CO}_2$ .

When a permeameter test is conducted, a fluid having much dissolved gas should be avoided. The accumulation of bubbles derived from the release of gases dissolved in the fluid can significantly affect the conductivity by effectively clogging pores. Keeping the fluid reservoir at a slightly higher temperature than the sample tube may inhibit the release of gas and obviate the need for a degassed fluid supply.

To summarize, permeameters can produce accurate results for packed samples over a fairly broad hydraulic conductivity range. It should be stressed, however, that the spatial variability of K necessarily limits the value of any single point estimate of K, however accurate it may be.

Estimates based on head gradient and specific discharge \*

If specific discharge and hydraulic gradient are known at a point in an isotropic material, Darcy's law can be used to calculate

$$K = \frac{-\vec{v}}{\nabla h} \quad . \quad (103)$$

The accuracy of a point estimate of K calculated from eq 103 depends, of course, on the accuracy of measurements of both  $\nabla h$  and  $\vec{v}$ . Hydraulic gradient ( $\nabla h$ ) can be determined with reasonable accuracy by comparing the heads observed in several piezometers in close proximity. Specific discharge ( $\vec{v}$ ) on the other hand, is not so easily estimated; the usual technique is to observe the time of travel of a tracer between two points in the medium that are a known distance apart. Unfortunately, tracers suffer from the effect of mechanical dispersion (see the Aquifer Dispersivity section) and their time of arrival at observation points may not be well defined. More will be said concerning specific discharge in a following section.

Estimates from tracer dilution in an auger hole or screened hole \*

For a given point in a porous material, specific discharge and piezometric head gradient combine to determine the hydraulic conductivity at that point. Gradient of head can generally be obtained with good accuracy from the measurement of head in neighboring boreholes (especially if two-dimensional flow and isotropy are assumed). In this section a tracer-dilution method used in a single hole for obtaining specific discharge is discussed.

Consider a tracer substance, introduced into a borehole, and mixed to a uniform concentration with the fluid in the hole. With time, the naturally existing horizontal movement of water through the borehole (or screened section of a cased hole) will diminish the concentration of the tracer. An analysis of the rate of dilution can yield a fairly accurate estimate for the specific discharge in the immediate neighborhood of the hole.

A number of different substances have been employed in tracer-dilution studies (e.g. tritium, fluorescein, various salts, isotopes). The desirability of a substance is largely a function of how easily it is detected. Determination of the tracer concentration at times after its introduction may be done by volumetric sampling or by use of a probe. Probes are to be preferred, since sampling perturbs the flow and consequently also the rate of dilution. Lewis et al. (1966) cite a number of tracer-dilution studies using either volumetric sampling or probes. Specific discharges of 1-8000 cm/day can be determined from these tests. The experimental procedure described by Lewis et al. (1966) uses fluorescein dye as the tracer and a volumetric sampling procedure in 19 to 51 mm screened well casings. Their results were obtained for fractured rocks with water tables up to 30.5 m from ground surface.

Advantages of tracer-dilution techniques center around the economy of using narrow-diameter holes and simple measurement equipment. Disadvantages include the problems of misrepresentation of the spatial variability of K by a single point value, false rates of dilution by vertical flows in the well casing, and unknown effective diameter of the screen caused by gravel packs or slumping of the hole during borehole construction. Sorption of tracers by the porous material is not a problem.

The use of packers, which effectively isolate sections of a hole or well screen, can eliminate the problem of vertical flows in the hole (packers are inflatable rubber bladders that are inserted into a well casing). At the same time they allow for estimating the conductivity of individual layers of material; the standard approach gives a value of K representative of that over the entire screened section.

#### Solution of the regional inverse problem

By definition, the effective management of a system depends upon the ability of the manager to foresee the results of various proposed strategies. For extensive aquifer systems, management often means determining the effects of proposed strategies on regional piezometric head.

A number of predictive methodologies have been developed for calculation of piezometric head in aquifers. These usually consist of a numeri-

cal solution of the differential equation of groundwater flow (Darcy's law - mass conservation). Inputs to the numerical solution procedures include initial or boundary conditions, and regional definitions of recharge and pumping, storage parameters, saturated thickness and hydraulic conductivity. Solution of the flow equation for piezometric head is referred to as solving the "direct" problem.

If the piezometric head in a region is known, but some of the inputs to the direct problem are unknown, then an "inverse" problem may be formulated to find those unknowns. Of particular interest here is the case in which hydraulic conductivity is unknown and is to be determined from the remaining inputs and the known head distribution. For unsteady flows, storage parameters may be determined along with hydraulic conductivity. At this point it will suffice to consider only steady flow and the attendant advantages and disadvantages of solving the associated inverse problem.

Solution of the inverse problem is complicated by the fact that, as just formulated, it is ill-posed. As a consequence, a multitude of different areal distributions of  $K$  will satisfy the conditions required of the solution. Since only one distribution of  $K$  can be the correct one, the inverse problem is ill-posed in the sense of not having a unique solution. In order to give the inverse problem a unique solution, additional conditions must be satisfied. For steady flow, uniqueness can be guaranteed if the distribution of  $K$  is required to have specific values of  $K$  at all points along a curve that intersects every streamline in the flow. The required values of  $K$  on that curve may be specified using one of the other methods described in this section. Theoretically, the effort involved is well rewarded, since as a result  $K$  may be determined over a wide area. An additional point to note is that specific discharge data may be used in place of the required values of  $K$ , since  $\vec{v}$  and  $K$  are related through Darcy's law and  $\nabla h$  is known.

From a practical standpoint, solution of the inverse problem cannot be relied on for producing accurate results. Although uniqueness can be guaranteed, there remain two serious drawbacks to this approach. First is the problem of sensitivity of  $K$  to small errors in measured piezometric head. Daly (1981) has shown that these errors can produce large and

spurious fluctuations in the resulting K distribution. The second problem concerns estimates of K for regions where there is little or no flow, i.e. where the gradient of piezometric head approaches zero. In those regions K becomes indeterminate. This is because it is fundamentally impossible to determine the hydraulic conductivity of a material unless it exerts a frictional resistance on a moving fluid.

Empirical equations based on physical analysis of samples

It is intuitively reasonable to assume that the conductivity of sands and gravels is related to the size of particles making up those materials. Before the turn of the century, Hazen postulated a simple relationship between conductivity and representative grain size

$$K = C d_{10}^2 \tag{104}$$

where  $d_{10}$ , determined by sieve analysis, is the "grain diameter" that determines the point where 10% of the total soil sample by weight is made up of particles of smaller grain diameter. If  $d_{10}$  is given in millimeters, and K is to be obtained in cm/s, then C varies from 0.9 to 1.2. Wenzel (1942) points out that C has its higher values for uniform, clean, loosely packed sands. Slichter (1899) develops a formula quite similar to that of Hazen. It must be emphasized that eq 104 applies only for loose filter sands.

A number of empirical formulas for K attempt to include porosity as a factor. The most well known of these is a formula called the Kozeny-Carman equation (Carman 1956):

$$K = d^2 \frac{\gamma}{\mu} \frac{n^3}{(1-n)^2} \cdot C \tag{105}$$

where d is an effective grain size;  $\gamma$  is the fluid specific weight, and  $\mu$  its viscosity; n is the porosity. The constant C varies from 0.0090 to 0.0045; Freeze and Cherry (1979) suggest 0.0056.

