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# **Catastrophic Glacier Advances**

by Johannes Weertman

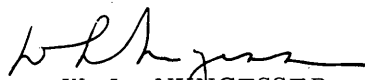
U. S. ARMY COLD REGIONS RESEARCH  
AND ENGINEERING LABORATORY  
Corps of Engineers  
Hanover, New Hampshire

## PREFACE

This report was prepared by Dr. Weertman\* under personal services contract with U. S. Army Cold Regions Research and Engineering Laboratory. The work was done under task 5010.01139, Mechanics of deformation of snow and ice, for the Research Division, J. A. Bender, chief.

The author wishes to thank Dr. J. F. Nye of the University of Bristol for suggesting that catastrophic glacier advances probably can be explained through a change in the "lubrication" at the bottom of a glacier.

This report has been reviewed and approved for publication by the Office of the Chief of Engineers, United States Army.



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Colonel, Corps of Engineers  
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<p>AD                                      Accession No.</p> <p>U. S. Army Cold Regions Research and Engineering Laboratory, Corps of Engineers, Hanover, N. H. CATASTROPHIC GLACIER ADVANCES – J. Weertman</p> <p>Research Report 102, Sept 1962, 8p - illus. DA Proj 8S66-02-001, CRREL task 5010.01139</p> <p>A theory is presented to explain sudden, major glacier advances, and formulas are offered to define the contributing factors. This theory was developed from a model for the sliding of glaciers which was expanded by taking into account the thickness of the water layer formed between the lower surface of the glacier and its bed. The following conditions are required for catastrophic sliding: (1) the glacier should be long (10-30 km) and its bottom surface should be at the melting point; (2) the water at the glacier bed should flow as a sheet of water with only negligible flow in stream channels; and (3) there must be an above average shear stress (ca. 2 bars) at the bed. The arrival of large kinematic waves could produce this abnormal stress. (4) The glacier bed should have relatively fewer large obstacles as compared with smaller ones. When these obstacles are smaller than the thickness of the water layer at the bottom of the (over)</p>	<p>UNCLASSIFIED</p> <p>I.      Glaciers--Velocity I.      Weertman, Johannes II.     U. S. Army Cold Regions Research and Engineering Laboratory</p>	<p>AD                                      Accession No.</p> <p>U. S. Army Cold Regions Research and Engineering Laboratory, Corps of Engineers, Hanover, N. H. CATASTROPHIC GLACIER ADVANCES – J. Weertman</p> <p>Research Report 102, Sept 1962, 8p - illus. DA Proj 8S66-02-001, CRREL task 5010.01139</p> <p>A theory is presented to explain sudden, major glacier advances, and formulas are offered to define the contributing factors. This theory was developed from a model for the sliding of glaciers which was expanded by taking into account the thickness of the water layer formed between the lower surface of the glacier and its bed. The following conditions are required for catastrophic sliding: (1) the glacier should be long (10-30 km) and its bottom surface should be at the melting point; (2) the water at the glacier bed should flow as a sheet of water with only negligible flow in stream channels; and (3) there must be an above average shear stress (ca. 2 bars) at the bed. The arrival of large kinematic waves could produce this abnormal stress. (4) The glacier bed should have relatively fewer large obstacles as compared with smaller ones. When these obstacles are smaller than the thickness of the water layer at the bottom of the (over)</p>	<p>UNCLASSIFIED</p> <p>I.      Glaciers--Velocity I.      Weertman, Johannes II.     U. S. Army Cold Regions Research and Engineering Laboratory</p>
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## SUMMARY

A theory is developed to explain catastrophic glacier advances, based on a previously developed glacier sliding theory (Weertman, 1957). It is found that catastrophic sliding is possible when the thickness of the water layer at the bottom of a glacier exceeds the size of the obstacles which normally control the velocity of sliding. The conditions which appear to be necessary for catastrophic advances to occur are: (1) The glacier should be long (10 - 30 km) and its bottom surface should be at the melting point. (2) The water at the glacier bed should flow as a sheet of water with only negligible flow in stream channels. (3) An above-average shear stress (of the order of 2 bar) should act at the bed. Such an abnormal stress could be produced by the arrival of large kinematic glacier waves. (4) The glacier bed should be smoother with respect to large protuberances and obstacles than to small hindrances.

The theory can be applied to explain the rapidly fluctuating velocity changes observed in ordinary glaciers. Kinematic water waves in the water layer at the bottom of a glacier can produce rapidly changing fluctuations in the surface velocity of the glacier.

