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COMPARISON BETWEEN MEASURED
AND
THEORETICAL TEMPERATURE PROFILES
OF THE
CAMP CENTURY, GREENLAND,
BOREHOLE

by
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PREFACE

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CONTENTS

	Page
Preface	ii
Summary	iv
Introduction	1
Theory	4
Discussion	9
Literature cited	12

ILLUSTRATIONS

Figure	
1. Measured and theoretical temperature profiles for the Camp Century borehole	1
2. Calculated values of shear strain rate as a function of depth	6
3. Calculated values of the horizontal velocity component as a function of depth	6
4. Plots of the quantities H , $V_x \partial T / \partial x$, and $(H - V_x \partial T / \partial x)$ as a function of depth	7
5. Integral of Dawson's integral versus y/L	9
6. Age of ice versus depth	10
7. Temperature profiles	11

TABLES

Table	
I. Camp Century borehole data	3
II. Numerical values of quantities appearing in eq 9 and 10 for curves 1 and 2 ..	8
III. Numerical values of quantities appearing in eq 9 and 10 for curves 3 and 4 ..	12

SUMMARY

Steady-state temperature profiles are calculated for the borehole drilled through the Greenland ice sheet at Camp Century. The profiles are found by modifying Robin's theory through the addition of several correction terms. One of these terms is the internal heating arising from creep deformation. The importance of this term was emphasized by Lliboutry. The new theoretical profiles do not differ appreciably from the profile derived from Robin's theory. The theoretical profiles do differ substantially from the Camp Century profile measured by Hansen. It is concluded that Hansen's observations are evidence that factors such as accumulation rate and the upper surface temperature are not in a long-term steady-state condition. Better agreement between theoretical and measured curves is obtained if it is assumed that the accumulation rate was about 40% smaller in the past and that the mean annual surface temperature varied by about 0.5C over the past 1000 years.

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Introduction

The temperature profile of the Greenland ice sheet that was measured in the deep drill hole at Camp Century (Hansen and Langway, 1966 and unpublished data) differs somewhat from the steady-state profile calculated from Robin's theory (Robin, 1955). Figure 1 shows the measured profile and a theoretical profile calculated from the Robin equation:

$$T = T_0 + (\pi/4)^{1/2} L(dT/dy)_{y=0} \operatorname{erf}(y/L) \quad (1a)$$

which can be rewritten as

$$T = T_u + (\pi/4)^{1/2} L(dT/dy)_{y=0} \{\operatorname{erf}(y/L) - \operatorname{erf}(h/L)\}. \quad (1b)$$

In these equations T is the temperature at a distance y above the bottom of an ice sheet whose thickness is h ; T_0 is the temperature at the bottom surface ($y=0$) and T_u is the temperature at the upper surface ($y=h$); $(dT/dy)_{y=0}$ is the thermal gradient at the bottom of the ice sheet; and L is a characteristic thermal diffusion distance given by the equation

Figure 1. Measured (by B.L. Hansen) and theoretical temperature profiles for the Camp Century borehole. Theoretical curve 1 calculated using local value of accumulation rate gradient; curve 2 calculated using average accumulation rate gradient.

$$L = (2\kappa h/V)^{1/2}. \quad (2)$$

In this equation κ is the coefficient of thermal diffusivity of ice and V is the vertical ice velocity (that is, the velocity in the y direction) at the top surface. This velocity is given by

$$V = (V_y)_{y=h} \approx -a + (V_x)_{y=h} dh/dx \quad (3)$$

where a is the accumulation rate and V_x and V_y are the ice velocities in the horizontal x direction and the vertical y direction respectively. (The coordinate system is chosen so that the third velocity $(V_z)_{y=h}$ is zero. The positive x direction is towards the edge of the ice sheet. The positive y direction is up.)

The theoretical profile was calculated from eq 1b and the measured values of the upper ice temperature and the thermal gradient at the bottom surface.* (The values of these measured quantities and others used in this paper are listed in Table I.)

As a first approximation Robin's equation gives a reasonable account of the temperature profile. There is a discrepancy of 2.7°C between the measured and the calculated temperature at the bottom of the ice sheet. The difference between the measured and the predicted value of $(T_u - T_0)$ thus amounts to about 25%. The theory fails to predict the temperature reversal near the top of the ice sheet (that is, the initial decrease rather than increase of the temperature with depth).

