THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS

Pt. II: A SIMPLIFIED METHOD OF CALCULATION

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August 1969

DA TASK 1T061102B52A02

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HANOVER, NEW HAMPSHIRE

PREFACE

This report was prepared by Mr. S. Lawrence Dingman, Research Hydrologist, Research Division, and Dr. Andrew Assur, Chief Scientist, U.S. Army Cold Regions Research and Engineering Laboratory.

The report was published under DA Task 1T061102B52A02, *Military Aspects of Cold Regions Research.*

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by

S. Lawrence Dingman and Andrew Assur

INTRODUCTION

In a previous report, Dingman *et al.* (1967) developed a method for calculating the temperature profile of a cooling river below a source of thermal pollution and the length of ice-free reach which could be maintained by such a source. Computer programs were used to calculate heat-loss rates based on mean daily values of meteorological parameters and to numerically integrate a complicated heat-loss expression. The present paper describes a simplified approach to the same problem, in which heat loss is calculated as a linear function of the difference between water temperature and air temperature, so that the integration can be performed analytically. A simplified but fairly general procedure for calculating water-air heat-loss rates on the basis of air temperature, windspeed, solar radiation, and general atmospheric conditions is also presented.

BASIC EQUATION

The analysis of Dingman *et al.* (1967) assumed a rectangular uniform channel carrying steady flow and subject to meteorological conditions which are constant in space and time. Heat load was also considered constant, and the heated water was assumed to mix instantaneously over the entire cross section (line source). A heat balance on a Eulerian volume element of the river below the heat source gave rise to the expression

$$x = -\rho_w v_w h C_p \int_{T_i}^{T_x} \frac{dT_w}{Q^*} \qquad (1)$$

where

$$T_i = T_{nat} + \frac{Q_p}{\rho_w v_w h w C_p} \qquad (2)$$

and x is distance below the heat source (cm), ρ_w is mass density of water (g cm^{-3}), v_w is river flow velocity (cm sec^{-1}), h is river depth (cm), C_p is specific heat of water (cal g^{-1} °C^{-1}), T_x is water temperature at x (°C), Q^* is heat-loss rate (cal sec^{-1} cm^{-2}), T_{nat} is the natural river temperature upstream from the heat source (°C), Q_p is the heat of pollution (cal sec^{-1}), w is the river width (cm), and T_i is the temperature of the river water at $x = 0$ (°C).

Q^* is the net heat-loss rate due to water-atmosphere exchanges by solar and long-wave radiation, convection, and evaporation, and is a complicated function of water temperature, air temperature, windspeed, humidity, cloud cover, and other meteorological factors. Here an approach common in heat-transfer problems is used, and Q^* is approximated by the expression

$$Q^* = Q_0 + q(T_w - T_a) \tag{3}$$

where Q_0 is heat loss from the surface when $T_w - T_a$ is zero (cal sec^{-1} cm^{-2}), T_w is water temperature, T_a is air temperature at the 2-m height (°C), and q is a conventional heat-transfer coefficient (cal sec^{-1} cm^{-2} °C^{-1}). Substituting eq 3 into eq 1, and introducing an "influence length," x^* (cm), where

$$x^* = \frac{\rho_w v_w h C_p}{q} \tag{4}$$

gives

$$x = -x^* \ln \frac{Q_0 + q(T_x - T_a)}{Q_0 + q(T_i - T_a)} \tag{5}$$

or

$$\exp\left(-\frac{x}{x^*}\right) = \frac{Q_0 + q(T_x - T_a)}{Q_0 + q(T_i - T_a)} . \tag{6}$$

Equation 6 is a convenient non-dimensional expression for the downstream decrease of water temperature below a thermal pollution source. It shows a simple exponential decay with distance.