# CATASTROPHIC GLACIER ADVANCES

by

J. Weertman

## INTRODUCTION

A rare but spectacular glacier phenomenon is the sudden, catastrophic advance which has been observed in a number of glaciers. As examples, the Black Rapids Glacier advanced 5 km in 5 months (Hance, 1937; Geist and Péwé, 1957; Post, 1960) and the Muldrow Glacier advanced almost 7 km in less than a year (Péwé, 1957; Post, 1960). These advances are thought to have been triggered off by the arrival of large kinematic waves in the lower reaches of the glaciers (Nye, 1960).

In this paper we present a theory for these rapid glacier advances. This theory is an extension of a glacier sliding theory (Weertman, 1957). The new feature introduced here is the effect of the thickness of the water layer which exists at the bottom of a temperate glacier.

Lliboutry (1959) has proposed a sliding mechanism which can be used to account for rapid advances. We have devoted a section to a critique of his mechanism, and conclude that his theory is inadequate for the present purpose, although it probably does account for avalanche sliding of ice slabs of moderate thickness. Another theory has been proposed by Robin (1955). He suggested that a cold glacier which was frozen to its bed would speed up if the bottom reached the pressure melting point and thus sliding became possible. Although Robin's theory cannot be ruled out, it does have difficulty in accounting for the very large velocity of catastrophic advances. The warming-up of the bottom should lead only to velocities found in an average temperate glacier and not to velocities which are two orders of magnitude larger.

## THEORY

### Review

In the theory of sliding that we proposed (Weertman, 1957), it was suggested that ice can move past protuberances on a glacier bed by either of two mechanisms - one involving pressure melting and the other creep rate enhancement through stress concentrations. If a glacier bed contains obstacles of only one size, the pressure melting mechanism leads to a sliding velocity  $S_1$  given by the equation\*

$$S_1 = C\tau r^2/L \quad (1)$$

where  $\tau$  is the shear stress acting parallel to the bed,  $L$  the average dimension of the protuberances,  $C$  a constant ( $C \cong 5.4 \text{ cm}^2/\text{bar-yr}$ ), and  $r$  a measure of the roughness of the bed. This last quantity is defined by letting the product  $rL$  equal the average distance of separation of obstacles of size  $L$ . A rough bed has a small value of  $r$ ; a smooth bed has a large value of  $r$ . The creep rate enhancement mechanism gives a sliding velocity  $S_2$  (when only one obstacle size is present), where

$$S_2 = C'\tau^n r^{2n} L/2^n. \quad (2)$$

In this equation  $n$  is a constant. At present, the best experimental value of  $n$  appears to be 3 (in our original paper we used 4.2), and  $C'$ , another constant, is approximately equal to  $0.017/\text{bar}^n\text{year}$ .

The velocity  $S_1$  increases with decreasing size of  $L$  whereas  $S_2$  increases as  $L$  increases. On any real glacier bed there will be protuberances and obstacles of all sizes. Because of the pressure melting mechanism, small obstacles are no real hindrance to sliding and, because of the creep rate enhancement, large protuberances offer no real barrier to the flow. Sliding velocity therefore is determined by protuberances of an intermediate size, namely, that size which corresponds to the same sliding velocity in

\*The notation has been changed from that used in the original paper.

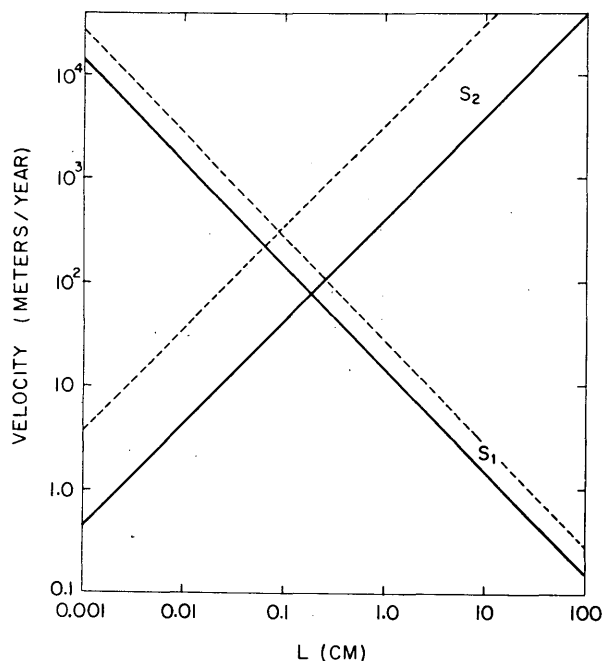


Figure 1. Sliding velocities  $S_1$  and  $S_2$  vs obstacle size for the case  $r = 16.6$ . The solid lines are for shear stress = 1 bar; the dashed lines for shear stress = 2 bar.