An equation that purports to include the distribution of particle sizes is that of Fair and Hatch (1933),

$$K = \frac{\gamma}{5\mu} \frac{n^3}{(1-n)^2} \frac{1}{\left(\frac{S}{100} \sum \frac{r_i}{d_i}\right)^2} \tag{106}$$

where  $S$  is a shape factor varying from 6.0 for spherical particles to 7.7 for angular grains. From a sieve analysis,  $r_i$  is the percent (by weight) of material held between sieves  $(i-1)$  and  $(i+1)$ ;  $d_i$  is the geometric mean of sieve mesh sizes  $(i-1)$  and  $(i+1)$ .

Krumbein and Monk (1943) include in their calculation formula for  $K$  a geometric mean diameter and the standard deviation of the particle size distribution of a sample. Masch and Denny (1966) present similar results in graphic form.

Other investigators have sought to include the degree of compaction (sample density) in the formula for  $K$ . Sample density was considered by Burmeister (1954) for a range of materials from coarse sand to coarse silt. His experimental results are presented graphically. The importance of sample density is shown by Slichter (1898) who points out that the hydraulic conductivity of materials composed of identical spheres may differ by a factor of more than 7, depending on the packing arrangement (sample density). The important effect of compaction cannot be overemphasized.

The empirical relationships referred to above can, at most, be recommended as rough estimators of  $K$ . Their application is limited to clean, unconsolidated materials with very few or no fines. Inaccuracy of the empirical equation approach simply reflects the difficulty of accounting for all factors controlling  $K$  in all types of porous material.

#### Estimates from borehole logging

In groundwater hydrology the term "geophysical survey technique" refers to any methodology designed to estimate a characteristic of subsurface materials or interstitial fluids by remote sensing. Borehole logging methods are those geophysical survey techniques conducted in unlined, drilled or augered holes. A particular borehole log is made by running one of a variety of special probes over the length of a hole, simultaneously recording the depth and response of the probe. Standard logging techniques include: electrical (resistance, resistivity, spontaneous potential, borehole fluid resistivity), nuclear (natural gamma, gamma-gamma, neutron), acoustic, caliper, temperature, flowmeter, and television monitoring (Keys and MacCary 1971).

A comparison of several different kinds of logs from the same borehole can yield information not readily apparent from a single log. An experienced interpreter can develop a valuable picture of subsurface materials and fluids. When results from several boreholes are compared, the horizontal extent of particular materials can often be observed and extrapolated.

Borehole logging is principally a mapping technique, and therein lies its great value as a tool for geohydrologic investigations. Although some researchers have postulated a functional relationship between  $K$  and the borehole resistivity formation factor (Archie 1942, Heigold et al. 1979), experimental data show such large scatter as to effectively eliminate borehole logs as a means of directly determining  $K$  with appreciable accuracy.

Other investigations have attempted an indirect approach by first correlating  $K$  with porosity  $n$  for the area under investigation. Porosity, which is somewhat more reliably predicted by various logs, is then the key to finding  $K$  (e.g. Bredehoeft 1964). Since a clear functional relationship between  $K$  and  $n$  cannot always be obtained, and because the accuracy of porosity determinations is not large, this procedure is not recommended. The additional lab work required to correlate  $K$  and  $n$  is another reason for discounting this approach.

#### Analysis of groundwater temperatures

Bair and Parizek (1978) describe a field study in which they related unexpected subsurface temperature changes to spatial variation of hydraulic conductivity. Taking measurements of temperature and comparing them with results calculated for a homogeneous material, they were able to discover areas where low temperature variability was due to high permeability and greater advection of heat by the moving groundwater. The results do not, however, attempt to quantitatively determine  $K$  as a function of space coordinates.

Stallman (1963) describes an inverse problem procedure using piezometric head and temperatures to determine the spatial variability of  $K$ . While theoretically sound, the approach suffers from all the drawbacks of "inverse problem" techniques mentioned earlier. In addition, Stallman's approach would require detailed information on thermal conductivities and



boundary conditions for the heat flow equation, which becomes involved in the solution procedure.

Neither of the above methods appears promising as an estimator of K.

Estimates from response of observation well to sea tides or surface water level changes

The fluctuation of water level at the boundary of an aquifer will cause a disturbance to the piezometric head distribution in the aquifer. The magnitude and time lag of the response is a function of aquifer storage parameters and hydraulic conductivity. Theoretically, if the initial fluctuation and the aquifer response are observed, the aquifer parameters may be obtained. This is another form of the inverse problem.

Fluctuations of aquifer boundary conditions may result from sea tides, changes in river stage or reservoir releases. It is safe to say that these conditions represent fairly uncommon situations and therefore limit the usefulness of the method.

Regardless of the problem of applicability, the method suffers from most of the pitfalls of the other inverse problem methods. Even if an entire aquifer is assumed to be homogeneous, there must be a large response in the aquifer to obtain appreciable accuracy in the estimated K. In transient response problems, such as this one, knowledge of the storage behavior of the aquifer is also required. This is cited by de Ritter and Wit (1965) as the reason why they failed to get reasonable results when they tried to use sea tide response. Although Carr and van der Kamp (1969) report good results, their one-dimensional flow situation cannot be considered indicative of the majority of aquifers subject to fluctuating boundary conditions.

Direction and magnitude of specific discharge vector

Knowledge of the spatial distribution of specific discharge  $\vec{v}$  is essential for predicting the fate of contaminants in groundwater. The most promising of the existing test procedures for obtaining  $\vec{v}$  involve the introduction and subsequent sensing or recovery of dissolved tracers. Included are single and multiple well tests using radioisotopes, electrolytes and dyes -- even heat has been suggested as a tracer.

Darcy's law provides a link between piezometric head gradient, hydraulic conductivity and specific discharge. Estimates of  $\vec{v}$  can be obtained from Darcy's law, subject to the accuracy of determinations of K and  $V_h$ . In isotropic materials the direction of  $\vec{v}$  is in the direction of  $V_h$ .

It should also be recalled that the average rate at which tracers move through the groundwater is the average linear velocity. Specific discharge and average linear velocity are related through the effective porosity as given in eq 42.

#### Point dilution methods\*

Consider an isolated screened section of a well (or unlined borehole) established in the zone of complete saturation. Inflatable packers are often used to seal off the section from the rest of the water column, and in some cases baffles are also used.

The basic operation of a point dilution test involves the one-time introduction of a tracer into the fluid in the screened section. As the natural background flow of groundwater continues, the concentration of tracer in the borehole fluid decreases due to the "flushing" action of water traversing the borehole. The borehole fluid is continuously mixed and the decline of tracer concentration is recorded.

The recorded dilution curve obtained from a point dilution test can be related to the magnitude of the background flow rate, namely  $|\vec{v}|$ . Applications of the point dilution test to field studies are reported by Raymond and Bierschenk (1957) using an electrolyte, and by Lewis et al. (1966) using fluorescein dye. The relationship used by Lewis et al. is

$$|\vec{v}| = \frac{\pi d}{8t} \ln \frac{C_0}{C} \quad (107)$$

where d is the well bore diameter,  $C_0$  is the concentration of tracer at time zero and C is the concentration at time t. If  $\ln(C_0/C)$  is plotted vs t,  $|\vec{v}|$  can be determined from the slope of a straight line fit through the data points.

