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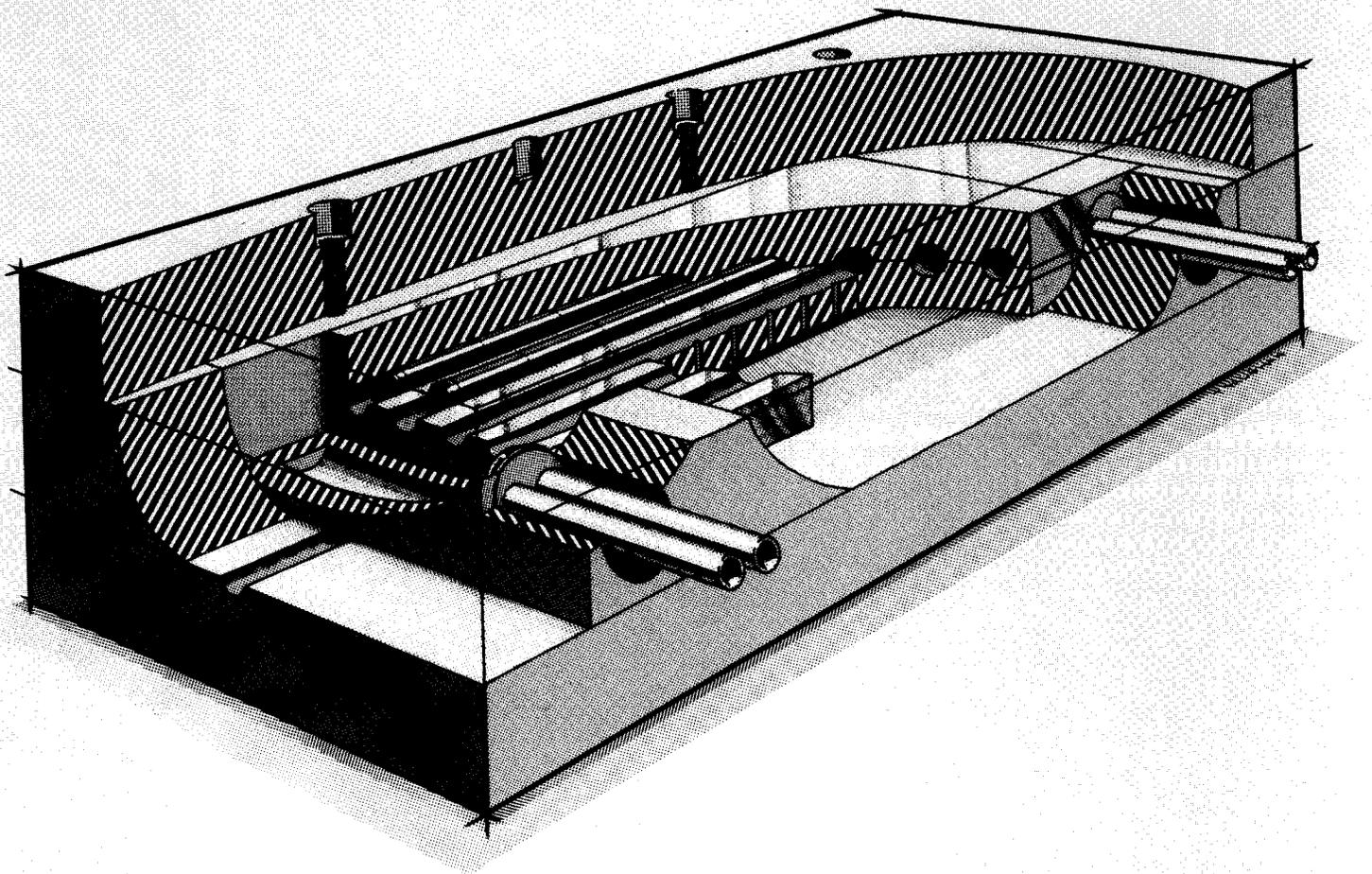
## REPORT 87-9



US Army Corps  
of Engineers

Cold Regions Research &  
Engineering Laboratory

### *Factors affecting water migration in frozen soils*



# CRREL Report 87-9

July 1987



## *Factors affecting water migration in frozen soils*

Xu Xiaozu, Joseph L. Oliphant and Allen R. Tice

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## **PREFACE**

This report was prepared by Xu Xiaozu, a visiting professor from the Lanzhou Institute of Glaciology and Cryopedology, Peoples Republic of China, Dr. J.L. Oliphant, former Research Scientist, and A.R. Tice, Physical Science Technician, of the Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. This paper reports work accomplished under a cooperative research agreement between CRREL and the Lanzhou Institute. Funding for this research was provided by DA project 4A161102AT24, *Research in Snow, Ice and Frozen Ground; Task A, Properties of Cold Regions Materials; Work Unit 002, Properties of Frozen Soils.*

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## NOMENCLATURE

$A$	coefficient
$B$	coefficient
$C$	constant
$D$	constant
$e$	density of soil water
$J$	flux
$J_i$	average flux in $i$ th segment
$J_{i-1}$	average flux in $i-1$ th segment
$P_w$	pressure or suction of pore water
$Q$	total flux
$T$	temperature
$T_f$	freezing point
$W$	volumetric water content of soil
$W_1^1, W_1^2$	total water content at times $\tau_1, \tau_2$
$W_0$	initial water content
$W_u$	unfrozen water content
$\Delta x$	length of segment
$\gamma_d$	dry density
$\mu_0$	chemical potential of pore water
$\mu_w$	soil-water potential
$\mu_w^1$	chemical potential of soil water
$\mu_{wf}$	soil-water potential of break point
$\rho$	density of soil water
$\tau$	time

# Factors Affecting Water Migration in Frozen Soils

XU XIAOZU, J.L. OLIPHANT AND A.R. TICE

## INTRODUCTION

Water migration in frozen soils under a temperature gradient not only changes the structure of frozen soil and the proportions of ice and unfrozen water, but also results in changes of the physical and mechanical properties, and causes ice segregation and frost heave. It is of great significance both for the explanation of cryogenic phenomena in permafrost areas and for engineering design in cold regions.

To evaluate and predict quantitatively the amount of water migration in frozen soils, much research work has been done, both theoretically and experimentally (Dirksen and Miller 1966, Hoekstra 1966, Miller et al. 1975, Anderson et al. 1978, Ershov 1979, Williams and Perfect 1979).

Most of the previous work has been concentrated on the investigation of the relationship between water migration and frost heave. The laboratory methods most commonly used are as follows.

A soil column surrounded by insulation is set up, either horizontally or vertically, and the temperature is kept below the freezing point at one end of the soil column and above at the other. Either a closed system (no external water supply) or an open system (having external water supply) is used. Results have shown that, in a partially frozen soil column, the amount of frost heave is much greater in the frozen portion than at the frozen/thawed boundary.

The most recent research is focused on the calculation of the amount of water migration in frozen soils, the determination of the hydraulic parameters of frozen soils and the explanation of driving forces. On the basis of the extended Darcy's law and the Clausius-Clapeyron equation, many mathematical models have been proposed using the methods of physicomathematics or thermodynamics (Xu 1982, O'Neill 1983), and several apparatus for determining the hydraulic parameters of frozen soils have been set up using methods that look at the stable or quasi-stable state (Ershov and Cheverev 1974, Burt and Williams 1976, Williams and Wood 1981, Nakano et al. 1982). A question remains, however, about the ap-

plicability of the extended Darcy's law to moisture flow in frozen soils.

The purpose of this report is to describe the changing characteristics of water migration in frozen soils in relation to the temperature, temperature gradient, dry density and time.

To obtain a deeper understanding of water migration in unsaturated frozen soils within a closed system and under small temperature gradients, we first carried out a series of tests on the soil-water potential and the unfrozen water content. We believe that the gradient of the soil-water potential is the driving force and that between the solid, liquid and gas phases, the unfrozen water (liquid phase) is the main path for water migration in frozen soils.