If  $S = 80$  m/yr under a shear stress of 1 bar,  $r_c$  is equal to 16.6. \* According to eq 3, these values of  $r_c$  and  $\tau$  require that  $L_c$  be equal to 0.18 cm.

Figure 1 shows a log-log plot of  $S_1$  and  $S_2$  vs  $L$  when  $r_c = 16.6$ . The intersection occurs at velocity  $S$  and obstacle size  $L_c$ . The solid lines in this figure are calculated for a  $\tau$  of 1 bar; the dashed lines for  $\tau$  at 2 bar. In the latter case  $S = 320$  m/yr and  $L_c = 0.09$  cm. Raising the stress thus decreases the controlling obstacle size.

In Figure 1 it was assumed that  $r$  is a constant and is independent of the size of  $L$ . If  $r$  is a function of  $L$ ,  $S_1$  and  $S_2$  no longer will follow straight lines on a log-log plot. For example, consider the case where  $r$  is given by a relationship of the type

$$r = a \left\{ 1 + b \left[ \exp(-L_0/L) \right] \left[ \log L/L_0 \right] \right\} \quad (5)$$

where  $a$ ,  $b$ , and  $L_0$  are constants. In this equation  $r$  is a constant for  $L < L_0$  and a slowly increasing function of  $L$  for  $L > L_0$ . The bed is smoother in the range of intermediate to large-sized protuberances. In Figure 2 curves of  $S_1$  and  $S_2$  are plotted for the case when  $a = 16.6$ ,  $b = (2 \log 10)^{-1}$  and  $L_0 = 0.01$  cm. The curves depart from those given in Figure 1 for values of  $L$  greater than  $L_0$ . The sliding velocity  $S$  and the obstacle size  $L_c$  are again determined by the intersection of two curves. As a result of making the bed smoother of obstacles greater than 0.01 cm, it can be seen that, under the same stress, sliding velocity is increased and obstacle size decreased.

#### Thickness of water layer at the bed of a glacier

Because of geothermal heat and heat produced by sliding, ice is continuously being melted at the bottom of a temperate glacier. One can expect, therefore, that a water layer will exist between the ice and the glacier bed. We now want to determine the thickness of this water layer.

\*In our original paper (Weertman, 1957), we used  $r = 4$ . This older value of  $r$  represents a glacier bed which appears to be much too rough.

eq 1 and 2. If  $S_1$  and  $S_2$  are set equal to each other, it is found that the obstacle size  $L_c$  controlling the sliding velocity is

$$L_c = \left[ 2^n C (C')^{-1} \tau^{1-n} (r_c)^{2-2n} \right]^{1/2} \quad (3)$$

and the sliding velocity is

$$S = \left[ C C' (\tau r_c^2)^{1+n} 2^{-n} \right]^{1/2} \quad (4)$$

where  $r_c L_c$  is the average separation of protuberances of size  $L_c$ . (In the original paper it was assumed that  $r$  is a constant for all values of  $L$ . Obviously the theory can be made more general by considering  $r$  to be a function of  $L$ .)

According to eq 4, the sliding velocity varies greatly with the roughness. Since values for roughness are not known, it is not possible to use this equation to predict actual sliding velocities, although it can be used to predict the stress dependence of the sliding velocity. By substituting typical values of the sliding velocity and shear stress into eq 4, one can work backwards to find what the value of  $r_c$  must be.



The volume  $\underline{W}$  of water produced per unit time and unit area at the glacier bottom is

$$\underline{W} = (Q + S\tau J^{-1}) / H \quad (5)$$

where  $Q$  = geothermal heat  $\cong 39$  cal/cm<sup>2</sup>-yr

$J$  = mechanical equivalent of heat = 41.8 bar-cm/cal, and

$H$  = latent heat of melting ice = 80 cal/cm<sup>3</sup> of water.

