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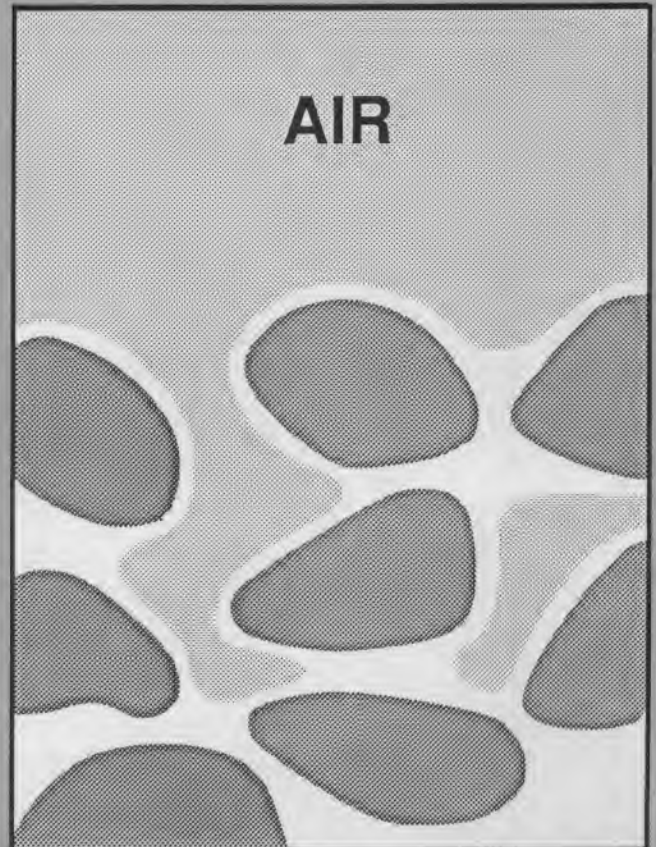
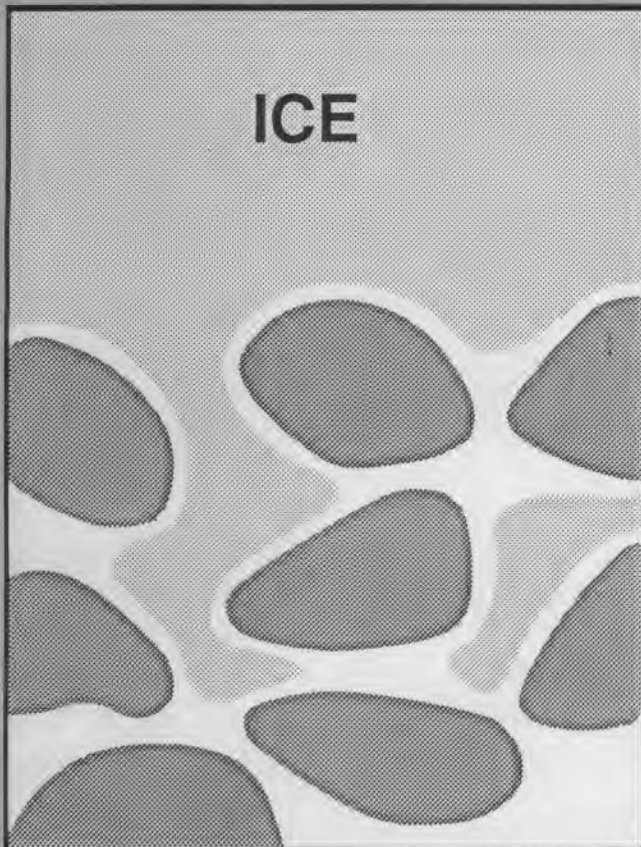
REPORT 88-16



**US Army Corps
of Engineers**

Cold Regions Research &
Engineering Laboratory

Comparison of soil freezing curve and soil water curve data for Windsor sandy loam



For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380, Metric Practice Guide, published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

Cover: This figure demonstrates the strong similarity between the state of air in ice-free soil and ice in air-free colloid-free soil. The two are indeed interchangeable through the ϕ variable discussed in this report.

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Patrick B. Black and Allen R. Tice

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PREFACE

This report was prepared by Dr. Patrick B. Black, Research Physical Scientist, and Allen R. Tice, Physical Science Technician, of the Geochemical Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding for this project was provided by DA Project 4A161102AT24, *Research in Snow, Ice and Frozen Ground*, Task A, *Properties of Cold Regions Materials*, Work Unit 002, *Properties of Frozen Soils*.

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Comparison of Soil Freezing Curve and Soil Water Curve Data for Windsor Sandy Loam

PATRICK B. BLACK AND ALLEN R. TICE

INTRODUCTION

The CRREL Outdoor Land Treatment Research Facility consists of six test beds containing two soil types. The soils' ice-free physical and chemical properties have been reported by Iskandar et al. (1979). Our report presents recently obtained laboratory-measured unfrozen water content data as a function of temperature, plotted as soil freezing curves (SFC), for all three horizons of the Windsor sandy loam present in the test beds. It then demonstrates how such a soil data base is related to the previously measured ice-free water content data as a function of matric suction, plotted as soil water curves (SWC). The theoretical basis for the similarity of the SFC and SWC is understood (Miller 1965, Koopmans and Miller 1966) but the value of relating different data, collected by different researchers at different times for the same field soil, needs to be shown to illustrate the usefulness of soil data bases.

This report first presents a generalized notation for the state of soil water and then reviews the similarity between the state of liquid water in unfrozen soil and air-free frozen soil in the generalized notation. Unfrozen water content data of the Windsor sandy loam measured by nuclear magnetic resonance (NMR) are compared to data on water content as a function of matric pressure presented by Iskandar et al. (1979) through the similarity relationships presented.

SOIL VARIABLE ϕ

For discussion of the state of water in soil-water systems, it is convenient to introduce the variable ϕ_{jk} :

$$\phi_{jk} = u_j - u_k, \quad (1)$$

the pressure difference between water in the two phases j and k (Black and Miller 1985, Black 1985). The matric pressure for ice-free soil is expressed as

$$\phi_{aw} = u_a - u_w \quad (2)$$

where the subscripts a and w refer to soil air and water pressures, respectively. Likewise, the state of water in an air-free frozen soil is expressed as

$$\phi_{iw} = u_i - u_w \quad (3)$$

where u_i is the ice pressure. Most often, though, the state of water in air-free frozen soil is expressed in terms of temperature, θ (in Celsius degrees). If the soil is devoid of solutes, the Clapeyron equation can be expressed as:

$$u_w - \frac{u_i}{\gamma_i} = \frac{h}{273} \theta \quad (4)$$

where γ_i is the specific gravity of ice and h is volumetric latent heat of fusion. Solving eq 3 and 4 for u_i gives the connection between water pressure u_w , ice temperature θ and ϕ_{iw} :

$$\phi_{iw} = (\gamma_i - 1)u_w - \left(\frac{\gamma_i h}{273}\right)\theta. \quad (5)$$

Another useful classification to employ in discussing the physical behavior of soil includes the concepts of adsorption space and capillary space. Adsorption space is that zone in which soil water is strongly affected by surface forces (real or virtual) emanating from the soil. Capillary space is the remaining zone in which soil water is not affected by soil force fields but is governed by the laws of surface tension. Granular soils contain mostly capillary water, while highly colloidal soils

are dominated by adsorption forces. The nature and formulation of the adsorption space are not addressed in this report but are employed only as a general classification scheme for soils.

SWC AND SFC SIMILARITY

Miller (1965) hypothesized that if the same states of soil moisture content and distribution are achieved by a freezing and thawing process as in a drying and wetting process, then the two states should be similar and interchangeable if the soil is either colloidal (adsorption space \gg capillary space) or colloid-free (adsorption space \ll capillary space). In the former case, the ice-free SWC data and air-free SFC data for the same soil at the same density and subjected to similar histories should be directly related (i.e. $\phi_{aw} = \phi_{iw}$). In the latter case, an additional correction factor that takes account of the differences in surface tension (σ_{aw} , σ_{iw}) is needed to bring the data into coincidence [i.e. $\phi_{aw} = (\sigma_{aw}/\sigma_{iw})\phi_{iw}$].

The experiments of Koopmans and Miller (1966) prove this hypothesis when the same soil is used to measure SWC and SFC data. They found that the transformation from temperature to ϕ_{iw} given by eq 5 was sufficient to directly relate SFC and SWC data for soils dominated by adsorption forces. In soils dominated by capillary space, their empirically determined ratio of air-water to ice-water surface tensions was

$$\frac{\phi_{aw}}{\phi_{iw}} = \frac{\sigma_{aw}}{\sigma_{iw}} = 2.20. \quad (6)$$

Table 1. Relationships between ϕ_{aw} and ϕ_{iw} for similar liquid water contents.

Adsorption space \ll capillary space

$$\sigma_{aw} = (\sigma_{aw}/\sigma_{iw})\phi_{iw} \quad (T1)$$

or

$$\phi_{aw} \text{ (kPa)} = (2.2) -1110 \frac{\text{kPa}}{^\circ\text{C}} \theta \text{ (}^\circ\text{C)} \quad (T2)$$

Adsorption space \gg capillary space

$$\phi_{aw} = \phi_{iw} \quad (T3)$$

or

$$\phi_{aw} \text{ (kPa)} = -1110 \frac{\text{kPa}}{^\circ\text{C}} \theta \text{ (}^\circ\text{C)} \quad (T4)$$

With a standard textbook value for σ_{aw} of $72.7 \times 10^{-3} \text{ J/m}^2$, their predicted σ_{iw} of $33.1 \times 10^{-3} \text{ J/m}^2$ agreed fairly well with estimates obtained independently of soil physics investigations (Hesstvedt 1964).

Equations 5 and 6 allow the interchange of ice-free SWC and air-free SFC data for soils that are dominated by either absorption space or by capillary space. Table 1 contains the relationships between ϕ_{aw} and ϕ_{iw} by assuming u_w to be zero (gauge) and choosing values for the physical constants in eq 5 for the two soil classifications.

MATHEMATICAL REPRESENTATION OF SWC AND SFC DATA

Brooks and Corey (1964) proposed a function to represent SWC data; from a large number of observations for ice-free soils, they found that the relative degree of saturation or dimensionless water content

$$S = \frac{W - W_d}{W_s - W_d} \quad (7)$$

could be reasonably described by the relationship:

$$S = \left(\frac{\phi_{aw}}{\phi_b} \right)^{-\delta} \quad (8)$$

where W , W_d and W_s are, respectively, the water contents (usually volumetric) at a given ϕ_{aw} , at the lower limit of drying and at saturation. ϕ_b is the air entry value and δ is a free parameter determined from a "curve fit" to the data and appears to be related to the pore-size distribution of the soil. Small values of δ are found to correspond to soils with a wide range of pore sizes; large δ values are obtained when grain sizes are nearly uniform. Equation 8 is valid for the range, $\phi_{aw} > \phi_b$; otherwise, $S = 1$ (i.e. $W = W_s$) for $\phi_{aw} < \phi_b$.

