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# Operator Variance in the Determination of the Plastic Limit

by G. E. H. Ballard and W. F. Weeks

U.S. ARMY MATERIEL COMMAND
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
Hanover, New Hampshire

# PREFACE

This paper constitutes an interim report accomplished partially under contract DA-11-190-ENG-64, (USA CRREL Project 5010.13101, Physics of Snow, Ice and Frozen Ground) with the Department of Earth Sciences, Washington University, St. Louis, Missouri. The experimental work was carried out in the soils laboratory at Washington University by G. E. H. Ballard, Research Fellow, and Dr. W. F. Weeks, Assistant Professor of Geology. Professor A. B. Cleaves was chairman of the research program on plastic limits. Equipment funds were provided by F. H. Elliott, President of Wabash Drilling, St. Louis, Missouri.

The following investigators participated directly by submitting plastic limit data: Professors Wm. D. Johns and H. M. Reitz of Washington University; W. J. Turnbull, Chief, Soils Division, and S. J. Knight, Chief, Army Mobility Research Center, U. S. Army Engineer Waterways Experiment Station; W. H. Price, Chief Research Engineer, and W. G. Holtz, Chief, Soils Engineering Branch, Bureau of Reclamation, U. S. Department of the Interior; and Professor H. Bolton Seed, Department of Civil Engineering, University of California. Without their cooperation, this study would have been impossible.

This report has been reviewed and approved for publication by Headquarters, U. S. Army Materiel Command.

W L Boyd

for

W. L. NUNGESSER
Colonel, CE
Commanding
USA CRREL

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## SUMMARY

An experiment was designed to investigate operator variance in the determination of the plastic limit of cohesive soils, independent of sample preparation and hydration time. A standard sample was prepared by dry-mixing commercial clay, "Grundite", with a pure silt. After the sample was hydrated for 3 weeks, five random samples were taken and five replicates performed on each sample. Analysis of variance (AOV) shows that there is no reason to doubt that the sample is homogeneous at the 0.05 level of significance. Random samples were then distributed to five zones of operators, where a zone is defined as a group of operators with similar backgrounds and engineering interests. Two operators from each group performed five replicates. The AOV model used in the analysis is mixed with the upper level fixed and the two lower levels random. The estimated "within" and "between" operator variances are 0.45 and 4.18 respectively. The estimated "between zones" contribution to the total sum of squares is negative. No correlation was found between an operator's internal variance and his deviation from the grand mean. On the basis of this experiment, no reason exists to doubt that an "untrained" operator can obtain results comparable to those of professional operators.

The major factor contributing to the total sample variance is the inconsistency between the individual operators who, although able to duplicate their own determinations, do not call the same end point. To minimize between operator variance, a readily prepared standard sample is suggested with which any operator can calibrate his plastic limit determinations against the expected national average.

# OPERATOR VARIANCE IN THE DETERMINATION OF THE PLASTIC LIMIT

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G. E. H. Ballard and W. F. Weeks

## INTRODUCTION

One of the authors (GB) has been engaged in a research program utilizing artificial soils to measure variations in the plastic limit which can be attributed to the clay content of the soil and the mineralogy of the clay. As some of the experimental data appeared to conflict with published data obtained from natural soils, it was desirable to be able to check the experimental results against results obtained by established laboratories. Furthermore, in order to provide a realistic extrapolation from the laboratory results to the field of applied soil mechanics, an accurate estimate of operator variance was necessary. To accomplish this a standard sample was prepared and its plastic limit determined by a varied group of operators.

## SAMPLE PREPARATION

The plastic limit standard sample was produced by dry-mixing "Grundite" and a pure silt. Grundite is commercially packaged for the foundry industry by the Illinois Clay Products Company, Joliet, Illinois; it is powdered Pennsylvanian shale, high in illite content, from Grundy County, Illinois. The silt was obtained by repeated (20) washings of a weathered high silt loess from St. Louis County, Missouri.

