EFFECTS OF STRATIGRAPHIC LAYERS ON WATER FLOW THROUGH SNOW

Samuel C. Colbeck

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PREFACE

This report was prepared by Dr. Samuel C. Colbeck, Geophysicist, Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. The work was performed under DA Project 4A061102B52E, *Research in Military Engineering and Construction*, Task 02, *Research in Snow, Ice and Frozen Soil*, Work Unit 009, *Physical Characteristics of Ice Masses*.

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ILLUSTRATIONS

NOTATION

d	diameter of the snow grains
h	thickness of the water saturated layer
h_i	thickness of the semipermeable layer
h_0	maximum value of h
H	thickness of the snowpack
H_j	thickness of a single layer of the snowpack
j	the jth layer
I	volume flux above a layer, dimensions of length per unit time
k	permeability, dimensions of length squared
k_i	permeability of the semipermeable layer
k_j	permeability of the jth layer
k_s	permeability of the snow
L	one-half of the length of a semipermeable layer
q_L	total discharge through a drain
R	largest pore size in the fine-textured or semipermeable layer
R'	largest pore size in the coarse-textured layer
S	maximum volume of water stored above an ice layer
S_+	larger value of S^* at a shock at some time
S_-	smaller value of S^* at a shock at some time
S^*	effective water saturation $(S_w - S_{wi})/(1 - S_{wi})$
S_w	water saturation (water volume/pore volume)
S_{wi}	irreducible value of water saturation
t	time
t_j	time required for a value of S^* to propagate across the jth layer
u	volume flux along an ice layer, dimensions of length per unit time
V	volume of water impounded above an ice layer
w_i	volume flux through a saturated layer between drains
W	total flow through a saturated layer between drains
x	coordinate along layer
z	vertical coordinate

NOTATION (Cont'd)

α	a constant, 5.47×10^6 m s^{-1}
θ	slope of a semipermeable layer
ξ	position of a shock front at some time
ρ	density
ϕ	porosity, pore volume/total volume
ϕ_e	effective porosity, $(1 - S_{wi})\phi$
ϕ_j	porosity of the jth layer
ϕ_s	porosity of the snow

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Introduction

Interest in the study of water flow through snow has recently intensified. The runoff from snow and the physical characteristics of snow have been observed for many years but only recently have efforts been made to predict either the movement of water within snow or the interaction between snow and water. In this paper the effects of layered structures on the flow of water through snow are discussed. While drainage channels of coarse grains and high permeability also affect the flow of water through snowpacks, the effects of stratigraphic layers are sufficiently complicated to require this separate investigation. First, water flow through layers of different textures is analyzed. Next, flow past semipermeable layers is analyzed in four cases:

1. Water flow to distinct drains
2. Saturated flow through layers of very low permeability
3. Simultaneous flow through drains and direct seepage through the layers
4. Simultaneous flow past sloping layers.

The nonhomogeneous nature of snowpacks and the rapid change of the structural features in time and space has been well established. A satisfactory understanding of the effects of inhomogeneous features is impeded by a lack of detailed and quantitative observations of their physical characteristics; hence a new generation of observations is needed. Although the effects of snow layers on water flow through snow cannot be completely evaluated until further experimental evidence is gathered, a theoretical basis for understanding their effects is given here.

Review of observations

Using common dye-tracing techniques, Gerdel (1948a) observed the impounding of water above ice layers and the development of drainage channels of increased permeability. Gerdel worked with seasonal snowpacks in the mountains, where such features are especially well developed; however, they are not unique to those environments. Sharp (1951a) described similar features in the firn of the Seward Glacier, and Hughes and Seligman (1939) reported their existence in the snow of the Great Aletsch Glacier.

More recently Wakahama and others (1968) described the features of a textured snowpack including the increased water saturation due to capillary effects at snow surfaces and the interfaces between layers of different texture. Langham (1971) developed a technique for tracing the flow of water through snowpacks without disturbing the surface flow. His observations clearly show the influence of semipermeable layers on shallow seasonal snowpacks. Flow through these layers appears to occur at discrete drains with spacings as close as every few tens of millimeters. When close spacings occur the layers have little effect on the flow of water except that some water retention could occur because of the capillary effect. Gerdel (1954), in referring to these layers as well-consolidated crusts, stated that "storage capacity is of a transient nature since the ice planes in wet snow rapidly disintegrate." In a later section the impounding and storing of water by these layers is described in terms of their physical characteristics.

