CONTRACT REPORT N-70-1

A NONLINEAR FINITE ELEMENT CODE FOR ANALYZING THE BLAST RESPONSE OF UNDERGROUND STRUCTURES

by

I. Farhoomand, E. Wilson

January 1970

Sponsored by Defense Atomic Support Agency

Conducted for U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

Under Contract No. DACA 39-67-0020

By Structural Engineering Laboratory, University of California, Berkeley, California

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ARMY-MRC VICKSBURG, MISS.

FOREWORD

The investigation described herein was performed under Contract No. DACA 39-67-0020 for the Nuclear Weapons Effects Division (NWED), U. S. Army Engineer Waterways Experiment Station (WES), under the sponsorship of the Defense Atomic Support Agency as part of Nuclear Weapons Effects Research (NWER) Subtask SC 210.

This contract was technically monitored by Mr. P. J. Rieck under the direct supervision of Mr. W. J. Flathau, Chief, Protective Structures Branch, and under the general supervision of Mr. G. L. Arbuthnot, Jr., Chief, Nuclear Weapons Effects Division, WES.

This study was conducted by Dr. Edward L. Wilson and Mr. Iraj Farhoomand of the University of California, Berkeley, California. Close technical cooperation was provided by Messrs. J. L. Kirkland and R. E. Walker and Dr. R. Froelich of NWED.

COL Levi A. Brown, CE, was Director of WES during the preparation of this report. Mr. F. R. Brown was Technical Director.

ABSTRACT

A nonlinear, axisymmetric, dynamic finite-element method of analysis computer program is developed. Elastic, two-dimensional problems can also be analyzed without loss of efficiency. An extensive description of the analytical procedures used in the code is given. A FØRTRAN IV listing of the computer code is presented along with information on utilizing the code to run problems.

Analytical results are compared with experimental data obtained from testing a modeled buried structure subjected to a blast loading.

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INTRODUCTION

Several attempts have been made to use the finite element method for the nonlinear dynamic analysis of granular materials[1,2,3]. These previous coded programs are not readily used by engineers for the dynamic analysis of complex structures. One of the objectives of this study is to develop a nonlinear computer program which is efficient and easy to use.

In Reference 4 the finite element method coupled with a stable step-by-step method of integration was used for the elastic dynamic analysis of two-dimensional plane strain solids. Reference 5 is a generalization of the material in Reference 4. It also contains a new coding technique which increases the capacity of the program. The purpose of this investigation is to extend the step-by-step dynamic analysis of axisymmetric structures to include nonlinearity of granular materials. It is worthy to note that elastic materials can also be treated by the nonlinear program without a loss of efficiency. In addition, the step-by-step integration method has been revised to provide better accuracy when nonlinear equations are involved.

DYNAMIC EQUILIBRIUM EQUATIONS

The force equilibrium of a system of structural elements is expressed by the following matrix equation:

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{u}}_t + \underset{\sim}{C}\,\dot{\underset{\sim}{u}}_t + \underset{\sim}{K}\,\underset{\sim}{u}_t = \underset{\sim}{R}_t \qquad (1)$$

where $\underset{\sim}{u}$, $\dot{\underset{\sim}{u}}$, and $\ddot{\underset{\sim}{u}}$ are vectors of nodal point displacements, velocities and accelerations at time "t". $\underset{\sim}{M}$ is the mass matrix assumed to be constant (independent of time). $\underset{\sim}{K}$ is the instantaneous stiffness matrix, i.e., it varies as a function of displacements. $\underset{\sim}{C}$ is the instantaneous damping matrix, and finally $\underset{\sim}{R}$ is the generalized external load vector applied at the nodal points of the system.

A formal mathematical development of the mass matrix $\underset{\sim}{M}$ is possible. Such an approach would result in a mass matrix with the same coupling properties as the stiffness matrix. However, if the physical lumped mass approximation is made, the mass matrix will be diagonal. The lumped mass approximation results in a small reduction in accuracy and considerable saving in computer storage and time. In this investigation one-fourth of the mass of each quadrilateral element is assumed to be concentrated at each of the four nodal points.

For most structures the exact form of the damping matrix $\underset{\sim}{C}$ is unknown. In the solution procedure the damping matrix may be completely arbitrary; however, there is little experimental justification for selecting specific damping coefficients. In the following analysis the damping matrix is ignored. This fact is specially justified for analysis of earth structures subjected to blast loading. Neglecting the damping matrix results in considerable simplification and saving in the computer time.

At any stage of the analysis, knowing the elastic constants of each element, the instantaneous stiffness matrix may be generated using the same procedure as used in the linear elastic analysis (reference 5). The assemblage of the stiffness matrix of the system is also the same.

STEP-BY-STEP INTEGRATION OF EQUILIBRIUM EQUATIONS

The dynamic equilibrium of the finite element system is given by equation (1). The basic concept of the solution of this set of second order differential equations is explained in references 4 and 5. However, in nonlinear analysis the overshooting problem (figure 1) makes the direct procedure inaccurate. To improve the accuracy of the method, neglecting the damping matrix $\underline{C}$, equation (1) is written as follows:

$$\underline{M}\,\underline{\ddot{u}}_t + \sum_{\tau=0}^{\tau=t} \underline{K}_\tau\,\underline{\Delta u}_\tau = \underline{R}_t \qquad (2)$$

where

$$\underline{\Delta u}_\tau = \underline{u}_\tau - \underline{u}_{\tau-\Delta t},$$

$$\underline{\Delta u}_0 = \underline{u}_0$$

and $\underline{K}_\tau$ is the instantaneous stiffness matrix for the displacement interval $\underline{\Delta u}_\tau$. The assumption is that $\underline{K}_\tau$ is constant for the displacement interval $\underline{\Delta u}_\tau$. The second term of the left hand side of equation (2) may be written as:

$$\sum_{\tau=0}^{\tau=t} \underline{K}_\tau\,\underline{\Delta u}_\tau = \underline{E}_t + \underline{K}_t\,\underline{\Delta u}_t \qquad (3)$$

where

$$\underline{E}_t = \sum_{\tau=0}^{\tau=t-\Delta t} \underline{K}_\tau\,\underline{\Delta u}_\tau \qquad (4)$$

The force vector E represents the forces carried by the elements of the system at the beginning of the time increment. The formation of

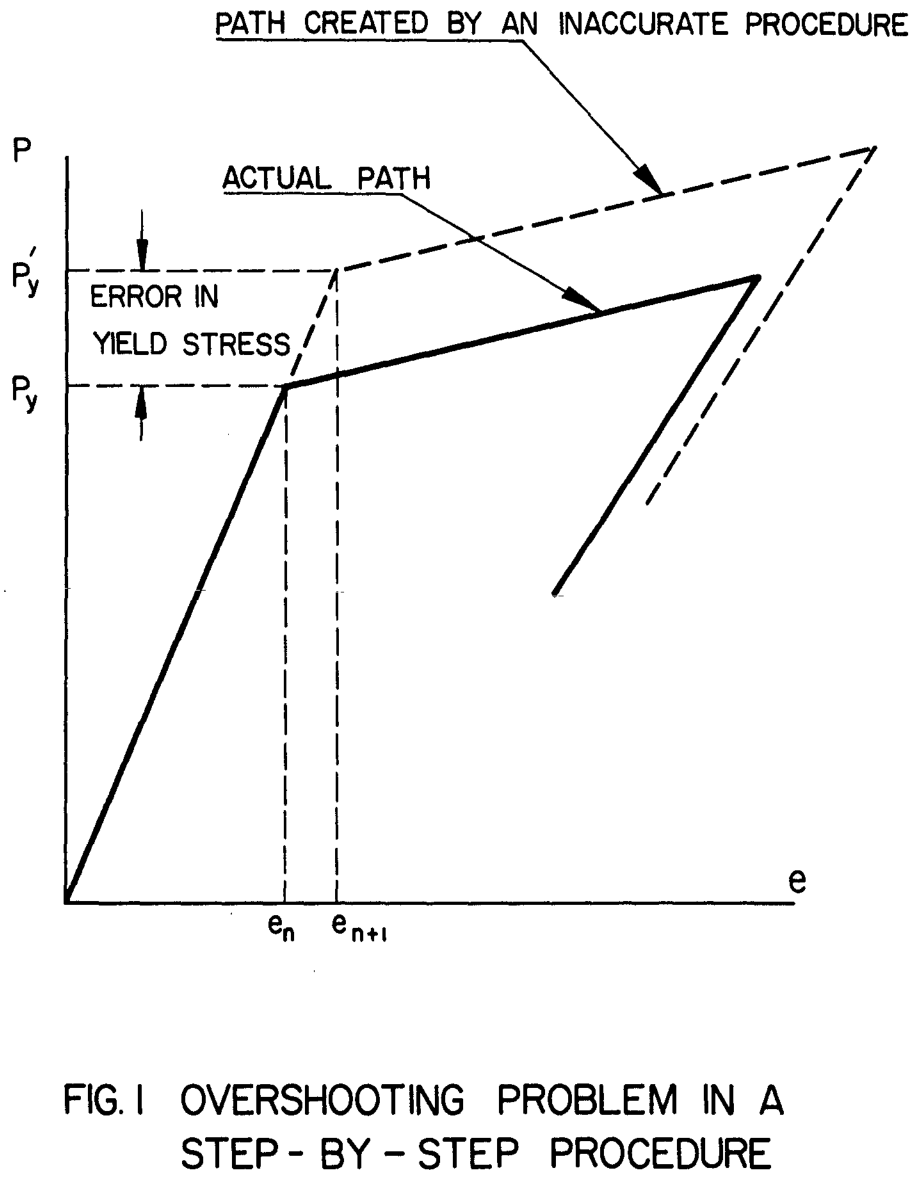

FIG. 1 OVERSHOOTING PROBLEM IN A STEP-BY-STEP PROCEDURE

force vector E for an axisymmetric finite element system is discussed in Appendix A. Using equation (3) the force equilibrium equation of a finite element system is expressed by:

$$\underline{M}\,\ddot{\underline{u}}_t + \underline{K}_t\,\Delta\underline{u}_t = \underline{R}_t - \underline{E}_t \tag{5}$$

This is the basic forces equilibrium equation which is used throughout this investigation.

The time discretization of equation (5) is approximated in the following manner: The acceleration of each point in the system is assumed to vary linearly within a small time interval, $2\Delta t$. This assumption leads to a parabolic variation of velocity and a cubic variation of displacement within the time interval $t-\Delta t$ and $t+\Delta t$.

A direct integration over the interval gives the following equations for acceleration and velocity at the end of the time interval:

$$\begin{aligned}\ddot{\underline{u}}_{t+\Delta t} &= \frac{1.5}{\Delta t^2}\,\Delta\underline{u}_{t+\Delta t} - \frac{3}{\Delta t}\,\dot{\underline{u}}_{t-\Delta t} - 2\,\ddot{\underline{u}}_{t-\Delta t}\\ \dot{\underline{u}}_{t+\Delta t} &= \frac{1.5}{\Delta t}\,\Delta\underline{u}_{t+\Delta t} - 2\,\dot{\underline{u}}_{t-\Delta t} - \Delta t\,\ddot{\underline{u}}_{t-\Delta t}\end{aligned} \tag{6}$$

The substitution of equation (6) into equation (5), results in a set of linear equations in terms of unknown vector $\Delta\underline{u}_{t+\Delta t}$. The solution of this set of equations yields the increment of displacements of the system at time $t+\Delta t$. The acceleration and velocities at time t may then be found from the following series of equations:

$$\ddot{\underline{u}}_{t+\Delta t} = \frac{1.5}{\Delta t^2}\,\Delta\underline{u}_{t+\Delta t} - \frac{3}{\Delta t}\,\dot{\underline{u}}_{t-\Delta t} - 2\,\ddot{\underline{u}}_{t-\Delta t} \tag{7}$$

$$\ddot{\underline{u}}_t = \frac{1}{2} \left(\ddot{\underline{u}}_{t+\Delta t} + \ddot{\underline{u}}_{t-\Delta t} \right) \qquad (8)$$

$$\dot{\underline{u}}_t = \dot{\underline{u}}_{t-\Delta t} + \frac{\Delta t}{2} \left(\ddot{\underline{u}}_t + \ddot{\underline{u}}_{t-\Delta t} \right) \qquad (9)$$

$$\dot{\underline{u}}_t = \underline{u}_{t-\Delta t} + \frac{\Delta t}{2} \dot{\underline{u}}_{t-\Delta t} + \frac{\Delta t^2}{3} \ddot{\underline{u}}_{t-\Delta t} + \frac{\Delta t^2}{6} \ddot{\underline{u}}_t \qquad (10)$$

Equation (8) is the essential factor in making the step-by-step method stable. In fact, it can be shown that all roots of the characteristics equation of the difference equation of the method lie between -1 and 1 for all sizes of time interval Δt. This means that regardless of the size of the time step the procedure is stable and the high frequency components do not cause the solution to blow up. However, the new procedure tends to introduce damping in the higher frequencies of the system. Fortunately this partial truncation of the higher modes is justified in many dynamic analyses. The selection of the time step and the finite element idealization for a particular problem will depend on the experience of the user with similar problems.

The step-by-step procedure, which is presented in a form which minimizes computer storage and execution time, is summarized in Table 1. The "effective" stiffness matrix is normally banded and its triangularized form is also banded. Therefore, a large amount of computer storage is not required. Since the stiffness matrix is different for each time step, it should be triangularized at every step. However, for weakly nonlinear problems, where the stiffness matrix might vary slightly, it may not be necessary to triangularize the matrix at each time step.

TABLE 1 SUMMARY OF STEP-BY-STEP PROCEDURE

I. Initial Calculation

A. Calculate the following constants:

$\tau = \Delta t$	$a4 = a0/a3$
$a0 = 3/\tau$	$a5 = 2/a3$
$a1 = .75/\tau^2$	$a6 = \tau/2$
$a2 = a0/2$	$a7 = \tau^2/6$
$a3 = 2a1$	$a8 = 2a7$

B. Form the stiffness matrix $\underline{K}$, diagonal mass matrix $\underline{M}$, and the internal force vector $\underline{E}$.

