TENSILE STRENGTH OF ICE UNDER TRIAXIAL STRESSES

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PREFACE

This report was prepared by F. Donald Haynes, Mechanical Engineer, Applied Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. The work was performed under DA Project 4A062112A894, *Engineering in Cold Environments*, Task 02, *Engineering Design Criteria*, Work Unit 005, *Crossing of Water Barriers*. The objective of this work unit is to develop methods, procedures and criteria for rapid strengthening of ice sheets in order to enable troops and equipment to cross ice-covered lakes, streams and estuaries.

Many USA CRREL personnel assisted in this project. The apparatus was designed by D. Garfield and Dr. I. Hawkes. D. Farrell proposed design modifications and was helpful in developing the testing technique. D. Nevel proposed design modifications, assisted greatly in providing consultation throughout the project, and reviewed this report. Dr. M. Mellor and Dr. I. Hawkes provided valuable consultation during the project and reviewed the report. The thin section photography was done by Dr. A. Gow. Apparatus components were made by R. Forrest and F. Gernhard. J. Ricard assisted in developing the sample preparation technique and J. Kalafut provided assistance with the instrumentation.

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NOMENCLATURE

Symbol	*Definition*	*Units*
A	Area	cm^2
D	Diameter of specimen	cm
e	Offset eccentricity	cm
P	Applied load	kg
r	Radial distance from center of specimen to a point	cm
R	Radius of specimen	cm
t	Time	second
α	Angle of loading	degrees
ℓ	Neck length	cm
ω	Electrical resistance	ohms
ρ_f	Fluid pressure	kg/cm^2
σ_c	Compressive stress	kg/cm^2
σ_{cu}	Compressive strength	kg/cm^2
σ_r	Radial compressive stress	kg/cm^2
σ_θ	Tangential tensile stress	kg/cm^2
σ_t	Tensile stress	kg/cm^2
σ_{tu}	Tensile strength	kg/cm^2

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INTRODUCTION

The biaxial and triaxial stress states are present in many strength studies of ice. Of particular interest to this investigation is the Brazil test. The Brazil test, the diametral compression of a solid disc, has been used to measure the tensile strength of ice and other materials. Figure 1 shows the Brazil test and stress distribution along the loading diameter. Mellor and Hawkes (1971) and Butkovich (1959) have made Brazil tests on ice and found values much lower than the uniaxial tensile strength.

The disc of ice fails in tension along the loaded diameter in the Brazil test. According to elastic theory the maximum tensile stress occurs at the center where the ratio of compression to tension is 3:1. In the past, it has been assumed that the compressive stress does not influence failure, although Mellor and Hawkes (1971) have questioned this assumption.

The purpose of this study is to investigate the effect of a compressive stress on the tensile strength of polycrystalline ice. The results will help to interpret tests with combined stress states and in particular the Brazil test.

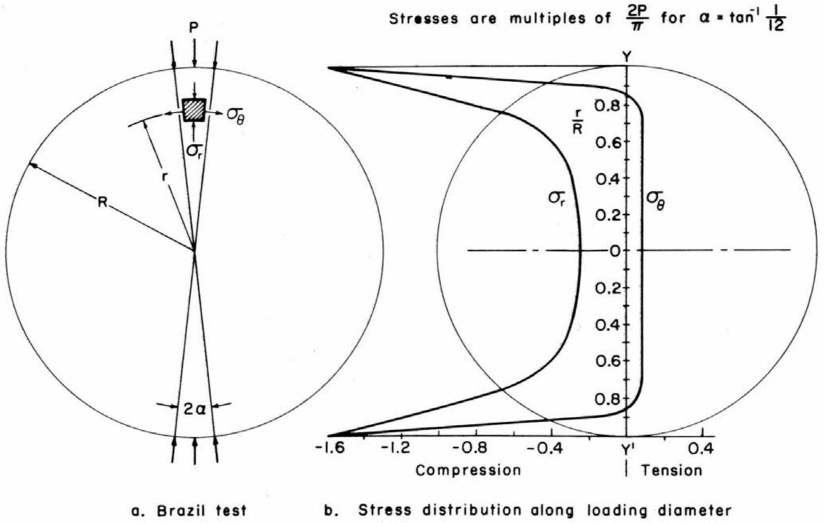
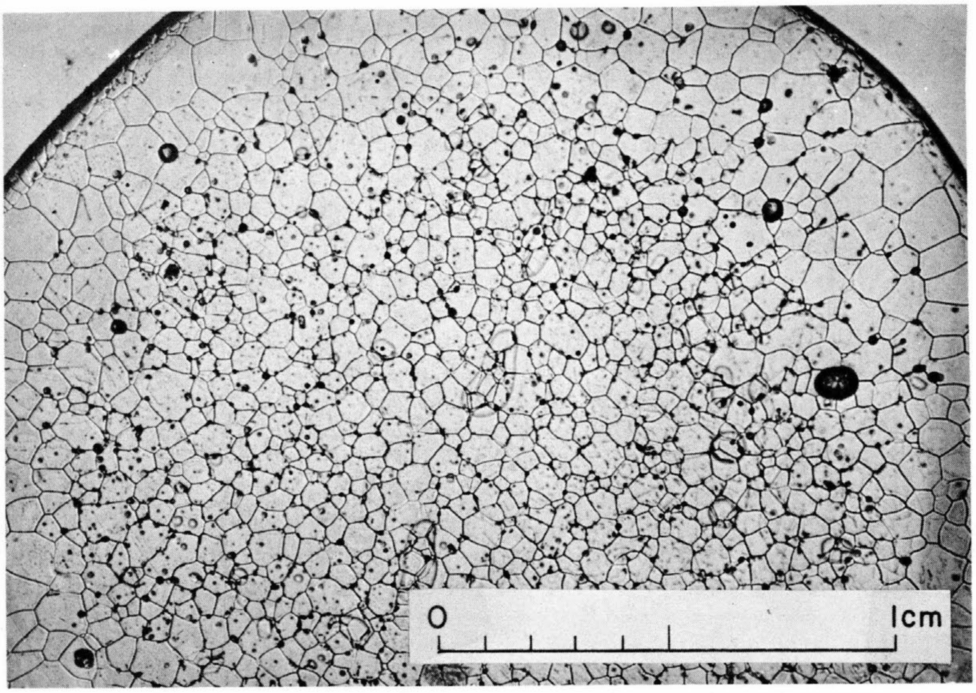
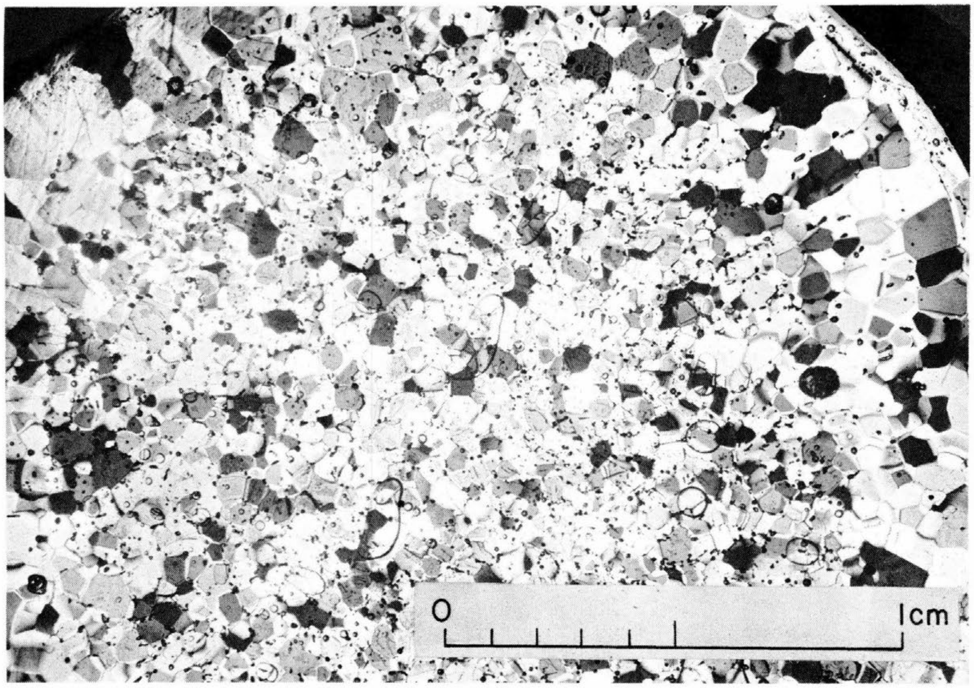