Robin's equation is an exact solution of the temperature profile at the center of an ice sheet that is in a steady state. For such an ice sheet neither the upper surface temperature nor the accumulation rate varies as a function of time when these quantities are averaged over time intervals of the order of 1 year. Any difference between a measured and a theoretical profile is a measure, of sorts, of the degree of the violation of the initial assumptions that led to the derivation of eq 1.

Long-term climatic change has been suggested in the past as one obvious violation of the initial assumptions. Robin's equation was derived under the assumption that ice moves only in the vertical direction. This assumption is correct at the center of an ice sheet. Away from the center the ice velocity also has a horizontal component. This velocity must be taken into account in the derivation of the temperature profile. The Camp Century borehole is, of course, far from the center of the ice sheet. Horizontal motion of the ice thus is a second process that could have caused the differences between the measured and the theoretical profiles of Figure 1.

If Robin's equation were corrected for the effect arising from horizontal ice motion, any remaining difference between the theoretical and measured profiles could reasonably be ascribed to past climatic change. Such a difference would be of great interest to paleoclimatic studies. Therefore, it is worthwhile to refine the theoretical profile calculation for the Camp Century borehole. It is the purpose of this paper to attempt to correct the theoretical profile of Figure 1 for the effect of horizontal ice motion. A correction also will be attempted for an effect pointed out recently by Lliboutry (1966): The creep deformation of ice gives rise to internal heating within the ice mass. This heat source, which is in addition to the geothermal heat that comes from beneath an ice sheet, should be included in the fundamental equation of heat transport.

Jenssen and Radok (1963) have already extended Robin's theory for horizontal ice motion. However, they were unable to obtain an analytic solution for this problem and had to give representative solutions obtained through use of a computer. It is not possible to take their results and apply them to the Camp Century borehole results.

* Since the temperature of the upper surface is fixed by the climate and the temperature gradient at the lower surface is determined by the geothermal heat flow, it seems logical to insert the values of these two quantities in eq 1b to obtain the theoretical profile rather than to use the other less basic parameters, the lower surface temperature and the temperature gradient at the upper surface.

Table 1. Camp Century borehole data.

Quantity	Description	Numerical value	Reference	Remarks
a	accumulation rate	32 g/cm ² yr (35 cm ³ /cm ² yr)	Crozaz and Langway, 1966	Average value over last 150 years (in equivalent of high density ice).
$V = (V_y)_{y=h}$	vertical ice velocity at upper surface	36 cm/yr		Calculated using eq 2
$V^* = (V_x)_{y=h}$	horizontal ice velocity at upper surface	3.3 m/yr	Mock, 1967, (unpublished)	
h	ice thickness	1387.4 m	Hansen and Langway, 1966	
dh/dx	surface slope	-3.6×10^{-3}	Mock, 1967 (unpublished)	
κ	thermal diffusivity of ice	0.0115 cm ² /sec	Carlsaw and Jaeger, 1959	
c	specific heat of ice	0.502 cal/°C g	Carlsaw and Jaeger, 1959	
ρ	density of high density glacier ice	0.917 g/cm ³	Langway, 1958	
$L = (2\kappa h/V)^{1/2}$	characteristic thermal diffusion distance	529 m		
$T_u = (T)_{y=h}$	upper surface temperature	-24.0°C	Hansen and Langway (unpublished)	Temperature at level below the seasonal fluctuations
$T_0 = (T)_{y=0}$	lower surface temperature	-13.0°C	Hansen and Langway, 1966	
$(dT/dx)_{y=h}$	horizontal temperature gradient at surface	0.022°C/km	Mock (unpublished)	
$(dT/dy)_{y=0}$	vertical temperature gradient at bottom	-1.77×10^{-4} °C/cm	Hansen and Langway, 1966 (unpublished)	
$(V_x dT/dx)_{y=h}$	horizontal temperature gradient at bottom surface	7.3×10^{-5} °C/yr		Calculated using eq 6
$(dT/dx)_{y=0}$	horizontal temperature gradient at bottom surface	0.057°C/km		
$(Q)_{y=0}$	heat generated per unit volume at bottom	6.75×10^3 ergs/cm ³ yr (1.6×10^{-4} cal/cm ³ yr)		Calculated using eq 8
$H = (Q \rho c)_{y=0}$		3.5×10^{-4} °C/yr		
da/dx	accumulation rate gradient	-0.38 cm ³ /cm ² yr/km ($+0.024$ cm ³ /cm ² yr/km)	Mock, 1968 Mock, in press	Local value at Camp Century (average value)
$a^{-1} da/dx$		-0.011/km ($+6.8 \times 10^{-4}$ /km)		Local value (average value)
$h^{-1} dh/dx$		-0.0026/km		
Q^*	activation energy of creep	0.60 eV (13.8 kcal/mole)	Ramseier, 1967	
n	stress exponent of steady-state creep	3.2	Glen, 1955	
$\rho g h d h/dx$	shear stress at bottom surface	0.45×10^8 dynes/cm ² (0.45 bars)		
$(\dot{\gamma}_{xy})_{y=0}$	shear strain rate at bottom surface	0.015/yr		Calculated using eq 7 and adjusting constant C so that $(V_x)_{y=h} = 3.3$ m/yr
J	mechanical equivalent of heat	4.18×10^7 ergs/cal		

