Vibration of Sphere for Determining the Dilatational Constants of Visco-Elastic Materials

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PREFACE

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This report has been reviewed and approved for publication by Headquarters, U.S. Army Materiel Command.

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CONTENTS

	Page
Preface -----	ii
Summary -----	iv
Introduction -----	1
Plane visco-elastic waves -----	1
Motions in a visco-elastic sphere -----	2
Dilatational constants -----	4
Discussion -----	6
References -----	7

ILLUSTRATIONS

Figure

1.	Finite sphere acted upon by an internal source -----	3
2.	Damping effect of $\tan \frac{\theta}{2}$ on amplitude ratio, R, and frequency ratio, $n \frac{\omega l}{C_1}$, for $n = 1$ -----	5
3.	Damping effect of $\tan \frac{\theta}{2}$ on amplitude ratio, R, and frequency ratio, $n \frac{\omega l}{C_1}$, for $n = 2$ -----	5
4.	Damping effect of $\tan \frac{\theta}{2}$ on amplitude ratio, R, and frequency ratio, $n \frac{\omega l}{C_1}$, for $n = 3$ -----	6

SUMMARY

The forced vibration problem of a finite sphere with an exciting source embedded inside it has been treated. The source is assumed to oscillate sinusoidally and the complex dilatational modulus is used to describe the motion inside the sphere. It is found that if the outer surface of the sphere is restrained from radial displacement, the ratio of the amplitude of a mid-point (between the outer surface of the sphere and the inner exciting source) to the amplitude of a point on the exciting source could be used as an indication of the dilatational properties of the sphere. Therefore, from the criterion of maximum amplitude ratio in conjunction with the frequency, the dilatational properties of a testing material can be determined. This method can be applied to visco-elastic fluids as well as solids.

VIBRATION OF SPHERE FOR DETERMINING THE DILATATIONAL CONSTANTS OF VISCO-ELASTIC MATERIALS

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Introduction

The dynamic properties of visco-elastic materials such as polymers have been studied extensively in recent years (Mason, 1960; Hillier and Kolsky, 1949; Lethersich, 1950). Most of the experimental work, however, was connected with the uniaxial longitudinal or torsional tests which lead to the determination of Young's modulus or shear modulus. The investigations concerning the dilatational modulus and its damping factor* seem to be very limited. This information is necessary when dealing with three-dimensional wave propagation problems which have been indicated (Kolsky, 1960) as obvious gaps in the study of linear visco-elasticity.

It has been shown that for a longitudinal sinusoidal disturbance the wave propagation constants can be related to the complex Young's modulus and, also, for a torsional disturbance a corresponding complex shear modulus is found. Based on this concept, the author has devised two testing methods (Lee, 1963) for the evaluation of these constants. The same concept is now extended to obtain information concerning the complex dilatational modulus of visco-elastic materials.

The dilatational wave is one of the body waves which travel within a medium of infinite extent. Any experimental consideration will have to satisfy this condition. Since no lateral free surfaces are involved when considering the dilatational wave propagation in a sphere, the above-mentioned condition is fulfilled. It is for this reason that a sphere is chosen for the analysis.

In the present work, the problem of a finite sphere with the exciting source embedded inside it has been treated. The outer surface of the sphere is assumed to have no radial displacement†. From the ratio of the amplitude of the mid-point (between the outer surface of the sphere and the exciting source) to the amplitude of a point on the exciting source, it is shown that the maximum value of this amplitude ratio in conjunction with the frequency determines the complex dilatational modulus of the material involved.

Although the experimental device for this method will not be as convenient as the methods (Lee, 1963) previously mentioned, the benefit of this study comes from obtaining direct information of these constants. And, also, it could offer an example for the analysis of the three-dimensional problems in visco-elasticity.

