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LAKE ERIE INTERNATIONAL JETPORT MODEL
FEASIBILITY INVESTIGATION

Report 17-6

APPLICATION OF THREE-DIMENSIONAL HYDRODYNAMIC
MODEL TO STUDY EFFECTS OF PROPOSED JETPORT
ISLAND ON THERMOCLINE STRUCTURE IN LAKE ERIE

by

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LAKE ERIE INTERNATIONAL JETPORT MODEL FEASIBILITY INVESTIGATION; APPLICATION OF THREE-DIMENSIONAL HYDRODYNAMIC MODEL TO STUDY EFFECTS OF PROPOSED JETPORT ISLAND ON THERMOCLINE STRUCTURE IN LAKE ERIE

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A previously developed three-dimensional, variable-density hydrodynamic model was applied to the Lake Erie area about Cleveland. This application was to investigate the effect of a proposed jetport island on the summer stratification pattern in the nearshore lake area and on the flushing characteristics of the Cuyahoga River outflow into the lake. Initial results obtained from the application of the model are presented.
FOREWORD

The wind-driven circulation study reported herein using a three-dimensional, variable-density numerical circulation model is a part of the model feasibility study being conducted by the U. S. Army Engineer Waterways Experiment Station (WES) for the Lake Erie Regional Transportation Authority (LERTA). The WES investigation is a portion of an airport feasibility study being undertaken for LERTA for evaluation of proposed airport sites, one of which is in Lake Erie near Cleveland, Ohio. Development of the numerical model applied in the study was sponsored by the U. S. Environmental Protection Agency (EPA). Additional support during preparation of this report was provided by the Large Lake Research Station (LLRS), Environmental Research Laboratory-Duluth (ERLD), EPA.

The study was performed initially by Dr. John F. Paul on a postdoctoral fellowship at Case Western Reserve University (CWRU), presently employed by CWRU and stationed at LLRS, ERLD, EPA; and Dr. Wilbert J. Lick, Chairman of the Department of Earth Sciences, Case Western Reserve University. Computer usage and assistance was provided in part by Dr. Richard T. Gedney and Mr. Frank B. Molls of the National Aeronautics and Space Administration, Lewis Research Center.

The application of the numerical model to the jetport study was performed under Contract No. DACW39-74-C-0080. The contract was monitored by Dr. D. L. Durham, Wave Dynamics Division, Hydraulics Laboratory, WES, under the general supervision of Dr. R. W. Whalin, Chief, Wave Dynamics Division, and Mr. H. B. Simmons, Chief, Hydraulics Laboratory. Grant Project Officer for the numerical model development were Mr. William L. Richardson, and Mr. David M. Dolan of LLRS, ERLD, EPA.

Successive Contracting Officers at WES were BG Ernest D. Peixotto, CE, and COL G. H. Hilt, CE, Directors of WES. Technical Director was Mr. F. R. Brown.
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LIST OF SYMBOLS

\( A_H \)  
Horizontal eddy viscosity

\( A_V \)  
Vertical eddy viscosity

\( b_0 \)  
Horizontal reference length

\( B_H \)  
Horizontal eddy diffusivity

\( B_V \)  
Vertical eddy diffusivity

\( f_1 \)  
Outer boundary condition for \( u \)

\( f_2 \)  
Outer boundary condition for \( v \)

\( f_3 \)  
Outer boundary condition for \( \Delta T \)

\( f(\Delta T) \)  
Equation of state

\( Fr \)  
Froude number

\( g \)  
Gravitational acceleration

\( g_1 \)  
River outflow boundary condition for \( u \)

\( g_2 \)  
River outflow boundary condition for \( v \)

\( g_3 \)  
River outflow boundary condition for \( \Delta T \)

\( h \)  
Bottom depth

\( h_0 \)  
Reference depth

\( k \)  
Dimensional Coriolis parameter

\( K \)  
Surface heat transfer coefficient

\( P \)  
Pressure

\( P_s \)  
Surface pressure

\( Pr \)  
Turbulent Prandtl number

\( Re \)  
Reynolds number

\( Ro \)  
Nondimensional Coriolis parameter
t  Time

$tw_x$  Surface wind stress in $x$ direction

$tw_y$  Surface wind stress in $y$ direction

$T$  Temperature

$T_E$  Equilibrium temperature

$u$  Velocity in $x$ direction

$u_0$  Reference velocity

$v$  Velocity in $y$ direction

$w$  Velocity in $z$ direction

$A_y$  Constant in variable $A_y$ term

$\beta$  Ratio of vertical to horizontal eddy diffusivities or constant in variable $A_y$ term