To test this hypothesis, we selected a clay (Morin clay) for our test material and determined its flux of water migration. Two sets of tests were done: one was conducted with different temperature gradients and dry densities, holding the experimental duration the same, and another with different experimental durations, keeping the temperature gradient and dry density the same.

## PRELIMINARY STUDIES

### Soil-water potential in unfrozen soils

#### *Factors affecting the soil-water potential*

From the point of view of thermodynamics, water migration in a soil-water system is caused by the water not being in equilibrium in the system. This results from a combination of many forces, including physical, physicochemical, mechanical and others arising during the water migration process.

Except for the gravitational water, the other types of water, i.e., adsorbed water and chemically bound water, are subjected to an action from surface energy of the mineral particles, or an action from the capillary forces and adsorption forces.

When water in the soil and mineral particles interacts in the soil-water system, the decrease in

free energy of bulk water can be expressed by the soil-water potential,  $\mu_w$ , which is the work required to transfer a unit mass of water under an isothermal condition,

$$\mu_w = \mu_w' - \mu_0 \quad (1)$$

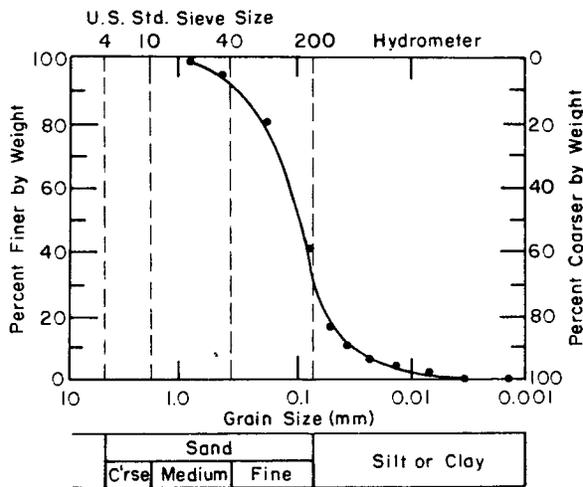
where  $\mu_w'$  is the chemical potential of soil water and  $\mu_0$  is the chemical potential of pure water.

Suppose that the density of water in a soil-water system is equal to that of the pore water, then the

soil-water potential is equal to the pressure or suction in the pore water in the equilibrium state, and can be expressed as

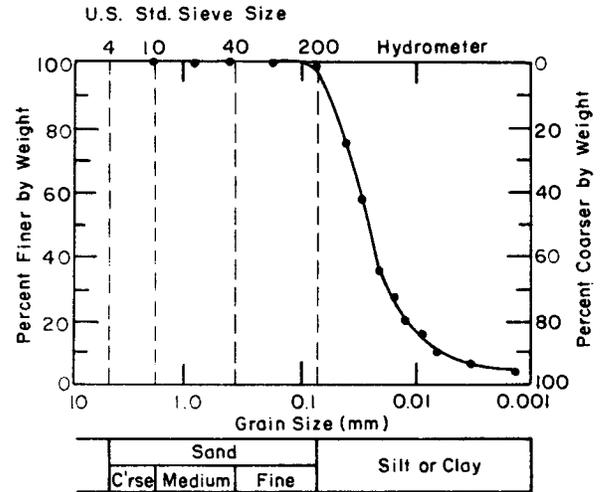
$$\mu_w = \frac{1}{\rho} P_w \quad (2)$$

where  $\rho$  = density of soil water and  $P_w$  = pressure, or suction, in the pore water in the soil-water system.



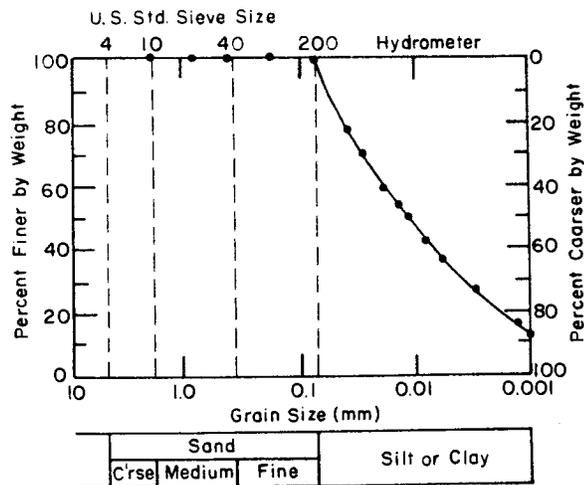
Organic Content 0, Liquid Limit 19.4, Specific Gravity 2.65

a. Graves sand.



Organic Content 0, Liquid Limit 28.3, Specific Gravity 2.66

b. Northwest silt.



Organic Content 0, Liquid Limit 38.3, Plastic Limit 22.8, Plastic Index 15.5  
Specific Gravity 2.72, Specific Surface Area 60

c. Morin clay.

Figure 1. Physical properties of soils used in this study.

Therefore, the factors affecting the soil-water potential, such as the water content, soil type, dry density and temperature, are investigated by the determination of soil-water characteristic curves. It has been shown (Williams 1967, Kaliuzhuyi 1980) that these curves obtained from the unfrozen soils are also applicable to frozen soils.

Three types of soils are used in this study. They are Graves sand, Northwest silt and Morin clay. The physical properties of these soils are shown in Figure 1.

*Influence of the water content on the soil-water potential.* As mentioned above, water in soils is subjected to an action from the surface energy of mineral particles. With the decrease in the degree of saturation, the capillary water is first removed, leaving the pellicular water. The forces to which water is subjected change from capillary forces to adsorption forces. As a result, the soil-water potential always decreases with a decrease in water content.

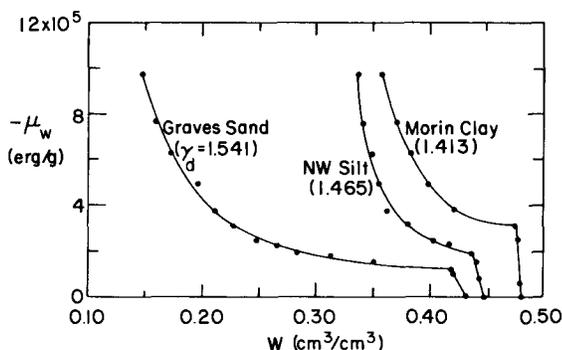


Figure 2. Effect of the water content (W) and soil type on soil-water potential.

Results shown in Figure 2 were obtained by an extraction method (Ingersoll 1981). They are in good agreement with the above statement. The process of the soil-water potential changing with the water content can be divided into three stages: the first is the stage of meniscus formation in big capillaries, where the soil-water potential decreases rapidly and linearly with decreasing water content. From the first stage to the second, there is an obvious break in the curve, called the break point or air entry point. The second is the stage of water extraction from the large capillaries, in which the soil-water potential decreases slowly, and the third is the stage of water extraction from small capil-

laries, in which the soil-water potential decreases rapidly again.

*Influence of the soil type on the soil-water potential.* The influence of the soil type implies both the influences of the mineral composition and the grain size of soil particles. However, here we only discuss the influence of the grain size distribution.

With the changing of the soil type from sand to clay, the amount of fine-grained particles increases. Consequently, the active specific surface area increases and the average radius of capillaries decreases. The two changes play a common role, only to different degrees, in increasing the bonded energy of water and decreasing the soil-water potential.

Again, from Figure 2 it can be seen clearly that at the same water content the sand has the highest value of the soil-water potential, the silt the middle value and the clay the lowest.

*Influence of the dry density on the soil-water potential.* If the soil type and the water content are kept the same, the soil-water potential increases with the increase in the dry density regardless of soil type (see Fig. 3). This is ascribable to the change of the soil structure.

