Research Report 126

ULTRASONIC PULSE MEASUREMENTS IN ANISOTROPIC LAKE ICE

by

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PREFACE

This is one of a series of reports on U. S. Army Snow Ice and Permafrost Research Establishment Project 022.01.034, Elastic and visco-elastic properties of snow and ice. The work described in this report consisted of in-situ measurements in lake ice with ultrasonic pulses.

The report was written by Dr. Roethlisberger, contract scientist. A. A. Wickham, U. S. Army Engineer Research and Development Detachment, assisted in the field work, and he as well as J. T. Long, ERDD, helped with the computations. The facilities of the SIPRE Keweenaw Field Station, Houghton, Mich., were used. Work on this project was performed for USA SIPRE's Basic Research Branch, Mr. J. A. Bender, Chief.

USA SIPRE was merged with Arctic Construction and Frost Effects Laboratory to form U. S. Army Cold Regions Research and Engineering Laboratory (USA CRREL), 1 February 1961. USA CRREL is an Army Materiel Command Laboratory.

Research Report 126 - Errata

Page 20, line 12: for PS read PH

Page 20, line 13: for SP read SV

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ILLUSTRATIONS

SUMMARY

Travel-time measurements of ultrasonic pulses were carried out in March 1960 on Lake Superior (Keweenaw Bay) near Baraga, Mich. The ice was about 45 cm thick and consisted of grains with vertical *c*-axis orientation with the exception of a surface layer of variable thickness. Ultrasonic pulses were transmitted and received by barium titanate cells of cylindrical and spherical shape.

The transducers were mounted at the surface and the distance was varied. A number of direct and reflected signals could be identified. Of the reflected events, the PS type were by far the clearest and strongest at distances many times the ice thickness, and thus best suited for ice thickness determination. In order to obtain satisfactory agreement between theoretical and measured travel times the anisotropy of the ice had to be taken into account. The elastic constants determined by Bass *et al.* (1957) gave reasonably good agreement between computed and measured travel times, but some discrepancies remain to be explained. Part of the observed reflections occurred on cracks in the ice. Using equipment with approx 100-kc signals the ice thickness was determined by the ultrasonic pulse method, destruction free, with an accuracy of 2-4 cm or 5-10%. For day-to-day comparisons the relative accuracy would be in the order of 0.5 cm.

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INTRODUCTION

The investigation of elastic waves propagated through a lake-ice sheet can follow different paths, depending on the mode of the waves and the wave length. In the present work ultrasonic vibrations were used with wave lengths considerably shorter than the thickness of the ice sheet. The purpose was to carry out seismic model studies *in situ* on a somewhat larger scale than is customary in laboratory seismics. The relatively large dimensions made it possible to use commercial concrete-testing equipment which can serve other projects as well and has the advantage of being rugged. One objective of the investigation was to find if ultrasonic reflection techniques can serve as a nondestructive means of measuring ice thickness. For accurate soundings the anisotropy of the ice had to be considered and the problem of the propagation of elastic waves in anisotropic ice had to be dealt with.

The ice cover of a lake is sometimes primarily formed of ice grains with their crystallographic *c*-axis in a vertical position while the three *a*-axes of the hexagonal lattice are randomly oriented in the horizontal plane. Such ice shows anisotropy which must be taken into account when observing a phenomenon which depends on the change of properties with direction as in the case of the propagation of elastic waves. In a crystalline material of the hexagonal system the propagation of elastic waves is of rotational symmetry and is independent of the direction of the *a*-axes in the plane perpendicular to *c*. The lake-ice sheet is therefore equivalent to one large ice crystal and it should be possible to apply single-crystal theory to polycrystalline lake ice with vertical *c*-axis orientation. The present work is an attempt to check if this can be done successfully. The considerable thickness and large extent of lake ice make it a unique material for investigations on the special type of rotationally symmetric anisotropy found in hexagonal crystals. Such an axisymmetric medium is said to show either uniaxial anisotropy or transverse isotropy.

FIELD MEASUREMENTS

The field work was carried out from 7 March to 18 March 1960 near Baraga, Michigan, on Keweenaw Bay, Lake Superior. Preceding measurements from 26 February to 4 March 1960 on a frozen pond at USA SIPRE's Keweenaw Field Station in Houghton, Michigan, helped to establish the measuring techniques. On Keweenaw Bay an area of about 10 m by 15 m was chosen about 1/4 mile offshore where the water was 5 m deep. The ice was about 30 cm thick and was covered with 10 to 20 cm of snow on an initial visit in late February. About two weeks before the measurements started the snow was removed from the working area in order to increase the freezing rate of the ice sheet. On 14 March the exposed area showed 46 cm of ice as against 33 cm where the snow had not been removed. However, the exposed area not only grew faster in thickness, but also developed fresh cracks almost every night, which interfered with the measurements.

The instrument used provided 1) high voltage pulses (up to 3000 v) for driving a piezoelectric transducer at the rate of about 60 pulses per second; 2) circuits to amplify the signal from piezoelectric receiver transducers; 3) an oscilloscope to display the instant of the transmission, the received signal and time marks on a dual beam cathode ray tube; and 4) exact time-measuring circuits controlled by a 100 kc quartz crystal.

The transducers, which could be used either as drivers or as receivers, were three different kinds of barium titanate cells: (1) hollow cylinders of dimensions ID = $\frac{1}{2}$ in., OD = $\frac{3}{4}$ in., length = 1 in., inside and outside silver-coated, wrapped in copper screen for shielding and encapsulated in epoxy resin forming cylinders of 15/16 in. diameter and $1\frac{1}{2}$ in. length topped by a $\frac{1}{2}$ in. high cone on the side of the cable (Roethlisberger, 1963, p. 191); (2) hollow spheres of approximately 1 in. diam and 1/8 in. wall thickness, uncoated; and (3) hollow spheres of approximately $\frac{1}{2}$ in. diam and 1/8 in. wall thickness, uncoated. The pulses consisted of short trains of resonant vibrations of the cells usually at frequencies of about 50 - 80 kc depending on the mode of vibration (radial, longitudinal) and presumably on the coupling between the cells and the surrounding ice. Higher frequencies of the order of 200 kc were occasionally observed, possibly related to harmonics, or to the thickness mode of resonant vibrations, or to a resonance in the ice (depth of cell below surface).

The instrument was set up in a pentagonal five-man tent. Power was provided by a 3.5-kw gasoline-powered generator mounted on a weasel (M 29 C) which was kept some 50 m away from the research area to reduce vibrations. The electric power was also used to heat the water needed to move the transducers.

The transducers were frozen into ice holes as closely matching the cells as possible. The holes for the cylinders were drilled with a 1-in. auger. For the 1-in. spheres semi-spherical holes were made either at the ice surface or at the bottom of a 1-in. augered hole by a steel ball with a handle welded to it, heated by submerging in hot water. In order to retrieve cells without breaking them and without appreciably disturbing the surrounding ice, a fine jet of hot water squeezed out of a rubber bulb was directed to the contact of cell and ice.

Most measurements were taken at the surface between two cells at various distances. Either cell was moved when the distance was changed, usually along a straight line. This line was kept as far away as possible from cracks. The measurements were taken at the longest distance first, in order to maintain an undisturbed surface between the cells. Because of the formation of fresh cracks the line had to be relocated during the investigation.

THEORETICAL TIME - DISTANCE CURVES

The problem of the propagation of elastic waves in anisotropic media was mathematically treated by Rudzki as far back as the end of the last century and, more completely, in 1911. More recent treatments followed by Musgrave (1954, 1959, 1960, 1961), Miller and Musgrave (1956), Postma (1955), Helbig (1958), Buchwald (1959), and Gassmann (1964). Beryl, which is very similar to ice in its elastic anisotropy, is often used as an example for hexagonal symmetry, i.e. uniaxial anisotropy.

Helbig's presentation served as a guide to set up programs for a Bendix G-15D electronic computer*. The following equations were used to compute ray velocities from the elastic constants c_{11}, c_{33}, c_{44}, c_{12} and c_{13} and the density ρ, with $c_{66} = (c_{11} - c_{12})/2$, $A = c_{13} + c_{44}$, $B = c_{11} - c_{44}$, $C = c_{33} - c_{44}$, and $E^2 = BC - A^2$:

$$\frac{\rho}{c_{66}} z_1^{(1)2} + \frac{\rho}{c_{44}} z_3^{(1)2} = 1 \tag{1}$$

$$\left.\begin{aligned} n_1^{(2)2} &= \frac{C\,w^{(2)} + A\,w^{(2)2}}{A\,c_{44} + (c_{11}c_{33} - c_{44}^2 - A^2)\,w^{(2)} + A\,c_{44}\,w^{(2)2}} \\ n_3^{(2)2} &= \frac{A + B\,w^{(2)}}{A\,c_{44} + (c_{11}c_{33} - c_{44}^2 - A^2)\,w^{(2)} + A\,c_{44}\,w^{(2)2}} \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned} z_1^{(2)} &= n_1^{(2)} \left[c_{44} + \frac{E^2}{C + 2Aw^{(2)} + Bw^{(2)2}}\right] \frac{1}{\sqrt{\rho}} \\ z_3^{(2)} &= n_3^{(2)} \left[c_{44} + \frac{E^2\,w^{(2)2}}{C + 2Aw^{(2)} + Bw^{(2)2}}\right] \frac{1}{\sqrt{\rho}} \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} n_1^{(3)2} &= \frac{C\,w^{(3)} + A\,w^{(3)2}}{A\,c_{33} + (c_{11}c_{33} - c_{44}^2 + A^2)\,w^{(3)} + A\,c_{11}\,w^{(3)2}} \\ n_3^{(3)2} &= \frac{A + B\,w^{(3)}}{A\,c_{33} + (c_{11}c_{33} - c_{44}^2 + A^2)\,w^{(3)} + A\,c_{11}\,w^{(3)2}} \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} z_1^{(3)} &= n_1^{(3)} \left[c_{11} - \frac{E^2}{C + 2\,Aw^{(3)} + B\,w^{(3)2}}\right] \frac{1}{\sqrt{\rho}} \\ z_3^{(3)} &= n_3^{(3)} \left[c_{33} - \frac{E^2\,w^{(3)2}}{C + 2\,Aw^{(3)} + B\,w^{(3)2}}\right] \frac{1}{\sqrt{\rho}} \end{aligned}\right\} \tag{5}$$