$\gamma$  Ratio of vertical to horizontal eddy viscosities

$\Delta T$  Temperature difference

$\Delta \rho$  Density difference

$\rho$  Density

$\rho_0$  Reference density

$\sigma$  Transformed vertical coordinate

$\Omega$  Velocity in $\sigma$ direction

( )  Refers to dimensional quantity
INTRODUCTION

This report is to describe the application of a previously-developed three-dimensional, variable-density hydrodynamic model (Paul and Lick 1973a, b, 1974a, b) to the nearshore Lake Erie area about Cleveland. The purpose of this application is to investigate the effect of a proposed jetport island in Lake Erie on the summer stratification pattern in the nearshore lake area and on the flushing characteristics of the Cuyahoga River outflow into the lake. The effects of such a proposed jetport island are investigated by comparing the model results with and without the jetport in the lake.

For the purpose of this investigation, the model is applied to two different-sized areas of interest: 1. a sixteen mile by sixteen mile area in the lake about Cleveland; and, 2. a two mile by six mile area around Cleveland Harbor. The larger area, hereafter referred to as the nearshore model, is used to investigate the summer stratification pattern in the lake, while the smaller one, referred to as the Cleveland Harbor model, is for investigating the flushing characteristics of the Cuyahoga River outflow. This report will concern itself with a discussion of the application of the model to these two areas of interest and of the initial results obtained from the model. Extensive results from the application are not included.

A brief outline of the present report is as follows. A summary of the equations used in the model, along with a discussion of the underlying assumptions, is first presented. The general numerical procedure used in the solution of the equations is then outlined. The particulars related to the physical areas modeled (grid layout, geometry) are presented next. Finally, some results are presented which qualitatively describe the effects of the jetport island.
The basic equations for the numerical model are derived from the time-dependent, three-dimensional equations of motion for a viscous, heat-conducting fluid. The geometry of the problem is as shown in Figure 1. The following assumptions are made:

1. The pressure is assumed to vary hydrostatically, and therefore
   \[ \frac{\partial p}{\partial z} = \rho g. \]

2. The rigid-lid approximation is made, i.e., \( w(z=0) = 0 \).

3. The Boussinesq approximation is made. This assumes that the density variations are small and can be neglected except in the gravity term.

4. Heat sources and/or sinks in the fluid are neglected.

5. Eddy coefficients are used to account for the turbulent and molecular diffusion effects in both the momentum and energy equations. The horizontal coefficient is assumed to be constant but the vertical coefficient is assumed to be dependent on the local vertical temperature gradient.

6. The variations in the bottom topography are assumed to be gradual.

The hydrostatic approximation is made because, for the cases on which the model will be used, variations in the vertical velocity are small enough that the neglected terms in the vertical momentum equation are small compared to the gravity term. The hydrostatic approximation has been used extensively in the modeling of oceanic basins and lakes (Crowley 1968, Bryan 1969, Gedney and Lick 1972, Simons 1971, 1972, Paskausky 1971) and it has been shown to be valid in the modeling of river discharges (Paul and Lick 1973b). A consequence of this approximation is that the order of the system of equations is reduced, and thus the computational effort required for a solution is reduced.

The rigid-lid approximation is used to damp out the surface gravity waves that would otherwise be present. The time scales associated with the gravity waves are
small compared with other relevant time scales in the model. The inclusion of these smaller time scales would greatly increase the computational time required for a solution. For the discharge of the Cuyahoga River into Lake Erie, it has been estimated (Paul and Lick 1973b) that the inclusion of surface gravity waves in the numerical model would increase the computational time by an order of magnitude. With this approximation, only the surface variations associated with gravity waves are neglected.

The Boussinesq approximation allows the fluid to be treated as incompressible. The density differences that are encountered in the application of the model are small, and therefore this approximation is valid. The coupling between the momentum and energy equations is retained. A consequence of the Boussinesq approximation is that the energy equation reduces to a balance between convection and diffusion.

All heat inputs and outputs to the model are assumed to occur at the boundaries of the model. As a consequence of this, heat transfer by radiation to the water is treated as a surface heat flux.