This water is continuously removed from the bed of the glacier. The driving force for the removal is the gradient in the hydrostatic pressure at the glacier bottom. If the slopes of the upper and lower surfaces of a glacier are small, the gradient in the hydrostatic pressure at the bed of a glacier is  $\rho g \alpha$ , (upon making the approximation that the density of ice equals the density of water) where  $\rho$  is the density of ice,  $g$  is the gravitational acceleration, and  $\alpha$  is the slope of the upper surface. (The slope of the bottom surface has no effect on the pressure gradient.) Now suppose that the water at the bottom of the glacier flows as a sheet\* of thickness  $\underline{D}$ . If  $\underline{x}$  is the distance from the head of the glacier, and if  $\underline{W}$  is essentially constant down the glacier, the average velocity  $\underline{V}$  of the water at distance  $\underline{x}$  is

$$\underline{V} = \underline{W}\underline{x}/\underline{D}. \quad (6)$$

From the theory of the mechanics of fluids (Rouse and Howe, 1953, p. 117), it can be shown that the average velocity  $\underline{V}$  of a fluid moving between plane parallel plates a distance  $\underline{D}$  apart under a pressure gradient of  $\rho g \alpha$  is given by

$$\underline{V} = \underline{D}^2 \rho g \alpha / 12\mu \quad (7)$$

where  $\mu$  is the viscosity of the fluid ( $\mu = 0.018$  dyne-sec/cm<sup>2</sup> =  $5.7 \times 10^{-16}$  bar-yr for water at 0C). The water formed at the bottom of a glacier will lift the glacier up until the water layer attains that thickness which will satisfy both eq 6 and 7. This thickness is given by

$$\underline{D} = (12\mu \underline{W}\underline{x}/\rho g \alpha)^{1/3}. \quad (8)$$

(Since eq 7 is derived for the case of flat plates and an actual glacier bed cannot be considered to be flat, eq 8 will determine a lower limit for  $\underline{D}$ . The actual value of  $\underline{D}$  will be somewhat larger.) In Figure 3 we have plotted  $\underline{D}$  as a function of shear stress  $\tau$  for three different values of  $\underline{x}$ . We assumed in these plots that  $r_c = 16.6$  and that  $\alpha = 3 \times 10^{-2}$ , a typical value for the slope of a glacier surface. Also shown on this plot are the values of the controlling obstacle size  $\underline{L}_c$  when the shear stress is 1 bar (and  $S = 80$  m/yr), and when the shear stress is 2 bar (and  $S = 320$  m/yr). It can be seen that for a normal glacier with a shear stress of about 1 bar acting on the bottom,  $\underline{L}_c$  is larger than the thickness of the water layer even for a glacier 30 km long. Thus normally the water layer will have no effect on the rate of sliding since it does not raise the ice above the controlling obstacle size. Only under unusual circumstances will  $\underline{D}$  be larger than  $\underline{L}_c$ . We consider such circumstances in the next section.

### Catastrophic advances

When a kinematic wave arrives at the lower reaches of a glacier it is reasonable to expect that the shear stress acting on the bottom of the glacier is raised somewhat above its normal value. However, one would not expect the stress to be raised to such values that, by eq 4, the value of  $\underline{S}$  is of the order of 10 km/yr. If in eq 4 a stress of

\*One might object that the water flowing at the bottom will move in narrow stream channels rather than as a sheet. Undoubtedly such is the case at the end of a glacier where the hydrostatic overburden is small. Elsewhere the high hydrostatic pressure at the bottom of a glacier would be expected to force ice to flow into, and fill up, any stream channel that formed and thus eliminate the stream. If the theory we have proposed (Weertman, 1961) for the formation of debris layers in cold ice caps is confirmed, these layers would be evidence that water flows as a sheet. However, a melt water stream that has become englacial or subglacial can remain so if it melts ice away from its channel walls as fast as they are closed in by the hydrostatic pressure. The melt water can carry with it the heat required for this melting if it enters the glacier at a temperature greater than 0C. If there are a number of subglacial melt water streams, the effective value of  $\underline{x}$  in the above analysis can be reduced.

