Research Report 103
FEBRUARY, 1963

Effective Thermal Conductivity of Ventilated Snow

by Yin-Chao Yen

U. S. ARMY COLD REGIONS RESEARCH
AND ENGINEERING LABORATORY
Hanover, New Hampshire

PREFACE

This is a report on work accomplished under USA CRREL subtask 5010.03136, Thermal properties of snow and ice. The investigation was performed by Dr. Yen, Chemical Engineer, as a part of the program of the Materials Research Branch.

The author wishes to express his gratitude to Mr. James A. Bender, Chief, Research Division. He also wishes to thank Mr. Earl D. Shaw for his help in setting up the experimental device and taking experimental data.

This report has been reviewed and approved for publication by the Commander, U. S. Army Materiel Command.

W. L. NUNGESSER
Colonel, CE
Commanding
USA CRREL

Manuscript received 19 September 1961
DA Task 8 X99-27-001-03

AD Accession No.

U. S. Army Cold Regions Research and Engineering Laboratory, Army Materiel Command, Hanover, N.H.
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1. Snow--Thermal conductivity
2. Heat transfer--Mathematical analysis

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SUMMARY

Thermal conductivities of unconsolidated snow particles with air flowing in a direction parallel but opposite to the energy flow have been investigated for the mass flow rate ranging approximately from 10 to 40×10^{-4} g/cm^2-sec based on the total cross-sectional area of the flask containing the snow sample. The results are interpreted as being the effective thermal conductivity of snow. In the experimental range for snow densities from 0.376 to 0.472 g/cm^3 and corresponding snow particle sizes of 0.07 to 0.22 cm nominal diameter, the results can be represented well by the following least-squares equation,

$$k_e = 0.0014 + 0.58\ G$$

where k_e is the effective thermal conductivity of snow in cal/cm-sec-C and G is the mass flow rate of dry air in g/cm^2-sec. When there is no flow, or $G = 0$, k_e reduces to a constant value of 0.0014 cal/cm-sec-C, equivalent to the thermal conductivity of snow k_e^o with motionless fluid. The value of 0.0014 is in good agreement with the data reported by Abel's (1893) and Kondrat'eva (1945).

EFFECTIVE THERMAL CONDUCTIVITY OF VENTILATED SNOW

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Introduction

Thermal conductivity of any material is defined by the equation:

$$Q = k_e^o \text{ grad } t$$

where Q is the flux of heat in cal/cm^2-sec and k_e^o is thermal conductivity (with stagnant fluid in the void space) in cal/cm-sec-C. Numerous investigators have determined thermal conductivities of snow using various techniques. Abel's (1893) was one of the first to undertake a fairly thorough study of the thermal conductivity of snow. For values of snow density ρ_s between 0.14 and 0.34 g/cm^3 inclusive, he concluded that

$$k_e^o = 0.0068\ \rho_s^2.$$

By comparing the thermal conductivity of water, Jansson (1901) reported the following relationship

$$k_e^o = 0.00005 + 0.0019\ \rho_s + 0.006\ \rho_s^4,$$

whereas Devaux (1933) concluded that

$$k_e^o = (0.7 + 70\ \rho_s^2) \times 10^{-5}$$

for values of ρ_s between 0.1 and 0.6 inclusive. Kondrat'eva (1945) reported results which are basically in agreement with those of Abel's for snow densities less than 0.35 g/cm^3. However, for values of ρ_s greater than 0.35, he reported that

$$k_e^o = 0.0085\ \rho_s^2\ .$$

The above relations represent a considerable variation of data.