Turbulence is important in most large scale geophysical fluid mechanical problems. The means of incorporating turbulence into numerical models has so far remained relatively simple. This is mainly due to a lack of basic understanding of the fundamental mechanisms involved in the turbulence associated with these problems. The use of constant eddy coefficients is a relatively simple and useful method for the inclusion of turbulence in numerical models (Paul and Lick 1973a, b, 1974a, b, Crowley 1968, Bryan 1969, Gedney and Lick 1972, Simons 1971, 1972, Paskausky 1971). However, it is known that the vertical temperature variation has an effect on the effective vertical eddy coefficient value (Kaplan 1974, Sundaram, et al 1969, 1970). For this reason, the vertical eddy coefficient is not taken to be constant but to be a function of the local vertical temperature gradient.
The present numerical model allows for variations in the depth of the basin which the outfall discharges into. A standard numerical procedure to fit the variable depth into a model is to vary the number of vertical points in the computational mesh according to the local depth (Bennett 1971). A seemingly more complicated procedure, although a lot simpler in many aspects, is to stretch the vertical coordinate with respect to the local depth. The equations are transformed according to

\[ x \leftrightarrow x, \]
\[ y \leftrightarrow y, \]
\[ \sigma \leftrightarrow z/h(x,y). \]

The equations to be solved are more complicated looking because of the appearance of the depth in the equation, but they are solved for a basin of constant depth in the transformed system. This greatly reduces the programming complexities of the model and makes the inclusion of depth variations simpler.

The assumption of gradual variations in the depth allows a reduced form for the transformed diffusion terms to be used. The transformation used is not conformal and so the transformed diffusion terms involve cross-derivatives of the spatial coordinates. The terms containing derivatives of the depth are neglected with respect to those terms containing only the depth. This approximation is used in meteorological problems when topographic variations are included (Phillips 1957, Smagorinsky, et al 1965).

The resulting system of transformed equations, in non-dimensional form, are the following:

\[
\frac{\partial u}{\partial t} + \text{Re} \left[ \frac{1}{h} \frac{\partial (hu)}{\partial x} + \frac{1}{h} \frac{\partial (hv)}{\partial y} + \frac{\partial \Omega}{\partial \sigma} \right] + \text{Rov} = - \frac{\partial P}{\partial x} + \frac{1}{h} \frac{\partial }{\partial x} \left( \frac{\partial \Omega}{\partial x} + \frac{3}{h} \frac{\partial u}{\partial y} + \frac{3}{y} \frac{\partial (hu)}{\partial y} \right)
\]

\[
\frac{\partial \dot{v}}{\partial t} + \text{Re} \left[ \frac{1}{h} \frac{\partial (hv)}{\partial x} + \frac{1}{h} \frac{\partial (hv)}{\partial y} + \frac{\partial \Omega}{\partial \sigma} \right] - \text{Rou} = - \frac{\partial P}{\partial y} + \frac{1}{h} \frac{\partial }{\partial y} \left( \frac{\partial \Omega}{\partial y} + \frac{3}{y} \frac{\partial v}{\partial x} + \frac{3}{x} \frac{\partial (hv)}{\partial x} \right)
\]

\[
\text{For} \quad \frac{\partial \Omega}{\partial \sigma} + \text{Re} \left[ \frac{1}{h} \frac{\partial (hu)}{\partial x} + \frac{1}{h} \frac{\partial (hv)}{\partial y} + \frac{\partial \Omega}{\partial \sigma} \right] - \text{Rou} = - \frac{\partial P}{\partial \sigma} + \frac{1}{h} \frac{\partial }{\partial \sigma} \left( \frac{\partial \Omega}{\partial \sigma} + \frac{3}{\sigma} \frac{\partial u}{\partial x} + \frac{3}{x} \frac{\partial (hu)}{\partial x} \right)
\]

\[
\text{For} \quad \frac{\partial \Omega}{\partial \sigma} + \text{Re} \left[ \frac{1}{h} \frac{\partial (hv)}{\partial x} + \frac{1}{h} \frac{\partial (hv)}{\partial y} + \frac{\partial \Omega}{\partial \sigma} \right] - \text{Rou} = - \frac{\partial P}{\partial \sigma} + \frac{1}{h} \frac{\partial }{\partial \sigma} \left( \frac{\partial \Omega}{\partial \sigma} + \frac{3}{\sigma} \frac{\partial v}{\partial x} + \frac{3}{x} \frac{\partial (hv)}{\partial x} \right)
\]
\[ \text{Pr} \frac{\partial T}{\partial t} + \text{Re} \left[ \frac{1}{h} \frac{\partial (hu \Delta T)}{\partial x} + \frac{1}{h} \frac{\partial (hv \Delta T)}{\partial y} + \frac{\partial \Delta T}{\partial x} \right] = \frac{1}{h} \frac{\partial}{\partial x} \left( h \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( h \frac{\partial T}{\partial y} \right) \]

where:

\[ \begin{align*}
\Delta \rho &= f(\Delta T), \\
\sigma &= z/h(x,y), \\
\Omega &= \frac{1}{h} \left[ w - \sigma (u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y}) \right] = \frac{\partial \sigma}{\partial t},
\end{align*} \]