The expressions $z_1^{(i)}$ and $z_3^{(i)}$ are components of the ray velocity in cylindrical coordinates, where $z_3^{(i)}$ is parallel to the crystallographic c-axis ($n_1^{(i)}$ and $n_3^{(i)}$ are similar components of the index vector). The index i, i=1, 2, 3, corresponds to three different types of waves, namely (1) a true shear wave

*Some misprints were discovered in his paper. The second line of his equation 74 should contain c_{33} instead of c_{11}. In equation 75, c_{11} should replace c_{44} in the numerator and $(B-\frac{E^2}{c_{33}})$ replace $(B+\frac{E^2}{c_{44}})$ in the denominator, and the whole expression must be multiplied by c_{11}/c_{33}.

S_1 polarized perpendicular to the plane through the ray and the c-axis, (2) and (3) a quasi-transverse wave S_2 and a quasi-longitudinal wave P respectively, polarized in the plane through the ray and the c-axis. The parameters $w^{(2)}$ and $w^{(3)}$ vary in the interval $0 \leq w^{(i)} \leq \infty$.

For numerical computation the elastic constants and the density had to be chosen. The following constants determined by Bass, Rossberg and Ziegler (1957) for single-crystal ice (at -16°C) were used:

$$c_{11} = 13.3 \times 10^{10} \quad \text{dyne/cm}^2$$
$$c_{33} = 14.2 \times 10^{10} \quad \text{"}$$
$$c_{44} = 3.06 \times 10^{10} \quad \text{"}$$
$$c_{12} = 6.3 \times 10^{10} \quad \text{"}$$
$$c_{13} = 4.6 \times 10^{10} \quad \text{"}$$

Assuming the density $\rho = 0.91$ g/cm^3, a fair agreement between the measured and computed velocities was obtained. With the constants determined by Jona and Scherrer (1951) the agreement was not as good, unless an impossibly high density was assumed.*

The result of the computation of ray velocity vs direction is given in Figure 1, where the wave surface is represented by curves in one quadrant of a plane containing the c-axis. The full wave surface is obtained by reflection across the abscissa (basal plane) and rotation about the c-axis. There are three sheets of the wave surface corresponding to the quasi-longitudinal wave P, the transverse wave S_1 polarized perpendicular to the plane of the graph, and the quasi-transverse wave S_2 polarized in the plane of the graph. The wave surface represents the envelope of plane waves of arbitrary orientation one time unit (sec) after passage through the origin and corresponds to a representation of ray velocities of plane waves in polar coordinates. It can also be interpreted, at least in isotropic material, as the wave front one time unit (sec) after a disturbance has originated at a point source, separating the disturbed from the undisturbed portion of the medium.

The next step was to compute travel times for reflected waves. Since the velocity vs direction relationship is not given in explicit form by the equations, but in terms of parameters, the computer programs were set up to create values for the parameters automatically. Large numbers of pairs of values for distance and travel time of the different reflected waves were computed, namely for the reflections PP, S_1S_1 and S_2S_2. The reflection was assumed to occur at the base of a layer of unit thickness as illustrated in Figure 2, the ice being oriented with the c-axis perpendicular to the layer. In addition to the above reflections the P-wave reflected as S_2, or S_2 reflected as P, were included in the analysis. For this type of reflection the condition had to be fulfilled that the horizontal components of the wave normals of the two waves are equal (Helbig, p. 188-191), in our case $n_1^{(2)} = n_1^{(3)}$. This was achieved by making $w^{(3)}$ the only independent parameter, computing $n_1^{(3)2}$

* The work of Brockamp and Querfurth (1964) was not available when the computations were carried out.

(eq 4), substituting it for $n_1^{(2)2}$ in eq 2, and solving for $w^{(2)}$. The solution reads:

$$w^{(2)} = \frac{C - n_1^{(3)2}(c_{11}c_{33} - c_{44}^2 - A^2) - \sqrt{\left[n_1^{(3)2}(c_{11}c_{33} - c_{44}^2 - A^2) - C\right]^2 - 4n_1^{(3)2}A^2c_{44}(n_1^{(3)2}c_{44} - 1)}}{2A(n_1^{(3)2}c_{44} - 1)} \qquad (6)$$

With the computed parameter $w^{(2)}$, the remaining equations containing $w^{(2)}$ were evaluated, and pairs of values of travel time and horizontal distance were computed from the resulting P and S_2 velocities and the directions of the rays for the layer of unit thickness.

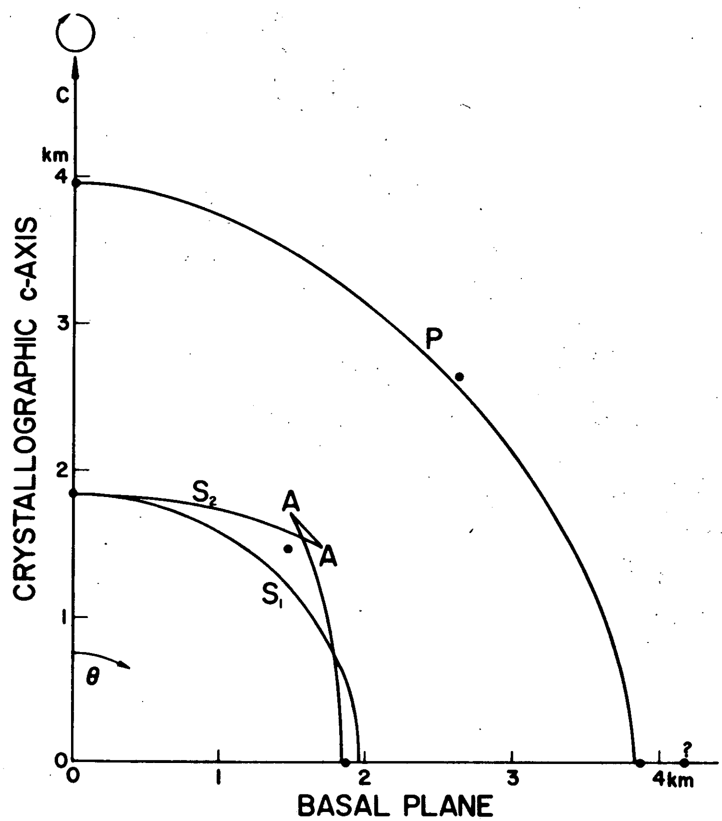


Figure 1. Theoretical wave surface for elastic waves in an ice crystal for the time unit of 1 sec. θ = angle of the ray with the crystallographic c-axis. A = cusps. Dots = observed velocities in lake ice.

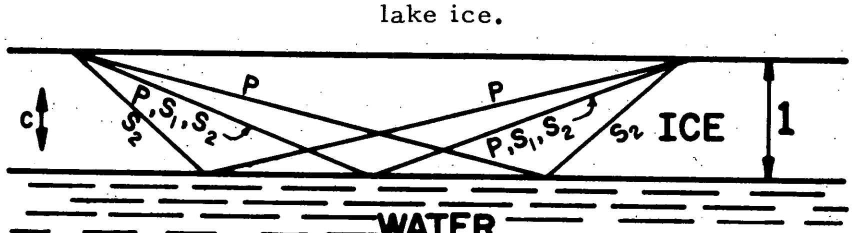


Figure 2. Single-reflection paths in an ice plate of unit thickness.

The results of the travel-time and distance computations are given in Figure 3. A number of multiple reflections have been added by either plotting the same numerical data as for the single reflection on a multiple scale or by adding the coordinates of two curves graphically. The S_2PS_2 "internal refraction" curve (Berckhemer and Oliver, 1955) was found by solving eq 6 for $n_1^{(3)} = \infty$. With dashed lines the curves for PP and PS reflections for isotropic ice are given for comparison, assuming $v_p = 2v_s = 3823$ m/sec for the velocities v_p and v_s of the P and S waves, respectively.