C. As a starting point, form and triangularize the effective stiffness matrix $\underline{\overline{K}}=\underline{K}+a3^* \underline{M}$.

D. Form the revised mass matrix $\underline{\overline{M}}=a3^* \underline{M}$.

II. For Each Time Increment

A. Form the effective load vector

$$\underline{\overline{R}}_{t+\Delta t} = \underline{R}_{t+\Delta t} - \underline{E}_{t-\Delta t} + \underline{\overline{M}}\,(a4\, \underline{\dot{U}}_{t-\Delta t} + a5\, \underline{\ddot{U}}_{t-\Delta t})$$

B. Back substitute to solve for the displacement vector $\underline{U}_{t+\Delta t}$

$$\underline{\overline{K}}\,\underline{\overline{U}}_{t+\Delta t} = \underline{\overline{R}}_{t+\Delta t}$$

C. Calculate acceleration, velocity, and displacement vectors at time "t":

$$\ddot{\underline{U}}_t = a1\ \underline{U}_{t+\Delta t} - a2\ \dot{\underline{U}}_{t-\Delta t} - .5\ \ddot{\underline{U}}_{t-\Delta t}$$

$$\dot{\underline{U}}_t = \dot{\underline{U}}_{t-\Delta t} + a6\ (\ddot{\underline{U}}_t + \ddot{\underline{U}}_{t-\Delta t})$$

$$\underline{U}_t + \underline{U}_{t-\Delta t} + \tau\ \dot{\underline{U}}_{t-\Delta t} + a7\ (\ddot{\underline{U}}_t + 2\ \ddot{\underline{U}}_{t-\Delta t})$$

D. Calculate strains, stresses, and the internal force vector $\underline{E}_t$ at time "t".

E. For each desired interval:

Calculate and triangularize the new effective stiffness matrix $\underline{\bar{K}}$.

F. Repeat for the next time step.

NONLINEAR MATERIAL BEHAVIOR

Many investigators have worked on nonlinear dynamic analysis of earth structures. Penzien[1] developed a lumped mass, one-dimensional model to study the response of semi-infinite soil layers in which the material had a hysteretic, bilinear stress-strain behavior. Ang[2] investigated a technique to solve nonlinear two and three dimensional dynamic analysis of soil media; however, his technique is cumbersome and impractical. Recently, Dibaj[3] developed a consistant formulation for the nonlinear dynamic analysis of earth structures based on plasticity rules and the finite element method. Dibaj's technique is also inefficient for practical purposes in the sense that to solve a practical problem the computer time and storage are large.

Most computer codes that attempt to account for nonlinear behavior are based on the three dimensional Prager-Drucker yield condition. The behavior of the system is then assumed to be piecewise linear, and the incremental elastic constants are evaluated for each time interval. Based on these constants the tangent stiffness is computed and the response of the system at the end of that interval is obtained. Although the procedure is straight forward, it requires a large amount of computer time. Furthermore, the incremental stress-strain relationship for practical soil materials cannot be accurately obtained from the Prager-Drucker yield condition

In reference 6, the stress-strain behavior of soils subjected to dynamic load is discussed. Among the prime factors which are important in the nonlinearity of the soil are volumetric change, hydrostatic pressure, second strain invariant, and shear stress. It is extremely difficult to use this experimental data directly in the computer program. Since, the nonlinear-hysteretic behavior of the bulk modulus appeared to be of major importance, a model was selected to accurately represents this property. Figure 2, shows a pressure volume strain relationship for a typical soil material. The bulk modulus is defined as the ratio of the incremental hydrostatic pressure to the incremental volumetric strain, or

$$K = \frac{\Delta p}{\Delta e}$$

Note that the bulk modulus is significantly different for loading and unloading. The strain e_f for a given maximum pressure is the volumetric strain which will cause the soil material to lose its incremental tensile stiffness. Therefore, if the strain, e, is less than e_f the average volume stress must vanish although the individual stresses may not be zero. In order to use the bulk modulus experimental data it was necessary to assume that the material is incremental isotropic. Or

$$\Delta\sigma = \underline{c}\Delta\underline{\varepsilon}$$

where

$$\underline{C} = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & K - \frac{2}{3}G & 0 \\ K - \frac{2}{3}G & K + \frac{4}{3}G & K - \frac{2}{3}G & 0 \\ K - \frac{2}{3}G & K - \frac{2}{3}G & K + \frac{4}{3}G & 0 \\ 0 & 0 & 0 & 2G \end{bmatrix} \qquad (11)$$

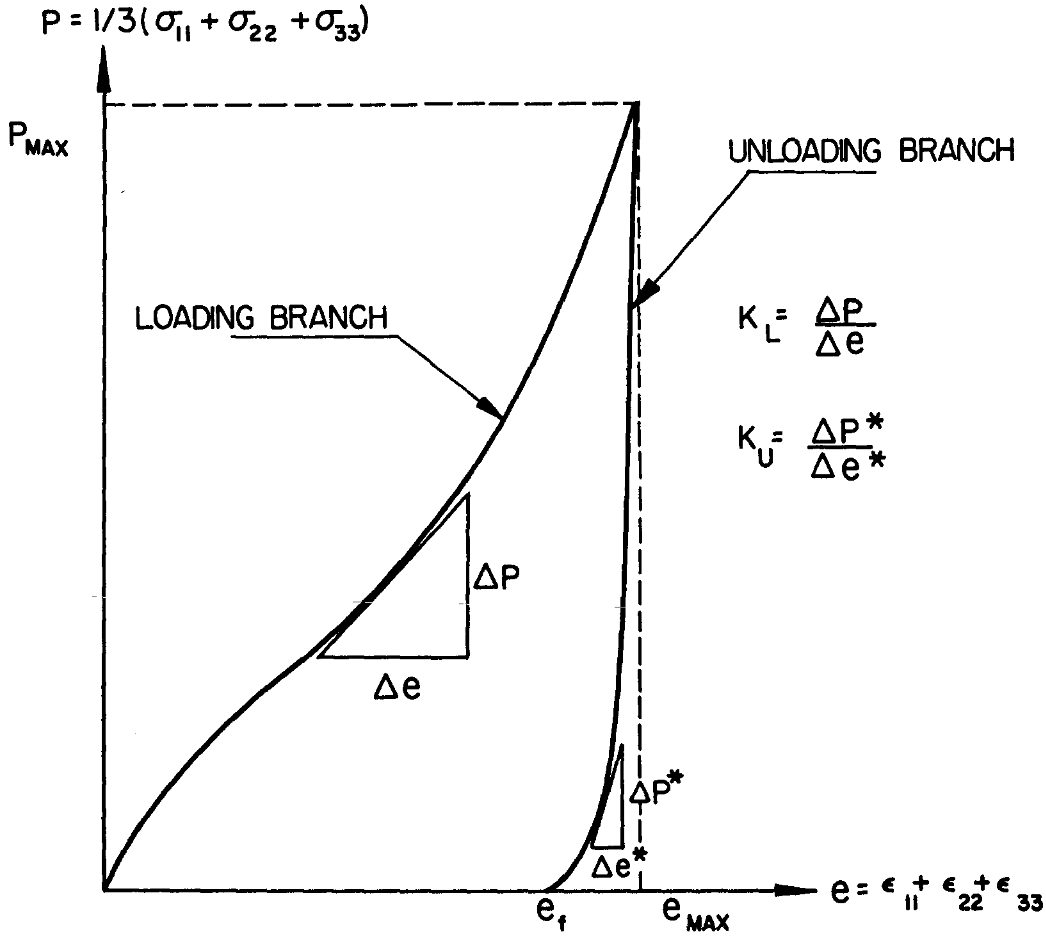

FIG. 2 STRESS-STRAIN DIAGRAM FOR A GRANULAR SOIL SAMPLE

In this formulation the shear modulus may be a function of pressure and may be found experimentally. In the next section of the report an alternate procedure will be given for the determination of the incremental shear modulus.

It is not necessary to express the material properties in mathematical form for the purpose of a numerical analysis. The following sequence of points which describe the stress-strain behavior may be used to define the input to the computer program

1. Volumetric change
2. Hydrostatic pressure.
3. Unloading bulk modulus.
4. Shear Modulus.
5. Strain at which tensile hydrostatic pressure is initiated in soil.

CONSISTENT FORMULATION OF NONLINEAR PROPERTIES

It is apparent that the nonlinear model discussed in the previous section has many limitations. However, it can be improved by considering in more detail a consistent mathematical formulation of the constitutive equations as suggested by Brown and Froelich (8)

Following reference (8), the internal energy may be written as:

$$W = W_1 + W_2$$

where

$$W_1 = f\ (\theta_1,\ S) \tag{12}$$

$$W_2 = -\ 2\sqrt{\tau_p}\ \ \theta_2 - \frac{\sqrt{\tau_p}}{2G_0}\ L_n\ (\ 1 + \frac{2G_0}{\sqrt{\tau_p}}\ \theta_2) \tag{13}$$

θ_1 and θ_2 are the first and second strain tensor invarients. S is entropy density. τ_p is the maximum shear stress soil can resist, and G_0 is the initial shear modulus. The stress tensor might be expressed by (reference 7):

$$\underline{\sigma} = A\ I_3 + B\ (\theta_1\ I_3 - \underline{\varepsilon}) + C \quad C\ of\ (\underline{\varepsilon}) \tag{14}$$

where I_3 is a 3x3 unit matrix, and

$$A = \frac{\partial w}{\partial \theta_1} = \frac{\partial w_1}{\partial \theta_1}, \quad B = \frac{\partial w}{\partial \theta_2} = \frac{\partial w_2}{\partial \theta_2} \tag{15}$$

in this case

$$C = \frac{\partial w}{\partial \theta_3} = \text{constant}$$

It is simple to see that equation 14 may also be written in the form of:

$$\underline{\sigma} = \left(\frac{1}{\theta_1} \frac{\partial w_1}{\partial \theta_1} + \frac{2}{3} \frac{\partial w_2}{\partial \theta_2} \right) \theta_1 \, I_3 - \frac{\partial w_2}{\partial \theta_2} \left(\underline{\varepsilon} - \frac{1}{3} \theta_1 \, I_3 \right) \tag{16}$$

comparing equations (11) and (16), one can conclude that it is possible to define incremental linear elastic constants in such a way that they satisfy nonlinearity, i.e.,

$$K_e = \frac{1}{\theta_1} \frac{\partial w_1}{\partial \theta_1}(\theta_1, s), \quad G_e = \frac{1}{2} \frac{\partial w_2}{\partial \theta_2}(\theta_2, \tau_p, G_0) \tag{17}$$

To define the tangent moduli for nonlinear analysis, variation of stress tensor should be expressed in terms of variation of strain tensor. Applying δ operator to equation (16) and neglecting second order terms, equation (16) will change to:

$$\delta \, \sigma = \left(\frac{\partial^2 w_1}{\partial \theta_1^{\,2}} + \frac{2}{3} \frac{\partial w_2}{\partial \theta_2} \right) \delta \, \theta_1 \, I_3 - \frac{\partial w_2}{\partial \theta_2} \, \delta \, \left(\underline{\varepsilon} - \frac{1}{3} \theta_1 \, I_3 \right) \tag{18}$$

Comparing equation (18) with the incremental strain stress relation, equation (11).

the tangent moduli are defined as follows:

$$K = \frac{\partial^2 w_1}{\partial \theta_1^2} \approx \frac{\Delta \frac{\partial w_1}{\partial \theta_1}}{\Delta \theta_1} \tag{19}$$

$$G = \frac{1}{2} \frac{\partial w_2}{\partial \theta_2} \tag{20}$$

Using equation (13), shear modulus G may be expressed by:

$$G = G_0/(1 + 2 G_0 \sqrt{\theta_2/\tau_p}) \tag{21}$$

Therefore the incremental shear modulus is not independent, but may be calculated directly; since the experimental determination of the shear limit, τ_p, is possible.

Equation (17) indicates that the bulk modulus is dependent on entropy; therefore S must be controlled during experiments in order to obtain the true behavior of K. Presently experimental tests do not give this information. Therefore, one must introduce simplication in order to use the current test results. Referring to Jackson's tests (reference 6), it appears that the maximum value of the first invarient of strain tensor may be considered as a measure of S. From experimental results one may plot hydrostatic pressure vs. volumetric change (figure 2), and express a bulk modulus by:

$$K = K(p, e_{max}, \dot{e}, e) \approx \Delta \left(\frac{\partial w_1}{\partial \theta_1} \right)/\Delta\theta_1 \tag{22}$$

Where $\dot{e}$ is the rate of change of e with respect to p, and

$$p = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}) \tag{23}$$

$$e = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \tag{24}$$

in which σ_{11} σ_{22} and σ_{33} are principle stresses, and ε_{11} ε_{22} and ε_{33} are principle strains.

Equation (22) implies that the loading and unloading bulk moduli are different (figure 2). This is an indication of the hysteretic behavior of the material and is accurately represented in the model.

APPLICATION

The validity of the finite element method as applied to the nonlinear dynamic analysis of axisymmetric systems is demonstrated by two examples.

The method of analysis is compared with an experimental study conducted by Jackson (reference 6). The model was a confined soil cylinder subjected to a blast load uniformly distributed on the model. Figure 3 shows the model, the finite element idealization, and the time variation of the blast load. The pressure-volume change of material of this same reference is constructed using a constant Poisson's ratio and is shown in figure 4. Vertical strain of the model was measured and plotted vs. time. A good agreement between the experimental and the finite element results is observed (figure 5). Specially, the permanent set, which linear analysis does not exhibit, is pronounced by the nonlinear analysis. The slight discrepancy is due to inexactness of the information adopted from reference 6.