Recently Murcray and Echols (1960) made observations on the flow of heat through cold snow. Their measurements were made under conditions of quite stable air temperature with no wind and no precipitation at a field station of the University of Alaska Geophysical Institute at Ballaines Lake, College, Alaska. They noted that the temperature gradient across the snow 0 to 6 in. above the ground was nearly twice that across the snow 6 in. to 1 ft above the ground. They considered this to be the exact opposite of what would be expected on the basis that the thermal conductivity of snow would increase rather than decrease with depth because of the increased density of the snow at the greater depth. The authors ruled out the possibility of this being caused by convective heat transfer, because there was an inverse temperature gradient in the atmosphere just above the snow. Also, they excluded the effect of vapor transfer in snow caused by temperature gradients on the basis that the temperature profile observed was in contrast to those reported by Kondrat'eva in 1945, and interpreted the observed phenomena as being due to radiation effects. In the study reported here, the effect of air flow (and the accompanying vapor transfer) on the thermal conductivity of snow has been investigated for the first time. The results obtained from this study may provide some information on the observations by Murcray and Echols in 1960.

Theory

Heat transfer studies can be carried out under either steady-state or transient conditions. Consider a bed on unconsolidated snow particles through which cold saturated air is flowing downward and heat is flowing upward. The unidirectional energy balance for the differential height of the snow bed dx can be represented as follows:

Energy input:

$$Gc_p(t-t_0) - k_e \frac{\partial t}{\partial x} + \frac{GM_w}{M}\left(\frac{p}{\pi - p}\right)\left[L_s + c_{pw}(t-t_0)\right]$$

= Energy output

$$= Gc_p\left[(t-t_0) - \frac{\partial}{\partial x}(t-t_0)\,dx\right] - k_e\left[\frac{\partial t}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial t}{\partial x}\right)dx\right]$$

$$+ \frac{GM_w}{M}\left[\frac{p}{\pi - p}\ \frac{\partial}{\partial x}\left(\frac{p}{\pi - p}\right)dx\right]\left[L_s + c_{pw}(t-t_0) - \frac{\partial}{\partial x}(t-t_0)\,dx\right]$$

+ Energy accumulation:

$$+ \rho_s\, c_{ps} \frac{\partial t}{\partial \theta}\,dx + c_{ps}(t-t_0)\frac{\partial \rho_s}{\partial \theta}\,dx \qquad (1)$$

where G = mass flow rate of dry air, g/cm^2-sec

c_p, c_{pw}, c_{ps} = specific heat of air, water vapor, and snow respectively, cal/g-C

t = temperature of the system, C

t_0 = temperature of the inlet air, C

k_e = effective thermal conductivity of snow, cal/cm-sec-C

M_w, M = molecular weight of water vapor and dry air respectively

L_s = latent heat of sublimation, cal/g

p = partial pressure of water vapor in air, mb

π = total pressure of the system, mb

x = distance measured in the direction of heat flow, cm

ρ_s = density of snow, g/cm^3

θ = time, sec.

The method of temperature measurement used in this experiment did not provide information on the small but finite temperature difference between the snow particles and the air stream surrounding them. However, in an experiment of this type, this temperature difference is usually considered to be negligible. It is apparent that the values of k_e calculated from eq 1 thus included the combined effects of heat transfer due to natural and forced convection in addition to the conduction through the ice network comprising the snow sample. For the steady state operation, the value of $\frac{\partial t}{\partial \theta}$ under the energy accumulation term is zero and eq 1 reduces to

$$k_e \frac{d^2t}{dx^2} + Gc_p \frac{dt}{dx} + \frac{GM_w}{M}\left(\frac{p}{\pi - p}\right) c_{pw} \frac{dt}{dx} + \frac{GM_w}{M} L_s \frac{d}{dx}\left(\frac{p}{\pi - p}\right)$$

$$+ \frac{GM_w}{M} c_{pw}\left(t - \frac{dt}{dx} dx\right)\frac{d}{dx}\left(\frac{p}{\pi - p}\right) = c_{ps}(t - t_0)\frac{\partial \rho_s}{\partial \theta} dx. \qquad (2)$$

