A NONLINEAR FINITE ELEMENT CODE FOR ANALYZING THE BLAST RESPONSE OF UNDERGROUND STRUCTURES

by

I. Farhoomand, E. Wilson

January 1970

Sponsored by Defense Atomic Support Agency

Conducted for U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

Under Contract No. DACA 39-67-0020

By Structural Engineering Laboratory, University of California, Berkeley, California

This document has been approved for public release and sale; its distribution is unlimited
A NONLINEAR FINITE ELEMENT CODE FOR ANALYZING THE BLAST RESPONSE OF UNDERGROUND STRUCTURES

by

I. Farhoomand, E. Wilson

January 1970

Sponsored by Defense Atomic Support Agency
NWER Subtask SC210

Conducted for U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi
Under Contract No. DACA 39-67-0020

By Structural Engineering Laboratory, University of California, Berkeley, California

This document has been approved for public release and sale; its distribution is unlimited
FOREWORD

The investigation described herein was performed under Contract No. DACA 39-67-0020 for the Nuclear Weapons Effects Division (NWED), U.S. Army Engineer Waterways Experiment Station (WES), under the sponsorship of the Defense Atomic Support Agency as part of Nuclear Weapons Effects Research (NWER) Subtask SC 210.

This contract was technically monitored by Mr. P. J. Rieck under the direct supervision of Mr. W. J. Flathau, Chief, Protective Structures Branch, and under the general supervision of Mr. G. L. Arbuthnot, Jr., Chief, Nuclear Weapons Effects Division, WES.

This study was conducted by Dr. Edward L. Wilson and Mr. Iraj Farhoosmand of the University of California, Berkeley, California. Close technical cooperation was provided by Messrs. J. L. Kirkland and R. E. Walker and Dr. R. Froelich of NWED.

COL Levi A. Brown, CE, was Director of WES during the preparation of this report. Mr. F. R. Brown was Technical Director.
ABSTRACT

A nonlinear, axisymmetric, dynamic finite-element method of analysis computer program is developed. Elastic, two-dimensional problems can also be analyzed without loss of efficiency. An extensive description of the analytical procedures used in the code is given. A FÔRTRAN IV listing of the computer code is presented along with information on utilizing the code to run problems.

Analytical results are compared with experimental data obtained from testing a modeled buried structure subjected to a blast loading.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>DYNAMIC EQUILIBRIUM EQUATIONS</td>
<td>2</td>
</tr>
<tr>
<td>STEP-BY-STEP INTEGRATION OF EQUILIBRIUM EQUATIONS</td>
<td>4</td>
</tr>
<tr>
<td>NONLINEAR MATERIAL BEHAVIOR</td>
<td>10</td>
</tr>
<tr>
<td>CONSISTENT FORMULATION OF NONLINEAR PROPERTIES</td>
<td>14</td>
</tr>
<tr>
<td>APPLICATION</td>
<td>18</td>
</tr>
<tr>
<td>COMPUTER PROGRAM</td>
<td>26</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>28</td>
</tr>
</tbody>
</table>

**APPENDIX A**  
Internal Force Vector for A Quadrilateral Element

**APPENDIX B**  
Description Of Input Data For Computer Program

**APPENDIX C**  
Fortran IV Listing of Computer Program

**APPENDIX D**  
Listing Of Input Data For Sample Problem
INTRODUCTION

Several attempts have been made to use the finite element method for the nonlinear dynamic analysis of granular materials\textsuperscript{1,2,3}. These previous coded programs are not readily used by engineers for the dynamic analysis of complex structures. One of the objectives of this study is to develop a nonlinear computer program which is efficient and easy to use.

In Reference 4 the finite element method coupled with a stable step-by-step method of integration was used for the elastic dynamic analysis of two-dimensional plane strain solids. Reference 5 is a generalization of the material in Reference 4. It also contains a new coding technique which increases the capacity of the program. The purpose of this investigation is to extend the step-by-step dynamic analysis of axisymmetric structures to include nonlinearity of granular materials. It is worthy to note that elastic materials can also be treated by the nonlinear program without a loss of efficiency. In addition, the step-by-step integration method has been revised to provide better accuracy when nonlinear equations are involved.
DYNAMIC EQUILIBRIUM EQUATIONS

The force equilibrium of a system of structural elements is expressed by the following matrix equation:

\[ M \ddot{u}_t + C \dot{u}_t + K u_t = R_t \]  \hspace{1cm} (1)

where \( u, \dot{u}, \) and \( \ddot{u} \) are vectors of nodal point displacements, velocities and accelerations at time "t". \( M \) is the mass matrix assumed to be constant (independent of time). \( K \) is the instantaneous stiffness matrix, i.e., it varies as a function of displacements. \( C \) is the instantaneous damping matrix, and finally \( R \) is the generalized external load vector applied at the nodal points of the system.

A formal mathematical development of the mass matrix \( M \) is possible. Such an approach would result in a mass matrix with the same coupling properties as the stiffness matrix. However, if the physical lumped mass approximation is made, the mass matrix will be diagonal. The lumped mass approximation results in a small reduction in accuracy and considerable saving in computer storage and time. In this investigation one-fourth of the mass of each quadrilateral element is assumed to be concentrated at each of the four nodal points.

For most structures the exact form of the damping matrix \( C \) is unknown. In the solution procedure the damping matrix may be completely arbitrary; however, there is little experimental justification for selecting specific damping coefficients. In the following analysis the damping matrix is ignored. This fact is
specially justified for analysis of earth structures subjected to blast loading. Neglecting the damping matrix results in considerable simplification and saving in the computer time.

At any stage of the analysis, knowing the elastic constants of each element, the instantaneous stiffness matrix may be generated using the same procedure as used in the linear elastic analysis (reference 5). The assemblage of the stiffness matrix of the system is also the same.
STEP-BY-STEP INTEGRATION OF EQUILIBRIUM EQUATIONS

The dynamic equilibrium of the finite element system is given by equation (1). The basic concept of the solution of this set of second order differential equations is explained in references 4 and 5. However, in nonlinear analysis the overshooting problem (figure 1) makes the direct procedure inaccurate. To improve the accuracy of the method, neglecting the damping matrix $C$, equation (1) is written as follows:

$$ M \ddot{u}_t + \sum_{\tau=0}^{\tau=t} K_\tau \Delta u_\tau = R_t $$

where

$$ \Delta u_\tau = u_\tau - u_{\tau-\Delta t}, $$
$$ \Delta u_0 = u_0 $$

and $K_\tau$ is the instantaneous stiffness matrix for the displacement interval $\Delta u_\tau$. The assumption is that $K_\tau$ is constant for the displacement interval $\Delta u_\tau$. The second term of the left hand side of equation (2) may be written as:

$$ \sum_{\tau=0}^{\tau=t} K_\tau \Delta u_\tau = E_t + K_\tau \Delta u_t $$

where

$$ E_t = \sum_{\tau=0}^{\tau=t-\Delta t} K_\tau \Delta u_\tau $$

The force vector $E$ represents the forces carried by the elements of the system at the beginning of the time increment. The formation of
FIG. 1 OVERSHOOTING PROBLEM IN A STEP-BY-STEP PROCEDURE
force vector \( E \) for an axisymmetric finite element system is discussed in Appendix A. Using equation (3) the force equilibrium equation of a finite element system is expressed by:

\[
M \ddot{u}_t + K_t \Delta u_t = R_t - E_t
\]

(5)

This is the basic forces equilibrium equation which is used throughout this investigation.

The time discretization of equation (5) is approximated in the following manner: The acceleration of each point in the system is assumed to vary linearly within a small time interval, \( 2\Delta t \). This assumption leads to a parabolic variation of velocity and a cubic variation of displacement within the time interval \( t-\Delta t \) and \( t+\Delta t \).

A direct integration over the interval gives the following equations for acceleration and velocity at the end of the time interval:

\[
\ddot{u}_{t+\Delta t} = \frac{1.5}{\Delta t^2} \Delta u_{t+\Delta t} - \frac{3}{\Delta t} \dot{u}_{t-\Delta t} - 2 \ddot{u}_{t-\Delta t}
\]

(6)

\[
\dot{u}_{t+\Delta t} = \frac{1.5}{\Delta t} \Delta u_{t+\Delta t} - 2 \dot{u}_{t-\Delta t} - \Delta t \ddot{u}_{t-\Delta t}
\]

The substitution of equation (6) into equation (5), results in a set of linear equations in terms of unknown vector \( \Delta u_{t+\Delta t} \). The solution of this set of equations yields the increment of displacements of the system at time \( t+\Delta t \). The acceleration and velocities at time \( t \) may then be found from the following series of equations:

\[
\ddot{u}_{t+\Delta t} = \frac{1.5}{\Delta t^2} \Delta u_{t+\Delta t} - \frac{3}{\Delta t} \dot{u}_{t-\Delta t} - 2 \ddot{u}_{t-\Delta t}
\]

(7)
\[ \ddot{u}_t = \frac{1}{2} ( \ddot{u}_{t+\Delta t} + \ddot{u}_{t-\Delta t} ) \quad (8) \]

\[ \dot{u}_t = \dot{u}_{t-\Delta t} + \frac{\Delta t}{2} ( \ddot{u}_t + \ddot{u}_{t-\Delta t} ) \quad (9) \]

\[ \ddot{u}_t = \ddot{u}_{t-\Delta t} + \frac{\Delta t}{2} \dot{u}_{t-\Delta t} + \frac{\Delta t^2}{3} \dddot{u}_{t-\Delta t} + \frac{\Delta t^2}{6} \dot{u}_t \quad (10) \]

Equation (8) is the essential factor in making the step-by-step method stable. In fact, it can be shown that all roots of the characteristics equation of the difference equation of the method lie between -1 and 1 for all sizes of time interval \( \Delta t \). This means that regardless of the size of the time step the procedure is stable and the high frequency components do not cause the solution to blow up. However, the new procedure tends to introduce damping in the higher frequencies of the system. Fortunately this partial truncation of the higher modes is justified in many dynamic analyses. The selection of the time step and the finite element idealization for a particular problem will depend on the experience of the user with similar problems.

The step-by-step procedure, which is presented in a form which minimizes computer storage and execution time, is summarized in Table 1. The "effective" stiffness matrix is normally banded and its triangularized form is also banded. Therefore, a large amount of computer storage is not required. Since the stiffness matrix is different for each time step, it should be triangularized at every step. However, for weakly nonlinear problems, where the stiffness matrix might vary slightly, it may not be necessary to triangularize the matrix at each time step.
I. Initial Calculation

A. Calculate the following constants:

\[ \tau = \Delta t \]
\[ a_0 = \frac{3}{\tau} \]
\[ a_1 = \frac{.75}{\tau^2} \]
\[ a_2 = \frac{a_0}{2} \]
\[ a_3 = 2a_1 \]
\[ a_4 = \frac{a_0}{a_3} \]
\[ a_5 = \frac{2}{a_3} \]
\[ a_6 = \frac{\tau}{2} \]
\[ a_7 = \frac{\tau^2}{6} \]
\[ a_8 = 2a_7 \]

B. Form the stiffness matrix \( K \), diagonal mass matrix \( M \), and the internal force vector \( F \).

C. As a starting point, form and triangularize the effective stiffness matrix \( K = K + a_3 F \).

D. Form the revised mass matrix \( \bar{M} = a_3 F \).

II. For Each Time Increment

A. Form the effective load vector

\[ \bar{R}_{t+\Delta t} = R_{t+\Delta t} - F_{t-\Delta t} + \bar{M} (a_4 \ddot{U}_{t-\Delta t} + a_5 \dddot{U}_{t-\Delta t}) \]

B. Back substitute to solve for the displacement vector \( \ddot{U}_{t+\Delta t} \)

\[ Kn\ddot{U}_{t+\Delta t} = \bar{R}_{t+\Delta t} \]

C. Calculate acceleration, velocity, and displacement vectors at time "t":

- 8 -
\[ \dddot{u}_t = a_1 \ddot{u}_{t+\Delta t} - a_2 \dot{u}_{t-\Delta t} + .5 \ddot{u}_{t-\Delta t} \]

\[ \dot{u}_t = \ddot{u}_{t-\Delta t} + a_6 (\ddot{u}_t + \ddot{u}_{t-\Delta t}) \]

\[ \bar{u}_t + \bar{u}_{t-\Delta t} + \gamma \ddot{u}_{t-\Delta t} + a_7 (\ddot{u}_t + 2 \ddot{u}_{t-\Delta t}) \]

D. Calculate strains, stresses, and the internal force vector \( \bar{E}_t \) at time "t".

E. For each desired interval:

Calculate and triangularize the new effective stiffness matrix \( \bar{K} \).

F. Repeat for the next time step.
NONLINEAR MATERIAL BEHAVIOR

Many investigators have worked on nonlinear dynamic analysis of earth structures. Penzien\(^1\) developed a lumped mass, one-dimensional model to study the response of semi-infinite soil layers in which the material had a hysteretic, bilinear stress-strain behavior. Ang\(^2\) investigated a technique to solve nonlinear two and three dimensional dynamic analysis of soil media; however, his technique is cumbersome and impractical. Recently, Dibaj\(^3\) developed a consistent formulation for the nonlinear dynamic analysis of earth structures based on plasticity rules and the finite element method. Dibaj's technique is also inefficient for practical purposes in the sense that to solve a practical problem the computer time and storage are large.