To find the total length of the ice-free reach X (cm) which would exist below a thermal pollution source the temperature difference $T_s - T_a$, where T_s is the freezing temperature of water, is substituted for $T_x - T_a$ in eq 5:

$$X = -x^* \ln \frac{Q_0 + q(T_s - T_a)}{Q_0 + q(T_i - T_a)} . \tag{7}$$

EVALUATING THE HEAT-TRANSFER COEFFICIENT

Equation 3 states that the heat-loss rate Q^* is a linear function of $T_w - T_a$. The value of the approach outlined here obviously depends on the extent to which this is true. Many previous studies of heat exchange between natural water bodies and the atmosphere have used expressions similar to eq 3 with some success. For example, the Canadian Joint Board of Engineers (1926), cited in Pruden *et al.* (1954), recommended

$$Q^* = 46.4(T_w - T_a) \tag{8}$$

where Q^* is in cal cm^{-2} day^{-1} and T_w and T_a are in °C for a reach of the St. Lawrence River and Pruden *et al.* (1954) developed a somewhat modified form

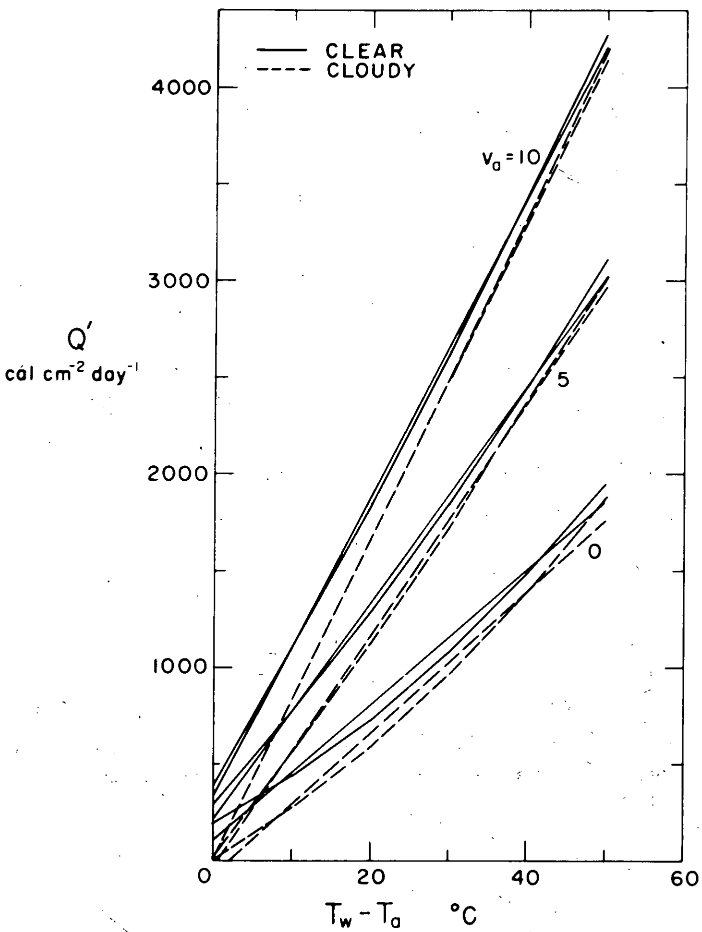

Figure 1. Total heat-loss rate, neglecting short-wave radiation, as a function of $T_w - T_a$*, with windspeeds at 0, 5 and 10 m sec*$^{-1}$.

$$Q^* = 183.6 + 15.5T_a + 43.1(T_w - T_a) \qquad (9)$$

also for a reach of the St. Lawrence River.

These equations, like many others of their type, are based on monthly averages of meteorological parameters in the areas for which they were developed. The heat-transfer coefficients and constants in these equations have "absorbed" the monthly (in these cases) averages and normal patterns of variation of all meteorologic factors in addition to air temperature which affect heat-loss rates. Thus such equations cannot be applied with confidence outside the region for which they were developed, nor can they be used to calculate heat-loss rates for time periods shorter than the averaging periods used to develop them.

To provide a more general basis for calculating the constants in eq 3, the computer program presented by Dingman *et al.* (1967, App. B) was used to construct graphs of total heat-loss rate neglecting solar radiation) Q' as a function of $T_w - T_a$ over a range of windspeeds and under two contrasting sets of general weather conditions (Fig. 1). (Appendix A lists the equations used in the computer program in calculating the individual heat balance components.) The windspeeds v_a were 0, 5, and 10 m sec^{-1} for: 1) clear sky with relative humidity of 50%; and 2) complete cloud cover, cloud height less than 500 m, and relative humidity of 100%. Water temperature was taken as 0C. The addition of heat to the water body as solar radiation Q_R is not considered in evaluating the heat-transfer coefficient but, as shown later, must be taken into account in calculating Q^*.