The predictive accuracy of eq 8 depends upon the value for W_d , the lower limit of drying. Unfortunately, data at very large values of ϕ_{aw} are seldom collected because of experimental complications requiring a value for W_d to be inferred by extending outside the range of data through numerical or graphical procedures. When optimal values for each parameter are determined by nonlinear optimization techniques for experimental data, negative values for W_d are obtained (Fields et al. 1984). Graphical interpolation methods (Brooks and Corey 1964, van Genuchten 1978) or rule of thumb guesses are biased at best.

Equation 8 can also be employed to represent SFC data (Black 1985, Black and Miller 1985) but the power curve relationship proposed by Anderson and Tice (1973) is most often used. Their relationship between gravimetric water content and the absolute value of temperature ($|\text{ }^\circ\text{C}|$) can be written more generally as (on a volumetric or gravimetric basis)

$$W = \alpha \phi_{iw}^\beta \quad (9)$$

where α and β are free parameters determined from a log-log regression of the data and the equations in Table 1 are used to relate temperature and ϕ_{iw} .

Implicit in eq 9 is the assumption that the minimum value of water content is zero (i.e. $W_d = 0$) at an infinitely cold temperature. This would seem to be reasonable and is corroborated by the successful fit of data to the equation (see App. A). Similarly, it would seem reasonable that W_d should also be zero at infinite ϕ_{aw} , but the relatively small magnitude (several bars) encountered in determining SWC data leaves the impression that there is finite water content even at very large ϕ_{aw} . When SFC data are included with SWC data, as is done below, eq 9 is sufficient to describe all the data greater than the air-entry value.

When W_d is assumed to be zero, eq 8 and 9 are equivalent and the free parameters in eq 9 take on physical significance. In this case:

$$\beta = -\delta; \quad \alpha = W_s \left(\frac{1}{\phi_b} \right)^\beta; \quad \text{when } W_d = 0$$

The air-entry value can be obtained from the simple log-log regression of data if W_s is known.

NMR MEASUREMENT OF UNFROZEN WATER CONTENT

Soil samples from all three horizons of the Windsor soil were collected. Remolded specimens were prepared to different gravimetric water contents and packed to different bulk densities in sealed plastic tubes at a volume of 8 cm³. These specimens were then placed in a constant temperature bath at approximately -6°C to induce freezing, at which time the bath temperature was increased to a temperature just below 0°C and allowed to equilibrate overnight.

A Praxis model PR-103 pulsed nuclear magnetic resonance (NMR) analyzer, factory-tuned to de-

tect only hydrogen, was used to determine unfrozen water contents. Each test specimen was removed sequentially from the bath, wiped dry and inserted in the NMR probe. After the four seconds required to record specimen temperature and first pulse NMR signal amplitude, the specimen was reinserted in the bath. When all observations had been completed at a given temperature, the bath temperature was decreased. Thermal equilibrium was attained after approximately 45 minutes and measurements repeated.

When the bath temperature reached -25°C, the cooling was stopped and a warming cycle started. Warming observations were made in a manner similar to those determined during cooling, and measurements stopped when specimens were completely thawed. Water contents were then determined gravimetrically and are presented in Table 2 along with dry bulk densities, volumetric water contents, porosities and relative degrees of saturation based upon the 8-cm³ specimen volume.

The unfrozen water contents at each temperature were determined from the measured first pulse NMR amplitude. The technique is described by Tice et al. (1978, 1981, 1982). Briefly, the ratio of the gravimetric water content to the first pulse amplitude of the ice-free case for each test specimen was determined. Unfrozen water contents were then deduced by multiplying the measured first pulse amplitude at the different temperatures by the above-determined ice-free ratio. The data and regression analysis for eq 9 are listed in Appendix A.

CHARACTERIZATION OF SWC

Volumetric water content (W_v) at various matric suctions, ϕ_{aw} , for the three horizons of the Wind-

Table 2. Initial physical characteristics for Windsor soil determined by Iskandar et al. (1979).

Soil horizon	W_{wT}	ρ_d	W_{vT}	G_s	e	S
A	32.7	1.376	45.0	2.63	47.7	94.3
B	22.1	1.582	35.0	2.69	41.2	85.0
C	23.5	1.534	36.0	2.73	43.8	82.2

W_{wT} = total gravimetric water content (mass water/mass soil) $\times 100$

W_{vT} = total volumetric water content (volume water/total volume) $\times 100$

ρ_d = dry density (mass soil/total volume)

G_s = specific gravity

e = porosity (volume pores/total volume) $\times 100$

S = relative degree of saturation (V_v/e) $\times 100$

sor soil were obtained from Figures 2, 3 and 4 of Iskandar et al. (1979) and listed in Appendix C. The initial physical characteristics are listed in Table 2.

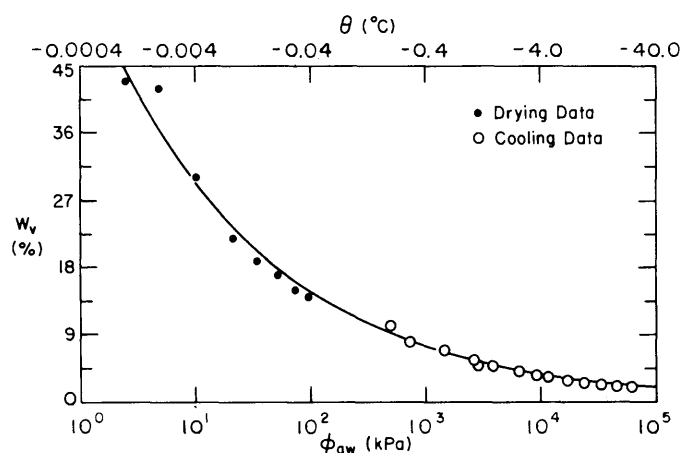
DISCUSSION

Ice-free SWC and air-free SFC data can be compared only on the basis of the same soil at the same bulk density and with similar histories for each (i.e. freezing and drying curves or wetting and warming curves can be compared). In more general terms, the soil must meet similar media requirements (Miller and Miller 1956). Since the same soil was used for both SFC and SWC measurements, only specimens with identical bulk den-

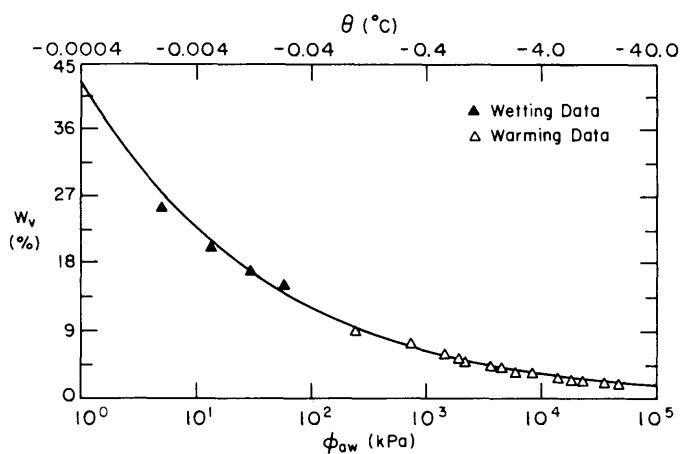
Table 3. Summary statistics of combined SFC and SWC data in terms of W_v (%) and ϕ_{aw} (kPa) fitted to eq 9 (i.e. $W_v = \alpha \phi_{aw}^\beta$).

Soil horizon		α	β	r^2	ϕ_b (kPa)
A	D/C	14.709	-0.300	0.987	20
	W/w	12.021	-0.274	0.991	
B	D/C	9.666	-0.301	0.978	8
	W/w	8.578	-0.268	0.963	
C	D/C	5.572	-0.389	0.921	10
	W/w	4.263	-0.328	0.965	

D/C = drying/cooling for SWC/SFC data
W/w = wetting/warming for SWC/SFC data
 $\phi_b = (\alpha/\text{porosity})^{1/\beta}$



a. Drying curve data from SWC of Iskandar et al. (1979). Cooling curve data from SFC, $\rho_d = 1.38 \text{ g/cm}^3$.



b. Wetting curve data from SWC of Iskandar et al. (1979). Warming curve data from SFC, $\rho_d = 1.38 \text{ g/cm}^3$.

Figure 1. Semilog plot of W_v vs ϕ_{aw} and θ for Windsor soil A horizon.

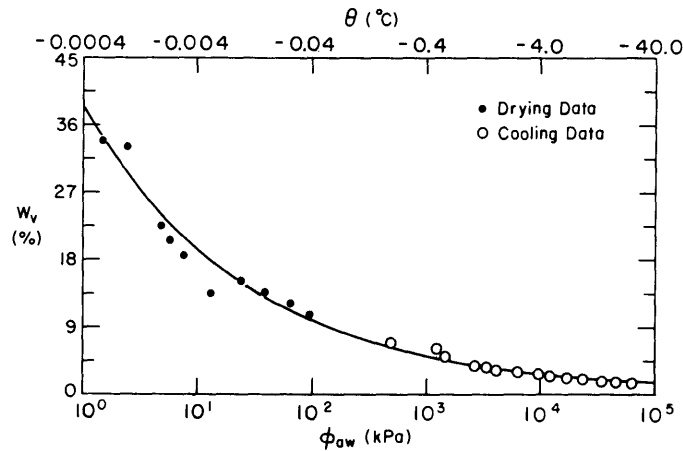
sities and complete initial saturation should be used. The two cases for which the dry bulk densities (ρ_d) are most similar are the warming and cooling data for the A horizon with $\rho_d = 1.38$ and the warming and cooling data for the B horizon with $\rho_d = 1.56$. In both cases, the high degree of saturation meets the air-free requirement. The dry bulk densities used for the unfrozen water content measurements in the C horizon were all greater than the density reported by Iskandar et al. (1979) of 1.534. The specimen that came closest, $\rho_d = 1.79$ and 85.8% saturation, was used.

Since the clay content of the Windsor soil is less than 1% (Iskandar et al. 1979), the transformation between SWC and SFC data is obtained with eq T2 of Table 1. Figures 1-3 are plots of transformed SFC and SWC data along with the best fit

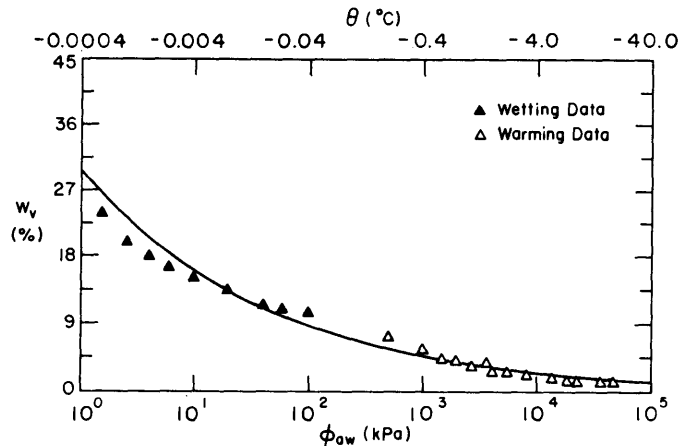
regression to eq 9. Table 3 contains a summary analysis for each plot. SWC data that appeared to be above the air-entry value ($\phi_{aw} < \phi_b$) were excluded in the regression.