Most computer codes that attempt to account for nonlinear behavior are based on the three dimensional Prager-Drucker yield condition. The behavior of the system is then assumed to be piecewise linear, and the incremental elastic constants are evaluated for each time interval. Based on these constants the tangent stiffness is computed and the response of the system at the end of that interval is obtained. Although the procedure is straightforward, it requires a large amount of computer time. Furthermore, the incremental stress-strain relationship for practical soil materials cannot be accurately obtained from the Prager-Drucker yield condition.

In reference 6, the stress-strain behavior of soils subjected to dynamic load is discussed. Among the prime factors which are important in the nonlinearity of the soil are volumetric change,
hydrostatic pressure, second strain invariant, and shear stress. It is extremely difficult to use this experimental data directly in the computer program. Since, the nonlinear-hysteretic behavior of the bulk modulus appeared to be of major importance, a model was selected to accurately represent this property. Figure 2, shows a pressure volume strain relationship for a typical soil material. The bulk modulus is defined as the ratio of the incremental hydrostatic pressure to the incremental volumetric strain, or

\[ K = \frac{\Delta p}{\Delta e} \]

Note that the bulk modulus is significantly different for loading and unloading. The strain \( e_f \) for a given maximum pressure is the volumetric strain which will cause the soil material to lose its incremental tensile stiffness. Therefore, if the strain, \( e \), is less than \( e_f \) the average volume stress must vanish although the individual stresses may not be zero. In order to use the bulk modulus experimental data it was necessary to assume that the material is incremental isotropic. Or

\[ \Delta \sigma = C \Delta \varepsilon \]

where

\[ C = \begin{bmatrix} K + \frac{4}{3} G & K - \frac{2}{3} G & K - \frac{2}{3} G & 0 \\ K - \frac{2}{3} G & K + \frac{4}{3} G & K - \frac{2}{3} G & 0 \\ K - \frac{2}{3} G & K - \frac{2}{3} G & K + \frac{4}{3} G & 0 \\ 0 & 0 & 0 & 2G \end{bmatrix} \]

(11)
FIG. 2  STRESS - STRAIN DIAGRAM FOR A GRANULAR SOIL SAMPLE
In this formulation the shear modulus may be a function of pressure and may be found experimentally. In the next section of the report an alternate procedure will be given for the determination of the incremental shear modulus.

It is not necessary to express the material properties in mathematical form for the purpose of a numerical analysis. The following sequence of points which describe the stress-strain behavior may be used to define the input to the computer program:

1. Volumetric change
2. Hydrostatic pressure.
3. Unloading bulk modulus.
4. Shear Modulus.
5. Strain at which tensile hydrostatic pressure is initiated in soil.
CONSISTENT FORMULATION OF NONLINEAR PROPERTIES

It is apparent that the nonlinear model discussed in the previous section has many limitations. However, it can be improved by considering in more detail a consistent mathematical formulation of the constitutive equations as suggested by Brown and Froelich (8).

Following reference (8), the internal energy may be written as:

\[ W = W_1 + W_2 \]

where

\[ W_1 = f(\theta_1, S) \quad (12) \]

\[ W_2 = -2 \sqrt{\tau_p} \theta_2 - \frac{\sqrt{\tau_p}}{2G_0} \ln \left( 1 + \frac{2G_0}{\sqrt{\tau_p}} \theta_2 \right) \quad (13) \]

\( \theta_1 \) and \( \theta_2 \) are the first and second strain tensor invariants. \( S \) is entropy density. \( \tau_p \) is the maximum shear stress soil can resist, and \( G_0 \) is the initial shear modulus. The stress tensor might be expressed by (reference 7):

\[ \dot{\sigma} = A I_3 + B (\theta_1 I_3 - \varepsilon) + C \quad \text{of} \ (\varepsilon) \quad (14) \]
where $I_3$ is a 3x3 unit matrix, and

$$\begin{align*}
A &= \frac{\partial w}{\partial \theta_1} = \frac{\partial w_1}{\partial \theta_1}, \quad B = \frac{\partial w}{\partial \theta_2} = \frac{\partial w_2}{\partial \theta_2}
\end{align*}$$

(15)

in this case

$$C = \frac{\partial w}{\partial \theta_3} = \text{constant}$$

It is simple to see that equation 14 may also be written in the form of:

$$\begin{align*}
\sigma &= \left( \frac{1}{\theta_1} \frac{\partial w_1}{\partial \theta_1} + \frac{2}{3} \frac{\partial w_2}{\partial \theta_2} \right) \theta_1 I_3 - \frac{\partial w_2}{\partial \theta_2} \left( \varepsilon - \frac{1}{3} \theta_1 I_3 \right)
\end{align*}$$

(16)

comparing equations (11) and (16), one can conclude that it is possible to define incremental linear elastic constants in such a way that they satisfy nonlinearity, i.e.,

$$K_e = \frac{1}{\theta_1} \frac{\partial w_1}{\partial \theta_1}(\theta_1, s), \quad G_e = \frac{1}{2} \frac{\partial w_2}{\partial \theta_2}(\theta_2, \tau_p, G_0)$$

(17)

To define the tangent moduli for nonlinear analysis, variation of stress tensor should be expressed in terms of variation of strain tensor. Applying $\delta$ operator to equation (16) and neglecting second order terms, equation (16) will change to:

$$\begin{align*}
\delta \sigma &= \left( \frac{\partial^2 w_1}{\partial \theta_1^2} + \frac{2}{3} \frac{\partial w_2}{\partial \theta_2} \right) \delta \theta_1 I_3 - \frac{\partial w_2}{\partial \theta_2} \delta \left( \varepsilon - \frac{1}{3} \theta_1 I_3 \right)
\end{align*}$$

(18)

Comparing equation (18) with the incremental strain stress relation, equation (11).
the tangent moduli are defined as follows:

$$
k = \frac{\partial^2 w_1}{\partial \theta_1^2} = \frac{\Delta \omega_1}{\Delta \theta_1}$$

\[ (19) \]

$$
G = \frac{1}{2} \frac{\partial w_2}{\partial \theta_2}
$$

\[ (20) \]

Using equation (13), shear modulus G may be expressed by:

$$
G = G_0/\left(1 + 2 G_0 \sqrt{\theta_2/\tau_p}\right)
$$

\[ (21) \]

Therefore the incremental shear modulus is not independent, but may be calculated directly; since the experimental determination of the shear limit, \(\tau_p\), is possible.

Equation (17) indicates that the bulk modulus is dependent on entropy; therefore \(S\) must be controlled during experiments in order to obtain the true behavior of \(K\). Presently experimental tests do not give this information. Therefore, one must introduce simplification in order to use the current test results. Referring to Jackson's tests (reference 6), it appears that the maximum value of the first invariant of strain tensor may be considered as a measure of \(S\). From experimental results one may plot hydrostatic pressure vs. volumetric change (figure 2), and express a bulk modulus by:

$$
K = K(p, e_{max}, \dot{e}, e) = \Delta \left( \frac{\partial w_1}{\partial \theta_1} \right) / \Delta \theta_1
$$

\[ (22) \]
Where $\dot{e}$ is the rate of change of $e$ with respect to $p$, and

$$p = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33})$$  \hspace{1cm} (23)$$

$$\epsilon = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$$  \hspace{1cm} (24)$$

in which $\sigma_{11}$, $\sigma_{22}$, and $\sigma_{33}$ are principle stresses, and $\epsilon_{11}$, $\epsilon_{22}$, and $\epsilon_{33}$ are principle strains.

Equation (22) implies that the loading and unloading bulk moduli are different (figure 2). This is an indication of the hysteretic behavior of the material and is accurately represented in the model.
The validity of the finite element method as applied to the nonlinear dynamic analysis of axisymmetric systems is demonstrated by two examples.

The method of analysis is compared with an experimental study conducted by Jackson (reference 6). The model was a confined soil cylinder subjected to a blast load uniformly distributed on the model. Figure 3 shows the model, the finite element idealization, and the time variation of the blast load. The pressure-volume change of material of this same reference is constructed using a constant Poisson's ratio and is shown in figure 4. Vertical strain of the model was measured and plotted vs. time. A good agreement between the experimental and the finite element results is observed (figure 5). Specially, the permanent set, which linear analysis does not exhibit, is pronounced by the nonlinear analysis. The slight discrepancy is due to inexactness of the information adopted from reference 6.

In another analysis, the results of an elastic and a nonlinear finite element analysis were compared with an experimental study of a structure buried in a soil material. This experimental study was conducted in the blast load simulator at Vicksburg, Mississippi. The stress-strain diagram used in the nonlinear analysis is shown in figure 6. Figure 7 shows the finite element idealization of the model and the time variation of the blast pressure which is applied on the model. A listing of the input data for this structure is given in Appendix D. In figure 8 the displacements at a point in the soil are
plotted. The experimental results indicate a permanent set in the material, whereas, the displacements from the elastic analysis return to zero. The results of the nonlinear analysis are also plotted on the same diagram, and are in good agreement with the experiment.
FIG. 3a  SOIL SAMPLE

FIG. 3b  FINITE ELEMENT IDEAL

FIG. 3c  TIME VARIATION OF BLAST PRESSURE

PRESSURE (PSI)

TIME (SEC.)
FIG. 4 STRESS-STRAIN BEHAVIOR OF A SOIL MATERIAL
FIG. 5 TIME VARIATION OF VOLUMETRIC STRAIN
FIG. 6  STRESS·STRAIN CURVE (VICKSBURG'S TEST)
FIG. 7a  TIME VARIATION OF PRESSURE

FIG. 7b  FINITE ELEMENT IDEALIZATION
FIG. 8 DISPLACEMENT VS. TIME IN SOIL MATERIAL
A Fortran IV listing of the computer program for the nonlinear dynamic analysis of axisymmetric structures is given in Appendix C. The program utilizes axisymmetric elements with quadrilateral cross-sections. The capacity of the program will depend on the storage of the computer used.

Within the program a method of dynamic storage allocation is used, therefore, for a given problem all required data is compressed into the smallest possible storage area. This also allows the capacity of the program to be increased or decreased by only changing one number within the program.

The operation of the program may be summarized by the following steps:

First.
Control information, material properties nodal point geometry and element data are read (or generated) by the computer.

Second.
For each element, an 8x8 incremental stiffness matrix and element mass matrix are formed. These are then added to the total stiffness and mass matrices of the system.

Third.
The step-by-step solution, as summarized in Table 1, is used to evaluate the displacements as a function of time. At
At specified time points displacement and stresses are printed. Also, at a different time interval new incremental element stiffnesses may be calculated.
REFERENCES


APPENDIX A

INTERNAL FORCE VECTOR FOR A QUADRILATERAL ELEMENT
APPENDIX A.

INTERNAL FORCE VECTOR FOR A QUADRILATERAL ELEMENT

INTRODUCTION

The purpose of this section is to present the development of the internal force vector for a quadrilateral element. Expression of virtual work is.

\[ W = \int_{vol} [\varepsilon]^T [\sigma] \, dV \]  
(A-1)

the same expression in terms of the displacements of discrete nodal points is.

\[ W = [d]^T [E] \]  
(A-2)

where

\[
\varepsilon = \begin{pmatrix}
\varepsilon_{rr} \\
\varepsilon_{zz} \\
\varepsilon_{\theta\theta} \\
\gamma_{rz}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial u}{\partial r} \\
\frac{\partial v}{\partial z} \\
u/r \\
\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}
\end{pmatrix},
\sigma = \begin{pmatrix}
\sigma_{rr} \\
\sigma_{zz} \\
\sigma_{\theta\theta} \\
\tau_{rz}
\end{pmatrix}
\]

\[ [d]^T = [u_1 \, v_1 \, u_2 \, v_2 \, u_3 \, v_3 \, u_4 \, v_4] \]

The strain vector can be related to the displacements at the nodal points by the operator \([B]\)

\[ [\varepsilon] = [B] [d] \]  
(A-3)
Substituting equation (A-3) into equation (A-1) and comparing the result with equation (A-2) we can write,

\[ [E] = \int_{\text{vol}} [B]^T [\sigma] \, dV \]

Or for constant thickness \( t \),

\[ [E] = t \int_{a} [B]^T [\sigma] \, dA \quad (A-4) \]

COORDINATE SYSTEMS

The coordinates \((R,Z)\) are cartesian while the natural coordinates \((S,T)\) may be skewed and are defined such that \( S \) and \( T \) vary from \(-1\) to \(1\), as shown in figure A-1. The \((R,Z)\) coordinates are given in terms of \((S,T)\) natural coordinates by the following interpolating functions:

\[
\begin{align*}
  r(s,t) &= \sum_{i=1}^{4} h_i r_i \\
  Z(s,t) &= \sum_{i=1}^{4} h_i Z_i \\
  h_1 &= (1-s)(1-t)/4 \\
  h_2 &= (1+s)(1-t)/4 \\
  h_3 &= (1+s)(1+t)/4 \\
  h_4 &= (1-s)(1+t)/4 
\end{align*}
\]

(A-5)