Figure 1. The Brazil test and stress distribution along the loading diameter. (After Fairhurst 1964.)



a. *Bubble distribution.*



b. *Grain structure.*

Figure 2. Magnified views of a thin section of a test specimen.

TEST SPECIMEN

The material used in this study was isotropic polycrystalline ice, similar to the ice tested by Hawkes and Mellor (1972). The ice is porous and similar to glacier ice.

The general method of preparing the ice specimens was to pack ice grains in a mold, saturate them with water and freeze them. The ice grains acted as nuclei for freezing and limited the amount of water required.

The ice grains were obtained by disaggregating snow. Snow was rubbed on snow and then the grains were allowed to pass through a U.S. Standard number 20 sieve but were caught on a number 40 sieve.

Lucite mold blocks split down the centerline were used to form the specimens. A dumbbell shape was achieved by placing Lucite inserts in the mold holes. To facilitate specimen removal, silicone grease was rubbed lightly on the insides of the holes before the inserts were fitted into the mold. After the mold was placed on a vibrator, snow was slowly added to three holes of the mold at −15°C. One hole of the mold was left open for the addition of water. When the snow was fully compacted, the mold was removed from the vibrator and left in an ambient temperature of −2.2°C for two hours.

Distilled, degassed water at 0°C was then added through the empty hole. It flowed through a passage on the base of the mold assembly and saturated the snow from the base upward. This method forced some of the air out of the compacted snow but many bubbles remained, as shown by the dark spots in Figure 2a.

After insulation was placed on the top and sides of the mold it was placed in an ambient temperature of −15°C for freezing. Directional freezing took place from the base upward, which minimized freezing strains.

Figure 2a shows bubble distribution in a specimen. There were a few large bubbles present and a region of nearly bubble-free ice on the outer periphery. The average bubble size was about 0.2 mm. The ice had a bulk density of 0.904 g/cm^3 and a melt water specific conductance of 1.72×10^{-5} ω^{-1} cm^{-1} at 26°C and 60 Hz.

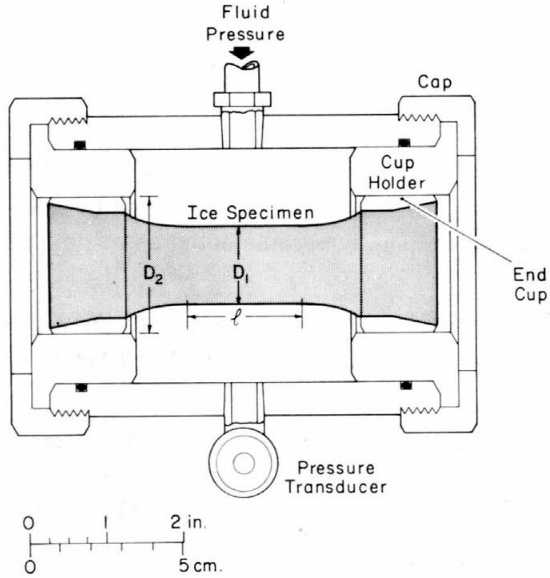
The grain structure is shown in Figure 2b. Polarized light was used to show that the grains were randomly oriented and had an average size of about 0.7 mm.

EQUIPMENT AND PROCEDURE

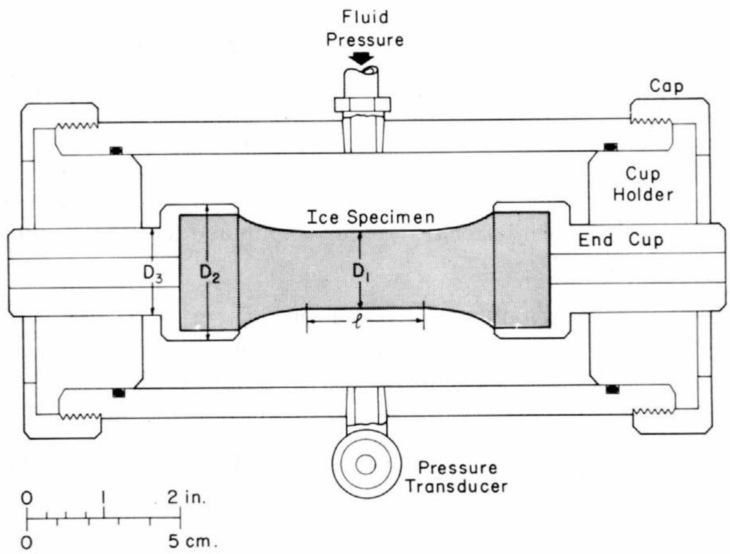
Test chamber

One of the major problems in uniaxial tensile testing of a brittle material is axial misalignment, which results in asymmetrical loading of the specimen. A ductile material will deform locally to minimize any bending stress but a brittle material will not deform locally so bending stresses contribute directly to the failure stress. The apparatus used in this study was designed to minimize any bending stress, provided the error of eccentricity was considered and minimized.

Baratta and Driscoll (1969, 1971) developed a device for testing a brittle material, POCO graphite, in uniaxial tension. A dumbbell specimen was placed in a cylindrical chamber which was then pressurized by pumping hydraulic fluid into the annular cavity until the specimen failed. Their results compared favorably with those of other tensile testing methods.



a. *Series 1 tests.*



b. *Series 2 tests.*

Figure 3. Test chambers.

The apparatus used in this study was proposed by Dr. Ivor Hawkes and designed by Donald Garfield and Dr. Hawkes at CRREL. It is similar to the device used by Baratta and Driscoll. The test chamber used in the series 1 tests, with compression:tension ratios of 0.21 and 0.53, is shown in Figure 3a. The cups were attached to an ice specimen by freezing them in place when the specimen was made in the Lucite mold. A 0.008-cm difference in diameter between the cup and the cup holder permitted fluid flow out of both ends of the test chamber during the test. This oil flow minimized the sliding friction between the cup and the cup holder.