In all cases it is evident that the curves of Q' vs $T_w - T_a$ are close to linear over the range $0 \leqslant (T_w - T_a) \leqslant 50$ and that the effect of complete low cloud cover is, as expected, to lower heat-loss rates. Regression techniques were used to approximate the curves of Figure 1, with the following results:

$$\text{Clear} \begin{cases} Q' = 105.18 + 35.08(T_w - T_a); & v_a = 0 \text{ m sec}^{-1} \quad (10) \\ Q' = 220.90 + 56.27(T_w - T_a); & v_a = 5 \text{ m sec}^{-1} \quad (11) \\ Q' = 336.62 + 77.47(T_w - T_a); & v_a = 10 \text{ m sec}^{-1} \quad (12) \end{cases}$$

$$\text{Cloudy} \begin{cases} Q' = -72.85 + 37.10(T_w - T_a); & v_a = 0 \text{ m sec}^{-1} \quad (13) \\ Q' = -27.44 + 59.99(T_w - T_a); & v_a = 5 \text{ m sec}^{-1} \quad (14) \\ Q' = 17.97 + 82.88(T_w - T_a); & v_a = 10 \text{ m sec}^{-1} \quad (15) \end{cases}$$

where Q' is in cal cm^{-2} day^{-1}.

In eq 10-15 the values of the slopes give the heat-transfer coefficient q which is effective at a given windspeed. Since incoming solar radiation lowers the heat-loss rate by the amount Q_R, this value must be subtracted from the intercept to give the value of Q_0 for a particular case. The values of the slopes and intercepts in eq 10-15 increase with increasing v_a for each set of general meteorological conditions. Figures 2 and 3 show that these relationships are very close to linear and can be approximated by

$$\text{Clear} \begin{cases} \alpha_0 = 105.18 + 23.14\, v_a \quad (16) \\ \beta_0 = 35.08 + 4.24\, v_a \quad (17) \end{cases}$$

$$\text{Cloudy} \begin{cases} \alpha_1 = -72.85 + 9.08\, v_a \quad (18) \\ \beta_1 = 37.10 + 4.58\, v_a \quad (19) \end{cases}$$

where α is the intercept and β is the slope, and the subscripts 0 and 1 refer to clear and cloudy conditions, respectively.

SAMPLE CALCULATION

The following sample calculation of the length of an ice-free reach is carried out for comparison with results from the computer program presented by Dingman *et al.* (1967, App. A). A power plant which discharges a heat of pollution (Q_p) of 5×10^8 cal sec^{-1} into a river of uniform width (w = 300 m), depth (h = 1.75 m), and velocity (v_w = 0.75 m sec^{-1}) is considered. Assuming complete mixing at the pollution site, this heat input causes a temperature rise of

$$\frac{Q_p}{\rho_w v_w h C_p w} =$$

$$\frac{(5)(10^8)}{(1 \text{ g cm}^{-3})(75 \text{ cm sec}^{-1})(175 \text{ cm})([3][10^4] \text{ cm})(1 \text{ cal g}^{-1}\,{}^\circ\text{C}^{-1})} = 1.3\text{C}$$