Since strains are defined by derivatives with respect to \((R,Z)\) and the displacement expansions are given in the \((S,T)\) system, the chain rule for differentiation must be used to calculate

\[
\frac{\partial s}{\partial r}, \frac{\partial s}{\partial z}, \frac{\partial t}{\partial r}, \text{ and } \frac{\partial t}{\partial z}
\]
Inverting the chain rule,
\[
\begin{pmatrix}
\frac{\partial a}{\partial s} \\
\frac{\partial a}{\partial t}
\end{pmatrix}
= \frac{1}{J} \begin{pmatrix}
\frac{\partial a}{\partial r} & \frac{\partial a}{\partial z} \\
\frac{\partial a}{\partial t} & \frac{\partial a}{\partial s}
\end{pmatrix}
\frac{1}{J}
\begin{pmatrix}
\frac{\partial r}{\partial s} & \frac{\partial z}{\partial s} \\
\frac{\partial t}{\partial s} & \frac{\partial z}{\partial t}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial a}{\partial r} \\
\frac{\partial a}{\partial z}
\end{pmatrix}
\]
Gives
\[
\begin{pmatrix}
\frac{\partial a}{\partial r} \\
\frac{\partial a}{\partial z}
\end{pmatrix}
= \frac{1}{J} \begin{pmatrix}
\frac{\partial a}{\partial r} & -\frac{\partial a}{\partial z} \\
-\frac{\partial a}{\partial t} & \frac{\partial a}{\partial s}
\end{pmatrix}
\frac{1}{J}
\begin{pmatrix}
\frac{\partial r}{\partial s} & \frac{\partial z}{\partial s} \\
\frac{\partial t}{\partial s} & \frac{\partial z}{\partial t}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial a}{\partial r} \\
\frac{\partial a}{\partial z}
\end{pmatrix}
\]
Where
\[
J = J(s,t) = \frac{\partial r}{\partial s} \frac{\partial z}{\partial t} - \frac{\partial r}{\partial t} \frac{\partial z}{\partial s}
\]
Since \(dA = J \, ds \, dt\) Equation (A-4) may be rewritten as.
\[
\begin{bmatrix}
 E
\end{bmatrix} = t \int_A \begin{bmatrix}
 B
\end{bmatrix}^T \begin{bmatrix}
 a
\end{bmatrix} \, J \, ds \, dt \quad (A-6)
\]

STRAIN DISPLACEMENT TRANSFORMATION [B]

Let nodal point values of the displacement \( u \) and \( v \) be given by
\[
\begin{align*}
[u]^T &= [u_1, u_2, u_3, u_4] \\
[v]^T &= [v_1, v_2, v_3, v_4]
\end{align*}
\]
The assumed displacement expansion uses the same interpolation functions as appeared in Eqs. A-5, i.e.
\[
\begin{align*}
    u(s,t) &= \sum_{i=1}^{4} h_i \, u_i \\
    v(s,t) &= \sum_{i=1}^{4} h_i \, v_i
\end{align*}
\]
The \( e_{\theta \theta} \) strain is given by
\[
e_{\theta \theta} = \sum_{i=1}^{4} \frac{h_i \, u_i}{r_i} = \sum_{i=1}^{4} G_i \, u_i
\]

- A 3 -
\( \varepsilon_{rr} \) may be obtained by differentiation

\[
\varepsilon_{rr} = \frac{\partial u}{\partial r} = \frac{\partial u}{\partial s} \frac{\partial s}{\partial r} + \frac{\partial u}{\partial t} \frac{\partial t}{\partial r}
\]

\[
= \frac{1}{J} \sum_{i=1}^{4} \sum_{j=1}^{4} u_i \left( \frac{\partial h_i}{\partial s} \frac{\partial h_j}{\partial t} - \frac{\partial h_i}{\partial t} \frac{\partial h_j}{\partial s} \right) z_j
\]

\[
= [u_j]^T [p] [z]/J.
\]

Where

\[
[p] = \sum_{i=1}^{4} \sum_{j=1}^{4} \left( \frac{\partial h_i}{\partial s} \frac{\partial h_j}{\partial t} - \frac{\partial h_i}{\partial t} \frac{\partial h_j}{\partial s} \right) = \frac{1}{8} \begin{bmatrix}
0 & 1-t & -s+t & -1+s \\
0 & 1+s & -S-t & 0 \\
0 & 1+t & & \\
\text{skew-symmetric} & 0 \\
\end{bmatrix}
\]

Similarly

\[
\varepsilon_{zz} = \frac{\partial v}{\partial z} = -[v] \frac{[p] [r]/J}{J}
\]

\[
\gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} = \frac{-[u] \frac{T}{[p] [r]} + [v] \frac{T}{[p] [z]}}{J}
\]

Let

\[
[p] [z] = [y] = \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{bmatrix} = \frac{1}{8J} \begin{bmatrix}
z_{24} - z_{34} & s & -z_{23} & t \\
-z_{13} + z_{34} & s & +z_{14} & t \\
-z_{24} + z_{12} & s & -z_{14} & t \\
z_{13} - z_{12} & s & +z_{23} & t
\end{bmatrix}
\]

And

\[
-[p] [r] = [x] = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} = \frac{1}{8J} \begin{bmatrix}
-r_{24} + r_{34} & s & +r_{23} & t \\
& & \text{and so on}
\end{bmatrix}
\]

- A 4 -
Then

\[
\begin{pmatrix}
\varepsilon_{rr} \\
\varepsilon_{zz} \\
\varepsilon_{\theta\theta} \\
\gamma_{rz}
\end{pmatrix}
= \begin{pmatrix}
y_1 & 0 & y_2 & 0 & y_3 & 0 & y_4 & 0 \\
0 & x_1 & 0 & x_2 & 0 & x_3 & 0 & x_4 \\
g_1 & 0 & g_2 & 0 & g_3 & 0 & g_4 & 0 \\
x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4
\end{pmatrix}
\begin{pmatrix}
u_1 \\
v_2 \\
v_3 \\
v_4
\end{pmatrix}
\]

or

\[ [\varepsilon] = [B] [d] \]

[B] is the strain displacement relationship required in Eq. A-6 to evaluate the internal force vector of the element.

COMPUTATION OF [E]

One point integration is used to evaluate the integral of Eq. A-6. Therefore Eq. A-6 is reduced to

\[ [E] = [B]^T \bigg|_{t=0} \ast [\sigma] \ast \text{volume} \bigg|_{s=0} \]

Where

\[ [B]^T \bigg|_{t=0} \text{ is the value of } [B]^T \text{ at point } t=0, s=0 \]

\[ [\sigma] \text{ is the stress vector at point } t=0, s=0 \]

and the volume is the total volume of the element.
APPENDIX B

DESCRIPTION OF INPUT DATA FOR COMPUTER PROGRAM
APPENDIX B

DESCRIPTION OF INPUT DATA FOR COMPUTER PROGRAM

The purpose of this computer program is to determine time-dependent displacements and stresses within elastic axisymmetric structures of arbitrary shape and materials. In order to define the computer input a two-dimensional cross-section of the axisymmetric structure must be idealized by a system of finite elements. Quadrilateral, triangular and one-dimensional membrane elements can be used. Elements in the system are identified by a sequence of numbers starting with one. Also, all nodal points are identified by a separate numbering sequence. The reference coordinate system to be used and a simple finite element representation of a structure is shown in Figure B-1.

The following sequence of punched cards numerically define the axisymmetric structure to be analyzed.

A. IDENTIFICATION CARD. (72 H)

Columns 1 to 72 contain information to be printed with results.

B. CONTROL CARD. (715, 4F10.0)

Columns 1 - 5 Number of nodal points (n)

6 - 10 Number of elements (k)

11 - 15 Number of different materials (m)

16 - 20 Number of time steps

21 - 25 Number of time increments between the print displacements and stresses
FIG. B-1 REFERENCE COORDINATE SYSTEM

FIG. B-2 PRESSURE BOUNDARY CONDITIONS
26 - 30 Number of load cards (L)
31 - 35 Number of boundary pressure cards (p)
36 - 40 Number of increments between the change of stiffness
41 - 50 Damping coefficient α
51 - 60 Damping coefficient β
61 - 70 Time increment Δt
71 - 80 Reference number to be added to all R ordinates

C. MATERIAL PROPERTY INFORMATION.

The following card must be supplied for each different material (I5,2F15.0)

Columns 1 - 5 Material identification number
5 - 20 Mass density of material
21 - 35 Thickness (for membrane shell elements)

D. STRESS-STRAIN INFORMATION.

To describe the stress-strain behavior of each material a sequence of six points are used. Therefore, Six cards with the following informations must be supplied for each different material. 6(5F10.0)

Columns 1 - 10 Volumetric change (ε_r + ε_z + ε_θ)
11 - 20 Average Stress [(σ_r + σ_z + σ_θ)/3.]
21 - 30 Unloading bulk modulus
31 - 40 Shear modulus
41 - 50 Ratio of permanent strain and the maximum strain

(ε_r/ε_max)

- B 3 -
The one-dimensional shell elements are restricted to linear materials. The material properties of the shell elements are also specified by the same input format - the unloading bulk modulus is defined as the modulus of elasticity and the shear modulus is defined as Poisson's ratio. The other information is not required.

D. NODAL POINT CARDS, (I5, F5.0, 2F10.0)

One card is required for each nodal point with the following information:

Columns 1 - 5 Nodal point number
         6 - 10 Boundary condition code "k"
         11 - 20 R-ordinate
         21 - 30 Z-ordinate

Specifications for code "k". If

  k = 0  load in the R-direction
         load in the Z-direction

  k = 1  zero displacement in the R-direction
         load in the Z-direction

  k = 2  load in the R-direction
         zero displacement in the Z-direction

  k = 3  zero displacement in the R-direction
         zero displacement in the Z-direction

Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated in equal intervals along a straight line between the defined nodal points. The boundary condition code is set equal to zero.
E. ELEMENT CARDS. (6I5)

Columns 1 - 5 Element number
6 - 10 Nodal point I
11 - 15 Nodal point J
16 - 20 Nodal point K
21 - 25 Nodal point L
26 - 30 Material Identification

The maximum difference "b" between these numbers is an indication of the band width. The execution time for the program will be proportional to this number squared.

For a right hand coordinate system the nodal point numbers I, J, K and L must be in sequence in a counter-clockwise direction around the element. Element cards must be in element number sequence. If element cards are omitted the program automatically generates the omitted information by incrementing by one the preceding I, J, K and L. The material identification for the generated cards is set equal to the corresponding value on the last card. The last element card must always be supplied. Triangular elements are also permissible; they are identified by repeating the last nodal point number (i.e. I, J, K, K). One dimensional membrane elements are identified by a nodal point numbering sequence of the form I, J, J, I.

F. PRESSURE CARDS (2I5, 3F10.0)

One card for each boundary element which is subjected to a normal pressure.

Columns 1 - 5 Nodal point I
6 - 10 Nodal point J
11 - 20 Pressure multiplier $P_i$
21 - 30 Pressure multiplier $P_j$
31 - 40 Arrival time of pressure at the center of the surface element

As shown in Figure B-2 the boundary element must be on the left as one progresses from I to J. Surface tensile force is input as a negative pressure.

G. LOAD CARDS (2F10.0)

These cards specify the normal pressure as a function of time in the form of straight line segments. One card is required for each point with the following information:

Columns 1 - 10 Time $t$
11 - 20 Normal pressure $p(t)$

OUTPUT INFORMATION

The following information is developed and printed by the program:

1. Reprint of input data
2. Pressure boundary conditions
3. Nodal point displacements, velocities and accelerations as a function of time
4. Stresses at the center of each element as a function of time
PROGRAM LIMITATIONS

The capacity of the program is limited by the dimension "d" of the "A" array in program DYNS.

\[4n (b + 1) + 18n + 2\ell + 7p + 14k + 32m\] must not be greater than d. The symbols n, \(\ell\), p and b have been defined previously and their values will depend on the particular structure to be analyzed. The maximum size which d can have will depend on the particular computer being utilized. For a computer with 32K storage the maximum value for d will be approximately 20000.
APPENDIX C

FORTRAN IV LISTING OF THE COMPUTER PROGRAM
PROGRAM DYN5\( INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT \)
COMMON NUMNP, MBAND, NT, NPKINT, NP, NUMPC, NUMMAT, RA, TT, DELT,
1 NST, ALFA, BETA, HED(12), A(33200)

C--------
C READ AND PRINT OF CONTROL INFORMATION
C--------
50 READ (6, 1000) HED, NUMNP, NUMEL, NUMMAT, NT, NPKINT, NP, NUMPC, NST, ALFA,
1 BETA, DELT, RA
WRITE (6, 2000) HED, NUMNP, NUMEL, NUMMAT, NT, NPKINT, NP, NUMPC, ALFA,
1 BETA, DELT, NST

C--------
C READ AND PRINT OF DATA
C--------
NEQ = 2 * NUMNP
N2 = 1 + NUMNP
N3 = N2 + NUMNP
N4 = N3 + NUMNP
N5 = N4 + 5 * NUMEL
N6 = N5 + NUMMAT
N7 = N6 + 30 * NUMMAT
N8 = N7 + NUMPC
N9 = N8 + NUMPC
N10 = N9 + NUMPC
N11 = N10 + NUMPC
N12 = N11 + NUMPC
N13 = N12 + NUMPC
N14 = N13 + NUMPC
N15 = N14 + NUMMAT
100 CALL DATAIN(A(1), A(N2), A(N3), A(N4), A(N5), A(N6), A(N7), A(N8), A(N9),
1 A(N10), A(N11), A(N12), A(N13), A(N14), A(N15))