A hydrometer analysis of the standard sample was performed. The cumulative gradation curve (Fig. 1) is linear on semi-logarithmic paper over its entire range: the computed coefficient of uniformity is 19, the coefficient of curvature is 0.54. The standard sample contains 40 percent in the size range 0.053 mm to 0.005 mm and 60 percent less than 0.005 mm. Forty five percent is less than 2 microns in size. Soluble salts* comprise 0.66 percent of the sample by weight and are primarily sulphates. X-ray defraction analysis of the less than two micron size shows this fraction to be predominantly illite with less than 10 percent kaolinite. The liquid limit of the standard sample is 49.0. The standard sample would therefore be classified as CL by the Unified Soil Classification System (Waterways Experiment Station, 1953).

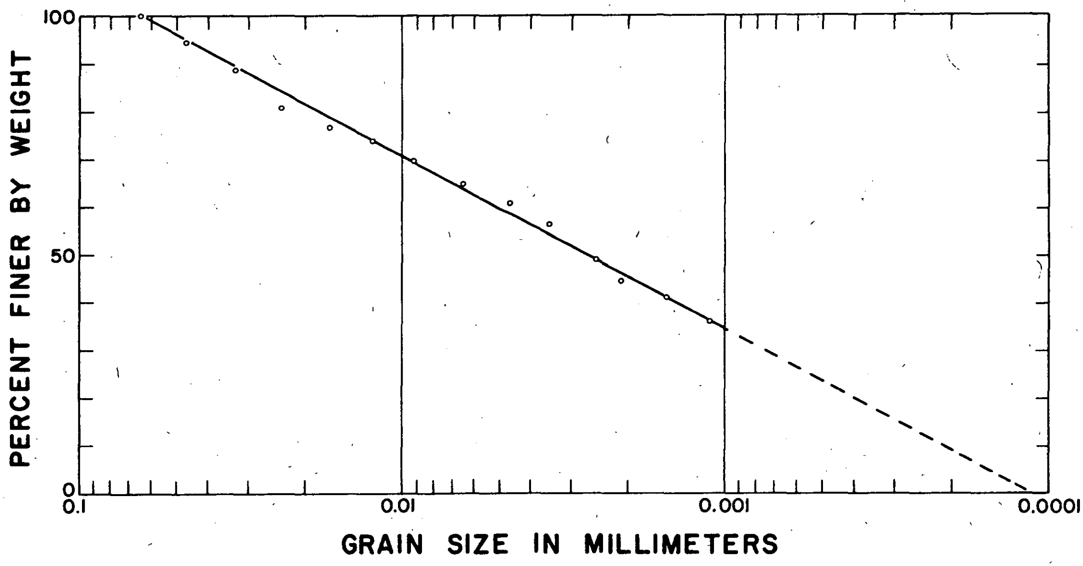

Figure 1. Grain size distribution of the standard sample.

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* Designation D-E-8 of the Earth Manual, First Edition, 1960, U. S. Department of Interior

To avoid some of the complications of pretreatment and hydration time, the standard sample was hydrated well above the plastic limit and wet mixed every day for a week. The sample was then sealed and allowed to stand for 2 weeks.

## SAMPLE HOMOGENEITY

Ten subsamples were split from the standard sample and forwarded to the various operators. Before shipment 5 of the 10 splits were selected at random and 5 limits were determined (GB) for each of the 5 splits. The individual limit determinations are given in Table I.

Table I. Individual plastic limit determinations used in the homogeneity test.

| Sub-samples | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Determinations | | | | | |
| 1 | 21.1 | 21.6 | 21.7 | 21.3 | 21.3 |
| 2 | 22.7 | 22.6 | 22.3 | 21.9 | 22.3 |
| 3 | 22.0 | 21.8 | 21.8 | 22.0 | 20.5 |
| 4 | 21.4 | 21.9 | 22.8 | 22.0 | 21.1 |
| 5 | 21.8 | 21.2 | 22.8 | 21.6 | 21.4 |

