Research Report 234

# ANALYTICAL AND EXPERIMENTAL STUDY OF A MELTING PROBLEM WITH NATURAL CONVECTION

by

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JULY 1967

U.S. ARMY MATERIEL COMMAND
COLD REGIONS RESEARCH & ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

DA Task 1VO14501B52A02

# PREFACE

This paper was prepared by Dr. Yin-Chao Yen, Head, Physical Sciences Branch, Research Division (Mr. James A. Bender, Chief), U.S. Army Cold Regions Research and Engineering Laboratory.

The author wishes to express his appreciation to PFC Thomas Nasman for his help in conducting the experiment and analyzing the experimental data.

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# NOMENCLATURE

| | |
|---|---|
| $C_p$ | heat capacity |
| g | gravitational acceleration |
| H(t) | heat flux |
| $H^+(t)$ | dimensionless heat flux given by eq 10 or 11 |
| k | thermal conductivity |
| h | heat transfer coefficient |
| L | latent heat of fusion |
| $N_{Nu}$ | Nusselt number |
| $N_{Pr}$ | Prandtl number |
| $N_{Ra}$ | Rayleigh number |
| $R_{\Delta T}$ | dimensionless parameter defined as $(T_s - T_m)/(T_m - T_0)$ |
| S(t) | solid liquid interface position |
| $S_c$ | transitional melting front |
| t | time |
| T | temperature |
| $T_m$ | melting temperature |
| $T_0$ | initial ice temperature |
| $T_s$ | temperature of the warm plate |
| x | distance |
| $\alpha$ | thermal diffusivity |
| $\beta$ | thermal expansion coefficient |
| $\sigma$ | dimensionless solid liquid interface position defined as $S/S_c$ |
| $\delta$ | dimensionless thermal boundary-layer thickness |
| $\phi$ | dimensionless parameter defined as $L/C_{p_2}(T_m - T_0)$ |
| $\rho$ | density |
| $\tau$ | dimensionless time defined as $\alpha_2 t/S_c^2$ |
| $\mu$ | viscosity |

Subscripts

| | |
|---|---|
| 1 | liquid phase |
| 2 | solid phase |

## SUMMARY

The correlation by O'Toole and Silveston (1959) of natural convection heat transfer for fluids confined between two parallel horizontal plates has been extended to the case involving phase change. The new correlation, which is applicable for melting from below, is

$$N_{Nu} = \frac{hS}{k_l} = 0.104\,(N_{Pr})^{0.084}\,(N_{Ra})^{[1-0.0114(\phi/R_{\Delta T})^{1.2}]/3}$$

where h is the heat transfer coefficient, S is the melting front, $k_l$ is the thermal conductivity of water, $\phi$ is defined as $L/C_{p_2}(T_m - T_0)$ and $R_{\Delta T}$ is defined as $(T_s - T_m)/(T_m - T_0)$. L is the latent heat of fusion; $C_{p_2}$ is the specific heat of ice; and $T_s$, $T_m$, and $T_0$ are the warm plate, melting and initial ice temperatures, respectively. The above expression is good for the water-ice system and is valid for $\phi$ varying from 7.30 to 24.50 and $R_{\Delta T}$ varying from 0.350 to 2.60.

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## INTRODUCTION

The problem of a horizontal layer of fluid heated from below has been studied extensively both theoretically and experimentally. A search for literature, however, failed to reveal any investigations related to the problem of melting from below with natural convection. Tien and Yen (1966) first presented an approximate analytical solution of this problem. In a recent paper, Yen et al. (1966) conducted a careful experimental investigation of the same problem to confirm the theoretical study. The experimental data were qualitatively in agreement with the results predicted by the analytical solutions. In the present analysis, the correlation by O'Toole and Silveston (1959) of natural convection heat transfer for fluids confined between two parallel horizontal plates has been extended to the case involving melting from below.