C--------
C FORM TOTAL STIFFNESS AND MASS MATRICES AND SOLVE STEP-BY-STEP
C--------
N16 = N15 + 2 * NP
N17 = N16 + NUMNP
N18 = N17 + NEQ
N19 = N18 + NEQ
N20 = N19 + NEQ
N21 = N20 + NEQ
N22 = N21 + NEQ
N23 = N22 + 5 * NUMEL
N24 = N23 + 4 * NUMEL
N25 = N24 + NEQ * MBAND
IF (N25 .LE. 33200) GO TO 200
WRITE (6, 1100)
STOP
200 DO 300 I = N16, N25
300 A(I) = 0.
CALL SOLVE(A(1), A(N2), A(N3), A(N4), A(N5), A(N6), A(N7), A(N8), A(N9),
1 A(N10), A(N11), A(N12), A(N13), A(N14), A(N15), A(N16), A(N17), A(N18),
2 A(N19), A(N20), A(N21), A(N22), A(N23), A(N24), NEQ)
GO TO 50

C--------
1000 FORMAT (12A6/B15.4F/15.4F/15.4F)
1100 FORMAT (25A5 DIMENSION OF A EXCEEDED)
2000 FORMAT (1H1 I12A6/
1 30H0 NUMBER OF NODAL POINTS------- I4 /
2 3OH0 NUMBER OF ELEMENTS-------- I4 /
3 3OH0 NUMBER OF DIFF. MATERIALS--- I4 /
4 3OH0 NUMBER OF TIME INCREMENTS--- I4 /
5 3OH0 PRINT INTERVAL--------------- I4 /
6 3OH0 NUMBER OF LOAD POINTS------ I4 /
7 3OH0 NUMBER OF PRESSURE CARDS---- I4 /
8 3OH0 DAMPING COEFFICIENT ALFA---- F10.5 /
9 3OH0 DAMPING COEFFICIENT BETA---- F10.5 /
0 3OH0 TIME INCREMENT-------------- F10.5/
1 3OH0 STIFFNESS CHANGE INTERVAL--- I4 )
END
SUBROUTINE STRESS (SIG, PRES, E, VOL, EE, MTYPE)
COMMON / LS4ARG/ LM(8), SS(4,8), XC, YC, S(0,8), C(4,4)
DIMENSION SIG(4), E(1), EE(5,6,1)

C-------
C VOLUME * STRESS. TO GET STRESSES ONE SHOULD DEVIDE BY THE VOL.
C THIS IS DONE AT THE END OF THE SUBROUTINE
C-------
C SHELL ELEMENT
C-------
DS=S(1,1)+S(2,1)+S(3,1)
IF (LM(4).NE.LM(6)) GO TO 300
ENU=EE(4,1,MTYPE)
E1=EE(3,1,MTYPE), (1.E-ENU**2)
C(1,1)=E1
C(2,2)=E1
C(1,2)=C(1,2)
C(2,1)=C(2,1)
SIG(1)=(SIG(1)+C(1,1)*S(1,1)+S(2,1)*C(1,2))*VOL
SIG(2)=(SIG(2)+C(2,1)*S(1,1)+C(2,2)*S(2,1))*VOL
GO TO 100
300 IF (C(1,1).NE.0.) GO TO 600
DO 550 I=1,4
550 SIG(I)=0.
RETURN
600 SIGP=(SIG(1)+SIG(2)+SIG(3))/3.
B=C(1,2)+.66666666666666*C(4,4)
PR =-PRES-SIGP
ST=ABS(PR/(B*DS))
DO 400 I=1,3
S(I,1)=S(I,1)*ST
DO 400 J=1,3
400 S(I,J+1)=C(I,J)-B
DO 500 I=1,3
500 SIG(I)=(SIG(I)+PK +S(I,2)*S(1,1)+S(I,3)*S(2,1)+S(I,4)*S(3,1))*VOL
SIG(4)=(SIG(4)+C(4,4)*S(4,1)*ST)*VOL
C-----INTERNAL FORCE
100 DO 200 I=1,4
DO 180 J=1,8
JJ=LM(J)
180 E(JJ)=E(JJ)+SS(I,J)*SIG(I)
700 SIG(I)=SIG(I)/VOL
200 CONTINUE
320 RETURN
END
SUBROUTINE SIGEPS (STRAIN, EE, MTYPE, C, PRES, EPSP)
DIMENSION EE(2,6,1), E(4), STRAIN(5, C(4,4))

C-------
C------- EPSM(1) IS THE LAST MAX. STRAIN OF THE UNLOADING BRANCH.
C-------
EPSP = STRAIN(1) + STRAIN(2) + STRAIN(3)
    IF (EPSP .LE. EPSM) GO TO 90
C------- COMPLETE CRACK
    PRES = 0.
    DO 80 I = 1, 4
    DO 80 J = 1, 4
    80 C(I, J) = 0.
    RETURN
90 EPSM = STRAIN(5)
    EPSP = EPSM
    IF (EPSP .GT. EPSM) EPSM = EPSP
    EPSA = EPSM
    DO 100 J = 2, 6
    IF (EPSA .LT. EE(I,I,MTYPE)) GO TO 200
100 CONTINUE
C------- FIND THE LOADING BULK OF THE CORNER POINT.
C 200 DE = EE(I,I,MTYPE) - EE(I-1,I,MTYPE)
    B = (EE(2,I,MTYPE) - EE(2,I-1,MTYPE))/DE
C------- FIND INFORMATION OF THE CORNER POINT.
C    R = (EPSA - EE(I,I,MTYPE))/DE
    DO 300 J = 1, 4
    300 E(J) = EE(J+1,I-1,MTYPE) + R*(EE(J+1,I,MTYPE) - EE(J, I-1,MTYPE))
    G = E(3)
    PRES = E(1)
    R = B
C------- CHECK FOR THE UNLOADING BRANCH.
C    IF (EPSP .GE. EPSM) GO TO 400
C------- ASSUME A BILINEAR BEHAVIOR FOR THE UNLOADING BRANCH
    EF = E(4) * EPSM
    EB = E(1) / (E(2) + E(2))
    IF (EF .GT. EPSP) GO TO 6
    ECON = EPSM - EB
    IF (EPSP .LT. ECON) GO TO 5
    B = E(2)
    PRES = E(1) - E(2) * (EPSM - EPSP)
    GO TO 7
5    B = E(1) / (2 * (EPSM - EF - EB))
    PRES = B * (EPSP - EF)
    GO TO 7
6    B = EPSP / EF * E(1) / (2 * (EPSM - EF - EB))
    PRES = 0.
    7 G = B * E(3) / R
400 CONTINUE
    STRAIN(5) = EPSM
C------- MATERIAL MATRIX.
C    C(4,4) = G
    G = (G + G) / 3.

-C 4-
C(1*1) = B + G + G
C(1*2) = B - G
C(1*3) = C(1*2)
C(2*1) = C(1*2)
C(2*2) = C(1*1)
C(2*3) = C(1*2)
C(3*1) = C(1*3)
C(3*2) = C(2*3)
C(3*3) = C(1*1)

RETURN
END
SUBROUTINE SOLVER(XZ, CODE, IX, KU, EE, HI, VI, VJ, T, INI, JNJ, H, P, 
 1 MASS, X0, X1, X2, B, EPS, SIG, A, NEQ) 
  COMMON NUMP, MBAND, NT, NPRINT, NP, NUMPC, NUMEL, NUMMAT, RA, TT, DELT, 
 1 NST, ALFA, BETA, 
  DIMENSION K(I), C(I), CODE(I), IX(I), KU(I), EE(I), HI(I), 
 1 X(I), VJ(I), T(I), INI(I), JNJ(I), P(I), MASS(I), X0(I), 
 2 X1(I), X2(I), B(I), A(NEQ), EPS(I), E(I), SIG(I), H(I), ELMAS(4) 
  COMMON /LS4ARK/ LM(8), SS(4, 8), XC, YC, S(8, 8), C(4, 4) 
  REAL MASS 

C ------
C INITIALIZATION
C -----
DO 40 I = 1, 4
DO 50 J = 1, 8
50 SS(I, J) = 0.
DO 40 J = 1, 4
40 C(I, J) = 0.

VOL = 0.

C CONSTANTS FOR THE STEP-BY-STEP SOLUTION
C -----
A1 = 3./DELT
A2 = 75./DELT**2
A3 = A1/2.
A4 = A1/A0
A5 = 2.*A2
A6 = DELT/2.
A7 = DELT**2/26.
A8 = A7 + A7

C FORM STIFFNESS AND MASS MATRIX OF THE SYSTEM
DO 375 N = 1, NUMEL
DO 199 I = 1, 4
J = I + 1
II = IX(I, N) + IX(I, N)
LM(II) = II
199 LM(J - 1) = II - 1

C FINE THE ELASTIC CONSTANTS.
C ------
I = IX(1, N)
J = IX(2, N)
K = IX(3, N)
L = IX(4, N)
MTYPE = IX(5, N)
IF(J, NE, K) CALL &IGEPS(EPS(I, N), EE, MTYPE, C, PRES, EPS)

C FORM ELEMENT STIFFNESS MATRICES
C -------
CALL STIFF(K, Z, CODE, IX(1, N), EE, A, NEQ, KU, MTYPE, VOL, H)
IF(J, NE, K) GO TO 444
RRR = VOL*KU(MTYPE)/4.
ELMA(1) = RRR
ELMA(2) = RRR
ELMA(3) = RRR
ELMA(4) = RRR
GO TO 454

- C 6 -
444 RM = B * XC
R12 = R(I) - R(J)
R13 = R(I) - R(K)
R14 = R(I) - R(L)
R23 = R(J) - R(K)
R24 = R(J) - R(L)
R34 = R(K) - R(L)
Z12 = Z(I) - Z(J)
Z13 = Z(I) - Z(K)
Z14 = Z(I) - Z(L)
Z23 = Z(J) - Z(K)
Z24 = Z(J) - Z(L)
Z34 = Z(K) - Z(L)
ROM = RM = TYPE / 72.
BR = (R34*Z12 - Z34*RB2)*ROM
AR = (VOL + VOL) / AC * ROM
CR = (R23*Z14 - Z23*R14)*ROM
ELMATT1 = AK*(KM + R13 + R(I)) - BK*(R(I) + R(I) + K(L))
CR*(R(I) + R(I) + K(J))
ELMATT2 = AK*(NM + R24 + R(J)) + BK*(R(J) + R(J) + K(R))
CR*(R(J) + R(J) + K(J))
ELMATT3 = AK*(NM + R(K) - R13) + BK*(R(J) + R(K) + K(R))
CR*(R(K) + R(K) + R(K))
ELMATT4 = AK*(RM + R(L) - R24) - BK*(R(K) + R(L) + R(L))
CR*(R(K) + R(L) + R(L))

454 CONTINUE
DO 350 I = 1, 4
II = IX(I*N)
350 MASS(II) = MASS(II) + ELMATT(I)
375 CONTINUE
C-------- INITIAL ACCELERATION
C-------- FORM THE EFFECTIVE STIFFNESS OF THE SYSTEM
II = 0
DO 400 I = 1, NEQ
II = I - II
IF(A(I*1).EQ.U*) GO TO 400
X2(I) = B(I) / MASS(I)
II = I*1
400 CONTINUE
C--------REVISE THE MASS MATRIX FOR SUBSEQUENT USE
DO 401 II = 1, N*MNP
401 MASS(II) = MASS(II) * AU
C--------INITIAL TRIANGULARIZATION
CALL TRIA (NEQ, MBAND, A)
C-------- STEP-BY-STEP SOLUTION
C--------
MM = NST - 1) * NUMEL
KK = 0
LL = 0
DO 500 NNN = 1, NT
TT = TT + DELT
CALL LOAD (T*P, B*INI, JNJ, HI, HJ, VI, VJ, IK)
II = 0
C-------- EFFECTIVE LOAD VECTOR
DO 460 I = 1, NEQ
II=I-II
B(I)=B(I)-E(I)+MASS(I)*(A4*X1(I)+A5*X2(I))
IF(A(I+1)*EQ.0.) B(I)=0.0
CONTINUE