Review of theory

Colbeck (1972) applied the theory of noncapillary, gravity drainage to snow, treating the snow as homogeneous. Sharp's (1951b) observations on the movement of diurnal waves of meltwater through the Seward Glacier firn were used as a basis for comparison between theory and experiment. The speed of propagation, magnitude, and shape of these waves were closely approximated by the waveforms calculated using the theory. From the speed of propagation of the waves the "effective permeability" of the firn was calculated as about one-half of the permeability measured in a laboratory (Kuroiwa 1968) on snow samples of similar density. The apparent decrease in permeability was attributed to the presence of stratigraphic layers of reduced permeability. The theory was capable of making accurate predictions of the movement of meltwater through snow if averaged parameters were used and the measurements were made on a larger scale than the structural features of the snow. Clearly the accuracy of the predictions decreases with increasing size of these features, suggesting that a more detailed examination of these structural features is needed.

The equation describing the thickness h of the saturated layer over a sloping boundary is (Colbeck in press b):

$$-\alpha k \frac{\partial}{\partial x}\left(h \frac{\partial h}{\partial x}\right) + \alpha k \theta \frac{\partial h}{\partial x} + \phi \frac{\partial h}{\partial x} = I . \qquad (1)$$

For a horizontal layer ($\theta = 0$) where the vertical influx I is zero and steady flow occurs, the thickness of the saturated layer assumes a parabolic profile (Polubarinova-Kochina 1962, p. 407). Langham (unpublished) applied this solution to the unsteady flow of water along an ice layer in snow with influx I along the layer. As Langham noted, some of the boundary conditions are contradictory and the water flux along the layer does not increase with distance as required where influx I is nonzero. When drains occur at regular spacings of 6 m, Langham predicted that delays of several hours would be caused in runoff. Another analysis of flow to drains in snow is given later and a general review of flow to drains is given by Bear and others (1968, p. 205).

Water flow through textured layers

From his observations in the Sierra Mountains of California, Gerdel (1948b) concluded that horizons of increased strength and density are common but layers of vitreous ice are rare. These horizons disaggregate quickly when heavy melting occurs; hence the Sierra snowpack is layered but has few impermeable horizons during intense melting. Where pore sizes are sufficiently large the dominant mode of flow is unsaturated flow through each layer; however, saturated conditions can occur with abrupt changes in grain size or density. It seems unlikely that the presence of a zone of saturation at the base of a fine-textured layer could cause a significant distortion of the flow field although the presence of a saturated layer may cause some distortion during the passage of meltwater waves at an air/snow interface (Colbeck in press a). The thickness of the water-saturated layer can be computed from the difference between the air entry values for two layers (Bear and others 1968, p. 48). At equilibrium,

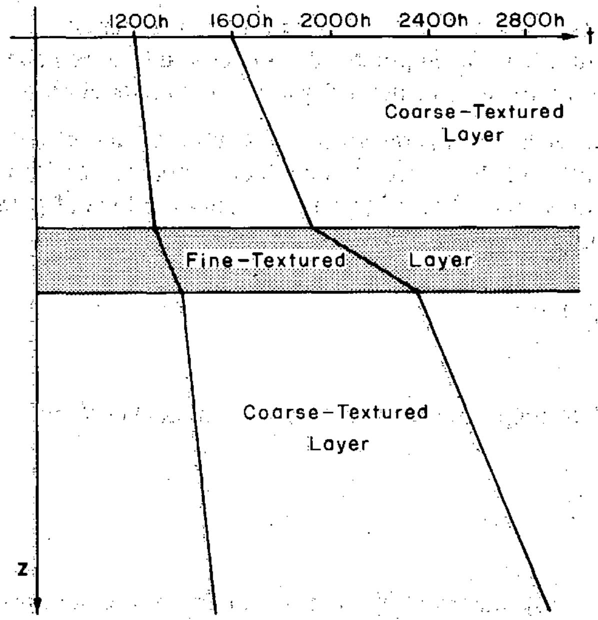

Figure 1. Idealized view of the propagation of values of S from the surface of a snowpack through layers of varying texture. The value originating at 1200h is larger in magnitude than the one originating at 1600h.*

$$h = 15.4 \left(\frac{1}{R} - \frac{1}{R'}\right) \quad \text{(mm)} \tag{2}$$

where R and R' are the largest pore sizes in the fine- and coarse-textured layers, respectively. The thickness of the saturated layer will generally be quite small and water flow through fine-textured layers is predominantly unsaturated flow.