Theoretically the specimen will break in tension at the point of smallest cross-sectional area. For the series 1 tests, the tensile strength is given by the expression

$$\sigma_{tu} = \rho_f \left(\frac{A_2 - A_1}{A_1} \right) \tag{1}$$

where ρ_f is the fluid pressure and A_1 and A_2 are the cross-sectional areas of the diameters D_1 and D_2 as shown in Figure 3a. The stress concentration factor is taken as unity in this study since Hawkes and Mellor (1972) found no significant stress concentration in their photoelastic study of a plastic model with a similar geometry. The geometry of the specimen (Fig. 3a) was designed to minimize stress concentration. The radial and tangential compressive stresses for both series 1 and series 2 tests are equal to the applied fluid pressure.

For the series 2 tests the apparatus was modified so that ratios greater than 0.53 could be obtained. The design modification was proposed by Dennis Farrell and Donald Nevel of CRREL. The higher ratios were obtained in the series 2 test chamber (Fig. 3b) by applying back pressure on the end caps which reduced the axial tensile stress. As before, the extended cups were attached to the ice specimen by freezing in place.

The tensile strength for the series 2 tests is given by

$$\sigma_{tu} = \rho_f \left(\frac{A_3 - A_1}{A_1} \right) \tag{2}$$

where ρ_f is the fluid pressure and A_1, A_2 and A_3 are the cross-sectional areas of the respective diameters as shown in Figure 3b. Since the area A_2 was fixed by the Lucite mold used in making the ice specimen, area A_3 was specified for four different cup sizes to give the stress ratios of 1:1, 2:1, 3:1 and 10:1.

Error of eccentricity

An error in the measured strength can result if the load is applied parallel to the specimen axis but offset from it. This error, E, is

$$E = \frac{\sigma_b}{\sigma_z} \tag{3}$$

where the bending stress $\sigma_b = 32Pe/\pi D^3$ and the axial stress $\sigma_z = 4P/\pi D^2$. Here P is the applied load, e is the offset eccentricity and D is the specimen diameter. Substitution for σ_b and σ_z in eq 3 gives an error of eccentricity equal to $E = 8e/D$.

The eccentricity was measured with a comparator as shown in Figure 4 and is equal to one-half of the run-out. Run-out is the maximum deflection measured in one turn of the specimen under the gage follower. The design of the loading apparatus permitted the eccentricity of the load to be measured from the specimen's geometry. Using eq 3, a maximum potential error of 3% was chosen for this study and 0.1 mm is the corresponding eccentricity cutoff value.

Equipment

A schematic of the entire apparatus is shown in Figure 5a. A three-phase, 2 hp a-c General Electric motor was used to drive a Vickers aircraft-type piston pump with a variable delivery rate up to 7.85 gal./min. A ½-gal. Parker Hannifin piston type accumulator assembled in the pressure line served as a bleed-off to achieve variable load rates and also minimized pressure surging and pressure pulsations.

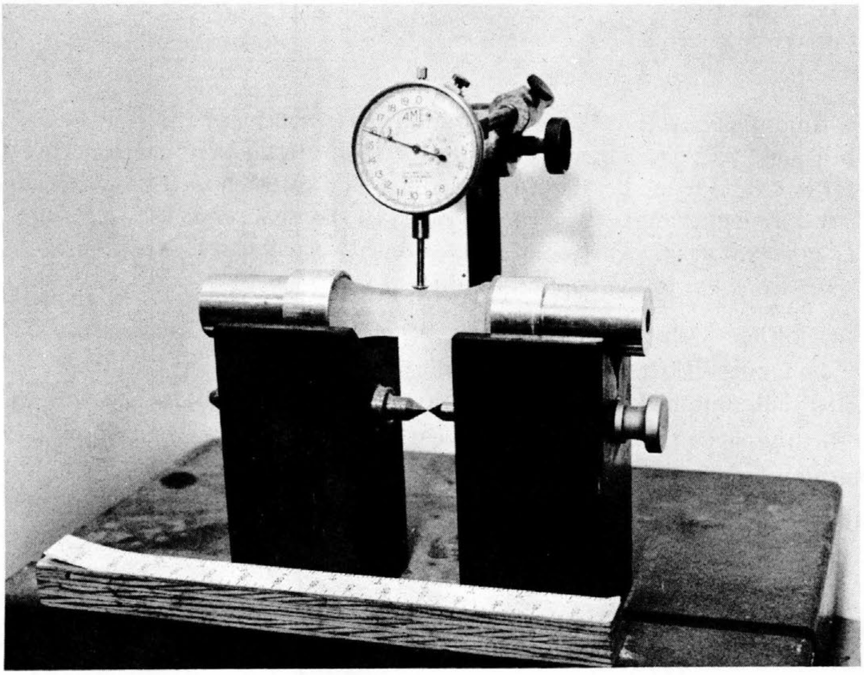


Figure 4. Measuring eccentricity of test specimen in a comparator.

A No. 500 Soil Moisture air compressor with a 42.2 kg/cm^2 rated capacity returned the piston to a position of zero oil volume in the accumulator after a test. Ice specimen deformation was measured with a Schaevitz 300 SS-L linear variable differential transformer/transducer with a range of $\pm$0.762 cm. The hydraulic pressure in the test chamber was recorded with a Baldwin-Lima-Hamilton pressure transducer rated at 35.2 kg/cm^2 capacity and calibrated to $\pm$0.1% of full scale. All fracture and deformation results were recorded on a Hewlett-Packard-Mosley two channel strip chart recorder.

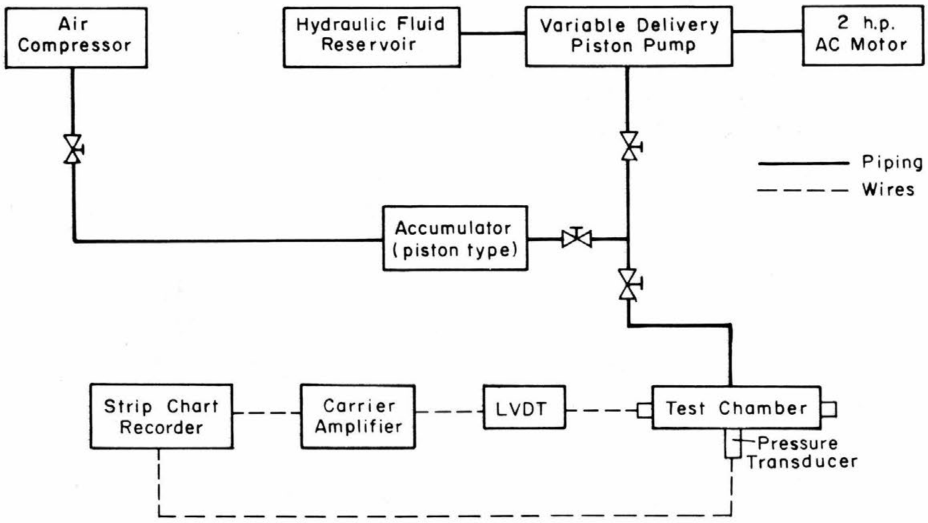
Figure 5b shows the series 2 test chamber ready for a test. The LVDT is assembled on the end of the chamber. The LVDT core was spring loaded in a unit assembly with the barrel so that it automatically returned to a zero setting after each test. The pressure transducer was attached to the test chamber at a 1.27-cm pipe port.