C SOLUTION AT END OF TIME STEP
C
CALL BACKS (NEQ*MBAND*A*B)
DO 480 I=1*NEQ
E(I)=U
ACC=A2*B(I)-A3*X1(I)-5*X2(I)
B(I)=DELTA*X1(I)+A7*(ACC+X2(I))+X2(I))
X0(I)=XU(I)+B(I)
X1(I)=X1(I)+A6*X2(I)+ACC
480 X2(I)=ACC
PRINT DISPRELACEMENT AND PREPARATION OF A NEW STIFFNESS METRIX
C----
LL=LL+1
IF (LL.NE.NPRINT) GO TO 499
LL=O
421 WRITE (6,2006) TT
DO 482 N=1*NUMNP
M=N+N
K=M-1
482 WRITE (6,2008) N,X0(K),X0(M),X1(K),X1(M),X2(K),X2(M),N
499 CONTINUE
MPRINT =O
DO 498 N=1*NUMEL
KK=KK+1
C----
C COMPUTE STRAIN, ELASTIC CONSTANT, STRESSES, INTERNAL FORCE, AND
C STIFFNESS
C----
CALL STRAIN (B*K,L,X(I,N),EPS(1,N),MTYPE,VOL,H)
IF (IX(2,N).NE.IX(3,N))
1CALL SIGEPS(EPS(1,N),EE,MTYPE,C, PRES,EPSP)
CALL STRESS (SIG(1,N),PRES,E,VOL,EE,MTYPE)
C
IF (KK.LE.MM) GO TO 170
IF (KK.GT.MM+1) GO TO 424
II=0
DO 425 I=1*NEQ
II=I-II
IF (A(I+1)*NE.0.) A(I+1)=MASS(I)
DO 425 J=2*MBAND
425 A(I+J)=0.
424 CALL STIFF (K*CODE,X(I,N),EE,A,NEW,U,MTYPE,VOL,H)
IF (N.NE.NUMEL) GO TO 170
KK=O
CALL TRIA (NEW*MBAND,A)
170 IF (LL.NE.0) GO TO 498
C------
C------CALCULATE THE PRINCIPLE STRESSES.
C------
CC=(SIG(1,N)+SIG(2,N))/2.
BB=CC-SIG(2,N)
CR=SQR(T(BB**2+SIG(4,N)**2))
SIGMAX=CC+CR
SIGMIN=CC-CR
IF(CR) 200,255,200
200 ANGLE=28.648*ATAN2(SIG(4,N),BB)
255 IF( MPRINT) 11U+105,110
105 WRITE (6,2000)
   MPRINT =50
110 MPRINT =MPRINT-1
305 WRITE (6,2001) N,XC,YC,(SIG(I,N),I=1,4)*SIGMAX,SIGMIN,ANGLE
C 498 CONTINUE
500 CONTINUE
RETURN
C---- --
2000 FORMAT (6H1EL.NU,7X,1HR,7X,1HZ,7X,5H$IG-R,7X,5H$IG-Z,7X
1 5H$IG-T,6X,6HTAU-KZ,5X,7H$IG-MAX,5X,7H$IG-MIN,7H ANGLE
22X 11HAVE, PRESS, 2X 11HVOL, CHANGE. )
2001 FORMAT (5*I5,1X,2F8.2,6E12.4,F6.2,2E13.4)
2006 FORMAT (6H1TIME T=Fl0.6/118 HONODAL POINT X-DISPLACEMENT Y-DISPLA
1CEMENT X-VELOCITY Y-VELOCITY X-ACCELERATION Y-ACCELERATI
20N NODAL POINT )
2007 FORMAT (F14.0)
2008 FORMAT (19*6E16.4,I9)
END
SUBROUTINE DATAIN(RZ,CODEIX,RU,EE,HJ,VI,VJ,T,INI,JNJ,H,P)
COMMON NUMNP,MABAND,NUMPRINT,NUMPC,NUMEL,NUMMAT,RA,TT
DIMENSION R(1),Z(1),CODE(1),IX(5),RU(1),EE(5),H(1)
1 HJ(1),VI(1),VJ(1),T(1),INI(1),JNJ(1),P(2),IE(5),H(1)
C-------
C READ AND PRINT OF MATERIAL PROPERTIES
C-------
59 M=1*NUMMAT
READ (5,1001) MTYPE,RO(1),H(1)
1 (EE(I,J),MTYPE,I=1,5),J=1,6)
WRITE (6,2000) MTYPE,RO(1),H(1)
59 WRITE (6,2012) (EE(I,J),MTYPE,I=1,5),J=1,6)
C-------
C READ AND PRINT OF NODAL POINT DATA
C-------
WRITE (6,2004)
L=U
60 READ (5,1002) N,CODE(N),R(N),Z(N)
R(N)=R(N)+RA
IF (L.EQ.U) GO TO 85
Z(N)=Z(N)-L
DR=(R(N)-R(L))/ZX
DZ=(Z(N)-Z(L))/ZX
85 NL=L+1
70 L=L+1
IF(N-L) 100,90,60
80 CODE(L)=0
R(L)=R(L-1)+DR
Z(L)=Z(L-1)+DZ
GO TO 70
90 WRITE (6,2002) (K,CODE(K),R(K),Z(K),K=NL,N)
IF(NUMNP-N) 110,160,60
100 WRITE (6,2009) N
STOP
110 CONTINUE
C-------
C READ AND PRINT OF ELEMENT NODES
C-------
WRITE (6,2001)
N=U
MBAND=U
130 READ (5,1003) M,(IE(I),I=1,5)
140 N=N+1
MB=U
DO 160 I=1+4
C------
C DETERMINATION OF BAND WITH
C------
DO 160 J=1,4
MM=ABS((IE(I)-IE(J))
IF(MM>GT,MB) MB=MM
160 CONTINUE
MB=2*MB+2
IF(MB>GT,MBAND) MBAND=MB
IF(M<GT,N) GO TO 145
DO 142 I=1+4
142 IX(I+N)=IX(I+N-1)+1
IX(5*N)=IX(5*N-1)

GO TO 150

145 DO 148 I=1,5
148 IX(I,N)=IE(I)

150 WRITE (6,2047) N*(IX(I,N),I=1,5),MB

C----- --

IF(N EQ NUMEL) GO TO 700
IF(N EQ M) GO TO 130
GO TO 140

C----- --

PRESSURE BOUNDARY CONDITIONS

C----- --

700 WRITE (6,2047)
DO 330 K=1,NUMPC
READ (5,1047) INI(K),JNJ(K),A,B,T(K)
I=INI(K)
J=JNJ(K)
DZ=(Z(I)-Z(J))/12.*U
DR=(R(J)-R(I))/12.*U
RXA=A*(R(I)+R(J))+B*(R(I)+3.*R(J))
IZX=A*(R(I)+R(J))+B*(R(I)+3.*R(J))
HI(K)=RX*DI
HJ(K)=ZK*DZ
V1(K)=RX*DR
VJ(K)=ZK*DR

330 WRITE (6,2047) I,J,A,B,H1(K),V1(K),HJ(K),VJ(K),T(K)

C----- --

READ AND PRINT OF LOAD DATA

C----- --

WRITE (6,2047)
DO 380 M=1,NP
380 READ (5,1047) (P(K,M),K=1,2)
WRITE (6,2047) ((P(K,M),K=1,2),M=1,NP)

C----- --

RETURN

1001 FORMAT (I5,2F15.0/(5F10.0))
1002 FORMAT (I5,F7.5/F210.0)
1003 FORMAT (6I5)
1004 FORMAT (2F10.0)
1007 FORMAT (2I5,/3F10.0)
2000 FORMAT (14H MATERIAL NUMBER = 15 , 10H DENSITY = E15.6 ,
1 13H THICKNESS = E15.6 /4X,29H STRAIN PRESSURE / UNLOADING BULK SHEAR M
30DULUS STRAIN SET RATIO / )
2001 FORMAT (49H ELEMENT NO. I J K L MATERIAL )
2002 FORMAT (17F10.2/2F10.3)
2003 FORMAT (11I3/4I6/2112)
2004 FORMAT (37H NODAL POINT TYPE X-ORD Y-ORD )
2005 FORMAT (2F15.7)
2007 FORMAT (2F15.7)
2009 FORMAT (26H NODAL POINT CARD ERROR N= 15)
2100 FORMAT (29H PRESSURE BOUNDARY CONDITIONS /
19X*1H1 *5X*1HJ *7X*4HP*/P*8X*4HP*/P*8X*2HH1*10X*2HVI*10X*2HHJ*10X*
2 2HVJ*11X*1HT)
2112 FORMAT (5E18.6)
2113 FORMAT (2I6/7F12.3)
END
SUBROUTINE STRAIN (X0,R,Z,IX,EPS,MTYPE,VOL,H)

DIMENSION X0(1),R(1),Z(1),IX(5),EPS(5),IX(4),Y(4),H(1)
COMMON / S4ANG / LM(8),SS(4,8),XC,YC,S(8,8),C(4,4)

DO 204 I=1,4
DO 205 K=1,8
500 SS(I,IK)=0.
J=I+1
II=IX(I)+IX(I)
LM(II)=II
200 LM(J-1)=II-1
I=IX(1)
J=IX(2)
K=IX(3)
L=IX(4)
MTYPE=IX(5)

C--------
C DISPLACEMENT STRAIN TRANSFORMATION MATRIX.
C--------

R13=R(I)-R(K)
R24=R(J)-R(L)
Z13=Z(I)-Z(K)
Z24=Z(J)-Z(L)
RM=(R(I)+R(J)+R(K)+R(L))/4.
YR=+Z(I)+Z(J)+Z(K)+Z(L)/4.
IF(JNE.K) GO TO 300

C------ Shell Element
XL=R24**2+Z24**2
SS(1,1)=R13/XL
SS(1,2)=Z13/XL
SS(1,3)=-SS(1,1)
SS(1,4)=-SS(1,2)
SS(2,1)=RM+RM
SS(2,3)=SS(2,1)
VOL=SQRT(XL)*H(MTYPE)+H(MTYPE))
GO TO 400

300 VOL=R13*Z24-Z13*R24
Y(1)=Z24/VOL
Y(2)=-Z13/VOL
Y(3)=-Y(1)
Y(4)=-Y(2)
X(1)=-R24/VOL
X(2)=R13/VOL
X(3)=-X(1)
X(4)=-X(2)
DO 100 I=1,4
II=II+1
JJ=II-1
SS(1,II)=Y(I)
SS(2,II)=X(I)
SS(3,II)=RM
SS(4,II)=Y(I)
100 SS(4,II)=X(I)
400 XC=.25/RM
VOL=VOL/2.*XC

C--------
C EVALUATION OF STRAIN.
C--------

-C 12-
DO 180 I=1*4
   S(I,1)=U.*
DO 180 J=1*8
   JJ=LM(J)
180  S(I,1)=S(I,1)+SS(J,J)*X0(JJ)
DO 190 I=1*4
190  EPS(I)=EPS(I)+S(I,1)
320 RETURN
END
SUBROUTINE ONED (R, Z, H, IX, VOL, MTYPE, EE)
COMMON / LS4ARG / LM(8), SS(4, 8), XC, YC, S(8, 8), C(4, 4)
DIMENSION R(1), Z(1), H(1), IX(5), ST(4, 8), EE(5, 6, 1)
DO 410 I = 1, 8
  DO 405 J = 1, 4
  405 ST(J, I) = 0.
  DO 410 J = 1, 8
  410 ST(I, J) = 0.

I = IX(1)
J = IX(2)
XC = (R(I) + R(J)) / 2.
YC = (Z(I) + Z(J)) / 2.
DX = R(J) - R(I)
DY = Z(J) - Z(I)
XL = SQRT(DX**2 + DY**2)
VOL = H(MTYPE) * A*L * XC
ENU = EE(4, 1, MTYPE)
E1 = EE(3, 1, MTYPE) / (1.0 - ENU**2)
C(1, 1) = E1
C(2, 2) = E1
C(1, 2) = ENU * E1
C(2, 1) = C(1, 2)

C---- STRAIN DISPLACEMENT RELATION
ST(1, 1) = -DX / XL**2
ST(1, 2) = -DY / XL**2
ST(1, 3) = -ST(1, 1)
ST(1, 4) = -ST(1, 2)
ST(2, 1) = -5 / XC
ST(2, 3) = -ST(2, 1)
DO 411 I = 1, 4
  DO 411 J = 1, 8
  411 SS(I, J) = 0.
  DO 412 I = 1, 2
    DO 412 J = 1, 4
      DO 412 K = 1, 2
        412 SS(I, J) = SS(I, J) + C(I, K) * ST(K, J)
  DO 414 J = 1, 4
    DO 414 I = 1, 4
      DO 414 K = 1, 2
        414 SS(I, J) = SS(I, J) + ST(K, I) * SS(K, J) * VOL
RETURN
END

- C 14 -
SUBROUTINE LOAD (T*P, B*INI, JNJ*HI, HJ*VI, VJ*IK)
COMMON NUMNP*MBAND, NT*NPRINT, NP*NUMPC, NUMEL*NUMMAT, RA*TT*DELT
DIMENSION T(1), P(2,1), B(1), INI(1), JNJ(1), HI(1), HJ(1), VI(1), VJ(1)

C
NEQ=NUMNP+NUMNP
DO 600 I=1,NEQ
600 B(I)=u
N=1
100 TAU=TT-T(N)
   IF(TAU) 500,200,200
200 IF(TAU,GE,P(1*IK),AND,TAU,LE,P(1*IK+1)) GO TO 300
   IF(TAU,GT,P(1*IK+1)) IK=IK+1
   IF(TAU,LT,P(1*IK)) IK=IK-1
   GO TO 200
300 D=P(1*IK+1)-P(1*IK)
   DH=P(2*IK+1)-P(2*IK)
   IF (TT,EQ,P(1*IK)) TAU=-DELT
   DT=TAU-P(1*IK)+DELT
   F=P(2*IK)+DT*DH/D
400 I=INI(N)+INI(N)
   II=I-1
   J=JNJ(N)+JNJ(N)
   JJ=J-1
   B(II)=B(II)+F*HI(N)
   B(JJ)=B(JJ)+F*HJ(N)
   B(I)=B(I)+F*VI(N)
   B(J)=B(J)+F*VJ(N)
500 N=N+1
   IF(N,GT,NUMPC) RETURN
   IF(T(N),EQ,T(N-1)) GO TO 400
   GO TO 100
END
SUBROUTINE STIFF (R,Z,CODE,IX,EE,A,NEW,KU,MTYPE,VOL,H)
COMMON / LS4ARG / LM(8),SS(4,8),XC,YC,S(8,8),C(4,4)
DIMENSION K(1),Z(1),CODE(1),IX(5),EE(5*6*1),A(NEW,1),RO(1),H(1)
I=IX(1)
J=IX(2)
K=IX(3)
L=IX(4)
IF (J.NE.K) GO TO 420
CALL ONED (R,Z,H,IX,VOL,MTYPE,EE)
GO TO 430
420 CONTINUE
CALL QUAD(K(1),K(J),K(K),K(L),Z(I),Z(J),Z(K),Z(L),XC,YC,VOL,C,
1 S*SS)
430 CONTINUE
C MODIFY FOR ZERO DISPLACEMENTS
C
DO 600 I=1,4
IJ=I+1
II=IX(I)
IF (CODE(II).EQ.0.0) GO TO 600
DO 570 J=1,8
S(IJ+J)=0.*
570 S(IJ+J)=0.*
DO 580 J=1,8
S(IJ-1+J)=0.*
580 S(IJ-1+J)=0.*
600 CONTINUE
C
DO 300 I=1,8
II=LM(I)
DO 300 J=1,8
JJ=LM(J)-II+1
IF (JJ.LT.1) GO TO 300
A(IJ+JJ)=A(IJ+JJ)+S(I+J)
300 CONTINUE
RETURN
END
SUBROUTINE BACK<(NN, MM, A, B)