The theory of water flow through homogeneous snow (Colbeck 1972) can be used to describe unsaturated flow through each layer where the thickness of the saturated layer is negligible. The speed of propagation (dz/dt) of a value of effective water saturation S^* is

$$\frac{dz}{dt} = 3\alpha k \phi_e^{-1} S^{*2} . \tag{3}$$

Shimizu (1970) found that permeability k varies as the square of grain size d and exponentially with density,

$$k = 7.7 \times 10^{-4} d^2 \exp(-7.8 \times 10^{-3} \rho), \tag{4}$$

and dz/dt should vary accordingly. The paths of propagation of two values of S^* starting at 1200h and 1600h are shown in Figure 1. The speed of propagation dz/dt decreases significantly in fine-grained or high-density layers as shown in this example. The value of S^* originating at 1200h is larger in magnitude than the corresponding value at 1600h and will propagate more rapidly, thus causing distortion of the trailing edge of the meltwater wave (i.e. the time difference between two values of S^* increases with depth). This distortion is greatly intensified by the fine-textured layer which, in this example, is equivalent to a layer with the same density as the surrounding snow, but with one-half the grain size. Much greater differences in texture generally occur within snowpacks, thus much greater distortions of the meltwater waves should also occur.

The effect of a layer of reduced permeability is similar to that of a diffuser which decreases the "effective permeability" of the entire snowpack or increases the "effective path length" along which the values of S^* propagate. For a layer j with known values of permeability k_j and effective porosity ϕ_j, the speed of propagation of S^* is

$$\left(\frac{dz}{dt}\right)_j = 3ak_j\phi_j^{-1}S^{*2}. \tag{5}$$

The time required for S^* to propagate across a layer of thickness H_j is

$$t_j = \frac{H_j\phi_j}{k_j}(3aS^{*2})^{-1}. \tag{6}$$

The time required for the propagation of S^* across n layers of the snowpack is

$$t = (3aS^{*2})^{-1}\,\overline{H\phi/k} \tag{7}$$

where

$$\overline{H\phi/k} = \sum_{j=1}^{n}\frac{H_j\phi_j}{k_j}. \tag{8}$$

The transit time of S^* to any depth in an unsaturated, layered snowpack can be calculated directly from eq 7 without resorting to step-by-step calculations of its propagation through each layer. Therefore, the construction of the trailing edge of a meltwater wave is straightforward if each value of S^* propagates without intersecting another value. When this happens, such as at the leading edge of a meltwater wave, a shock front is formed and eq 7 is no longer applicable.

There are two methods by which the leading edge of the wave can be constructed; both invoke the principle of continuity. Using the derivation of the shock-front speed given by Sheldon and Cardwell (1959) and the flux-concentration relationship found by Colbeck and Davidson (in press), the speed of propagation of the shock front is

$$(d\xi/dt)_j = (ak_j/\phi_j)(S_+^2 + S_+\,S_- + S_-^2). \tag{9}$$

The large and small values of S^* which join to form the shock (S_+ and S_-) can be calculated from eq 7. The position of the shock at any time can be calculated from eq 9 by reconstructing the entire history of the shock, i.e. by starting at the surface and iterating to the desired depth.

There is a more direct method of constructing the shock front at any depth after the position of each value of S^* which left the surface has been located at that depth. The volume of water leaving the surface must pass through each succeeding depth; thus the area under the wave of flux (or S^*) versus time is invariant with depth. The shock front must arrive at the time necessary to maintain material balance and, by trial and error, the correct position of the shock front at any depth can be found on the graph of S^* versus time. This method is readily adaptable to diurnal waves of meltwater and was used in the study of these waves passing through the Seward Glacier firn (Colbeck 1972).

Water flow past semipermeable layers

In the context used here, semipermeable layers have either small, water-saturated pores and/or distinct drains separated by a large distance compared to the thickness of the layer. Before intense melting and infiltration begin, flow to drains is predominantly two-dimensional. During heavy melting, however, rapid disintegration of the layers occurs (Gerdel 1954), suggesting a transition to a vertical mode of flow or a combination of both vertical and horizontal flow. These modes are examined separately and then collectively in the next three sections.

The most common method of formation of ice layers is thought to be the freezing of percolating water rather than a surface phenomenon. In particular, in buried layers of fine-textured snow, water percolating from the surface is retained by the small pores. If refreezing occurs, these layers of fine texture become layers of semipermeable or impermeable ice. Therefore, it is expected that layers of wind crust and other stratigraphic discontinuities are preferred locations for the formation of ice layers.

A substantial amount of work has been done on multilayered and two-dimensional flow in both soil physics and petroleum engineering and this previous work is cautiously applied to snow. Snow grains grow rapidly when saturated with water (Wakahama 1968), hence, in a water-saturated layer above an impermeable ice band, the grain size and permeability increase significantly, causing an increased tendency for lateral flow.