\* A promising method that has the potential to become a standard.

A more detailed analysis of the dilution problem was presented by Drost et al. (1968). Their result is given by the following:

$$\left| \vec{v} \right| = \frac{V}{\alpha F t} \ln \frac{C_0}{C} \quad (108)$$

where  $V$  is the dilution volume of the isolated borehole segment,  $F$  is the vertical cross-sectional area of the dilution volume perpendicular to the flow, and  $\alpha$  is given by

$$\alpha = \alpha_0 [1 - f(\text{Re})] \quad (109)$$

where  $f(\text{Re})$  is a function of the Reynolds number  $\text{Re}$ . Drost et al. do not determine  $f(\text{Re})$  except to say that  $0 \leq f(\text{Re}) \leq 1$ , and that  $f(\text{Re})$  may be determined experimentally. In nearly all field situations the flow will be laminar and  $f(\text{Re})$  may be assumed zero. The parameter  $\alpha_0$  depends upon the geometry and hydraulic properties of well screen, gravel pack and aquifer material (with "hydraulic conductivities"  $K_1$ ,  $K_2$  and  $K_3$  respectively). If  $K_1 \gg K_2 \gg K_3$ , only the geometry of the well cross section is needed to accurately determine  $\alpha$ . Unfortunately, the refinements of Drost et al. may be rendered useless unless the well screen-gravel pack is very carefully constructed to rigid specifications. In all cases the omission of a gravel pack would promote confidence in the results of a point dilution test. Again, augering, with or without screened casing, presents a more desirable alternative for monitor well construction. Grisak et al. (1977) present the results of a field study using eq 108.

A comparison of eq 108 and 109 shows that the formula of Lewis et al. (1966) assumes  $\alpha = 2$ , corresponding to laminar flow, the absence of gravel pack, and neglect of well screen thickness and accompanying head loss.

While the magnitude of  $\vec{v}$  is found by applying the above formulas, the direction of flow cannot be determined from the point dilution test as described above. A number of special probes have been developed, however, which, subsequent to the point dilution test, are able to sense the direction of travel of radioactive tracers that have left the borehole. The instruments described by Payne et al. (1965), Selecki and Filipek (1966), and Drost et al. (1968) are collimated detectors that scan the compass direc-

tions and record the intensity of radiation entering the borehole. The particular direction at which the detector senses the most intense radiation is clearly the direction of movement of the tracer and the groundwater. Drost et al. suggest the use of tracers with a relatively short half-life (2-30 days) and high specific activity (those detectable through intervening soil). They also suggest procedures that promote the adsorption of tracer onto soil particles; this facilitates clearer definition of the tracer flow direction. Isotopes with short half-lives preclude danger to public health since the rate of flow of groundwater is usually quite slow.

A two-well point dilution technique for determining both the magnitude and direction of  $\vec{v}$  has been proposed by Saleem (1969). One of the wells is called the master well and is used for performing point dilution tests. The second well, used in a portion of the test procedure, is pumped at a steady rate so as to perturb the flow field at the master well.

Saleem's approach is to first observe the dilution of a tracer under natural background conditions; this determines  $|\vec{v}|$ . Then the second well is pumped at a steady rate (typically small) that perturbs the piezometric head gradient at the master well at the same order of magnitude as the natural gradient. A second point dilution test is made in the master well and a new value  $|\vec{v}'|$  is determined. Simple geometry is then applied to obtain the direction of  $\vec{v}$ . One advantage of the procedure is that it does not require the use of radioactive tracers. Homogeneity of the material around both wells is assumed, however.

In an earlier section, some of the advantages and problems of point dilution tests were presented. Given the availability of equipment, the technique has the potential for giving accurate spatial definition of flow field at modest cost. Detailed drawings of down-hole equipment and more specific test procedures are provided in the references, especially Drost et al. (1968) and Saleem (1969). Good reviews of earlier work are provided by Halevy et al. (1967) and Lewis et al. (1966).

#### Recovery of tracers from down gradient wells

The most direct method of determining groundwater flow velocity is to observe the time of travel of a tracer between two points a known distance

apart. Given the direction of flow (from a contour map of  $\nabla h$ ) the magnitude of the apparent average linear velocity can be obtained. From this result, and a known effective porosity, the specific discharge magnitude can be calculated.

Tracer recovery tests have been conducted by various investigators since the late 19th century. Earlier, in connection with the definition of dispersion coefficients, the work of Slichter (1905) was described. Others (e.g. Dole 1906, Pearson and White 1967, Aulenbach et al. 1978, Muir and Coplen 1981) have used the same approach in tracking a wide variety of tracers.

The most common problems plaguing tracer recovery tests are lack of precise definition of flow pattern; dispersion and dilution of the tracer; adsorption onto, or absorption into, porous material grains; and the typically slow rate of movement of groundwater and tracer. A precise description of flow pattern is required in order to properly place recovery wells. It has not been uncommon for tracer clouds to completely bypass recovery wells and remain undetected despite short distances between those wells and the tracer source.

As Slichter (1905) observed, a tracer does not travel through a porous material without some mixing and dispersion. This makes tracer concentration, observed at a recovery well, slowly increase to a maximum and then slowly decrease as the tracer cloud moves by. Identification of time of travel is thus subject to error. Adsorption and absorption, and later release of tracer, also tend to introduce error in observed time of travel.

The final difficulty (slow rates of movement) may result in very long times for a tracer recovery test. Estimates of very small rates of flow may be considerably in error.

#### Use of temperature logs

The use of groundwater temperature as a tracer has been suggested by Keys and Brown (1978). Unfortunately, the heat content of groundwater is very much affected by "absorption," and the method suffers from all of the remaining difficulties associated with tracer recovery tests. As stated earlier, however, groundwater temperature sensing may provide some valuable indication of the spatial variability of conductivity and hence  $\vec{v}$  as well.

### Flow direction from piezometric head contour maps \*

Flow patterns produced from contour maps of head usually assume two-dimensional flow. The accuracy of the pattern depends on the density of sample points and the interpolation scheme used to draw the contours. For regions where  $V_h$  varies quickly with respect to both magnitude and direction, there is no substitute for a dense array of sample points.

Many contour plotting routines are available on computers. An approach using a number of randomly located sample points is described by Pinder et al. (1981). In all cases flow lines and equipotentials must be orthogonal to one another. Equipotentials must be drawn so as to correspond with sources of constant head (bodies of water) and be perpendicular to no-flow boundaries.

### Transmissivity

Transmissivity  $T$  is an aquifer property that is defined only for groundwater flow that is two-dimensional, horizontal and saturated. As was shown in the Definition of Parameters section, it is the product  $\bar{K}b$ , where  $\bar{K}$  is an average hydraulic conductivity taken over the saturated thickness  $b$ .

Fundamental principles show clearly that  $T$  cannot be estimated without "sampling" the hydraulic conductivity over the entire saturated thickness. Consequently, nearly all estimation procedures based on hydraulics involve the use of fully penetrating wells that are screened over a major part of the saturated thickness.