Inspection of Figures 1-3 and the high r^2 values (> 0.9) for the regression fit clearly indicates the success of eq 9 in describing both SWC and SFC data for this soil and the suitability of eq T1 of Table 1 for relating the two sets of soil data. The worst case ($r^2 = 0.921$) is the drying/cooling data for the C horizon (Fig. 3a). The data sets were not at the same bulk densities, which violates the requirement for comparison. Additional efforts failed to produce a soil specimen with as low a density ($\rho_d = 1.534$) as reported by Iskandar et al. (1979).

The analysis presented in this report extends the

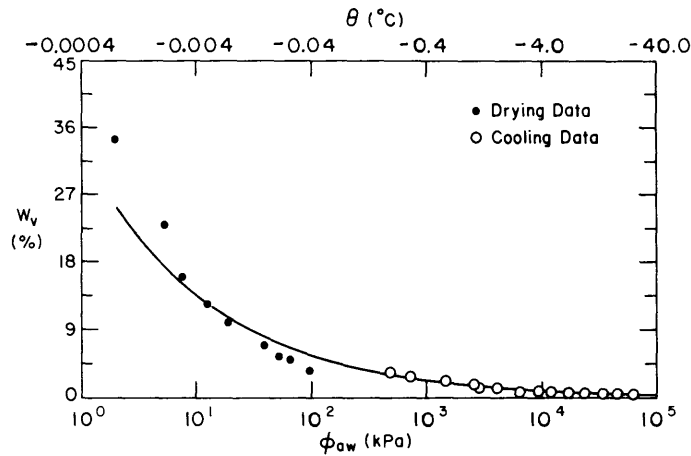


a. Drying curve data from SWC of Iskandar et al. (1979). Cooling curve data from SFC, $\rho_d = 1.56 \text{ g/cm}^3$.

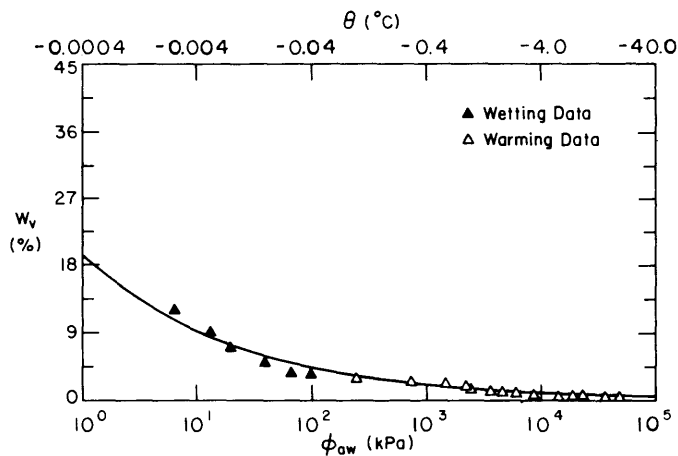


b. Wetting curve data from SWC of Iskandar et al. (1979). Warming curve data from SFC, $\rho_d = 1.56 \text{ g/cm}^3$.

Figure 2. Semilog plot of W_v vs ϕ_{aw} and θ for Windsor soil B horizon.



a. Drying curve data from SWC of Iskandar et al. (1979). Cooling curve data from SFC, $e_d = 1.79 \text{ g/cm}^3$.



b. Wetting curve data from SWC of Iskandar et al. (1979). Warming curve data from SFC, $e_d = 1.79 \text{ g/cm}^3$.

Figure 3. Semilog plot of W_v vs ϕ_{aw} and θ for Windsor soil C horizon.

range of the SFC data to higher temperatures than are easily obtained with present techniques. Correspondingly, one may extend the range of the SWC data to a suction of greater than 60 MPa. In addition, the new interpretation of the free parameters in the power curve relationship allows determination of the air-entry value from air-free SFC data.

It is of interest to note that the values for α and β determined from the SFC data alone are not appreciably different from the values determined from the combined SWC and SFC data. This indicates that SFC data alone are good indicators of the behavior of this type of granular soil and may be employed where SWC data do not exist.

CONCLUSIONS

In this report, data for the unfrozen water content as a function of temperature (SFC) were measured and analyzed in terms of a power curve relationship. These data were then related to ice-free water content as a function of matric suction data (SWC), collected nearly a decade earlier, through the modified Clapeyron equation and surface tension adjustments. It was shown that when the soils were strictly similar (i.e. when the same bulk density was used for the SWC and SFC data), then a single relationship between moisture content and the ϕ -variable obtained from SWC data

and air-free SFC data described these data, demonstrating the validity of the procedure for granular, colloid-free soil.

The method can expand the utility of data bases for soil physical properties. When data are carefully collected, and experimental conditions duly noted, the results compiled for one intention may be used by others for different objectives at later times.

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APPENDIX A: SOIL FREEZING CURVE DATA

Table A1. Regression parameters to eq 9 (i.e. $W_w = \alpha\theta^\beta$) for unfrozen water content data listed below on a percent weight basis as determined by NMR and initial water contents and bulk densities.

	α	β	r^2	W_{wT}	ρ_d	W_{vT}	e	S		
A	w	4.081	-0.253	0.997	9.98	1.63	16.24	38.0	42.7	
	c	4.758	-0.307	0.993						
	w	4.015	-0.259	0.993	15.25	1.62	24.68	38.4	64.3	
	c	4.608	-0.300	0.993						
	w	4.014	-0.277	0.996	26.34	1.48	38.98	43.7	89.1	
	c	4.814	-0.326	0.979						
	w	3.568	-0.282	0.984	28.65	1.54	44.10	41.4	106.4	
	c	4.155	-0.317	0.986						
	w	3.653	-0.281	0.983	32.60	1.38	44.85	47.5	94.4	*
c	4.130	-0.309	0.996							
B	w	2.523	-0.322	0.990	6.65	1.74	11.59	35.3	32.8	
	c	2.697	-0.325	0.984						
	w	2.185	-0.291	0.995	11.77	1.78	21.00	33.8	62.1	
	c	2.357	-0.311	0.970						
	w	2.100	-0.304	0.972	15.45	1.79	27.64	33.5	82.6	
	c	2.375	-0.331	0.997						
	w	2.789	-0.354	0.982	24.58	1.55	38.11	42.4	89.9	
	c	2.911	-0.348	0.980						
	w	2.597	-0.364	0.965	25.28	1.56	39.42	42.0	93.8	*
c	2.564	-0.356	0.980							
C	w	0.892	-0.357	0.938	8.98	1.75	15.70	35.9	44.7	
	c	0.983	-0.405	0.987						
	w	0.872	-0.393	0.963	11.25	1.80	20.27	34.1	59.5	
	c	0.897	-0.391	0.973						
	w	0.881	-0.396	0.989	13.24	1.79	23.71	34.4	68.9	
	c	0.853	-0.399	0.988						
	w	0.925	-0.384	0.920	16.54	1.79	29.54	33.4	85.8	*
	c	0.989	-0.419	0.984						

Key: W_{wT} - total gravimetric water content
(mass water/mass soil)*100
 W_{vT} - total volumetric water content
(volume water/total volume)*100
 ρ_d - dry density (mass soil/total volume) (g/cc)
e - porosity (volume pores/total volume)*100
S - relative degree of saturation (W_w/e)*100
w - warming curve
c - cooling curve
A,B,C - soil horizon
* - used in SWC/SFC comparison

Date: 12/31/87 Time: 15:40:09.41 Operator: PBB

Experiment name: MCAWA
Experiment number:

Wet density: 1.99
Dry density: 1.78
Gravimetric water content(Ww): 11.77 +/- .02
Volumetric water content(Wv): 21.00

Theoretical NMR equation: $TNMR = 3.2828E2 + -5.3714E-1 * (-C)$
r-squared: .99

$$Ww = 2.181*(-C)**(-.290)$$

r-squared: .995
Standard Deviation: .004
F-value: 2.6219E3
Significance: .000

Date: 12/31/87 Time: 15:44:34.65 Operator: PBB

Experiment name: MCACA
Experiment number:

Wet density: 1.99
Dry density: 1.78
Gravimetric water content(Ww): 11.77 +/- .02
Volumetric water content(Wv): 21.00

Theoretical NMR equation: $TNMR = 3.2752E2 + -4.2052E-1 * (-C)$
r-squared: .86

$$Ww = 2.357*(-C)**(-.311)$$

r-squared: .970
Standard Deviation: .027
F-value: 3.8808E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	4.1	4.0	.2	.2	3.8	3.6	.2
.3	3.0	3.1	.2	.5	3.5	3.0	.2
.5	2.7	2.7	.2	.6	2.8	2.8	.2
.7	2.5	2.4	.2	.9	2.1	2.4	.2
.9	2.3	2.2	.2	1.2	2.1	2.2	.2
1.4	2.0	2.0	.2	1.5	1.9	2.1	.2
1.8	1.9	1.8	.2	2.4	1.8	1.8	.2
2.6	1.6	1.6	.2	3.8	1.5	1.6	.2
3.7	1.5	1.5	.2	4.7	1.5	1.5	.2
5.7	1.3	1.3	.2	6.8	1.3	1.3	.2
7.8	1.2	1.2	.2	9.7	1.2	1.2	.2
9.6	1.1	1.1	.2	13.7	1.1	1.0	.2
14.5	1.0	1.0	.2	18.7	1.0	.9	.2
19.4	1.0	.9	.2	25.0	.9	.9	.2
.5	2.7	2.7	.2				

Date: 12/31/87 Time: 16:03:57.69 Operator: PBB

Experiment name: MCAWB
Experiment number:

Wet density: 1.86
Dry density: 1.74
Gravimetric water content(Ww): 6.65 +/- .02
Volumetric water content(Wv): 11.59

Theoretical NMR equation: $TNMR = 1.7584E2 + -2.8652E-1 * (-C)$
r-squared: .86

$$Ww = 2.513*(-C)**(-.320)$$

r-squared: .989
Standard Deviation: .012
F-value: 1.1219E3
Significance: .000

Date: 12/31/87 Time: 16:07:11.96 Operator: PBB

Experiment name: MCACC
Experiment number:

Wet density: 1.93
Dry density: 1.55
Gravimetric water content(Ww): 24.58 +/- .02
Volumetric water content(Wv): 38.11

Theoretical NMR equation: $TNMR = 5.3915E2 + -7.9089E-1 * (-C)$
r-squared: .99

$$Ww = 2.911*(-C)**(-.348)$$

r-squared: .980
Standard Deviation: .027
F-value: 5.8845E2
Significance: .000

11

-C	Ww	Ww-calc	Uncertainty
.2	4.3	4.1	.2
.4	3.5	3.4	.2
.5	3.1	3.1	.2
.7	2.8	2.8	.2
1.0	2.4	2.5	.2
1.4	2.4	2.3	.2
1.7	1.9	2.1	.2
2.4	1.8	1.9	.2
3.5	1.7	1.7	.2
5.8	1.3	1.4	.2
7.7	1.3	1.3	.2
9.5	1.3	1.2	.2
14.6	1.1	1.1	.2
19.4	1.0	1.0	.2
.5	3.1	3.2	.2