The equilibrium temperature is defined as the temperature at the surface for which there is no heat transfer.

\[ \rho_0 = \bar{\rho}(T_E) \text{ and} \]

\[ \begin{align*}
u_0 &= \text{reference velocity}, \\
b_0 &= \text{horizontal reference length}, \\
h_0 &= \text{vertical reference length}, \\
A_H &= \text{horizontal eddy viscosity}, \\
A_V &= \text{vertical eddy viscosity}, \\
B_H &= \text{horizontal eddy diffusivity}, \\
B_V &= \text{vertical eddy diffusivity}, \\
T_E &= \text{equilibrium temperature} \\
k &= \text{Coriolis parameter}, \\
f(\Delta T) &= \text{equation of state and} \\
\bar{()} &= \text{refers to dimensional quantity}.\]
The conservative form of the convective terms is used as this has been found by Arakawa (1966) to be advantageous for numerical computations. The density is taken as only a function of temperature. The energy equation is nondimensionalized in terms of temperature differences. The effect of round-off error will be less in the evaluation of the derivatives if the differences are used.

The rigid-lid condition is difficult to apply in a numerical solution of the above system of equations. To alleviate this difficulty, an additional equation, a Poisson equation for the pressure, which contains the rigid-lid condition, can be derived. This is accomplished by taking the divergence of the vertically integrated horizontal momentum equations and using the vertically integrated continuity and hydrostatic pressure equations. The Poisson equation is:

\[
\frac{\partial}{\partial x} \left( \frac{2}{\rho} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{2}{\rho} \frac{\partial P}{\partial y} \right) = -h \frac{\partial^2}{\partial t^2} (\nabla \cdot \mathbf{u})_{\text{c=0}}
\]

\[+ (\frac{b_v}{h_0})^2 \frac{\partial}{\partial x} \left( \frac{1}{h} \frac{\partial u}{\partial x} \right)_{\text{c=1}} - \frac{1}{h} \frac{\partial}{\partial x} (\frac{\partial u}{\partial \sigma})_{\text{c=0}} \]

\[+ (\frac{b_v}{h_0})^2 \frac{\partial}{\partial y} \left( \frac{1}{h} \frac{\partial v}{\partial y} \right)_{\text{c=1}} - \frac{1}{h} \frac{\partial}{\partial y} (\frac{\partial v}{\partial \sigma})_{\text{c=0}} \]

\[= \frac{\partial}{\partial x} \left[ \int_0^l \left( \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \frac{\partial}{\partial \sigma} \left( \frac{\partial h}{\partial x} \Delta \rho \right) \right) \right] d\sigma \]

\[- \frac{1}{h} \frac{\partial}{\partial y} \left( \frac{\partial}{\partial \sigma} \left( \frac{\partial h}{\partial x} \Delta \rho \right) \right) \]

\[= \frac{\partial}{\partial x} \left[ \int_0^l \left( \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial}{\partial \sigma} \left( \frac{\partial h}{\partial x} \Delta \rho \right) \right) \right] d\sigma \]

\[- \frac{1}{h} \frac{\partial}{\partial y} \left( \frac{\partial}{\partial \sigma} \left( \frac{\partial h}{\partial x} \Delta \rho \right) \right) \]

\[= \frac{\partial}{\partial x} \left( \frac{\partial}{\partial \sigma} \left( \frac{\partial h}{\partial x} \Delta \rho \right) \right) \]

\[= \frac{\partial}{\partial x} \left( \frac{\partial}{\partial \sigma} \left( \frac{\partial h}{\partial x} \Delta \rho \right) \right) \]
The term $P_s$ is the integration constant resulting from the vertical integration of the hydrostatic pressure equation and is the surface pressure, i.e., the pressure at the surface $z = 0$. It is a function of both $x$ and $y$. This surface pressure can be interpreted in terms of the equivalent height of water above or below the surface $z = 0$ required to provide the prescribed pressure. In this way, surface displacements of water, neglecting gravity waves, can be compared between this rigid-lid model and the equivalent free surface model.

The vertical velocity at the surface $z = 0$ has not been set to zero in the right-hand-side of the Poisson equation for the surface pressure. This is because a corrective procedure (Paul and Lick 1973b) is used in the numerical solution to eliminate accumulative error in the satisfaction of the continuity equation.