## THEORETICAL CONSIDERATIONS

For the classical Stefan's problem, the following differential equations describe the temperature change in the liquid and solid phases respectively:

$$\frac{\partial T_1}{\partial t} = \alpha_1 \frac{\partial^2 T_1}{\partial x^2} \quad x<S(t) \qquad (1)$$

$$\frac{\partial T_2}{\partial t} = \alpha_2 \frac{\partial^2 T_2}{\partial x^2} \quad x>S(t) \qquad (2)$$

with the following initial and boundary conditions:

$$\text{at } t = 0, \; T_2 = T_0, \; \text{for all } x \geq 0, \; S = 0 \qquad (3)$$

$$x = S(t), \; T_1 = T_2 = T_m \qquad (4)$$

and at the moving interface

$$-k_1 \frac{\partial T_1}{\partial x} = -k_2 \frac{\partial T_2}{\partial x} + L\rho \frac{ds}{dt} \qquad (5)$$

$$\text{at } x \rightarrow \infty \qquad T_2 = T_0 \qquad (6)$$

$$x = 0, \qquad T_1 = T_s \qquad (7)$$

where $T_1$, $T_2$, $T_m$, $T_s$ and $T_0$ are the liquid, solid, melting, heat source and initial ice sample temperatures respectively; $k_1$, $\alpha_1$, $k_2$ and $\alpha_2$ are the corresponding thermal conductivities and diffusivities of the liquid and solid phases; and x is the distance measured from the heat source.

Complete solutions of eq 1 and 2 with conditions 3 through 7 can be found in Carslaw and Jaeger (1959). Equation 1 accounts only for the case in which the heat transfer in the liquid phase is purely by conduction. In the actual situation, however, it is clear that since the liquid resulting from melting forms the lower part of the system and is subject to a negative temperature gradient (for melting, $T_s \geq T_m$), the system is unstable hydrodynamically for most liquids whose density decreases as temperature increases. Therefore, the liquid phase resulting from the melting resembles the classical Rayleigh's problem. However, it should be borne in mind that the present problem is far more complicated and differs from the previously mentioned classical problem in a number of fundamental ways, such as in the nonlinear and time-dependent temperature gradient, moving interface, and density inversion somewhere between the warm surface and the water-ice interface. For two rigid boundaries, the onset of instability begins when the Rayleigh number exceeds its critical value, which is reported to be approximately 1708. For the present case, with a moving upper boundary which advances upward as melting progresses, it is clear from the definition of the Rayleigh number, $g\rho_1^2\beta C_{p_1} S^3(T_s-T_m)/\mu k$, that it increases as the melting proceeds (since the value of S increases as the melting progresses). Accordingly the heat transfer in the liquid phase changes mode from conduction to convection as soon as the Rayleigh number reaches the critical number. The transitional melting front, $S_c$, which characterizes the change of heat transfer mode is

$$S_c = [1708\,\mu_1 k_1 / g\,\beta_1\,\rho_1^2\,C_{p_1}\,(T_s - T_m)]^{\frac{1}{3}}. \tag{8}$$

It should be recognized that the value of 1708 in eq 8 is also taken only as an approximation since in the present problem the system involves neither two rigid boundaries nor one rigid and one free boundary. It is understood then that as soon as the transitional melting front $S_c$ is reached, eq 1 will no longer describe the heat transfer mode in the liquid phase. To simplify the analysis of this hydrodynamically unstable problem, it is assumed that the rate at which the temperature distribution reaches its steady state in the liquid phase is much greater than the melting rate. If this is so, the problem reduces to a much simpler melting problem in which a prescribed heat flux is imposed on the liquid-solid interface.