In this paper a much less ambitious calculation than that attempted by Jenssen and Radok will be attempted. Instead of trying to find the general temperature distribution of an ice sheet we will try instead to obtain a plausible correction to Robin's theory only for the special case of the Camp Century borehole.

Theory

Under steady-state conditions the heat transport equation is*

$$\kappa(\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2) - (V_x \partial T/\partial x + V_y \partial T/\partial y + V_z \partial T/\partial z) + H = 0. \quad (4)$$

Here V_y is the vertical velocity, V_x the horizontal velocity, V_z the other horizontal velocity, and $H = Q/\rho c$, where Q is the internal heat generated per unit volume, ρ is the density of ice, and c is the specific heat of ice. It is assumed that κ and ρ are constants. This assumption breaks down in the firn layer at the upper surface.[†] However, it can be shown that the error in the temperature profile that results from the violation of this assumption is quite small.

By proper choice of the coordinate system V_z can be set equal to zero.** The thermal gradients in the horizontal directions are so small that only negligible heat transport in the horizontal direction occurs by conduction. Mass transport of ice produces the major transport of heat in this direction. Therefore, eq 4 can be simplified to

$$\kappa \partial^2 T/\partial y^2 + (V_y/h)(\partial T/\partial y) - V_x(\partial T/\partial x) + H = 0. \quad (5)$$

In this equation use has been made of the relationship $V_y \approx -V_y/h$.

Equation 1 is the solution of this equation when the last two terms on the left-hand side are zero. If these two terms are known as a function of vertical distance y the equation can be solved to give the theoretical profile. Consider now what we know or can reasonably deduce about these two terms.

* The adiabatic heating of ice has been ignored in eq 4. For ice with no porosity the adiabatic heating that occurs when the pressure is changed suddenly from atmospheric (1 bar) to that existing at the bottom of the Camp Century borehole (130 bars) is less than 0.01°C. The ice of the Greenland ice sheet contains entrapped gas bubbles and consequently the adiabatic heating is somewhat greater than this value. In samples of ice taken from a 360-m borehole at Site 2, Greenland, close to Camp Century, Langway (1967) has found that the pore spaces close off from each other at about a 70-m depth. The ice density at this depth is 0.828 g/cm³. The theoretical ice density is 0.92 g/cm³. Since the pores are filled with entrapped air their volume at depths below 70 m is inversely proportional to the overburden pressure. The work done per unit volume of ice in compressing the gas within the pores is approximately $[(0.92 - 0.828)/0.92]P_0 \log(P/P_0)$, where P is the pressure at depth and P_0 is the pressure at which the pores close off ($P_0 \approx 5$ bars). The maximum heating of ice that can occur from this work for $P = 130$ bars is only of the order of 0.02°C.