-C	Ww	Ww-calc	Uncertainty
.3	4.4	4.4	.2
.5	4.1	3.6	.2
.7	3.3	3.3	.2
1.1	2.8	2.8	.2
1.4	2.6	2.6	.2
1.7	2.3	2.4	.2
2.6	2.0	2.1	.2
3.9	1.7	1.8	.2
5.0	1.6	1.7	.2
7.0	1.4	1.5	.2
9.7	1.2	1.3	.2
13.9	1.3	1.2	.2
18.7	1.1	1.1	.2
25.3	1.0	.9	.2

Date: 12/31/87 Time: 16:10:24.81 Operator: PBB

Date: 12/31/87 Time: 16:14:13.35 Operator: PBB

Experiment name: MCAWC
Experiment number:

Experiment name: MCACD
Experiment number:

Wet density: 1.93
Dry density: 1.55
Gravimetric water content(Ww): 24.58 +/- .02
Volumetric water content(Wv): 38.11

Wet density: 1.95
Dry density: 1.56
Gravimetric water content(Ww): 25.28 +/- .02
Volumetric water content(Wv): 39.42

Theoretical NMR equation: $TNMR = 5.3858E2 + -7.0359E-1 * (-C)$
r-squared: .99

Theoretical NMR equation: $TNMR = 5.1214E2 + -5.0069E-1 * (-C)$
r-squared: .99

$$Ww = 2.755*(-C)**(-.348)$$

$$Ww = 2.564*(-C)**(-.356)$$

r-squared: .972
Standard Deviation: .037
F-value: 4.5329E2
Significance: .000

r-squared: .980
Standard Deviation: .026
F-value: 5.8557E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty
.3	4.7	4.3	.2
.5	3.7	3.6	.2
.6	3.3	3.3	.2
.8	3.0	3.0	.2
1.1	2.9	2.7	.2
1.5	2.5	2.4	.2
1.8	2.0	2.2	.2
2.4	1.9	2.0	.2
3.5	1.7	1.8	.2
5.9	1.4	1.5	.2
7.7	1.3	1.4	.2
9.6	1.2	1.3	.2
14.8	1.1	1.1	.2
19.6	1.1	1.0	.2
.5	3.2	3.6	.2

-C	Ww	Ww-calc	Uncertainty
.2	4.3	4.4	.2
.5	3.9	3.4	.2
.6	3.0	3.1	.2
1.1	2.3	2.5	.2
1.4	2.3	2.3	.2
1.7	2.0	2.1	.2
2.6	1.8	1.8	.2
4.0	1.6	1.6	.2
5.0	1.4	1.4	.2
7.0	1.3	1.3	.2
9.6	1.1	1.1	.2
13.9	1.0	1.0	.2
18.6	.9	.9	.2
25.3	.8	.8	.2

Date: 12/31/87 Time: 16:17:23.78 Operator: PBB

Date: 01/02/88 Time: 14:37:06.47 Operator: PBB

Experiment name: MCAWD
Experiment number:

Experiment name: MCACA
Experiment number: 1

Wet density: 1.95
Dry density: 1.56
Gravimetric water content(Ww): 25.28 +/- .02
Volumetric water content(Wv): 39.42

Wet density: 1.99
Dry density: 1.78
Gravimetric water content(Ww): 11.77 +/- .02
Volumetric water content(Wv): 21.00

Theoretical NMR equation: $TNMR = 5.1195E2 + -4.7169E-1 * (-C)$
r-squared: .99

Theoretical NMR equation: $TNMR = 3.2752E2 + -4.2052E-1 * (-C)$
r-squared: .86

$$Ww = 2.607 * (-C) ** (-.366)$$

$$Ww = 2.357 * (-C) ** (-.311)$$

r-squared: .969
Standard Deviation: .044
F-value: 4.0795E2
Significance: .000

r-squared: .970
Standard Deviation: .027
F-value: 3.8808E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty
.2	4.8	4.3	.2
.4	3.7	3.6	.2
.6	3.0	3.2	.2
.8	2.8	2.8	.2
1.1	2.4	2.6	.2
1.5	2.6	2.2	.2
1.7	1.8	2.1	.2
2.3	1.8	1.9	.2
3.4	1.6	1.7	.2
5.7	1.3	1.4	.2
7.7	1.1	1.2	.2
9.5	1.1	1.1	.2
14.8	1.0	1.0	.2
19.6	1.0	.9	.2
.5	3.5	3.4	.2

-C	Ww	Ww-calc	Uncertainty
.2	3.8	3.6	.2
.5	3.5	3.0	.2
.6	2.8	2.8	.2
.9	2.1	2.4	.2
1.2	2.1	2.2	.2
1.5	1.9	2.1	.2
2.4	1.8	1.8	.2
3.8	1.5	1.6	.2
4.7	1.5	1.5	.2
6.8	1.3	1.3	.2
9.7	1.2	1.2	.2
13.7	1.1	1.0	.2
18.7	1.0	.9	.2
25.0	.9	.9	.2

Date: 01/02/88 Time: 14:39:22.03 Operator: PBB

Date: 01/02/88 Time: 14:40:24.04 Operator: PBB

Experiment name: MCAWA
Experiment number: 2

Experiment name: MCACB
Experiment number: 3

Wet density: 1.99
Dry density: 1.78
Gravimetric water content(Ww): 11.77 +/- .02
Volumetric water content(Wv): 21.00

Wet density: 1.86
Dry density: 1.74
Gravimetric water content(Ww): 6.65 +/- .02
Volumetric water content(Wv): 11.59

Theoretical NMR equation: $TNMR = 3.2828E2 + -5.3714E-1 * (-C)$
r-squared: .99

Theoretical NMR equation: $TNMR = 1.7565E2 + -2.5730E-1 * (-C)$
r-squared: .82

$$Ww = 2.181*(-C)**(-.290)$$

$$Ww = 2.697*(-C)**(-.325)$$

r-squared: .995
Standard Deviation: .004
F-value: 2.6219E3
Significance: .000

r-squared: .984
Standard Deviation: .019
F-value: 7.2544E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	4.1	4.0	.2	.2	4.4	4.2	.2
.3	3.0	3.1	.2	.5	3.7	3.4	.2
.5	2.7	2.7	.2	.6	3.2	3.1	.2
.7	2.5	2.4	.2	1.0	2.5	2.7	.2
.9	2.3	2.2	.2	1.3	2.4	2.5	.2
1.4	2.0	2.0	.2	1.6	2.2	2.3	.2
1.8	1.9	1.8	.2	2.5	1.9	2.0	.2
2.6	1.6	1.6	.2	3.8	1.6	1.7	.2
3.7	1.5	1.5	.2	4.8	1.6	1.6	.2
5.7	1.3	1.3	.2	6.9	1.4	1.4	.2
7.8	1.2	1.2	.2	9.6	1.3	1.3	.2
9.6	1.1	1.1	.2	13.7	1.2	1.2	.2
14.5	1.0	1.0	.2	18.6	1.1	1.0	.2
19.4	1.0	.9	.2	25.1	1.0	.9	.2
.5	2.7	2.7	.2				

Date: 01/02/88 Time: 14:41:25.99 Operator: PBB

Experiment name: MCAWB
Experiment number: 4

Wet density: 1.86
Dry density: 1.74
Gravimetric water content(Ww): 6.65 +/- .02
Volumetric water content(Wv): 11.59

Theoretical NMR equation: $TNMR = 1.7584E2 + -2.8652E-1 * (-C)$
r-squared: .86

$$Ww = 2.513*(-C)**(-.320)$$

r-squared: .989
Standard Deviation: .012
F-value: 1.1219E3
Significance: .000

Date: 01/02/88 Time: 14:42:33.39 Operator: PBB

Experiment name: MCACC
Experiment number: 5

Wet density: 1.93
Dry density: 1.55
Gravimetric water content(Ww): 24.58 +/- .02
Volumetric water content(Wv): 38.11

Theoretical NMR equation: $TNMR = 5.3915E2 + -7.9089E-1 * (-C)$
r-squared: .99

$$Ww = 2.911*(-C)**(-.348)$$

r-squared: .980
Standard Deviation: .027
F-value: 5.8845E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty
.2	4.3	4.1	.2
.4	3.5	3.4	.2
.5	3.1	3.1	.2
.7	2.8	2.8	.2
1.0	2.4	2.5	.2
1.4	2.4	2.3	.2
1.7	1.9	2.1	.2
2.4	1.8	1.9	.2
3.5	1.7	1.7	.2
5.8	1.3	1.4	.2
7.7	1.3	1.3	.2
9.5	1.3	1.2	.2
14.6	1.1	1.1	.2
19.4	1.0	1.0	.2
.5	3.1	3.2	.2

-C	Ww	Ww-calc	Uncertainty
.3	4.4	4.4	.2
.5	4.1	3.6	.2
.7	3.3	3.3	.2
1.1	2.8	2.8	.2
1.4	2.6	2.6	.2
1.7	2.3	2.4	.2
2.6	2.0	2.1	.2
3.9	1.7	1.8	.2
5.0	1.6	1.7	.2
7.0	1.4	1.5	.2
9.7	1.2	1.3	.2
13.9	1.3	1.2	.2
18.7	1.1	1.1	.2
25.3	1.0	.9	.2

Date: 01/02/88 Time: 14:49:35.82 Operator: PBB

Date: 01/02/88 Time: 14:50:40.63 Operator: PBB

Experiment name: MCAWC
Experiment number: 6

Experiment name: MCACD
Experiment number: 7

Wet density: 1.93
Dry density: 1.55
Gravimetric water content(Ww): 24.58 +/- .02
Volumetric water content(Wv): 38.11

Wet density: 1.95
Dry density: 1.56
Gravimetric water content(Ww): 25.28 +/- .02
Volumetric water content(Wv): 39.42

Theoretical NMR equation: $TNMR = 5.3858E2 + -7.0358E-1 * (-C)$
r-squared: .99

Theoretical NMR equation: $TNMR = 5.1214E2 + -5.0069E-1 * (-C)$
r-squared: .99

$$Ww = 2.755*(-C)**(-.348)$$

$$Ww = 2.564*(-C)**(-.356)$$

r-squared: .972
Standard Deviation: .037
F-value: 4.5329E2
Significance: .000

r-squared: .980
Standard Deviation: .026
F-value: 5.8557E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.3	4.7	4.3	.2	.2	4.3	4.4	.2
.5	3.7	3.6	.2	.5	3.9	3.4	.2
.6	3.3	3.3	.2	.6	3.0	3.1	.2
.8	3.0	3.0	.2	1.1	2.3	2.5	.2
1.1	2.9	2.7	.2	1.4	2.3	2.3	.2
1.5	2.5	2.4	.2	1.7	2.0	2.1	.2
1.8	2.0	2.2	.2	2.6	1.8	1.8	.2
2.4	1.9	2.0	.2	4.0	1.6	1.6	.2
3.5	1.7	1.8	.2	5.0	1.4	1.4	.2
5.9	1.4	1.5	.2	7.0	1.3	1.3	.2
7.7	1.3	1.4	.2	9.6	1.1	1.1	.2
9.6	1.2	1.3	.2	13.9	1.0	1.0	.2
14.8	1.1	1.1	.2	18.6	.9	.9	.2
19.6	1.1	1.0	.2	25.3	.8	.8	.2
.5	3.2	3.6	.2				