In another analysis, the results of an elastic and a nonlinear finite element analysis were compared with an experimental study of a structure buried in a soil material. This experimental study was conducted in the blast load simulator at Vicksburg, Mississippi. The stress-strain diagram used in the nonlinear analysis is shown in figure 6. Figure 7 shows the finite element idealization of the model and the time variation of the blast pressure which is applied on the model. A listing of the input data for this structure is given in Appendix D. In figure 8 the displacements at a point in the soil are plotted. The experimental results indicate a permanent set in the material, wheareas, the displacements from the elastic analysis return to zero. The results of the nonlinear analysis are also plotted on the same diagram, and are in good agreement with the experiment.

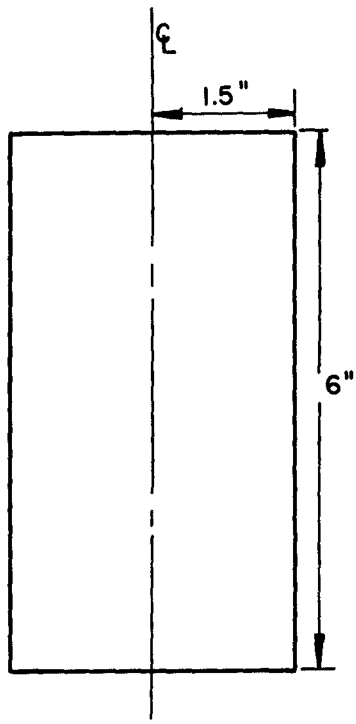

FIG. 3a SOIL SAMPLE

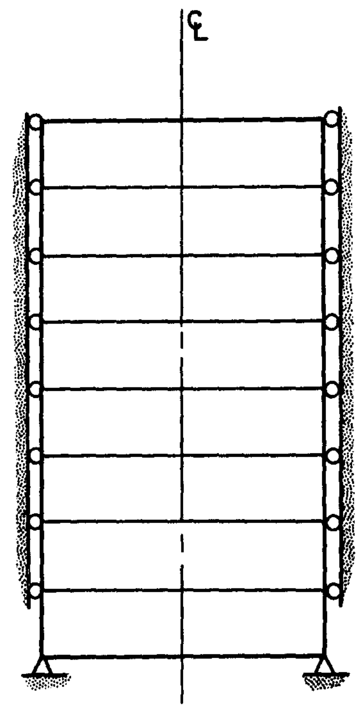

FIG. 3b FINITE ELEMENT IDEAL

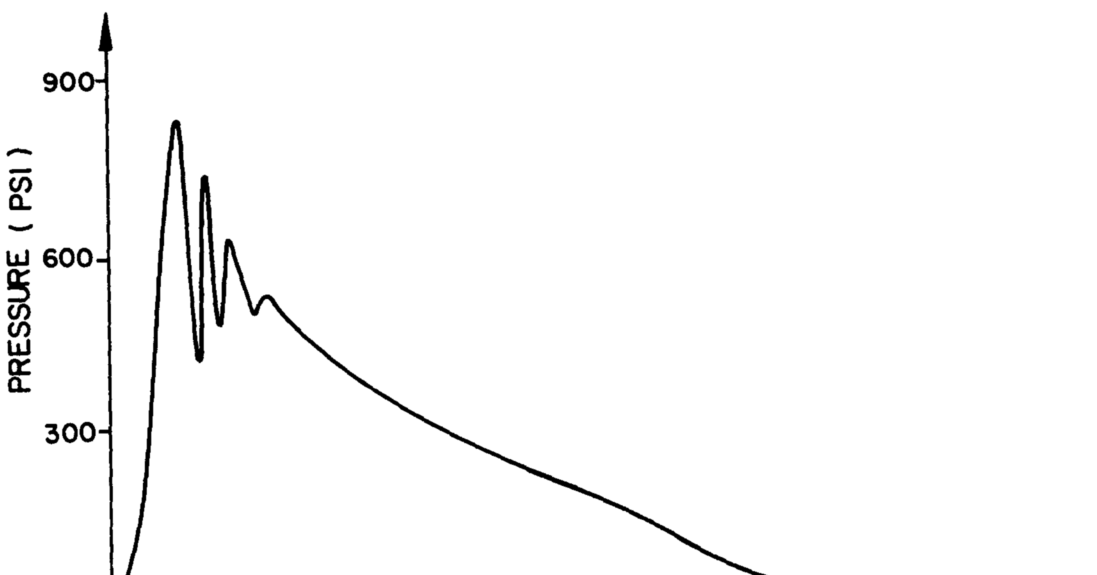

FIG. 3c TIME VARIATION OF BLAST PRESSURE

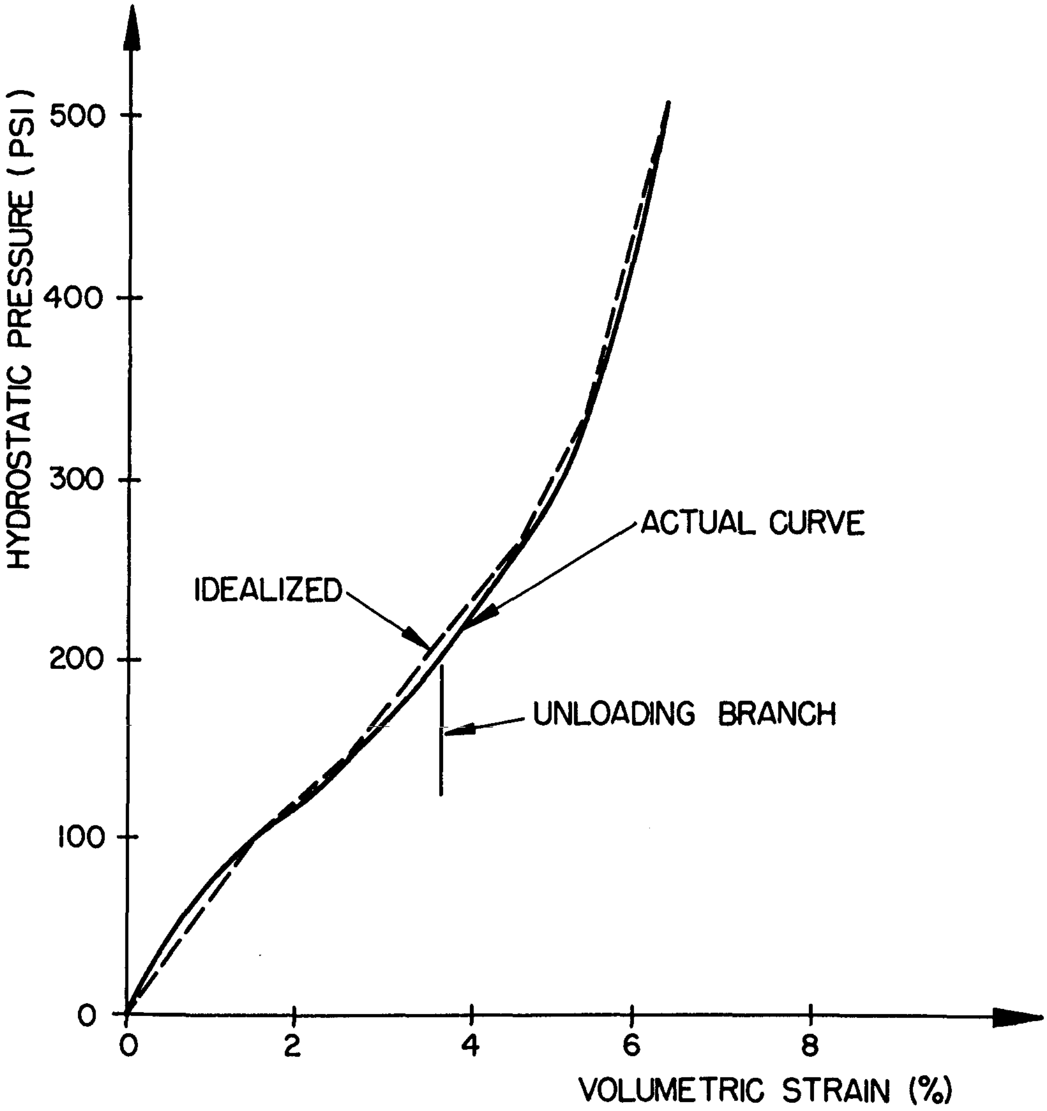

FIG. 4 STRESS-STRAIN BEHAVIOR OF A SOIL MATERIAL

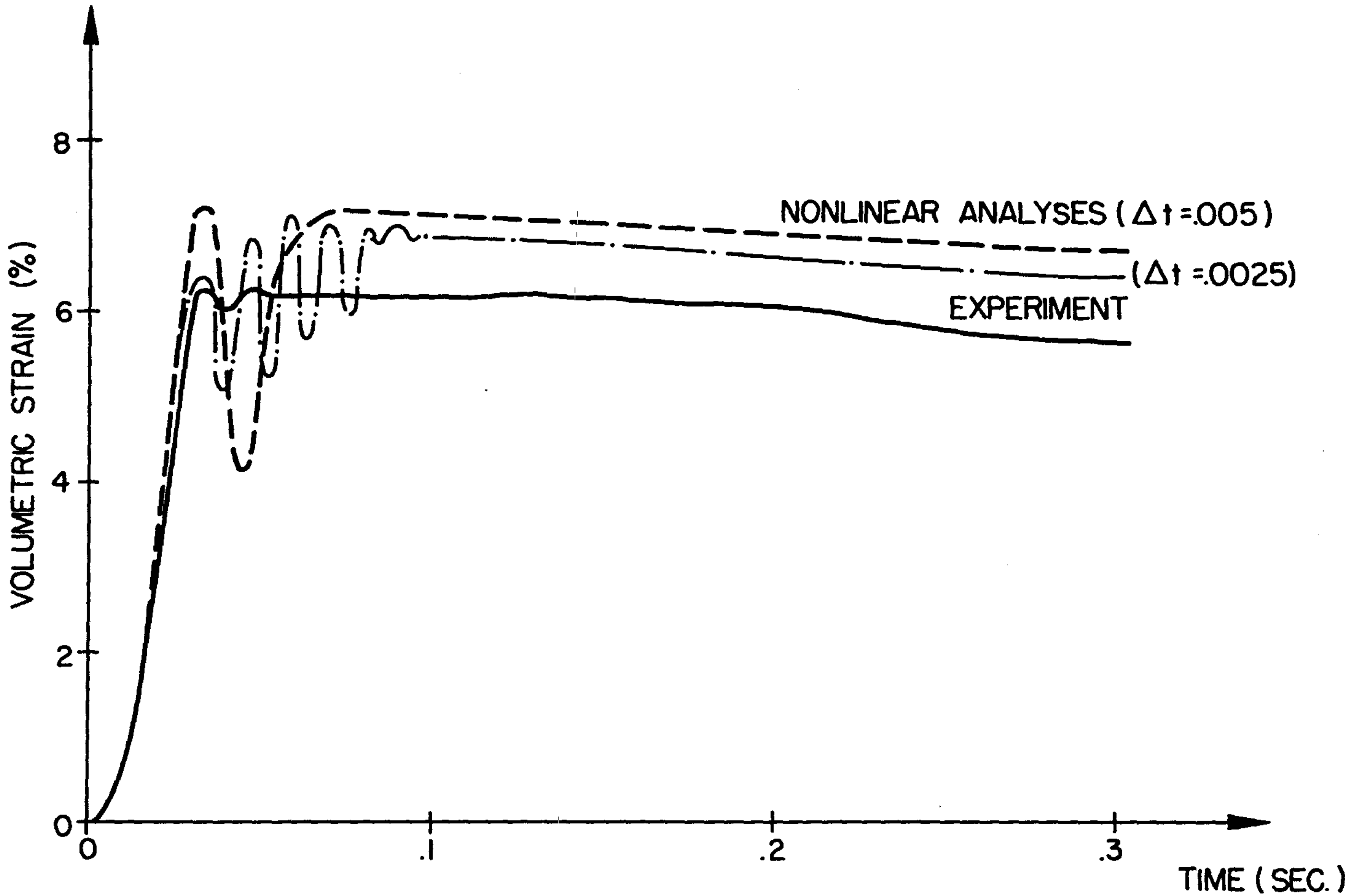

FIG. 5 TIME VARIATION OF VOLUMETRIC STRAIN

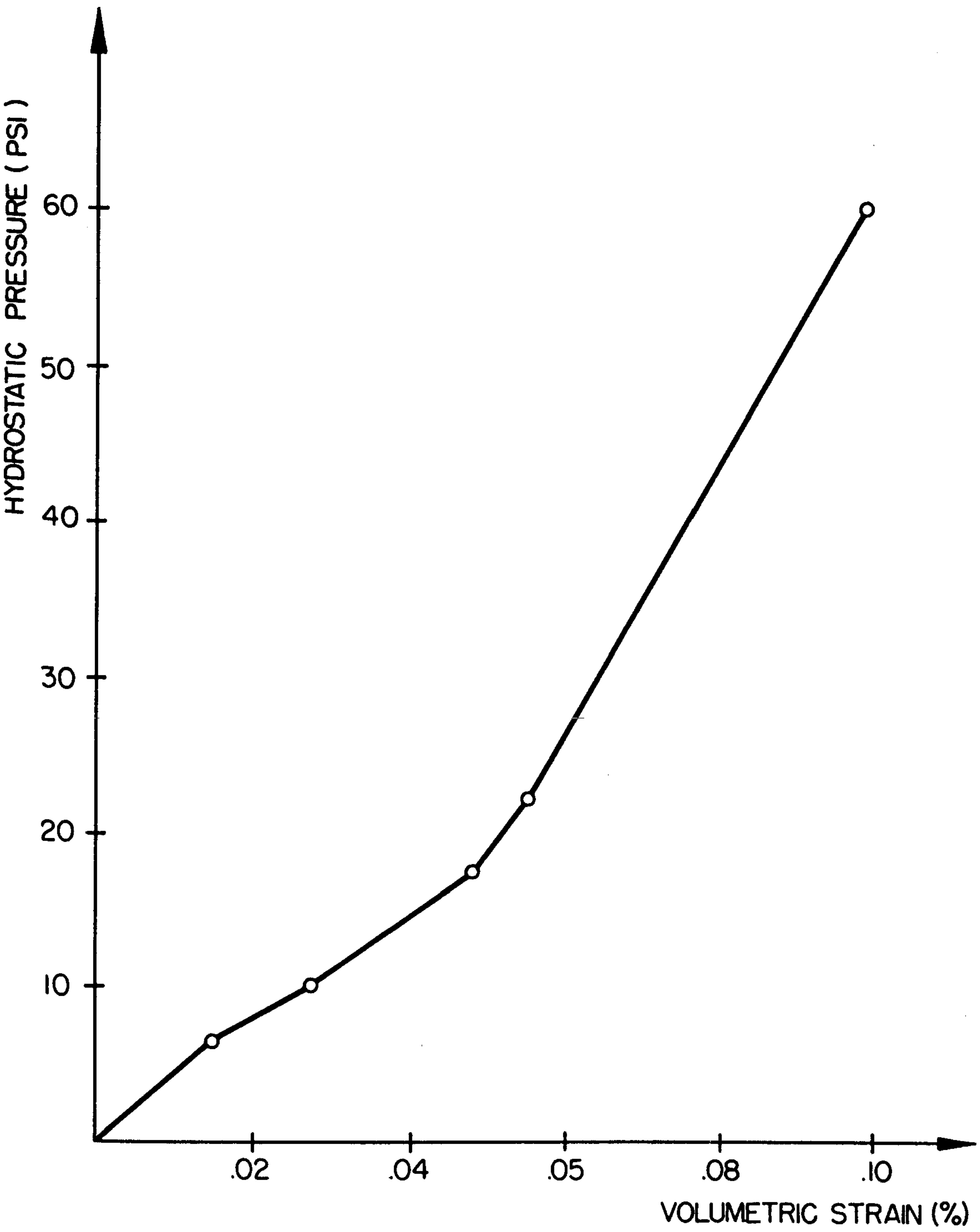

FIG.6 STRESS · STRAIN CURVE (VICKSBURG'S TEST)