† If κ is not a constant the first term on the left-hand side of eq 4 is replaced by $\nabla \cdot (\kappa \nabla T)$. The thermal diffusivity varies appreciably within the top 50 m of the ice sheet. This variation of κ does not introduce an appreciable error in the theoretical profiles we are considering because the major heat transport in this upper region of the ice sheet occurs through motion of matter rather than by the flow of heat down temperature gradients.

** There is some indication from inclinometer (B.L. Hansen, personal communication) that the velocity vector in the horizontal direction of the Camp Century borehole does not point in the same direction at the surface as at depth. If future measurements indicate that the differences in direction are large it will be necessary to include the $V_z \partial T/\partial z$ term in further corrections to the theoretical profile.

The horizontal temperature gradient $(dT/dx)_{y=h}$ has been measured at the surface (Mock, 1968). Its value at depth can be estimated through the use of eq 1b. Since $h/L > 1$ for the Camp Century borehole the horizontal temperature gradient is approximately

$$\partial T/\partial x \approx dT_u/dx + [(T - T_u)/2][h^{-1}dh/dx - (V^{-1}dV/dx)] \quad (6)$$

where T is given by eq 1b.

Ordinarily T_u and V decrease towards the center of an ice sheet. Thus dT_u/dx and dV/dx are positive quantities. The slope is always negative. Therefore, dT_0/dx , in general, is smaller than dT_u/dx . The horizontal temperature gradient dT_0/dx at the bottom surface can even be negative (that is, the bottom surface of an ice sheet can be warmer near the center than at the edge).

At Camp Century the term $V^{-1}dV/dx$ is anomalous in sign. (It is assumed that $V^{-1}dV/dx \approx a^{-1}da/dx$.) Its sign is negative rather than positive. Its magnitude is larger than that of the term $h^{-1}dh/dx$. The isohyets (lines of constant rate of accumulation) have a peculiar pattern in the Camp Century vicinity. They make the high angle of about 70° with respect to the surface contour lines rather than lie approximately tangent to them. If the local pattern of the isohyets is disregarded the average value of $a^{-1}da/dx$ over a distance of about 200 km upslope from Camp Century has a normal sign. In this paper temperature profiles will be calculated using both the local, anomalous, value of $a^{-1}da/dx$ and the average, more normal, value.

The ice velocity in the horizontal direction is estimated by integrating the (engineering) shear strain rate $\dot{\epsilon}_{xy}$. Since the longitudinal strain rate is small the shear strain rate is given by Glen's law

$$\dot{\epsilon}_{xy} = C \exp(-Q^*/RT) [\rho g(h-y)dh/dx]^n \quad (7)$$

where T is the temperature in degrees absolute, C is a constant, n is another constant that is approximately equal to 3, Q^* is the activation energy of creep, R is the gas constant, and the expression $\rho g(h-y)dh/dx$ is equal to the shear stress at vertical distance y .

A plot of normalized shear strain rate versus vertical distance y is given in Figure 2. The shear strain rate was calculated using $n = 3.2$ in eq 7. Normalization was accomplished by dividing the shear strain rate by the shear strain rate for $T = 260K (-13C)$ and a shear stress $\rho gh dh/dx = 0.45$ bars.

If the shear strain rate given in Figure 2 is integrated (numerically) the horizontal velocity V_x at vertical distance y can be obtained. A plot of V_x versus y is shown in Figure 3. It is assumed in this plot that because the bottom surface is frozen to the base $(V_x)_{y=0} = 0$. The constant C of eq 7 was adjusted so that $(V_x)_{y=h}$ is equal to the measured value of 3.3 m/yr.

The value of the term $V_x \partial T/\partial x$ for any value of y can be obtained through the use of Figure 3 to give V_x and eq 6 to give $\partial T/\partial x$. Figure 4 shows a plot of this term as a function of vertical distance y . Also shown is a plot of $H = Q/\rho c$, where Q is the internal energy produced by creep deformation. This is the term that Lliboutry suggested may be of importance to the temperature profile. We have assumed that, to a good approximation,

$$Q = \dot{\epsilon}_{xy} \sigma / J \quad (8)$$

where the shear creep rate $\dot{\epsilon}_{xy}$ is given by eq 7 and Figure 2, σ is the shear stress at depth y and is approximately equal to $\rho g(h-y)dh/dx$, and J is the mechanical equivalent of heat. In eq 8 it is assumed that the contribution to internal heating from the longitudinal creep rate is negligible.