Then assuming a two-level nested statistical model

$$y_{ij} = \mu + \alpha_i + \epsilon_{j(i)}$$

where $i = 1 \ldots I$, $j = 1 \ldots J$, $y_{ij}$ is a measured plastic limit, $\mu$ is the unknown population mean, and $\alpha_i$ and $\epsilon_{j(i)}$ are normally distributed random variables with means 0 and variances $\sigma_\alpha^2$ and $\sigma_\epsilon^2$ associated with the groups and samples respectively, an analysis of variance was calculated as shown in Table II (Graybill, 1961, p. 337). The numbers given in brackets are the calculated values. The notation is that commonly used in statistical papers; a dot in subscript indicates summation over the replaced subscript, and a superscript bar and hat indicate a mean and an "estimator of" respectively. The best estimator ($\hat{\sigma}_\alpha^2$) of the between sub-sample variance is 0.0585, while the best estimator ($\hat{\sigma}_\epsilon^2$) of the within sub-sample variance is 0.286. Assuming as a null hypothesis that $\sigma_\alpha^2 = 0$, we obtain 2.02 as the F ratio. The tabulated value of F(4, 20, 0.05) is 2.866. Therefore, the null hypothesis cannot be rejected at the 5% level of significance and it may be assumed that the sample is homogeneous. The 0.95 confidence intervals on the variances are $0.137 < \sigma_\epsilon^2 < 0.498$ and $0 < \sigma_\alpha^2 < 0.899$. The best estimator of the sample plastic limit is $\bar{y}_{..} = 21.80$. The 0.95 confidence limits on this mean are $21.37 < \mu < 22.22$.

Table II. Mathematical model for homogeneity test.

| Source of variation | Degrees of freedom | Mean square | Average mean square |
|---|---|---|---|
| Between sub-samples | I-1<br>[4] | $S_\alpha^2 = \dfrac{J\sum_i(\bar{y}_{i.}-\bar{y}_{..})^2}{I-1}$<br>[0.578] | $= \hat{\sigma}_\epsilon^2 + J\hat{\sigma}_\alpha^2$ |
| Within | I(J-1)<br>[20] | $S_\epsilon^2 = \dfrac{\sum_i\sum_j(y_{ij}-\bar{y}_{i.})^2}{I(J-1)}$<br>[0.286] | $= \hat{\sigma}_\epsilon^2$ |

## DISTRIBUTION OF SAMPLES

Five zones of influence — considered to be representative of the major professional categories operating in the United States — were selected (a zone is defined as a group of operators with similar backgrounds and engineering interests). The cooperating organizations were the Department of Civil Engineering of the University of California, the U. S. Army Corps of Engineers, the Bureau of Reclamation, H. M. Rietz and Associates, and the Earth Science Department of Washington University. Since the operators were advised ahead of time that their names would not appear with particular results, the first four zones are not identified directly.

The last zone (#5) has been designated "untrained" and consisted of Dr. Wm. D. Johns and the second author (WW). These operators have had little or no experience with soils engineering tests. They were provided with subsamples and copies of the ASTM and Earth Manual designations for determining the plastic limit. No further instructions on performing the determination were offered.

Each operator was requested to determine only five plastic limits and to submit all five. As a result no selectivity on the part of the operator is assumed. The cooperating zones were independent and were not advised of the other participating zones. The zones of influence therefore range from the "untrained" through the practicing engineering concern and the university to the major sub-divisions of governmental interest.

## ANALYSIS

The experiment consists of 5 defined zones of interest. Two operators were selected from each zone, and 5 replicates were performed by each operator. Assuming a mixed model with fixed and random effects (Graybill, 1961, p. 396) we may write

$$y_{ijk} = \mu + a_i + \beta_{j(i)} + \epsilon_{k(ij)}$$

$$i = 1 \ldots I, \; j = 1 \ldots J, \; k = 1 \ldots K$$

where $y_{ijk}$ is any plastic limit determination, $\mu$ is the unknown population mean, and $a_i$ is a constant associated with the $i^{th}$ zone of influence ($\Sigma a_i = 0$). The $\beta_{j(i)}$ and $\epsilon_{k(ij)}$ are assumed to be normally distributed with means of 0 and variances of $\sigma_\beta^2$ and $\sigma_\epsilon^2$ and are associated with "operators" and "determinations within operators" respectively. The AOV table for this model is presented in Table III. The numerical results of the AOV calculations are given in brackets in this table. The results of the plastic limit determinations are shown in Table IV.