Date: 01/02/88 Time: 14:51:45.77 Operator: PBB

Date: 01/02/88 Time: 14:53:00.86 Operator: PBB

Experiment name: MCAWD
Experiment number: 8

Experiment name: MCACE
Experiment number: 9

Wet density: 1.95
Dry density: 1.56
Gravimetric water content(Ww): 25.28 +/- .02
Volumetric water content(Wv): 39.42

Wet density: 2.07
Dry density: 1.79
Gravimetric water content(Ww): 15.45 +/- .02
Volumetric water content(Wv): 27.64

Theoretical NMR equation: $TNMR = 5.1195E2 + -4.7169E-1 * (-C)$
r-squared: .99

Theoretical NMR equation: $TNMR = 4.2548E2 + -1.0696E0 * (-C)$
r-squared: .99

$$Ww = 2.607 * (-C) ** (-.366)$$

$$Ww = 2.375 * (-C) ** (-.331)$$

r-squared: .969
Standard Deviation: .044
F-value: 4.0795E2
Significance: .000

r-squared: .997
Standard Deviation: .003
F-value: 3.7911E3
Significance: .000

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-C	Ww	Ww-calc	Uncertainty
.2	4.8	4.3	.2
.4	3.7	3.6	.2
.6	3.0	3.2	.2
.8	2.8	2.8	.2
1.1	2.4	2.6	.2
1.5	2.6	2.2	.2
1.7	1.8	2.1	.2
2.3	1.8	1.9	.2
3.4	1.6	1.7	.2
5.7	1.3	1.4	.2
7.7	1.1	1.2	.2
9.5	1.1	1.1	.2
14.8	1.0	1.0	.2
19.6	1.0	.9	.2
.5	3.5	3.4	.2

-C	Ww	Ww-calc	Uncertainty
.2	4.0	3.9	.2
.4	3.2	3.2	.2
.6	2.8	2.8	.2
1.1	2.2	2.3	.2
1.4	2.2	2.1	.2
1.6	2.1	2.0	.2
2.6	1.6	1.7	.2
3.9	1.5	1.5	.2
4.8	1.4	1.4	.2
6.9	1.3	1.3	.2
9.6	1.1	1.1	.2
13.7	1.0	1.0	.2
18.5	.9	.9	.2
25.2	.8	.8	.1

Date: 01/02/88 Time: 14:54:10.17 Operator: PBB

Experiment name: MCAWE
Experiment number: 10

Wet density: 2.07
Dry density: 1.79
Gravimetric water content(Ww): 15.45 +/- .02
Volumetric water content(Wv): 27.64

Theoretical NMR equation: $TNMR = 4.2492E2 + -9.8287E-1 * (-C)$
r-squared: .99

$$Ww = 2.100*(-C)**(-.304)$$

r-squared: .974
Standard Deviation: .019
F-value: 4.8266E2
Significance: .000

Date: 01/02/88 Time: 14:57:31.58 Operator: PBB

Experiment name: MCBCA
Experiment number: 11

Wet density: 1.90
Dry density: 1.75
Gravimetric water content(Ww): 8.98 +/- .02
Volumetric water content(Wv): 15.70

Theoretical NMR equation: $TNMR = 2.5466E2 + -3.0848E-1 * (-C)$
r-squared: .72

$$Ww = .983*(-C)**(-.405)$$

r-squared: .987
Standard Deviation: .003
F-value: 8.9589E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty
.2	3.8	3.3	.2
.3	3.0	2.9	.2
.5	2.5	2.6	.2
.7	2.3	2.3	.2
1.0	2.0	2.1	.2
1.5	1.9	1.9	.2
1.7	1.6	1.8	.2
2.4	1.7	1.6	.2
3.3	1.4	1.5	.2
5.8	1.2	1.2	.2
7.6	1.1	1.1	.2
9.5	1.0	1.1	.2
14.6	.9	.9	.2
19.5	.9	.8	.2
.5	2.6	2.6	.2

-C	Ww	Ww-calc	Uncertainty
.2	1.8	1.8	.2
.4	1.6	1.4	.2
.6	1.2	1.2	.2
.9	.9	1.0	.2
1.1	.9	.9	.2
1.6	.8	.8	.2
2.8	.6	.7	.2
4.2	.6	.5	.2
4.8	.5	.5	.2
7.0	.5	.4	.2
9.6	.4	.4	.2
13.8	.4	.3	.2
18.6	.3	.3	.2
25.4	.3	.3	.2

Date: 01/02/88 Time: 14:59:57.91 Operator: PBB

Date: 01/02/88 Time: 15:05:50.09 Operator: PBB

Experiment name: MCBWA
Experiment number: 12

Experiment name: MCBCB
Experiment number: 13

Wet density: 1.90
Dry density: 1.75
Gravimetric water content(Ww): 8.98 +/- .02
Volumetric water content(Wv): 15.70

Wet density: 2.01
Dry density: 1.80
Gravimetric water content(Ww): 11.25 +/- .02
Volumetric water content(Wv): 20.27

Theoretical NMR equation: $TNMR = 2.5558E2 + -4.5201E-1 * (-C)$
r-squared: .97

Theoretical NMR equation: $TNMR = 3.1244E2 + -5.4034E-1 * (-C)$
r-squared: .99

$$Ww = .975*(-C)**(-.403)$$

$$Ww = .897*(-C)**(-.391)$$

r-squared: .647
Standard Deviation: .155
F-value: 2.3826E1
Significance: .000

r-squared: .973
Standard Deviation: .006
F-value: 4.3362E2
Significance: .000

19

-C	Ww	Ww-calc	Uncertainty
.2	1.8	1.7	.2
.3	1.3	1.5	.2
.6	1.2	1.2	.2
.9	1.0	1.0	.2
1.0	.9	1.0	.2
1.5	.6	.8	.2
1.8	.8	.8	.2
2.5	.6	.7	.2
3.4	.5	.6	.2
5.8	.5	.5	.2
7.6	.4	.4	.2
9.6	.4	.4	.2
14.7	.4	.3	.2
19.7	.3	.3	.2
.5	2.7	1.3	.2

-C	Ww	Ww-calc	Uncertainty
.2	1.8	1.6	.2
.3	1.5	1.4	.2
.6	1.2	1.1	.2
.9	.8	.9	.2
1.2	.8	.8	.2
1.6	.7	.7	.2
2.8	.5	.6	.2
4.2	.5	.5	.2
4.8	.5	.5	.2
7.0	.4	.4	.2
9.6	.4	.4	.2
13.7	.4	.3	.2
18.6	.3	.3	.2
25.3	.2	.3	.2

Date: 01/02/88 Time: 15:08:41.13 Operator: PBB

Date: 01/02/88 Time: 15:11:44.14 Operator: PBB

Experiment name: MCBWB
Experiment number: 14

Experiment name: MCBCC
Experiment number: 15

Wet density: 2.01
Dry density: 1.80
Gravimetric water content(Ww): 11.26 +/- .02
Volumetric water content(Wv): 20.29

Wet density: 2.03
Dry density: 1.79
Gravimetric water content(Ww): 13.24 +/- .02
Volumetric water content(Wv): 23.71

Theoretical NMR equation: $TNMR = 3.1244E2 + -5.4034E-1 * (-C)$
r-squared: .99

Theoretical NMR equation: $TNMR = 3.7091E2 + -3.4233E-1 * (-C)$
r-squared: .57

$$Ww = .950*(-C)**(-.434)$$

$$Ww = .853*(-C)**(-.399)$$

r-squared: .641
Standard Deviation: .162
F-value: 2.3164E1
Significance: .000

r-squared: .988
Standard Deviation: .003
F-value: 9.8528E2
Significance: .000

20

-C	Ww	Ww-calc	Uncertainty
.2	1.6	1.9	.2
.3	1.3	1.6	.2
.6	1.2	1.2	.2
.8	1.1	1.0	.2
1.0	.9	.9	.2
1.5	.7	.8	.2
1.8	.6	.7	.2
2.5	.6	.6	.2
3.4	.5	.6	.2
5.8	.4	.4	.2
7.6	.4	.4	.2
9.5	.4	.4	.2
14.6	.3	.3	.2
19.6	.3	.3	.2
.5	2.7	1.3	.2

-C	Ww	Ww-calc	Uncertainty
.2	1.8	1.7	.2
.2	1.5	1.5	.2
.5	1.2	1.1	.2
.9	.8	.9	.2
1.1	.8	.8	.2
1.5	.7	.7	.2
2.7	.6	.6	.2
3.9	.5	.5	.2
4.8	.4	.5	.2
6.9	.3	.4	.2
9.6	.4	.3	.2
13.6	.3	.3	.2
18.5	.3	.3	.2
25.3	.2	.2	.2

Date: 01/02/88 Time: 15:14:14.52 Operator: PBB

Experiment name: MCBWC
Experiment number: 16

Wet density: 2.03
Dry density: 1.79
Gravimetric water content(Ww): 13.24 +/- .02
Volumetric water content(Wv): 23.71

Theoretical NMR equation: $TNMR = 3.7035E2 + -2.5605E-1 * (-C)$
r-squared: .54

$$Ww = .953*(-C)**(-.433)$$

r-squared: .672
Standard Deviation: .148
F-value: 2.6645E1
Significance: .000

Date: 01/02/88 Time: 15:18:42.89 Operator: PBB

Experiment name: MCB CD
Experiment number: 17

Wet density: 2.08
Dry density: 1.79
Gravimetric water content(Ww): 16.54 +/- .02
Volumetric water content(Wv): 29.54

Theoretical NMR equation: $TNMR = 4.2484E2 + -6.4985E-1 * (-C)$
r-squared: .98

$$Ww = .989*(-C)**(-.419)$$

r-squared: .984
Standard Deviation: .005
F-value: 7.2919E2
Significance: .000

21

-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.2	1.8	2.2	.2	.2	1.9	1.8	.2
.3	1.4	1.6	.2	.3	1.6	1.5	.2
.6	1.2	1.2	.2	.6	1.3	1.2	.2
.7	1.0	1.1	.2	1.1	1.0	1.0	.2
1.0	.9	.9	.2	1.2	.8	.9	.2
1.4	.9	.8	.2	1.7	.8	.8	.2
1.8	.7	.7	.2	2.7	.6	.6	.2
2.5	.6	.6	.2	3.9	.5	.6	.2
3.4	.5	.6	.2	4.9	.5	.5	.2
5.7	.4	.4	.2	7.0	.4	.4	.2
7.6	.4	.4	.2	9.7	.4	.4	.2
9.5	.3	.4	.2	13.9	.3	.3	.2
14.6	.3	.3	.2	18.7	.3	.3	.2
19.6	.3	.3	.2	25.5	.3	.3	.2
.5	2.6	1.3	.2				