The LVDT measured the axial deformation over the entire specimen length. In order to obtain the deformation in the neck of the specimen, the simplifying assumption was made that the deformation in a section of the specimen was proportional to its cross-sectional area. Deformation in the neck was found to be 0.293 of the total deformation. Hawkes and Mellor (1972) used a value of 0.3 after calculations gave a value of 0.3 and a value of 0.29 was found by experiment. A value of 0.3 was used for data reduction in this study.

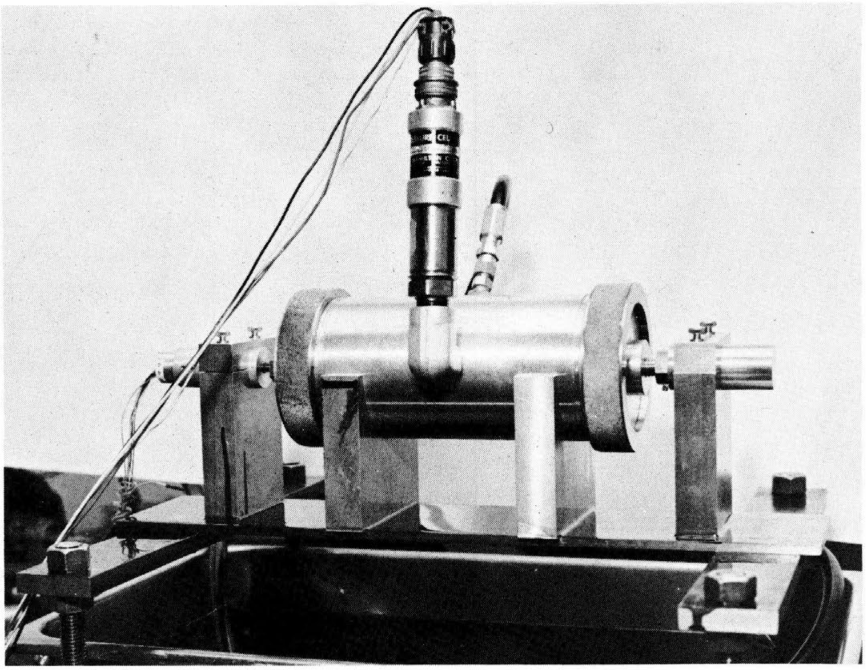
Procedure

After each specimen was removed from the mold its diameter was measured with vernier calipers to $\pm$0.00254 cm. Following this the eccentricity of each specimen was determined.

To protect the surface of the ice specimen from penetration of the hydraulic fluid and possible loss in strength, a rubber membrane was placed on each specimen tested. The membrane had a thickness of 0.01 cm and had a tight fit on each end of the gage length of the specimen. Inspection of the specimen after fracture showed that no fluid had touched the ice over the gage length.



a. *Schematic.*



b. *Apparatus assembled for a test.*

Figure 5. Test apparatus.

After the diameter was measured, the eccentricity checked and the rubber membrane placed on the specimen, it was inserted into the test chamber. Electrical connections to the LVDT and the pressure transducer were made. The needle valve to the accumulator was opened. The cam plate on the piston pump was adjusted to give a desired time to failure value. Except for the recorder and the air compressor, the entire apparatus was kept in a coldroom where the temperature was maintained at −7.0 ± 1.0°C.

A remote control on the pump motor was used to start a test. The test chamber was hydraulically pressurized until the specimen fractured, which was indicated on the recorder by a momentary drop in pressure followed by a rapid increase. The test specimen was removed immediately following a test so that the mode of fracture could be examined.

RESULTS

The results of 141 tests are given in Appendix A. A summary of the results is given in Table I. The summary includes only the valid data. Reasons for invalidating data include large eccentricity, specimen flawed before a test, fractures not occurring on the neck section and poor axial alignment in the test chamber.

Table I. Summary of results for valid triaxial tensile tests.
Bubbly polycrystalline ice, density 0.904 g/cm^3, temperature $-7 \pm 1°C$.

Compression: tension stress ratio	*No. of tests*	*Tensile strength (kg/cm^2)*			*Compressive stress (kg/cm^2)*	*Average strain rate (sec^{-1}) $\times 10^{-5}$*	*Average stress rate (kg/cm^2 sec)*	*Average time to failure (sec)*
		Max	*Mean*	*Min*	*Mean*			
0.21:1	18	22.4	18.52	14.17	3.55	1.42	0.59	31.56
0.53:1	21	16.53	12.58	7.31	6.67	3.73	0.24	51.99
1.016:1	10	14.4	10.72	6.50	10.90	0.31	0.36	29.57
2.065:1	6	9.8	8.18	6.17	16.90	2.66	0.27	30.20
3.155:1	5	6.93	6 36	5.93	20.08	*	0.28	22.16
10.14:1	1	*	1.71	*	17.30	*	0.05	33.20

*Not measured.

Tensile strength

The tensile strength found at five compression:tension stress ratios is plotted in Figure 6. Results of the 10.14:1 ratio are not plotted because only one valid test was obtained. As shown in the figure there is a definite effect of compressive stress on tensile strength. The average values at each ratio are given as well as the individual test points. All mean values are lower than the uniaxial average value found by Hawkes and Mellor (1972).

A time-to-failure distinction is made for each test point plotted. The results show a tendency towards a higher tensile strength for a longer time to failure. The higher strength results for a lower stress rate are probably the effect of stress relaxation in the specimen. If a test is run rapidly, the ice responds elastically. The stress concentration caused by the nonhomogeneous test material is realized in an elastic manner. If a test is run slowly, there is sufficient time to allow the ice to creep. This creep relaxes the stress concentrations previously mentioned. Hence, the test specimen requires more load to increase the stress concentration to the level needed for crack initiation. This increased load will give a high nominal failure stress.