The densification process causes a change in pore size, especially for large pores, while the originally small pores do not change their size significantly. As usual, the pellicular water is in the larger pores and the capillary water is in the smaller pores. Therefore, the change in the pore size results in the transition of water from the pellicular to the capillary, and so increases the soil-water potential.

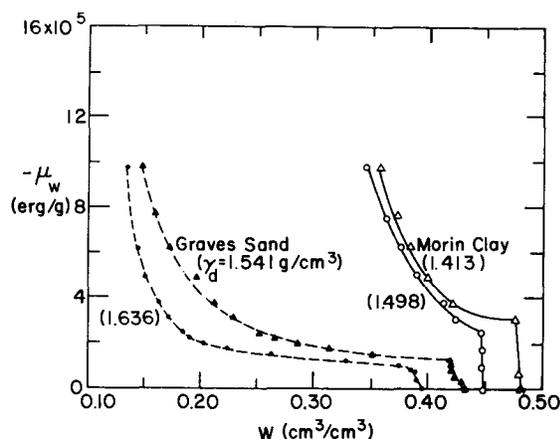


Figure 3. Effect of the dry density on soil-water potential.

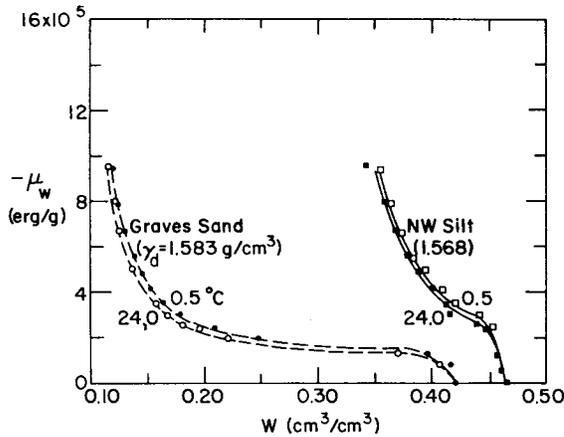


Figure 4. Effect of the temperature on soil-water potential.

*Influence of temperature on the soil-water potential.* Figure 4 shows the curves of the soil-water potential versus the water content for two kinds of soils at the temperatures 0.5°C and 24°C respectively. For each kind of soil, the two curves at different temperatures are quite parallel. This shows that the influence of temperature on the soil-water potential has nothing to do with the water content. Also, the soil-water potential at 24°C is higher than that at 0.5°C, which is reasonable because the kinetic energy of water molecules is higher at high temperatures. The higher the kinetic energy of the water molecules, the lower the bonded energy of water, and the lower the bonded energy of water, the higher the soil-water potential.

Since the relationship between the soil-water potential and the temperature is important for water migration in frozen soils, we will discuss it in more detail in the *Discussion* section of this report.

#### Relationship between the soil-water potential and the water content

If we take the break point on the curve (see Fig. 2) as a starting point, the relationship between the

soil-water potential and the water content can be well expressed by a simple power curve

$$\mu_w = A W^{-B} \quad (3)$$

where  $\mu_w$  = absolute value of the soil-water potential

$W$  = volumetric water content of the soil

$A$  and  $B$  = coefficients specific to each soil.

The values of the coordinates of the break point and coefficients  $A$  and  $B$  for those soils tested are listed in Table 1.

By introducing the break point coordinates into eq 3, we have

$$\frac{\mu_w}{\mu_{wf}} = \left(\frac{W_f}{W}\right)^B \quad (4)$$

where  $\mu_{wf}$  and  $W_f$  are the soil-water potential and the water content of the break point.

Equation 4 can be rewritten as

$$\mu_w = \mu_{wf} W_f^B W^{-B}. \quad (5)$$

Combining eq 3 and 5, we have

$$A = \mu_{wf} W_f^B. \quad (6)$$

#### Unfrozen water content in frozen soils

The presence of unfrozen water in frozen soils is attributable to the soil-water potential mentioned above. The unfrozen water content in frozen soils not only depends on the composition and properties of both the mineral particles and soil solution, but also depends on the environment of the soil-water system—for example, the temperature and pressure that the system is under. Previous work has established the relationship between the unfrozen water content and temperature (Anderson and Tice 1973, Banin and Anderson 1974, Kononov 1979, Tice et al. 1982). On the basis of this previ-

Table 1. Values of the break point coordinates and coefficients  $A$  and  $B$ .

Soil type	$\gamma_d$ (g/cm <sup>3</sup> )	Saturated water content (cm <sup>3</sup> /cm <sup>3</sup> )	Break point coordinate		Coefficient in eq 3	
			$W_f$ (cm <sup>3</sup> /cm <sup>3</sup> )	$\mu_{wf}$ (erg/g)	A	B
Graves sand	1.541	0.431	0.418	$-1.2 \times 10^5$	$-9.7173 \times 10^3$	2.3844
Northwest silt	1.465	0.447	0.437	$-1.8 \times 10^5$	$-0.2157 \times 10^3$	7.6171
Morin clay	1.413	0.476	0.476	$-3.1 \times 10^5$	$-7.5904 \times 10^3$	4.9973

ous work, we investigated the influences of three factors—the initial water content, dry density and salt concentration—on the unfrozen water content in frozen soils by the nuclear magnetic resonance technique reported by Tice et al. (1982).

*Factors affecting the unfrozen water content in frozen soils*

*Influence of the initial water content on the unfrozen water content.* Experimental results show that the unfrozen water content in frozen Morin clay decreases with the increase in the initial water content at a given temperature and that the difference increases with increasing temperature (see Fig. 5). For the range of the initial water content from 4.81% of dry soil weight to 41.16%, the unfrozen water content ( $W_u$ ) difference changes with temperature as follows:

$\Delta W_u$ (%)	$t$ (°C)
1.0	< -10
2.0	< - 1.0
3.0	< - 0.2

All the curves shown in Figure 5 are quite parallel for the change in the initial water content.

When the initial water content reaches or exceeds the value of saturation in frozen soils, ice lenses are formed. The bottom curve shown in Figure 5 is obtained from pure ice made from distilled water. It shows that the pure ice still has unfrozen water in it when the temperature is higher

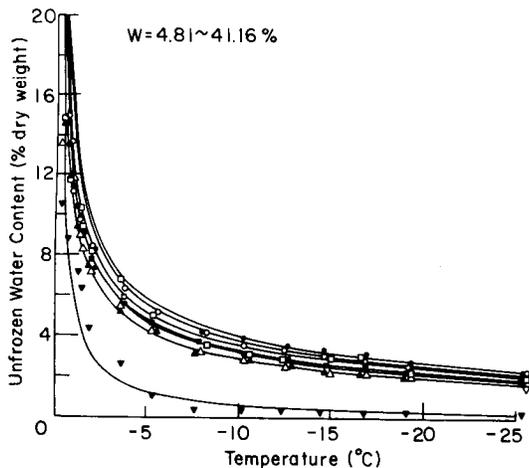


Figure 5. Effect of the water content of Morin clay on the unfrozen water content.

than  $-6^{\circ}\text{C}$ . Therefore, it is possible for unfrozen water to pass through the ice lens, even though the amount of unfrozen water in ice lenses may be much less than that in frozen soils.

*Influence of the dry density on the unfrozen water content.* Figure 6 shows the unfrozen water content at different temperatures when the dry density changes from  $0.997$  to  $1.714 \text{ g/cm}^3$  and the initial water content remains at approximately 19% by weight. It can be seen that the unfrozen water content is relatively constant at a given temperature and within a wide range of dry densities. For example, there is less than 1% difference in the unfrozen water content at temperatures below  $-1.0^{\circ}\text{C}$ .