Water flow to drains. In this section an impermeable boundary with drains at regular intervals is examined. For nonsteady flow over a horizontal boundary, eq 1 reduces to

$$-\alpha k \frac{\partial}{\partial x}\left(h \frac{\partial h}{\partial x}\right) + \phi \frac{\partial h}{\partial t} = I. \tag{10}$$

Langham's (unpublished) use of the parabolic solution has been discussed. When no accretion occurs, Boussinesq (as quoted by Polubarinova-Kochina 1962, p. 515) solved eq 10 by separation of variables, finding an elliptical profile of height which is time-dependent. For the elliptical profile the flow of water through the saturated layer increases linearly with distance as required by continuity and all of the boundary conditions are physically consistent. Nevertheless, Boussinesq's elliptic profile is only an approximate solution in the case of finite accretion. The elliptic profile

$$(h/h_0)^2 + (x/L)^2 = 1 \tag{11}$$

where the maximum value of thickness h_0 varies with time and L is fixed by the geometry of the ice layer is shown in Figure 2. The flux of water along the boundary is

$$q = \alpha k\,(h_0/L)^2\, x\,. \tag{12}$$

The maximum thickness of the water-saturated layer for quasi-steady conditions is

$$h_0 \approx \sqrt{\frac{I}{\alpha k}}\, L\,. \tag{13}$$

The volume of water impounded per unit width of ice layer is

$$V = 0.5\,\pi h_0 L \tag{14}$$

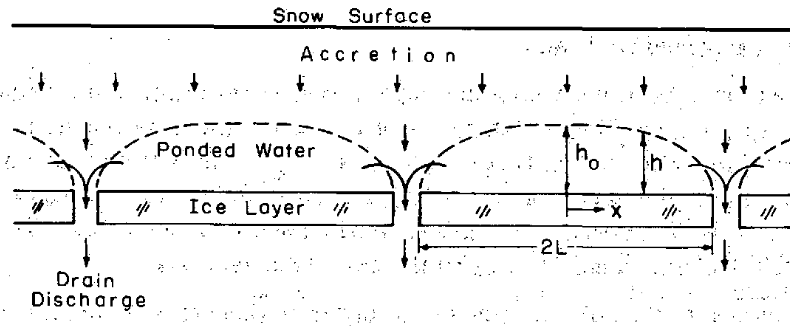

Figure 2. Two-dimensional view of water flow to a drain. Only the accretion from above and the discharge through the drains are shown.

or, by eq 8,

$$V = 0.5\pi\phi\sqrt{\frac{I}{ak}}\,L^2. \tag{15}$$

The maximum volume of water S delayed by this ponding action is the difference between the maximum and minimum values of V or, for any 24-hour cycle,

$$S = \frac{0.5\pi\phi L^2}{\sqrt{ak}}\,(I_{max}^{1/2} - I_{min}^{1/2}) \tag{16}$$

where I_{max} and I_{min} are the maximum and minimum values of accretion respectively. Therefore the volume of water delayed by an ice layer decreases with depth because the difference between I_{max} and I_{min} steadily decreases with depth. Also, where several ice layers occur, the effect of each subsequent layer is less important than the effect of the preceding layer because the waveform is diffused by each layer.

Consider snow with a density of 350 kg m^{-3} and a grain size of 1 mm as an example. The permeability is calculated from eq 4 whence (d, k and I are given in units of millimeters and seconds):

$$S = 0.186\,L^2\,(I_{max}^{1/2} - I_{min}^{1/2}). \tag{17}$$

Using the values of I (0.00142 and 0.00014 mm s^{-1}) measured in homogeneous snow at a depth of 2.05 m on a day of fair-weather melting (Colbeck and Davidson in press),

$$S = 0.0048\,L^2 \tag{18}$$

where the total volume of water flowing through the area of length $2L$ and unit width was $0.104\,L\ m^3\ d^{-1}$. The dye-tracing evidence discussed above suggests that spacings of less than 0.1 m occur in a "ripe" snowpack. With this spacing about 0.2% of the total volume of flow is delayed by the ice layer, an insignificant amount. At a spacing of 1 m, about 2% of the water is delayed, and at 10 m, 25% of the water is delayed. At the 1-m spacings, drainage of the stored water would take about 4800 seconds (1.33 hours). The delay in runoff for the wider spacings is significant in terms of the transit time of water flowing through a 2- or 3-m snowpack. In addition to causing the daily waves to diffuse, ice layers with wider spacings would average the flow between successive days and could even store a significant amount of meltwater early in the season for release later. However, these effects are limited to the start of the melting season since the presence of a saturated layer results in the disintegration of the impermeable boundary, thus forming new flow paths.