Table III. Mathematical model for analysis of operator variance.

| Source of variation | Degrees of freedom | Mean square | Average mean square |
|---|---|---|---|
| Between zones | I - 1<br>[4] | $S_a^2 = \dfrac{JK \sum_i (\bar{y}_{i..} - \bar{y}_{...})^2}{I - 1}$<br>[12.337] | $= \hat{\sigma}_\epsilon^2 + K\hat{\sigma}_\beta^2 + \dfrac{JK}{I-1}\sum(a_i - \bar{a}_.)^2$ |
| Between operations within zones | I (J - 1)<br>[5] | $S_\beta^2 = \dfrac{K \sum_i \sum_j (\bar{y}_{ij.} - \bar{y}_{i..})^2}{I (J - 1)}$<br>[21.349] | $= \hat{\sigma}_\epsilon^2 + K\hat{\sigma}_\beta^2$ |
| Determinations within operators | IJ (K - 1)<br>[40] | $S_\epsilon^2 = \dfrac{\sum_i \sum_j \sum_k (y_{ijk} - \bar{y}_{ij.})^2}{IJ (K - 1)}$<br>[0.448] | $= \hat{\sigma}_\epsilon^2$ |

Table IV: Individual plastic limit determinations used in the analysis of operator variance.

| Zones | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Operator 1 | 21.7 | 21.6 | 23.3 | 17.3 | 22.0 |
| | 22.3 | 21.2 | 22.6 | 17.3 | 19.4 |
| | 22.2 | 19.8 | 23.6 | 16.9 | 19.4 |
| | 22.0 | 21.5 | 22.9 | 17.0 | 20.0 |
| | 22.4 | 21.4 | 23.3 | 17.1 | 20.3 |
| Operator 2 | 20.9 | 20.6 | 23.5 | 24.2 | 23.2 |
| | 21.3 | 22.4 | 22.5 | 23.3 | 22.0 |
| | 21.3 | 22.0 | 23.9 | 23.8 | 21.9 |
| | 21.0 | 21.3 | 22.9 | 22.2 | 23.0 |
| | 21.1 | 23.0 | 23.9 | 22.2 | 22.0 |

Table V: Statistical parameters associated with individual operators.

| ZONE | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Sample number | 592-3 | 592-1 | 592-2 | 592-9 | 592-10 |
| Mean | 22.1 | 21.1 | 23.1 | 17.1 | 20.2 |
| Standard deviation | 0.26 | 0.71 | 0.38 | 0.17 | 1.07 |
| Deviation from $\bar{y}\ldots$ | 0.5 | 0.5 | 1.5 | 4.5 | 1.4 |
| Sample number | 592-5 | 592-7 | 592-6 | 592-4 | 592-8 |
| Mean | 21.1 | 21.9 | 23.3 | 23.1 | 22.4 |
| Standard deviation | 0.17 | 0.90 | 0.58 | 0.86 | 0.59 |
| Deviation from $\bar{y}\ldots$ | 0.5 | 0.1 | 1.7 | 1.5 | 0.8 |

The best estimator of the "within operator" variance $(\sigma_\epsilon^2)$ is 0.448, while the best estimator of the "between operators within zones" variance $(\sigma_\beta^2)$ is 4.180. To test the hypothesis that $\sigma_\beta^2 = 0$, we can calculate the F = 47.654 which, when compared with the tabulated value F(5, 40, 0.05), clearly indicates that there is appreciably more variation between the operators than between replicate measurements by a given operator. The 0.95 confidence limits on these variances are $0.274 < \sigma_\epsilon^2 < 0.665$ and $1.445 < \sigma_\beta^2 < 25.58$.

The mean square for the "between zones" is less than the mean square "between operators within zones". This could not happen even if $\Sigma(a_i - \bar{a}.)^2$ were equal to 0. There is obviously no evidence against the hypothesis that $\Sigma(a_i - \bar{a}.)^2 = 0$, and we may conclude that there is no reason to doubt that the zone means are equal.

The grand sample mean is $\bar{y}\ldots = \hat{\mu} = 21.56$. The individual operator means, standard deviations, and deviations from the grand mean $(\bar{y}_{ij.} - \bar{y}\ldots)$ are given in Table V.

No significant correlation exists between operator standard deviation and $(\bar{y}_{ij.} - \bar{y}\ldots)$ indicating that justification does not exist for eliminating the untrained zone 5 (Fig. 2). It would follow from this experiment that little or no training is required for a well educated person, using only the ASTM and Earth Manual designations as a guide, to obtain results comparable to those obtained by professional operators.