The following boundary conditions are used with the above system of equations:

River outflow

$u = g_1(y, z)$

$v = g_2(y, z)$

$\Delta T = g_3(y, z)$

Shore

$u = 0$

$\frac{\partial \Delta T}{\partial n} = 0$

Bottom

$v = 0$

$w = 0$

$\frac{\partial \Delta T}{\partial z} = 0$

Surface

$\frac{\partial v}{\partial z} = twy$

$\frac{\partial \Delta T}{\partial z} = K\Delta T$
\[ w = 0 \]

**Outer Boundary**

either \( \frac{\partial u}{\partial n} = 0 \) or \( u = f_1 \)

\( \frac{\partial v}{\partial n} = 0 \) \( v = f_2 \)

\( \frac{\partial \Delta T}{\partial n} = 0 \) \( \Delta T = f_3 \)

**Pressure Conditions**

\( \frac{\partial P}{\partial n} \) integrated \( x \) or \( y \) momentum equation,

specify pressure level at one point.

The functional forms \( g_1 \), \( g_2 \), and \( g_3 \) are the specified velocity and temperature profiles across the river outfall. The bottom and shore are taken as no-slip, impermeable, insulated surfaces. A heat transfer condition proportional to the temperature difference (Paul and Lick 1973b) and a wind-dependent stress are imposed at the surface. The normal derivative pressure boundary conditions are derived from the appropriate vertically integrated momentum equation. The pressure must be specified at one point to make its solution unique. The boundary conditions applied at the outer \( x \) and \( y \) boundaries are either that the normal derivatives of the velocity and temperature are zero, or that the velocity and temperature are specified.
The general arrangement of variables in the grid system is identical to that used previously by Paul and Lick (1974b). The horizontal velocities are defined at integral nodal points, the temperature is defined at half-integral nodal points in the horizontal and integral nodal points in the vertical, and the surface pressure is defined at half-integral nodal points in the horizontal. Figure 2 indicates typical horizontal and vertical grid sections and shows the relative positions of the various variables.

The finite difference approximations to the equations are derived by integrating the equations over nodal cells. Either the mid-point or trapezoidal integration rule is used to evaluate these integrals. Typical nodal cells are indicated in Figure 3. Part of the rationale behind the arrangement of the variables is to provide cells which lend themselves to easy use of the integration rules.

In the derivation of the finite difference equations, variables are sometimes required at points where they are not defined. In these circumstances, the undefined quantity is taken as the simple average of the neighboring values.

The Euler explicit time scheme is used exclusively in the present model. This is where the time derivative is written as a simple forward time difference and the rest of the equation is evaluated with the previously calculated values. The three time level explicit time scheme used by Paul and Lick (1973b) has been found not to be as stable as the Euler explicit scheme. Even the use of the filtering procedure suggested by Bryan (1969) for the three time level scheme did not provide enough stability when inertia dominated flows are calculated.

The following scheme is used for the solution of the difference equations:

1. It is assumed that values from the previous time step are available.
2. The surface pressure is calculated with the right-hand-side of the equation evaluated from the previous time step values.

3. The temperature is calculated by the explicit time scheme.

4. The density is calculated from the equation of state.

5. The horizontal velocities are calculated by the explicit time scheme.

6. The vertical velocity is calculated by vertically integrating the continuity equation from the bottom.

7. The present time step is now complete.

At each time step the Poisson equation for the surface pressure is solved by the error vector propagation (EVP) method (Roache 1971). This is an efficient direct method of solving the equation in non-rectangular geometry; however, discretion in its use is required because of the sensitivity of the results to the precision of the arithmetic used in the calculations.

The forcing term in the Poisson equation for the surface pressure involves a time derivative of the vertical velocity at the surface. The rigid-lid condition is that this velocity is zero. However, by the numerical vertical integration of the continuity equation, some non-zero value for the vertical velocity at the surface is obtained. This deviation from zero is an indication that the continuity equation is not satisfied exactly by the difference solution. This error in the satisfaction of the continuity equation can grow in time if it is not accounted for. The source of this error is the inability to solve the Poisson equation for the surface pressure exactly.

The corrective procedure devised by Hirt and Harlow (1967) is used to correct for this error. The procedure involves the term \( \frac{\partial}{\partial t} (\Omega(z=0)) \). This term is written as a simple backward time difference with the present time level set to zero and the previous time level set to the value calculated from the previous integration of the continuity equation. This procedure was effective in previous calculations on river discharge problems and ther-
mal plumes (Paul and Lick 1973a, b, 1974a, b).