The values of the unfrozen water content in those samples with higher dry densities are greater than those with lower dry densities at temperatures below  $-2.0^{\circ}\text{C}$ ; this is reversed at temperatures above  $-2.0^{\circ}\text{C}$  and is ascribable to the influence of pore size (see Table 2). At temperatures below  $-2.0^{\circ}\text{C}$ , the unfrozen water content increases with the increase in the dry density because the soil-water potential is higher at high dry density. At temperatures above  $-2.0^{\circ}\text{C}$ , the film water in large pores is not frozen because its freezing point is lower than that of the capillary water. As the dry density increases, the transition of water from the film to the capillary will result in decreasing the unfrozen water content.

*Influence of the salt concentration on the unfrozen water content.* To change the salt concentration of the solution in the soil-water system, we used solutions of sodium chloride with three dif-

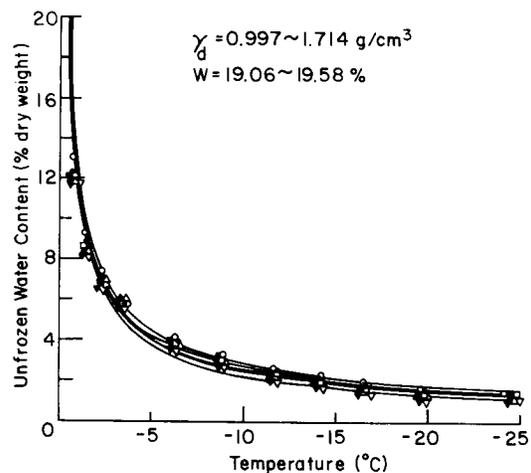


Figure 6. Effect of the dry density of Morin clay on the unfrozen water content.

**Table 2. Values of the unfrozen water content ( $W_u$ ) calculated from eq 7 at different dry densities.**

$\gamma_d$ (g/cm <sup>3</sup> )	W		$W_u$ at various temperatures (%)						
	by weight (%)	by volume (cm <sup>3</sup> /cm <sup>3</sup> )	-0.5°C	-1.0°C	-2.0°C	-5°C	-10°C	-15°C	-20°C
0.997	19.47	0.194	18.64	11.68	7.33	3.95	2.48	1.89	1.56
1.079	19.58	0.211	18.90	12.01	7.64	4.19	2.67	2.05	1.69
1.263	19.39	0.245	17.61	11.44	7.44	4.20	2.73	2.12	1.78
1.331	19.36	0.258	17.37	11.33	7.39	4.20	2.74	2.13	1.79
1.432	19.26	0.276	17.95	11.57	7.46	4.18	2.70	2.09	1.74
1.581	19.23	0.304	16.66	11.09	7.38	4.31	2.87	2.26	1.91
1.643	19.34	0.318	17.37	11.45	7.55	4.35	2.87	2.25	1.89
1.714	19.06	0.327	16.06	10.78	7.24	4.28	2.87	2.27	1.93

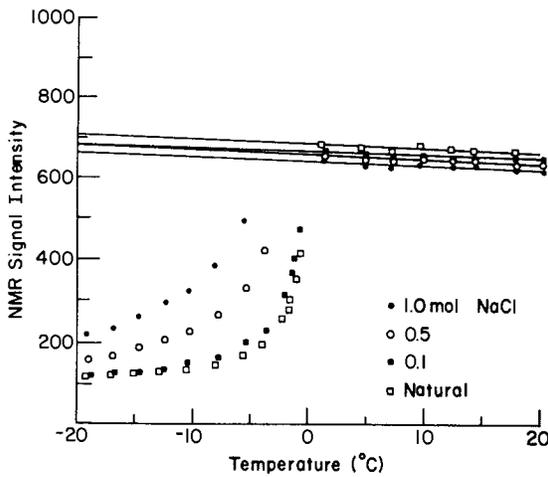
ferent concentrations of 0.1, 0.5 and 1.0 mol, respectively, and mixed these with the oven-dried Morin clay. The initial water content was kept at approximately 24%.

The results show that the unfrozen water content increases greatly with the increase in the salt concentration, and that there is a good linear relationship between them (Fig. 7 and 8) because the freezing point decreases linearly with the increase in the molality (Fig. 9).

*Prediction of the unfrozen water content by two-point or one-point methods*

Anderson and Tice (1972) presented the following equation describing the relationship between the unfrozen water content and the negative temperature

$$W_u = C T^{-D} \quad (7)$$



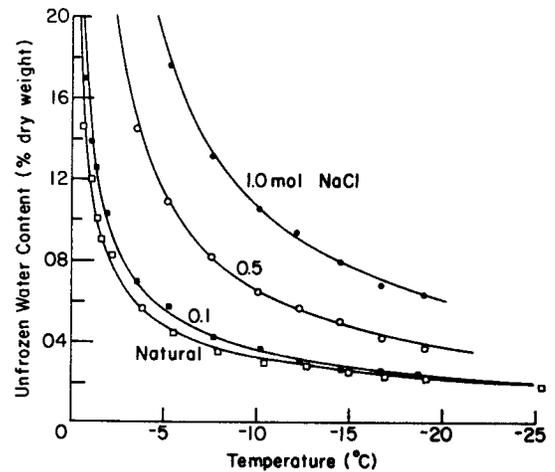
**Figure 7. Effect of the salt concentration of Morin clay on the NMR signal.**

where  $W_u$  = unfrozen water content (% by weight)

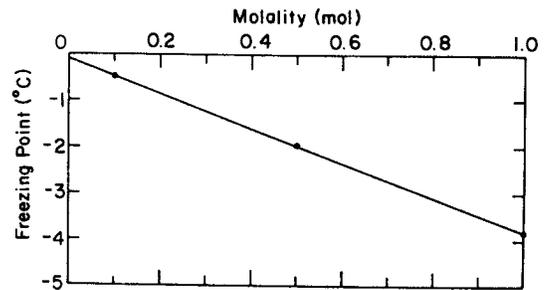
$T$  = temperature below freezing (°C)

$C$  and  $D$  = constants characteristic of each soil.

Equation 7 can be expressed as a straight line on log-log paper. On this straight line there is a start-



**Figure 8. Relationship between molality of Morin clay and the unfrozen water content.**



**Figure 9. Relationship between molality of Morin clay and initial freezing point.**

**Table 3. Values of constants  $C$  and  $D$  in eq 7, and other parameters.**

Soil type	$W_o$ (%)	$\gamma_d$ (g/cm <sup>3</sup> )	Salt concentration (mol)	$T_f$ (°C)	Constant	
					C	D
Morin clay	4.81	1.401	—	-7.09	14.4099	-0.5602
	9.26	1.545	—	-2.04	13.8692	-0.5673
	14.38	1.602	—	-0.79	12.6011	-0.5512
	19.44	1.671	—	-0.41	11.8345	-0.5552
	25.77	1.532	—	-0.24	11.7509	-0.5541
	30.90	1.545	—	-0.19	11.8075	-0.5741
	36.07	1.416	—	-0.11	10.4109	-0.5564
	41.16	1.343	—	-0.09	10.6700	-0.5675
	19.47	0.997	—	-0.47	11.6878	-0.6733
	19.58	1.079	—	-0.47	12.0125	-0.6538
	19.39	1.263	—	-0.43	11.4438	-0.6221
	19.36	1.331	—	-0.42	11.3301	-0.6166
	19.26	1.432	—	-0.45	11.5740	-0.6328
	19.23	1.581	—	-0.39	11.0903	-0.5867
	19.34	1.643	—	-0.42	11.4507	-0.6013
	19.06	1.714	—	-0.37	10.7822	-0.5746
	19.27	1.473	0.1 NaCl	-0.72	15.8527	-0.5958
	19.05	1.472	0.5 NaCl	-2.25	34.4404	-0.7322
	19.06	1.544	1.0 NaCl	-3.80	50.7306	-0.7327
	19.13	1.812	1.99 SW	-6.79	85.8868	-0.7840
24.71	1.614	0.1 NaCl	-0.47	15.3221	-0.6246	
24.12	1.715	0.5 NaCl	-1.94	40.7384	-0.7909	
23.69	1.648	1.0 NaCl	-3.79	69.8565	-0.8109	
Northwest silt	16.67	1.523	—	-0.13	5.2752	-0.5675
Fairbanks silt	40.46	—	—	-0.02	6.8988	-0.4735
Graves sand	18.08	1.543	—	-0.02	1.6945	-0.6104

ing point with the coordinates of the initial water content ( $W_o$ ) and the freezing point ( $T_f$ ). Then, the slope of the line is

$$D = \frac{\ln W_o - \ln W_u}{\ln T - \ln T_f} \quad (8)$$

From eq 8 we have the following expression, which is exactly the same as Konovalov's (1979) equation:

$$\frac{W_o}{W_u} = \left(\frac{T}{T_f}\right)^D \quad (9)$$

Equation 9 can be rewritten as

$$W_u = W_o T_f^D T^{-D} \quad (10)$$

Comparing eq 7 and 10, we have

$$C = W_o T_f^D \quad (11)$$

To predict the unfrozen water content in frozen soils by eq 10, the only constant we need to deter-

mine is the slope of the straight line,  $D$ , if the initial water content and the freezing point of a soil are given.