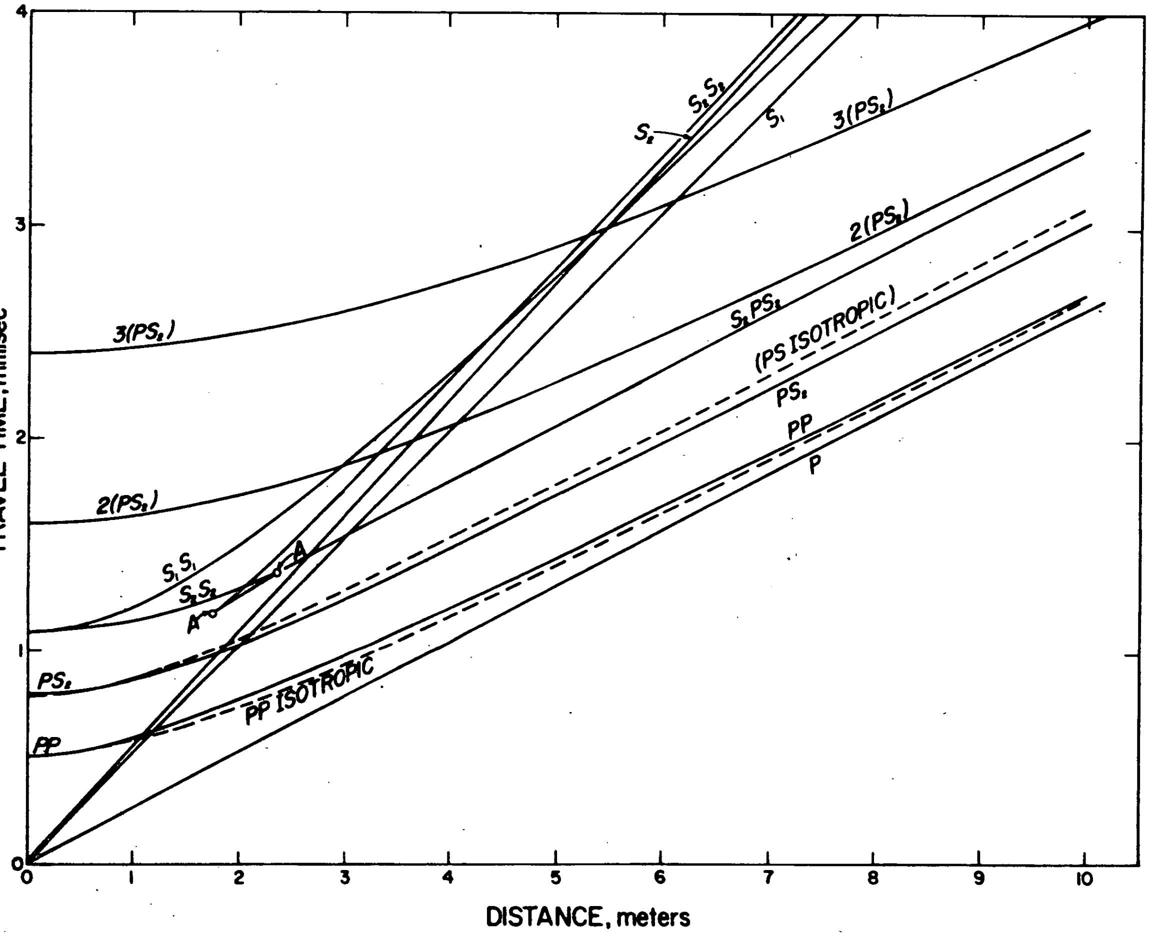


Figure 3. Theoretical travel time vs distance graph for ice of 1 m thickness. Solid lines for vertical c-axes orientation, dashed lines for polycrystalline isotropic ice. A = cusps.

A particular feature of the wave surface diagram are the cusps* marked "A" in Figure 1. Corresponding cusps are seen in the reflection diagram of Figure 3. Since the computation of the wave surface is based on the assumption of planar waves, it is not certain what the physical meaning of the cuspoidal rings would be when working with spherical or cylindrical transducers. Our measurements were not sufficiently detailed to give experimental evidence of the occurrence of the cusps.

*Cuspoidal rings of the three-dimensional wave surface.

The main effort was directed towards measuring a variety of events, especially different types of reflections, when the transducers were mounted at the surface of the ice. No exact determination of velocities vs direction was attempted because of lack of time. However, three direct measurements of velocity in three different directions were carried out. This was done at the end of the investigation when the air temperature became quite high and it was difficult to freeze the cells to the ice. A small trench with one vertical wall about 35 cm high was cut into the ice with a chain saw. The $\frac{1}{2}$ in. spherical cells were used on the vertical wall at variable distances along three straight lines, vertical, horizontal and at about 45°. One cell was kept in place. The horizontal line was situated at about 35 cm depth from the surface. However, the more reliable velocity value for the horizontal direction was obtained as a by-product of the reflection survey with cells at the surface.

The results of the velocity measurements are given in Figures 4 to 6. The straight travel-time lines do not go through the origin because no zero-time correction was applied (the zero-time correction is the time measured when the cells touch each other). While the onsets of the P-wave line up on a straight line, the determination of the S velocity is not as good. In the case of the horizontal propagation, the abnormally large zero correction makes the whole measurement doubtful, especially since the P velocity deviates from the one found over much longer distances at the ice surface (Fig. 7). The velocities figured out on the time-distance graphs are plotted as dots in Figure 1 for comparison with the theoretical values based on the elastic constants of single-crystal ice given by Bass et al. (p. 4).

No attempt has been made to account for the ice temperature, which was not measured. A fair estimate is about -10C for the top layer of the ice for the bulk of measurements (Fig. 7), and -1 to -5C for Figures 4 - 6. The temperature effect on velocities in anisotropic lake ice has been found to be small by Brockamp and Querfurth (1964) except for temperatures above about -0.5C.

During the main investigation the time was measured of all the events which seemed to indicate the arrival of a new wave. This was usually an abrupt increase of amplitude, but sometimes also a sudden change of phase after a long regular train of vibrations, or the beginning of such a regular train of vibrations of a particular frequency. The cells showed only one or two predominant resonance frequencies, but harmonics or some other high frequencies sometimes appeared. No identification as to wave type was attempted in the field, although the first few events were quite obvious after a few rough calculations. Later all the measured values were plotted on a single time-distance graph together with theoretical curves, which helped to identify the events. Figure 7 is a simplified version of this graph, where points which were very close together are plotted as one and identified as double or multiple points. The theoretical curves were taken from Figure 3 after appropriate scale reductions, assuming ice thicknesses of 40 cm and 44 cm for the solid and dashed lines respectively.

For a number of cell setups permanent recordings were obtained by photographing the oscilloscope screen with a polaroid camera. In Figures 8a-s typical recordings are reproduced. With a few exceptions they are arranged in the order of increasing distance between drive and receiver. The events are identified on the pictures where identification was possible.

The recordings served primarily to observe changes with time and to compare signals of different setups at equal distance. Figure 8o and 8p are recordings from the same cells in the same place, taken in the afternoon of one day and the morning of the next one. Cracks developed overnight and they caused new events (marked B on Figure 8p and Figure 7). The line along which the reflections were investigated had to be moved away from the cracks afterwards.

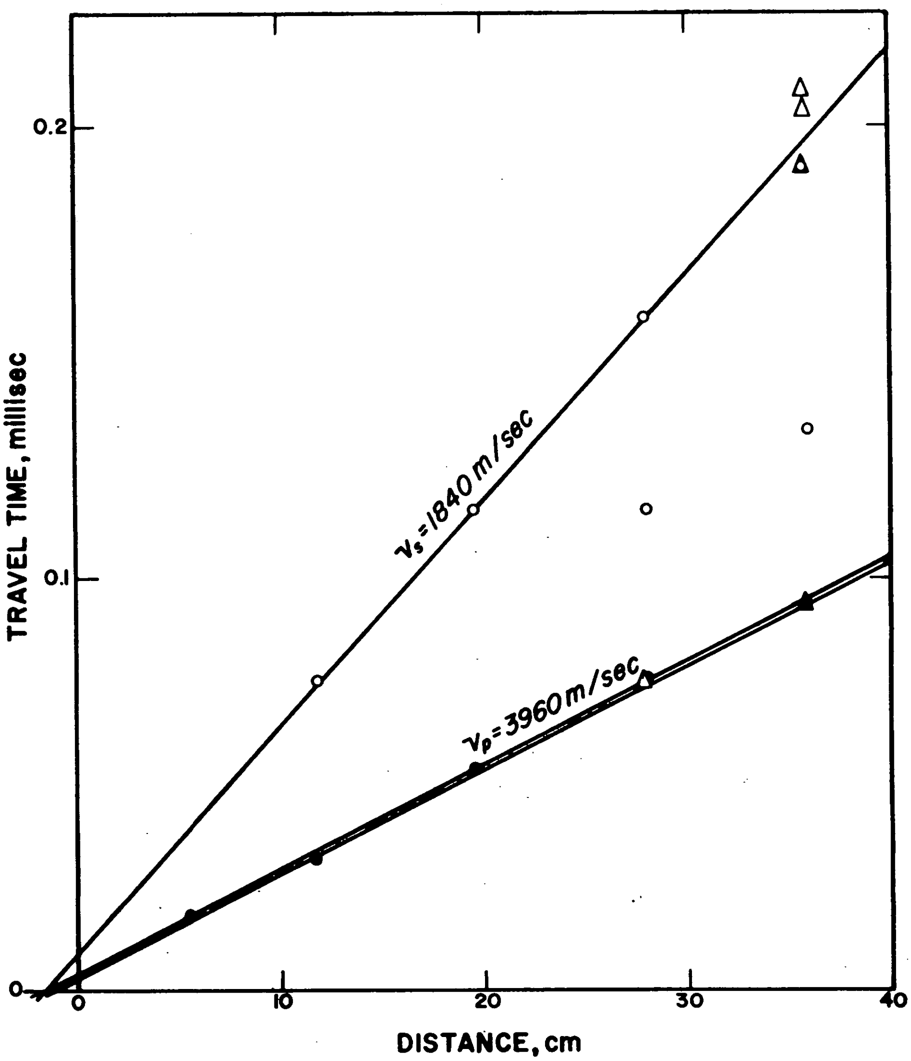


Figure 4. Travel time vs distance for velocity measurements along vertical line.

Figure 8o and 8q give a comparison of recordings from spherical and cylindrical cells at about equal distance. In recording 8f a combination of a 1-in. driver and a $\frac{1}{2}$-in. receiver was tried; it gave a better PP reflection than two 1-in. cells. If the function of a pair of cells is exchanged, the driver becomes the receiver and vice versa, the recording hardly changes, as demonstrated by Figure 8m and 8n.