After the temperatures are calculated by the explicit time scheme, they are checked to see if static stability is satisfied, i.e., if the temperatures decrease monotonically downward (assuming that density increases with decreasing temperature). When a static instability is encountered, an infinite mixing procedure is used (Paul and Lick 1973b). This procedure just averages the temperature over any unstable region.
APPLICATION OF THE MODEL

The particular grid layout used depends on the actual geometry to be described. The shoreline used in the model is determined by where the water depths become less than some certain value. Zero depth is not chosen as the shoreline because the vertical coordinate transformation used is singular for zero depths. This use of non-zero depth shorelines does not appreciably affect the boundary used and is not a restriction on the model. For both areas modeled, a depth of twenty feet was chosen as the shoreline. For the nearshore model, this is reasonable because of the extremely small area neglected compared to the total area modeled. For the Cleveland Harbor model, this is reasonable because of the minimum depths maintained in the harbor by dredging operations.

The vertical eddy coefficient is taken as dependent on the local vertical temperature gradient. This is similar to the form suggested by Sundaram, et al (1969, 1970) and is identical to that used in a previous application of this model (Paul and Lick 1974b). The expression for the vertical eddy coefficient $A_v$ is:

$$A_v = \alpha + \beta \frac{\partial T}{\partial z}$$

where $\alpha$ and $\beta$ are constants dependent on the local conditions of the physical system modeled. The constant $\alpha$ is chosen so that in the absence of vertical temperature gradients, the eddy coefficient is equal to that which would be used for a constant eddy coefficient.

The stress imposed on the water surface due to the wind action is calculated from the formulae developed by Wilson (1960). These formulae have been successfully used in numerical calculations of wind-driven circulations in lakes (Haq and Lick 1974, Sheng and Lick 1975) and in a previous application of this present model to a power plant outfall (Paul and Lick 1974b).
Application to the Nearshore Model

For the nearshore model, a sixteen mile by sixteen mile area in Lake Erie near Cleveland is chosen. This area is similar to that used for a constant density, free-surface model of the Lake Erie area about Cleveland (Sheng and Lick 1975). A constant horizontal grid spacing of one mile is used. The jetport island used in the model is two miles by three miles, located five miles off Cleveland in approximately fifty feet of water. The horizontal grid layouts without and with the jetport island are shown in Figures 4 and 5, respectively. The arrangement of the variables in the horizontal grid without the jetport island is shown in Figure 6. Seven grid points are used in the vertical dimension. Arrangement of the variables in the vertical sections is indicated in Figure 7. The bottom topography used in the model is the same as that used by Sheng and Lick (1975) and was obtained from the Lake Erie Survey Charts. The bottom topography is shown in Figure 8.

The boundary conditions for the open water boundaries of the model are the following. Along the outer x boundary in the lake (boundary 2 of Figure 9), the velocities and temperature are specified. The pressure boundary condition is obtained from the vertically integrated x momentum equation. The velocity and temperature values are obtained from an application of the numerical model to either the Central Basin of Lake Erie or the entire lake (Gedney, Molls and Paul 1975). Note that in the present application of the model, these outer boundary values remain constant in time. This is a limitation of the application of the model; however, the model will give good qualitative results for time scales such that the relative changes in the outer boundary values would be small. The model does have the capability to handle time-dependent boundary conditions. For the present appli-
cation this aspect was not used. Along the two y boundaries in the lake (boundaries 1 and 3 of Figure 9), it is assumed that the variables are smoothly varying, i.e., the first normal derivatives of the velocities and the temperature are zero and the second normal derivative of the pressure is zero.
For the Cleveland Harbor model, a two mile by six mile area around Cleve-
land Harbor is chosen. A variable-spacing grid system is used in this applica-
tion. A fine grid, 125 feet in the y direction and 140 feet in the x direction, is used in the vicinity of the river entrance into the harbor. The grid spacing gradually increases as the distance from the river entrance increases until a grid spacing of \( \frac{1}{4} \) mile is reached in both horizontal directions. The horizontal grid layout used is shown in Figure 10. The arrangement of variables in the horizontal and vertical sections is identical to that used in the nearshore model. Five grid points are used in the vertical dimension. The bottom topography used is taken from the Cleveland Harbor Chart of the Lake Survey Center. The bottom topography is shown in Figure 11.

The boundary conditions for the open water boundaries of the model are the following. Along the outer x boundary (boundary 3 of Figure 12), the velocities, temperature, and pressure are specified. These values are obtained from the results of the nearshore model, either with or without the jetport island included. As in the application to the nearshore model, these boundary values remain constant in time. The comments concerning this procedure in the nearshore model are relevant here also. Along the two y boundaries in the lake (boundaries 1 and 3 of Figure 12), it is assumed that the variables are smoothly varying.
RESULTS

The nearshore and Cleveland Harbor models discussed in the previous sections were run with typical summer conditions for time periods of less than one day. A limited number of results are presented to indicate the qualitative effects of the jetport island on the stratification pattern in the lake and on the flushing characteristics of the Cuyahoga River outflow into the lake.