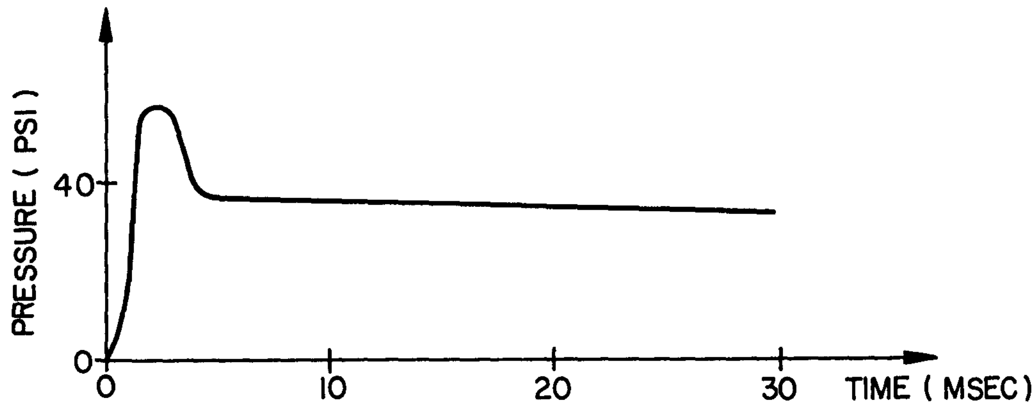

FIG. 7a TIME VARIATION OF PRESSURE

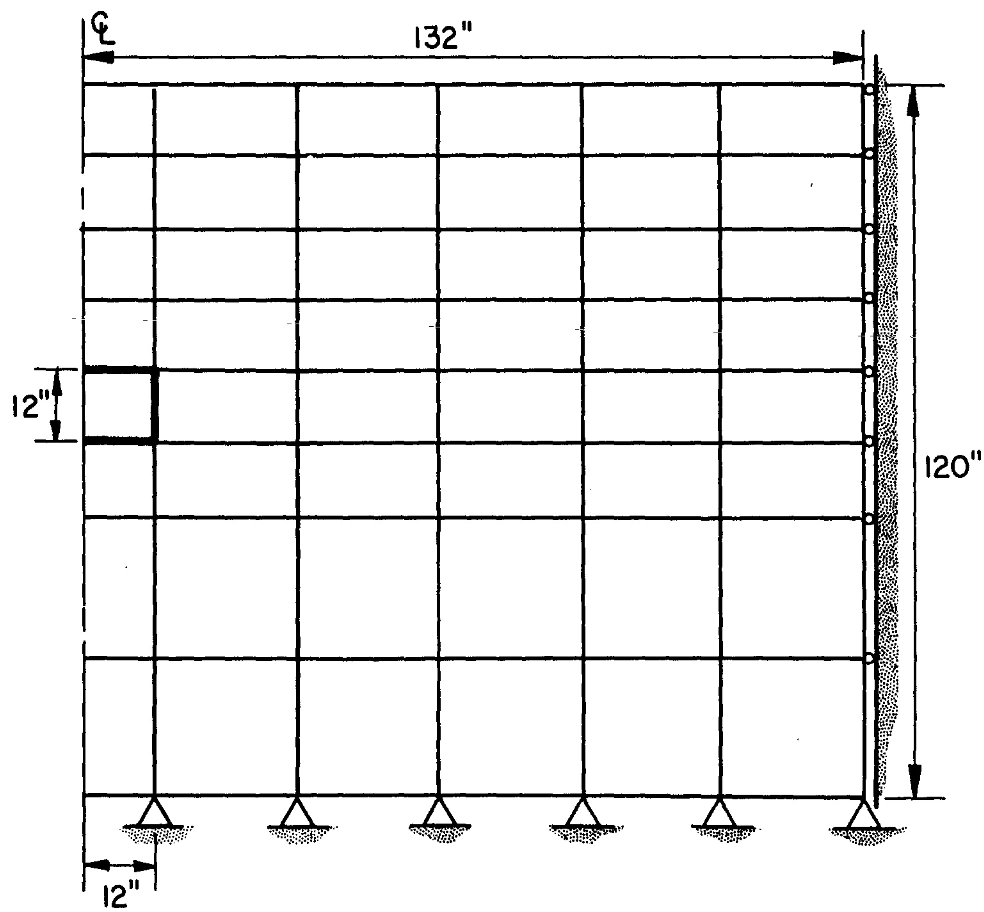

FIG. 7b FINITE ELEMENT IDEALIZATION

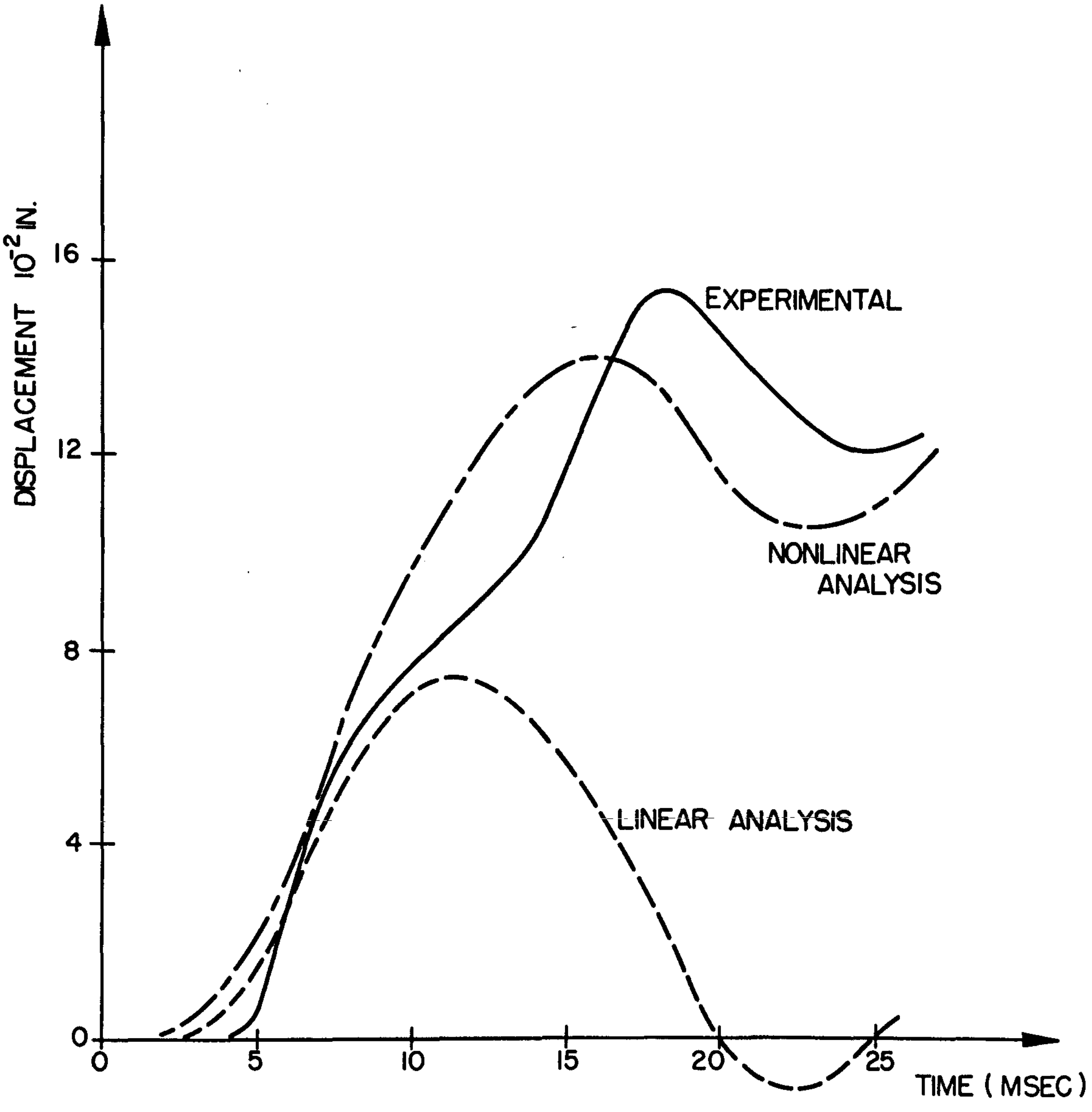

FIG. 8 DISPLACEMENT VS. TIME IN SOIL MATERIAL

COMPUTER PROGRAM

A Fortran IV listing of the computer program for the nonlinear dynamic analysis of axisymmetric structures is given in Appendix C. The program utilizes axisymmetric elements with quarilateral cross-sections. The capacity of the program will depend on the storage of the computer used.

Within the program a method of dynamic storage allocation is used, therefore, for a given problem all required data is compressed into the smallest possible storage area. This also allows the capacity of the program to be increased or decreased by only changing one number within the program.

The operation of the program may be summarized by the following steps:

First.

Control information, material properties nodal point geometry and element data are read (or generated) by the computer.

Second.

For each element, an 8x8 incremental stiffness matrix and element mass matrix are formed. These are then added to the total stifness and mass matrices of the system.

Third.

The step-by-step solution, as summarized in Table 1, is used to evaluate the displacements as a function of time. At

At specified time points displacement and stresses are printed. Also, at a different time interval new incremental element stiffnesses may be calculated.

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APPENDIX A

INTERNAL FORCE VECTOR FOR A QUADRILATERAL ELEMENT

APPENDIX A.

INTERNAL FORCE VECTOR FOR A QUADRILATERAL ELEMENT

INTRODUCTION

The purpose of this section is to present the development of the internal force vector for a quadrilateral element. Expression of virtual work is.

$$W = \int_{vol} [\varepsilon]^T \, [\sigma] \, dV \tag{A-1}$$

the same expression in terms of the displacements of descrete nodal points is.

$$W = [d]^T \, [E] \tag{A-2}$$

where

$$\underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{zz} \\ \varepsilon_{\theta\theta} \\ \gamma_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{\partial v}{\partial z} \\ u/r \\ \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \end{Bmatrix} , \underline{\sigma} = \begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \\ \tau_{rz} \end{Bmatrix}$$

$$[d]^T = [u_1 \;\; v_1 \;\; u_2 \;\; v_2 \;\; u_3 \;\; v_3 \;\; u_4 \;\; v_4]$$

The strain vector can be related to the displacements at the nodal points by the operator [B]

$$[\varepsilon] = [B] \, [d] \tag{A-3}$$

Substituting equation (A-3) into equation (A-1) and comparing the result with equation (A-2) we can write,

$$[E] = \int_{vol} [B]^T [\sigma] \, dV$$

Or for constant thickness t,

$$[E] = t \int_{a} [B]^T [\sigma] \, dA \qquad \text{(A-4)}$$

COORDINATE SYSTEMS

The coordinates (R,Z) are cartesian while the natural coordinates (S,T) may be skewed and are defined such that S and T vary from - 1 to 1, as shown in figure A-1. The (R,Z) coordinates are given in terms of (S,T) natural coordinates by the following interpolating functions:

$$r\,(s,t) = \sum_{i=1}^{4} h_i \, r_i \qquad Z\,(s,t) = \sum_{i=1}^{4} h_i \, Z_i$$

$$\begin{aligned} h_i &= (1-s)(1-t)/4 \qquad & h_3 &= (1+s)(1+t)/4 \\ h_2 &= (1+s)(1-t)/4 \qquad & h_4 &= (1-s)(1+t)/4 \end{aligned} \qquad \text{(A-5)}$$

Since strains are defined by derivatives with respect to (R,Z) and the displacement expantions are given in the (S,T) system, the chain rule for differentiation must be used to calculate

$$\frac{\partial s}{\partial r}, \frac{\partial s}{\partial z}, \frac{\partial t}{\partial r}, \text{ and } \frac{\partial t}{\partial z}$$