Estimation of the heat flux

In previous papers by Tien and Yen (1966) and Yen et al. (1966), the heat flux expression was derived from a correlation by O'Toole and Silveston (1959) for the natural convection heat transfer for fluids confined between two parallel horizontal plates. The correlation used for these previous papers was

$$N_{Nu} = \frac{hS}{k_1} = 0.104\,(N_{Pr})^{0.084}\,(N_{Ra})^{0.305} \tag{9}$$

in which $N_{Nu}$, $N_{Pr}$ and $N_{Ra}$ are Nusselt, Prandtl and Rayleigh numbers, respectively; h is the heat transfer coefficient; and S is the melting front. Without any modification to take into account the fact that this was a problem with a moving upper boundary due to melting and a time-dependent temperature gradient, eq 9 was used to derive an expression for dimensionless heat flux (Tien and Yen, 1966) defined as

$$H^+(t) = \frac{S_c}{k_2\,(T_m - T_0)}\,H(t) = R_{\Delta T}\,\frac{k_1}{k_2}\,\sigma^{-0.085}. \tag{10}$$

In the recent paper by Yen et al. (1966), the foregoing expression was used to describe the heat flux at the water-ice interface. The conduction equation for the solid phase was then solved with the heat balance integral technique. From that study, it was concluded that the results computed from analytical solutions were about 15% higher than the corresponding experimental data for $T_s$ = 18.8C, $T_0$ = -4.8C and $\tau - \tau_c$ = 1500. It was also noted that as the value of $T_s$ lowered the discrepancy increased. The cause of these differences was then considered to be the fact that there is a 4C layer somewhere between the warm surface and the water-ice interface. (The existence of the 4C layer has not been investigated experimentally.) As $T_s$ was reduced, the distance between the 4C water layer and the interface could be increased. Due to this increase, the rate of heat transfer from the 4C water layer to the melting front, and subsequently the rate of melting, were decreased.

To apply the correlation developed by O'Toole and Silveston (1959) to the moving boundary problem, it was decided that in the present analysis an arbitrary constant B instead of 0.305 should be used as the exponent to the Rayleigh number in eq 9. Following an approach similar to that used in the previous papers by the present author, it can be shown that the dimensionless heat flux will have the following form:

$$H^{+}(t) = \frac{S_c}{k_2 (T_m - T_0)} H(t) = R_{\Delta T} \frac{k_1}{k_2} \sigma^{-(1-3B)} \tag{10}$$

or

$$H^{+}(t) = R_{\Delta T} \frac{k_1}{k_2} \sigma^{-A} \tag{11}$$

where A = 1-3B. A is an unknown value which is presumed to be dependent on thermal parameters $\phi$ and $R_{\Delta T}$. After obtaining the expression for the heat flux function for the water-ice interface (see eq 11), the heat conduction equation in the solid phase was solved and can be summarized by the following pair of nonlinear differential equations:

$$\frac{d\eta}{d\tau} = \frac{4}{\phi} \left[ \frac{3(1+\phi)}{\eta} - R_{\Delta T} \sigma^{-A} \left(\frac{k_1}{k_2}\right) \right] \tag{12}$$

$$\frac{d\sigma}{d\tau} = \frac{1}{\phi} \left[ R_{\Delta T} \sigma^{-A} \left(\frac{k_1}{k_2}\right) - \frac{3}{\eta} \right] \tag{13}$$

where $\eta$ is defined as $\delta - \sigma$ or the difference between the dimensionless thermal boundary layer thickness $\delta$ and dimensionless liquid-solid interface $\sigma = S/S_c$; $\phi$ is defined as $L/C_{p_2} (T_m - T_0)$ in which L is the latent heat of fusion, $C_{p_2}$ is the specific heat of ice, and $R_{\Delta T}$ is defined as $(T_s - T_m)/(T_m - T_0)$.