Plane visco-elastic waves

The equation of motion in rectangular coordinates may be abbreviated in tensor notations

$$\rho \frac{\partial^2 U_i(\omega)}{\partial t^2} = \frac{\partial \sigma_{ij}(\omega)}{\partial x_j}, \quad (1.1)$$

where $U_i(\omega)$ and $\sigma_{ij}(\omega)$ are the components of the displacement vector and the stress tensor, respectively, and ρ is the density of the medium. For the sinusoidal disturbance, the stress-strain tensor of a linear visco-elastic medium is expressible by

* These are referred to the equivalent terms, M^* and $\tan \theta$, as described on p. 2 and 3.

† The same problem of a free outer surface sphere has also been considered. Since the result involves the shear modulus in addition to the dilatational modulus, it is not presented at this time.

$$\sigma_{ij}(\omega) = \lambda(i\omega) \Delta(\omega) \delta_{ij} + 2G(i\omega) \epsilon_{ij}(\omega) , \quad (1.2)$$

where

$$\lambda(i\omega) = \lambda'(\omega) + i\lambda''(\omega), \quad G(i\omega) = G'(\omega) + iG''(\omega) \quad (1.3a, b)$$

are the complex factors, similar to Lamé constants, and $\Delta(\omega)$ is the dilatation:

$$\Delta(\omega) = \epsilon_{11} + \epsilon_{22} + \epsilon_{33} . \quad (1.3c)$$

δ_{ij} is the Kronecker delta and ϵ_{ij} are the components of the strain tensor.

Using eq 1.2 and the strain-displacement relation

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) , \quad (1.4)$$

eq 1.1 can be further written to

$$\rho \frac{\partial^2 U_i}{\partial t^2} = \left[\lambda(i\omega) + G(i\omega) \right] \frac{\partial \Delta(\omega)}{\partial x_i} + G(i\omega) \nabla^2 U_i . \quad (1.5)$$

In order to obtain the expression for the dilatational wave in a visco-elastic medium, we use the condition that if the displacements U_i satisfy the relation

$$U_i = \frac{\partial \phi}{\partial x_i} , \quad (1.6)$$

with ϕ being a scalar potential function, then the irrotational condition will be satisfied. From eq 1.6 the following is obvious:

$$\Delta(\omega) = \nabla^2 \phi \quad \text{and} \quad \frac{\partial \Delta(\omega)}{\partial x_i} = \nabla^2 U_i . \quad (1.7a, b)$$

Hence, the plane dilatational wave equation for a linear visco-elastic medium subjected to a sinusoidal disturbance is obtained from the substitution of eq 1.7 b in eq 1.5:

$$\rho \frac{\partial^2 U_i}{\partial t^2} = \left[\lambda(i\omega) + 2G(i\omega) \right] \nabla^2 U_i . \quad (1.8)$$

Also, the plane distortional wave equation is

$$\rho \frac{\partial^2 U_i}{\partial t^2} = G(i\omega) \nabla^2 U_i , \quad (1.9)$$

by letting $\Delta(\omega) = \text{zero}$ in eq 1.5.

Motions in a visco-elastic sphere

When dealing with problems of spherical symmetry, it may be advantageous to use spherical coordinates and rewrite eq 1.8 to the form

$$\rho \frac{\partial^2 U_r}{\partial t^2} = \left[\lambda(i\omega) + 2G(i\omega) \right] \left(\frac{\partial^2 U_r}{\partial r^2} + \frac{2}{r} \frac{\partial U_r}{\partial r} - 2 \frac{U_r}{r^2} \right) , \quad (2.1)$$

where U_r is the displacement along the radial direction and r is the distance from the center of the sphere. The particular solution of eq 2.1, which may be useful in describing the motion in a finite sphere when undergoing harmonic oscillation, can be assumed of the form

$$U_r = \left\{ A \left[\frac{\beta}{r} \cos \beta r - \frac{1}{r^2} \sin \beta r \right] + B \left[\frac{\beta}{r} \sin \beta r + \frac{1}{r^2} \cos \beta r \right] \right\} \cos \omega t, \quad (2.2)$$