CAMP CENTURY BOREHOLE TEMPERATURES

Figure 2. Calculated values of shear strain rate as a function of depth.

Figure 3. Calculated values of the horizontal velocity component as a function of depth.

Figure 4. Plots of the quantities H , $V_x \partial T / \partial x$, and $(H - V_x \partial T / \partial x)$ as a function of depth. Curve 1 calculated using local value of accumulation rate gradient; curve 2 with average value of this gradient.

Figure 4 also shows the sum $H - V_x \partial T / \partial x$ as a function of y . This curve can be represented approximately by the relationships

$$\begin{aligned} H - V_x \partial T / \partial x &= A_1 - A_2 y / y_0 & \text{for } 0 < y < y_0 \\ &= B & \text{for } y_0 < y \end{aligned} \quad (9)$$

where the values of the constants A_1 , A_2 , B , and y_0 are listed in Table II.

An analytical solution of eq 5 for the temperature profile can be found if eq 9 is used to evaluate the term $H - V_x \partial T / \partial x$. The solution is equal to $T(y) = \theta_0(y) + \theta_1(y)$ for the region $0 < y < y_0$ and $T(y) = \theta_0^*(y) + \theta_2(y)$ for the region $y_0 < y$, where the functions θ_0 , θ_1 , etc., are given by

$$\theta_0 = \theta_{u0} + (\pi/4)^{1/2} L (d\theta_0/dy)_{y=y_0} \exp(y_0^2/L^2) [\operatorname{erf}(y/L) - \operatorname{erf}(h/L)] \quad (10a)$$

$$\begin{aligned} \theta_1 &= -(B/\kappa) \int_{y_0}^y \exp(-t^2/L^2) \int_0^t \exp(s^2/L^2) ds dt \\ &\quad - (BL^2/\kappa) \int_{y_0/L}^{y/L} \exp(-t^2) \int_0^t \exp(s^2) ds dt \end{aligned} \quad (10b)$$

Table II. Numerical values of quantities appearing in eq 9 and 10 for curves 1 and 2.

Quantity	(Curve 1) Numerical value using local value of $a^{-1} da/dx$	(Curve 2) Numerical value using average value of $a^{-1} da/dx$
A_1	$3.5 \times 10^{-4} \text{ } ^\circ\text{C/yr}$	$3.5 \times 10^{-4} \text{ } ^\circ\text{C/yr}$
A_2	$4.23 \times 10^{-4} \text{ } ^\circ\text{C/yr}$	$4.23 \times 10^{-4} \text{ } ^\circ\text{C/yr}$
B	$-7.3 \times 10^{-5} \text{ } ^\circ\text{C/yr}$	$-7.3 \times 10^{-5} \text{ } ^\circ\text{C/yr}$
y_0	260 m	400 m
$(d\theta^*/dy)_{y=0}$	$-2.4 \times 10^{-4} \text{ } ^\circ\text{C/cm}$	$-2.2 \times 10^{-4} \text{ } ^\circ\text{C/cm}$
$(d\theta/dy)_{y=0}$	$-1.5 \times 10^{-4} \text{ } ^\circ\text{C/cm}$	$-1.15 \times 10^{-4} \text{ } ^\circ\text{C/cm}$
$(\theta_0 + \theta_1)_{y=y_0}$	-20.6C	-22.06C
θ_{b0}^*	-15.7C	-15.05C
θ_{u0}	-24.9C	-24.8C

$$\theta_0^* = \theta_{b0}^* + (\pi/4)^{1/2} L (d\theta_0^*/dy)_{y=y_0} \text{erf}(y/L) \quad (10c)$$

$$\begin{aligned} \theta_2 &= -\kappa^{-1} \int_0^y [\exp(-t^2/L^2) \int_0^t (A_1 - A_2 s/y_0) \exp(s^2/L^2) ds] dt \\ &= (A_2 L^2 y/2y_0\kappa) - (A_1 L^2/\kappa) \int_0^{y/L} [\exp(-t^2) \int_0^t \exp(s^2) ds] dt. \end{aligned} \quad (10d)$$