The structure of the ice sheet was only sketchily studied. At the beginning of the investigation a rough check had been made on the fabrics of 3-in. cores and in all places the c-axis was approximately vertical. These preliminary tests were done at some depth below the surface. The orientation was checked on thin slabs of ice held between two sheets of crossed polaroid. A number of checks were also carried out in the research area itself after the ultrasonic measurements were finished. An elongated slab from the area where the velocity measurements were made was inspected and three cores were taken along the main reflection line (Fig. 9, 10). It was found that a thin layer at the surface showed a different orientation from the ice underneath, probably with arbitrary orientation of the c-axis (no measurements on the universal stage were carried out).

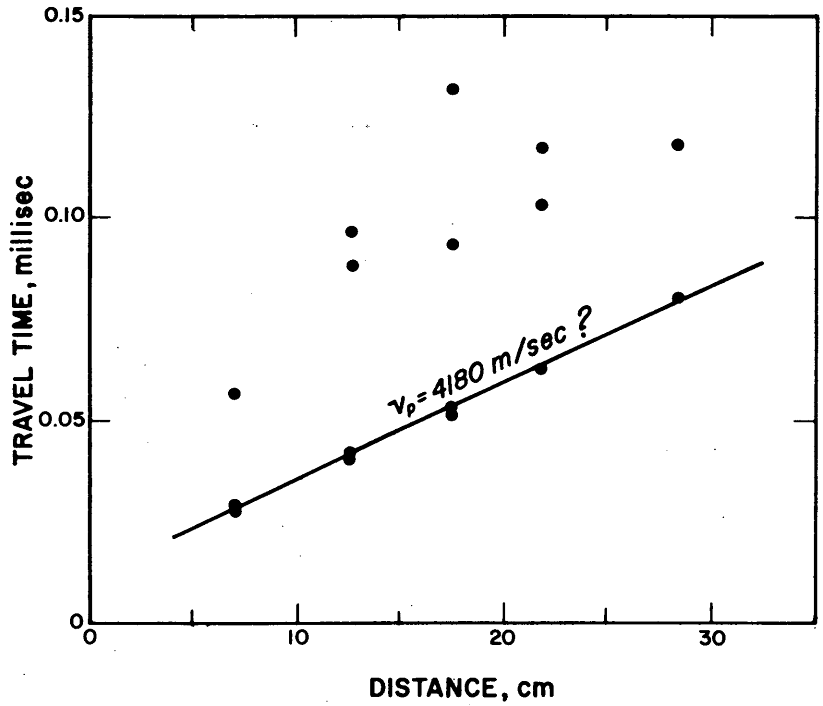


Figure 5. Travel time vs distance for velocity measurements along horizontal line.

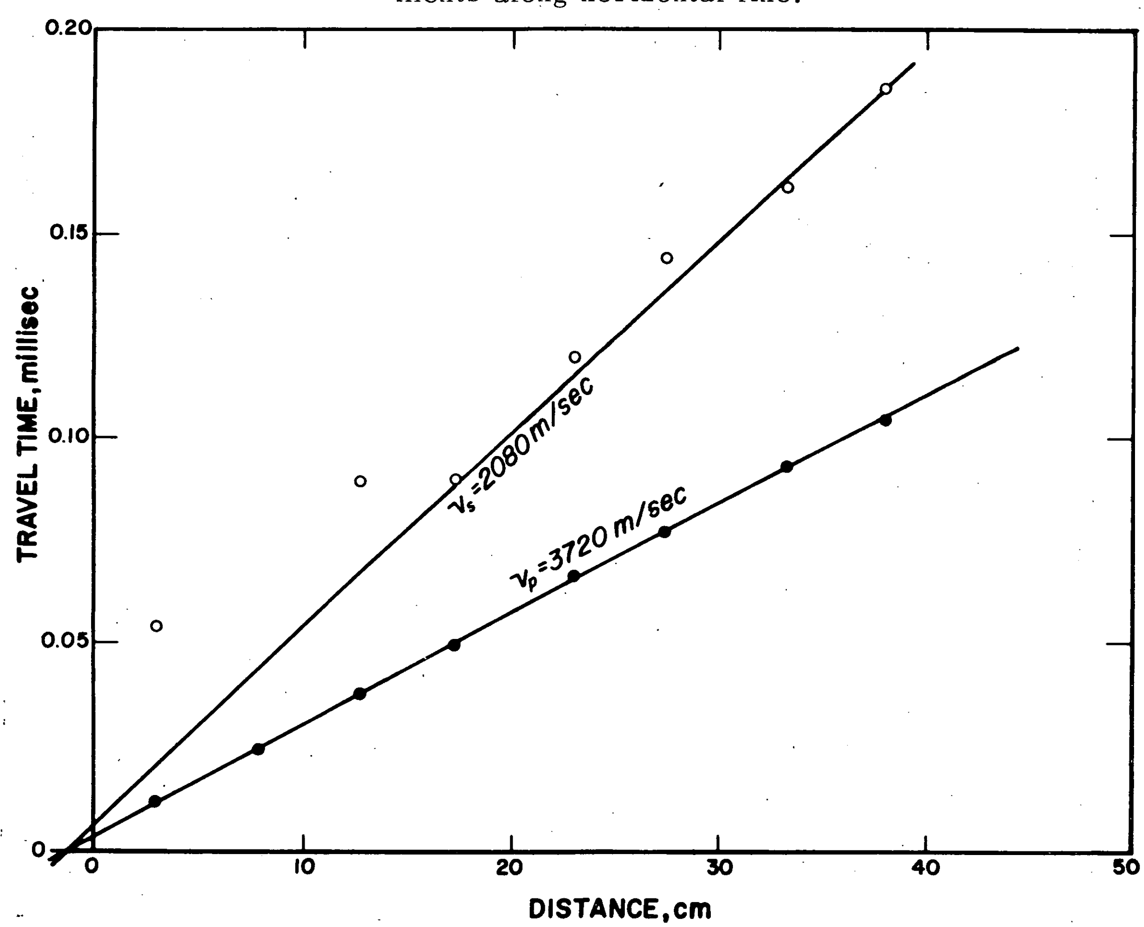


Figure 6. Travel time vs distance for velocity measurements along line dipping 45°.

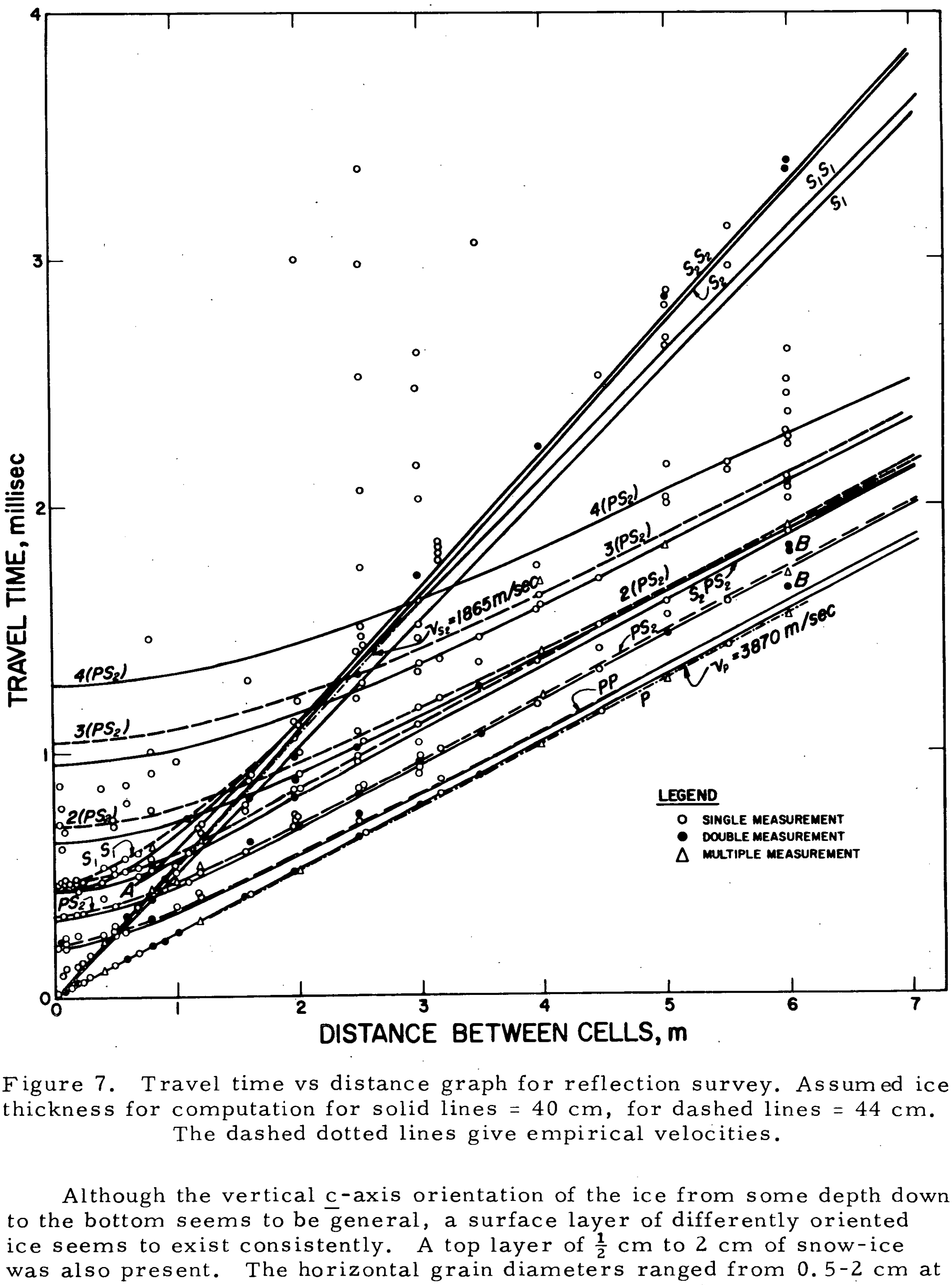
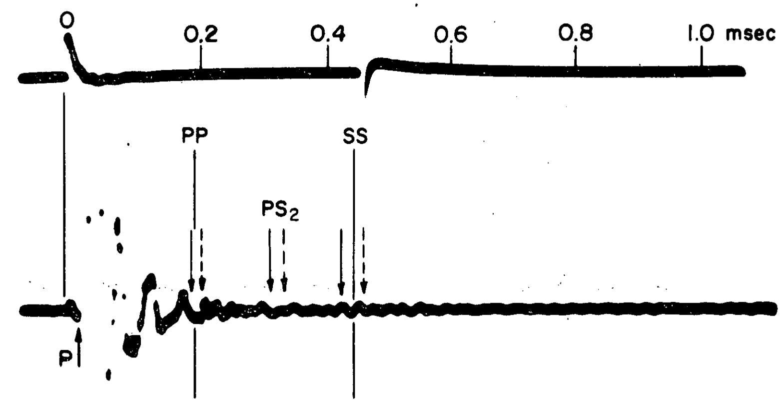
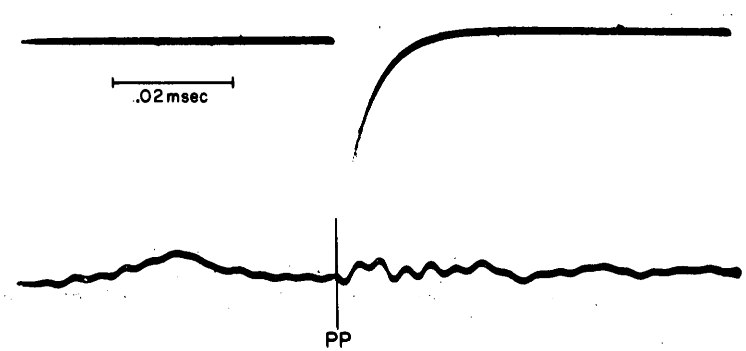