## EXPERIMENTAL AND ANALYTICAL RESULTS

The experimental apparatus and procedure are described in detail in a paper by Yen et al. (1966). In all experiments, bubble-free, homogeneous ice samples were prepared before the melting experiment. Caution was always taken to eliminate any possible entrainment of air during the coupling operation of the assembled melting chamber and the heat source. This was necessary to get reliable and reproducible results. As in previous papers, Figures 1-3 are plots of dimensionless melting front $\sigma$ versus dimensionless time $\tau - \tau_c$. It should be realized that when $\tau - \tau_c = 0$, $\sigma = 1$. In the figures, black dots represent the experimental data. Equations 12 and 13 were integrated simultaneously by the method of Runge-Kutta. For each pair of $R_{\Delta T}$ and $\phi$, a trial and error procedure was used to discover a specific value of A that would give the best fit to the experimental data of the corresponding set values of $R_{\Delta T}$ and $\phi$. The computed results along with the values of A are indicated by crosses in Figures 1-3. Table I is a summary of the experimental parameters and the values of A. Altogether 15 experiments were conducted. Three original ice temperatures, $T_0$, were used, i.e., -6.5, -13.0 and -22C, respectively. Ten heat source temperatures, $T_s$, varying from 7.72 to 18.80C, were employed. From Figures 1-3 it can be seen that the computed results give a slightly higher value than the experimental data for small values of dimensionless time, $\tau - \tau_c$, while for large values of $\tau - \tau_c$ the computed results show a little lower value than the experimental data. However, in general, the results from theory and experiment are in close agreement.

Table I. Summary of experimental parameters and values of A.

| Exp no | $T_0$(°C) | $T_s$(°C) | $\phi$ | $R_{\Delta T}$ | A |
|---|---|---|---|---|---|
| 13 | -22.00 | + 7.72 | 7.30 | 0.350 | 0.440 |
| 17 | -22.00 | +10.05 | 7.30 | 0.460 | 0.320 |
| 14 | -22.00 | +13.10 | 7.30 | 0.600 | 0.230 |
| 11 | -13.00 | + 7.72 | 12.50 | 0.600 | 0.450 |
| 3-5 | -13.00 | +10.60 | 12.50 | 0.820 | 0.250 |
| 8 | -13.00 | +14.02 | 12.50 | 1.100 | 0.220 |
| 1,2 | -13.00 | +18.80 | 12.50 | 1.450 | 0.140 |
| 18 | - 6.50 | + 7.75 | 24.50 | 1.200 | 0.450 |
| 19 | - 6.50 | + 9.83 | 24.50 | 1.500 | 0.300 |
| 4,5 | - 6.50 | +13.00 | 24.50 | 2.000 | 0.240 |
| 6,7 | - 6.50 | +18.00 | 24.50 | 2.600 | 0.200 |

Evaluation of the functional relationship between A, $R_{\Delta T}$ and $\phi$

From Figures 1-3, it can be observed that for a specific value of $\phi$ the value of A increases as $R_{\Delta T}$ decreases. For the same $R_{\Delta T}$, it can be seen that the value of A increases as $\phi$ increases (see Fig. 1 and 2, for instance, when $R_{\Delta T} = 0.60$, $A = 0.230$, for $\phi = 7.30$ and $A = 0.450$ for $\phi = 12.50$). It is thought that the functional relationship between A, $R_{\Delta T}$ and $\phi$ can be expressed as

$$A = a\phi^m (R_{\Delta T})^n \tag{14}$$

where constant a and exponents m and n are to be determined. A log-log plot of A vs $R_{\Delta T}$ with $\phi$ as parameter is shown in Figure 4. Without exception, the values of A for different $\phi$ fall into parallel lines. The slope of the lines or the value of n is found to be -1.2. Therefore eq 14 can be rewritten as

$$A = a\,\phi^{m}(R_{\Delta T})^{-1.2}\,. \qquad (15)$$

A similar log-log plot of $A(R_{\Delta T})^{1.2}$ vs $\phi$ also gives a linear relationship (Fig. 5). The slope or the value of m is 1.2. The intercept or the value of a is 0.0114. Finally the value of A can be correlated by the following expression:

$$A = 0.0114\left(\frac{\phi}{R_{\Delta T}}\right)^{1.2}. \qquad (16)$$

## CONCLUSION

From the relationship A = 1-3B given earlier in the paper, and with an expression for A given by eq 16, values of B can be expressed in terms of $\phi$ and $R_{\Delta T}$ by