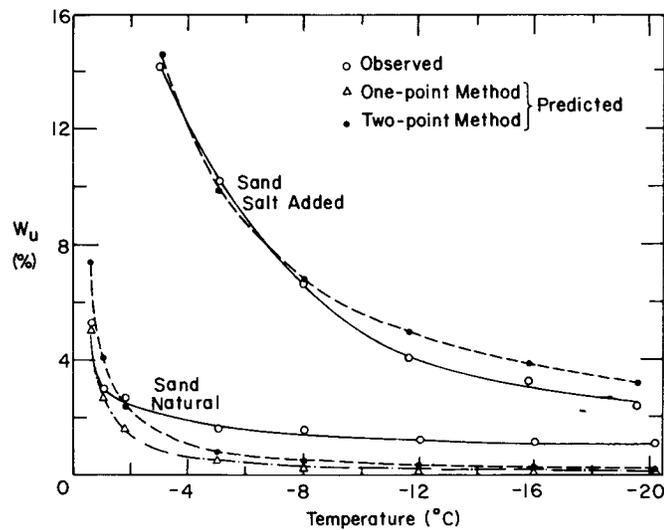
From Table 3 we can see that the value of  $D$  changes with the soil type and salt concentration, and does not change very much for a given soil, even though the initial water content and the dry density vary over a wide range. Therefore, by the measurements of two freezing points at two different initial water contents, we can calculate the constant  $D$  by eq 8, and then predict the unfrozen water content by eq 10. This is called the two-point method.

Again, from eq 10, we have

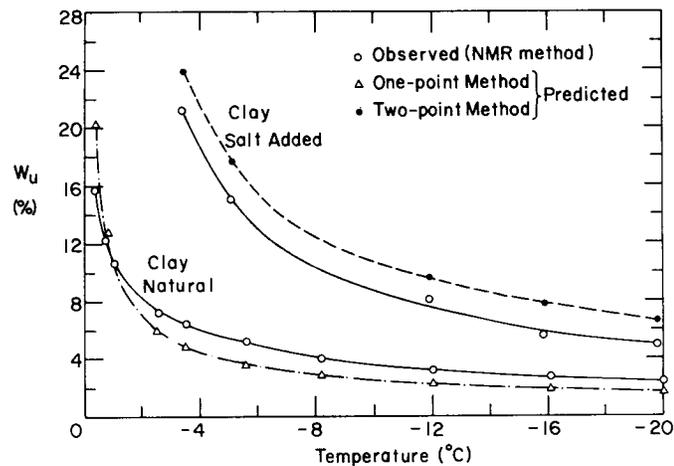
$$W_u = W_o T_f^D \quad (12)$$

if  $T = -1^\circ\text{C}$ . Then,

$$D = \frac{\ln \frac{W_u | T = -1}{W_o}}{\ln T_f} \quad (13)$$



a. Dartmouth sand with salt added and no salt added.



b. Morin clay.

Figure 10. Comparison of observed and predicted curves.

The unfrozen water content in frozen soils only needs to be determined once at  $-1^{\circ}\text{C}$ . Then, we can calculate  $D$  by eq 13. This is called the one-point method.

Comparisons of both the observed and the predicted results from the sand and clay are shown in Figure 10. Errors of predicting the unfrozen water content are 1 to 3% on the average for the two-point method and 1% or so for the one-point method.

## TEMPERATURE GRADIENT EXPERIMENT

The apparatus used in this test was described in detail in a separate paper (Oliphant et al. 1983); however, to give readers a clear picture of it, we will describe it briefly.

The apparatus consists of two aluminum blocks of the same size—30.5 cm long, 11.2 cm wide and 2.54 cm thick. Each block has five semicircular

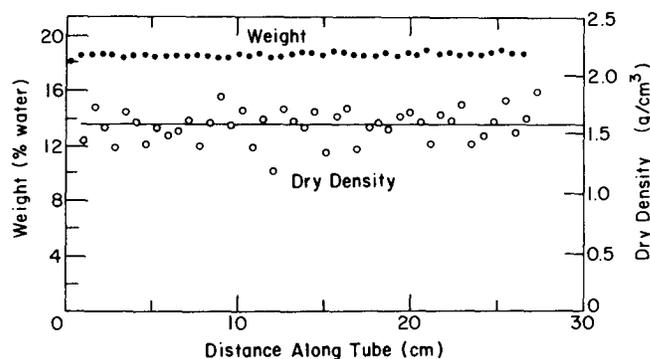


Figure 11. Moisture distribution before testing.

grooves that are spaced 1 cm apart along one of its faces. When the two blocks are bolted together, five holes, 0.95 cm in diameter, are formed, into which the plastic soil columns are placed. At each end of the two blocks there is a reservoir connected to a circulating refrigerated bath to keep temperature constant, with an accuracy of  $\pm 0.02^\circ\text{C}$ . The two reservoirs are also made up of aluminum, and are 11.2 cm long, 5.08 cm wide and 5.08 cm thick.

The whole apparatus is surrounded by foam insulation and set up horizontally in a constant-temperature room with the ambient temperature maintained close to  $0^\circ\text{C}$ . Thermocouples are inserted into the two blocks along the middle groove, 2.54 cm apart, to monitor the temperature distribution both horizontally and vertically.

As mentioned earlier, we used oven-dried Morin clay for these experiments. The clay was mixed with distilled water to obtain an initial water content of approximately 20%, and then placed in a sealed bottle for 2 or 3 days to allow moisture equilibration. Then, it was packed carefully into the plastic tubes, which have 0.95-cm o.d., 0.792-cm i.d. and are 27.8 cm long. Two rubber stoppers approximately 0.5 cm long were inserted into both ends of each tube to prevent moisture loss. The packed soil columns were then put aside for 1 or 2 days at room temperature to allow moisture equilibration in the tubes. Afterwards, they were removed to the coldroom to be frozen.

We define frozen soil as a soil in which ice and unfrozen water coexist. To keep the soil specimen always in the frozen state during testing, we fixed the temperature at the warm end of the soil column at about or lower than  $-0.6^\circ\text{C}$ . This is because the freezing point of Morin clay at the initial

water content of 20% is about  $-0.4^\circ\text{C}$  (see Table 3). The temperature selected for the cold end of the soil column depended on the required temperature gradient.

To start the test, we put five blank soil columns into the blocks until the expected linear temperature gradient was established. After that, we exchanged the blank ones for the five clay-filled columns. Several days later, these soil columns were quickly removed to the coldroom and each was sectioned into 44 segments. The total water content of each segment was determined gravimetrically.