DIMENSION A(I), B(I)

MMM = MM - 1
N = 0

270 N = N + 1
C = B(N)
IF (A(N) .NE. 0.0) B(N) = B(N) / A(N)
IF (N .EQ. NN) GO TO 300
IL = N + 1
IH = MINU(NN, N + MMM)
M = N
DO 285 I = IL, IH
M = M + NN
285 B(I) = B(I) - A(M) * C
GO TO 270

300 IL = N
N = N - 1
IF (N .EQ. 0) RETURN
IH = MINU(NN, N + MMM)
M = N
DO 400 I = IL, IH
M = M + NN
400 B(N) = B(N) - A(M) * B(I)
GO TO 300

END
SUBROUTINE TRIA(NEQ,M,A)
DIMENSION A(1)
NE=NEQ-1
MN=M-1
MM=MN*NEQ
MK=NEQ-MN
DO 300 N=1,NE
NT=N-MK
IF(NT.GT.0) MM=MM-NEQ
IF(A(N).EQ.0.0) GO TO 300
L=N
IL=N+NEQ
IH=N+MM
DO 200 I=IL,IH,NEQ
L=L+1
J=L
C=A(I)/A(N)
DO 100 K=1,IH,NEQ
A(J)=A(J)-C*A(K)
100 J=J+NEQ
A(I)=C
200 CONTINUE
300 CONTINUE
RETURN
END
SUBROUTINE QUAD (K1,K2,K3,K4,Z1,Z2,Z3,Z4,KM,ZM,VOL,D,WK,WS)

FORMS STIFFNESS MATRIX WK, CENTROIDAL STRESS MATRIX WS
FOR A FIVE POINT AXISYMMETRIC NODE'S QUADRILATERAL USING
A FOUR POINT INTEGRATION FORMULA.
CONSTANT SHEAR STRAIN INTRODUCES INCOMPATIBILITY

DIMENSION WK(10,8),QS(4,8),D(4,4),TQ(4),WC(4,10),SS(4),QQ(10,10)
DATA SS,-1.,1.,1.,1., / , TQ /-1.,-1.,1.,1.,1., /

DO 6 I=1,10
DO 6 J=1,10
6 QQ(I,J)=0.
R12=R1-R2
R13=R1-R3
R14=R1-R4
R23=R2-R3
R24=R2-R4
R34=R3-R4
Z12=Z1-Z2
Z13=Z1-Z3
Z14=Z1-Z4
Z23=Z2-Z3
Z24=Z2-Z4
Z34=Z3-Z4
VOL=R13*Z24-R24*Z34
RM=(R1+R2+R3+R4)/4.
LM=(Z1+Z2+Z3+Z4)/4.
IF (D(I,J).EQ.W) GO TO 888
Y5=Z24/VOL
X6=R13/VOL
X7=R24/VOL
Y8=Z13/VOL
X5=-X7
Y6=-Y8
Y7=-Y5
X8=-X6
DO 30 II=1,4
S=SS(II)*W.577350269189626
T=TQ(II)*W.577350269189626
XJ =VOL+S*(K34*Z12-K12*Z34)+T*(R23*Z14-K14*Z23)
XJAC=XJ/8.0
SM=1.*S
SP=1.*S+5.
TM=1.*T
TP=1.*T+T
H1=W.25*SM*TM
H2=W.25*SP*TM
H3=W.25*SP*TP
H4=W.25*SM*TP
R=H1*R1+H2*R2+H3*R3+H4*R4
G1=H1/R
G2=H2/R
G3=H3/R
G4=H4/R
GC=SM*SP*TM*TP/R
X1=(-R24+R34*G4*R23*T)/XJ
X2=( R13-R34*G5-R14*T)/XJ

- C 19 -
\[ X_3 = \frac{R_{24} - R_{12}* + R_{14}* + R_{14}*T}{XJ} \]
\[ X_4 = \frac{R_{13} + R_{12} + R_{23} + R_{23}T}{XJ} \]
\[ Y_1 = \frac{Z_{24} - Z_{34} + Z_{23} + Z_{23}T}{XJ} \]
\[ Y_2 = \frac{Z_{13} + Z_{23} + Z_{14} + Z_{14}T}{XJ} \]
\[ Y_3 = \frac{Z_{24} + Z_{12} - Z_{14} + Z_{14}T}{XJ} \]
\[ Y_4 = \frac{Z_{13} - Z_{12} + Z_{23} + Z_{23}T}{XJ} \]
\[ RS = 0.25*(R_{21} + R_{12} + R_{23} + R_{23}T - R_{14} - R_{14}T) \]
\[ ZL = 0.25*(Z_{11} + Z_{11} + Z_{21} + Z_{21}T - Z_{34} + Z_{34}T) \]
\[ RT = 0.25*(R_{21} - R_{21} + R_{23} - R_{23}T) \]
\[ ZL = 0.25*(Z_{11} - Z_{11} + Z_{21} - Z_{21}T) \]
\[ X_3 = -2u*\frac{(T*SM*SP*RS - S*TM*TP)}{XJAC} \]
\[ Y_4 = 2u\frac{(T*SM*SP*ZS - S*TM*TP)}{XJAC} \]
\[ FAC = XJAC*R \]

DO 10 I=1+4
D1=D(I+1)*FAC
D2=D(I+2)*FAC
D3=D(I+3)*FAC
D4=D(I+4)*FAC
QC(I+1)=D1*Y1+D4*X5+D3*G1
QC(I+3)=D1*Y2+D4*X6+D3*G2
QC(I+5)=D1*Y3+D4*X7+D3*G3
QC(I+7)=D1*Y4+D4*X8+D3*G4
QC(I+9)=D1*Y5+D4*X9+D3*G5
QC(I+2)=D2*X1+D4*Y5
QC(I+4)=D2*X2+D4*Y6
QC(I+6)=D2*X3+D4*Y7
QC(I+8)=D2*X4+D4*Y8
QC(I+1U)=D2*XC
10 CONTINUE
DO 20 I=1+1U
D1=QC(I+1)
D2=QC(I+2)
D3=QC(I+3)
D4=QC(I+4)
QQ(I+1)=QQ(I+1)*D1*Y1+D4*X5+D3*G1
QQ(I+3)=QQ(I+3)*D1*Y2+D4*X6+D3*G2
QQ(I+5)=QQ(I+5)*D1*Y3+D4*X7+D3*G3
QQ(I+7)=QQ(I+7)*D1*Y4+D4*X8+D3*G4
QQ(I+9)=QQ(I+9)*D1*Y5+D4*X9+D3*G5
QQ(I+2)=QQ(I+2)*D2*X1+D4*Y5
QQ(I+4)=QQ(I+4)*D2*X2+D4*Y6
QQ(I+6)=QQ(I+6)*D2*X3+D4*Y7
QQ(I+8)=QQ(I+8)*D2*X4+D4*Y8
QQ(I+1U)=QQ(I+1U)+D2*XC
20 CONTINUE
30 CONTINUE

FORM STRESS MATRIX QS AT CENTROID (RM,2M) OF ELEMENT

DO 40 I=1+4
D1=D(I+1)
D2=D(I+2)
D3=D(I+3)/(4.0*RM)
D4=D(I+4)
T1 = (D1*R24 - D4*R24)/VOL
T2 = (-D1*R13 + D4*R13)/VOL
T3 = (-D2*R24 + D4*R24)/VOL
T4 = (-D2*R13 - D4*R13)/VOL
QC(I1) = D3 + T1
QC(I3) = D3 + T2
QC(I5) = D3 - T1
QC(I7) = D3 - T2
QC(I9) = 4.0*D3
QC(I2) = T3
QC(I4) = T4
QC(I6) = -T3
QC(I8) = -T4
QC(I0) = U0
40 CONTINUE

C ELIMINATE CENTRE NODE

C DO 50 N = 1, 2
L = 10 - N
M = L + 1
DO 50 I = 1, L
C = QQ(I*M)/QQ(M*M)
DO 50 J = 1, L
50 QQ(I, J) = QQ(I, J) - C*QQ(M, J)

C RELOCATE STRESS, STIFFNESS AND LOAD MATRICES

C 888 CONTINUE

C DO 70 J = 1, 8
DO 70 I = 1, 4
QK(I, J) = QQ(I, J)
70 QK(I+4, J) = QQ(I+4, J)
VOL = VOL*RM/2.
RETURN
END
APPENDIX D

LISTING OF INPUT DATA FOR SAMPLE PROBLEM
TEST FOR THE NONLINEAR AXISYMMETRIC ANALYSIS. VICKSBURG'S EXAMPLE.

63 50 2 20 1 11 6 1
1 .000164
 2.000155 6.5
 3.0002910
 4.0016 40
 5.001855 50
 6.00260

2.000000 17500. 874
 3.000000 13500. 874
 4.000000 10500. 874
 5.000000 12000. 874
 6.000000 21000. 874
 7.000000 27000. 874

1 .3
 2 .3
 3 .3
 4 .3
 5 .3
 6 .3
 7 .3
 8 .1
 9 .0
 10 .1

13.0
 14.1
 15.1
 16.0
 17.1
 18.0
 19.1
 20.1
 21.1
 22.1
 23.0
 24.1
 25.1
 26.1
 27.0
 28.1
 29.1
 30.0
 31.1
 32.1
 33.1
 34.1
 35.1
 36.1
 37.0
 38.1
 39.1
 40.1
 41.0
 42.1
 43.1
 44.0
 45.1
 46.0
 47.1
 48.0
 49.1
 50.1
 51.0
 52.1
 53.1
 54.1
 55.0
 56.1
 57.1
 58.0
 59.1
 60.1
 61.1
 62.1
 63.1

- D1 -
<table>
<thead>
<tr>
<th>30</th>
<th>36</th>
<th>37</th>
<th>44</th>
<th>43</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>41</td>
<td>42</td>
<td>49</td>
<td>48</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>43</td>
<td>44</td>
<td>51</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>48</td>
<td>49</td>
<td>56</td>
<td>55</td>
<td>1</td>
</tr>
<tr>
<td>42</td>
<td>50</td>
<td>51</td>
<td>58</td>
<td>57</td>
<td>1</td>
</tr>
<tr>
<td>47</td>
<td>55</td>
<td>56</td>
<td>63</td>
<td>62</td>
<td>1</td>
</tr>
<tr>
<td>48</td>
<td>36</td>
<td>37</td>
<td>37</td>
<td>36</td>
<td>2</td>
</tr>
<tr>
<td>49</td>
<td>37</td>
<td>37</td>
<td>30</td>
<td>37</td>
<td>2</td>
</tr>
<tr>
<td>50</td>
<td>29</td>
<td>30</td>
<td>30</td>
<td>29</td>
<td>2</td>
</tr>
<tr>
<td>63</td>
<td>62</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>61</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>60</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>59</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>58</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>57</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>.001</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.002</td>
<td>54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.003</td>
<td>54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.004</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.005</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.011</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.019</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.030</td>
<td>34</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.040</td>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.050</td>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Address</td>
<td>No. of Copies</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>--------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Army</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chief of Engineers, Department of the Army, Washington, D. C. 20314</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: ENGME-S</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGME</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGCW-E</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGCW-Z</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGMC-E</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGMC-EM</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGAS-I</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGAS-I, Library</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENGNA</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chief of Research and Development, Headquarters, Department of the Army, Washington, D. C. 20310</td>
<td>3 copies of Form 1473</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Director of Army Technical Information</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chief of Research and Development, Department of the Army, Washington, D. C. 20310</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Atomic Office</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRDES</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division Engineers, U. S. Army Engineer Divisions, Continental United States</td>
<td>Cy to ea</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commandant, U. S. Army Air Defense School, Fort Bliss, Tex. 79906</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commandant, U. S. Army Command &amp; General Staff College, Fort Leavenworth, Kans. 66027</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Archives</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commandant, Army War College, Carlisle Barracks, Pa. 17013</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Library</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding General, Aberdeen Proving Ground, Aberdeen, Md. 21005</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Director, Ballistic Research Laboratories</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding General, The Engineer Center, Fort Belvoir, Va. 22060</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Assistant Commandant, Engineer School</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding General, U. S. A. Electronics Command, Fort Monmouth, N. J. 07703</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: AMSEL-GG-DD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding General, USA Missile Command, Huntsville, Ala. 35809</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding General, USA Munition Command, Dover, N. J. 07801</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding General, U. S. Continental Army Command, Fort Monroe, Va. 23351</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Address</td>
<td>No. of Copies</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>---------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: AMCRD-DE-N</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding Officer, Picatinny Arsenal, Dover, N. J. 07801</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: ORDBR-TK</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding Officer, U. S. Army Aviation Materiel Laboratories, Fort Eustis, Va. 23604</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding Officer, U. S. Army Combat Developments Command, Institute of Nuclear Studies, Fort Bliss, Tex. 79916</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding Officer, U. S. Army Mobility Equipment Research and Development Center Fort Belvoir, Va. 22060</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Technical Documents Center, Building 315</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commanding Officer, U. S. Army Nuclear Defense Laboratory, Edgewood Arsenal Edgewood, Md. 21040</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Technical Library</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Department of the Army, CE Ballistic Missile Construction Office, P. O. Box 4187 Norton AFB, Calif. 92409</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Director of Civil Defense, Office of the Secretary of the Army, Washington, D. C. 20310</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Mr. George Sisson (RE-ED)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Director, Nuclear Cratering Group, U. S. Army Corps of Engineers, Lawrence Radiation Laboratory, P. O. Box 808, Livermore, Calif. 94550</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Director, U. S. Army Corps of Engineers, Coastal Engineering Research Center Washington, D. C. 20016</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Mr. T. Saville, Jr.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Director, U. S. Army Corps of Engineers, Ohio River Division Laboratories, 5851 Mariemont Avenue, Cincinnati, Ohio 45227</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technical Library</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USAMERDC, Bldg 314</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fort Belvoir, Va. 22060</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Director, U. S. Army CRREL, P. O. Box 282, Hanover, N. H. 03755</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Mr. K. Boyd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Director, U. S. Army Construction Engineering Research Laboratory, P. O. Box 4005, Champaign, Ill. 61820</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATTN: Library</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2
Address