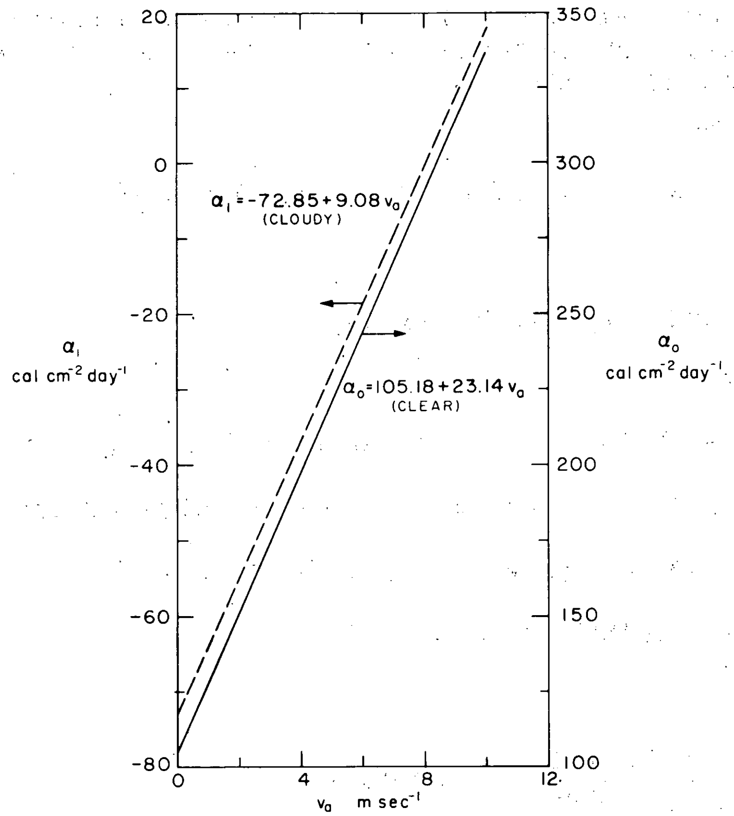

Figure 2. Intercepts in eq 10-15 as functions of windspeed.

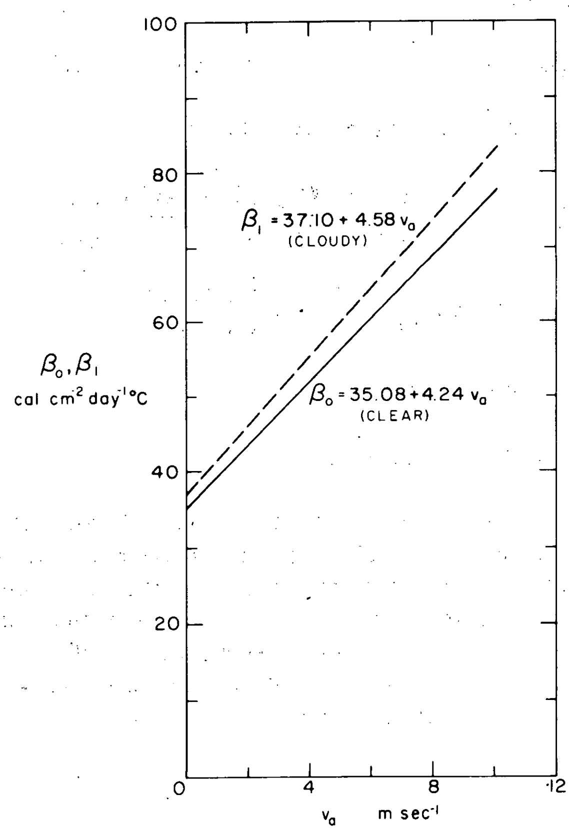

Figure 3. Slopes in eq 10-15 as functions of windspeed.

above the natural water temperature of 0C. Weather conditions above the river are: cloud cover = 0; relative humidity = 50%; windspeed = 2 m sec^{-1}; air temperature = -25C; heat gain due to solar radiation (Q_R) = 85 cal cm^2 day^{-1}.

Using eq 15

$$\alpha_0 = 105.18 + 23.14(2) = 151.46 \text{ cal cm}^{-2} \text{ day}^{-1} = Q'$$

and

$$Q_0 = Q' - Q_R = 66.46 \text{ cal cm}^{-2} \text{ day}^{-1}.$$

Equation 17 gives

$$\beta_0 = 35.08 + 4.24(2) = 43.56 \text{ cal cm}^{-2} \text{ day}^{-1} \text{ °C}^{-1} = q.$$

For the calculation of x^* it is convenient to change the units of q to cal cm^{-2} sec^{-1} $°C^{-1}$:

$$q = (43.56 \text{ cal cm}^{-2} \text{ day}^{-1} \text{ °C}^{-1})([1.157][10^{-5}] \text{day sec}^{-1})$$

$$= (5.04)(10^{-4}) \text{ cal cm}^{-2} \text{ sec}^{-1} \text{ °C}^{-1}.$$

Then

$$x^* = \frac{(1)(75)(175)(1)}{(5.04)(10^{-4})} = (2.604)(10^7) \text{ cm}.$$

The units of Q_0 are now similarly changed:

$$Q_0 = (66.46)(1.157)(10^{-5}) = (7.69)(10^{-4}) \text{ cal cm}^{-2} \text{ sec}^{-1}.$$