In view of the deterioration of ice layers in the presence of a saturated layer of water, a more general solution of eq 10 is not necessary. The predictions of this model become less accurate as the number of drains through the ice layers increases because of the interactions between the drains. At the close spacings commonly observed, it is probably better to attribute a finite permeability to the ice layers and apply Darcy's law for flow through saturated media. In the next section ice layers are treated as water-saturated layers, and in the following section a combination of the flow through and around these layers is considered.

Saturated flow through semipermeable layers. For saturated flow it is assumed that the semipermeable layer is saturated all of the time and that flow only occurs when a finite thickness of ponded water exists above it. Assuming the surrounding snow has pore sizes up to 2 mm, the size of the largest pore R in the semipermeable layer is given by

$$\frac{15.4}{R} \leq 7.7 + h_i , \tag{19}$$

where h_i, the thickness of the semipermeable layer, is assumed to be greater than or equal to the thickness of the layer of ponded water h. For a layer 20 mm thick,

$$R \leq 0.56 \text{ mm}.$$

If larger pores occur at some interval the flow would be two-dimensional.

Under saturated conditions of flow when $h > 0$ the appropriate form of Darcy's law is (Amyx and others 1960, p. 74)

$$w_i = ak_i \left(\frac{h}{h_i} + 1 \right) \tag{20}$$

where w_i is the volume flux through the semipermeable layer. When no ponding occurs, no flow occurs. For a rate of accretion I above the layer, continuity requires that

$$I - w_i = \phi_s \frac{\partial h}{\partial t} . \tag{21}$$

where ϕ_s is the porosity of the snow overlying the semipermeable layer.

For one-dimensional flow the thickness of the layer of ponded water is described by

$$I = ak_i \left(\frac{h}{h_i} + 1 \right) + \phi_s \frac{\partial h}{\partial t} \tag{22}$$

where, for the overlying layer of snow, I can be computed from the theory of gravity drainage. Equation 22 can then be solved for any particular situation when I is specified as a function of time. To illustrate the general principles involved, the steady-state solution for various combinations of the parameters is shown in Figure 3. For any given ice layer, k_i and h_i are specified, and for any

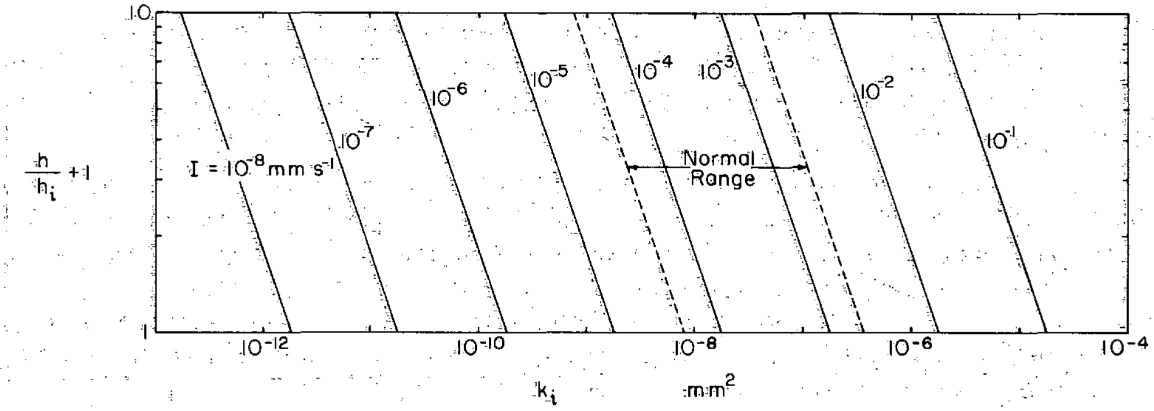

Figure 3. Isolines of steady-state flow through a semipermeable layer. If h is less than the required value then dh/dt > 0; if h is greater than the value dh/dt < 0.

given rate of accretion I is specified. If the thickness of the layer of impounded water is less than the value required for steady flow h must be increasing and if the thickness is greater than the value required for steady flow h must be decreasing.

Values of permeability for homogeneous snow generally exceed 10^{-4} mm^2 and values of water flow are generally less than 2×10^{-3} mm s^{-1}. From Figure 3 it is obvious that ponding would not occur in snowpacks undergoing drainage of meltwater waves unless the semipermeable layers have values of permeability of less than 5×10^{-7} mm^2, or about 0.5% of the minimum permeability of snow. During periods of heavy melting, values of water flow measured in glacial firn (Sharp 1951) and homogeneous snow (Colbeck and Davidson in press) are generally between 0.7×10^{-4} mm s^{-1} and 2.10^{-3} mm s^{-1}. From this normal range of values, shown in Figure 3, it is apparent that when $k_i \leq 6 \times 10^{-9}$ mm^2, a layer of impounded water will exist throughout normal periods of melting. While this value of permeability is less than 10^4 times that of snow, it is not as low as values frequently encountered in flow of petroleum through sandstone reservoirs (Muskat 1946, p. 103). Kuroiwa (pers. comm.) measured values of permeability of about 10^{-6} mm^2 in ice on the surface of a temperate glacier. If this value of permeability is characteristic of ice in equilibrium with water, no ponding could occur except at unusually high flow rates where $[(h/h_i) + 1]$ is large.