where A and B are the constants to be determined from boundary conditions and

$$\beta = (1 - i \tan \frac{\theta}{2}) \frac{\omega}{C_1} , \quad (2.3a)$$

C_1 = the dilatational wave velocity

$$= \left(\frac{M^*}{P} \right)^{\frac{1}{2}} \sec \frac{\theta}{2} , \quad (2.3b)$$

$$M^* = \{ [\lambda'(\omega) + 2G'(\omega)]^2 + [\lambda''(\omega) + 2G''(\omega)]^2 \}^{\frac{1}{2}} , \quad (2.3c)$$

$$\tan \theta = \frac{\lambda''(\omega) + 2G''(\omega)}{\lambda'(\omega) + 2G'(\omega)} , \quad (2.3d)$$

and ω = the angular frequency of the oscillation.

Let us now use eq 2.2 to consider the forced vibration in a finite visco-elastic sphere, Figure 1, due to the action of a time varying forcing function, $f(t)$, on a concentric spherical cavity. In the present study the forcing function will be assumed to vary sinusoidally with time. The normal stresses induced on the surface of the cavity are to be uniformly distributed and can be expressed by

$$\sigma_0(\omega) \cos \omega t \quad (2.4)$$

for a given frequency. Let the radius of the cavity be "a" and that of the sphere "b". If there is no radial displacement at the outer surface, then the boundary conditions for the sphere at its inner and outer surfaces are

$$\sigma_{r=a} + \sigma_0 \cos \omega t = 0 \quad (2.5a)$$

and

$$U_{r=b} = 0. \quad (2.5b)$$

Noting that

$$\sigma_r = \left\{ [\lambda(i\omega) + 2G(i\omega)] \frac{\partial U_r}{\partial r} + 2\lambda(i\omega) \frac{U_r}{r} \right\} \quad (2.6)$$

and substituting eq 2.2 in 2.5 a and b, we obtain

$$U_r = \frac{\sigma_0 a^3}{D_0 r^2} \left\{ (\beta^2 b r + 1) \sin \beta(b-r) - \beta(b-r) \cos \beta(b-r) \right\} \cos \omega t , \quad (2.7)$$

where

$$D_0 = [\rho \omega^2 a^2 \beta b - 4 G(i\omega) \beta(b-a)] \cos \beta(b-a) + [4 G(i\omega)(\beta^2 a b + 1) - \rho \omega^2 a^2] \sin \beta(b-a). \quad (2.8)$$

It is seen that the amplitude of the displacement inside the sphere depends upon the stress input and the dilatational and distorsional properties of the medium.

If we consider, however, the amplitude ratio of U_r and U_a , we find

$$R = \frac{U_r}{U_a} = \frac{a^2 [(\beta^2 b r + 1) \sin \beta(b-r) - \beta(b-r) \cos \beta(b-r)]}{r^2 [(\beta^2 a b + 1) \sin \beta(b-a) - \beta(b-a) \cos \beta(b-a)]} \quad (2.9)$$

depending only on the dilatational constants of the sphere. Thus, R may be utilized for the purpose of determining these constants.

Figure 1. Finite sphere acted upon by an internal source.

Dilatational constants

For the convenience of establishing the criterion for calculation, we choose U_r to be at a point midway between the inner and outer surfaces of the sphere. Let $r = (n+1)a$ and $b = (2n+1)a$, $n = 1, 2, 3, \dots$. The amplitude of R in eq 2.9 can then be expressed by

$$|R| = \frac{[(H-J)^2 + (I+K)^2]^{\frac{1}{2}}}{(n+1)^2 [(L-N)^2 + (M+O)^2]^{\frac{1}{2}}}^{\frac{1}{2}}, \quad (3.1)$$