Substituting the values of x^*, Q_0, q, T_s, T_i, and T_a into eq 7

$$X = -(2.604)(10^7) \ln \frac{(7.69)(10^{-4}) + (5.04)(10^{-4})(25)}{(7.69)(10^{-4}) + (5.04)(10^{-4})(26.3)}$$

$$X = -(2.604)(10^7) \ln \frac{(7.69)(10^{-4}) + (1.26)(10^{-2})}{(7.69)(10^{-4}) + (1.33)(10^{-2})}$$

$$X = -(2.604)(10^7) \ln 0.9474$$

$$X = (1.409)(10^6) \text{ cm} = 14.09 \text{ km}.$$

The computer-calculated value of X for this case is 13.18 km. Other calculations by this simplified method also differed by less than 10% from the computer-calculated values (see Table I). The simplified method tends to slightly overestimate ice-free reach lengths when the initial polluted water temperature is above about 1C. This is as expected, since the heat-loss rates calculated by this method are based on a water temperature of 0C, whereas in the actual situation (and in the computer calculation), heat-loss rates are somewhat higher because water temperature is greater than 0C throughout the open reach.

Table I. Comparison of ice-free reach lengths calculated by simplified method with those calculated by method of Dingman *et al.* (1967).

	Trial 1	*Trial 2*	*Trial 3*	*Trial 4*
Q_p (cal sec^{-1})	5×10^8	10^8	3×10^8	8×10^8
w (m)	300	60	200	100
h (m)	1.75	0.6	5	3
v_w (m sec^{-1})	0.75	0.6	1	1.5
T_{nat} (°C)	0	0	0	0
Q_{CL} (cal cm^{-2} day^{-1})†	98.55	98.55	270.31	270.31
C †	0	0	10	10
H (m)†			500	500
e_a (mb)†	0.403	1.43	0.189	4.21
v_a (m sec^{-1})	2	6	4	8
T_a (°C)	-25	-10	-40	-5
X (km) (Dingman *et al.*, 1967)	13.18	15.44	6.21	186.6
X (km) (Simplified method)	14.09	15.97	6.16	202.5

† See Appendix A for definitions.

DISCUSSION

Using eq 10-19, calculations can now be made of heat-loss rates, temperature profiles below a thermal pollution source in a cooling river, and the length of ice-free reach maintained in a river by a thermal pollution source, under fairly general conditions and without recourse to a computer. Equations 10-15, and hence eq 16-19, are derived considering that $T_w = 0C$. This is true only at the downstream end of an ice-free reach, but little error is introduced by this assumption unless $T_w - T_a$ is small relative to T_i. Use of heat-loss equations other than those listed in Appendix A would change the constants in eq 10-19; however, the equations in Appendix A have proved adequate in several field tests, and Dingman *et al.* (1967) found good agreement between observed lengths of ice-free reaches and those calculated using the equations in the one case they tested.

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APPENDIX A: SYMBOLS AND EQUATIONS USED IN CALCULATING HEAT-LOSS RATES

Symbols

Symbol	*Meaning*	*Units*
C	Cloud cover	dimensionless ($0 \leqslant C \leqslant 10$)
H	Cloud height	m
Q_a	Heat added to river by long-wave radiation	cal cm^{-2} day^{-1}
Q_{ar}	Long-wave radiation reflected at river surface	cal cm^{-2} day^{-1}
Q_B	Net heat lost from river as long-wave radiation	cal cm^{-2} day^{-1}
Q_{bs}	Long-wave back radiation from river surface	cal cm^{-2} day^{-1}
Q_{CL}	Incoming clear-sky solar radiation	cal cm^{-2} day^{-1}
Q_E	Heat lost from river by evaporation	cal cm^{-2} day^{-1}
Q_H	Heat lost from river by convection to atmosphere	cal cm^{-2} day^{-1}
Q_R	Heat added to river by solar radiation	cal cm^{-2} day^{-1}
Q_{RI}	Solar radiation incident at river surface	cal cm^{-2} day^{-1}
Q_{RR}	Solar radiation reflected at river surface	cal cm^{-2} day^{-1}
T_a	Air temperature (2 m height)	°C
T_w	Water surface temperature	°C
a	(See eq A8)	
b	(See eq A9)	
e_a	Vapor pressure of air (2 m height)	mb
e_{sw}	Saturation vapor pressure at temperature T_w	mb
k_n	(See eq A13)	
σ	Stefan-Boltzmann constant (1.171)(10^{-7})	cal cm^{-2} day^{-1} °K^{-4}

Equations

(See Dingman *et al.*, 1967 for discussion.)