Army (Continued)

District Engineer, U. S. Army Engineer District, Omaha, 6012 U. S. Post Office and Court House
215 N. 17th Street, Omaha, Nebr. 68101
ATTN: MROGS-B

President, U. S. Army Air Defense Board, Fort Bliss, Tex. 79906

Superintendent, U. S. Military Academy, West Point, N. Y. 10996
ATTN: Library

U. S. Army Engineer Division, Missouri River, P. O. Box 103, Downtown Station
Omaha, Nebr. 68101
ATTN: Mr. Ken Lane

Navy

Commander-in-Chief, Pacific, FPO, San Francisco 94129

Commander-in-Chief, U. S. Atlantic Fleet, U. S. Naval Base, Norfolk, Va. 23511

Chief of Naval Operations, Navy Department, Washington, D. C. 20350
ATTN: OP-75
OP-03EG

Chief of Naval Research, Navy Department, Washington, D. C. 20390
ATTN: Code 811

Commandant of the Marine Corps, Navy Department, Washington, D. C. 20380
ATTN: Code A04E

Commander, Naval Facilities Engineering Command, Navy Department, Washington, D. C. 20370
ATTN: Code 04
Code 03

Commander, Naval Ordnance Systems Command, Washington, D. C. 20360

Commander, Naval Ship Engineering Center, Washington, D. C. 20360
ATTN: Code 6115

Commanding Officer, Nuclear Weapons Training Center, Atlantic Naval Base, Norfolk, Va. 23511
ATTN: Nuclear Warfare Department

Commanding Officer, Nuclear Weapons Training Center, Pacific, Naval Station, North Island
San Diego, Calif. 92136

Commanding Officer & Director, Naval Electronics Laboratory, San Diego, Calif. 92152

Commanding Officer & Director, Naval Ship Research and Development Center
Carderock, Md. 20007

Commanding General, Marine Corps Development and Education Command, Quantico, Va. 22134
ATTN: Director, Development Center

No. of Copies

1
1
2
1
1
2
1
1
1
1
1
1
2
1
1
2
<table>
<thead>
<tr>
<th>Address</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commander, U. S. Naval Civil Engineering Laboratory, Port Hueneme, Calif. 93041</td>
<td>2</td>
</tr>
<tr>
<td>ATTN: Code L31</td>
<td></td>
</tr>
<tr>
<td>Commander, U. S. Naval Civil Engineer Corps Officer School, U. S. Naval Construction Battalion Center, Port Hueneme, Calif. 93041</td>
<td>1</td>
</tr>
<tr>
<td>Commanding Officer, U. S. Naval Damage Control Training Center, Naval Base Philadelphia, Pa. 19112</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: ABC Defense Course</td>
<td></td>
</tr>
<tr>
<td>Commanding Officer, U. S. Naval Weapons Evaluation Facility, Kirtland Air Force Base Albuquerque, N. Mex. 87117</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Code WEVS</td>
<td></td>
</tr>
<tr>
<td>Commanding Officer, U. S. Naval Weapons Laboratory, Dahlgren, Va. 22448</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: TE</td>
<td></td>
</tr>
<tr>
<td>Commander, U. S. Naval Oceanographic Office, Suitland, Md. 20023</td>
<td>1</td>
</tr>
<tr>
<td>Commander, U. S. Naval Ordnance Laboratory, Silver Spring, Md. 20910</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: EA</td>
<td>1</td>
</tr>
<tr>
<td>EU</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>Commander, U. S. Naval Ordnance Test Station, China Lake, Calif. 93555</td>
<td>1</td>
</tr>
<tr>
<td>Director, U. S. Naval Research Laboratory, Washington, D. C. 20390</td>
<td>1</td>
</tr>
<tr>
<td>President, U. S. Naval War College, Newport, R. I. 02840</td>
<td>1</td>
</tr>
<tr>
<td>Special Projects, Navy Department, Washington, D. C. 20360</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: SP-272</td>
<td></td>
</tr>
<tr>
<td>Superintendent, U. S. Naval Postgraduate School, Monterey, Calif. 93940</td>
<td>1</td>
</tr>
<tr>
<td>Underwater Explosions Research Division, Naval Ship Research and Development Center Norfolk Naval Shipyard, Portsmouth, Va. 23511</td>
<td>1</td>
</tr>
<tr>
<td>Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Dayton, Ohio 45433</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Mr. Frank Janik, Jr.</td>
<td></td>
</tr>
<tr>
<td>Air Force Institute of Technology, AFIT-L, Building 640, Wright-Patterson AFB, Ohio 45433</td>
<td>1</td>
</tr>
<tr>
<td>Commander, Air Force Logistics Command, Wright-Patterson AFB, Ohio 45433</td>
<td>2</td>
</tr>
<tr>
<td>ATTN: SCTSW</td>
<td></td>
</tr>
<tr>
<td>Address</td>
<td>No. of Copies</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Air Force Technical Applications Center, Department of the Air Force,</td>
<td>1</td>
</tr>
<tr>
<td>Washington, D. C. 20333</td>
<td></td>
</tr>
<tr>
<td>Air Force Weapons Laboratory, Kirtland AFB, N. Mex. 87117</td>
<td>2</td>
</tr>
<tr>
<td>ATTN: Library</td>
<td></td>
</tr>
<tr>
<td>WLDC</td>
<td>1</td>
</tr>
<tr>
<td>WLDC/R. W. Henny</td>
<td></td>
</tr>
<tr>
<td>Director, Air University Library, Maxwell AFB, Ala. 36112</td>
<td>2</td>
</tr>
<tr>
<td>Commander, Strategic Air Command, Offutt AFB, Nebr. 68113</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: OAWS</td>
<td></td>
</tr>
<tr>
<td>Commander, Tactical Air Command, Langley AFB, Va. 23365</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Document Security Branch</td>
<td></td>
</tr>
<tr>
<td>Space and Missile Systems Organization, Norton AFB, Calif. 92409</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: SAMSO (SMQNM)</td>
<td></td>
</tr>
<tr>
<td>Headquarters, USAF, Washington, D. C. 20330</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: AFRSTG</td>
<td></td>
</tr>
<tr>
<td>Director, Air Research and Development Command Headquarters, USAF</td>
<td>1</td>
</tr>
<tr>
<td>Washington, D. C. 20330</td>
<td></td>
</tr>
<tr>
<td>ATTN: Combat Components Division</td>
<td></td>
</tr>
<tr>
<td>Director of Civil Engineering, Headquarters, USAF, Washington, D. C.</td>
<td>1</td>
</tr>
<tr>
<td>20330</td>
<td></td>
</tr>
<tr>
<td>ATTN: AFOCE</td>
<td></td>
</tr>
<tr>
<td>Director, U. S. Air Force Project RAND, Via: U. S. Air Force Liaison</td>
<td>1</td>
</tr>
<tr>
<td>Office, The RAND Corporation, 1700 Main Street, Santa Monica, Calif.</td>
<td></td>
</tr>
<tr>
<td>90406</td>
<td></td>
</tr>
<tr>
<td>ATTN: Library</td>
<td></td>
</tr>
<tr>
<td>Dr. Harold L. Brode</td>
<td>1</td>
</tr>
<tr>
<td>Dr. Olen A. Nance</td>
<td></td>
</tr>
<tr>
<td>Other DOD Agencies</td>
<td></td>
</tr>
<tr>
<td>Administrator, National Aeronautics &amp; Space Administration, 400</td>
<td>1</td>
</tr>
<tr>
<td>Maryland Avenue, S. W. Washington, D. C. 20546</td>
<td></td>
</tr>
<tr>
<td>Assistant to the Secretary of Defense (Atomic Energy), Washington, D.</td>
<td>1</td>
</tr>
<tr>
<td>C. 20301</td>
<td></td>
</tr>
<tr>
<td>Commandant, Armed Forces Staff College, Norfolk, Va. 23511</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Library</td>
<td></td>
</tr>
<tr>
<td>Commandant, National War College, Washington, D. C. 20310</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Class Rec. Library</td>
<td></td>
</tr>
<tr>
<td>Commandant, The Industrial College of the Armed Forces, Fort McNair</td>
<td>1</td>
</tr>
<tr>
<td>Washington, D. C. 20310</td>
<td></td>
</tr>
</tbody>
</table>
### Other DOD Agencies (Continued)

<table>
<thead>
<tr>
<th>Address</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commander, Test Command, DASA, Sandia Base, Albuquerque, N. Mex. 87115</td>
<td>2</td>
</tr>
<tr>
<td>ATTN: TCCOM, TCDT</td>
<td></td>
</tr>
<tr>
<td>Commander, Field Command, DASA, Sandia Base, Albuquerque, N. Mex. 87115</td>
<td>2</td>
</tr>
<tr>
<td>Defense Documentation Center (DDC), Cameron Station, Alexandria, Va. 22314</td>
<td>20</td>
</tr>
<tr>
<td>(NO TOP SECRET TO THIS ADDRESS)</td>
<td></td>
</tr>
<tr>
<td>ATTN: Mr. Myer Kahn</td>
<td></td>
</tr>
<tr>
<td>Director, Defense Atomic Support Agency, Washington, D. C. 20301</td>
<td>5</td>
</tr>
<tr>
<td>ATTN: SPSS</td>
<td></td>
</tr>
<tr>
<td>Director of Defense Research and Engineering, Washington, D. C. 20301</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Technical Library</td>
<td></td>
</tr>
<tr>
<td>Mr. Frank J. Thomas</td>
<td>1</td>
</tr>
<tr>
<td>Director, Advanced Research Projects Agency, Washington, D. C. 20301</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: NTDO</td>
<td></td>
</tr>
<tr>
<td>Director, Defense Intelligence Agency, Washington, D. C. 20301</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: DIAAP-1K2</td>
<td></td>
</tr>
<tr>
<td>DIA-AP8B-1</td>
<td>1</td>
</tr>
<tr>
<td>Director, Weapons Systems Evaluation Group, Washington, D. C. 20305</td>
<td>1</td>
</tr>
<tr>
<td>Langley Research Center, NASA, Langley Field, Hampton, Va. 23365</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Mr. Philip Donely</td>
<td></td>
</tr>
<tr>
<td>Manager, Albuquerque Operations Office, USAEC, P. O. Box 5400, Albuquerque, N. Mex. 87115</td>
<td>1</td>
</tr>
<tr>
<td>Manager, Nevada Operations Office, USAEC, P. O. Box 1676, Las Vegas, Nev. 89101</td>
<td>1</td>
</tr>
<tr>
<td>National Aeronautics &amp; Space Administration, Man-Spacecraft Center, Space Technology Division, Box 1537, Houston, Tex. 77001</td>
<td>1</td>
</tr>
<tr>
<td>National Military Command System Support Center, Pentagon BE 685, Washington, D. C. 20301</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Technical Library</td>
<td></td>
</tr>
<tr>
<td>U. S. Atomic Energy Commission, Washington, D. C. 20545</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Chief, Classified Tech Lib, Tech Information Service</td>
<td></td>
</tr>
<tr>
<td>U. S. Documents Officer, Office of the United States National Military Representative—SHAPE APO New York 09055</td>
<td>1</td>
</tr>
</tbody>
</table>