The input parameters for the test cases are shown in Table I. The wind direction and speed, the ambient lake temperature, and the thermocline location were provided by the U.S. Army Waterways Experiment Station as the typical summer conditions. The horizontal eddy coefficient was chosen as a typical value for lake calculations (Sheng and Lick 1975), the vertical eddy coefficient was chosen as a typical value for Lake Erie under the prescribed wind speed (Gedney 1971), and the equation of state was taken as a least-squares curve fit of temperature vs. density data (Weast 1972). For the test cases, the water surface is taken as insulated; this condition should not affect the results over the time scale of the calculations.
Results for the Nearshore Model

The results for the nearshore model without the jetport island are shown in Figures 19 to 30, and the results with the jetport island are shown in Figures 37 to 48. Results are presented for times of 7.4 hours and 14.8 hours of real time simulation. The initial conditions used for the test cases are indicated in Figures 13 to 18 and 31 to 36.

Comparing the horizontal velocity plots (Figures 19-21, 25-27, 37-39, 43-45), there is the initial indication that the jetport island influences only a limited localized area (2 to 3 miles from the island). However, examination of the horizontal isotherm plots (Figures 22-24, 28-30, 40-42, 46-48) indicates that the influence of the jetport island on the temperature structure in the lake can extend out quite a few miles and that the area of influence increases with time (about 3 to 4 miles after 7.4 hours and about 6 to 8 miles after 14.8 hours). This latter aspect is indicated in Figures 40-42 and 46-48, which are for times of 7.4 hours and 14.8 hours, respectively. More detailed examination of the velocity plots also reveals that the jetport influence is more than a localized effect. This large area of influence is due to the upwelling of cold water on the eastern edge of the island and the downwelling of warm water on the western edge. These upwellings and downwellings result in changes to the stratification structure in that area of the lake. It should be remembered that this is a variable-density model and that changes in the temperature structure do cause changes in the velocity pattern. For previous calculations with a constant-density, free surface model (Sheng and Lick, 1975), it was found that the jetport island only exerted an influence over a distance of one to two miles into the lake.

These results are for a constant direction wind. Changes in wind direction will result in changes in the location of the upwelling and downwelling.
regions around the island. Wind direction changes and wind speed changes would modify the size and shape of the region influenced by the jetport island.

The results presented here are only preliminary and are used to indicate the qualitative influence of the jetport island on the lake. These results show that there will be upwellings and downwellings around the island; the magnitude and location of these will depend on the wind direction and speed. These upwellings and downwellings will result in mixing between the hypolimnion and the epilimnion in this region of the lake and may possibly cause erosion of the thermocline in this area. It should be remembered that the results of the model are influenced by the values chosen for the parameters. In particular, it has been shown (Paul, Chen and Lick 1975) that the parameter values for the vertical eddy diffusivity can have a large effect on the vertical temperature structure. Further experimentation with the numerical model is required to obtain detailed knowledge of the influence of the vertical diffusivity term on the results. Other items that should be investigated are the value of the horizontal eddy diffusivity, the effect of heat transfer at the water surface, the effect of maintaining constant temperature and velocity values along the outer boundary and the effect of increasing the region of computation.
Results for the Cleveland Harbor Model

The results for the Cleveland Harbor model are shown in Figures 49 to 52 for 4 hours of simulation time. The outer boundary values for the results were interpolated from the nearshore results without the jetport island after 14.8 hours of simulation time. The initial conditions are zero velocities and the temperatures extrapolated from the specified values along the outer boundary. The river was specified with a discharge of 4.8 m³/sec, a maximum velocity of 3.0 cm/sec and a uniform temperature of 75°F (24°C).

The results for the horizontal currents at the surface and at 20 ft. from the surface are shown in Figures 49 and 50. The currents in the harbor are predominantly wind driven; the increase in the surface current magnitude in the harbor along the wind direction is due to the importance of horizontal diffusion in the momentum balance. Water enters the harbor at depth through both entrances. The river flow is small compared to the wind driven currents but its effect in bringing in warm water can be seen in the isotherm plots in Figures 51 and 52. Because the river water is lighter than the harbor water (which is almost of uniform temperature), it rises and spreads over the surface as it enters the harbor.