The average moisture flux was calculated by the following expression:

$$J_i = J_{i-1} + \frac{(W_i^2 - W_i^1)\Delta x \gamma_d}{\tau_2 - \tau_1} \quad (14)$$

where  $J_{i-1}$  and  $J_i$  = average flux of the  $i-1$ th and  $i$ th segments, respectively ( $\text{g}/\text{cm}^2 \text{ s}$ )

$W_i^1$  and  $W_i^2$  = total water content at times  $\tau_1$  and  $\tau_2$  respectively, in weight fraction

$\Delta x$  = length of the segment (cm)

$\gamma_d$  = dry density of the soil ( $\text{g}/\text{cm}^3$ ).

By using the apparatus and following the experimental procedure mentioned above, we found that the moisture distribution in the soil column was sometimes not uniform before testing (Fig. 11). The temperature gradient in the horizontal direction is quite linear (Fig. 12) and stable during testing, but there is still a temperature difference occurring in the vertical direction within  $0.1^\circ$  to approximately  $0.4^\circ\text{C}$ , which depends on the temperature gradient.

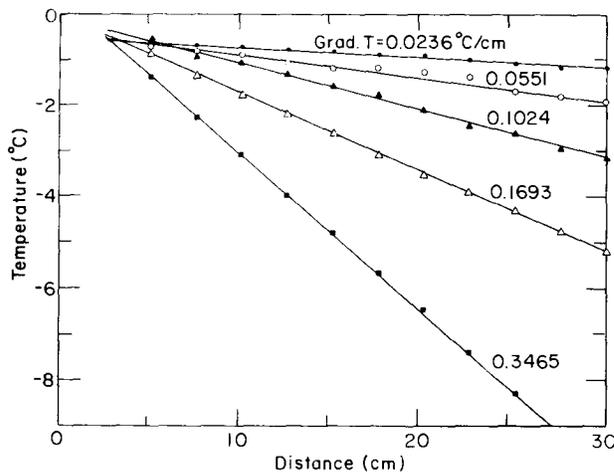


Figure 12. Temperature distribution in horizontal direction.

### Water content profiles after migration

Figure 13 illustrates the distribution curves of the total water content in frozen Morin clay under different temperature gradients for 5 days. From these figures, it can be seen that unfrozen water in the frozen soil columns migrates from the warm end (segment 1) to the cold (segment 44), and that each soil column can be divided into two sections according to the relationship between the total water contents before and after migration. The section adjacent to the warm side is called the "drying part," in which the final total water content is less than its initial water content, and the remaining section of the soil column is called the "wetting part," in which the final total water content is higher than the initial.

The length of the drying part becomes shorter and shorter, and the degree of drying becomes higher and higher as the temperature gradient goes up.

There are two peaks in the wetting part: one occurring at the front of this part and the other at the end. The front peak gradually becomes sharper and sharper as the temperature gradient goes up.

Figure 14 illustrates the total water content profiles of the soil columns under the temperature gradient equal to 0.1693 °C/cm for 2, 5 and 10 days respectively. It shows that the front peak of the wetting part moves towards the cold end of the soil column as the test duration increases.

Water migration in the liquid phase in the frozen soil can be explained as follows. In these experiments, the temperature gradient can be con-

sidered as an external force that causes a soil-water potential gradient for water migration in the frozen soil column. The soil-water potential at the place with higher temperature is greater than that at the place with lower temperatures. Unfrozen water in frozen soil migrates in the direction of decreasing temperature under the soil-water potential gradient; thus, the thermodynamic equilibrium between ice and unfrozen water at the place with higher temperature is upset by the unfrozen water decreasing. The potential of unfrozen water is lower than that of ice. A new potential gradient occurs in the direction from ice to unfrozen water and causes a transition from ice to unfrozen water, so the total water content at the place with higher temperature is decreasing. On the other hand, the thermodynamic equilibrium between ice and unfrozen water at the cold end is destroyed by the increasing unfrozen water, resulting in the phase transition of unfrozen water to ice. This process passes through the test from the very beginning to the end and along the whole length of the soil columns. In the closed system, i.e., under the case without an external water supply, water migration will be stopped when the stored water in the dry part of the soil column is exhausted.

### Factors affecting the flux

The flux of water migration can be calculated by eq 14 according to the total water content measurements. In this case, we only used one soil type, Morin clay, so that the flux can be considered as a function of the following four factors:

$$J = f(\tau, T, \gamma_d, \text{grad}T) \quad (15)$$

where  $\tau$  = time, or test duration

$\gamma_d$  = the dry density of the soil (g/cm<sup>3</sup>)

$T$  = temperature (°C)

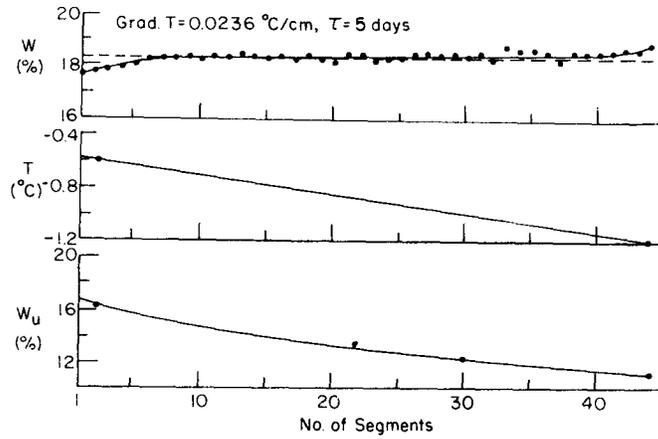
$\text{grad}T$  = temperature gradient (°C/cm).

### Influence of time

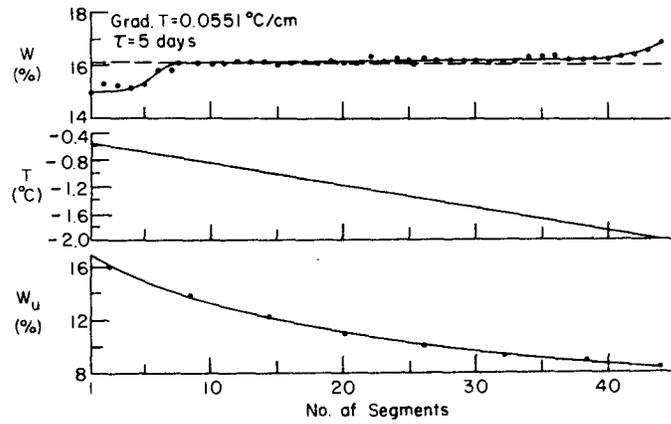
To investigate the relationship between the flux and time, four tests with different durations of 2, 5, 10 and 16 days, respectively, were conducted under a given set of conditions:  $\text{grad}T = 0.1693$  °C/cm,  $\gamma_d = 1.45$  to 1.47 g/cm<sup>3</sup>, and  $T = -0.9$ ° to approximately 5.2°C.

Figure 15 shows that the flux decays with time and that there is a good relationship between them (see Fig. 16), i.e., the flux is inversely proportional to the square root of time

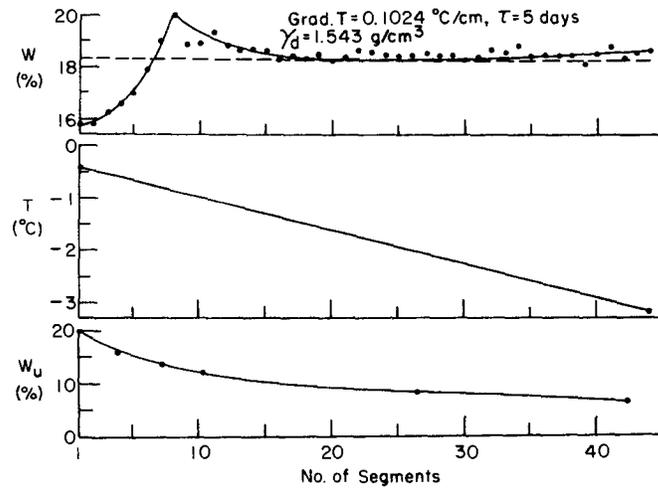
$$J = (A\tau^{-1/2} + B)_{T, \text{grad}T, \gamma_d} \quad (16)$$



a.  $\text{grad}T = 0.0236 \text{ }^\circ\text{C/cm}$ .