Date: 01/02/88 Time: 15:21:18.16 Operator: PBB

Date: 01/02/88 Time: 15:25:09.90 Operator: PBB

Experiment name: MCBWD
Experiment number: 18

Experiment name: MCCA
Experiment number: 19

Wet density: 2.08
Dry density: 1.79
Gravimetric water content(Ww): 16.54 +/- .02
Volumetric water content(Wv): 29.54

Wet density: 1.82
Dry density: 1.38
Gravimetric water content(Ww): 32.60 +/- .02
Volumetric water content(Wv): 44.85

Theoretical NMR equation: $TNMR = 4.2502E2 + -6.7850E-1 * (-C)$
r-squared: .98

Theoretical NMR equation: $TNMR = 6.7555E2 + -1.5195E0 * (-C)$
r-squared: 1.00

$$Ww = 1.004*(-C)**(-.422)$$

$$Ww = 4.130*(-C)**(-.309)$$

r-squared: .596
Standard Deviation: .205
F-value: 1.9179E1
Significance: .001

r-squared: .996
Standard Deviation: .013
F-value: 3.0354E3
Significance: .000

22

-C	Ww	Ww-calc	Uncertainty
.1	1.7	2.5	.2
.3	1.5	1.6	.2
.6	1.3	1.2	.2
.9	1.1	1.1	.2
1.0	.9	1.0	.2
1.5	.8	.8	.2
1.9	.7	.8	.2
2.5	.7	.7	.2
3.5	.5	.6	.2
5.8	.4	.5	.2
7.7	.4	.4	.2
9.6	.4	.4	.2
14.7	.3	.3	.2
19.7	.3	.3	.2
.5	2.8	1.4	.2

-C	Ww	Ww-calc	Uncertainty
.2	7.4	7.1	.2
.3	5.9	6.0	.2
.6	5.1	4.9	.2
1.1	4.1	4.1	.2
1.2	3.7	3.9	.2
1.6	3.6	3.6	.2
2.7	3.1	3.1	.2
3.8	2.7	2.7	.2
4.8	2.6	2.6	.2
6.9	2.2	2.3	.2
9.7	2.0	2.0	.2
13.7	1.8	1.8	.2
18.7	1.7	1.7	.2
25.4	1.6	1.5	.2

Date: 01/02/88 Time: 15:27:41.27 Operator: PBB

Experiment name: MCCWA
Experiment number: 20

Wet density: 1.82
Dry density: 1.38
Gravimetric water content(Ww): 32.60 +/- .02
Volumetric water content(Wv): 44.85

Theoretical NMR equation: $TNMR = 6.7499E2 + -1.4333E0 * (-C)$
r-squared: .99

$$Ww = 3.558*(-C)**(-.270)$$

r-squared: .951
Standard Deviation: .112
F-value: 2.5090E2
Significance: .000

Date: 01/02/88 Time: 15:30:46.15 Operator: PBB

Experiment name: MCCCB
Experiment number: 21

Wet density: 1.98
Dry density: 1.54
Gravimetric water content(Ww): 28.65 +/- .02
Volumetric water content(Wv): 44.10

Theoretical NMR equation: $TNMR = 6.4718E2 + -1.4805E0 * (-C)$
r-squared: .98

$$Ww = 4.155*(-C)**(-.317)$$

r-squared: .986
Standard Deviation: .042
F-value: 8.4688E2
Significance: .000

23

-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	6.7	6.8	.2	.2	7.1	6.4	.2
.3	5.5	5.0	.2	.3	5.8	5.9	.2
.6	4.5	4.1	.2	.6	5.0	5.0	.2
.8	4.0	3.8	.2	.9	4.1	4.2	.2
.9	3.7	3.6	.2	1.2	3.8	3.9	.2
1.5	3.3	3.2	.2	1.7	3.4	3.5	.2
1.9	3.1	3.0	.2	2.6	3.0	3.1	.2
2.5	2.6	2.8	.2	4.0	2.6	2.7	.2
3.5	2.6	2.5	.2	4.8	2.5	2.5	.2
5.8	2.2	2.2	.2	6.9	2.2	2.2	.2
7.6	2.0	2.1	.2	9.7	2.0	2.0	.2
9.6	1.9	1.9	.2	13.7	1.8	1.8	.2
14.6	1.7	1.7	.2	18.6	1.7	1.6	.2
19.6	1.6	1.6	.2	25.4	1.5	1.5	.2
.5	3.4	4.3	.2				

Date: 01/02/88 Time: 15:33:43.72 Operator: PBB

Experiment name: MCCWB
Experiment number: 22

Wet density: 1.98
Dry density: 1.54
Gravimetric water content(Ww): 28.65 +/- .02
Volumetric water content(Wv): 44.10

Theoretical NMR equation: $TNMR = 6.4645E2 + -1.3661E0 * (-C)$
r-squared: .99

$$Ww = 3.457*(-C)**(-.268)$$

r-squared: .938
Standard Deviation: .134
F-value: 1.9683E2
Significance: .000

Date: 01/02/88 Time: 15:36:26.25 Operator: PBB

Experiment name: MCCCC
Experiment number: 23

Wet density: 1.87
Dry density: 1.48
Gravimetric water content(Ww): 26.34 +/- .02
Volumetric water content(Wv): 38.98

Theoretical NMR equation: $TNMR = 6.2949E2 + -1.6453E0 * (-C)$
r-squared: 1.00

$$Ww = 4.814*(-C)**(-.326)$$

r-squared: .979
Standard Deviation: .085
F-value: 5.6382E2
Significance: .000

24

-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	6.6	6.6	.2	.3	8.2	7.3	.2
.3	5.2	4.8	.2	.3	6.7	6.9	.2
.5	4.6	4.1	.2	.6	5.7	5.8	.2
.7	3.8	3.8	.2	1.0	4.7	4.8	.2
.9	3.7	3.5	.2	1.2	4.5	4.5	.2
1.4	3.4	3.1	.2	1.5	4.0	4.2	.2
1.8	3.0	2.9	.2	2.6	3.5	3.5	.2
2.4	2.7	2.7	.2	3.7	3.1	3.1	.2
3.5	2.4	2.5	.2	4.7	2.9	2.9	.2
5.7	2.2	2.2	.2	6.9	2.5	2.6	.2
7.6	2.0	2.0	.2	9.6	2.3	2.3	.2
9.6	1.8	1.9	.2	13.7	2.0	2.0	.2
14.6	1.7	1.7	.2	18.6	1.9	1.9	.2
19.6	1.6	1.6	.2	25.4	1.7	1.7	.2
.5	3.1	4.2	.2				

Date: 01/02/88 Time: 15:43:36.59 Operator: PBB

Experiment name: MCCWC
Experiment number: 24

Wet density: 1.87
Dry density: 1.48
Gravimetric water content(Ww): 26.34 +/- .02
Volumetric water content(Wv): 38.98

Theoretical NMR equation: $TNMR = 6.2912E2 + -1.5882E0 * (-C)$
r-squared: 1.00

$$Ww = 3.839*(-C)**(-.258)$$

r-squared: .913
Standard Deviation: .288
F-value: 1.3590E2
Significance: .000

Date: 01/02/88 Time: 15:46:22.19 Operator: PBB

Experiment name: MCCCD
Experiment number: 25

Wet density: 1.86
Dry density: 1.62
Gravimetric water content(Ww): 15.25 +/- .02
Volumetric water content(Wv): 24.68

Theoretical NMR equation: $TNMR = 3.8319E2 + -6.4083E-1 * (-C)$
r-squared: .96

$$Ww = 4.608*(-C)**(-.300)$$

r-squared: .993
Standard Deviation: .023
F-value: 1.7567E3
Significance: .000

25

-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	8.1	7.6	.2	.2	7.6	7.3	.2
.2	6.0	5.7	.2	.3	6.4	6.4	.2
.5	5.1	4.6	.2	.6	5.6	5.4	.2
.7	4.4	4.2	.2	.9	4.5	4.7	.2
.9	4.1	3.9	.2	1.1	4.2	4.4	.2
1.4	3.8	3.5	.2	1.5	4.1	4.1	.2
1.8	3.5	3.3	.2	2.6	3.4	3.5	.2
2.5	3.1	3.0	.2	3.7	3.1	3.1	.2
3.5	2.8	2.8	.2	4.8	2.8	2.9	.2
5.6	2.4	2.5	.2	6.9	2.6	2.6	.2
7.6	2.2	2.3	.2	9.6	2.3	2.3	.2
9.5	2.1	2.1	.2	13.6	2.2	2.1	.2
14.6	1.9	1.9	.2	18.5	1.9	1.9	.2
19.6	1.8	1.8	.2	25.1	1.8	1.8	.2
.5	2.9	4.6	.2				

Date: 01/02/88 Time: 15:49:45.41 Operator: PBB

Experiment name: MCCWD
Experiment number: 26

Wet density: 1.86
Dry density: 1.62
Gravimetric water content(Ww): 15.25 +/- .02
Volumetric water content(Wv): 24.68

Theoretical NMR equation: $TNMR = 3.8356E2 + -6.9803E-1 * (-C)$
r-squared: .99

$$Ww = 3.835*(-C)**(-.240)$$

r-squared: .897
Standard Deviation: .276
F-value: 1.1331E2
Significance: .000

Date: 01/02/88 Time: 15:54:42.67 Operator: PBB

Experiment name: MCCCE
Experiment number: 27

Wet density: 1.79
Dry density: 1.63
Gravimetric water content(Ww): 9.98 +/- .02
Volumetric water content(Wv): 16.24

Theoretical NMR equation: $TNMR = 2.4288E2 + -4.7760E-1 * (-C)$
r-squared: .92

$$Ww = 4.758*(-C)**(-.307)$$

r-squared: .993
Standard Deviation: .028
F-value: 1.6648E3
Significance: .000

26

-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	7.6	7.3	.2	.2	8.1	7.8	.2
.3	5.7	5.2	.2	.3	6.5	6.7	.2
.5	5.0	4.6	.2	.6	5.9	5.6	.2
.7	4.5	4.2	.2	1.0	4.6	4.8	.2
.9	4.1	3.9	.2	1.2	4.3	4.5	.2
1.3	3.8	3.6	.2	1.5	4.2	4.2	.2
1.9	3.5	3.3	.2	2.6	3.6	3.5	.2
2.5	3.2	3.1	.2	3.7	3.1	3.2	.2
3.5	3.0	2.8	.2	4.7	2.9	3.0	.2
5.7	2.5	2.5	.2	6.9	2.7	2.6	.2
7.6	2.3	2.4	.2	9.6	2.3	2.4	.2
9.5	2.2	2.2	.2	13.7	2.1	2.1	.2
14.5	2.0	2.0	.2	18.6	2.0	1.9	.2
19.5	1.9	1.9	.2	25.2	1.8	1.8	.2
.5	2.9	4.6	.2				