It will be noticed that the results of operator 1 of zone 4 are considerably lower than the values obtained by the other operators. This difference so impressed some readers of an early draft of this paper that they suggested that these results should be discarded because they "evidently do not represent results obtained by a properly trained operator." Inasmuch as the only criterion of "proper" training here is agreement with other operators, such judgment is possible only in hindsight. Because arbitrary discarding of data without independent justification is extremely dangerous, we have included these results in our analysis. It is, however, of interest to point out that even if these results are discarded, an analysis similar to that used in Table II will show that the null hypothesis that the "between operator" variance $\sigma_{\alpha}^2 = 0$ can be rejected at the 5 percent level of significance [$F = 11.81$ as compared to $F(8, 36, 0.05) \sim 2.21$]. In this case our estimates of the "between operator" variance ($\sigma_{\alpha}^2$) and the "within operator" variance ($\sigma_{\epsilon}^2$) would be 1.07 and 0.49 respectively and our conclusions remain unchanged.

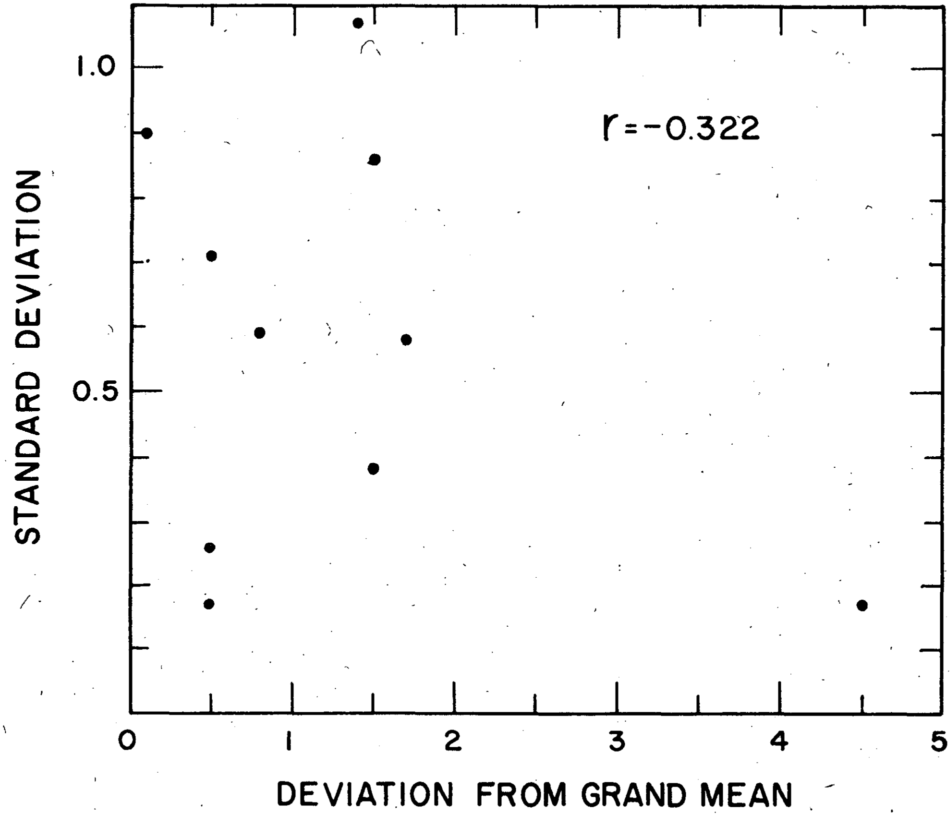

Figure 2. Operator standard deviation vs operator deviation from the grand mean ($r \doteq$ sample correlation coefficient.)

## CONCLUSIONS AND SUGGESTED STANDARDIZATION

The analysis of variance just presented indicates that the individual reproducibility of both experienced and untrained operators in determining a plastic limit appears to be adequate for most engineering and research purposes. Because there is no reason to doubt that the zone means are equal, there is no evidence that operators from a specific laboratory will have a common interpretation of the boundary between the plastic and solid state which is significantly different from the interpretation of another laboratory.

The major factor contributing to the total sample variance is the individual operator. The determination of a plastic limit, therefore, appears to be a matter of individual interpretation of the plastic-solid state boundary. Each operator decides when this boundary is reached. Once this decision is made, the operator is able to reproduce closely what he considers to be the limit. Unfortunately, there does not appear to be sufficient agreement between operators regarding the precise definition of the limit; the range of operator means is from 17.1 to 23.1 (a deviation of 35% from the lowest operator mean). Therefore, it can be concluded that at present the determination of the plastic limit is not adequately standardized.