### Slug and bailing tests\*

Slug tests for obtaining transmissivity involve the sudden, measured increase of piezometric head at a well; bailing tests involve a sudden, measured decrease of head. Both tests depend on the observation of the return of piezometric head to its equilibrium value. Occasionally, slug-bail tests are conducted using a series of individual increases and decreases of head.

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\* A promising method that has the potential to become a standard.

Changes in piezometric head may be made by adding or withdrawing water, inserting or withdrawing a weighted float, or by varying the pressure within a sealed casing. The pressure tests described by Bredehoeft and Papadopulos (1980) may greatly decrease the time required for observations.

The results of an analysis of the hydraulics of slug tests apply equally well to bail tests, a decrease in head being simply a negative increase. Therefore, the following may be considered to apply to both cases.

The analyses of Skibitzke (1954), Ferris and Knowles (1954), and Ferris et al. (1962) are based on the same assumptions used to derive the classical Theis equation. The well is assumed to be a fully penetrating line source in an aquifer of thickness  $m$ , infinite areal extent, and characterized by constant transmissivity  $T$  and storage coefficient  $S$ . Initially, the piezometric head is at equilibrium, then at time zero a volume of water  $q$  is added to the well. At time  $t$  the perturbation to the initial piezometric head (observed in the well of radius  $r_w$ ) is

$$s = \frac{q}{4\pi Tt} e^{-\frac{r_w^2 S}{4Tt}} \quad (110)$$

As  $t$  becomes large this can be approximated as

$$s \approx \frac{q}{4\pi Tt} \quad (111)$$

Ferris and Knowles (1954) use eq 111 to determine  $T$  from field data. Their result compares favorably with the result of a pumping test conducted at the same site.

Papadopulos and Cooper (1967) present a modified form of the Theis solution which treats the pumping well not as a line source, but as a cavity of finite diameter having measurable storage capacity. Their solution is a significant improvement, especially for large diameter wells penetrating materials of low hydraulic conductivity.

Beginning from the modified Theis equation, Cooper et al. (1967) obtain the solution for piezometric head resulting from addition of a slug of water to a well of finite diameter. With reference to Figure 9,

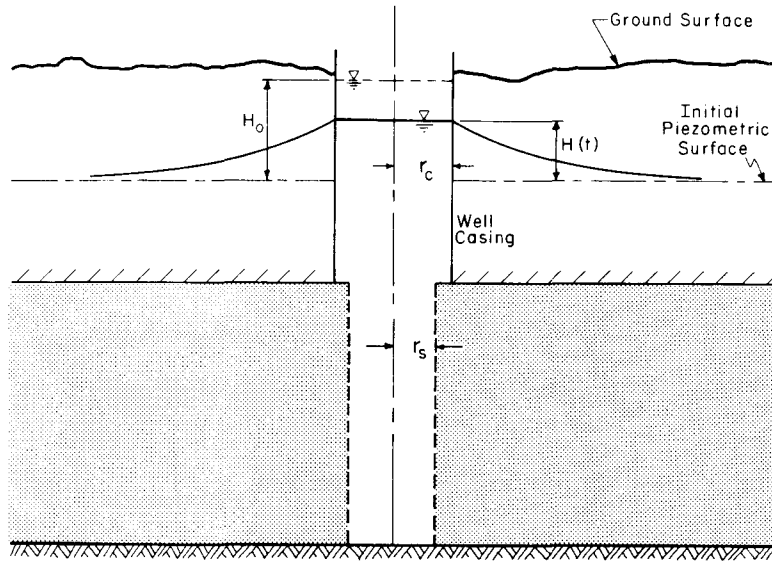


Figure 9. Slug test (after Cooper et al. 1967).

$$H(t) = \frac{8}{\pi^2} H_0 \alpha \int_0^{\infty} e^{-\beta u^2 / \alpha} \frac{du}{u \Delta(u)} \quad (112)$$

where  $\alpha = r_s^2 S / r_c^2$ ,  $\beta = Tt / r_c^2$ , and

$$\Delta(u) = [uJ_0(u) - 2\alpha J_1(u)]^2 + [uY_0(u) - 2\alpha Y_1(u)]^2. \quad (113)$$

The functions  $J_0(u)$  and  $J_1(u)$  are Bessel functions of the first kind, while  $Y_0(u)$  and  $Y_1(u)$  are Bessel functions of the second kind. Cooper et al. (1967), Papadopoulos et al. (1973), and Bredehoeft and Papadopoulos (1980) express eq 112 more conveniently as

$$H(t)/H_0 = F(\beta, \alpha) \quad (114)$$

where  $F(\beta, \alpha)$  is tabulated in those references (calculated by evaluating the integral of eq 112).

In order to obtain transmissivity (and with much less accuracy, storage coefficient), the well-known graphic "type-curve" method can be applied. Bredehoeft and Papadopoulos (1980) suggest two separate type-curve



methods, the choice between them depending upon the value of  $\alpha$ . (The parameter  $\alpha$  is unknown at the outset of the analysis, but it may be presumed and later calculated and compared with the presumed value.) The two type-curve methods will now be briefly described.

For  $\alpha$  which turns out to be less than 0.1, the following procedure is recommended. First, on a sheet of semilog paper, a family of type curves  $F(\beta, \alpha)$  vs  $\beta$  is plotted, each individual curve corresponding to a specific value of  $\alpha$ , e.g.  $10^{-1}$ ,  $10^{-2}$ , ...  $10^{-10}$ . (The parameter  $\beta$  is plotted along the logarithmic axis.) Next, on an identical sheet of semilog paper, the field data are plotted,  $H(t)/H_0$  vs  $t$ , with  $t$  along the logarithmic axis. The field data plot is then overlain on the first such that the logarithmic axes coincide. Then the field data plot is moved in the direction of the logarithmic axes until the points of the field data curve lie on one of the underlying type curves. This last step is made easier by use of a thin tracing paper for the field data plot. Next, record the  $\beta$  and  $t$  values of any two overlapping points along the logarithmic axes, and record the  $\alpha$  value of the type curve underlying the field data. These values and the defining equations for  $\alpha$  and  $\beta$  are sufficient to establish  $T$  and  $S$ . Unfortunately, as noted by the preceding references, the estimate of storage coefficient by this method is not accurate. It turns out that field data may, in many cases, be fit to type curves having values of  $\alpha$  differing by orders of magnitude; this uncertainty is directly reflected in the estimate of  $S$ . The uncertainty in the estimate of  $T$  is, however, much less. Papadopoulos et al. (1973) show that near  $\alpha = 10^{-5}$  an uncertainty in  $\alpha$  of two orders of magnitude corresponds to an uncertainty in  $T$  of only 30%.