Tensile strength as a function of axial strain rate is plotted in Figure 7. Hawkes and Mellor (1972) found that the uniaxial tensile strength was relatively insensitive to strain rate over the range of 10^{-6} to 10^{0} sec^{-1}. For ratios of 0.21 and 0.53 the results tend to agree with the results of Hawkes

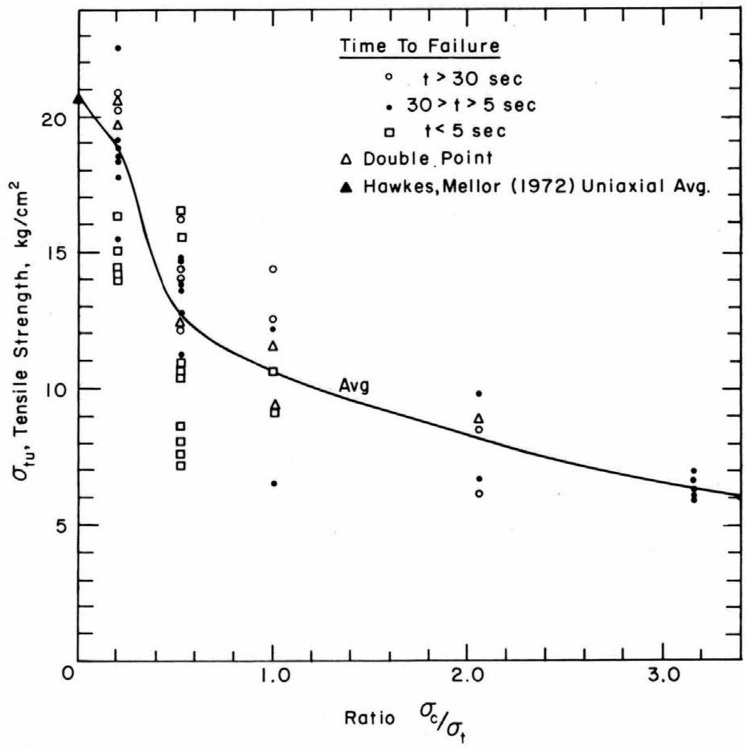


Figure 6. Tensile strength of polycrystalline ice at five compression:tension stress ratios.

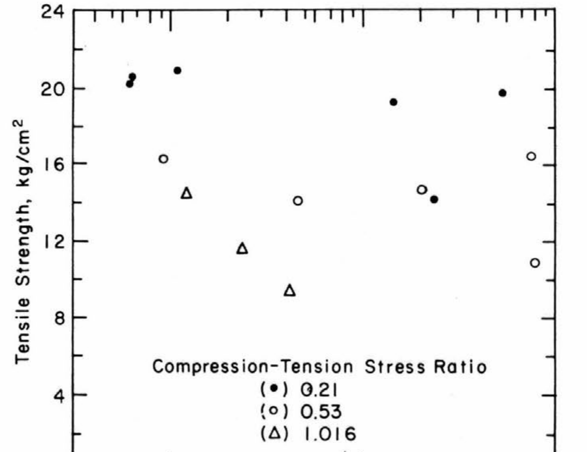
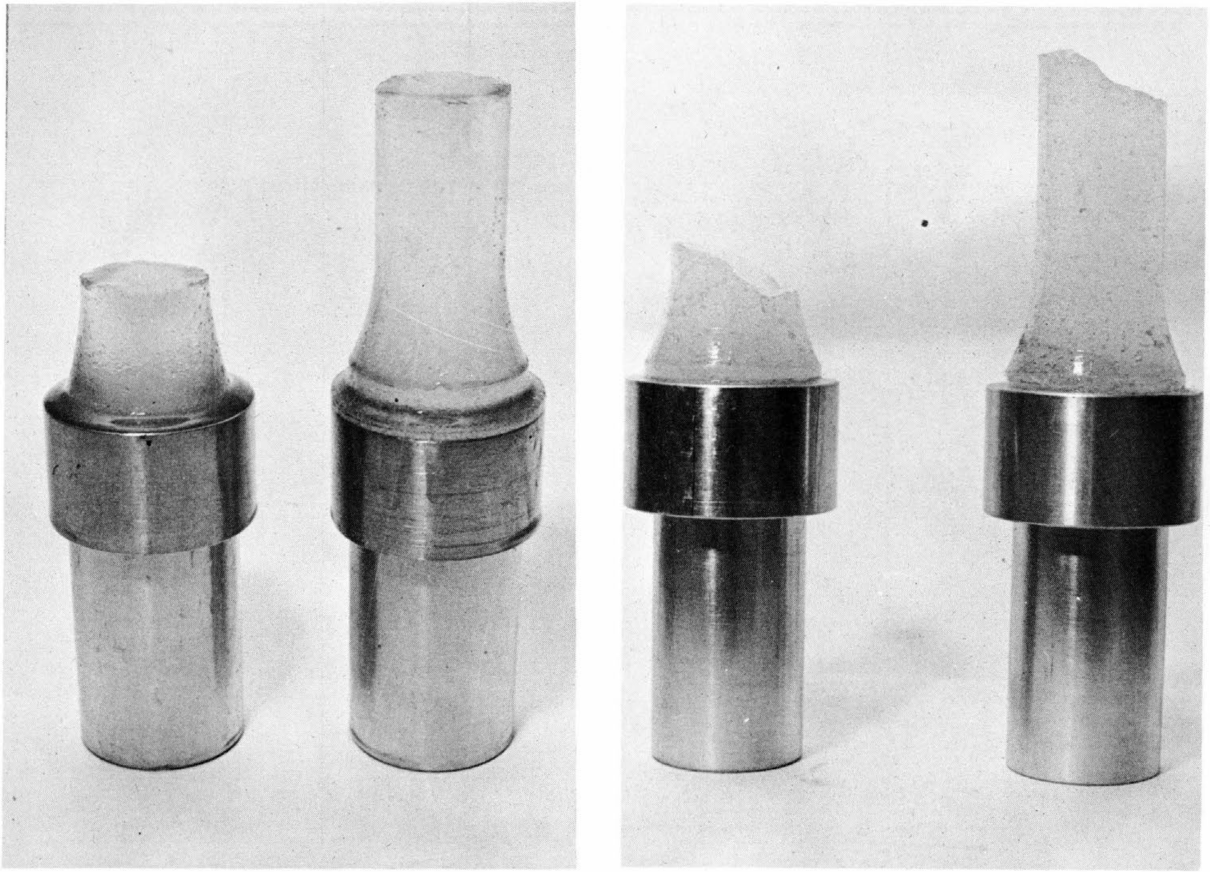


Figure 7. Tensile strength as a function of axial strain rate.



a. Tensile. *b. Irregular.*

Figure 8. Fracture of test specimens.

and Mellor. However, the limited data obtained at the ratio of 1.016:1 show that the tensile strength is sensitive to strain rate. This indicates that at higher ratios there may be a greater stress relaxation effect as previously discussed. Because of the higher fluid pressures used it was difficult to obtain valid strain measurements at the higher ratios.

Fracture

The specimen was removed from the test chamber immediately after each test so that the mode of fracture could be determined. Figure 8a shows a typical tensile fracture found at all six ratios. All of the tests at ratios 0.21:1, 0.53:1 and 1.016:1 produced a tensile fracture. At ratios 2.065:1 and 3.155:1, however, over half of the tests produced an irregular failure (Fig. 8b). Hawkes and Mellor (1970) have explained the possibility of an irregular fracture by propagating crack arrest or deflection, resulting in an irregular separation, or by a coalescence of cracks.

Figure 9 shows a fracture pattern on a thin section of a specimen. After test 23 of series 2 the specimen was united with a water drop bond to permit a study of the pattern. The photograph shows that the fracture is either transcrystalline or intercrystalline or a combination of both.