Determination of the plastic limit may be compared with the determination of an indefinite end-point in a chemical titration. Fortunately for the chemist, his problem can be resolved in two ways:

1. The calculated amount of titrating solution necessary to achieve the end-point in a standard solution can be introduced and the state of the resulting solution observed. The analyst then attempts to reproduce this end-point in subsequent titrations.

2. The analyst selects some state of the "end-point" that he believes he can definitely reproduce and calculates a correction factor based on the known composition of the standard solution that he is titrating.

Unfortunately, neither of these techniques is available to the student of soil mechanics. At present no absolute method of determining the "true" plastic limit of a specific sample exists. However, considering the large operator variance, it does appear advisable to attempt to establish a "standard" sample, even if the "standard" has no absolute significance. The standard sample, if possible, should meet the following requirements:

1. It should be an artificial soil, the components of which should be readily available anwhere in the country.

2. The percentage of each component should be defined in a range of minimum plastic-limit variation with respect to mixing errors.

3. The "defining" plastic-limit determinations should be performed by an operator who has a low variance and whose determinations can be considered as a "reasonable" estimate of the mean of other investigators.

A standard sample has been prepared by mixing 2 parts of Grundite to 1 part of washed silt (passing US sieve #230, retained on US sieve #270, opening sizes are 62 and 53μ respectively). Figure 3 shows the relationship of the plastic limit to the clay content of an artificial soil in which the clay fraction is a Grundite-derived illite (Ballard, 1962). Since Grundite contains approximately 25% silt, the 2 to 1 mix of Grundite and silt places the standard sample in the vicinity of 48% clay in Figure 3, the point where mix errors have a minimum influence.

Ten plastic limit determinations were performed (GB) on this standard sample. The individual measurements are presented in Table VI and give a mean of 20.58 and a variance of 0.284.

Table VI: Plastic limit determinations on the standard sample.

| | | | | |
|---|---|---|---|---|
| 21.2 | 21.1 | 20.9 | 20.7 | 19.7 |
| 19.9 | 20.4 | 20.7 | 21.1 | 20.1 |