For  $\alpha$  that turns out to be greater than 0.1, Bredehoeft and Papadopoulos (1980) note that the technique as just described produces large uncertainty for both  $T$  and  $S$ . They conclude that under many conditions only the product of  $T$  and  $S$  may be obtained with appreciable accuracy. The suggested approach for the case of  $\alpha > 0.1$  is to attempt to determine  $T$  and  $S$  separately; if this cannot be done with small uncertainty, estimate  $TS$ . The tables provided in the reference allow a family of type curves, one for each value of  $\alpha$  (e.g. 0.1, 0.2, 0.5, 1.0, ...), to be plotted. The type curves are plotted as  $F(\beta, \alpha)$  vs  $\alpha\beta$  on semilog paper, with  $\alpha\beta$  on the log

axis. Field data are then plotted as a curve of  $H(t)/H_0$  vs  $t$ , with  $t$  on the log axis. Type curve matching is done as before and an attempt is made to fit the data to a specific type curve. If this can be done with good accuracy, then the values  $\alpha$ ,  $\alpha\beta$  and  $t$  can be used to get  $T$  and  $S$  separately. If, on the other hand, the uncertainty in  $\alpha$  is large, then only  $TS$  may be obtained with appreciable accuracy. This is done by making an approximate curve match and recording the values  $t$  and  $\alpha\beta$  of two overlapping points on the logarithmic axes. Note that

$$\alpha\beta = \frac{r_s^2 TSt}{r_c^4} . \quad (115)$$

A number of the assumptions underlying the analysis of slug test hydraulics have already been stated. From the point of view of evaluating the slug test as a field method, well construction is by far the most important aspect to consider. It is certain that the largely unpredictable effects of irregular borehole cross section and gravel pack may render the test quite incapable of yielding reliable estimates of the properties of surrounding aquifer material. Even if the geometry and hydraulic properties of the well screen and gravel pack were known exactly, the slug test as described above could not take this information into account. More serious is the fact that the gravel pack prevents the measurement of aquifer properties by masking their effect on the well. As pointed out by Ferris et al. (1962) the transmissivity determined by the slug test is representative only of the material close to the well. This is a consequence of the simple idea that the hydraulic properties of a material cannot be known unless the material can exert frictional resistance on a moving fluid. This can also be seen from the slug test equations which show that nearly all head loss is experienced within very small distances from the well, especially when the storage coefficient is appreciable.

In summary, the slug test has the potential for good accuracy when the well screen is highly permeable and in intimate contact with undisturbed aquifer material. The presence of a gravel pack, however, even one that is carefully placed, effectively prevents measurement of aquifer properties by this method.

### Solution of the regional inverse problem

In an earlier section the inverse problem approach for determining hydraulic conductivity was explained. An even more popular version of the problem is that one that attempts to determine transmissivity. The only difference between the two problems is that the first assumes knowledge of saturated thickness. Unfortunately, all of the difficulties attendant to the problem for K also apply for determining T. Thus, while it is possible, in theory, to determine  $T(x,y)$  by working the inverse problem, instability hampers its use as an accurate, practical tool (Neuman 1975, Yakowitz and Duckstein 1980, Daly 1981).

Despite the inherent difficulties of the method, a number of papers on the inverse problem have appeared in the literature (Stallman 1956, Emsellem and de Marsily 1971, Birtles and Morel 1979, Neuman and Yakowitz 1979, Yeh and Yoon 1981). In each case, however, some additional information about the smoothness of the spatial distribution of T must be assumed in order to control instability. From the point of view of contaminant transport, it is questionable whether such assumptions are justifiable.

### Estimates from response of an observation well to changes in surface water levels

The comments of the earlier subsection with the same title as above in the Hydraulic Conductivity section apply directly to methods for estimating transmissivity by observing water level response in a well due to changes in water level in a hydraulically connected body of surface water. Both theory and field application are described by Ferris (1950, 1951).

### Volumetric flow rate

Volumetric flow rate (and mass flow rate of contaminants) can be calculated from estimates of specific discharge (see the Direction and Magnitude of Specific Discharge Vector section).

### Total porosity

Total porosity is the ratio of void space to the total volume of a sample of porous material.

### Laboratory measurements of bulk and particle densities\*

Total porosity  $n$  may be determined from the equation

$$n = 1 - \frac{\rho_T}{\rho_s} \quad (116)$$

where  $\rho_T$  is the bulk density of a sample, and  $\rho_s$  is an average particle density.

Bulk density can be determined in the laboratory by first measuring the volume of a sample of porous material and then drying the sample until it assumes a constant mass. The bulk density is then the ratio of the mass of the dried sample to its original volume.

In the field, a coring device can be used to obtain a measured volume of material, although the extent of possible compaction must be considered.

At or very near the ground surface, samples can be taken with a trowel, small shovel or spoon. The volume of the sample is found by measuring the volume of the remaining excavation. The volume of the excavation can be obtained by refilling the hole with sand. The sand used for this is held in a container whose initial and final sand volumes can be easily measured. Alternatively, the excavation volume can be measured by filling a balloon, placed in the hole, with water. If the balloon is filled to the level of the initial soil surface, the volume of water in the balloon is equal to the volume of the excavation (Blake 1965).

For a consolidated chunk of porous material, Archimedes' principle can be applied to determine volume. Weighing the chunk in air ( $W_1$ ) and then in water ( $W_2$ ) (presuming that water can be kept out of the sample pores) we see that

$$\frac{W_1 - W_2}{\gamma_w} = V_{\text{sample}} \quad (117)$$

where  $\gamma_w$  is the specific weight of water. In practice, a thin coating of paraffin is applied to the sample before weighing in water; this prevents

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\* A promising method that has the potential to become a standard.

the absorption of water. The paraffin is applied by dipping the chunk into a container of that material held at a temperature slightly higher than its melting point. The volume of the paraffin coating must be subtracted from the volume of sample plus paraffin as determined from eq 117. Thus

$$\frac{W_3 - W_p}{\gamma_w} + \frac{W_5 - W_3}{\gamma_p} = V_{\text{sample}} \quad (118)$$

where

$W_3$  = weight in air of sample plus paraffin

$W_4$  = weight in water of sample plus paraffin

$W_5$  = weight in air of sample

$\gamma_p$  = specific weight of paraffin coating.

A detailed procedure for this technique is given by Stearns (1927).

Once bulk density is determined, the remaining unknown in eq 116 is the particle density. Particle density is determined using a pycnometer. A pycnometer is simply a volumetric flask that can be filled and refilled with water to precisely equal volumes. Particle density is determined by another application of Archimedes' principle. The procedure depends upon four separate weighings of the pycnometer. First, the empty pycnometer is weighed ( $W_1$ ). Then it is filled to roughly 20% of its volume with a sample of dry material and weighed a second time ( $W_2$ ). The pycnometer with sample is filled with water, precautions being taken to remove all air from the sample. The pycnometer is weighed a third time ( $W_3$ ) and then completely emptied. Finally, it is filled to its previous volume with water and weighed a fourth time ( $W_4$ ). Particle density is found from

$$\rho_s = \rho_w \frac{(W_2 - W_1)}{(W_4 - W_3) + (W_2 - W_1)} \quad (119)$$

where  $\rho_w$  is the density of water at the temperature under which the test is run. Details of the procedure are given by Stearns (1927), Lambe (1951) and Blake (1965).

Estimates based on identification  
with materials of known total porosity

Manger (1963) has tabulated the bulk density and porosity (or effective porosity) of more than 900 types of sediments and sedimentary rocks.