Date: 01/02/88 Time: 15:57:10.37 Operator: PBB

Experiment name: MCCWE
Experiment number: 28

Wet density: 1.79
Dry density: 1.63
Gravimetric water content(Ww): 9.98 +/- .02
Volumetric water content(Wv): 16.24

Theoretical NMR equation: $TNMR = 2.4251E2 + -4.2020E-1 * (-C)$
r-squared: .97

$$Ww = 3.923*(-C)**(-.236)$$

r-squared: .916
Standard Deviation: .226
F-value: 1.4253E2
Significance: .000

Date: 01/04/88 Time: 14:08:57.90 Operator: PBB

Experiment name: MCAWA
Experiment number: 2

Wet density: 1.99
Dry density: 1.78
Gravimetric water content(Ww): 11.77 +/- .02
Volumetric water content(Wv): 21.00

Theoretical NMR equation: $TNMR = 3.2828E2 + -5.3714E-1 * (-C)$
r-squared: .99

$$Ww = 2.185*(-C)**(-.291)$$

r-squared: .995
Standard Deviation: .004
F-value: 2.4583E3
Significance: .000

-C	Ww	Ww-calc	Uncertainty
.1	7.8	7.4	.2
.3	5.8	5.3	.2
.5	4.9	4.7	.2
.7	4.5	4.2	.2
.9	4.2	4.0	.2
1.3	3.7	3.7	.2
1.9	3.5	3.4	.2
2.5	3.1	3.2	.2
3.5	3.0	2.9	.2
5.7	2.6	2.6	.2
7.7	2.4	2.4	.2
9.5	2.3	2.3	.2
14.6	2.1	2.1	.2
19.6	2.0	1.9	.2
.5	3.2	4.7	.2

-C	Ww	Ww-calc	Uncertainty
.1	4.1	4.1	.2
.3	3.0	3.1	.2
.5	2.7	2.7	.2
.7	2.5	2.4	.2
.9	2.3	2.2	.2
1.4	2.0	2.0	.2
1.8	1.9	1.8	.2
2.6	1.6	1.7	.2
3.7	1.5	1.5	.2
5.7	1.3	1.3	.2
7.8	1.2	1.2	.2
9.6	1.1	1.1	.2
14.5	1.0	1.0	.2
19.4	1.0	.9	.2

Date: 01/04/88 Time: 14:12:30.02 Operator: PBB

Experiment name: MCAWB
Experiment number: 4

Wet density: 1.86
Dry density: 1.74
Gravimetric water content(Ww): 6.65 +/- .02
Volumetric water content(Wv): 11.59

Theoretical NMR equation: $TNMR = 1.7584E2 + -2.8652E-1 * (-C)$
r-squared: .86

$$Ww = 2.523*(-C)**(-.322)$$

r-squared: .990
Standard Deviation: .011
F-value: 1.1652E3
Significance: .000

Date: 01/04/88 Time: 14:17:08.00 Operator: PBB

Experiment name: MCAWC
Experiment number: 6

Wet density: 1.93
Dry density: 1.55
Gravimetric water content(Ww): 24.58 +/- .02
Volumetric water content(Wv): 38.11

Theoretical NMR equation: $TNMR = 5.3858E2 + -7.0359E-1 * (-C)$
r-squared: .99

$$Ww = 2.789*(-C)**(-.354)$$

r-squared: .982
Standard Deviation: .024
F-value: 6.5970E2
Significance: .000

28

-C	Ww	Ww-calc	Uncertainty
.2	4.3	4.1	.2
.4	3.5	3.4	.2
.5	3.1	3.1	.2
.7	2.8	2.8	.2
1.0	2.4	2.5	.2
1.4	2.4	2.3	.2
1.7	1.9	2.1	.2
2.4	1.8	1.9	.2
3.5	1.7	1.7	.2
5.8	1.3	1.4	.2
7.7	1.3	1.3	.2
9.5	1.3	1.2	.2
14.6	1.1	1.1	.2
19.4	1.0	1.0	.2

-C	Ww	Ww-calc	Uncertainty
.3	4.7	4.4	.2
.5	3.7	3.6	.2
.6	3.3	3.3	.2
.8	3.0	3.0	.2
1.1	2.9	2.7	.2
1.5	2.5	2.4	.2
1.8	2.0	2.3	.2
2.4	1.9	2.0	.2
3.5	1.7	1.8	.2
5.9	1.4	1.5	.2
7.7	1.3	1.4	.2
9.6	1.2	1.3	.2
14.8	1.1	1.1	.2
19.6	1.1	1.0	.2

Date: 01/04/88 Time: 14:20:35.51 Operator: PBB

Date: 01/04/88 Time: 14:47:27.46 Operator: PBB

Experiment name: MCAWD
Experiment number: 8

Experiment name: MCAWE
Experiment number: 10

Wet density: 1.95
Dry density: 1.56
Gravimetric water content(Ww): 25.28 +/- .02
Volumetric water content(Wv): 39.42

Wet density: 2.07
Dry density: 1.79
Gravimetric water content(Ww): 15.45 +/- .02
Volumetric water content(Wv): 27.64

Theoretical NMR equation: $TNMR = 5.1195E2 + -4.7169E-1 * (-C)$
r-squared: .99

Theoretical NMR equation: $TNMR = 4.2492E2 + -9.8287E-1 * (-C)$
r-squared: .99

$$Ww = 2.597*(-C)**(-.364)$$

$$Ww = 2.100*(-C)**(-.304)$$

r-squared: .965
Standard Deviation: .049
F-value: 3.3216E2
Significance: .000

r-squared: .972
Standard Deviation: .021
F-value: 4.1663E2
Significance: .000

29

-C	Ww	Ww-calc	Uncertainty
.2	4.8	4.3	.2
.4	3.7	3.5	.2
.6	3.0	3.2	.2
.8	2.8	2.8	.2
1.1	2.4	2.5	.2
1.5	2.6	2.2	.2
1.7	1.8	2.1	.2
2.3	1.8	1.9	.2
3.4	1.6	1.7	.2
5.7	1.3	1.4	.2
7.7	1.1	1.2	.2
9.5	1.1	1.1	.2
14.8	1.0	1.0	.2
19.6	1.0	.9	.2

-C	Ww	Ww-calc	Uncertainty
.2	3.8	3.3	.2
.3	3.0	2.9	.2
.5	2.5	2.6	.2
.7	2.3	2.3	.2
1.0	2.0	2.1	.2
1.5	1.9	1.9	.2
1.7	1.6	1.8	.2
2.4	1.7	1.6	.2
3.3	1.4	1.5	.2
5.8	1.2	1.2	.2
7.6	1.1	1.1	.2
9.5	1.0	1.1	.2
14.6	.9	.9	.2
19.5	.9	.8	.2

Date: 01/04/88 Time: 14:52:29.33 Operator: PBB

Experiment name: MCBWA
Experiment number: 12

Wet density: 1.90
Dry density: 1.75
Gravimetric water content(Ww): 8.98 +/- .02
Volumetric water content(Wv): 15.70

Theoretical NMR equation: $TNMR = 2.5558E2 + -4.5201E-1 * (-C)$
r-squared: .97

$$Ww = .892*(-C)**(-.357)$$

r-squared: .938
Standard Deviation: .012
F-value: 1.8280E2
Significance: .000

Date: 01/04/88 Time: 14:55:10.37 Operator: PBB

Experiment name: MCBWB
Experiment number: 14

Wet density: 2.01
Dry density: 1.80
Gravimetric water content(Ww): 11.25 +/- .02
Volumetric water content(Wv): 20.27

Theoretical NMR equation: $TNMR = 3.1244E2 + -5.4034E-1 * (-C)$
r-squared: .99

$$Ww = .872*(-C)**(-.393)$$

r-squared: .963
Standard Deviation: .007
F-value: 3.1654E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.2	1.8	1.5	.2	.2	1.6	1.6	.2
.3	1.3	1.3	.2	.3	1.3	1.4	.2
.6	1.2	1.1	.2	.6	1.2	1.1	.2
.9	1.0	.9	.2	.8	1.1	1.0	.2
1.0	.9	.9	.2	1.0	.9	.9	.2
1.5	.6	.8	.2	1.5	.7	.7	.2
1.8	.8	.7	.2	1.8	.6	.7	.2
2.5	.6	.6	.2	2.5	.6	.6	.2
3.4	.5	.6	.2	3.4	.5	.5	.2
5.8	.5	.5	.2	5.8	.4	.4	.2
7.6	.4	.4	.2	7.6	.4	.4	.2
9.6	.4	.4	.2	9.5	.4	.4	.2
14.7	.4	.3	.2	14.6	.3	.3	.2
19.7	.3	.3	.2	19.6	.3	.3	.2

Date: 01/04/88 Time: 15:00:38.77 Operator: PBB

Experiment name: MCBWC
Experiment number: 16

Wet density: 2.03
Dry density: 1.79
Gravimetric water content(Ww): 13.24 +/- .02
Volumetric water content(Wv): 23.71

Theoretical NMR equation: $TNMR = 3.7035E2 + -2.5605E-1 * (-C)$
r-squared: .54

$$Ww = .881*(-C)**(-.396)$$

r-squared: .989
Standard Deviation: .002
F-value: 1.0723E3
Significance: .000

Date: 01/04/88 Time: 15:04:16.44 Operator: PBB

Experiment name: MCBWD
Experiment number: 18

Wet density: 2.08
Dry density: 1.79
Gravimetric water content(Ww): 16.54 +/- .02
Volumetric water content(Wv): 29.54

Theoretical NMR equation: $TNMR = 4.2502E2 + -6.7850E-1 * (-C)$
r-squared: .98

$$Ww = .925*(-C)**(-.384)$$

r-squared: .920
Standard Deviation: .019
F-value: 1.3872E2
Significance: .000

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-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.2	1.8	1.9	.2	.1	1.7	2.1	.2
.3	1.4	1.4	.2	.3	1.5	1.4	.2
.6	1.2	1.1	.2	.6	1.3	1.1	.2
.7	1.0	1.0	.2	.9	1.1	1.0	.2
1.0	.9	.9	.2	1.0	.9	.9	.2
1.4	.9	.8	.2	1.5	.8	.8	.2
1.8	.7	.7	.2	1.9	.7	.7	.2
2.5	.6	.6	.2	2.5	.7	.6	.2
3.4	.5	.5	.2	3.5	.5	.6	.2
5.7	.4	.4	.2	5.8	.4	.5	.2
7.6	.4	.4	.2	7.7	.4	.4	.2
9.5	.3	.4	.2	9.6	.4	.4	.2
14.6	.3	.3	.2	14.7	.3	.3	.2
19.6	.3	.3	.2	19.7	.3	.3	.2

Date: 01/04/88 Time: 15:07:04.29 Operator: PBB

Experiment name: MCCWA
Experiment number: 20

Wet density: 1.82
Dry density: 1.38
Gravimetric water content(Ww): 32.60 +/- .02
Volumetric water content(Wv): 44.85