Figure 7. Travel time vs distance graph for reflection survey. Assumed ice thickness for computation for solid lines = 40 cm, for dashed lines = 44 cm. The dashed dotted lines give empirical velocities.

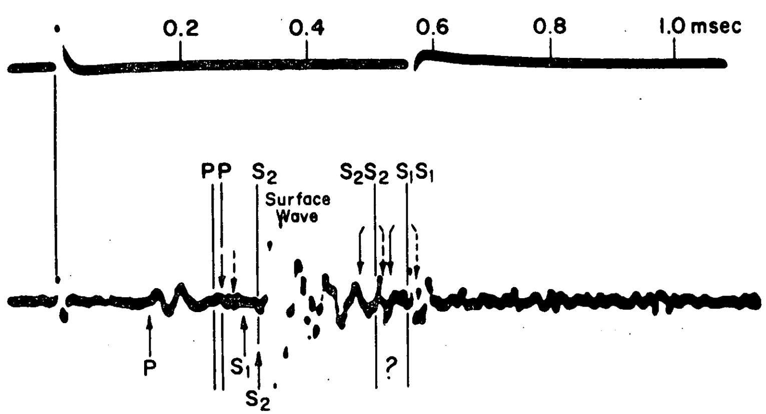
Although the vertical c-axis orientation of the ice from some depth down to the bottom seems to be general, a surface layer of differently oriented ice seems to exist consistently. A top layer of $\frac{1}{2}$ cm to 2 cm of snow-ice was also present. The horizontal grain diameters ranged from 0.5-2 cm at 3-cm depth to 2-10 cm at 15-cm depth. No grain-size observations were made in the ice with uniform c-axis orientation.



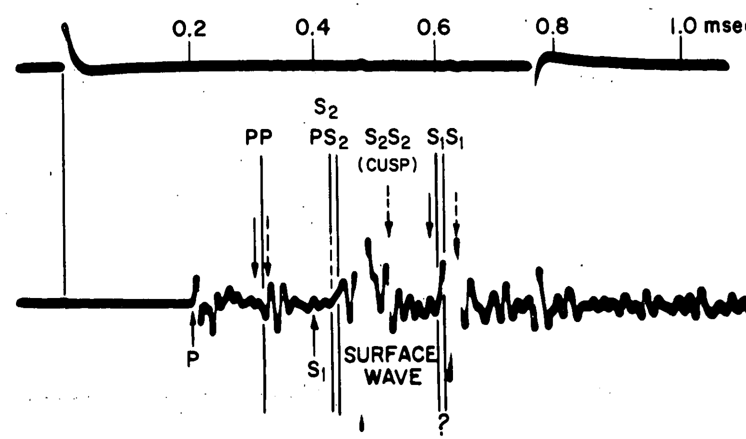
8a. Transducers: cylinders 3.9 cm apart. At SS high frequency signal not visible on reproduced recording.



8b. Section of Figure 8a containing PP in expanded scale.

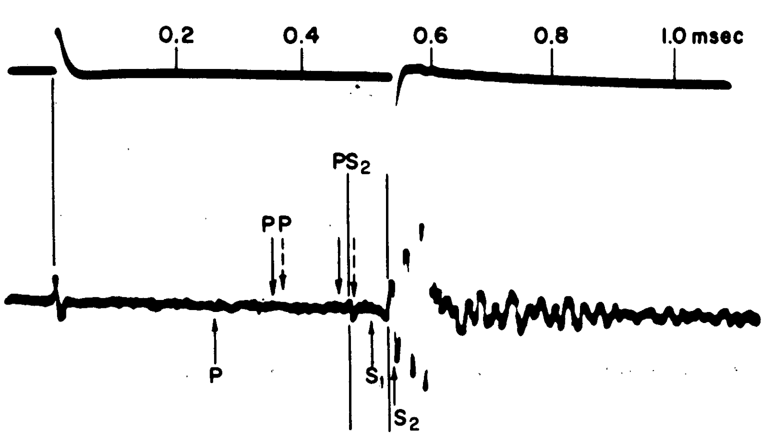


8c. Cylinders 58.3 cm apart.

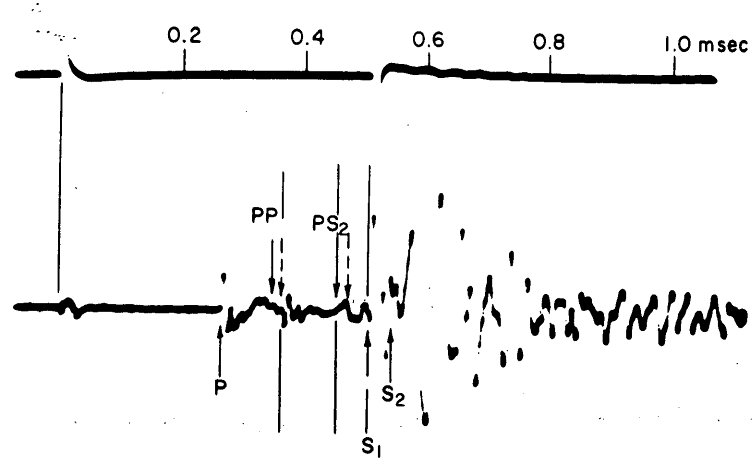


8d. 1-in. spheres 79.9 cm apart.

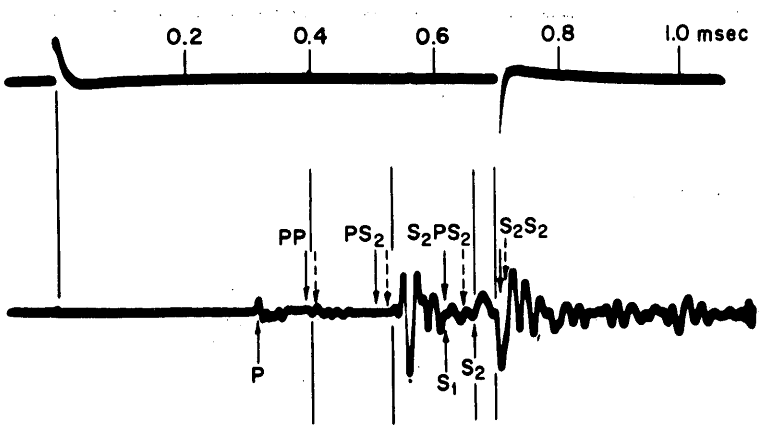
Figure 8. Soniscope registrations. Upper trace: time marker with approximate time scale added. Lower trace: received ultrasonic signal; measured events marked with long vertical lines, theoretical events for ice thicknesses of 40 cm and 44 cm marked with solid and dotted arrows respectively (except o and q, where the theoretical arrows are adjusted to match the P wave and one later event).