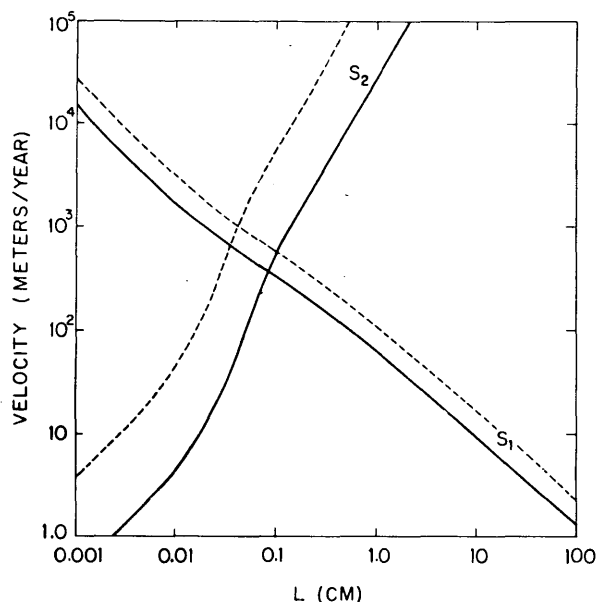


Figure 2. Sliding velocities  $\underline{S}_1$  and  $\underline{S}_2$  vs obstacle size for a glacier bed which is smoother with respect to large than to small obstacles (see text). The solid curves are for shear stress = 1 bar; the dashed curves for shear stress = 2 bar.

1 bar leads to a sliding velocity of 80 m/yr, a stress of 11 bar would be required to give the observed velocity of catastrophic advances, approximately 10 km/yr. This is a very unreasonable stress. Suppose, however that the stress is raised by a factor of 2. The controlling obstacle size will be reduced by a factor of 2, and, moreover, the thickness of the water layer at the bottom will be increased by approximately a factor of 2. According to Figure 3, the possibility exists, particularly if the glacier is long, that the controlling obstacle size  $\underline{L}_C$  will be smaller than the thickness of the water layer. If this situation should occur, it is obvious that  $\underline{L}_C$  no longer will control the sliding rate. An obstacle size of the order of the water thickness will take over this function. The new sliding velocity can be determined by substituting  $\underline{D}$ , given by eq 8, for the term  $\underline{L}$  in eq 2 and using eq 5 for  $\underline{W}$  with  $\underline{S}_2$  substituted for  $\underline{S}$ . When the geothermal heat is small compared to the heat of sliding, the new velocity of sliding  $\underline{S}_3$  is given by

$$\underline{S}_3 = (C' \tau^n r^{2n} / 2^n)^{3/2} (12 \mu x \tau / JH \rho g \alpha)^{1/2} \quad (9)$$

and the thickness of the water layer by

$$\underline{D}_3 = (C' \tau^n r^{2n} / 2^n)^{1/2} (12 \mu x \tau / JH \rho g \alpha)^{1/2}. \quad (10)$$

This sliding velocity has a much more sensitive dependence on both stress and roughness than the ordinary sliding velocity. The stress dependence is 5th power for  $n = 3$  and the roughness dependence is 9th power. Table I lists values of  $\underline{S}_3$  and  $\underline{D}_3$  for various values of  $\underline{x}$  and for  $r = 16.6$  and  $r = 25$  (a value 50% larger than 16.6) when  $\tau$  is 2 bar.

Table I. Values of  $\underline{S}_3$  and  $\underline{D}_3$  for different values of  $\underline{x}$ .  $\tau = 2$  bar.

$\underline{x}$ (km)	$r = 16.6$		$r = 25$	
	$\underline{S}_3$ (km/yr)	$\underline{D}_3$ (cm)	$\underline{S}_3$ (km/yr)	$\underline{D}_3$ (cm)
1	-	-	9.2	0.23
10	0.8	0.23	31	0.77
30	1.4	0.40	53	1.3

It can be seen from this table and Figure 1 that if  $r$  remains constant at 16.6, the sliding velocity can be increased by about an order of magnitude as a result of a stress increase which make  $D > L_c$ . This increase really is not great enough to explain catastrophic advances. However, if the bed is smoother with respect to larger obstacle sizes, such as in Figure 2, then it can be seen from the table that very much greater increases in the sliding velocity are possible. A truly catastrophic advance could be achieved. Thus under rather special conditions catastrophic sliding can occur in the model we have proposed if the thickness of the water layer at the bottom of a glacier is taken into account. The conditions for this type of advance can be summarized as follows:

(1) The glacier should be long\* (10-30 km) and its bottom surface should be at the melting point.

(2) Only negligible water flow should occur in stream channels at the glacier bed. The water layer should move as a sheet.