The constants θ_{u0} , θ_{b0}^* , $(d\theta_0^*/dy)_{y=y_0}$, and $(d\theta_0^*/dy)_{y=y_0}$ are chosen so that at the upper surface the the calculated temperature is equal to the measured temperature (that is, $\theta_0(h) + \theta_1(h) = T_u$); at $y=y_0$ there is no discontinuity in temperature or temperature gradient (that is, $\theta_0(y_0) + \theta_1(y_0) = \theta_0^*(y_0) + \theta_2(y_0)$ and $(d\theta_0^*/dy + d\theta_1/dy)_{y=y_0} = (d\theta_0^*/dy + d\theta_2/dy)_{y=y_0}$); and at the bottom surface the calculated temperature gradient is equal to the measured temperature gradient (that is, $(d\theta^*/dy)_{y=0} = (dT/dy)_{y=0}$). Table II lists the values of these various constants that are so obtained.

The function $\exp(-t^2) \int_0^t \exp(s^2) ds$ that appears in these equations is known as Dawson's

integral and is tabulated in Abramowitz and Stegun (1964). The integral of Dawson's integral,

$\int_0^{y/L} [\exp(-t^2) \int_0^t \exp(s^2) ds] dt$, can be found by numerical integration with the use of Table II

and by integrating a series expansion of Dawson's integral. Figure 5 shows a plot of the integral of Dawson's integral.

Figure 5. Integral of Dawson's integral versus y/L .

Discussion

The temperature profiles calculated from the theory just presented are plotted in Figure 1. Both of the new theoretical profiles lie within 0.5°C of the curve obtained from Robin's theory. This result is perhaps not surprising. Robin's theory is applicable at the center of an ice sheet where the horizontal ice velocity is zero at all depths. At Camp Century the horizontal velocity at the surface is a slow 3.3 m/yr. Thus, an essential assumption of Robin's theory is almost satisfied. Another reason for the close similarity to Robin's profile is the fact that the effect of the $V_x \partial T / \partial x$ term of eq 5 is partly compensated by the effect of the H term. The $V_x \partial T / \partial x$ term causes the predicted temperature T_0 at the bottom surface to decrease by about an additional 0.5°C whereas (for curve 2) the H term causes T_0 to increase by about 1.3°C. The internal heating term H is as important as the horizontal mass transport term. (Lliboutry has correctly stressed that it should not be neglected in the determination of temperature profiles.)

Overall, the refinement in theory has not substantially reduced the disagreement between the predicted and the observed temperature profiles. At the bottom surface the disagreement between the measured and the predicted values of T_0 has been reduced only from 2.7°C for Robin's theory to 2.0°C for curve 2. A temperature reversal near the top surface that was not predicted in Robin's theory is found in the new theory. However the predicted reversal is only 0.14°C for curve 2, whereas the observed reversal is 0.6°C. (That is, as the depth increases the temperature decreases by 0.6°C before it begins to increase.) We feel that both the 2°C disagreement and the disagreement in the temperature reversal are large enough to warrant the firm conclusion that the temperature profile of the Camp Century borehole differs significantly from a steady-state profile. The reader may think, offhand, that a 2°C difference between a predicted and a measured profile is inconsequential. However, as will be seen, very substantial changes in quantities appearing in the various equations in this paper are required to improve by 2°C the fit between the measured and predicted profiles.

Figure 6. Age of ice versus depth calculated from equation $t = (h/a) \log(h/y)$ for present accumulation rate of $a = 35 \text{ cm}^3/\text{cm}^2 \text{ yr}$ and an assumed paleo-accumulation rate of $22.4 \text{ cm}^3/\text{cm}^2 \text{ yr}$.

One really does not expect that quantities such as the accumulation rate and upper surface temperature should have remained constant over a time period comparable to the age of the ice at the greater depths. Figure 6 shows a plot of the age of the ice at various depths. The relationship is estimated from the equation $t = (h/a) \log(h/y)$, where t is the age of the ice at vertical distance y . This equation can be found from Nye's theory (Nye, 1963) on the rate of thinning of steady-state ice sheets. (This age is a minimum age. If the fact is taken into account that h increases and a decreases as x is decreased, the age at a given value of y is increased over the age given by the equation.) The ice at the greater depths must have been deposited as snow during the end of the Pleistocene period when the climate differed from that of the present time.