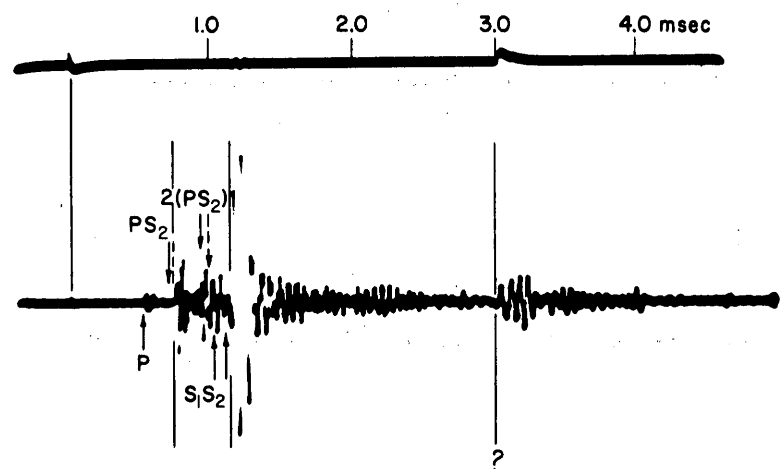
8e. Cylinders 98.7 cm apart.



8f. Transmitter = 1-in. sphere, receiver = $\frac{1}{2}$-in. sphere, 100.7 cm apart.

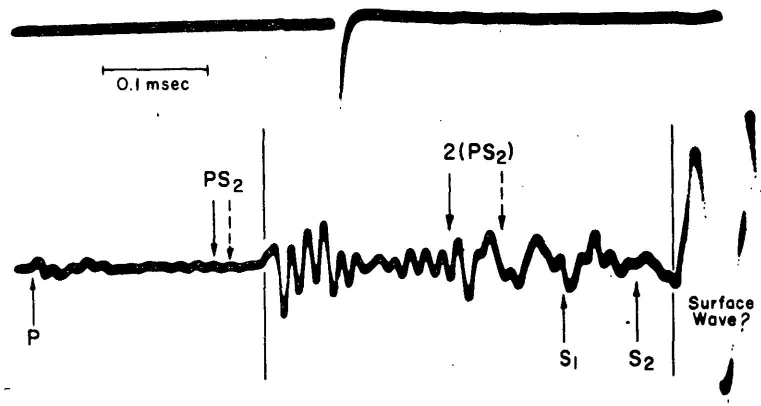


8g. 1-in. spheres 119.7 cm apart.

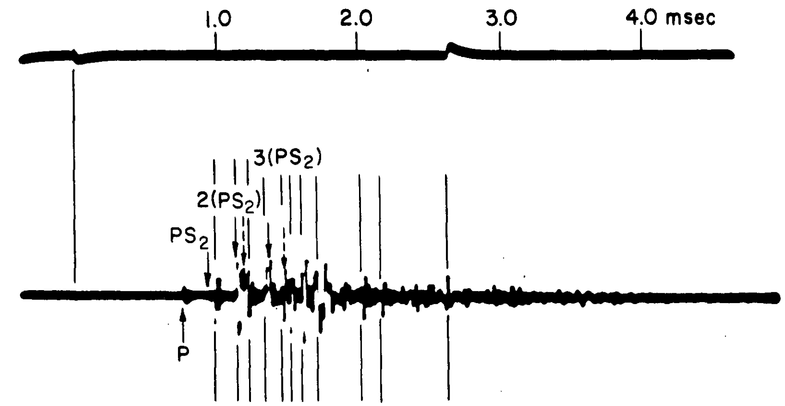


8h. 1-in. spheres 200.5 cm apart.

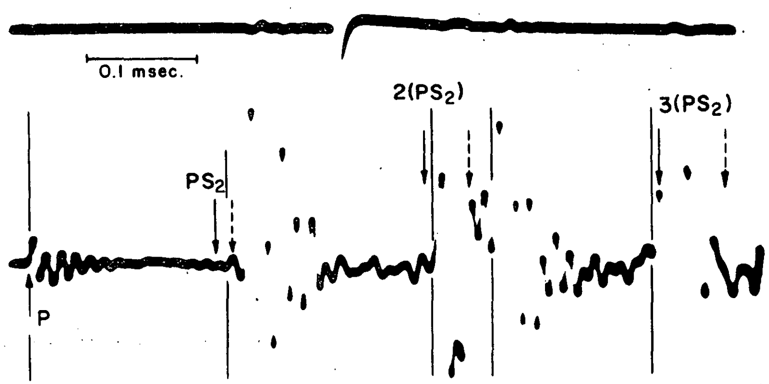
Figure 8 (Cont'd). Soniscope registrations.



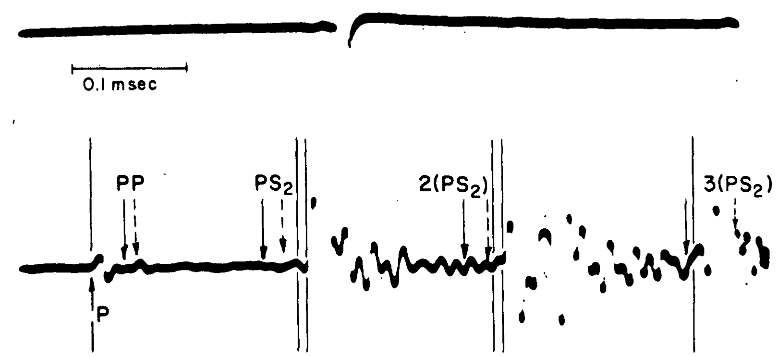
8 i. Section of Figure 8h in expanded scale.



8 j. 1-in. spheres 299.8 cm apart.

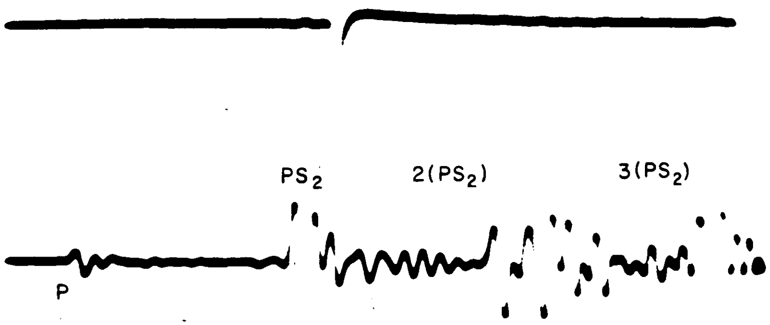


8k. Section of Figure 8 j in expanded scale.

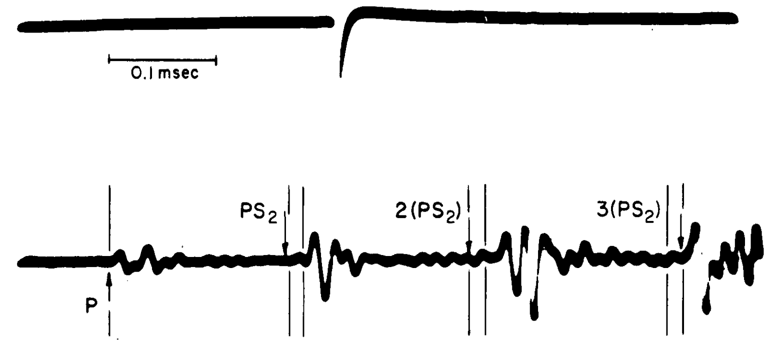


8m. 1-in; spheres 399.5 cm apart. Expanded scale.

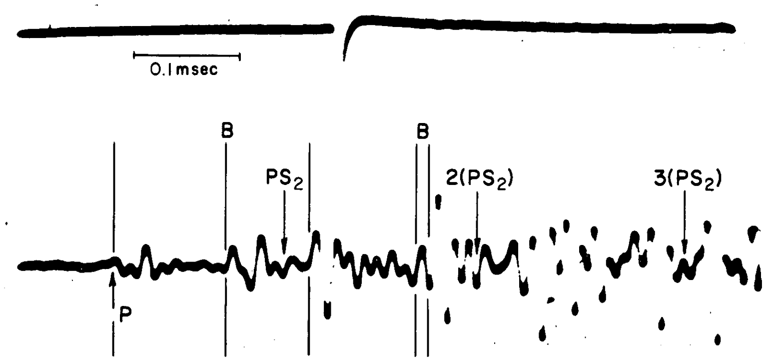
Figure 8 (Cont'd). Soniscope registrations.



8n. Same as in Figure 8m, transmitter and receiver reversed.

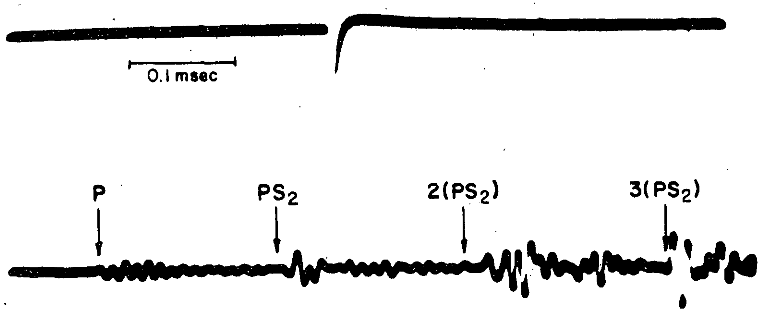


8o. 1-in. spheres 601.5 cm apart. The arrows mark the theoretical positions of events for ice of 41 cm thickness.

