Research Report 216

SLIDING OF NON-TEMPERATE GLACIERS

by

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PREFACE

This report was prepared by Dr. J. Weertman* under personal service contract with U. S. Army Cold Regions Research and Engineering Laboratory. The work was done for the Research Division, J. A. Bender, Chief.

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SUMMARY

It is shown that if the bottom surface of a non-temperate glacier is at the melting point the glacier normally will slide despite the fact that obstacles in the bed may protrude into cold ice.

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Introduction

Field observations have shown that temperate glaciers slide over their beds and cold glaciers do not.* Recently Lliboutry (1966) has made the interesting suggestion that a glacier or ice sheet whose bottom surface is at the melting point but whose temperature elsewhere is below the melting point would not be able to slide. He points out that in this situation irregularities and protuberances of the bed project into the cold ice above the bed and concludes, not unreasonably, that the regelation mechanism of sliding does not operate.

Important consequences would follow should Lliboutry's conclusion be correct in general. Any glacier so situated that the mean annual temperature of its upper surface is well below the melting point is either a cold glacier or a glacier which is below the melting point everywhere except in the region of the bed. Hence it would appear that a measurement of the temperature of the upper surface of a glacier at a depth below which the seasonal temperature fluctuations are attenuated (that is, at a depth of about 9 m) would give information sufficient to determine whether or not the glacier is sliding.†

It appears worthwhile, therefore, to investigate in greater detail the effect pointed out by Lliboutry. It will be shown in this paper that the temperature gradient normal to the bed is an important parameter in determining whether sliding can or cannot occur in a glacier whose bottom surface is at the melting point. Only if a rather large temperature gradient exists will sliding be prevented. Since the temperature gradient of a glacier whose bottom surface is at the melting point is expected to be small, it is concluded that sliding usually will occur in such a glacier.

Theory

Consider a glacier bed whose protuberances and irregularities project into cold ice. A cross section of such a bed is shown in Figure 1. It will be assumed that the glacier bed is rough and irregular both in the direction of ice flow and in the cross direction. (This is the assumption made in my theory of sliding (Weertman, 1957; 1964). In Lliboutry's theory (1959; 1965) the bed is considered to be rough in the direction of sliding but not in the other direction.)

Assume that at the melting point a perfectly flat rock-ice interface cannot support a shear stress across it because of the formation of a water film at the interface. Support for tangential stresses at the bed of a temperate glacier arises from the blocking action of the protuberances. Sliding of a temperate glacier occurs by movement of ice and water around the obstacles. The two main mechanisms (Weertman, 1957; 1964) leading to this mass flow are (1) an enhanced creep flow (arising from stress concentrations) of ice around the irregularities and (2) the classical regelation mechanism.

In the case of a non-temperate glacier, as pictured in Figure 1, the enhanced creep flow mechanism does not become inoperative simply because the obstacles

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* A cold glacier is a glacier whose temperature everywhere is below the melting point. Every portion of a temperate glacier is at the melting point. The term "melting point" is understood to mean the melting or freezing temperature of ice at existing hydrostatic pressure.

† This statement would have to be modified if a layer of temperate ice of finite thickness were to form at the bottom of a glacier by the mechanism discussed by Lliboutry (1966).

penetrate cold ice. Since part of the rock-ice boundary is at the melting point and thus has difficulty in supporting tangential stresses, stress concentrations will occur in the vicinity of obstacles. Therefore an enhanced creep flow can occur around the protuberances. However, if this is the only sliding mechanism which operates, the speed of sliding will be negligible over a bed that has a full spectrum of obstacle sizes. The sliding velocity arising from enhanced creep flow diminishes as the obstacle size decreases.

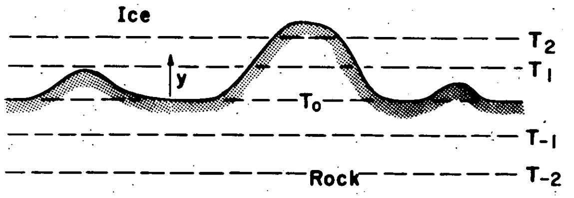

Figure 1. Glacier bed, no shear stress, ($T_2 < T_1 < T_0 < T_{-1} < T_{-2}$).

The regelation mechanism of sliding is simply the melting of ice on the upstream (higher pressure-lower melting point) side of an obstacle, the flowing of the resultant melt water around the obstacle, and the refreezing of this water on the downstream (lower pressure-higher melting point) side of the obstacle. Since heat conduction through and around obstacles controls this mechanism the sliding velocity arising from it is larger the smaller are the obstacles. Obviously this mechanism will not operate if the ice is frozen to rock.

Suppose for the moment that no shear stress is exerted across the bed of the glacier shown in Figure 1 and suppose that the thermal diffusion coefficients of ice and rock are equal (a good approximation). Suppose further that no ice is being melted from the bottom surface. In this situation the isotherms will be parallel lines, as shown in Figure 1. The temperature T_0 in this figure is taken to be the melting point of ice under a hydrostatic pressure equal to the overburden pressure. The temperature gradient $T' = dT/dy$, where T is temperature and y is the direction normal to the bed, will be given by the equation:

$$T' = H/D \tag{1}$$

where H is the geothermal heat escaping from the earth in unit time through a unit area and D is the thermal conductivity of ice and rock (which, for simplicity, are assumed to equal each other).