Inverting the chain rule,

$$\begin{Bmatrix} \frac{\partial}{\partial s} \\ \\ \frac{\partial}{\partial t} \end{Bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial s} & \frac{\partial z}{\partial s} \\ \\ \frac{\partial r}{\partial t} & \frac{\partial z}{\partial t} \end{bmatrix} \begin{Bmatrix} \frac{\partial}{\partial r} \\ \\ \frac{\partial}{\partial z} \end{Bmatrix}$$

Gives

$$\begin{Bmatrix} \frac{\partial}{\partial r} \\ \\ \frac{\partial}{\partial z} \end{Bmatrix} = \frac{1}{J} \begin{bmatrix} \frac{\partial z}{\partial t} & \frac{-\partial z}{\partial s} \\ \\ \frac{-\partial r}{\partial t} & \frac{\partial r}{\partial s} \end{bmatrix} \begin{Bmatrix} \frac{\partial}{\partial s} \\ \\ \frac{\partial}{\partial t} \end{Bmatrix}$$

Where

$$J = J\ (s,t) = \frac{\partial r}{\partial s} \frac{\partial z}{\partial t} - \frac{\partial r}{\partial t} \frac{\partial z}{\partial s}$$

Since $dA = J\ ds\ dt$ Equation (A-4) may be rewritten as.

$$[E] = t \int_A [B]^T [\sigma]\ J\ ds\ dt \qquad \text{(A-6)}$$

STRAIN DISPLACEMENT TRANSFORMATION [B]

Let nodal point values of the displacement u and v be given by

$$[u]^T = [u_1, u_2, u_3, u_4]$$

$$[v]^T = [v_1, v_2, v_3, v_4]$$

The assumed displacement expansion uses the same interpolation functions as appeared in Eqs. A-5, i.e.

$$u\ (s,t) = \sum_{i=1}^{4} h_i\ u_i \qquad v\ (s,t) = \sum_{i=1}^{4} h_i\ v_i$$

The $\varepsilon_{\theta\theta}$ strain is given by

$$\varepsilon_{\theta\theta} = \sum_{i=1}^{4} \frac{h_i\ u_i}{r_i} = \sum_{i=1}^{4} G_i\ u_i$$

ε_{rr} may be obtained by differentiation

$$\varepsilon_{rr} = \frac{\partial u}{\partial r} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial r} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial r}$$

$$= \frac{1}{J} \sum_{i=1}^{4} \sum_{j=1}^{4} u_i \left(\frac{\partial h_i}{\partial s}\frac{\partial h_j}{\partial t} - \frac{\partial h_i}{\partial t}\frac{\partial h_j}{\partial s}\right) z_j$$

$$= [u]^T [p] [z]/J.$$

Where

$$[P] = \sum_{i=1}^{4} \sum_{j=1}^{4} \left(\frac{\partial h_i}{\partial s}\frac{\partial h_j}{\partial t} - \frac{\partial h_i}{\partial t}\frac{\partial h_j}{\partial s}\right) = \frac{1}{8}\begin{bmatrix} 0 & 1-t & -s+t & -1+s \\ & 0 & 1+s & -S-t \\ & & 0 & 1+t \\ \text{skew-symmetric} & & & 0 \end{bmatrix}$$

Similarly

$$\varepsilon_{zz} = \frac{\partial v}{\partial z} = - [V]^T [p] [r]/J$$

$$\gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} = \frac{-[u]^T [p] [r] + [v]^T [p] [z]}{J}$$

Let

$$\frac{[P] [z]}{J} = [y] = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \frac{1}{8J} \begin{pmatrix} z_{24} - z_{34}\, s - z_{23}\, t \\ -z_{13} + z_{34}\, s + z_{14}\, t \\ -z_{24} + z_{12}\, s - z_{14}\, t \\ z_{13} - z_{12}\, s + z_{23}\, t \end{pmatrix}$$

And

$$-\frac{[P] [r]}{J} = [x] = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \frac{1}{8J} \begin{pmatrix} -r_{24} + r_{34}\, s + r_{23}\, t \\ \text{and so on} \\ \\ \\ \end{pmatrix}$$

Then

$$\begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{zz} \\ \varepsilon_{\theta\theta} \\ \gamma_{rz} \end{Bmatrix} = \begin{bmatrix} y_1 & 0 & y_2 & 0 & y_3 & 0 & y_4 & 0 \\ 0 & x_1 & 0 & x_2 & 0 & x_3 & 0 & x_4 \\ g_1 & 0 & g_2 & 0 & g_3 & 0 & g_4 & 0 \\ x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

or

$$[\varepsilon] = [B]\,[d]$$

[B] is the strain displacement relationship required in Eq. A-6 to evaluate the internal force vector of the element.

COMPUTATION OF [E]

One point integration is used to evaluate the integral of Eq. A-6. Therefore Eq. A-6 is reduced to

$$[E] = [B]^T \Big|_{\substack{t=o \\ s=o}} * [\sigma] * \text{volume}$$

Where

$[B]^T \big|_{\substack{t=o \\ s=o}}$ is the value of $[B]^T$ at point t=o, s=o , $[\sigma]$ is the stress vector at point t=o, s=o , and the volume is the total volume of the element.

APPENDIX B

DESCRIPTION OF INPUT DATA FOR COMPUTER PROGRAM

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DESCRIPTION OF INPUT DATA FOR COMPUTER PROGRAM

The purpose of this computer program is to determine time-dependent displacements and stresses within elastic axisymmetric structures of arbitrary shape and materials. In order to define the computer input a two-dimensional cross-section of the axisymmetric structure must be idealized by a system of finite elements. Quadrilateral, triangular and one-dimensional membrane elements can be used. Elements in the system are identified by a sequence of numbers starting with one. Also, all nodal points are identified by a separate numbering sequence. The reference coordinate system to be used and a simple finite element representation of a structure is shown in Figure B-1.

The following sequence of punched cards numerically define the axisymmetric structure to be analyzed.

A. IDENTIFICATION CARD. (72 H)

Columns 1 to 72 contain information to be printed with results.

B. CONTROL CARD. (7I5, 4F10.0)

Columns	1 - 5	Number of nodal points (n)
	6 - 10	Number of elements (k)
	11 - 15	Number of different materials (m)
	16 - 20	Number of time steps
	21 - 25	Number of time increments between the print displacements and stresses

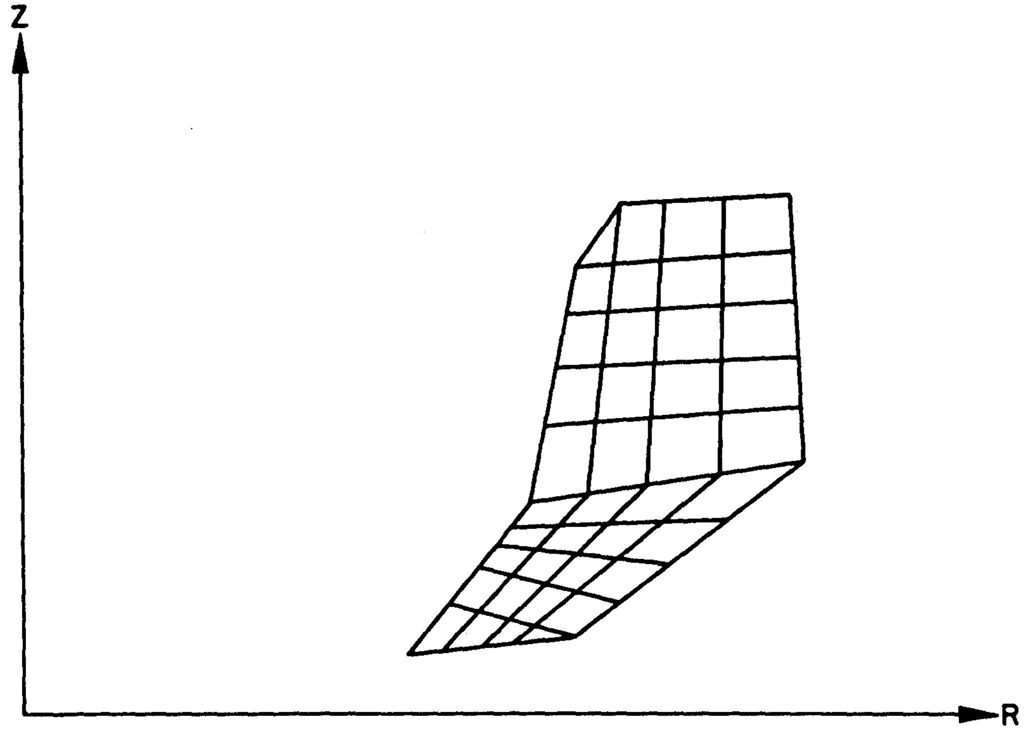

FIG. B-1 REFERENCE COORDINATE SYSTEM

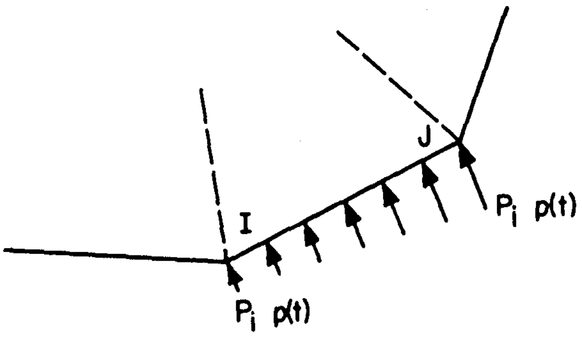

FIG. B-2 PRESSURE BOUNDARY CONDITIONS

26 - 30 Number of load cards (ℓ)

31 - 35 Number of boundary pressure cards (p)

36 - 40 Number of increments between the change of stiffness

41 - 50 Damping coefficient α

51 - 60 Damping coefficient β

61 - 70 Time increment Δ^t

71 - 80 Reference number to be added to all R ordinates

C. MATERIAL PROPERTY INFORMATION.

The following card must be supplied for each different material (I5,2F15.0)

Columns 1 - 5 Material identification number

5 - 20 Mass density of material

21 - 35 Thickness (for membrane shell elements)

D. STRESS-STRAIN INFORMATION.

To describe the stress-strain behavior of each material a sequence of six points are used. Therefore, Six cards with the following informations must be supplied for each different material. 6(5F10.0)

Columns 1 - 10 Volumetric change $(\varepsilon_r + \varepsilon_z + \varepsilon_\theta)$

11 - 20 Average Stress $[(\sigma_{rr} + \sigma_{zz} + \sigma_{\theta\theta})/3.]$

21 - 30 Unloading bulk modulus

31 - 40 Shear modulus

41 - 50 Ratio of permanent strain and the maximum strain $(\varepsilon_f/\varepsilon_{max})$

The one-dimensional shell elements are restricted to linear materials. The material properties of the shell elements are also specified by the same input format - the unloading bulk modulus is defined as the modulus of elasticity and the shear modulus is defined as Poisson's ratio. The other information is not required.

D. NODAL POINT CARDS, (I5, F5.0, 2F.10.0)

One card is required for each nodal point with the following information:

Columns	1 - 5	Nodal point number
	6 - 10	Boundary condition code "k"
	11 - 20	R-ordinate
	21 - 30	Z-ordinate

Specifications for code "k". If

k = 0	load in the R-direction load in the Z-direction
k = 1	zero displacement in the R-direction load in the Z-direction
k = 2	load in the R-direction zero displacement in the Z-direction
k = 3	zero displacement in the R-direction zero displacement in the Z-direction

Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated in equal intervals along a straight line between the defined nodal points. The boundary condition code is set equal to zero.

E. ELEMENT CARDS. (6I5)

Columns	1 - 5	Element number	
	6 - 10	Nodal point I	The maximum difference "b" between these numbers is an indication of the band width. The execution time for the program will be proportional to this number squared.
	11 - 15	Nodal point J	
	16 - 20	Nodal point K	
	21 - 25	Nodal point L	
	26 - 30	Material Identification	

For a right hand coordinate system the nodal point numbers I, J, K and L must be in sequence in a counter-clockwise direction around the element. Element cards must be in element number sequence. If element cards are omitted the program automatically generates the omitted information by incrementing by one the preceeding I, J, K and L. The material identification for the generated cards is set equal to the corresponding value on the last card. The last element card must always be supplied. Triangular elements are also permissible; they are identified by repeating the last nodal point number (i.e. I, J, K, K). One dimensional membrane elements are identified by a nodal point numbering sequence of the form I, J, J, I.

F. PRESSURE CARDS (2I5, 3F10.0)

One card for each boundary element which is subjected to a normal pressure.

Columns	1 - 5	Nodal point I
	6 - 10	Nodal point J
	11 - 20	Pressure multiplier P_i

21 - 30 Pressure multiplier P_j

31 - 40 Arrival time of pressure at the center of the surface element

As shown in Figure B-2 the boundary element must be on the left as one progresses from I to J. Surface tensile force is input as a negative pressure.

G. LOAD CARDS (2F10.0)

These cards specify the normal pressure as a function of time in the form of straight line segments. One card is required for each point with the following information:

Columns 1 - 10 Time t

11 - 20 Normal pressure p (t)

OUTPUT INFORMATION

The following information is developed and printed by the program:

1. Reprint of input data
2. Pressure boundary conditions
3. Nodal point displacements, velocities and accelerations as a function of time
4. Stresses at the center of each element as a function of time

PROGRAM LIMITATIONS

The capacity of the program is limited by the dimension "d" of the "A" array in program DYNS.

$[4n\ (b + 1) + 18n + 2\ell + 7p + 14k + 32m]$ must not be greater than d. The symbols n, ℓ, p and b have been defined previously and their values will depend on the particular structure to be analyzed. The maximum size which d can have will depend on the particular computer being utilized. For a computer with 32K storage the maximum value for d will be approximately 20000.