$$B = \frac{1-0.0114\,(\phi/R_{\Delta T})^{1.2}}{3}. \qquad (17)$$

By substituting the expression for B in eq 9, replacing the constant exponent 0.305 as proposed earlier, the correlation for natural convection heat transfer for fluids resulting from melting and confined by a rigid lower boundary and a moving upper boundary can finally be developed. The final correlation which involves the Prandtl and Rayleigh numbers and thermal parameters $\phi$ and $R_{\Delta T}$ is

$$N_{Nu} = \frac{hS}{k_l} = 0.104\,(N_{Pr})^{0.084}\,(N_{Ra})^{[1-0.0114(\phi/R_{\Delta T})^{1.2}]/3} \qquad (18)$$

which is applicable for $\phi$ varying from 7.30 to 24.50 and $R_{\Delta T}$ varying from 0.350 to 2.60.

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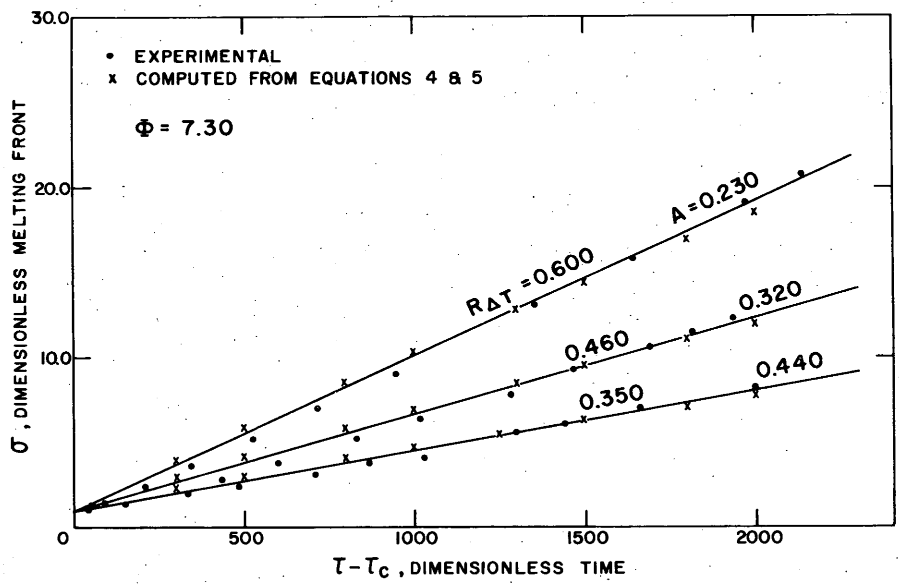


Figure 1. Relation between dimensionless melting front $\sigma$ and dimensionless time $\tau - \tau_c$ for the case $\phi = 7.30$.