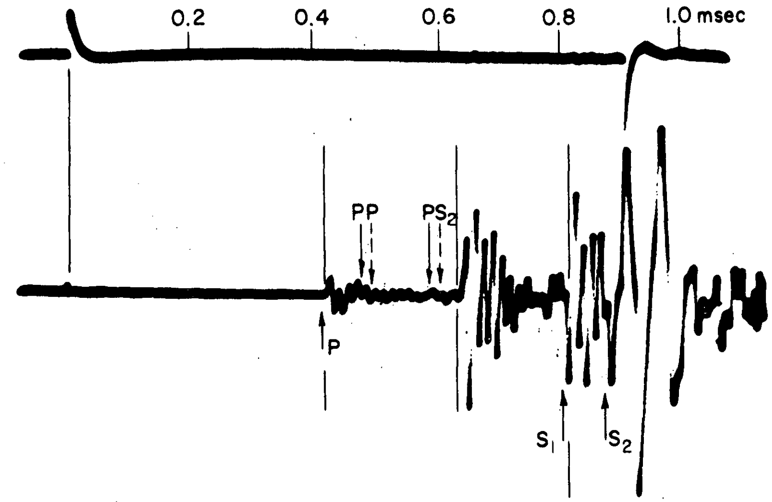


8p. Same setup as for Figure 8o after a new crack has developed (signals B, compare plottings in Figure 7).

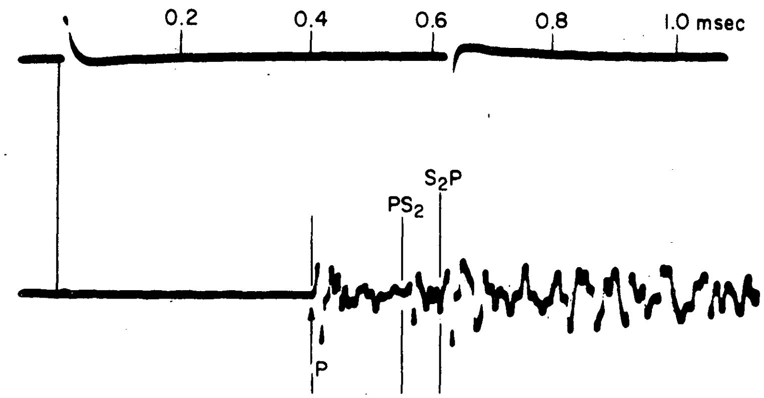
Figure 8 (Cont'd). Soniscope registrations.



8q. Cylinders 603.0 cm apart. The arrows mark the theoretical positions of events for ice of 41.3 cm thickness.



8r. 1-in. spheres 160.4 cm apart.



8s. 1-in. spheres, receiver 16 cm deep, horizontal distance from transmitter 158.0 cm.

Figure 8 (Cont'd). Soniscope registrations.

The ice thickness was measured with the SIPRE ice-thickness kit and was found to be:

On 11 March: 45.7 cm

On 18 March: 49.1, 49.5, and 50.1 cm.

The mean value for the week from 14 to 18 March was about 48 cm. By extrapolation one obtains about 44-45 cm for the time from 8 to 11 March. The mean thickness for the whole measuring period (8-18 March) was about 46 cm.

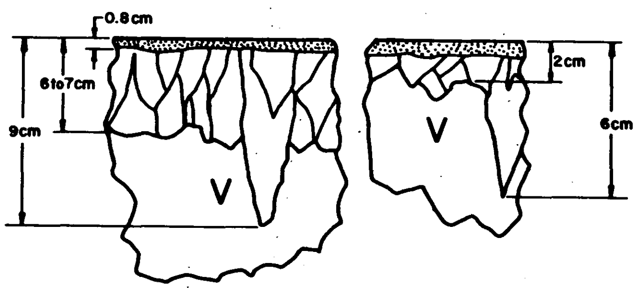


Figure 9. Vertical sections through ice near the surface. Dots: snow-ice, V: vertical c-axes.

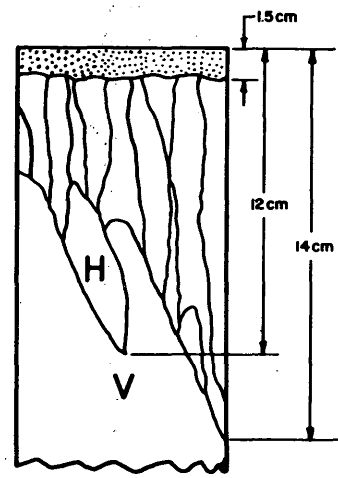


Figure 10. Vertical section through 3-in. core near the surface. Dots: snow-ice, H: c-axis horizontal, V: c-axes vertical.

DISCUSSION

Figure 7 shows which waves were consistently picked up in the reflection survey. They were direct P, direct S_2 and perhaps S_1 signals, surface waves, single-reflected PP for short distances not much over twice the ice thickness, single-reflected PS (plus SP), single-reflected S_2S_2 and S_1S_1 for short distances, double-reflected PS and equivalent combinations*, and higher multiples of the PS/SP type. The correctness of the interpretation of the PS/SP event could be demonstrated by setting one cell deeper into the ice than the other, to a depth of 1/3 of the ice thickness. The travel time of PS then becomes different from the one for SP, and the PS/SP event splits up on the record into two events accordingly. In Figures 8r and 8s recordings are shown for about equal distance, but with both cells at the surface in the case of 8r and with the receiver set deeper in 8s. Both events PS_2 and S_2P are clearly visible in the latter case. The internally refracted wave S_2PS_2 was

*For the double reflection, six combinations exist: PSPS, PPSS, PSSP, SPSP, SSPP and SPPS, which will all carry a changing fraction of total energy when the angle of incidence (the distance) is changed.

perhaps observed in some of the measurements, unless the few points fitting the line (Fig. 7) originated from reflections on cracks. The recordings in Figures 8a-s give an idea of the strength of the events. At long distance single and multiple PS/SP and the surface wave arrivals were by far the strongest signals observed.

The reflections were used to determine the ice thickness. In an anisotropic material this is not possible by simple arithmetic, except for very small spreads (distance between transducers), since travel time vs distance is not given explicitly but by parametric expressions. The graphical presentation for unit ice thickness (Fig. 3) can be used to determine the actual thickness of the ice sheet. The problem consists simply of determining the scale factor by which the coordinates must be changed so that the measured event falls on the appropriate curve. A straight line through the origin is drawn through the measured point plotted on the time-distance graph of Figure 3, and is brought to intersect the theoretical curve. The ratio of either of the coordinates of the observed point divided by the corresponding coordinate of the intersection gives the ice thickness in meters. In single measurements the proper identification of the reflection events is frequently quite problematic.

A large number of observed points are plotted in Figure 7. The theoretical curves have been reduced in scale in this case to fit the observation rather than determining a great many scale factors from each point. Two families of curves have been drawn with solid and dashed lines corresponding to ice thicknesses of 40 and 44 cm respectively. A fair number of points fall between the two lines or close to them, so that ice thickness between 40 and 44 cm is indicated by the measurements. This is less than the mean thickness of 46 cm measured directly. The fact that the cells were mounted about 1 to 4 cm deep is probably the main reason for finding too small a thickness.

The especially sharp reflections of the PS type at long spreads deserve particular attention. At these long distances the standard thickness determination as described above yields low accuracy results because of the low angle of the intersection between the straight line through the origin and the theoretical curves. The situation can be largely improved by using travel time differences between the P and PS events instead of total travel times, because these time differences become quite insensitive to distance, or total travel time, as the distance increases. At large distances the difference between the travel time of P and PS (and between multiple reflections of type PS) approaches a constant value of 0.376 millisec per meter of ice, and the ice thickness determination becomes very simple. The observed travel time difference in millisec has only to be divided by 0.376 msec/m to obtain the ice thickness in meters. At lesser distances this figure is larger, e.g., for single reflections approximately 4 msec/m at a distance of ten times the ice thickness. If the approximate ice thickness is known it may be computed more accurately by simple division by a factor measurable from Figure 3. To increase the accuracy a separate diagram of theoretical values for the travel-time difference between P and PS is given in Figure 11, including for comparison the same information for isotropic ice (polycrystalline ice with random crystallographic orientation; $v_p = 3823$ m/sec, $v_s = 1911$ m/sec). The same diagram can be used for multiple reflections when observed distance and time difference are divided by the multiplicity n, which is the number of PS paths in the ice sheet. Accurate ice thicknesses are found from Figure 11 by trial and error, or by the computation of an additional graph as described below.