Figure 2 shows qualitatively what can happen to the isotherms around an obstacle of Figure 1 when a shear stress exerted across the bed of the glacier causes motion of ice towards the right.

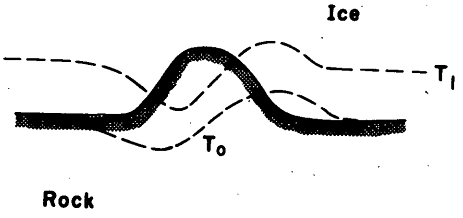

Figure 2. Glacier bed, shear stress across the bed.

The shear stress pushes ice against the upstream side of the obstacle and increases the hydrostatic pressure on the ice there. The converse is true for the ice in contact with the downstream side of the obstacle. The melting temperature of ice is reduced when the pressure is increased. The lowering of the melting point ΔT is given by:

$$\Delta T = C \Delta P \tag{2}$$

where ΔP is the change in pressure and $C = 7.4 \times 10^{-3}$ °C/bar. If in Figure 1 the height of an obstacle is h the temperature difference between the top of the obstacle and the bottom will be:

$$\Delta T = T'h. \tag{3}$$

If this difference is less than $C \Delta P$, the ice in Figure 2 in contact with the upstream side of the obstacle will be at the melting temperature despite the fact that the obstacle protrudes into "cold" ice. Hence ice can melt on the upstream side and the water thus produced can move to the downstream side and bring this side also to the melting point. Hence all the ice in contact with the obstacle will be at the melting point. Therefore the regelation mechanism can operate despite the fact that the glacier is not temperate and obstacles protrude into cold ice.

The pressure increase ΔP is estimated easily if the bed contains obstacles of only one size. For the moment suppose that this is the case. If the average spacing between obstacles is L, there is one obstacle within an area L^2 on the bed. Let the cross sectional area of the obstacle in the direction of flow be A. It might be expected that $A \sim h^2$. If σ is the shear stress acting on the bed, ΔP will be of the order of:

$$\Delta P \approx \sigma \beta L^2/A \approx \sigma \beta L^2/h^2 \;. \qquad (4)$$

where β is a constant approximately equal to 1/3 to 1/6.

By combining eq 2, 3 and 4 we find that any obstacle whose height is less than

$$h \approx \sigma C\beta L^2/T'h^2 \qquad (5)$$

will be surrounded by ice at the melting point.

The temperature gradient T' will always be of the order of magnitude or less than the value which is required to transport the geothermal heat. Thus:

$$h \approx CD\beta L^2/h^2H \;. \qquad (6)$$

Consider what might be typical for the maximum value of h. The constants have the values: $C = 7.4 \times 10^{-3}$ °C/bar; D = 0.005 cal/cm-sec-°C; $H \cong 40$ cal/cm²-year; and $\beta \cong 1/3$ to 1/6. A typical value for the shear stress at the bottom of a glacier is $\sigma \cong 1$ bar. The ratio L/h is a measure of the roughness of the bed of the glacier. If it is assumed that L/h = 15, which is the value required to obtain agreement between theoretical and observed sliding velocities (Weertman, 1964), the value of h given by eq 6 is 10 to 20 m.

This value of h is many orders of magnitude larger than the "controlling obstacle size" to sliding* which is estimated (Weertman, 1964) to be of the order of 3 mm. Since h is so large it is clear that, for a glacier bed containing a full spectrum of obstacle sizes, the regelation mechanism will operate on all obstacles from the very smallest to those equal to the controlling obstacle size. The creep enhancement mechanism will cause flow around the larger obstacles. Thus sliding can occur in the same manner as in a temperate glacier.

Conclusion

It is concluded that if the bottom surface of a glacier is at the melting point the glacier normally will slide despite the fact that obstacles in the bed may protrude into cold ice.

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* The creep rate enhancement mechanism permits ice to flow easily around large obstacles in the bed and the regelation mechanism permits fast movement around the smaller obstacles. Hence the major resistance to slippage in a temperate glacier arises from an obstacle of intermediate size — the "controlling obstacle size." Ice flow around obstacles larger than the controlling obstacle size occurs predominantly by the creep enhancement mechanism and mass movement around the smaller obstacles can be attributed almost entirely to the regelation mechanism.

LITERATURE CITED

Lliboutry, L. (1959) Une théorie du frottement du glacier sur son lit (A theory of the friction of a glacier on its bed), Annales de Géophysique, vol. 15, p. 250-65 (text in French).

____________ (1965) Traité de glaciologie, Tome 2 (Glaciers - Variations du climat-Sols gelés) (Treatise on glaciology, Volume 2. Glaciers - Variations in climate-frozen ground). Paris: Masson et Cie (text in French).

____________ (1966) Bottom temperatures and basal low-velocity layer in an ice sheet, Journal of Geophysical Research, vol. 71, p. 2535-43.

Weertman, J. (1957) On the sliding of glaciers, Journal of Glaciology, vol. 3, p. 33-38.

____________ (1964) The theory of glacier sliding, Journal of Glaciology, vol. 5, p. 287-303.

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13. ABSTRACT

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