His compilation includes remarks on the experimental procedure used for each determination, the number of samples of each sediment or rock type, and the average, minimum and maximum values observed in analyzing the samples. Values of porosity are also tabulated by Morris and Johnson (1967) and Davis (1969).

#### Borehole logging

Keys and MacCary (1971), in their discussion of applications of borehole geophysics to water resources investigations, suggest several logging techniques for estimating porosity. For each technique, log response must be calibrated against values of porosity determined from samples taken to the lab.

Gamma-gamma and neutron logs are suggested as those most useful for estimating total porosity. Gamma-gamma logs record the backscatter of gamma rays originating from a source contained in a borehole probe. The intensity of backscatter is proportional to the bulk density of the surrounding porous material. Calibration of the response of the material enables the probe to measure bulk density. Cores or cuttings can be analyzed to estimate particle density. Total porosity can be found by substitution of bulk and particle densities in eq 116.

Neutron logs respond to the density of hydrogen nuclei in material surrounding the borehole. The density of hydrogen nuclei is assumed to be a measure of pore water content. With calibration, the neutron log can be used to measure water content and porosity of saturated materials. A disadvantage of the technique is that the log cannot differentiate between interstitial water and chemically bound water.

Borehole logging does not appear to offer an accurate way of measuring total porosity. The approach would appear best suited for determining relative porosities and locating the most porous layers encountered over the length of an individual hole or several holes in the same geologic setting.

#### Average linear velocity

Average linear velocity should be determined from specific discharge and effective porosity according to eq 42. The Direction and Magnitude of

Specific Discharge Vector and the Effective Porosity sections should be consulted for techniques for estimating those parameters.

#### Storage coefficient (confined aquifers)

Steady groundwater flow in an aquifer implies an unchanging distribution of piezometric head. If disturbances to such an initial condition occur (from pumping, for example), the flow becomes unsteady. Unsteady flow problems are solved by determining the change of head throughout an aquifer as a function of time.

The properties of a porous material that affect both transmission speed and damping of disturbances are critically important for the analysis of unsteady flows. The ability of a porous material to damp out disturbances to the piezometric head is defined in terms of storage parameters and hydraulic conductivity. In the Storage Coefficient-Specific Yield section, storage was related to the volume of water released (or taken up) by a unit volume of porous material, given a unit change in piezometric head.

For confined aquifers, storage is only a function of fluid and matrix compressibility (eq 53). For unconfined flows (those bounded above by a phreatic surface), changes in storage due to compressibility are very small compared to those resulting from movement of the phreatic surface and actual saturation-desaturation of pores.

Storage capacity for confined aquifers is defined in terms of storage coefficient, while for unconfined flows specific yield is the appropriate parameter (see the Specific Yield section).

Storage coefficient is often involved as a by-product of field methods for determining transmissivity, consequently several of the tests described below have been presented in some detail in the Transmissivity section.

#### Slug and bail test

Although primarily a test for estimating transmissivity, slug tests may be used to estimate storage coefficient  $S$ . As the type curve procedure demonstrates, however, values of  $S$  may be uncertain by several orders of magnitude; thus "determination of  $S$  by this method has questionable reliability" (Papadopoulos and Cooper 1967).

The pressurized slug test described by Bredehoeft and Papadopoulos (1980) is recommended for wells in very fine materials. Again, however, the uncertainty in the determined value of S is comparable with the conventional slug test.

Estimates based on analysis of  
cyclic disturbances to piezometric head

Observation of the movement and damping of cyclic disturbances to piezometric head provides an opportunity for determining S. The source of disturbances may be natural (earth tides, sea tides) or man-induced (reservoir releases, cyclic pumping-recharge).

Ferris (1951) describes a procedure for analyzing the response of an aquifer to cyclic variations of river stage, the river being either hydraulically connected to the aquifer or not hydraulically connected, but exerting influence by changing the weight of overburden. His analysis uses a one-dimensional model that predicts the time lag and damping of the cyclic disturbances. By observing the time lag at various distances from the river, both T and S can be determined. Accuracy is shown to be improved by increasing the magnitude and period of the disturbance, but no simple relationship between accuracy and test conditions is given. Carr and van der Kamp (1969) analyze the same problem for response to sea tides. Their analysis includes a correction for the inability of a well of finite diameter to respond immediately to the surrounding change of head, this being a result of the storage capacity of the well itself. Both Ferris (1951) and Carr and van der Kamp (1969) demonstrate the use of their techniques with actual field data.

Bredehoeft (1967) analyzes the response of confined aquifers to earth tides. He says that very sensitive pressure transducers ( $\pm 0.5$  mm water) that produce data from observation wells could be used to determine storage coefficient. It is doubtful, however, that such small changes in head could be separated from background "noise," the normal head changes that accompany the use of a developed aquifer.

A more promising method of observing response to cyclic pumping-recharge is presented by Black and Kipp (1981). Their two-well procedure involves periodic pumping and subsequent recharge of water from an



"excitation" well, and recording of head in a nearby observation well. The advantages of the procedure include the researcher's ability to select the period and amplitude of the disturbance, and that there is no net discharge from the excitation well. The authors caution, however, that their method would not be useful for "high storage/low hydraulic conductivity media."

#### Inverse problem solutions

Earlier, the solution of inverse problems for transmissivity were described. When the analyzed aquifer has undergone a period of unsteady flow it is possible in theory to determine T and S from pumping-recharge data and records of the time variation of piezometric head. Although several such models have been proposed, none can claim to have satisfactorily overcome the problem of instability discussed in earlier sections. Consequently, the inverse problem approach is not recommended for determining storage coefficient.

#### Specific yield (unconfined aquifers)

Specific yield is the ratio of 1) the volume of water that can be ultimately drained by gravity from a sample of saturated porous material to 2) the total volume of the sample. As a consequence of its definition, specific yield is occasionally called drainable porosity.

It has long been recognized that specific yield is a time-dependent quantity. The reason for this is that complete drainage does not happen simultaneously with decrease of phreatic surface. The experiments of Prill et al. (1965) demonstrate that even for sandy materials long times are required for drainage to reach an equilibrium, perhaps months or years. In practice, this problem can be alleviated by extrapolating the ultimate drainage volume from data collected during the period of most rapid drainage. As a rule the necessary period of observation is much shorter than the time required to reach equilibrium.

The existence of a capillary fringe extending upward from a phreatic surface complicates both the measurement of and the utility of specific yield. The fringe, ranging in thickness from centimeters to more than a meter, is a region where pore water is held against gravity by surface tension.

Above the capillary fringe, when drainage of a saturated material ceases, the difference between total porosity  $n$  and specific retention  $S^r$  is the specific yield  $S^*$ , or

$$S^* = n - S^r . \quad (120)$$

Specific retention is defined as the ratio of the volume of residual water remaining after complete drainage to the total volume of a porous material sample.

Within the capillary fringe, complete drainage does not occur and eq 120 cannot be applied. When a phreatic surface is lowered, water that is not held by capillarity drains from the material just above the new phreatic surface, while capillary water is transmitted down from the top of the old capillary fringe. The sum of waters from these two sources is the apparent specific yield of aquifer material.