APPENDIX C

FORTRAN IV LISTING OF THE COMPUTER PROGRAM

```
      PROGRAM DYNS(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON NUMNP,MBAND,NT,NPRINT,NP,NUMPC,NUMEL,NUMMAT,RA,TT,DELT,
     1 NST ,ALFA,BETA,HED(12),A(33200)
C--------
C     READ AND PRINT OF CONTROL INFORMATION
C--------
   50 READ  (5,1000) HED,NUMNP,NUMEL,NUMMAT,NT,NPRINT,NP,NUMPC,NST,ALFA,
     1BETA,DELT,RA
      WRITE (6,2000) HED,NUMNP,NUMEL,NUMMAT,NT,NPRINT,NP,NUMPC,ALFA,
     1 BETA,DELT,NST
C-------
C     READ AND PRINT OF DATA
C-------
      NEQ=2*NUMNP
      N2=1+ NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+5*NUMEL
      N6=N5+NUMMAT
      N7=N6+30*NUMMAT
      N8=N7+NUMPC
      N9=N8+NUMPC
      N10=N9+NUMPC
      N11=N10+NUMPC
      N12=N11+NUMPC
      N13=N12+NUMPC
      N14=N13+NUMPC
      N15=N14+NUMMAT
  100 CALL DATAIN(A(1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9)
     1 ,A(N10),A(N11),A(N12),A(N13),A(N14),A(N15))
C--------
C     FORM TOTAL STIFFNESS AND MASS MATRICES AND SOLVE STEP-BY-STEP
C--------
      N16=N15+2*NP
      N17=N16+NUMNP
      N18=N17+NEQ
      N19=N18+NEQ
      N20=N19+NEQ
      N21=N20+NEQ
      N22=N21+NEQ
      N23=N22+5*NUMEL
      N24=N23+4*NUMEL
      N25=N24+NEQ*MBAND
      IF (N25.LE. 33200) GO TO 200
      WRITE (6,1100)
      STOP
  200 DO 300 I=N16,N25
  300 A(I)=0.
      CALL SOLVE(A(1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
     1 A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18)
     2 ,A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),NEQ)
      GO TO 50
C-------
 1000 FORMAT (12A6/8I5,4F10.0)
 1100 FORMAT (25H0 DIMENSION OF A EXCEEDED)
 2000 FORMAT (1H1 12A6/
     1 30H0 NUMBER OF NODAL POINTS------ I4 /
```

```
2 30H0 NUMBER OF ELEMENTS----------- I4 /
3 30H0 NUMBER OF DIFF. MATERIALS--- I4 /
4 30H0 NUMBER OF TIME INCREMENTS--- I4 /
5 30H0 PRINT INTERVAL-------------- I4 /
6 30H0 NUMBER OF LOAD POINTS------- I4 /
7 30H0 NUMBER OF PRESSURE CARDS---- I4 /
8 30H0 DAMPING COEFFICIENT ALFA---- F10.5 /
9 30H0 DAMPING COEFFICIENT BETA---- F10.5 /
0 30H0 TIME INCREMENT-------------- F10.5/
1 30H0 STIFFNESS CHANGE INTERVAL---  I4 )
 END
```

```
      SUBROUTINE STRESS (SIG,PRES,E,VOL,EE,MTYPE)
      COMMON / LS4ARG/ LM(8),SS(4,8),XC,YC,S(8,8),C(4,4)
      DIMENSION SIG(4),E(1),EE(5,6,1)
C-------
C     VOLUME  * STRESS . TO GET STRESSES ONE SHOULD DEVIDE BY THE VOL.
C     THIS IS DONE AT THE END OF THE SUBROUTINE
C--------
C---- --
C     SHELL ELEMENT
C-------
      DS=S(1,1)+S(2,1)+S(3,1)
      IF (LM(4).NE.LM(6)) GO TO 300
      ENU=EE(4,1,MTYPE)
      E1 =EE(3,1,MTYPE)/(1.-ENU**2)
      C(1,1)=E1
      C(2,2)=E1
      C(1,2)=E1*ENU
      C(2,1)=C(1,2)
      SIG(1)=(SIG(1)+C(1,1)*S(1,1)+S(2,1)*C(1,2))*VOL
      SIG(2)=(SIG(2)+C(2,1)*S(1,1)+C(2,2)*S(2,1))*VOL
      GO TO 100
  300 IF (C(1,1).NE.0.) GO TO 600
      DO 550 I=1,4
  550 SIG(I)=0.
      RETURN
  600 SIGP=(SIG(1)+SIG(2)+SIG(3))/3.
      B=C(1,2)+.6666666666666*C(4,4)
      PR  =-PRES-SIGP
      ST=ABS(PR/(B*DS))
      DO 400 I=1,3
      S(I,1)=S(I,1)*ST
      DO 400 J=1,3
  400 S(I,J+1)=C(I,J)-B
      DO 500 I=1,3
  500 SIG(I)=(SIG(I)+PR  +S(I,2)*S(1,1)+S(I,3)*S(2,1)+S(I,4)*S(3,1))*VOL
      SIG(4)=(SIG(4)+C(4,4)*S(4,1)*ST)*VOL
C-----INTERNAL FORCE
  100 DO 200 I=1,4
      DO 180 J=1,8
      JJ=LM(J)
  180 E(JJ)=E(JJ)+SS(I,J)*SIG(I)
  700 SIG(I)=SIG(I)/VOL
  200 CONTINUE
  320 RETURN
      END
```

```
      SUBROUTINE SIGEPS (STRAIN,EE,MTYPE,C,    PRES,EPSP)
      DIMENSION EE(5,6,1),E(4),STRAIN(5),C(4,4)
C--------
C------EPSM(1) IS THE LAST MAX. STRAIN OF THE UNLOADING BRANCH.
C-------
      EPSP=STRAIN(1)+STRAIN(2)+STRAIN(3)
      IF(EPSP.LE.0.) GO TO 90
C-----COMPLET CRACK
      PRES=0.
      DO 80 I=1,4
      DO 80 J=1,4
   80 C(I,J)=0.
      RETURN
   90 EPSM=STRAIN(5)
      EPSP=-EPSP
      IF(EPSP.GT.EPSM) EPSM=EPSP
      EPSA=EPSM
C
      DO 100 I=2,6
      IF(EPSA.LT.EE(1,I,MTYPE)) GO TO 200
  100 CONTINUE
C------FIND THE LOADING BULK OF THE CORNER POINT.
C
  200 DE=EE(1,I,MTYPE)-EE(1,I-1,MTYPE)
      B= (EE(2,I,MTYPE)-EE(2,I-1,MTYPE))/DE
C------FIND INFORMATION OF THE CORNER POINT.
C
      R=(EPSA-EE(1,I-1,MTYPE))/DE
      DO 300 J=1,4
  300 E(J)=EE(J+1,I-1,MTYPE)+R*(EE(J+1,I,MTYPE)-EE(J 1,I-1,MTYPE))
      G=E(3)
      PRES=E(1)
      R=B
C------CHECK FOR THE UNLOADING BRANCH.
C
      IF (EPSP.GE.EPSM) GO TO 400
C-----ASSUME A BILINEAR BEHAVIOR FOR THE UNLOADING BRANCH
      EF=E(4)*EPSM
      EB=E(1)/(E(2)+E(2))
      IF(EF.GT.EPSP) GO TO 6
      ECON=EPSM-EB
      IF(EPSP.LT.ECON) GO TO 5
      B=E(2)
      PRES=E(1)-E(2)*(EPSM-EPSP)
      GO TO 7
    5 B=E(1)/(2.*(EPSM-EF-EB))
      PRES=B*(EPSP-EF)
      GO TO 7
    6 B=EPSP/EF*E(1)/(2.*(EPSM-EF-EB))
      PRES=0.
    7 G=B*E(3)/R
  400 CONTINUE
      STRAIN(5)=EPSM
C------ MATERIAL MATRIX.
C
      C(4,4)=G
      G=(G+G)/3.
```

```
      C(1,1)=B+G+G
      C(1,2)=B-G
      C(1,3)=C(1,2)
      C(2,1)=C(1,2)
      C(2,2)=C(1,1)
      C(2,3)=C(1,2)
      C(3,1)=C(1,3)
      C(3,2)=C(2,3)
      C(3,3)=C(1,1)
C
      RETURN
      END
```

```
      SUBROUTINE SOLVE(R,Z,CODE,IX,RO,EE,HI,HJ,VI,VJ,T,INI,JNJ,H,P,
     1 MASS,X0,X1,X2,B,E,EPS,SIG,A,NEQ)
      COMMON NUMNP,MBAND,NT,NPRINT,NP,NUMPC,NUMEL,NUMMAT,RA,TT,DELT,
     1 NST ,ALFA,BETA
      DIMENSION R(1),Z(1),CODE(1),IX(5,1),RO(1),EE(5,6,1),HI(1),
     1  HJ(1),VI(1),VJ(1),T(1),INI(1),JNJ(1),P(2,1),MASS(1),X0(1),
     2 X1(1),X2(1),B(1),A(NEQ,1),EPS(5,1),E(1),SIG(4,1),H(1),ELMASS(4)
      COMMON / LS4ARG / LM(8),SS(4,8),XC,YC,S(8,8),C(4,4)
      REAL MASS
C-------
C     INITIALIZATION
C---- --
      DO 40 I=1,4
      DO 50 J=1,8
   50 SS(I,J)=0.
      DO 40 J=1,4
   40 C(I,J)=0.
      VOL=0.
C     ---
C     CONSTANTS FOR THE STEP-BY-STEP SOLUTION
C     ---
      A1=3./DELT
      A2=.75/DELT**2
      A3=A1/2.
      A0 = A2+ A2
      A4=A1/A0
      A5 = 2./A0
      A6=DELT/2.
      A7=DELT**2/6.
      A8=A7+A7
C-----FORM STIFFNESS AND MASS MATRIX OF THE SYSTFM
      DO 375 N=1,NUMEL
      DO 199 I=1,4
      J=I+I
      II=IX(I,N)+IX(I,N)
      LM(J)=II
  199 LM(J-1)=II-1
C-------
C     FINE THE ELASTIC CONSTANTS.
C--------
      I=IX(1,N)
      J=IX(2,N)
      K=IX(3,N)
      L=IX(4,N)
      MTYPE=IX(5,N)
      IF(J.NE.K)  CALL SIGEPS (EPS(1,N),EE,MTYPE,C,    PRES,EPSP)
C---- ---
C     FORM ELEMENT STIFFNESS MATRICES
C--------
      CALL STIFF (R,Z,CODE,IX(1,N),EE,A,NEQ,RO,MTYPE,VOL ,H)
      IF (J.NE.K)  GO TO 444
      RRR=VOL*RO(MTYPE)/4.
      ELMASS(1)=RRR
      ELMASS(2)=RRR
      ELMASS(3)=RRR
      ELMASS(4)=RRR
      GO TO 454
```

```
  444 RM=8.*XC
      R12=R(I)-R(J)
      R13= R(I)-R(K)
      R14=R(I)-R(L)
      R23=R(J)-R(K)
      R24=R(J)-R(L)
      R34=R(K)-R(L)
      Z12=Z(I)-Z(J)
      Z13=Z(I)-Z(K)
      Z14=Z(I)-Z(L)
      Z23=Z(J)-Z(K)
      Z24 =Z(J)-Z(L)
      Z34=Z(K)-Z(L)
      ROM=RO(MTYPE)/72.
      BR=(R34*Z12-Z34*RB2)*ROM
      AR=(VOL+VOL)/XC *ROM
      CR=(R23*Z14-Z23*R14)*ROM
      ELMASS(1)=AR*(RM+R13+R(I))-BR*(R(I)+R(I)+R(L)) CR*(R(I)+R(I)+R(J))
      ELMASS(2)=AR*(RM+R24+R(J))+BR*(R(J)+R(J)+R(K)) CR*(R(I)+R(J)+R(J))
      ELMASS(3)=AR*(RM+R(K)-R13)+BR*(R(J)+R(K)+R(K)) CR*(R(K)+R(K)+R(L))
      ELMASS(4)=AR*(RM+R(L)-R24)-BR*(R(I)+R(L)+R(L)) CR*(R(K)+R(L)+R(L))
  454 CONTINUE
      DO 350 I=1,4
      II=IX(I,N)
  350 MASS(II)=MASS(II)+ELMASS(I)
  375 CONTINUE
C--------
C     INITIAL ACCELERATION
C--------
      TT=P(1,1)
      IK=1
      CALL LOAD (T,P,B,INI,JNJ,HI,HJ,VI,VJ,IK)
C-----FORM THE EFECTIVE STIFFNESS OF THE SYSTEM
      II=0
      DO 400 I=1,NEQ
      II=I-II
      IF(A(I,1).EQ.0.) GO TO 400
      X2(I)=B(I)/MASS(II)
      A(I,1)=A(I,1)+A0*MASS(II)
  400 CONTINUE
C-----REVISE THE MASS METRIX FOR SUBCIQUENT USE
      DO 401 II=1,NUMNP
  401 MASS(II)=MASS(II)*A0
C-----INITIAL TRIANGULARIZATION
      CALL TRIA (NEQ,MBAND,A)
C-------
C     STEP-BY-STEP SOLUTION
C--------
      MM=(NST-1)*NUMEL
      KK=0
      LL=0
      DO 500 NNN=1,NT
      TT=TT+DELT
      CALL LOAD (T,P,B,INI,JNJ,HI,HJ,VI,VJ,IK)
      II=0
C-----EFECTIVE LOAD VECTOR
      DO 460 I=1,NEQ
```

```
      II=I-II
      B(I)=B(I)-E(I)+MASS(II)*(A4*X1(I)+A5*X2(I))
      IF(A(I,1).EQ.0.0) B(I)=0.0
  460 CONTINUE
C
C     SOLUTION AT END OF TIME STEP
C
      CALL BACKS (NEQ,MBAND,A,B)
      DO 480 I=1,NEQ
      E(I)=0.
      ACC=A2*B(I)-A3*X1(I)-.5*X2(I)
      B(I)=DELT*X1(I)+A7*(ACC+X2(I)+X2(I))
      X0(I)=X0(I)+B(I)
      X1(I)=X1(I)+A6*(X2(I)+ACC)
  480 X2(I)=ACC
C     PRINT DISPLACEMENT AND PREPARATION OF A NEW STIFFNESS METRIX
C---- --+
      LL=LL+1
      IF (LL.NE.NPRINT ) GO TO 499
      LL=0
  421 WRITE (6,2006) TT
      DO 482 N=1,NUMNP
      M=N+N
      K=M-1
  482 WRITE (6,2008) N,X0(K),X0(M),X1(K),X1(M),X2(K),X2(M),N
  499 CONTINUE
      MPRINT =0
      DO 498 N=1,NUMEL
      KK=KK+1
C---- --
C     COMPUTE STRAIN, ELASTIC CONTANT,STRESSES, INTERNAL FORCE, AND
C     STIFFNESS
C---- --
      CALL STRAIN (B,R,Z,IX(1,N),EPS(1,N),MTYPE,VOL,H)
      IF (IX(2,N).NE.IX(3,N))
     1CALL        SIGEPS(EPS(1,N),EE,MTYPE,C,     PRES,EPSP)
      CALL STRESS (SIG(1,N),PRES,E,VOL,EE,MTYPE)
C
      IF( KK.LE.MM) GO TO 170
      IF (KK.GT.MM+1) GO TO 424
      II=0
      DO 425 I=1,NEQ
      II=I-II
      IF (A(I,1).NE.0.) A(I,1)=MASS(II)
      DO 425 J=2,MBAND
  425 A(I,J)=0.
  424 CALL STIFF (R,Z,CODE,IX(1,N),EE,A,NEQ,RO,MTYPE,VOL ,H)
      IF (N.NE.NUMEL) GO TO 170
      KK=0
      CALL TRIA (NEQ,MBAND,A)
  170 IF (LL.NE.0) GO TO 498
C-------
C------CALCULATE THE PRINCIPLE STRESSES.
C--------
      CC=(SIG(1,N)+SIG(2,N))/2.
      BB=CC-SIG(2,N)
      CR=SQRT(BB**2+SIG(4,N)**2)
```

```
      SIGMAX=CC+CR
      SIGMIN=CC-CR
      IF(CR) 200,255,200
  200 ANGLE=28.648*ATAN2(SIG(4,N),BB)
  255 IF( MPRINT) 110,105,110
  105 WRITE (6,2000)
      MPRINT =50
  110 MPRINT =MPRINT-1
  305 WRITE (6,2001) N,XC,YC,(SIG(I,N),I=1,4),SIGMAX,SIGMIN,ANGLE
     1 ,PRES,EPSP
C
  498 CONTINUE
  500 CONTINUE
      RETURN
C---- -+
 2000 FORMAT (6H1EL.NO  7X  1HR  7X  1HZ  7X  5HSIG-R  7X  5HSIG-Z  7X
     1 5HSIG-T  6X  6HTAU-RZ  5X  7HSIG-MAX  5X  7HSIG-MIN  7H  ANGLE
     22X  11HAVE. PRESS.  2X  11HVOL. CHANGE. )
 2001 FORMAT ( I5,1X,2F8.2,6E12.4,F6.2,2E13.4)
 2006 FORMAT (6H1TIME T=F10.6/118 HONODAL POINT X-DISPLACEMENT  Y-DISPLA
     1CEMENT    X-VELOCITY      Y-VELOCITY  X-ACCELERATION  Y-ACCELERATI
     2ON  NODAL POINT )
 2007 FORMAT (F10.0)
 2008 FORMAT (I9,6E16.4,I9)
      END
```

```
      SUBROUTINE DATAIN(R,Z,CODE,IX,RO,EE,HI,HJ,VI,VJ,T,INI,JNJ,H,P)
      COMMON NUMNP,MBAND,NT,NPRINT,NP,NUMPC,NUMEL,NUMMAT,RA,TT
      DIMENSION R(1),Z(1),CODE(1),IX(5,1),RO(1),EE(5,6,1),HI(1),
     1  HJ(1),VI(1),VJ(1),T(1),INI(1),JNJ(1),P(2,1),IE(5),H(1)
C--------
C   READ AND PRINT OF MATERIAL PROPERTIES
C-------
      DO 59 M=1,NUMMAT
      READ  (5,1001) MTYPE,RO(MTYPE),H(MTYPE),
     1 ((EE(I,J,MTYPE),I=1,5),J=1,6)
      WRITE (6,2000) MTYPE,RO(MTYPE),H(MTYPE)
   59 WRITE (6,2012) ((EE(I,J,MTYPE),I=1,5),J=1,6)
C--------
C     READ AND PRINT OF NODAL POINT DATA
C--------
      WRITE (6,2004)
      L=0
   60 READ (5,1002) N,CODE(N),R(N),Z(N)
      R(N)=R(N)+RA
      IF (L.EQ.0) GO TO 85
      ZX=N-L
      DR=(R(N)-R(L))/ZX
      DZ=(Z(N)-Z(L))/ZX
   85 NL=L+1
   70 L=L+1
      IF(N-L) 100,90,80
   80 CODE(L)=0.0
      R(L)=R(L-1)+DR