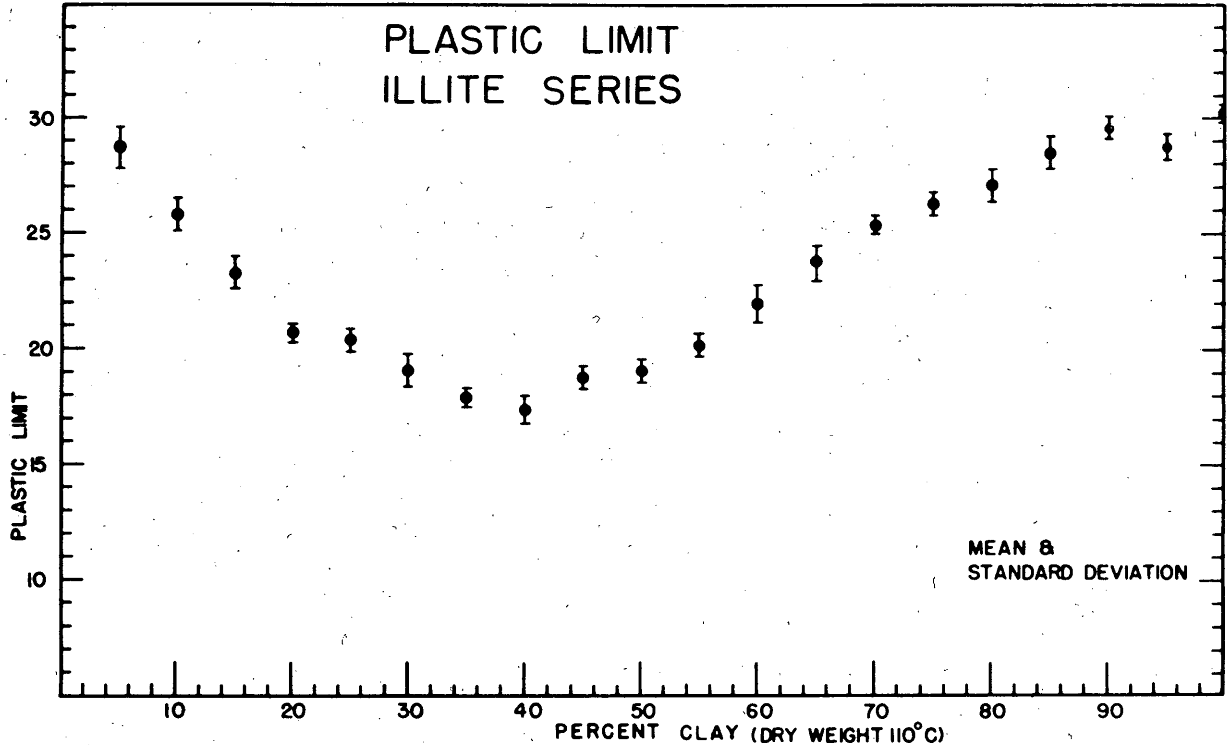

Figure 3. Plastic limit vs clay content in a series of artificial soils.

The 0.95 confidence intervals on this mean are $20.20 < \mu < 20.96$. Since Ballard's average plastic limit for the homogeneity test (21.80) was not statistically different from the grand mean of the other operators (21.56), we feel that the value of 20.6 will serve as an adequate "standard" value. This value has no particular significance as an absolute measure of the "true" boundary between the plastic and the solid state, but it should serve as a representative measure of the average value that would be obtained by other workers in soil mechanics. The slight increase in the mean over the values shown in Figure 3 is due to minor components, primarily organic carbon (Odel, Thornburn and McKenzie, 1960), which have been eliminated in the pure illite series. The increased accuracy that might be gained by the use of a pure-clay sample is less important than the ready availability of the Grundite-derived illite.

The following procedure is suggested for use by an operator who wishes to "standardize" his plastic limit determination:

1. The standard sample should be prepared and hydrated above the plastic limit at least 24 hr prior to determination.

2. Five plastic limits should be determined and the results averaged.

3. The 0.95 confidence limits on the difference in population means ($\mu_1 - \mu_2$) should be calculated as follows:

$$(\bar{y}_1 - \bar{y}_2) \pm t_{(0.025,\ n_1 + n_2 - 2)} \left[ S_0 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)^{\frac{1}{2}} \right]$$

where

$$S_0^2 = \frac{\sum_{j=1}^{n_1} (y_{1j} - \bar{y}_1)^2 + \sum_{j=1}^{n_2} (y_{2j} - \bar{y}_2)^2}{n_1 + n_2 - 2}$$

In the case we are considering, $\bar{y}_1 = 20.6$, $\sum_{j=1}^{10} (y_{1j} - \bar{y}_1)^2 = 2.56$, $t(0.025, 13) = 2.160$,

and $n_1$ and $n_2$ equal 10 and 5 respectively.

Thus our relation reduces to

$$(20.6 - \bar{y}_2) \pm 1.184 \left[ \frac{2.56 + \sum_{j=1}^{5} y_{2j}^2 - \frac{(\sum_{j=1}^{5} y_{2j})^2}{5}}{13} \right]^{\frac{1}{2}}$$

where $\bar{y}_2$ is the operators mean, $\sum_{j=1}^{5} y_{2j}^2$ is the sum of the squares of the operator's

plastic limit determinations,

and $(\sum_{j=1}^{5} y_{2j})^2$ is the sum of the operator's plastic limits squared.

If these confidence limits contain zero, adequate standardization has been achieved.

In conclusion we would like to state that although the standard sample we have suggested has several desirable characteristics, it is not necessarily an "optimum" standard. It might, for instance, be possible to prepare standards from completely synthetic components that could easily be subjected to rigid quality controls. Certainly several standards with different plasticity indices and using different clay groups should be available. It is hoped that this brief discussion will stimulate further interest in the general problem of providing well defined standards for soil property testing.

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U. S. Army Cold Regions Research and Engineering Laboratory, Army Materiel Command, Hanover, N. H.
OPERATOR VARIANCE IN THE DETERMINATION OF THE PLASTIC LIMIT – G. E. H. Ballard and W. F. Weeks

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