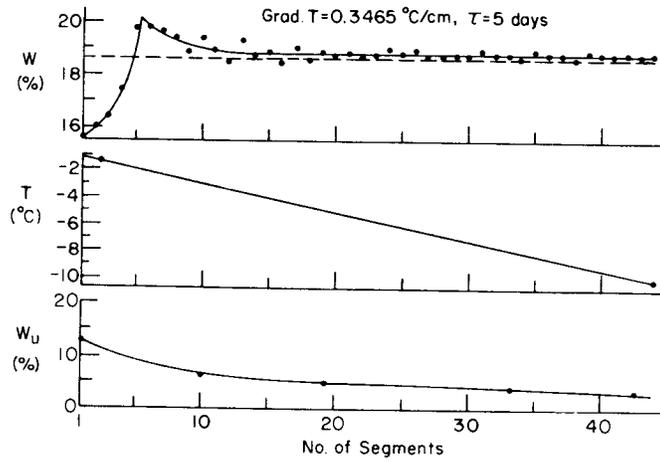


b.  $\text{grad}T = 0.0551 \text{ }^\circ\text{C/cm}$ .



c.  $\text{grad}T = 0.1024 \text{ }^\circ\text{C/cm}$ .

Figure 13. Water content profiles at test conclusion.



d.  $\text{grad}T = 0.3465 \text{ } ^\circ\text{C}/\text{cm}$ .

Figure 13 (cont'd). Water content profiles at test conclusion.

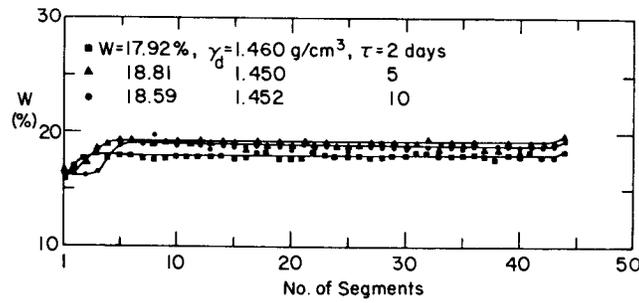


Figure 14. Water content profile for different test durations.

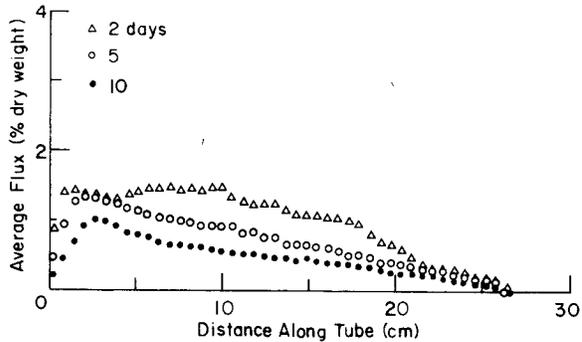


Figure 15. Influence of testing duration on the flux.

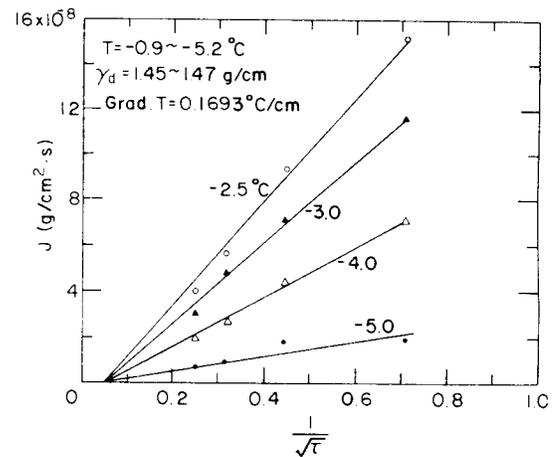


Figure 16. Relationship between the flux and test duration.

$$Q = (A\tau^{1/2} + B)T_{\text{grad}}T_{\gamma_d} \quad (17)$$

where  $Q$  = the total flux;  $A$  and  $B$  are empirical coefficients.

Figure 16 also shows that the slope of the straight line changes with temperature. The higher the temperature, the steeper the slope.

It should be noted that there is a contradiction in eq 16 and 17. Theoretically, when  $\tau = 0$ ,  $J$  or  $Q$  should be zero, too. But experimentally, there is still a coefficient  $B$ .

#### Influence of temperature

Figure 17 shows the flux versus temperature curves for different dry densities and temperature gradients, but within the same test duration of 5 days. From Figure 17, it can be seen clearly that the flux decreases with decreasing temperature, but the curve shapes change with temperature gradient. At lower temperature gradients, for instance less than 0.1 °C/cm, the curves are in the convex shape and then, with the increase in temperature gradient, the curve shapes become gradually linear and concave.

There are two possible explanations of the phenomena mentioned above. One is that the lower temperature gradient causes the lower soil-water potential gradient. As a result, the first peak of the wetting part in the soil column occurs slowly; in other words, the length of the drying part is greater. Another reason could be the "end influence," which means that the flux must be zero at the end of each soil column, no matter how great the flux is close to the end.

To investigate the end influence, a second, parallel test was conducted. Two soil columns, having approximately the same dry density and initial water content, were tested under the same temperature gradient and for the same test duration. One

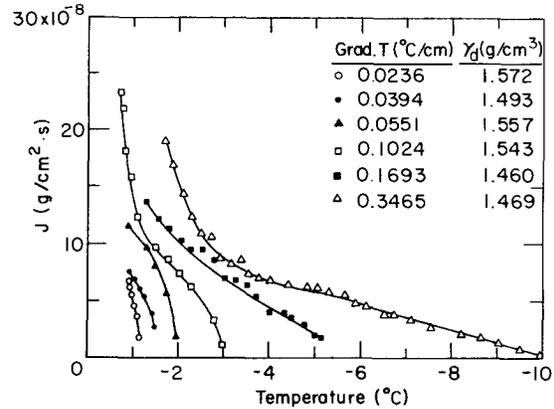


Figure 17. Influence of temperature on the flux.

of the soil columns was complete and the other was cut into halves. Results (Fig. 18) show that the flux in the first half column is fairly close to that of the front part in the complete one, but quite different from that of the end part; the flux in the secondary half column is just the extension of that of the front part of the complete column. Therefore, it can be concluded that the flux at the end of each soil column is reduced by end effects. Omitting the flux at the end part, we obtain a good relationship between the flux and temperature as

$$J = (A T^{-B})_{\text{grad}T, \gamma_d, \tau} \quad (18)$$

The values of coefficients  $A$  and  $B$  in eq 18 obtained for different temperature gradients, dry densities and test durations, are listed in Table 4.

#### Influence of the dry density

From Figure 19 it can be seen that the flux at first increases with the increase of the dry density.

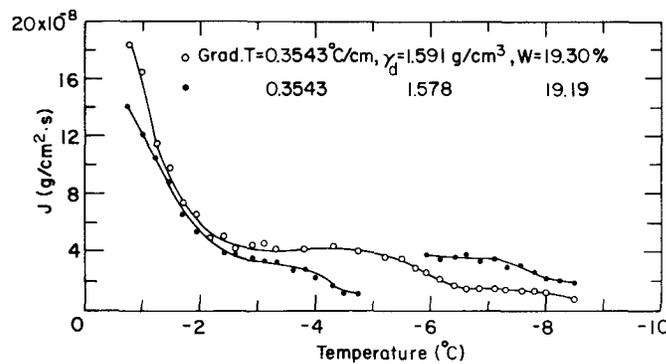


Figure 18. End influence of soil column on the flux.