where

$$\left. \begin{aligned} H &= [Q(a^2 - \gamma^2) + 1] \sin a \cosh \gamma - 2Qa\gamma \cos a \sinh \gamma \\ I &= [Q(a^2 - \gamma^2) + 1] \cos a \sinh \gamma + 2Qa\gamma \sin a \cosh \gamma \\ J &= a \cos a \cosh \gamma + \gamma \sin a \sinh \gamma \\ K &= a \sin a \sinh \gamma - \gamma \cos a \cosh \gamma \\ L &= [P(a^2 - \gamma^2) + 1] \sin 2a \cosh 2\gamma - 2Pa\gamma \cos 2a \sinh 2\gamma \\ M &= [P(a^2 - \gamma^2) + 1] \cos 2a \sinh 2\gamma + 2Pa\gamma \sin 2a \cosh 2\gamma \\ N &= 2a \cos 2a \cosh 2\gamma + 2\gamma \sin 2a \sinh 2\gamma \\ O &= 2a \sin 2a \sinh 2\gamma - 2\gamma \cos 2a \cosh 2\gamma \\ P &= \frac{2n+1}{n^2}, \quad Q = (n+1)P \\ a &= n \frac{\omega a}{C_1}, \quad \gamma = a \tan \frac{\theta}{2} \end{aligned} \right\} \quad (3.2)$$

$$\underline{n = 1}$$

To make use of eq 3.1, we first consider the case of $n = 1$, which gives $b = 3a$.

If f_{ro} is the fundamental frequency when R reaches its maximum, $R_{\max}$, we find, from eq 3.1, that

$$C_1 = 3.6168 a f_{ro} (1 + \tan^2 \frac{\theta}{2}) \quad (3.3)$$

and

$$\tan \frac{\theta}{2} = \frac{0.12082}{R_{\max}} \quad (3.4)$$

From the relationship as given by eq 2.3b for C_1 and M^* , eq 3.3 can be further deduced to be the more useful form

$$M^* = 13.081 a^2 f_{ro}^2 \rho (1 + \tan^2 \frac{\theta}{2}) \quad (3.5)$$

Thus, it is seen that the dilatational constants M^* and $\tan \frac{\theta}{2}$ can be determined from the measurable items f_{ro} and $R_{\max}$ by the use of eq 3.4 and 3.5.

 $n = 2$ and 3

When a larger sphere size is desirable, we next consider the cases when $n = 2$ and 3 , corresponding to $b = 5a$ and $b = 7a$, respectively.

In a similar manner, it is found that

$$\tan \frac{\theta}{2} = \frac{0.058949}{R_{\max}}, \quad (3.6)$$

$$M^* = 44.937 a^2 f_{ro}^2 \rho \left(1 + \tan^2 \frac{\theta}{2}\right), \quad (3.7)$$

for

$$n = 2,$$

and also

$$\tan \frac{\theta}{2} = \frac{0.0341545}{R_{\max}}, \quad (3.8)$$

$$M^* = 92.555 a^2 f_{ro}^2 \rho \left(1 + \tan^2 \frac{\theta}{2}\right), \quad (3.9)$$

for

$$n = 3.$$

To obtain a graphic view of how R varies with $\tan \frac{\theta}{2}$ and $n \frac{\omega a}{C_1}$, we plot eq 3.1 for these three different cases of $n = 1, 2$, and 3 , shown in Figures 2, 3, and 4, respectively. They all seem to indicate that the presence of $\tan \frac{\theta}{2}$ damped the amplitude of R and shifted the peak points.

Figure 2. Damping effect of $\tan \frac{\theta}{2}$ on amplitude ratio, R , and frequency ratio, $n \frac{\omega l}{C_1}$, for $n = 1$.

Figure 3. Damping effect of $\tan \frac{\theta}{2}$ on amplitude ratio, R , and frequency ratio, $n \frac{\omega l}{C_1}$, for $n = 2$.

Figure 4. Damping effect of $\tan \frac{\theta}{2}$ on amplitude ratio, R , and frequency ratio, $n \frac{\omega l}{C_1}$, for $n = 3$.

Discussion

(1) Figures 2, 3, and 4 all indicate that the magnitude of R for constant $\tan \frac{\theta}{2}$ decreases rapidly with increased value of n and that their peaks are subsequently flattened.