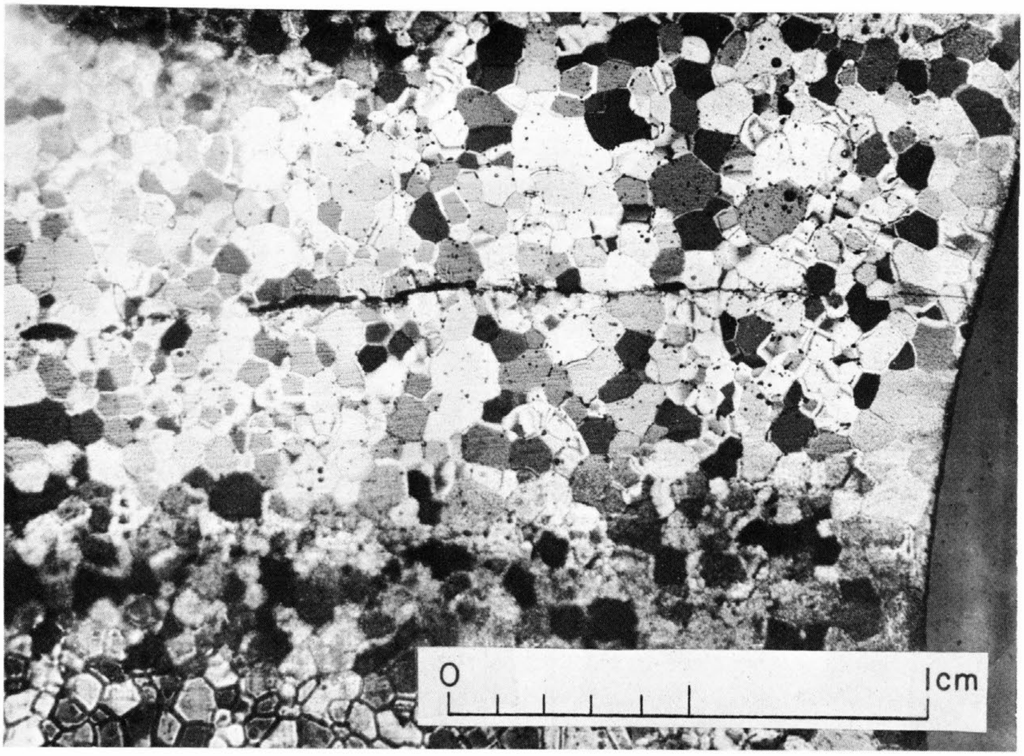


Figure 9. Fracture pattern of a specimen.

DISCUSSION

Uniaxial tensile tests are difficult to perform and therefore only a few tests have been reported. Hawkes and Mellor (1972) conducted uniaxial tensile and compression tests on polycrystalline ice. They found the mean uniaxial tensile strength for this type of ice to be 20.6 kg/cm^2 and the mean compressive strength to be 66.7 kg/cm^2. These results give a compression:tension strength ratio of 3.24:1.

Uniaxial tensile strength

A comparison of the test results with the uniaxial results of Hawkes and Mellor (1972) shows that the uniaxial strength value is approached with a low ratio (0.21:1). This result demonstrates the validity of the experimental technique. The smaller the ratio the closer a uniaxial stress state is approximated. For example, Baratta and Driscoll (1971) used a ratio of 0.05:1 and found a close correlation with uniaxial results. If the specimen geometry were adapted to give a low ratio, it is believed a close correlation with uniaxial results would be found. Therefore, it is possible that the uniaxial tensile strength could be found with the present apparatus.

It has been shown by Gow (1973) that the uniaxial compressive strength of bubbly ice is quite sensitive to porosity. A similar effect might be expected with tensile strength, especially if pores tend to be elongated normal to the fracture surface. Therefore, biaxial tests on ice of different porosities could produce different results although the general trend should be the same.

Brazil test

Mellor and Hawkes (1971) obtained an average Brazil test value of about 4.6 kg/cm^2 for bubbly ice at –7°C. Butkovich (1959) obtained a mean Brazil tensile strength of about 4 kg/cm^2 for bubbly ice at –5°C. These results indicate that the Brazil test gives strength values much lower than the uniaxial tensile strength.

An analysis of the Brazil test was done by Fairhurst (1964). He used the Griffith fracture criterion where a compression:tension stress ratio of 3:1 or greater is used. Fairhurst's results indicate that failure may occur away from the center of the disc and that the tensile strength found in such cases is lower than the true value. He concluded that the uniaxial tensile strength is underestimated for materials with a low compression:tension strength ratio. This conclusion agrees with the data cited.

The test results do not support the assumption that the compressive stress has no effect upon failure in the Brazil test. In this study compressive stress had a definite effect on the tensile strength of ice. This fact supports the view of Mellor and Hawkes (1971) that the Griffith failure criterion does not apply for ice in the Brazil test, since the compressive stress component must influence failure.

The tensile test conducted in this study is quite dissimilar to the Brazil test. However, since the compression:tension stress ratio is similar, a comparison can be made. The mean tensile strength value found in this study at a ratio of 3.155:1 was 6.36 kg/cm^2. This result would indicate that the Brazil test should give a tensile strength value about one-third the uniaxial value for ice. If failure in the Brazil test occurs away from the center of the disc, the compression:tension ratio becomes larger, and the Brazil result could be less than one-third the uniaxial strength. The Brazil test could give a consistent index of tensile strength if failure always occurred at the same location along the loaded diameter but this is doubtful.

Mellor and Hawkes (1971) have shown that the Brazil test can be used successfully to measure the uniaxial tensile strength of most rocks. However, the compression:tension strength ratio is greater than 8:1 for most rocks but it is only 3.24:1 for polycrystalline ice. According to Griffith's theory, the compressive stress does not affect the outcome for an ideal Brazil test if the compression: tension strength ratio is greater than 8:1. Results indicate that the Griffith criterion does apply for rocks in the Brazil test.

COMPARISON WITH THEORY

A comparison with a few of the prominent biaxial failure theories for brittle materials is made to determine which one is most applicable to the test results. Justification for comparison with biaxial criteria is based on the fact that fracture initiation is assumed to be independent of the intermediate principal stress by most of these theories.

The Coulomb-Mohr criterion (see Timoshenko 1956) is a straight-line envelope expressed by

$$\frac{\sigma_1}{\sigma_{tu}} - \frac{\sigma_2}{\sigma_{cu}} = 1 \qquad (4)$$

where σ_1 and σ_2 are the principal stresses and σ_{tu} and σ_{cu} are the uniaxial tensile and compressive strengths respectively. Mohr assumed that the intermediate stress did not influence failure. This criterion is shown in Figure 10. According to the theories plotted, a biaxial stress state outside the