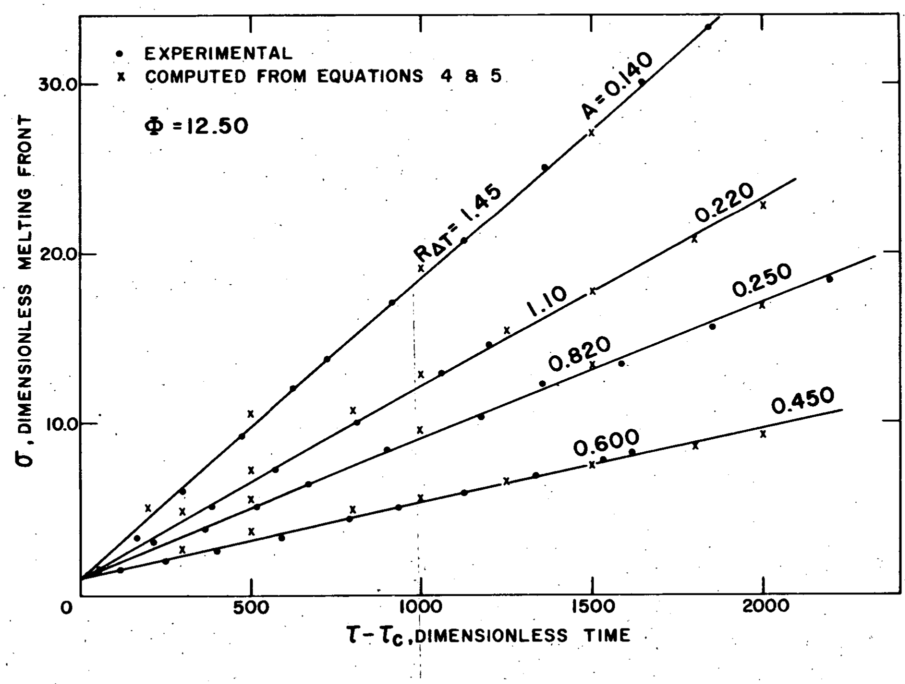


Figure 2. Relation between dimensionless melting front $\sigma$ and dimensionless time $\tau - \tau_c$ for the case $\phi = 12.50$.

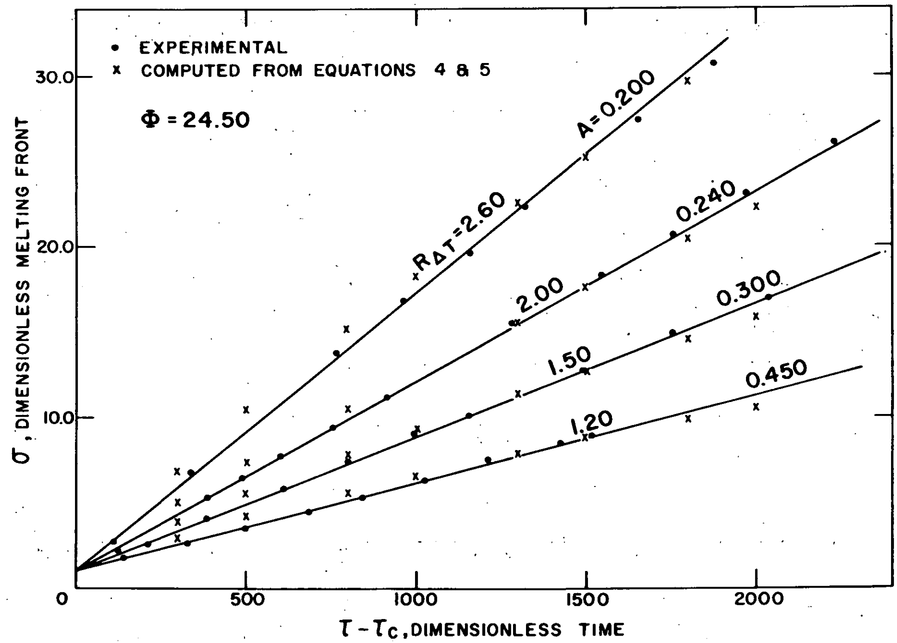


Figure 3. Relation between dimensionless melting front $\sigma$ and dimensionless time $\tau - \tau_c$ for the case $\phi = 24.50$.

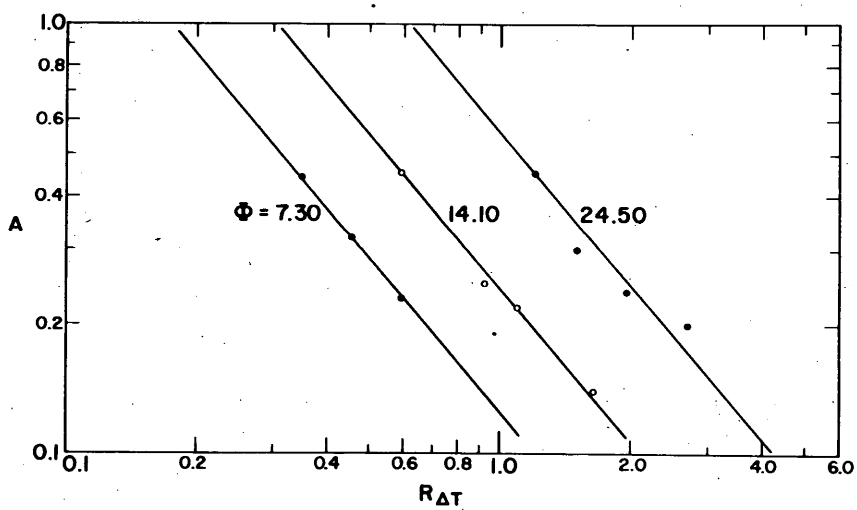


Figure 4. Relation between A and $R_{\Delta T}$.