Thus, the choice of the n value, which governs the relationship between the size of the sphere and its cavity, depends upon the $\tan \frac{\theta}{2}$ value of the material being tested. When the internal friction of a material is likely to be small, it is preferable to use the large n value for the purpose of flattening† the sharp peak of $R_{\max}$. On the other hand, small n values should be used for materials of large internal friction.

(2) It does not appear that n can be extended indefinitely. When n becomes sufficiently large, the R curves will become very flat and close together. The criterion of $R_{\max}$ may not be suitable for use in accurately determining $\tan \frac{\theta}{2}$ and M^* .

(3) It should be mentioned that this method can also be applied for determining the dilatational constants of a visco-elastic fluid. When considering a visco-elastic fluid, however, it is customary to use the bulk modulus $K(i\omega)$ and the shear modulus $G(i\omega)$, which are related to $M(i\omega)$ by

$$M(i\omega) = K(i\omega) + \frac{4}{3} G(i\omega). \quad (4.1)$$

Therefore, in this sense, $M(i\omega)$ alone is not sufficient to give independent value of both $K(i\omega)$ and $G(i\omega)$. To do this, an additional test, such as for $G(i\omega)$ (Meister *et al.*, 1960) will be needed.

† It makes the determination of $R_{\max}$ less critical.

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<p>AD Accession No.</p> <p>U. S. Army Cold Regions Research and Engineering Laboratory, Army Materiel Command, Hanover, N.H. VIBRATION OF SPHERE FOR DETERMINING THE DILATATIONAL CONSTANTS OF VISCO-ELASTIC MATERIALS — Tung-Ming Lee</p> <p>Research Report 132, Feb 1964, 7p - illus. DA Task 8S-66-02-001-02, CRREL Task 5010.21206 Unclassified Report</p> <p>The forced vibration problem of a finite sphere with an exciting source embedded inside it has been investigated. The source is assumed to oscillate sinusoidally and the complex dilatational modulus is used to describe the motion inside the sphere. It is found that if the outer surface of the sphere is restrained from radial displacement, the ratio of the amplitude of a mid-point (between the outer surface of the sphere and the inner exciting source) to the amplitude of a point on the exciting source could be used as an indication of the dilatational properties of the sphere. Therefore, from the criterion of maximum amplitude ratio in conjunction with the frequency, the dilatational properties of a testing material can be determined. This method can be applied to visco-elastic fluids as well as solids.</p>	<p>UNCLASSIFIED</p> <p>I. Solids--Mechanical properties I. Lee, Tung-Ming II. U. S. Army Cold Regions Research and Engineering Laboratory</p>	<p>AD Accession No.</p> <p>U. S. Army Cold Regions Research and Engineering Laboratory, Army Materiel Command, Hanover, N.H. VIBRATION OF SPHERE FOR DETERMINING THE DILATATIONAL CONSTANTS OF VISCO-ELASTIC MATERIALS — Tung-Ming Lee</p> <p>Research Report 132, Feb 1964, 7p - illus. DA Task 8S-66-02-001-02, CRREL Task 5010.21206 Unclassified Report</p> <p>The forced vibration problem of a finite sphere with an exciting source embedded inside it has been investigated. The source is assumed to oscillate sinusoidally and the complex dilatational modulus is used to describe the motion inside the sphere. It is found that if the outer surface of the sphere is restrained from radial displacement, the ratio of the amplitude of a mid-point (between the outer surface of the sphere and the inner exciting source) to the amplitude of a point on the exciting source could be used as an indication of the dilatational properties of the sphere. Therefore, from the criterion of maximum amplitude ratio in conjunction with the frequency, the dilatational properties of a testing material can be determined. This method can be applied to visco-elastic fluids as well as solids.</p>	<p>UNCLASSIFIED</p> <p>I. Solids--Mechanical properties I. Lee, Tung-Ming II. U. S. Army Cold Regions Research and Engineering Laboratory</p>
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