Specific yield can be measured directly by determining the volume of complete drainage from a column of porous material not containing a capillary fringe. Indirectly, but more commonly,  $S^*$  can be found by determining the moisture content of material in the column, computing  $S^r$ , and using eq 120.

Specific retention can be computed by first weighing a known volume of completely drained material ( $W_1$ ), then drying and reweighing ( $W_2$ ), thus

$$S_r = \frac{W_1 - W_2}{\gamma_w V_T} \quad (121)$$

where  $\gamma_w$  is the specific weight of water and  $V_T$  is the known sample volume. Alternatively,  $S^r$  can be computed from residual water content determined by a calibrated neutron probe (Jones and Skibitzke 1956, Keys and MacCary 1971.)

Saturation and subsequent drainage  
of sample columns in the laboratory\*

Saturation and drainage of a laboratory column of porous material is the most direct means of measuring specific yield. In addition to the above considerations, Meinzer (1932) cautions that evaporation should be prevented and tests should be run at constant temperature.

\* A promising method that has the potential to become a standard.

Determination of specific retention in the field  
after a significant lowering of water table \*

After a significant natural or man-induced lowering of the water table, the specific retention of material above the capillary fringe is determined by sampling (Ellis and Lee 1919) or by neutron probe (Johnson and Kunkel 1963, Jones and Schneider 1969, Hanson 1973).

Moisture tension technique \*

The pore water remaining in a volume of material after drainage exists under a condition of negative hydraulic head or positive "soil water tension." Soil water tension is related to moisture content. As a material becomes drier (desaturates), tension increases. Like hydraulic head, tension can be expressed as either a height of water or a pressure (eq 17).

Johnson (1967) states that a value of soil water tension between 0.10 and 0.33 atmospheres indicates essentially complete drainage. The basic principle of moisture tension techniques is to desaturate a laboratory sample until its soil water tension is in this range. Subsequent determination of the moisture content of the sample can be used to get specific retention and, with knowledge of porosity, specific yield.

In the laboratory a sealed pressure cylinder is the principal part of a device for regulating the soil moisture tension of a porous material sample. The base of the cylinder is a porous ceramic plate capable of passing water but not air. A typical experiment consists of loading a known volume of sample in the cylinder, saturating the sample, and pressurizing the cylinder to the desired soil moisture tension. When water flow from the cylinder reaches an equilibrium, the sample is removed and analyzed for moisture content. Details and results of the procedure are given by Prill and Johnson (1967).

Centrifuge methods \*

Johnson et al. (1963) describe a detailed study of the use of a centrifuge for desaturation of samples. Residual water content (after 1 hour of centrifuging at 1000 times gravity) is used to compute specific retention.

Estimates based on identification with materials  
of known specific yield or specific retention

Both particle size analysis and geologic classification are possible means for inferring the specific retention and specific yield of an aquifer material. Johnson (1967) presents a graph from which specific yield can be estimated on the basis of sand, silt and clay fractions. Morris and Johnson (1967) give an extensive list of specific yields for various sediments.

Considerable caution is advised in using such results with too much confidence. Clearly, specific yield is a function not only of particle size and composition, but, perhaps more importantly, it is a function of the degree of compaction and particle arrangement.

Use of approaches developed to measure storage coefficient

Many of the approaches using groundwater hydraulics to determine storage coefficient can be applied in the case of unconfined aquifers where specific yield is to be estimated. The adaptation simply requires  $S^*$  to be substituted for  $S$ . In order for the application to be valid, it is necessary that flow in the unconfined aquifer be essentially horizontal (hydrostatic pressure distribution) and water table fluctuations be small compared to saturated thickness (Stallman 1961).

Analysis of the response of an unconfined aquifer to cyclic disturbances of head can yield estimates of specific yield. These methods have been described in some detail in the Estimates Based on Analysis of Cyclic Disturbances to Piezometric Head section. Ferris (1951) applies the technique to actual field data for an unconfined aquifer. Because of the nature of the phenomenon, the earth-tide approach of Bredehoeft (1967) cannot be applied to unconfined aquifers.

Slug-bail tests could be applied to the unconfined aquifer case although the previous comments still apply; that is, slug-bail tests cannot be considered reliable predictors of storage parameters.

Inverse problems can be formulated for unsteady flows in unconfined aquifers. Theoretically, historical records of head and recharge-discharge could be used to compute a spatial distribution of specific yield. Unfortunately, the fundamental difficulties of the inverse problem approach remain, and the method cannot be recommended as a practical tool.

#### Effective porosity

As defined in the Definition of Parameters section, effective porosity  $\phi$  is a measure of the total void space available for fluid flow through a volume of porous material. Its upper limit is the total porosity, though it is more nearly equal to specific yield. The primary use of effective porosity is for computation of average linear velocity from specific discharge (see eq 42). Since average linear velocity is an average speed at which contaminants move through porous materials,  $\phi$  is an important transport parameter.

By exploitation of the apparently close agreement between effective porosity and specific yield, the laboratory tests described in the Specific Yield section can be applied to estimate  $\phi$ . Tables of specific yield-effective porosity (Manger 1963, Morris and Johnson 1967) can also be used, but, as always, estimates based on a table are generally subject to appreciable uncertainty.

#### Tracer-breakthrough curve techniques\*

If specific discharge  $\vec{v}$  and average linear velocity  $\vec{v}^*$  are known, then

$$\phi = \frac{\vec{v}}{\vec{v}^*} \quad (122)$$

Specific discharge can be calculated from hydraulic conductivity and piezometric head gradient through Darcy's law, or it may be found using one of the procedures described in the Direction and Magnitude of Specific Discharge Vector section. Average linear velocity can be determined by observing the time of travel of the peak concentration of a tracer moving in the direction of  $\vec{v}_h$ . Keeley and Scalf (1969) describe a field application of

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\* A promising method that has the potential to become a standard.

this approach for computing storage capacity of the Ogallala aquifer near Bushland, Texas.

#### Electrical resistivity logs

For many years electrical resistivity logs have been used by the oil industry as a means of estimating the porosity of materials encountered over the length of unlined boreholes (Archie 1942). More recently, others (e.g. Patten and Bennett 1963, Keys and MacCary 1971) have evaluated this technique for groundwater investigations.

The primary objective of electric logging is to compute the resistance of earth materials as a function of depth in a borehole. This is done by measuring the magnitude of electrical current flowing in a circuit consisting of 1) a current source, one terminal of which is connected to an earth ground, the other to a borehole probe; 2) the probe, which transmits current to the wall of an unlined hole; and 3) the earth material between the probe-borehole wall contact and the earth ground.

An analysis of current flow in the borehole logging circuit demonstrates that nearly all of the voltage drop takes place in earth materials very near the probe. Thus, the computed resistance is essentially that of material near the probe location.

The electrical resistance of a saturated porous material is largely a function of the amount of interconnected pore space in the material, i.e. the effective porosity. A calibration between resistivity and effective porosity could be used to estimate the aquifer parameter.

For the ideal case, upon which the theory of electric logging is based, the borehole probe is assumed to be a point source embedded in an electrically homogeneous material of infinite extent. In practice the borehole, borehole fluid (drilling mud, native or foreign water), and invasion by the borehole fluid complicate the electrical properties of a material. The resulting nonhomogeneity makes interpretation of resistivity logs more uncertain.