Early in the melt season values of k_i are low and two-dimensional flow occurs. As the ice layers deteriorate and k_i increases, the mode of flow should change to more seepage through the ice layers and less flow along them. The permeability of 10^{-8} mm^2 is the division between the one-dimensional and two-dimensional flow regimes for periods of heavy runoff.

Two-dimensional flow. Semipermeable layers are known to be discontinuous but no definitive studies of their areal extent have been made. Their characteristic length probably varies according to the conditions of their formation and decreases as the melt season progresses. The thickness, length and permeability of these layers must be known before a detailed study of the dominant mode of flow past them can be made. Nevertheless, a general treatment of the problem is given and some limiting conclusions are drawn.

Darcy's law describes the horizontal flow of the impounded water above the ice layer,

$$u = -ak_s \frac{\partial h}{\partial x} \tag{23}$$

and the flow through the ice layer,

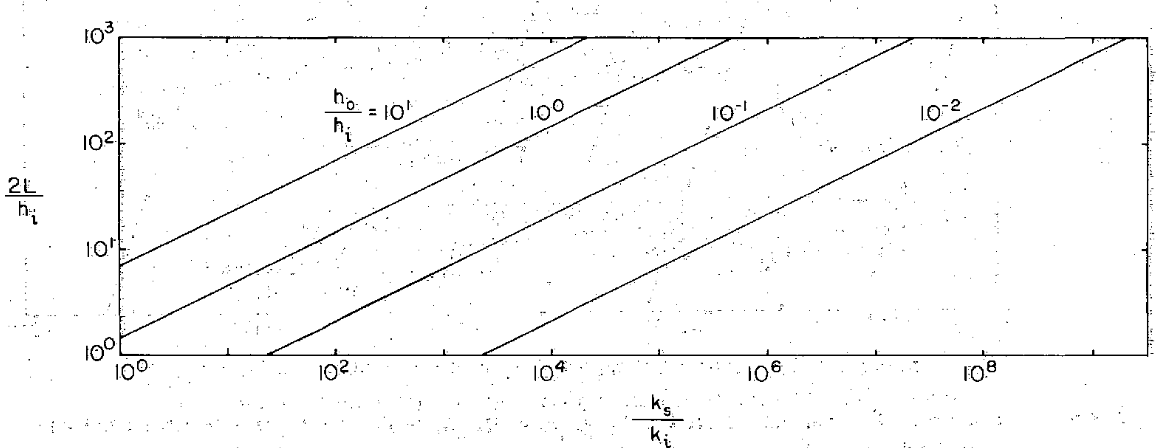

Figure 4. Isolines of flow that is equally divided between flow around an ice layer and seepage through the ice layer.

$$w_i = ak_i\left(\frac{h}{h_i} + 1\right). \tag{20}$$

Assuming that the elliptic profile is a good approximation, the total discharge (q_L) through each drain is given by

$$q_L = 2ak_s h_0^2 L^{-1}. \tag{24}$$

The total flow through the ice layer between the drains is

$$W = 2Lak_i\left(\frac{\pi h_0}{4h_i} + 1\right) \tag{25}$$

and the ratio of the two modes of flow is

$$\frac{W}{q_L} = \frac{1}{16}\left(\frac{k_i}{k_s}\right)\left(\frac{2L}{h_i}\right)^2\left(\pi + \frac{4h_i}{h_0}\right)\left(\frac{h_i}{h_0}\right). \tag{26}$$

The ratio of the vertical to the horizontal flow varies as the ratio of the permeabilities in the ice (k_i) and overlying snow (k_s), as the square of the ratio of the length of the ice layer ($2L$) to its thickness (h_i), and as a function of the ratio of the thickness of the ice layer (h_i) to the maximum thickness of the layer of saturated water (h_0). Only h_i/h_0 varies rapidly with time; hence for any given geometry and permeability only h_0 must be specified in order to characterize the dominant mode of the flow.

Figure 4 shows, for any given k_s/k_i and $2L/h_i$, the values of h_i/h_0 for which the flow of water is divided evenly between horizontal flow to drains and direct seepage through the ice layer. If the thickness of the ponded water h_0 is greater than that required for equilibrium, then the dominant mode of flow is two-dimensional in character, i.e. both horizontal and vertical flow increase with h_0 but the horizontal component increases more rapidly. If either mode of flow is dominant, the flow can be treated by the appropriate method discussed in the previous sections.