It is interesting to consider what changes in past values of quantities such as the accumulation rate can best improve the fit between the theoretical and measured temperature profiles.

It can be seen from Figure 1 that the predicted temperature reversal near the top surface is not large enough and the predicted temperature at the bottom surface is not warm enough. If the horizontal ice velocity had been larger in the past a larger temperature reversal would have been produced near the top surface. A larger horizontal ice motion would lead to increased internal heating. The internal heating could lead to a higher bottom temperature. Curve 3 in Figure 7 shows the temperature profile that would result if the average surface velocity at Camp Century during the past had been three times the present surface velocity. This velocity would have occurred if the ice thickness remained the same but the surface slope had been 1.41 times larger than the present surface slope. The various constants appearing in eq 9 and 10 that are found for this increased velocity and slope are listed in Table III.

Figure 7. Temperature profiles. Curve 2 is the same as curve 2 of Fig. 1. Curve 3 is for a horizontal velocity three times that used to calculate curve 2. Curve 4 is for an accumulation rate $1/(1.25)^2$ times the accumulation used for curve 2. Also shown is the temperature profile measured by B.L. Hansen.

It can be seen in Figure 7 that an increase in surface velocity improves the fit between the measured and predicted bottom temperatures. A large temperature reversal also is produced. However, the disagreement between the measured and the predicted profiles over most values of y is increased by a considerable amount. It hardly seems possible to explain the discrepancy between the curves in Figure 1 by postulating that in the past the ice motion of the Greenland ice sheet was much more vigorous than it is at present.

Another way to increase the value of the predicted temperature at the bottom surface is to assume that in the past the accumulation rate was smaller than it is at the present time. Figure 7 also shows a temperature profile calculated under the assumption that the average accumulation rate in the past was $22.4 \text{ cm}^3/\text{cm}^2 \text{ yr}$, instead of the present value of $35 \text{ cm}^3/\text{cm}^2 \text{ yr}$. Table III lists pertinent data for this profile. The calculated temperature profile agrees with the measured profile to within 0.5°C over the entire range of values of y . The only serious disagreement is that the predicted value of the temperature reversal near the top surface is of the order of 0.1°C rather than the observed 0.6°C .

The temperature reversal near the top surface cannot be satisfactorily explained by changes in either the horizontal ice velocity or the accumulation rate. Another explanation of this feature of the temperature profile is based on the supposition that the reversal is caused by a change in the mean annual temperature of about 0.5°C during the last 1000 years.

The difference between the theoretical steady-state temperature profile and the observed profile can be accounted for reasonably if the mean annual temperature varied by about 0.5°C during

Table III. Numerical values of quantities appearing in eq 9 and 10 for curves 3 and 4.

The average value of $a^{-1} da/dx$ was used in the calculations.

Quantity	Curve 3	Curve 4
A_1	3 times value of curve 2	same as curve 2
A_2	"	"
B	"	"
y_0	same as curve 2	"
$(d\theta^*/dy)_{y=0}$	$-2.99 \times 10^{-4} \text{C/cm}$	$-2.33 \times 10^{-4} \text{C/cm}$
$(d\theta/dy)_{y=y_0}$	$-1.43 \times 10^{-4} \text{C/cm}$	$-1.38 \times 10^{-4} \text{C/cm}$
$(\theta_0 + \theta_1)_{y=y_0}$	-23.0C	-20.5C
θ_{b0}^*	-13.8C	-13.2C
θ_{u0}	-26.4C	-25.0C
L	same as curve 2	661 m (1.25 times value of curve 2)
a	"	$22.4 \text{ cm}^3/\text{cm}^2 \text{ yr}$ ($1(1.25)^2$ times value of curve 2)
V_x	3 times value of curve 2	same as curve 2
dh/dx	1.41 times value of curve 2	"

the last 1000 years and if the average paleo-accumulation rate over the last 10,000 to 15,000 years was about 40% smaller than the present accumulation rate. Unfortunately paleo-accumulation rates are known near Camp Century (at Site 2, which is further inland than Camp Century) only over the last 1000 years. Langway (1967) has reported the following accumulation rates at Site 2: 42.3 g/cm² yr for the period 1954–1957; 34.2 g/cm² yr for around 1773; 37.4 g/cm² yr for around 1513; 41.1 g/cm² yr for around 1233; and 41.6 g/cm² yr for around 934. The rate shows about 20% variation. There is no evidence of a general decrease in the rate with time. However, for longer periods of time, which are closer to the end of the Pleistocene, it would not be unreasonable to expect a difference in accumulation rate of the order of 40%.