(3) There must be an above-average shear stress ( $\sim 2$  bar) at the bed. The arrival of large kinematic waves could produce this abnormal stress.

(4) The glacier bed should be smoother with respect to large protuberances than with respect to small ones.

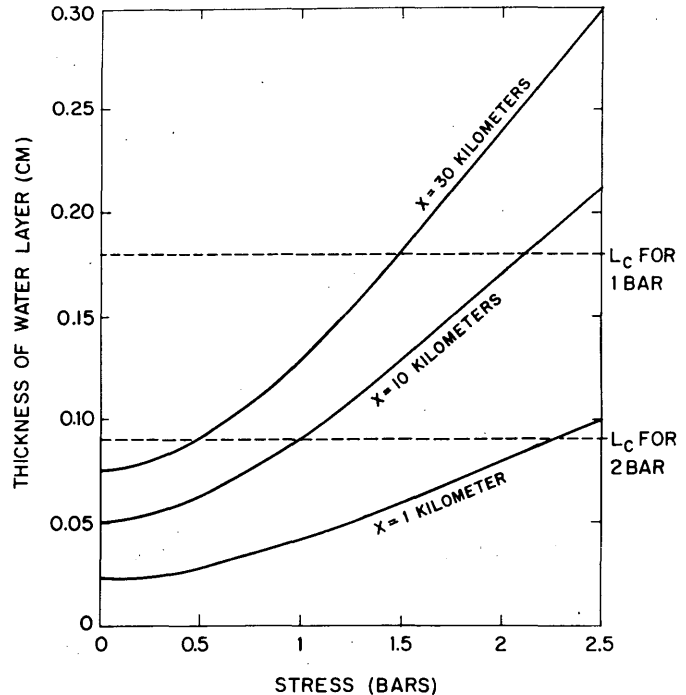


Figure 3. Thickness of water layer at the bottom of a glacier vs shear stress at bottom at three different distances from the head of the glacier.  $r = 16.6$ .  $L_c$  is the controlling obstacle size.

### CRITIQUE OF LLIBOUTRY'S MECHANISM

Lliboutry (1959) has advanced an interesting sliding mechanism. He points out that if a glacier is sliding fast enough, portions of it will lose contact with the bed. This situation is illustrated in Figure 4. Lliboutry analyzed his model using a glacier bed which is rough and irregular in only one direction. We shall now analyze his mechanism for a more realistic bed which is rough in two directions. In Figure 4 we assume an obstacle size of average dimension  $L$  and, as before, an average separation  $rL$  between the obstacles. However, the average distance which a point on the bottom ice surface must move after it crosses one obstacle before it meets another is not  $rL$  but  $r^2L$ . (N.B., if one threw pennies at random on the floor and then drew an arbitrary straight line starting from one penny, it is highly improbable that this line would meet another penny at the average distance  $rL$  but very probable that it would at a distance  $r^2L$ .)

If the ice surface touches the bed only at the top of the protuberances, the stress there is of the order of  $pghr^2$ , where  $h$  is the ice thickness. This stress above the obstacles will cause creep to occur at these points and the ice will flow down and around

\*A large ice sheet, because of its large values of  $x$ , could have extensive regions where eq 9 for  $S_3$  determines the sliding velocity. Large surface velocities can be avoided if either the shear stress is low ( $\sim \frac{1}{3}$  to  $\frac{1}{2}$  bar) or if the roughness of the bed is larger than that of a normal glacier. Since a large ice sheet may find it difficult to "sweep its bed clean," the roughness could very well be larger.

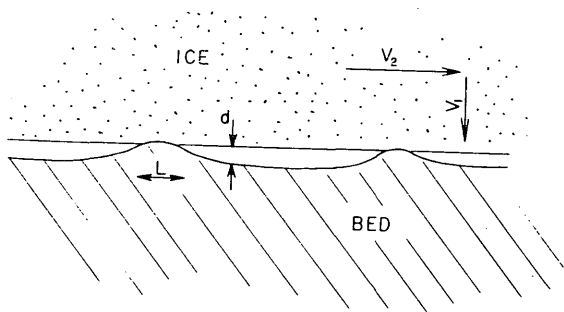


Figure 4. Llibouty's sliding model in which the bottom glacier surface touches the bed only at the tops of obstacles.