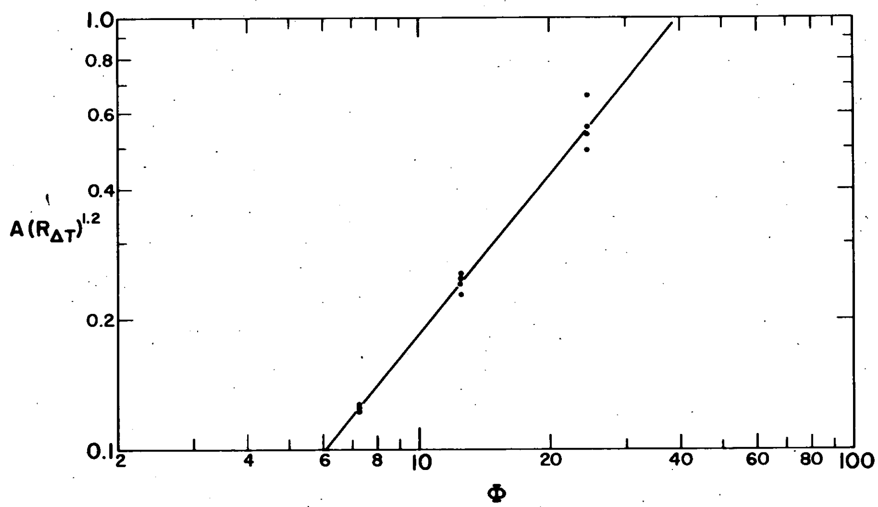


Figure 5. Relation between $A(R_{\Delta T})^{1.2}$ and $\phi$.

**DOCUMENT CONTROL DATA - R & D**

*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

| | |
|---|---|
| 1. ORIGINATING ACTIVITY *(Corporate author)*<br>U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, N.H. | 2a. REPORT SECURITY CLASSIFICATION<br>Unclassified<br>2b. GROUP |

3. REPORT TITLE

AN ANALYTICAL AND EXPERIMENTAL STUDY OF A MELTING PROBLEM WITH NATURAL CONVECTION

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Research Report

5. AUTHOR(S) *(First name, middle initial, last name)*

Yin-Chao Yen

| 6. REPORT DATE | 7a. TOTAL NO. OF PAGES | 7b. NO. OF REFS |
|---|---|---|
| July 1967 | 13 | 4 |

| | |
|---|---|
| 8a. CONTRACT OR GRANT NO.<br>b. PROJECT NO.<br>c. DA Task 1VO14501B52A02<br>d. | 9a. ORIGINATOR'S REPORT NUMBER(S)<br>Research Report 234<br>9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* |

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited

| 11. SUPPLEMENTARY NOTES | 12. SPONSORING MILITARY ACTIVITY |
|---|---|
| | U.S. Army Cold Regions Research and Engineering Laboratory |

13. ABSTRACT

The correlation by O'Toole and Silveston (1959) of natural convection heat transfer for fluids confined between two parallel horizontal plates has been extended to the case involving phase change. The new correlation, which is applicable for melting from below in a water-ice system, is described with special focus on theoretical considerations, estimation of heat flux, and the experimental and analytical results. In all experiments, bubble-free, homogeneous ice samples were prepared beforehand to assure reliable and reproducible results. In general, the results from theory and experiment are in close agreement.

| 14. KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Ice--Melting | | | | | | |
| Ice--Phase transition | | | | | | |
| Ice--Thermodynamic properties--Mathematical analysis | | | | | | |
| Heat transfer--Mathematical analysis | | | | | | |