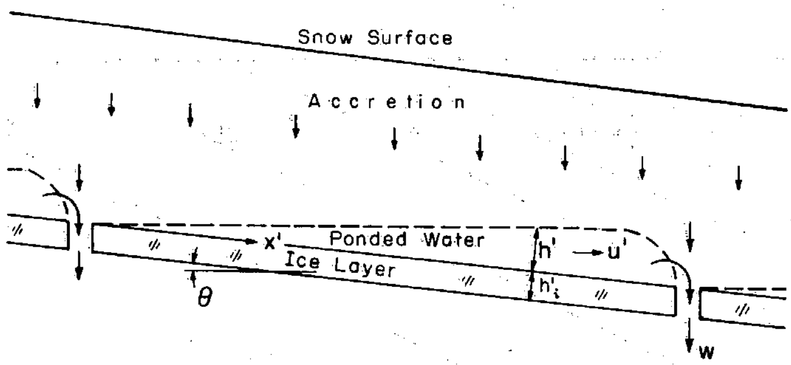

Figure 5. Two-dimensional view of water flow past a sloping ice layer. Only the accretion from above and the discharge through the drains are shown.

Some limiting conditions on the mode of flow through snowpacks can be determined by noting that large values of h_0/h_i have not been observed in "ripe" snowpacks. Thus if lateral flow through a saturated layer is significant, k_i/k_s must be small and/or $2L/h_i$ must be small. When the spacings between the drains are large, the ice layer must be essentially impermeable or lateral flow negligible. When k_s/k_i is not sufficiently large, the flow occurs by direct seepage through the ice layer. For most cases, it is likely that

$$10^{-1} \leq h_0/h_i \leq 10^1. \tag{27}$$

Then for a large ice layer where

$$2L/h_i = 10^3 \tag{28}$$

one-dimensional flow will dominate if k_s/k_i is less than 10^5 and two-dimensional flow will dominate if k_s/k_i is greater than 10^7. For normal values of k_s in water-saturated snow, it is apparent that two-dimensional flow over large ice layers will occur only if k_i is less than 5×10^{-11} mm^2. This value is significantly less than that measured by Kuroiwa (pers. comm.) on ice and is about equal to the permeability of a fine-grained shale. For ice layers of intermediate sizes where

$$2L/h_i = 50 \tag{29}$$

one-dimensional flow will dominate unless (k_s/k_i) is greater than 10^4. Assuming normal values of k_s for water-saturated snow, k_i must be less than 5×10^{-8} mm^2. This value is also less than Kuroiwa's value for the permeability of temperate-glacier ice. Although this discussion is tentative since the permeability of an ice layer has never been measured, it does suggest that water can flow directly through ice layers during periods of heavy melting and that, for the purpose of constructing a model of runoff from snow, it may be sufficient to construct a model of one-dimensional flow.

Sloping-ice layers. The flow of water over a sloping layer is depicted in Figure 5. Flow directly through the layer and along the surface in two directions is possible although flow downslope will occur more readily. The equation describing the thickness of the water layer can be easily formulated in terms of the sloping coordinates, however a solution would be difficult to obtain even for the simplest case. Therefore, the flow will be characterized in terms of measurable parameters.

The Darcian flow per unit area along the layer is

$$u = -\alpha k_s \left(\cos\theta \, \frac{\partial h}{\partial x} - \sin\theta\right) \tag{30}$$

and the vertical flow through the layer (for $h \neq 0$) is

$$w_i = \alpha k_i \left(\frac{h}{h_i} + 1 \right) \tag{31}$$

When $\cos \theta \, (\partial h / \partial x) - \sin \theta$ assumes positive and negative values at the uphill and downhill ends of the layer respectively, flow occurs to both uphill and downhill drains. When θ is large, k_i is large or the accretion is small, then only downslope and direct infiltration are likely. Inasmuch as this mode of flow should dominate for any significant slope, this case is considered here. For significantly large values of θ it is assumed that

$$\cos \theta \frac{\partial h}{\partial x} << \sin \theta .$$

With this approximation, the ratio of the total flow through the layer to the flow along the layer at each point is

$$\frac{W}{q_L} = \frac{k_i}{k_s} \frac{2L}{h_i} \left(\frac{h_i}{h} + 1 \right) \cot \theta . \tag{32}$$

In flow over sloping layers, the ratios of the length to the thickness, and of the thickness of the ice layer to that of the water layer are less significant than for a flat layer. For sloping layers, the relative importance of flow to the drains is not as dependent upon the thickness of the water layer. The functional dependence on θ is most important at small angles since $\cot \theta$ decreases very rapidly up to angles of 10° to 20°. At a slope of 10° and intermediate values of the other parameters, the flow along the ice layer is an order of magnitude greater than for a flat layer. When the slope increases from 10° to 45°, the flow along the layer increases by only a factor of 5.7. Therefore, a small shift from the horizontal causes the largest increase in the flow along the layer.