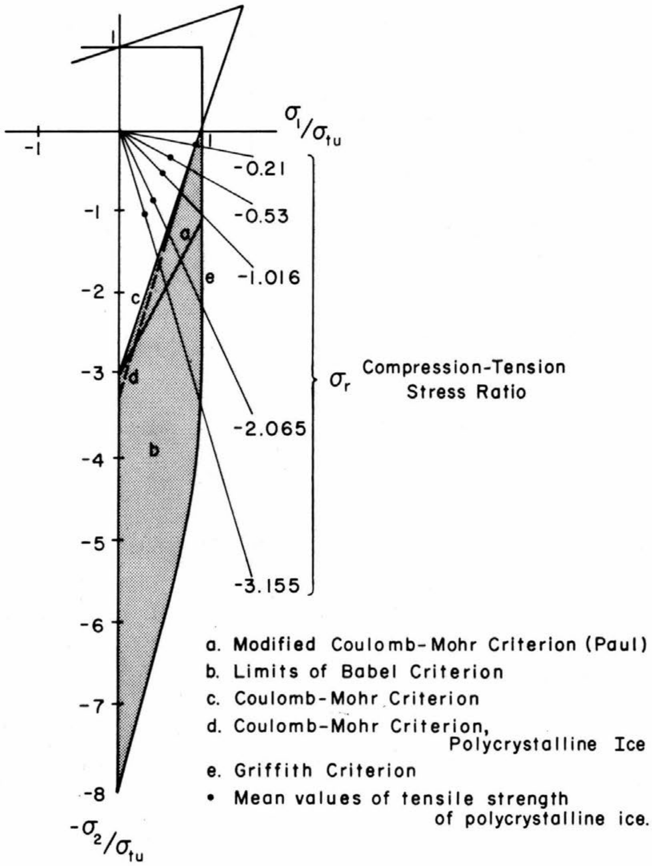


Figure 10. Biaxial failure envelopes for brittle materials. (After Babel 1966.)

envelope will cause fracture. The Coulomb-Mohr criterion for polycrystalline ice was formed by using the uniaxial results of Hawkes and Mellor (1972).

A modification of the Coulomb-Mohr criterion was made by Paul (1961) to accommodate data on gray cast iron. A tension cutoff is proposed, which extends to a ratio near unity.

Griffith (1924) proposed a criterion which assumes that failure is determined by sharp discontinuities present in brittle materials. He assumed that failure will occur when a critical tensile stress on a boundary of a discontinuity is reached. The Griffith criterion is usually used for interpreting Brazil test results.

A theory for the biaxial fracture strength of brittle materials was proposed by Babel (1966). He tested hollow cylindrical zirconia specimens under a biaxial stress state. His theory for fracture is the region between the Coulomb-Mohr criterion and the Griffith criterion as shown in Figure 10. When the flaw is a sharp crack, like that found in glass, Babel uses the Griffith criterion. When the flaw is spherical, like that found in cast iron, he uses the Coulomb-Mohr criterion. Intermediate points correspond to flaw shape transition.

A theory of fracture initiation under a triaxial stress state has been developed by Murrell and Digby (1970). They considered a Griffith type brittle solid with ellipsoidal cavities. Fracture initiation was shown to be independent of the intermediate principal stress. Their theory includes the following criteria: 1) the fracture is tensile as long as the tensile stress, normal to the fracture surface, is greater than the other principal stresses, and 2) when the compressive stress becomes greater than the tensile stress there is a transition from tensile to shear fracture.

The results of this study are compared with all the theories cited in Figure 10. At the 0.21:1 ratio the mean value of the tensile strength is only about 2% below the Coulomb-Mohr line but at the higher ratios it is considerably lower. Despite these discrepancies the Coulomb-Mohr criterion is the best approximation to the test results. Since the flaws in the test material are nearly spherical, the results agree with Babel's analysis that failure with spherical flaws is best predicted by the Coulomb-Mohr criterion. The test results do not agree with the tension cutoff value of slightly larger than one, proposed by Paul. The results do indicate that if a cutoff does exist for ice it does not extend beyond a ratio of 0.21:1.

The test results showed a transition from a tensile to an irregular fracture for ratios greater than unity. This result shows some agreement with the Murrell and Digby prediction of shear fracture under similar ratios.

CONCLUSIONS

The results of this study indicate that compressive stress has a definite effect on the tensile strength of polycrystalline ice. The tensile strength at a compression:tension stress ratio of 3 is about one-third the uniaxial value. These results support the evidence that the Brazil test underestimates the tensile strength of ice. They also indicate that the Brazil test value for ice can be no greater than one third the uniaxial tensile strength.

A comparison of the test results with several theories in the compression:tension quadrant indicates a lower fracture strength than has been predicted by any theory. However, the Coulomb-Mohr criterion is the best approximation to the test results.

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APPENDIX A. TEST RESULTS

Bubbly, polycrystalline ice, density 0.904 g/cm^3, temperature –7 ± 1°C. Test specimen: Dumbbell, neck diameter 2.54 cm, neck length 3.8 cm, gage length 8.3 cm, no entry for strain and time to failure means not measured.

Test no.	*Compression: tension stress ratio*	*Run out eccentricity (cm × 10^{-3})*	*Tensile strength (kg/cm^2)*	*Axial strain ΔL (cm × 10^{-5})*	*Time to failure (sec)*	*Data rejection criterion*
			Series I			
1	0.21	55.8	8.58			Large eccentricity
2	0.21	101.6	21.8			Large eccentricity
3	0.21	35.6	24.4			Large eccentricity
4	0.21	177.8	20.8			Large eccentricity
5	0.21	63.5	20.1			Large eccentricity
6	0.53	27.9				Large eccentricity
7	0.53	66.0	9.4			Large eccentricity
8	0.53	99.0	12.3			Large eccentricity
9	0.53	68.5	15.3			Large eccentricity
10	0.53	122.0	19.5			Large eccentricity
11	0.53	33.0	18.6			Large eccentricity
12*	0.53	15.2	14.4		51	
13	0.53	17.8	12.2		38.4	Broke at end cap
14*	0.53	7.6	13.9		62	
15*	0.53	12.7	13.8		16	
16	0.53	33.0	10.8		16	Large eccentricity
17	0.53	15.2	4.0		4.3	Crack near one end
18*	0.53	17.8	12.7		8	
19	0.53	40.6	12.7		12	Large eccentricity
20	0.53	12.7	1.4		1	Crack in specimen
21*	0.53	10.2	7.4		0.8	
22*	0.53	20.3	8.02		1	
23*	0.53	17.8	8.65		1	
24	0.53	7.6	7.63		0.8	No membrane used
25	0.53	25.4	6.96		4	Large eccentricity
26	0.53	38.0	15.9		5	Large eccentricity
27	0.53	15.2	13.4		9.5	Did not break
28*	0.53	10.2	10.3		3	
29	0.53	10.2	11.2		10	Specimen broke in three pieces
30	0.53	10.2	13.6		10	Flaw in specimen
31†	0.53	88.8	9.38		12	Large eccentricity
32†	0.53	35.5	13.7		10.6	Large eccentricity
33*†	0.53	12.7	13.6		9.5	
34†	0.53	38.1	7.04		4.3	Large eccentricity
35	0.21	63.5	12.4		1.5	Large eccentricity
36	0.21	30.5	17.7		2	Large eccentricity
37*	0.21	7.6	16.3		1.5	
38	0.21	27.9	13.7		1.3	Large eccentricity
39*	0.21	10.2	14.7		1.5	
40	0.21	101.6	18.6		4.5	Large eccentricity
41	0.21	20.3	4.34		0.5	Crack in specimen
42	0.21	27.9	14		1.5	Large eccentricity
43	0.21	12.7	7.23		1	Flaw in specimen
44*	0.21	20.3	19.16		11.5	

*Valid data.