### Other Agencies

<table>
<thead>
<tr>
<th>Address</th>
<th>No. of Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerospace Corporation, 1111 E. Mill Street, San Bernardino, Calif. 92408</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. M. B. Watson</td>
<td></td>
</tr>
<tr>
<td>Agbabian-Jacobsen Associates, Engineering Consultants, 8939 South Sepulveda Boulevard Los Angeles, Calif. 90045</td>
<td>1</td>
</tr>
<tr>
<td>Address</td>
<td>No. of Copies</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Applied Theory, Inc., 1728 Olympic Blvd, Santa Monica, Calif. 90404</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. John G. Trulio</td>
<td></td>
</tr>
<tr>
<td>AVCO Corporation, Research and Advanced Development Division, 201 Lowell Street Wilmington, Mass. 01887</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Mr. R. E. Cooper</td>
<td></td>
</tr>
<tr>
<td>Battelle Memorial Institute, 505 King Avenue, Columbus, Ohio 43201</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. P. N. Lamori</td>
<td></td>
</tr>
<tr>
<td>Bell Telephone Laboratories, Inc., Whippany Road, Whippany, N. J. 07981</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Mr. R. W. Mayo</td>
<td></td>
</tr>
<tr>
<td>The Boeing Company, P. O. Box 3707, Seattle, Wash. 98124</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Technical Library</td>
<td></td>
</tr>
<tr>
<td>Corrugated Metal Pipe Institute, Crestview Plaza, Port Credit, Ontario, Canada</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Mr. W. A. Porter</td>
<td></td>
</tr>
<tr>
<td>Defence Research Establishment, Suffield, Ralston, Alberta, Canada</td>
<td>1</td>
</tr>
<tr>
<td>General Research Corporation, P. O. Box 3587, Santa Barbara, Calif. 93105</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Mr. Benjamin Alexander</td>
<td></td>
</tr>
<tr>
<td>Denver Mining Research Center, Building 20, Denver Federal Center, Denver, Colo. 80225</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Leonard A. Obert</td>
<td></td>
</tr>
<tr>
<td>Dynamic Science Corporation, 1900 Walker Avenue, Monrovia, Calif. 91016</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. J. C. Peck</td>
<td></td>
</tr>
<tr>
<td>Edgerton, Germeshausen &amp; Grier, Inc., 95 Brookline Avenue, Boston, Mass. 02129</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: D. F. Hansen</td>
<td></td>
</tr>
<tr>
<td>Engineering Physics Company, 12721 Twinbrook Parkway, Rockville, Md. 20852</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Vincent J. Cushing</td>
<td></td>
</tr>
<tr>
<td>Mr. W. Daneck</td>
<td></td>
</tr>
<tr>
<td>General American Transportation Corporation, General American Research Division</td>
<td>1</td>
</tr>
<tr>
<td>7449 North Natchez Avenue, Niles, Ill. 60648</td>
<td></td>
</tr>
<tr>
<td>ATTN: Dr. G. L. Neidhardt</td>
<td></td>
</tr>
<tr>
<td>General Electric Company, Missile and Space Vehicle Department, Valley Forge Space Technology Center, Goddard Boulevard, King of Prussia, Pa. 19406</td>
<td>1</td>
</tr>
<tr>
<td>General Electric Company, TEMPO, 816 State Street, Santa Barbara, Calif. 93101</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Mr. Warren Chan (DASIAC)</td>
<td></td>
</tr>
<tr>
<td>Address</td>
<td>No. of Copies</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>IIT Research Institute, 10 West 35th Street, Chicago, Ill. 60616</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. T. Schiffman</td>
<td></td>
</tr>
<tr>
<td>Kondner Research, Downes Road, Parkton, Md. 21120</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. R. L. Kondner</td>
<td></td>
</tr>
<tr>
<td>Lockheed Missile and Space Company, Lockheed Aircraft Corporation, 111 Lockheed Way Sunnyvale, Calif. 94086</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. R. E. Meyerott</td>
<td></td>
</tr>
<tr>
<td>Los Alamos Scientific Laboratory, P. O. Box 1663, Los Alamos, N. Mex. 87544</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Report Librarian</td>
<td></td>
</tr>
<tr>
<td>Ministry of Defense, MEXE, Christchurch, Hampshire, England</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Philip S. Bulson</td>
<td></td>
</tr>
<tr>
<td>Mr. Bruce T. Boswell</td>
<td></td>
</tr>
<tr>
<td>The Mitre Corporation, Route 62 and Middlesex Turnpike, Bedford, Mass. 01730</td>
<td>1</td>
</tr>
<tr>
<td>Physics International Company, 2700 Merced Street, San Leandro, Calif. 94577</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Charles Godfrey</td>
<td></td>
</tr>
<tr>
<td>Mr. Fred M. Sauer</td>
<td></td>
</tr>
<tr>
<td>Research Analysis Corporation, Document Control Supervisor, McLean, Va. 22101</td>
<td>1</td>
</tr>
<tr>
<td>Dr. John S. Rinehart, Senior Research Fellow (R.2), IER/ESSA, Boulder, Colo. 80302</td>
<td>1</td>
</tr>
<tr>
<td>Sandia Laboratories, P. O. Box 5800, Albuquerque, N. Mex. 87115</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Classified Document Division for Dr. M. L. Merritt</td>
<td></td>
</tr>
<tr>
<td>Southwest Research Institute, 8500 Culebra Road, San Antonio, Tex. 78228</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Robert C. DeHart</td>
<td></td>
</tr>
<tr>
<td>Systems, Science and Software, P. O. Box 1620, La Jolla, Calif. 92037</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. K. D. Pyatt, Jr.</td>
<td></td>
</tr>
<tr>
<td>TRW Space Technology Laboratories, One Spce Park, Redondo Beach, Calif. 90278</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Millard Barton</td>
<td></td>
</tr>
<tr>
<td>Mr. M. V. Anthony</td>
<td></td>
</tr>
<tr>
<td>Mr. J. L. Merritt</td>
<td></td>
</tr>
<tr>
<td>URS Corporation, 1811 Trousdale Drive, Burlingame, Calif. 94010</td>
<td>2</td>
</tr>
<tr>
<td>ATTN: Mr. Harold Mason</td>
<td></td>
</tr>
<tr>
<td>U. S. Department of the Interior, Geological Survey, Geologic Division, Branch of Engineering Geology, 345 Middlefield Road, Menlo Park, Calif. 94025</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Harold W. Olsen</td>
<td></td>
</tr>
<tr>
<td>Paul Weidlinger, Consulting Engineer, 110 East 59th Street, New York, N. Y. 10022</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. M. L. Baron</td>
<td></td>
</tr>
<tr>
<td>Address</td>
<td>No. of Copies</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>University of Arizona, Tucson, Ariz. 85721</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Donald A. DaDeppo, Department of Civil Engineering</td>
<td></td>
</tr>
<tr>
<td>Professor Bruce G. Johnston, Dept of Civil Engineering</td>
<td></td>
</tr>
<tr>
<td>Dr. George Howard, College of Engineering</td>
<td></td>
</tr>
<tr>
<td>University of California, Lawrence Radiation Laboratory, P. O. Box 808</td>
<td>2</td>
</tr>
<tr>
<td>Livermore, Calif. 94550</td>
<td></td>
</tr>
<tr>
<td>ATTN: Technical Information Division</td>
<td></td>
</tr>
<tr>
<td>University of Colorado, School of Architecture, Boulder, Colo. 80304</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor G. K. Vetter</td>
<td></td>
</tr>
<tr>
<td>University of Detroit, Department of Civil Engineering, 4001 West McNichols Road Detroit, Mich. 48221</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor W. J. Baker</td>
<td></td>
</tr>
<tr>
<td>University of Florida, Department of Mechanical Engineering, Gainesville, Fla. 32603</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor John A. Samuel</td>
<td></td>
</tr>
<tr>
<td>Florida State University, Department of Engineering Science, Tallahassee, Fla. 32302</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. G. L. Rogers</td>
<td></td>
</tr>
<tr>
<td>University of Illinois, Urbana Campus, Department of Civil Engineering, Urbana, Ill. 61801</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor N. M. Newmark</td>
<td></td>
</tr>
<tr>
<td>Professor S. L. Paul</td>
<td>1</td>
</tr>
<tr>
<td>Professor M. T. Davisson</td>
<td>1</td>
</tr>
<tr>
<td>Professor G. K. Sinnamon</td>
<td>1</td>
</tr>
<tr>
<td>Professor W. J. Hall</td>
<td>1</td>
</tr>
<tr>
<td>Professor A. J. Hendron, Jr.</td>
<td>1</td>
</tr>
<tr>
<td>Professor M. A. Sozen</td>
<td>1</td>
</tr>
<tr>
<td>Iowa State University of Science and Technology, Ames, Iowa 50010</td>
<td>2</td>
</tr>
<tr>
<td>ATTN: Professor Glen Murphy</td>
<td></td>
</tr>
<tr>
<td>Lehigh University, Bethlehem, Pa. 18015</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. J. F. Libsch, Materials Research Center</td>
<td></td>
</tr>
<tr>
<td>Dr. D. A. Van Horn, Department of Civil Engineering</td>
<td></td>
</tr>
<tr>
<td>University of Massachusetts, Department of Civil Engineering, Amherst, Mass. 01002</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. M. P. White</td>
<td></td>
</tr>
<tr>
<td>Massachusetts Institute of Technology, Division of Sponsored Research, 77 Massachusetts Avenue, Cambridge, Mass. 02139</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Robert J. Hansen</td>
<td></td>
</tr>
<tr>
<td>Dr. Robert V. Whitman</td>
<td></td>
</tr>
<tr>
<td>University of Michigan, Civil Engineering Department, Ann Arbor, Mich. 48104</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor Frank E. Richart, Jr., Consultant</td>
<td></td>
</tr>
<tr>
<td>Address</td>
<td>No. of Copies</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Dr. George B. Clark, Director, Rock Mechanics Research Group, University of Missouri at Rolla, Rolla, Mo. 65401</td>
<td>1</td>
</tr>
<tr>
<td>University of New Mexico, Eric H. Wang Civil Engineer Research Facility, Albuquerque, N. Mex. 87106</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Eugene Zwoyer</td>
<td></td>
</tr>
<tr>
<td>University of New Mexico, Eric H. Wang Civil Engineering Research Facility, P. O. Box 188 University Station, Albuquerque, N. Mex. 87106</td>
<td>2</td>
</tr>
<tr>
<td>Nova Scotia Technical College, School of Graduate Studies, Halifax, Nova Scotia, Canada</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. G. G. Meyerhof</td>
<td></td>
</tr>
<tr>
<td>Pennsylvania State University, University Park, Pa. 16802</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor G. Albright, Dept of Architectural Engineering</td>
<td></td>
</tr>
<tr>
<td>Professor Richard Kummer, 101 Eng. A</td>
<td>1</td>
</tr>
<tr>
<td>Purdue University, School of Civil Engineering, Civil Engineering Building, Lafayette, Ind. 47907</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor M. B. Scott</td>
<td></td>
</tr>
<tr>
<td>Rensselaer Polytechnic Institute, Troy, N. Y. 12180</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Clayton Oliver Dohrenwend, Security Officer, Mason House</td>
<td></td>
</tr>
<tr>
<td>Rice University, Department of Civil Engineering, Houston, Tex. 77001</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor A. S. Veletsos</td>
<td></td>
</tr>
<tr>
<td>San Jose State College, Department of Civil Engineering, San Jose, Calif. 95114</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Franklin J. Agardy</td>
<td></td>
</tr>
<tr>
<td>University of Texas, Balcones Research Center, Austin, Tex. 78712</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. J. Neils Thompson</td>
<td></td>
</tr>
<tr>
<td>Utah State University, Department of Mechanical Engineering, Logan, Utah 84321</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor R. K. Watkins</td>
<td></td>
</tr>
<tr>
<td>University of Washington, Seattle, Wash. 98105</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: C. H. Norris, Department of Civil Engineering</td>
<td></td>
</tr>
<tr>
<td>Dr. A. B. Arons, Department of Physics</td>
<td>1</td>
</tr>
<tr>
<td>Professor William Miller, Department of Civil Engineering, 307 More Hall</td>
<td></td>
</tr>
<tr>
<td>The George Washington University, Nuclear Defense Design Center, School of Engineering and Applied Science, Washington, D. C. 20006</td>
<td>1</td>
</tr>
<tr>
<td>Worcester Polytechnic Institute, Department of Civil Engineering, Worcester, Mass. 01609</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Dr. Carl Koozitz</td>
<td></td>
</tr>
<tr>
<td>Northern Arizona University, Box 5753, Flagstaff, Arizona 86001</td>
<td>1</td>
</tr>
<tr>
<td>ATTN: Professor Sandor Popovics</td>
<td></td>
</tr>
<tr>
<td>University of California, Berkeley, Dept. of Civil Engineering, Berkeley, Calif. 94720</td>
<td>25</td>
</tr>
<tr>
<td>ATTN: Dr. E. L. Wilson</td>
<td></td>
</tr>
</tbody>
</table>
A nonlinear, axisymmetric, dynamic finite-element method of analysis computer program is developed. Elastic, two-dimensional problems can also be analyzed without loss of efficiency. An extensive description of the analytical procedures used in the code is given. A FORTRAN IV listing of the computer code is presented along with information on utilizing the code to run problems. Analytical results are compared with experimental data obtained from testing a modeled buried structure subjected to a blast loading.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blast effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computer programs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finite element method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subsurface structures</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>