Clay content is another significant source of potential error in interpreting borehole logs. The high concentration of mobile ions in elemental clay structures gives clay a relatively low resistance. When

reflected in the log, this makes effective porosity appear to be large (Patten and Bennett 1963).

In summary it would seem that the best use of electric logs is as a tool for locating the most porous layers in a single well or group of wells in the same geologic environment.

#### Aquifer dispersivity-dispersion coefficients

As explained in the Aquifer Dispersivity section, dispersion is a transport mechanism used to describe observed mass flux not accounted for by simple advection. The earlier section also described how, at the micro-scale, dispersion results from flow around material grains, while at the macroscale, dispersion was a means of compensating for an imprecise knowledge of the spatial variability of hydraulic conductivity. In this section only the field scale dispersion flux is considered. Laboratory scale dispersion has little effect in field scale problems (Ogata 1961a, b, 1970, Simpson 1962, Harleman et al. 1963, Klotz and Moser 1974).

In an isotropic material,

$$\begin{aligned} D_L &= \alpha_1 \left| \vec{v}^* \right| \\ D_T &= \alpha_2 \left| \vec{v}^* \right| \end{aligned} \quad (123)$$

where  $D_L$  and  $\alpha_1$  are the longitudinal dispersion coefficient and aquifer dispersivity, while  $D_T$  and  $\alpha_2$  are the transverse values. The quantity  $\left| \vec{v}^* \right|$  is the magnitude of the average linear velocity vector. As explained in the Aquifer Dispersivity section, the dispersion coefficients are analogous to the diffusion coefficients of Fick's law; they govern the magnitude of mass flux due to concentration gradient.

Early in the development of dispersion theory, researchers postulated that the dispersivities were intrinsic properties of a porous material, just as is porosity or conductivity. For laboratory scale experiments this indeed seems to be the case (as the results of Harleman et al. [1963] and Klotz and Moser [1974] demonstrate). On the field scale, however, an imprecise knowledge of the spatial variability of hydraulic conductivity causes an "apparent" dispersion which is orders of magnitude greater than

would be predicted on the basis of values of  $\alpha$  determined in the lab. Consequently, laboratory determination of aquifer dispersivity is of little or no value in estimating field scale dispersion (Daly 1979).

#### Calibration of transport models\*

Because of the inherent difficulties in determining field scale dispersivities, it is standard modeling practice to assume isotropy and homogeneity with respect to dispersion. Thus, only two dispersivities,  $\alpha_1$  and  $\alpha_2$ , are input to most transport models.

In the absence of a reliable field procedure for determining  $\alpha_1$  and  $\alpha_2$ , calibration is a common means of arriving at acceptable values of those parameters. To make the task simpler it is also standard practice to assume that  $\alpha_2$  is some fixed fraction of  $\alpha_1$ ,

$$\alpha_2 = \beta \alpha_1 \quad 0 \leq \beta \leq 1 . \quad (124)$$

Model calibration usually proceeds by fixing  $\beta$ , then varying  $\alpha_1$  until a reasonable agreement between observed and predicted contaminant distributions is obtained. If this is not possible, a different  $\beta$  is selected and calibration is repeated.

Bredehoeft and Pinder (1973) conducted a contaminant modeling study at Brunswick, Georgia. Their final calibrated values were  $\beta = 0.3$ , and  $\alpha_1 = 200$  ft (61 m). Pinder's (1973) study of an aquifer on Long Island, New York, used final values of  $\beta = 0.2$  and  $\alpha_1 = 70$  ft (21 m). Konikow and Bredehoeft's (1974) model of the alluvial aquifer of the Arkansas River near La Junta, Colorado, used  $\beta = 0.3$  and  $\alpha_1 = 100$  ft (30 m).

#### Single well recharge-recovery, and radial injection tracer tests

A longitudinal dispersivity coefficient can be determined, at least in theory, from an analysis of the fate of an injected tracer. Tracer concentration, observed at several observation wells near the injection well, can be used along with an analytic solution to the radial tracer injection problem. Since  $\alpha_1$  appears in the solution for the predicted concentration as a function of radial distance from the injection well, the observed

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\* A promising method that has the potential to become a standard.



concentration should lead to  $\alpha_1$  (Hoopes and Harleman 1967, Pickens et al. 1981).

A single well injection followed by recovery of an injected tracer by pumping is a similar approach. In this case, the analytic solution can be compared to observed concentrations of tracer in the pumped water (Fried et al. 1974).

The primary problems with the radial tracer techniques is that they ignore porous material nonhomogeneity and adsorption-absorption of contaminant. It has already been explained how nonhomogeneity is the principal cause of observed dispersion. The additional unaccounted for interaction of porous material and tracer can lead to unexpected changes in the concentration of tracer in flowing groundwater.

#### Inverse problem for dispersivities

Recently, Umari et al. (1981) developed a computer model for working an inverse problem whose solution is a set of aquifer dispersivities. While they claim that their model, suitably constrained, is "fast, stable and accurate," it is questionable whether the necessary input data could be collected so as to eliminate uncertainty in calculated dispersivities. Umari et al. do not apply their technique to a field problem to evaluate its performance.

#### CONCLUSIONS

A reliable prediction of the trajectory of contaminants in groundwater depends upon our appreciating, and taking into account, the effects of spatial variability of aquifer parameters. Traditionally, groundwater hydrology tended to emphasize the "black box" approach, that is, one which largely ignores the inner workings of a system (the "box"). As a consequence, aquifer parameters tended to be used as a means of describing the influence of one element of a system on another (well on a stream, recharge on a well, etc.). This has been done without much regard for the geologic structure of materials between elements.

For contaminant transport, the black box approach is inadequate for describing the operation of an aquifer system. In order to confidently

predict the speed, direction and ultimate fate of a contaminant plume, a clear and detailed picture of the groundwater flow pattern must be obtained. Such a determination of flow pattern depends on the spatial detail of flow parameters. It is therefore more desirable to have moderately accurate parameter estimates at many points than to have very accurate estimates at only a few points. In this same connection, the value of geologic mapping and inference should not be underestimated as a tool for determining the trajectory of contaminant plumes. It is not likely that such information could be easily represented solely in terms of the aquifer parameters.

Parameter identification surveys that seek to determine the fate of contaminants should strive to measure those quantities that are most directly related to the flow pattern. Thus it is more desirable to directly measure average linear velocity than to measure separately conductivity, head gradient and effective porosity (then calculate  $\vec{v}^*$  from those values). Tracer tests and borehole dilution tests are particularly promising in this regard.

With regard to observation points, it appears that the simple piezometer may provide an alternative to the partially or fully penetrating screened well. If wells are used, packers may be employed to obtain information on vertical segments of the well. Gravel packs, while essential to the construction of pumped wells, should be avoided if many of the tests presented here are to yield reliable results. When pumping is not anticipated, it is desirable that well screens (and piezometers) be in intimate contact with aquifer material. Investigation of the feasibility of such construction is recommended.

For each of the aquifer parameters considered in the Parameter Estimation Techniques section of this report, the methodologies are presented in the order of their perceived value. An asterisk is added to the title of each procedure that is recommended for consideration as a standard method.

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