      Z(L)=Z(L-1)+DZ
      GO TO 70
   90 WRITE (6,2002) (K,CODE(K),R(K),Z(K),K=NL,N)
      IF(NUMNP-N) 100,110,60
  100 WRITE (6,2009) N
      STOP
  110 CONTINUE
C--------
C     READ AND PRINT OF ELEMENT NODES
C--------
      WRITE (6,2001)
      N=0
      MBAND=0
  130 READ (5,1003) M,(IE(I),I=1,5)
  140 N=N+1
      MB=0
      DO 160 I=1,4
C---- --
C     DETERMINATION OFBAND WITH
C---- --
      DO 160 J=I,4
      MM=IABS(IE(I)-IE(J))
      IF(MM.GT.MB) MB=MM
  160 CONTINUE
      MB=2*MB+2
      IF(MB.GT.MBAND) MBAND=MB
      IF(M.EQ.N) GO TO 145
      DO 142 I=1,4
  142 IX(I,N)=IX(I,N-1)+1
```

```
      IX(5,N)=IX(5,N-1)
      GO TO 150
  145 DO 148 I=1,5
  148 IX(I,N)=IE(I)
  150 WRITE (6,2003) N,(IX(I,N),I=1,5),MB
C---- --
      IF(N.EQ.NUMEL) GO TO 700
      IF(N.EQ.M) GO TO 130
      GO TO 140
C---- --
C     PRESSURE BOUNDARY CONDITIONS
C---- --
  700 WRITE (6,2010)
      DO 330 K=1,NUMPC
      READ (5,1007) INI(K),JNJ(K),A,B,T(K)
      I=INI(K)
      J=JNJ(K)
      DZ=(Z(I)-Z(J))/12.0
      DR=(R(J)-R(I))/12.0
      RX=A*(3.0*R(I)+R(J))+B*(R(I)+R(J))
      ZX=A*(R(I)+R(J))+B*(R(I)+3.0*R(J))
      HI(K)=RX*DZ
      HJ(K)=ZX*DZ
      VI(K)=RX*DR
      VJ(K)=ZX*DR
  330 WRITE (6,2013) I,J,A,B,HI(K),VI(K),HJ(K),VJ(K),T(K)
C---- --
C        READ AND PRINT OF LOAD DATA
C---- ---
      WRITE (6,2007)
      DO 380 M=1,NP
  380 READ(5,1004) (P(K,M),K=1,2)
      WRITE (6,2005) ((P(K,M),K=1,2),M=1,NP)
C---- ---
      RETURN
 1001 FORMAT (I5,2F15.0/(5F10.0))
 1002 FORMAT  (I5,F5.0,2F10.0)
 1003 FORMAT (6I5)
 1004 FORMAT (2F10.0)
 1007 FORMAT (2I5,3F10.0)
 2000 FORMAT (19H  MATERIAL NUMBER =  I5 ,10H DENSITY = E15.6 ,
     1   13H  THICKNESS =  E15.6 /   4X,
     290H       STRAIN              PRESSURE        UNLOADING BULK    SHEAR M
     3ODULUS    STRAIN SET RATIO         / )
 2001 FORMAT (49H1ELEMENT NO.      I      J      K      L     MATERIAL  )
 2002 FORMAT (I7, F10.2,2F10.3)
 2003 FORMAT (1I13,4I6,2I12)
 2004 FORMAT (37H1NODAL POINT TYPE     X-ORD      Y-ORD )
 2005 FORMAT (2F15.7)
 2007 FORMAT  (27H1          TIME       PRESSURE P)
 2009 FORMAT (26H0NODAL POINT CARD ERROR N= I5)
 2010 FORMAT (29H1PRESSURE BOUNDARY CONDITIONS/
     115X,1HI,5X,1HJ,7X,4HPI/P,8X,4HPJ/P,8X,2HHI,10X,2HVI,10X,2HHJ,10X,
     2 2HVJ,11X,1HT)
 2012 FORMAT (5E18.6)
 2013 FORMAT (2I6,7F12.3)
      END
```

```
      SUBROUTINE STRAIN (XO,R,Z,IX,EPS,MTYPE,VOL,H)
      DIMENSION XO(1),R(1),Z(1),IX(5)   ,EPS(5)  ,X(4),Y(4),H(1)
      COMMON / LS4ARG / LM(8),SS(4,8),XC,YC,S(8,8),C(4,4)
      DO 200 I=1,4
      DO 500 IK=1,8
  500 SS(I,IK)=0.
      J=I+I
      II=IX(I)+IX(I)
      LM(J)=II
  200 LM(J-1)=II-1
      I=IX(1)
      J=IX(2)
      K=IX(3)
      L=IX(4)
      MTYPE=IX(5)
C---- --
C     DISPLACEMENT STRAIN TRANSFORMATION MATRIX.
C--------
      R13=R(I)-R(K)
      R24=R(J)-R(L)
      Z13=Z(I)-Z(K)
      Z24=Z(J)-Z(L)
      RM=1./(R(I)+R(J)+R(K)+R(L))
      YC=(Z(I)+Z(J)+Z(K)+Z(L))/4.
      IF(J.NE.K) GO TO 300
C-----SHELL ELEMENT
      XL=R24**2+Z24**2
      SS(1,1)=R13/XL
      SS(1,2)=Z13/XL
      SS(1,3)=-SS(1,1)
      SS(1,4)=-SS(1,2)
      SS(2,1)=RM+RM
      SS(2,3)=SS(2,1)
      VOL=SQRT(XL)*(H(MTYPE)+H(MTYPE))
      GO TO 400
  300 VOL=R13*Z24-Z13*R24
      Y(1)=Z24/VOL
      Y(2)=-Z13/VOL
      Y(3)=-Y(1)
      Y(4)=-Y(2)
      X(1)=-R24/VOL
      X(2)=R13/VOL
      X(3)=-X(1)
      X(4)=-X(2)
      DO 100 I=1,4
      II=I+I
      JJ=II-1
      SS(1,JJ)=Y(I)
      SS(2,II)=X(I)
      SS(3,JJ)=RM
      SS(4,II)=Y(I)
  100 SS(4,JJ)=X(I)
  400 XC=.25/RM
      VOL=VOL/2.*XC
C-------
C     EVALUATION OF STRAIN.
C--------
```

```
      DO 180 I=1,4
      S(I,1)=0.
      DO 180 J=1,8
      JJ=LM(J)
  180 S(I,1)=S(I,1)+SS(I,J)*X0(JJ)
      DO 190 I=1,4
  190 EPS(I)=EPS(I)+S(I,1)
  320 RETURN
      END
```

```
      SUBROUTINE ONED (R,Z,H,IX,VOL,MTYPE,EE)
      COMMON / LS4ARG / LM(8),SS(4,8),XC,YC,S(8,8),C(4,4)
      DIMENSION R(1),Z(1),H(1),IX(5),ST(4,8) ,EE(5,6,1)
      DO 410 I=1,8
      DO 405 J=1,4
  405 ST(J,I)=0.
      DO 410 J=1,8
  410 S(I,J)=0.
      I=IX(1)
      J=IX(2)
      XC=(R(I)+R(J))/2.
      YC=(Z(I)+Z(J))/2.
      DX=R(J)-R(I)
      DY=Z(J)-Z(I)
      XL=SQRT(DX**2+DY**2)
      VOL=H(MTYPE)*XL*XC
      ENU=EE(4,1,MTYPE)
      E1=EE(3,1,MTYPE)/(1.-ENU**2)
      C(1,1)=E1
      C(2,2)=E1
      C(1,2)=ENU*E1
      C(2,1)=C(1,2)
C-----STRAIN DISPLACEMENT RELATION
      ST(1,1)=-DX/XL**2
      ST(1,2)=-DY/XL**2
      ST(1,3)=-ST(1,1)
      ST(1,4)=-ST(1,2)
      ST(2,1)=.5/XC
      ST(2,3)=ST(2,1)
      DO 411 I=1,4
      DO 411 J=1,8
  411 SS(I,J)=0.
      DO 412 I=1,2
      DO 412 J=1,4
      DO 412 K=1,2
  412 SS(I,J)=SS(I,J)+C(I,K)*ST(K,J)
      DO 414 J=1,4
      DO 414 I=1,4
      DO 414 K=1,2
  414 S(I,J)=S(I,J)+ST(K,I)*SS(K,J)*VOL
      RETURN
      END
```

```
      SUBROUTINE LOAD (T,P,B,INI,JNJ,HI,HJ,VI,VJ,IK)
      COMMON NUMNP,MBAND,NT,NPRINT,NP,NUMPC,NUMEL,NUMMAT,RA,TT,DELT
      DIMENSION T(1),P(2,1),B(1),INI(1),JNJ(1),HI(1),HJ(1),VI(1),VJ(1)
C
      NEQ=NUMNP+NUMNP
      DO 600 I=1,NEQ
  600 B(I)=0.
      N=1
  100 TAU=TT-T(N)
      IF(TAU) 500,200,200
  200 IF(TAU.GE.P(1,IK).AND.TAU.LE.P(1,IK+1)) GO TO 300
      IF (TAU.GT.P(1,IK+1)) IK=IK+1
      IF (TAU.LT.P(1,IK)) IK=IK-1
      GO TO 200
  300 D=P(1,IK+1)-P(1,IK)
      DH=P(2,IK+1)-P(2,IK)
      IF (TT.EQ.P(1,1)) TAU=-DELT
      DT=TAU-P(1,IK) +DELT
      F=P(2,IK)+DT*DH/D
  400 I=INI(N)+INI(N)
      II=I-1
      J=JNJ(N)+JNJ(N)
      JJ=J-1
      B(II)=B(II)+F*HI(N)
      B(JJ)=B(JJ)+F*HJ(N)
      B(I)=B(I)+F*VI(N)
      B(J)=B(J)+F*VJ(N)
  500 N=N+1
      IF (N.GT.NUMPC) RETURN
      IF(T(N).EQ.T(N-1)) GO TO 400
      GO TO 100
      END
```

```
      SUBROUTINE STIFF (R,Z,CODE,IX,EE,A,NEQ,RO,MTYPE,VOL,H)
      COMMON / LS4ARG / LM(8),SS(4,8),XC,YC,S(8,8),C(4,4)
      DIMENSION R(1),Z(1),CODE(1),IX(5),EE(5,6,1),A(NEQ,1),RO(1),H(1)
      I=IX(1)
      J=IX(2)
      K=IX(3)
      L=IX(4)
      IF (J.NE.K) GO TO 420
      CALL ONED (R,Z,H,IX,VOL,MTYPE,EE)
      GO TO 430
  420 CONTINUE
C
      CALL QUAD(R(I),R(J),R(K),R(L),Z(I),Z(J),Z(K),Z(L),XC,YC,VOL,C,
     1 S,SS)
  430 CONTINUE
C
C     MODIFY FOR ZERO DISPLACEMENTS
C
      DO 600 I=1,4
      IJ=I+I
      II=IX(I)
      IF (CODE(II).EQ.0.0) GO TO 600
      IF (CODE(II).EQ.1.0) GO TO 580
      DO 570 J=1,8
      S(IJ,J)=0.
  570 S(J,IJ)=0.
  580 IF (CODE(II).EQ.2.0) GO TO 600
      DO 590 J=1,8
      S(IJ-1,J)=0.
  590 S(J,IJ-1)=0.
  600 CONTINUE
C
      DO 300 I=1,8
      II=LM(I)
      DO 300 J=1,8
      JJ=LM(J)-II+1
      IF (JJ.LT.1) GO TO 300
      A(II,JJ)=A(II,JJ)+S(I,J)
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BACKS(NN,MM,A,B)
C
      DIMENSION A(1),B(1)
C
      MMM=MM-1
      N=0
  270 N=N+1
      C=B(N)
      IF(A(N).NE.0.0) B(N)=B(N)/A(N)
      IF(N.EQ.NN) GO TO 300
      IL=N+1
      IH=MIN0(NN,N+MMM)
      M=N
      DO 285 I=IL,IH
      M=M+NN
  285 B(I)=B(I)-A(M)*C
      GO TO 270
C
  300 IL=N
      N=N-1
      IF(N.EQ.0) RETURN
      IH=MIN0(NN,N+MMM)
      M=N
      DO 400 I=IL,IH
      M=M+NN
  400 B(N)=B(N)-A(M)*B(I)
      GO TO 300
C
      END
```

```
      SUBROUTINE TRIA(NEQ,M,A)
      DIMENSION A(1)
      NE=NEQ-1
      MN=M-1
      MM=MN*NEQ
      MK=NEQ-MN
      DO 300 N=1,NE
      NT=N-MK
      IF(NT.GT.0) MM=MM-NEQ
      IF(A(N).EQ.0.0) GO TO 300
      L=N
      IL=N+NEQ
      IH=N+MM
      DO 200 I=IL,IH,NEQ
      L=L+1
      J=L
   90 C=A(I)/A(N)
      DO 100 K=I,IH,NEQ
      A(J)=A(J)-C*A(K)
  100 J=J+NEQ
      A(I)=C
  200 CONTINUE
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE QUAD (R1,R2,R3,R4,Z1,Z2,Z3,Z4,RM,ZM,VOL,D,QK,QS)
C
C     FORMS STIFFNESS MATRIX QK, CENTROIDAL STRESS MATRIX QS
C              FOR A FIVE POINT AXISYMMETRIC IRON'S QUADRILATERAL USING
C     A FOUR POINT INTEGRATION FORMULA.
C     CONSTANT SHEAR STRAIN INTRODUCES INCOMPATIBILITY
      DIMENSION QK(8,8),QS(4,8),D(4,4),TT(4),QC(4,10),SS(4),QQ(10,10)
      DATA SS/ -1.,1.,1.,-1. / , TT /-1.,-1.,1.,1. /
C
      DO 6 I=1,10
      DO 6 J=1,10
    6 QQ(I,J)=0.
      R12=R1-R2
      R13=R1-R3
      R14=R1-R4
      R23=R2-R3
      R24=R2-R4
      R34=R3-R4
      Z12=Z1-Z2
      Z13=Z1-Z3
      Z14=Z1-Z4
      Z23=Z2-Z3
      Z24=Z2-Z4
      Z34=Z3-Z4
      VOL=R13*Z24-R24*ZB3
      RM=(R1+R2+R3+R4)/4.0
      ZM=(Z1+Z2+Z3+Z4)/4.0
      IF (D(1,1).EQ.0.) GO TO 888
      Y5=Z24/VOL
      X6=R13/VOL
      X7=R24/VOL
      Y8=Z13/VOL
      X5=-X7
      Y6=-Y8
      Y7=-Y5
      X8=-X6
      DO 30 II=1,4
      S=SS(II)*0.577350269189626
      T=TT(II)*0.577350269189626
      XJ =VOL+S*(R34*Z12-R12*Z34)+T*(R23*Z14-R14*Z23)
      XJAC=XJ/8.0
      SM=1.0-S
      SP=1.0+S
      TM=1.0-T
      TP=1.0+T
      H1=0.25*SM*TM
      H2=0.25*SP*TM
      H3=0.25*SP*TP
      H4=0.25*SM*TP
      R=H1*R1+H2*R2+H3*R3+H4*R4
      G1=H1/R
      G2=H2/R
      G3=H3/R
      G4=H4/R
      GC=SM*SP*TM*TP/R
      X1=(-R24+R34*S+R23*T)/XJ
      X2=( R13-R34*S-R14*T)/XJ
```

```
      X3=( R24-R12*S+R14*T)/XJ
      X4=(-R13+R12*S-R23*T)/XJ
      Y1=( Z24-Z34*S-Z23*T)/XJ
      Y2=(-Z13+Z34*S+Z14*T)/XJ
      Y3=(-Z24+Z12*S-Z14*T)/XJ
      Y4=( Z13-Z12*S+Z23*T)/XJ
      RS=0.25*(-TM*R1+TM*R2+TP*R3-TP*R4)
      ZS=0.25*(-TM*Z1+TM*Z2+TP*Z3-TP*Z4)
      RT=0.25*(-SM*R1-SP*R2+SP*R3+SM*R4)
      ZT=0.25*(-SM*Z1-SP*Z2+SP*Z3+SM*Z4)
      XC=-2.0*(T*SM*SP*RS-S*TM*TP*RT)/XJAC
      YC= 2.0*(T*SM*SP*ZS-S*TM*TP*ZT)/XJAC
      FAC=XJAC*R
C
C     FORM STIFFNESS QK
C
      DO 10 I=1,4
      D1=D(I,1)*FAC
      D2=D(I,2)*FAC
      D3=D(I,3)*FAC
      D4=D(I,4)*FAC
      QC(I,1)=           D1*Y1+D4*X5+D3*G1
      QC(I,3)=           D1*Y2+D4*X6+D3*G2
      QC(I,5)=           D1*Y3+D4*X7+D3*G3
      QC(I,7)=           D1*Y4+D4*X8+D3*G4
      QC(I,9)=           D1*YC      +D3*GC
      QC(I,2)=           D2*X1+D4*Y5
      QC(I,4)=           D2*X2+D4*Y6
      QC(I,6)=           D2*X3+D4*Y7
      QC(I,8)=           D2*X4+D4*Y8
      QC(I,10)=          D2*XC
   10 CONTINUE
      DO 20 I=1,10
      D1=QC(1,I)
      D2=QC(2,I)
      D3=QC(3,I)
      D4=QC(4,I)
      QQ(1,I)=QQ(1,I)+D1*Y1+D4*X5+D3*G1
      QQ(3,I)=QQ(3,I)+D1*Y2+D4*X6+D3*G2
      QQ(5,I)=QQ(5,I)+D1*Y3+D4*X7+D3*G3
      QQ(7,I)=QQ(7,I)+D1*Y4+D4*X8+D3*G4
      QQ(9,I)=QQ(9,I)+D1*YC      +D3*GC
      QQ(2,I)=QQ(2,I)+D2*X1+D4*Y5
      QQ(4,I)=QQ(4,I)+D2*X2+D4*Y6
      QQ(6,I)=QQ(6,I)+D2*X3+D4*Y7
      QQ(8,I)=QQ(8,I)+D2*X4+D4*Y8
      QQ(10,I)=QQ(10,I)+D2*XC
   20 CONTINUE
   30 CONTINUE
C
C     FORM STRESS MATRIX QS AT CENTROID (RM,ZM) OF ELEMENT
C
      DO 40 I=1,4
      D1=D(I,1)
      D2=D(I,2)
      D3=D(I,3)/(4.0*RM)
      D4=D(I,4)
```

```
      T1=( D1*Z24-D4*R24)/VOL
      T2=(-D1*Z13+D4*R13)/VOL
      T3=(-D2*R24+D4*Z24)/VOL
      T4=( D2*R13-D4*Z13)/VOL
      QC(I,1)=D3+T1
      QC(I,3)=D3+T2
      QC(I,5)=D3-T1
      QC(I,7)=D3-T2
      QC(I,9)=4.0*D3
      QC(I,2)= T3
      QC(I,4)= T4
      QC(I,6)=-T3
      QC(I,8)=-T4
      QC(I,10)=0.0
   40 CONTINUE
C
C     ELIMINATE CENTRE NODE
C
      DO 50 N=1,2
      L=10-N
      M=L+1
      DO 50 I=1,L
      C=QQ(I,M)/QQ(M,M)
      DO 50 J=1,L
   50 QQ(I,J)=QQ(I,J)-C*QQ(M,J)
C
C     RELOCATE STRESS, STIFFNESS AND LOAD MATRICES
C
  888 CONTINUE
      DO 70 J=1,8
      DO 70 I=1,4
      QK(I,J)=QQ(I,J)
   70 QK(I+4,J)=QQ(I+4,J)
      VOL=VOL*RM/2.
      RETURN
      END
```