For this simulation, the harbor flow is not affected significantly by the lake flow. Specifying the outer boundary conditions from the nearshore results with the jetport island does not noticeably alter the flow in the harbor. This is because the boundary conditions are not significantly different for these two cases. The fact that the boundary conditions are not significantly different is only due to the particular wind direction used in the model. For some wind directions, the jetport island could significantly alter the lake conditions just out from the harbor.

The results presented here are only preliminary and are used to indicate what flow might be expected in the harbor. This model will probably require
more numerical experimentation than the nearshore model. In particular, the follow-
ing should be investigated further: the effect of the horizontal and vertical eddy
diffusivities, the effect of river discharge flow rate, the effect of the western
connection for the harbor with the lake (which was neglected here), and the effect
of the boundary conditions.
REFERENCES


Kaplan, S. 1974. Unpublished material on eddy diffusivities in a pond. Dept. of Earth Sciences, Case Western Reserve University, Cleveland, Ohio.


\[ b_0 \]

\[ h_o \]

\[ B_V = A_V \]

\[ B_H = A_H \]

\[ u_o \]

epilimnion temperature 75°F (24°C)

hypolimnion temperature 55°F (13°C)

thermocline depth 30 ft (915 cm)

wind 12 mph (536 cm/sec) from south

\[ Re = \frac{u_o b_o}{A_H} \]

387 (1290)*

\[ Ro = \frac{k b_o}{A_H} \]

6.66 x 10³ (2.21 x 10⁴)*

\[ Fr = \frac{u_o}{\nu g h_o} \]

9.6 x 10⁻³

*Values used for Cleveland Harbor Model.

Parameters Used in Application of Model

Table I
Geometry for the Model

Figure 1
Arrangement of Variables in Grid sections

Figure 2
Typical Nodal Cells for Grid System

Figure 3
Horizontal Grid Layout for Nearshore Model Without Jetport Island

Figure 4
Horizontal Grid Layout for Nearshore Model with Jetport Island

Figure 5
Arrangement of Variables in the Horizontal Grid for the Nearshore Model Without Jetport Island

Figure 6
vertical plane at nodal-horizontal section
(section A-A or A'-A')

vertical plane at half-nodal-horizontal section
(section B-B or B'-B')

Arrangement of Variables in Vertical Sections for the Nearshore Model

Figure 7
Bottom Topography Contours for Nearshore Model

Figure 8
Boundaries for Nearshore Model

Figure 9
Horizontal Grid Layout for Cleveland Harbor Model

Figure 10
Bottom Topography Contours for Cleveland Harbor Model

Figure 11
Open Water Boundaries for Cleveland Harbor Model

Figure 12
Initial Conditions for Surface Isotherms

Figure 16
Initial Conditions for Isotherms at 20 ft Depth

Figure 17
Initial Conditions for Isotherms at 40 ft Depth

Figure 18
Velocities at 20 ft Depth for 7.4 Hours

Figure 20
Velocities at 40 ft Depth for 7.4 Hours

Figure 21
Surface Isotherms for 7.4 Hours

Figure 22
Isotherms at 20 ft Depth for 7.4 Hours

Figure 23
Isotherms at 40 ft Depth for 7.4 Hours

Figure 24
Velocities at 20 ft Depth for 14.8 Hours

Figure 26
Velocities at 40 ft Depth for 14.8 Hours

Figure 27
Surface Isotherms for 14.8 Hours

Figure 28
### Isotherms at 20 ft Depth for 14.8 Hours

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**Figure 29**
Isotherms at 40 ft Depth for 14.8 Hours
Figure 30
Initial Conditions for Velocities at 40 ft Depth

Figure 33
Initial Conditions for Surface Isotherms

Figure 34
Initial Conditions for Isotherms at 20 ft Depth

Figure 35
Initial Conditions for Isotherms at 40 ft Depth

Figure 36
Velocities at 20 ft Depth for 7.4 Hours

Figure 38
Velocities at 40 ft Depth for 7.4 Hours

Figure 39
Surface Isotherms for 7.4 Hours

Legend

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Figure 40
Isotherms at 20 ft Depth for 7.4 Hours

Figure 41
Isotherms at 40 ft Depth for 7.4 Hours

Figure 42
Velocities at 20 ft Depth for 14.8 Hours

Figure 44
Velocities at 40 ft Depth for 14.8 Hours

Figure 45
Surface Isotherms for 14.8 Hours

Figure 46
Isotherms at 20 ft Depth for 14.8 Hours

Figure 47
Isotherms at 40 ft Depth for 14.8 Hours
Figure 48
Surface Velocities for 4.0 Hours
Figure 49
Surface Isotherms for 4 Hours

Figure 51
Isotherms at 20 ft Depth for 4 Hours

Figure 52