Hansen (personal communication) has pointed out that clear ice layers (that is, air bubble-free layers) were found in the cores taken from about 600 m above the bottom of the ice sheet. He has suggested that these layers may have been formed during the climatic optimum of 5000 to 6000 years ago. If the mean annual surface temperature were warmer during this period, during the summer melting of firn followed by refreezing of the melt water into clear ice layers could have occurred. He also has suggested that the existence of a higher mean annual temperature during this period will help explain the 2C temperature discrepancy between the theoretical steady-state temperature profile and the measured one.

Literature cited

- Abramowitz, M. and Stegun, I.E. (editors) (1964). Handbook of mathematical functions. National Bureau of Standards, Applied Mathematics Series 55, p. 319.
- Carslaw, H.S. and Jaeger, J.C. (1959) Conduction of heat in solids. Oxford: Clarendon Press, 2nd Edition, p. 497.

- Crozaz, G. and Langway, C.C., Jr. (1966) Dating Greenland firn-ice cores with Pb-210. *Earth and Planetary Science Letters*, vol. 1, p. 194-196.
- Glen, J.W. (1955) The creep of polycrystalline ice. *Proceedings of the Royal Society (London)*, vol. 228A, p. 519-538.
- Hansen, B.L. and Langway, C.C., Jr. (1966) Deep core drilling in ice and core analysis at Camp Century, Greenland, 1961-1966. *Antarctic Journal of the United States*, vol. 1, p. 207-208.
- Jenssen, D. and Radok, U. (1963) Heat conduction in thinning ice sheets. *Journal of Glaciology*, vol. 4, p. 387-397.
- Langway, C.C., Jr. (1958) A 400 meter deep ice core in Greenland, Preliminary report. *Journal of Glaciology*, vol. 3, p. 216-217.
- _____ (1967) Stratigraphic analysis of a deep ice core from Greenland. U.S. Army Cold Regions Research and Engineering Laboratory (USA CRREL) Research Report 77.
- Lliboutry, L. (1966) Bottom temperatures and basal low-velocity layer in an ice sheet. *Journal of Geophysical Research*, vol. 71, p. 2535-2543.
- Mock, S.J. (1967) Accumulation patterns on the Greenland ice sheet. USA CRREL Research Report 233. Also *Journal of Glaciology* (in press).
- _____ (1968) Snow accumulation studies on the Thule Peninsula, Greenland. USA CRREL Research Report 238. Also *Journal of Glaciology*, vol. 7, no. 49.
- Nye, J.F. (1963) Correction factor for accumulation measured by the thickness of the annual layers in an ice sheet. *Journal of Glaciology*, vol. 4, p. 785-788.
- Ramseier, R.O. (1967) Self-diffusion in ice monocrystals. USA CRREL Research Report 232.
- Robin, G. deQ. (1955) Ice movement and temperature distribution in glaciers and ice sheets. *Journal of Glaciology*, vol. 2, p. 523-532.

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13. ABSTRACT Steady-state temperature profiles are calculated for the borehole drilled through the Greenland ice sheet at Camp Century. The profiles are found by modifying Robin's theory through the addition of several correction terms. One of these terms is the internal heating arising from creep deformation. The importance of this term was emphasized by Lliboutry. The new theoretical profiles do not differ appreciably from the profile derived from Robin's theory. The theoretical profiles do differ substan- tially from the Camp Century profile measured by Hansen. It is concluded that Hansen's observations are evidence that factors such as accumulation rate and the upper surface temperature are not in a long-term steady-state condition. Better agreement between theoretical and measured curves is obtained if it is assumed that the accumulation rate was about 40% smaller in the past and that the mean annual surface temperature varied by about 0.5C over the past 1000 years.			

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