Discussion

The examination of the effects of stratigraphic features on the flow of water has provided a better understanding of the movement of water through snowpacks. The existence of these features has been documented by many workers but measurements of their permeability, thickness and areal extent are lacking. Large variations in the properties of these layers are known to occur in time and space; hence a general treatment of water flow is necessary. The effects of stratigraphic horizons on the flow of water are not completely identified but rather are defined in terms of the probable values of the pertinent parameters.

The flow of water through unsaturated layers is described in terms of the theory of gravity drainage through porous media. The propagation of values of volume flux (or effective-water saturation) through a snowpack can be predicted for an unsaturated snowpack of any layered composition. The propagation of a known waveform throughout the snowpack can be predicted by constructing as many of the characteristics as necessary (by assuming that no shock front occurs) and then using the conservation of mass to locate the position of the shock front. This technique assigns effective parameters to the snowpack as a whole and, when unsaturated flow occurs, gives accurate predictions of the movement of water using the simple concepts of one-dimensional flow.

When a layer of reduced permeability exists and the accumulation of water above that layer produces a "perched water table," lateral flow along the layer is likely since large pores or open drains are usually present. In this case, flow at the local scale must be considered two-dimensional and the usefulness of the one-dimensional theory is reduced. This situation does not preclude the use of the one-dimensional theory on a scale which is large compared to the areal extent of the ice layer; however, and much work on the problem of predicting water movement in stratified snowpacks remains. On the local scale, the relative importance of water flow through and around ice layers is examined by adopting Boussinesq's elliptical solution for water flow to drains. While this is only an approximate solution, it provides a basis for evaluating the effects of the thickness, length and permeability of an ice layer on the distortion of the flow field. In particular it is shown that the storage of water above an impermeable layer is significant when distinct drains occur at spacings of 1.0 m or greater.

In view of the rapid deterioration of ice layers during intense melting, the treatment of these layers as porous media is reasonable. At low permeabilities where pores are small, only saturated flow through the layers occurs. From the analysis given here it is shown that ponding of water above semipermeable layers in snow does not occur unless the permeability of the layers is less than 5×10^{-7} mm^2 or about 0.5% of that for snow. Also, when the permeability of the layer is less than 6×10^{-9} mm^2, continuous storage of water should occur during periods of intense runoff. This value of permeability is quite small compared to that for either snow or ice on the surface of a temperate glacier.

Although direct infiltration and lateral flow both increase with the thickness of the water layer, the relative importance of the lateral mode of flow increases with the thickness of the water layer. As a general rule during periods of heavy runoff, the flow is predominantly vertical when the permeability of the ice layer k_i exceeds 10^{-8} mm. As the ratio of length to thickness increases, the critical value of k_i decreases. For example, when the layer is 10 m long but only 10 mm thick, the critical value of k_i is about 5×10^{-11} mm. The tendency for lateral flow to occur increases rapidly if the layer is tilted and the significance of the other parameters for a tilted layer is greatly reduced. For typical values of these parameters, the ratio of flow along the layer to flow through the layer increases by an order of magnitude when the angle of slope increases from 0° to 10°. The effect of increasing the slope further is less pronounced.

The theoretical work presented here provides a basis for predicting the unsaturated flow of water through layered snow and for understanding the effect of semipermeable layers on the character of the flow. The problems of predicting the time dependent movement of meltwater waves around ice layers and through vertical drainage channels have not been discussed. While the theoretical analysis of these problems is possible, solution of the resulting equations would be difficult. At best the solutions would be academic exercises until more descriptive experimental work has been completed. Much further knowledge of the permeability, thickness and areal extent of these layers is needed. Further, a description of the metamorphism of these features must be made before a satisfactory understanding of their effects can be achieved.

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13. ABSTRACT

The flow of water through layered snowpacks is discussed. A method for predicting flow through unsaturated layers is given. The flow along ice layers and through ice layers is analyzed in terms of the slope, permeability, thickness and length of the layers. It is shown that the permeability of ice layers required to cause large flow diversions is quite small. The effect of slope is large even at small angles.

14. Key Words

- Drainage
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- Hydrology
- Ice
- Percolation
- Snow
- Snow cover
- Snowmelt
- Surface water runoff

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