†Specimens annealed 1½ weeks.

Test no.	Compression: tension stress ratio	Run out eccentricity (cm × 10^{-3})	Tensile strength (kg/cm^2)	Axial strain ΔL (cm × 10^{-5})	Time to failure (sec)	Data rejection criterion
			Series I (Cont'd)			
45	0.21	40.6	25.1		21.6	Large eccentricity
46	0.21	279.0	13.1		3.6	Large eccentricity
47	0.21	279.0	11.6		1.5	Large eccentricity
48	0.21	35.5	19.8		15	Large eccentricity
49	0.21	7.6	11.9		18	Flaw in specimen
50	0.21	165.0	20.3		51	Broke at end cap
51	0.21	30.4	22.4		32.5	Large eccentricity
52*	0.21	20.3	18.5		15	
53*	0.21	12.7	18.8		17	
54	0.21	15.2	9.8		2	Broke at end cap
55*	0.21	15.2	17.7		5	
56	0.21	7.6	23.3		32	Broke at both end caps
57	0.21	12.7	20.2	3680.0	25	Broke at end cap
58*	0.21	12.7	19.8	6320.0	40.5	
59*	0.21	15.2	22.4		5	
60	0.21	40.6	16.7		2	Broke at end cap
61	0.21	10.2	9.8	40.0	0.1	Surface flaw
62	0.21	17.8	15.5		5	Broke at end cap
63	0.21	40.6	25.1	320.0	15	Large eccentricity
64	0.21	38.1	15		3	Broke at end cap
65	0.21	25.4	9.8		0.5	Large eccentricity
66	0.21	12.7	15.5	90.0	15	Broke at end cap
67	0.21	15.2	11.9	140.0	4	Broke at end cap
68	0.21	10.2	14	220.0	2	Broke at end cap
69*	0.21	12.7	20.6		66	
70*	0.21	20.3	20.6	230.0	120	
71*	0.21	20.3	20.9	240.0	78	
72	0.21	10.2	18.7		99	Flaw in specimen
73	0.21	33.0	20.2		108	Large eccentricity
74	0.21	27.9	20.6		84	Large eccentricity
75*	0.21	20.3	14.2	135.0	2	
76*	0.21	12.7	20.3	173.0	93	
77	0.21	7.6	17.7	330.0	147	Flaw in specimen
78*	0.21	20.3	19.2	22.4	5.5	
79	0.21	35.5	9.6		0.5	Large eccentricity
80*	0.21	20.3	19.8		3	
81*	0.21	5.1	16.6		4	
82*	0.21	15.2	15		0.75	
83*	0.53	17.8	12.3		238.8	
84	0.53	12.7	4.1		0.75	Flaw in specimen
85*	0.53	15.2	13.6		14	
86*	0.53	20.3	8.75		1	
87	0.53	25.4	10.9	6.3	194	Large eccentricity
88*	0.53	20.3	14.7	620.0	11	
89*	0.53	12.7	14.8		14.5	
90*	0.53	12.7	14.2	330.0	25	
91*	0.53	10.2	12.3		186	
92*	0.53	7.6	16.5	438.0	2	
93*	0.53	10.2	15.6		2.25	
94*	0.53	10.2	10.9	224.0	1	
95*	0.53	10.2	12		126	
96*	0.53	15.2	16.3	825.0	318	

*Valid data.

Test no.	Compression: tension stress ratio	Run out eccentricity (cm × 10^{-3})	Tensile strength (kg/cm^2)	Axial strain ΔL (cm × 10^{-5})	Time to failure (sec)	Data rejection criterion
			Series II			
1*	1.016	15.2	6.5		5	
2*	1.016	15.2	9.45		30	
3*	1.016	10.2	12.5		43	
4*	1.016	20.3	9.08		2.6	
5*	1.016	10.2	10.6		2.9	
6**	1.016	12.7	4.12		20.1	Poor axial alignment
7*	1.016	12.7	9.45	755.0	64	
8*	1.016	12.7	12.1		6.4	
9*	1.016	12.7	11.6	336.0	48.6	
10*	1.016	12.7	14.4	156.0	48.2	
11*	1.016	20.3	11.5		45	
12**	2.065	20.3	8.95		12.8	
13**	2.065	25.4	4.96		48.6	Large eccentricity
14*	2.065	15.2	8.95		29	
15*	2.065	12.7	6.68	1720.0	23.4	
16	2.065	17.8	4.67		40	Poor axial alignment
17**	2.065	20.3	4.78		43.3	Poor axial alignment
18**	2.065	25.4	7.43		42.4	Large eccentricity
19*	2.065	20.3	8.52		43.5	
20	2.065	10.2	6.17		45.1	
21**	2.065	15.2	6.21		41.2	Poor axial alignment
22**	2.065	10.2	11.9		53.7	Broke at end cap
23**	2.065	12.7	14.6		31	Broke at end cap
24**	2.065	10.2	>17		31.7	Broke at end cap
25*	2.065	12.7	9.8		27.6	
26**	3.155	10.2	4.14		13.6	Poor axial alignment
27*	3.155	10.2	6.3		24.8	
28*	3.155	20.3	6.6		22	
29**	3.155	20.3	6.36		30.6	Poor axial alignment
30**	3.155	15.2	7.06		29.3	Poor axial alignment
31**	3.155	20.3	10.7		24.4	Poor axial alignment
32*	3.155	7.6	6.04		23.4	
33**	3.155	10.2	>11		25.3	Broke at end cap
34	3.155	20.3	>11		26.4	Broke at end cap
35*	3.155	10.2	5.93		21.8	
36*	3.155	10.2	6.93		18.8	
37**	10.14	15.2	0.77		9.0	Poor axial alignment
38**	10.14	10.2	0.92		26.4	Broke at end cap
39	10.14	12.7	1.52		32.5	Broke at end cap
40	10.14	12.7	1.48		26.6	Poor axial alignment
41**	10.14	10.2	1.18		22.8	Poor axial alignment
42*	10.14	10.2	1.71		33.8	
43	10.14	10.2	1.09		33.4	Broke at end cap
44**	10.14	10.2	1.14		24.8	Broke at end cap
45**	10.14	10.2	1.07		33.2	Broke at end cap

*Valid data.
**Irregular break.

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13. ABSTRACT

An investigation was conducted to determine the effect of a compressive stress on the tensile strength of bubbly polycrystalline ice. One hundred forty-five tests were made in an apparatus of novel design. A cylindrical dumbbell specimen was stressed in axial tension and radial and tangential compression by a hydraulic system which minimized bending stresses. Compression-tension ratios ranging from 0.21 to 10.14 were used for the tests. Tensile strength was found to decrease with an increase in the ratio. At the ratio of 3.155 the tensile strength is about one third the uniaxial value. The test results support the evidence that the Brazil test underestimates the tensile strength for ice. They also indicate that the Brazil test value for ice can be no greater than one third the uniaxial tensile strength. A comparison of the experimental results with a few prominent biaxial failure theories indicates a lower tensile strength than predicted by any theory. However, the best approximation to the results is the Coulomb-Mohr criterion.

14. Key Words

Compressive stress
Ice
Tensile strength
Tension tests
Triaxial

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