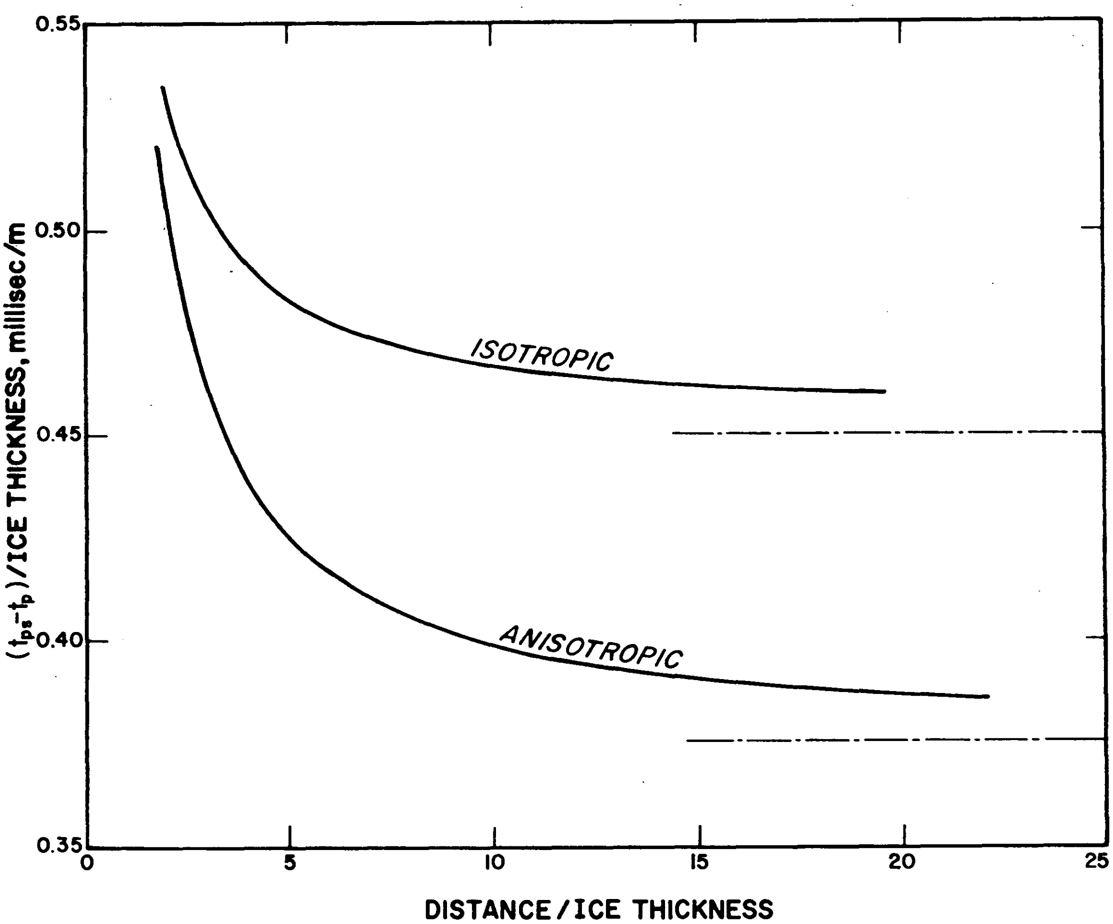


Figure 11. Theoretical values for travel-time differences $t_{ps} - t_p$ for unit ice thickness.

A particular case has been analyzed where the transducers were placed at a distance of 6.03 m, the longest distance included in the investigation, and where the PS-type reflections were especially clear and numerous. Figure 8q is a recording of the P wave and the first three PS reflections of the chosen case, illustrating the exceptional sharpness of the signals. A number of four PS reflections were used for the thickness determination. The diagram of Figure 11 provided theoretical travel-time differences between P and PS for the fixed distance of 6.03 m between cells and for variable ice thickness from 36 to 46 cm. Travel-time differences vs ice thickness for the single and the first three multiple PS-reflections were plotted for the isotropic and anisotropic case. It was sufficient to compute only a very few points for each individual case, since the connecting curves turned out to be almost straight lines (Fig. 12). The ice thicknesses which correspond to the measured travel-time differences were finally taken from the diagram (Table I). The consistency of the results is somewhat better in the isotropic than in the anisotropic case. However, the assumption of isotropic ice would give an impossibly small ice thickness. The true ice thickness of 9 March, the day the ultrasonic measurements at 6.03 m were carried out, was probably close to 44 cm (p. 16), a figure which is matched closely by the 42 cm obtained with the assumption of anisotropic ice.

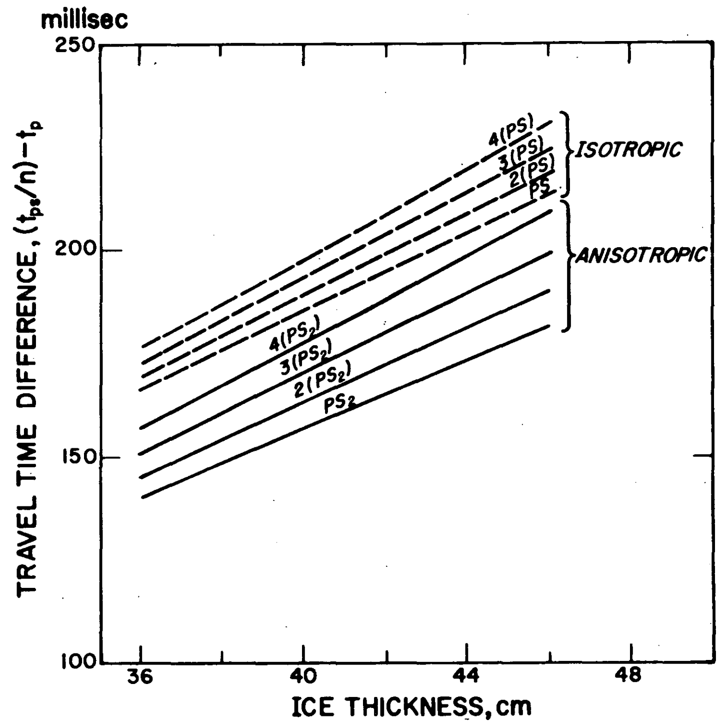


Figure 12. Travel-time difference vs ice thickness for the measurements at 6.03 m distance between transducers. (Respective record see Figure 8q).

Table I. Ice-thickness determination with PS reflections (in cm).

Assumption of character of ice	Multiplicity n of reflection: n = 1 (single PS)	n = 2 (double PS)	n = 3	n = 4	Average
isotropic	36.4	37.4	36.8	37.6	37.0
anisotropic	42.8	42.9	41.3	41.3	42.1

A striking feature of the results obtained with anisotropic theory is the tendency of higher multiples to indicate smaller ice thickness than the single PS reflection. This tendency was consistently observed and is also apparent in Figure 7. It cannot be readily explained. If corrections were made for the depth of the cells from the surface this tendency would be even stronger.

There are various reasons for discrepancies between the theoretical and observed travel time, however: (1) The ice thickness changed over the period of measurements, causing part of the scatter in Figure 7. (2) The ice varied slightly in thickness over the area, as may be seen from the three values measured about 3 m apart on 18 March. (3) In addition the thickness of the snow-ice, and especially of the surface ice of deviating orientation, varied from place to place. (4) The cells were mounted at variable depth from 1 to 4 cm. If the cells were at a different distance from the reflecting plane, then the PS and SP signals would arrive at slightly different times and interference might have occurred, possibly masking the very first break of the signal. (5) The temperature effect might have caused a velocity gradient perpendicular to the surface, causing curved rays. (6) The elastic constants and the density chosen to compute the theoretical curves were not perfect for the lake ice under investigation. Specifically, in the horizontal direction the velocities of 3870 m/sec for the P wave and 1865 m/sec for the S_2 wave (dash-dot lines in Fig. 7) would agree better with the observations than 3823 m/sec and 1834 m/sec obtained by computation. (The observed velocities are higher than the ones reported by Brockamp and Querfurth, 1964, if assuming -10°C in the top layer of the ice by 0.8% for PS and 2.7% for SP, probably due to the effect of the deviating orientation of ice crystals in the surface layer and due to experimental error).

The accuracy of the ice thickness determination cannot be firmly stated, because direct measurements by coring methods could not be carried out simultaneously with the ultrasonic measurements without intolerable damage to the ice. A rough estimate of the accuracy is 2-4 cm, or 5-10% of 40 cm when using the scale factor (Fig. 7). A lesser relative error would undoubtedly have been obtained with this method on a thicker ice sheet. Using only the reflections of the PS type at long transducer spreads, a discrepancy of 4% was observed between single and multiple reflections. The average ice thickness obtained from the single and first three multiple reflections deviated about 5% from a value extrapolated from direct observations made 2 days after the ultrasonic measurements. Using the single reflection alone and applying cell-depth corrections the deviation is substantially smaller, in the order of 0.5 cm. An accuracy of the same amount could be expected for relative measurements, say day-to-day growth observations. A temperature effect might have to be considered, but from the figures of Brockamp and Querfurth (1964) it can be estimated to be less than 1% for temperature changes of a few centigrades below -1°C, and in the order of a few % between -1 and 0°C.

CONCLUSIONS

From the observation of ultrasonic pulses sent through lake ice between ceramic transducers placed at the surface of the ice it was found that different types of reflections could be obtained which in turn could be used to measure the ice thickness. At distances as large as fifteen times the ice thickness, especially sharp signals of PS reflections were obtained (Fig. 8o-8q). The ice was found to be anisotropic with vertical crystallographic _c_-axes except for a relatively thin layer at the surface. Neglecting the anisotropy for the interpretation of the PS-type reflections would cause an error of 15%.

The finding of the very strong PS signals is a contribution to knowledge of the propagation of elastic waves in anisotropic material. It is also of interest for the seismologist operating on glaciers, where a strongly preferred orientation of the _c_-axes is quite common. [Fairly strong PS-reflections were observed near the edge of the Greenland ice sheet in the Thule area (Roethlisberger, 1959).] On lake ice the method provides means to measure ice thickness in a fixed place without damaging the ice and without disturbing the water underneath.

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13. ABSTRACT

Travel-time measurements of ultrasonic pulses were carried out in March 1960 on Lake Superior (Keweenaw Bay) near Baraga, Mich. The ice was about 45cm thick and consisted of grains with vertical c-axis orientation with the exception of a surface layer of variable thickness. Ultrasonic pulses were transmitted and received by barium titanate cells of cylindrical and spherical shape. The transducers were mounted at the surface and the distance was varied. Several direct and reflected signals could be identified. Of the reflected events, the PS type were the clearest and strongest at distances many times the ice thickness, and thus best suited for ice thickness determination. In order to obtain satisfactory agreement between theoretical and measured travel times the anisotropy of the ice had to be taken into account. The elastic constants determined by Bass et al. (1957) gave reasonably good agreement between computed and measured travel time, but some discrepancies remain to be explained. Part of the observed reflections occurred on cracks in the ice. Using equipment with approximately 100-kc signals the ice thickness was determined by the ultrasonic pulse method, destruction free, with an accuracy of 2-4 cm or 5-10%. For day-to-day comparisons the relative accuracy would be in the order of 0.5cm.

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