Solar radiation

$$Q_R = Q_{RI} - Q_{RR} \tag{A1}$$

where

$$Q_{RI} = Q_{CL}[0.35 + 0.061(10 - C)] \quad \text{(List, 1963)} \tag{A2}$$

$$Q_{RR} = 0.108\,Q_{RI} - (6.766)(10^{-5})\,Q_{RI}^2 \quad \text{(Dingman et al., 1967)} \tag{A3}$$

Long-wave radiation

$$Q_B = Q_{bs} - Q_a + Q_{ar} \qquad \text{(A4)}$$

where

$$Q_{bs} = 0.970\,\sigma\,T_w^4 \qquad \text{(A5)}$$

$$Q_a = (0.68 + 0.036\sqrt{e_a})\,\sigma\,T_a^4 \quad \text{(clear sky)} \quad \text{(Anderson, 1954)} \qquad \text{(A6)}$$

$$Q_a = (a + b\,e_a)\,\sigma\,T_a^4 \quad \text{(cloudy sky)} \quad \text{(Anderson, 1954)} \qquad \text{(A7)}$$

$$a = 0.740 + 0.025C\ \exp[(-1.92)(10^{-4})H] \quad \text{(Anderson, 1954)} \qquad \text{(A8)}$$

$$b = (4.9)(10^{-3}) - (5.4)(10^{-4})C\ \exp[(-1.97)(10^{-4})H] \quad \text{(Anderson, 1954)} \qquad \text{(A9)}$$

$$Q_{ar} = 0.03\,Q_a\,. \qquad \text{(A10)}$$

Convection and evaporation

$$Q_H = (k_n + 3.9\,v_a)(T_w - T_a) \quad \text{(Rimsha and Donchenko, 1957)} \qquad \text{(A11)}$$

$$Q_E = (1.56\,k_n + 6.08\,v_a)(e_{sw} - e_a) \quad \text{(Rimsha and Donchenko, 1957)} \qquad \text{(A12)}$$

where

$$k_n = 8.0 + 0.35(T_w - T_a) \quad \text{(Rimsha and Donchenko, 1957).} \qquad \text{(A13)}$$

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
U.S. Army Cold Regions Research and Engineering Laboratory
Hanover
New Hampshire 03755

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE
THE EFFECTS OF THERMAL POLLUTION ON RIVER ICE CONDITIONS
II. A SIMPLIFIED METHOD OF CALCULATION

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(First name, middle initial, last name)*
S. Lawrence Dingman and Andrew Assur

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 1969	11	5

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c. DA Task 1T061102B52A02

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
Research Report 206, Pt. II

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT
This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
U.S. Army Cold Regions Research and Engineering Laboratory
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13. ABSTRACT

In part I of this study, an expression relating distance x below a thermal pollution source to water temperature T_w under certain conditions was developed:

$$x = C \int_{T_i}^{T_x} dT_w/Q^*$$

where C is a constant, T_i and T_x are the water temperatures at $x = 0$ and x, respectively, and Q^* is the heat-loss rate. Q^* was represented therein by a complicated expression requiring numerical integration of the above relation. Here, the more complete equation is shown to be well approximated by $Q^* = Q_0 + q(T_w - T_a)$, where T_a is air temperature, Q_0 is heat-loss rate when $T_w - T_a = 0$, and q is a heat-transfer coefficient. It is further shown that for a given set of general meteorologic conditions, q is a linear function of windspeed v_a and Q_0 is a linear function of v_a and net incoming solar radiation Q_R. Thus by specifying general meteorologic conditions v_a, Q_R, and $T_w - T_a$, one can evaluate the simple linear relation for Q^*, and the expression relating x and T_w can be easily determined by analytical integration. This relation takes the form of an exponential-decay curve. Results using these approximations are within 10% of those calculated by numerical integration for the cases tested.

14. KEY WORDS

Stream pollution	Thermal power plants
Thermal pollution	Waterways (transportation)

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