Theoretical NMR equation: $TNMR = 6.7499E2 + -1.4333E0 * (-C)$
r-squared: .99

$$Ww = 3.653*(-C)**(-.281)$$

r-squared: .983
Standard Deviation: .042
F-value: 7.0073E2
Significance: .000

Date: 01/04/88 Time: 15:09:52.86 Operator: PBB

Experiment name: MCCWB
Experiment number: 22

Wet density: 1.98
Dry density: 1.54
Gravimetric water content(Ww): 28.65 +/- .02
Volumetric water content(Wv): 44.10

Theoretical NMR equation: $TNMR = 6.4645E2 + -1.3661E0 * (-C)$
r-squared: .99

$$Ww = 3.568*(-C)**(-.282)$$

r-squared: .984
Standard Deviation: .036
F-value: 7.5771E2
Significance: .000

32

-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	6.7	7.2	.2	.1	6.6	7.0	.2
.3	5.5	5.2	.2	.3	5.2	5.0	.2
.6	4.5	4.2	.2	.5	4.6	4.2	.2
.8	4.0	3.9	.2	.7	3.8	3.9	.2
.9	3.7	3.7	.2	.9	3.7	3.6	.2
1.5	3.3	3.3	.2	1.4	3.4	3.2	.2
1.9	3.1	3.1	.2	1.8	3.0	3.0	.2
2.5	2.6	2.8	.2	2.4	2.7	2.8	.2
3.5	2.6	2.6	.2	3.5	2.4	2.5	.2
5.8	2.2	2.2	.2	5.7	2.2	2.2	.2
7.6	2.0	2.1	.2	7.6	2.0	2.0	.2
9.6	1.9	1.9	.2	9.6	1.8	1.9	.2
14.6	1.7	1.7	.2	14.6	1.7	1.7	.2
19.6	1.6	1.6	.2	19.6	1.6	1.5	.2

Date: 01/04/88 Time: 15:12:27.15 Operator: PBB

Experiment name: MCCWC
Experiment number: 24

Wet density: 1.87
Dry density: 1.48
Gravimetric water content(Ww): 26.34 +/- .02
Volumetric water content(Wv): 38.98

Theoretical NMR equation: $TNMR = 6.2912E2 + -1.5882E0 * (-C)$
r-squared: 1.00

$$Ww = 4.014*(-C)**(-.277)$$

r-squared: .996
Standard Deviation: .014
F-value: 3.1023E3
Significance: .000

Date: 01/04/88 Time: 15:15:24.01 Operator: PBB

Experiment name: MCCWD
Experiment number: 26

Wet density: 1.86
Dry density: 1.62
Gravimetric water content(Ww): 15.25 +/- .02
Volumetric water content(Wv): 24.68

Theoretical NMR equation: $TNMR = 3.8356E2 + -6.9803E-1 * (-C)$
r-squared: .99

$$Ww = 4.015*(-C)**(-.259)$$

r-squared: .993
Standard Deviation: .019
F-value: 1.8076E3
Significance: .000

33

-C	Ww	Ww-calc	Uncertainty	-C	Ww	Ww-calc	Uncertainty
.1	8.1	8.4	.2	.1	7.6	8.0	.2
.2	6.0	6.1	.2	.3	5.7	5.6	.2
.5	5.1	4.9	.2	.5	5.0	4.9	.2
.7	4.4	4.4	.2	.7	4.5	4.5	.2
.9	4.1	4.1	.2	.9	4.1	4.1	.2
1.4	3.8	3.6	.2	1.3	3.8	3.7	.2
1.8	3.5	3.4	.2	1.9	3.5	3.4	.2
2.5	3.1	3.1	.2	2.5	3.2	3.2	.2
3.5	2.8	2.8	.2	3.5	3.0	2.9	.2
5.6	2.4	2.5	.2	5.7	2.5	2.6	.2
7.6	2.2	2.3	.2	7.6	2.3	2.4	.2
9.5	2.1	2.2	.2	9.5	2.2	2.2	.2
14.6	1.9	1.9	.2	14.5	2.0	2.0	.2
19.6	1.8	1.8	.2	19.5	1.9	1.9	.2

Date: 01/04/88 Time: 15:18:36.41 Operator: PBB

Experiment name: MCCWE

Experiment number: 28

Wet density: 1.79
Dry density: 1.63
Gravimetric water content(Ww): 9.98 +/- .02
Volumetric water content(Wv): 16.24

Theoretical NMR equation: $TNMR = 2.4251E2 + -4.2020E-1 * (-C)$
r-squared: .97

$$Ww = 4.081*(-C)**(-.253)$$

r-squared: .997
Standard Deviation: .009
F-value: 3.8581E3
Significance: .000

-C	Ww	Ww-calc	Uncertainty
.1	7.8	8.0	.2
.3	5.8	5.6	.2
.5	4.9	4.9	.2
.7	4.5	4.4	.2
.9	4.2	4.1	.2
1.3	3.7	3.8	.2
1.9	3.5	3.5	.2
2.5	3.1	3.2	.2
3.5	3.0	3.0	.2
5.7	2.6	2.6	.2
7.7	2.4	2.4	.2
9.5	2.3	2.3	.2
14.6	2.1	2.1	.2
19.6	2.0	1.9	.2

APPENDIX B: ERROR ANALYSIS

Temperature

Sample temperatures were determined from a thermocouple immersed in the constant temperature bath to the depth of the soil samples and measured with a NESLAB DR-2 digital readout to an accuracy of $\pm 0.1^\circ\text{C}$:

$$\Delta(\text{Temperature}) = \pm 0.1^\circ\text{C}.$$

Gravimetric water content

Gravimetric water contents were determined by measuring the difference between wet sample and oven-dried sample mass:

$$W_w = \frac{M_{\text{wet}} - M_{\text{dry}}}{M_{\text{dry}} - M_{\text{tare}}}$$

where M_{wet} = mass of wet soil and tare

M_{dry} = soil mass (oven-dried) plus tare

M_{tare} = mass of the tare.

The error involved is obtained by application of the chain rule:

$$\Delta W_w = \frac{\partial W_w}{\partial M_{\text{wet}}} \Delta M_{\text{wet}} + \frac{\partial W_w}{\partial M_{\text{dry}}} \Delta M_{\text{dry}} + \frac{\partial W_w}{\partial M_{\text{tare}}} \Delta M_{\text{tare}}.$$

The partial differentials are found to be

$$\frac{\partial W_w}{\partial M_{\text{wet}}} = \frac{1}{M_{\text{dry}} - M_{\text{tare}}}$$

$$\frac{\partial W_w}{\partial M_{\text{dry}}} = - \frac{W_w + 1}{M_{\text{dry}} - M_{\text{tare}}}$$

and

$$\frac{\partial W_w}{\partial M_{\text{tare}}} = \frac{W_w}{M_{\text{dry}} - M_{\text{tare}}}$$

with

$$\Delta M_{\text{total}} = \pm 0.001 \text{ g}$$

$$\Delta M_{\text{soil}} = \pm 0.001 \text{ g}$$

and

$$\Delta M_{\text{tare}} = \pm 0.001 \text{ g}.$$

Unfrozen water content

The unfrozen water content is determined from the ratio of measured NMR signal to the “theoretical” NMR signal of supercooled water at that temperature, both adjusted for background NMR signal, multiplied by the gravimetric water content of the ice-free soil:

$$UWC = \frac{NMR - BKG}{TNMR - BKG} W_w$$

where UWC is the calculated unfrozen water content, NMR is the signal measured at a temperature, TNMR is the calculated NMR signal for a theoretical supercooled soil water at that temperature obtained from a linear regression of the above-0°C NMR data and BKG is the background NMR signal. The uncertainty is determined by applying the chain rule:

$$\Delta UWC = \frac{\partial UWC}{\partial NMR} \Delta NMR + \frac{\partial UWC}{\partial BKG} \Delta BKG + \frac{\partial UWC}{\partial TNMR} \Delta TNMR + \frac{\partial UWC}{\partial W_w} \Delta W_w.$$

Solving for each partial gives

$$\frac{\partial UWC}{\partial NMR} = \frac{W_w}{TNMR - BKG}$$

$$\frac{\partial UWC}{\partial BKG} = \frac{NMR - TNMR}{(TNMR - BKG)^2} W_w$$

$$\frac{\partial UWC}{\partial TNMR} = - \frac{NMR - BKG}{(TNMR - BKG)^2} W_w$$

and

$$\frac{\partial UWC}{\partial W_w} = \frac{NMR - BKG}{TNMR - BKG} .$$

The uncertainty of each term associated with the NMR signal is found, after years of observation, to be

$$\Delta NMR = \pm 2 \text{ counts}$$

$$\Delta BKG = \pm 2 \text{ counts}$$

and because of the near perfect linear correlation of above 0°C data, the uncertainty of the “theoretical” supercooled signal is assumed to be the same as the others:

$$\Delta TNMR = \pm 2 \text{ counts.}$$

APPENDIX C: SOIL WATER CURVE DATA

Table C1. Soil water curve (SWC) data for three horizons of the Windsor soil from Iskandar et al. (1979).

<i>Soil horizon</i>	<i>Drying</i>		<i>Wetting</i>	
	<i>Suction, ϕ_{aw} (bars)</i>	<i>Volumetric water content, W_v</i>	<i>Suction, ϕ_{aw} (bars)</i>	<i>Volumetric water content, W_v</i>
A	0.000	0.450	0.588	0.150
	0.025	0.430	0.294	0.170
	0.049	0.420	0.132	0.200
	0.103	0.300	0.049	0.255
	0.216	0.220	0.000	0.420
	0.348	0.190		
	0.529	0.170		
	0.745	0.150		
	0.980	0.140		
	B	0.000	0.350	0.980
0.015		0.340	0.588	0.110
0.025		0.330	0.402	0.115
0.049		0.225	0.196	0.135
0.059		0.205	0.098	0.150
0.078		0.185	0.059	0.165
0.137		0.135	0.039	0.180
0.245		0.150	0.025	0.200
0.402		0.135	0.015	0.240
0.667		0.120	0.000	0.335
0.980		0.105		
C		0.000	0.360	0.980
	0.020	0.345	0.662	0.037
	0.054	0.230	0.392	0.050
	0.078	0.160	0.196	0.070
	0.127	0.125	0.132	0.090
	0.196	0.100	0.064	0.120
	0.392	0.070	0.000	0.300
	0.529	0.055		
	0.667	0.050		
	0.980	0.035		

A facsimile catalog card in Library of Congress MARC format is reproduced below.

Black, Patrick B.

Comparison of soil freezing curve and soil water curve data for Windsor sandy loam / by Patrick B. Black and Allen R. Tice. Hanover, N.H.: U.S. Army Cold Regions Research and Engineering Laboratory; Springfield, Va.: available from National Technical Information Service, 1988.

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1. Frozen soils. 2. Nuclear magnetic resonance. 3. Soil water. 4. Unfrozen water content. I. Tice, Allen R. II. United States Army. Corps of Engineers. III. Cold Regions Research and Engineering Laboratory. IV. Series: CRREL Report 88-16.