**Table 4. Values of coefficients  $A$  and  $B$  in eq 18, and other parameters.**

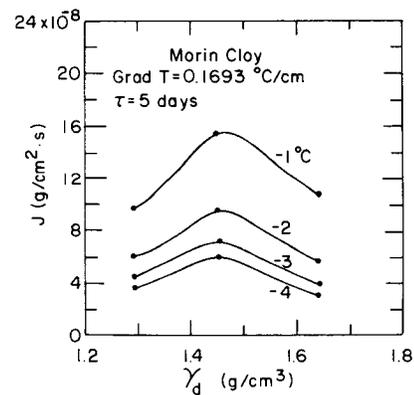
W (%)	$\gamma_d$ (g/cm <sup>3</sup> )	gradT (°C/cm)	$\tau$ (day)	T (°C)	Point	J = A T <sup>-B</sup>		
						A × 10 <sup>-4</sup>	B	Residual
18.37	1.572	0.0236	5	-0.91 ~ -1.02	8	5.7400	1.8491	0.1345
18.54	1.383	0.0276	5	-0.64 ~ -0.92	18	4.2541	1.5233	0.2775
18.43	1.473	0.0276	5	-0.64 ~ -0.99	22	6.6387	0.8500	3.3798
18.38	1.493	0.0394	5	-0.89 ~ -1.30	17	6.8936	0.9197	0.7397
18.10	1.581	0.0394	10	-0.89 ~ -1.25	16	8.8158	1.1342	0.3764
18.06	1.591	0.0394	21	-0.72 ~ -1.25	23	3.8801	0.7988	0.3027
16.19	1.261	0.0551	5	-0.81 ~ -1.41	19	5.2688	1.1816	1.5146
16.08	1.557	0.0551	5	-0.91 ~ -1.61	22	11.6524	0.7273	4.8765
19.66	1.538	0.0630	10	-0.58 ~ -1.35	21	9.7729	0.4977	3.8493
19.82	1.550	0.0630	15	-0.74 ~ -1.54	22	7.7385	1.5038	1.5146
18.52	1.147	0.1024	5	-0.64 ~ -2.07	24	9.9279	0.3134	19.5277
18.61	1.314	0.1024	5	-0.64 ~ -2.07	24	11.4902	0.5021	8.1171
18.34	1.543	0.1024	5	-0.64 ~ -2.07	23	14.9033	1.1085	17.7278
17.92	1.460	0.1693	2	-2.46 ~ -3.80	14	31.9381	0.8646	1.3857
18.82	1.450	0.1693	5	-1.02 ~ -3.80	28	15.4163	0.6824	10.5542
18.59	1.452	0.1693	10	-1.13 ~ -3.90	28	11.6791	0.8285	1.4293
18.04	1.453	0.1693	16	-1.23 ~ -3.50	23	10.5257	1.2731	1.3293
17.64	1.465	0.1693	16	-1.33 ~ -4.01	27	10.1175	1.1028	0.6958
17.71	1.475	0.1693	16	-1.33 ~ -3.91	27	10.0092	1.2272	1.0747
13.90	1.514	0.1732	10	-1.76 ~ -4.29	25	16.7113	1.2006	3.0150
14.80	1.446	0.1732	10	-1.76 ~ -3.76	20	8.6353	0.6279	0.3743
18.63	1.469	0.3465	5	-1.65 ~ -5.66	20	32.4343	1.1140	6.8124
17.45	1.640	0.1693	5	-1.03 ~ -4.74	37	10.8448	0.8957	15.8024
17.57	1.638	0.1693	5	-0.92 ~ -4.10	31	11.1044	0.9668	10.5753
16.77	1.292	0.1693	5	-1.02 ~ -4.84	38	9.6400	0.6866	2.8564

When the dry density reaches a certain value, however, the flux reaches a maximum, and after that, the flux decreases with the increase in the dry density.

The change of the dry density does not have much influence on the unfrozen water content as we have demonstrated in Figure 6, but it does influence the soil microstructure. When the dry density is low, there are many large pores in the soil column, so that the continuity of the unfrozen water film among soil particles is poor. In this case, increasing the dry density may offer more opportunities for unfrozen water passing through these paths because of the increasing continuity of the unfrozen water films. Therefore, there is an optimal density for water migration in frozen soils. After that, if the dry density keeps increasing, soil particles are in contact with each other more and more, they cut off the path for water migration and cause a decrease in the flux.

*Influence of temperature gradient*

The temperature gradient causes the driving force—the soil-water potential gradient—for



*Figure 19. Influence of the dry density on the flux.*

water migration in frozen soils. Figure 20 shows that the flux is directly proportional to the temperature gradient if the temperature, the dry density and the test duration are all kept the same. Then, we have

$$J = (A + B \text{ grad}T)_{T, \gamma_d, \tau} \quad (19)$$

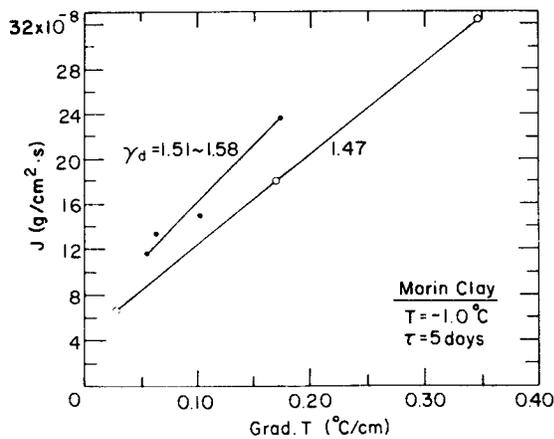


Figure 20. Relationship between the flux and temperature gradient.

So far, we have proved experimentally that the extended Darcy's law is still applicable for water migration in frozen soils, even if the temperature gradient goes down to as low as 0.0236 °C/cm.

## DISCUSSION

### Relationship between the soil-water potential and temperature

We have described the relationship between the soil-water potential and the water content in eq 3 to 6 and the relationship between the unfrozen water content and the temperature in eq 7 to 11. Combining those equations mentioned above, we can obtain the equations describing the relationship between the soil-water potential and temperature as follows:

$$\mu_w = M T^N \quad (20)$$

$$M = \mu_{wf} W_f^B W_o^{-B} T_f^{-BD} \quad (21)$$

$$N = BD \quad (22)$$

Equation 20 shows that the soil-water potential decreases with the decreasing temperature in frozen soils.

### Technical problems with the water migration test

Water migration in frozen soils is not only dependent on the properties of frozen soils and the soil-water potential gradient, but is also largely controlled by the microstructure.

There are two technical problems in our tests. One is a temperature difference in the vertical direction and another is a density difference along the soil column. Both of them affect the results of tests to different degrees and sometimes lead to failure.

To solve the first problem, we suggest that the reservoirs be enlarged and the insulation material be thickened. To solve the second problem, a way to pack the soil columns more evenly along their length is required.

## CONCLUSIONS

1. Relationships between the soil-water potential and the water content, between the unfrozen water content and temperature, and between the soil-water potential and temperature, can be represented by a power law equation.

2. The unfrozen water content in frozen Morin clay changes with the initial water content and the dry density only within a range of 3% of dry soil weight, and increases linearly with the increase in the molality because of the linear freezing point depression.

3. The curves of the unfrozen water content versus temperature are quite parallel with the change in the initial water content and rotate a little bit counterclockwise with the change in the dry density.

4. The flux of water migration in unsaturated, frozen Morin clay is directly proportional to the temperature gradient, inversely proportional to the square root of the test duration, decreases with the decrease of temperature in the power law form, and changes with the dry density.

5. The behavior of water migration in unsaturated, frozen soils is something like that in the unsaturated, unfrozen soils. Darcy's law is still applicable to frozen soil, even though the temperature gradient goes down to as low as 0.0236 °C/cm.

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