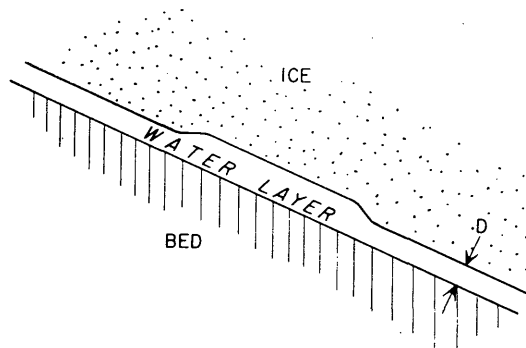


Figure 5. The water layer at the bottom of a glacier with a region of extra thickness. Irregularities in the bed have been ignored in this figure.

the protuberances. The creep rate will be of the order of  $C'(\rho g h r^2 / 2)^n$  and will occur over a distance of the order of the protuberance size (Weertman, 1957). Thus the ice surface flows down at a velocity  $V_1$  given approximately by

$$V_1 = C' L (\rho g h r^2 / 2)^n. \quad (11)$$

Suppose the velocity of sliding is equal to  $V_2$ . If the ice surface is to remain free of the bed except where the ice touches the protuberances, it is necessary that the time it takes a point on the ice surface to move from one obstacle to another ( $r^2 L / V_2$ ) be less than the time it takes for the ice surface to flow down a distance of the order of the height  $d$  ( $d \sim L$ ) of a protuberance. This latter time is above  $L / V_1$ . Therefore the speed of sliding  $V_2$  must be such that

$$V_2 \geq r^2 V_1 = r^2 C' L (\rho g h r^2 / 2)^n. \quad (12)$$

If a glacier is 300 m thick and its bed has a roughness factor  $r = 16.6$ , the sliding velocity must be greater than  $(3.3 \times 10^{11} / \text{yr}) L$  for Llibouty's mechanism to operate. If  $L$  is 10 cm, this velocity is  $3.3 \times 10^7$  km/yr. Therefore it seems very unlikely that Llibouty's mechanism is applicable whenever there is appreciable hydrostatic pressure at the bottom of a glacier. It could work in a region very near the edge or snout of a glacier, where the pressure is small. Its most likely application is to the problem of the avalanching of thin ice slabs where high sliding velocities are encountered.

#### RAPIDLY VARYING VELOCITY CHANGES OF A GLACIER

One confusing glacial phenomenon which has been repeatedly noted but never satisfactorily explained is the day-to-day and even hour-to-hour fluctuations in the surface velocity of glaciers (Sharp, 1954). The catastrophic advance theory proposed in an earlier section can be used to explain these fluctuations. Consider Figure 5, which shows the water layer at the bottom surface of a glacier. Suppose, as is shown, that there is a region in which this layer is thicker than normal. The length of this region is supposed to be much longer than the thickness of the glacier so that the extra thickness is accommodated by a slight elastic displacement of the glacier. The water in this extra thick region moves faster than in a normal region (see eq 7). One has, therefore, the necessary condition for a traveling or kinematic wave to move down the water layer.\* It is necessary only that the lower ice surface be capable of an elastic upward displacement so that the extra thick region can shift its position. If this region is long enough it will always be possible to obtain this elastic displacement. The analysis of a kinematic wave in the water layer will be identical to that developed for kinematic waves

\*Not to be confused with kinematic waves traveling through the ice of a glacier, which were mentioned earlier.

traveling through the ice of a glacier (Finsterwalder, 1907; Nye, 1958, 1959, 1960; Weertman, 1958). The traveling wave will have a velocity three times the average water velocity. If  $\underline{D}$  is equal to 0.05 cm, the wave velocity is 0.8 km/day when  $\alpha = 3 \times 10^{-2}$ .

Fluctuations in the surface velocity of a glacier can be brought about in the following manner. Suppose that at the head of a glacier some extra water came into the glacier bed.\* This extra water will start a kinematic wave in the water layer at the glacier's bed. If the thickness of the wave is larger than  $L_c$ , the controlling obstacle size, the sliding velocity will be increased in the region of the kinematic wave. However, the period of this increased sliding velocity will be short since the velocity of the kinematic water wave is much faster than the velocity of ice in a glacier. One needs only to postulate a number of frequently occurring kinematic water waves passing down the bed of a glacier to account for the rapidly changing velocity fluctuations at the surface of the glacier.

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\*For example, melt water can flow down the side of a glacier, in the water layer between the ice and the bedrock, if its edge is higher than its center.

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