APPENDIX D

LISTING OF INPUT DATA FOR SAMPLE PROBLEM

```
TEST FOR THE NONLINEAR AXISYMMETRIC ANALYSIS.  VICKSBURG'S EXAMPLE.
   63    50     2    20     1    11     6     1                        .0015
    1 .000164
                                2000000.  17500.    .874
 .000155    6.5                 2000000.  13500.    .874
 .00029     10.                 2000000.  10500.    .874
 .0016       40.                2000000.  12000.    .874
 .001855     50.                2000000.  21000.    .874
 .002       60.                 2000000.  27000.    .874
    2   .0007                        .5
                               30000000.   .3
                               30000000.   .3
                               30000000.   .3
                               30000000.   .3
                               30000000.   .3
                               30000000.   .3
    1    3.         0.          0.
    2    3.        12.          0.
    3    3.        36.          0.
    4    3.        60.          0.
    5    3.        84.          0.
    6    3.       108.          0.
    7    3.       132.          0.
    8    1.         0.         24.
    9    0.        12.         24.
   14    1.       132.         24.
   15    1.         0.         48.
   16    0.        12.         48.
   21    1.       132.         48.
   22    1.         0.         60.
   23    0.        12.         60.
   28    1.       132.         60.
   29    1.         0.         72.
   30    0.        12.         72.
   35    1.       132.         72.
   36    1.         0.         84.
   37    0.        12.         84.
   42    1.       132.         84.
   43    1.         0.         96.
   44    0.        12.         96.
   49    1.       132.         96.
   50    1.         0.        108.
   51    0.        12.        108.
   56    1.       132.        108.
   57    1.         0.        120.
   58    0.        12.        120.
   63    1.       132.        120.
    1     1     2     9     8     1
    6     6     7    14    13     1
    7     8     9    16    15     1
   12    13    14    21    20     1
   13    15    16    23    22     1
   18    20    21    28    27     1
   19    22    23    30    29     1
   24    27    28    35    34     1
   25    30    31    38    37     1
   29    34    35    42    41     1
```

30	36	37	44	43	1
35	41	42	49	48	1
36	43	44	51	50	1
41	48	49	56	55	1
42	50	51	58	57	1
47	55	56	63	62	1
48	36	37	37	36	2
49	37	30	30	37	2
50	29	30	30	29	2
63	62	1.		1.	
62	61	1.		1.	
61	60	1.		1.	
60	59	1.		1.	
59	58	1.		1.	
58	57	1.		1.	

.001	14.
.002	54.
.003	54.
.004	39.
.005	38.
.011	36.
.019	35.
.030	34.
.040	32.
.050	31.

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4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final report

5. AUTHOR(S) (First name, middle initial, last name)

Iraj Farhoomand
Edward Wilson

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
January 1970	70	8

8a. CONTRACT OR GRANT NO. DACA 39-67-0020	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) U. S. Army Engineer Waterways Experiment Station Contract Report N-70-1
d.	

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A nonlinear, axisymmetric, dynamic finite-element method of analysis computer program is developed. Elastic, two-dimensional problems can also be analyzed without loss of efficiency. An extensive description of the analytical procedures used in the code is given. A FØRTRAN IV listing of the computer code is presented along with information on utilizing the code to run problems. Analytical results are compared with experimental data obtained from testing a modeled buried structure subjected to a blast loading.

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