The change of snow density with time, $\frac{\partial \rho_s}{\partial \theta}$, under the steady state operation is very difficult to measure, and no attempt has been made in this study. However, in connection with the investigation of heat transfer by water-vapor transfer, this quantity has been obtained analytically and found to be negligible (Yen, 1962). Furthermore, terms 3 and 5 in eq 2 are much smaller than terms 2 and 4, and consequently eq 2 can be simplified into the following form:

$$Gc_p \frac{dt}{dx} + \frac{GM_w L_s}{M} \frac{d}{dx}\left(\frac{p}{\pi - p}\right) + k_e \frac{d^2t}{dx^2} = 0. \qquad (3)$$

Based on the experimental data reported by Bader et al. (1939), the partial pressure of water vapor in air can be reasonably replaced by the saturation vapor pressure of snow p_s. Furthermore, the saturation vapor pressure in this study is much lower than the total pressure of the system π. From the above considerations, eq 3 becomes

$$Gc_p \frac{dt}{dx} + \frac{GM_w L_s}{M\pi} \frac{dp_s}{dx} + k_e \frac{d^2t}{dx^2} = 0. \qquad (4)$$

Yosida (1950) represented the saturation vapor pressure, p_s, of snow by the following exponential function

$$p_s = 6.1\, e^{0.0857t} \qquad (5)$$

where p_s is the pressure in millibars, and t is the temperature in deg C. Substituting eq 5 directly into eq 4 will result in a nonlinear second order differential equation and its solution in a closed form will be difficult to obtain even in the simplest case. However, a linear approximation of eq 5 in the temperature range covered in this study ($-17\,C < t < -7C$) can be made as indicated below:

$$p_s = 6.1\, e^{0.0857t} \cong 4.636 + 0.195\, t. \qquad (6)$$

Thus $dp_s/dx = 0.195\, dt/dx$. Hence, eq 4 takes the form

$$\frac{G\left(c_p + 0.195 \frac{M_w L_s}{M\,\pi}\right)}{k_e} \frac{dt}{dx} + \frac{d^2t}{dx^2} = 0. \qquad (7)$$

Eq 7 involves the assumption that the mean values of thermal conductivity k_e, specific heat c_p, latent heat of sublimation L_s, and total pressure of the system π

are constants in the snow bed. With the boundary conditions

$$x = 0 \qquad t = t_0$$

$$x = L \qquad t = t_L$$

the solution to eq 7 can be written as

$$\frac{t - t_0}{t_0 - tL} = 1 - \frac{1 - e^{-\alpha x}}{1 - e^{-\alpha L}}, \tag{8}$$

where

$$\alpha = \frac{G\left(c_p + 0.195\, \frac{M_w L_s}{M\,\pi}\right)}{k_e}. \tag{9}$$

Apparatus and experimental procedure

The test procedure for obtaining the effective axial thermal conductivity of the snow consisted of taking steady-state temperature measurements along the edge and axis of a cylindrical bed of snow. Figure 1 shows the simplified flow diagram of the apparatus used. The entire apparatus, with the exception of the micromanometer, the wet test meter, and the Leeds and North potentiometer, was placed in a refrigerated room maintained at approximately -20C. The micromanometer, wet test meter, and the potentiometer were placed in an adjoining warmer room so that readings could be taken without disturbing the cold room. To prevent the formation of ice crystals in the discharge line as the warmer saturated air leaving the hot end of the test apparatus cooled to the existing cold room temperature, a heat tape was wrapped around that part of the discharge line in the cold room. Cold-room air was compressed in the cold room where the experimental set-up was located and then passed through a surge tank to minimize the fluctuations of the air pressure from the compressor. A pancake-type pressure regulator was used to obtain the desired flow rate.

Since the temperature of the cold room could not be maintained any closer than $\pm$ 1C, the air was passed through a constant-temperature bath where the temperature could be maintained at slightly above room temperature to an accuracy of $\pm$ 0.05C.

The air from the cold room, which was initially saturated with water vapor, became dehydrated as a result of being compressed and cooled to the room temperature and then re-expanded and passed through the constant-temperature bath. For this reason it was necessary to pass it through a water vapor saturator to re-saturate it. The saturator was a cylindrical tank about 8-in. in diam and 2-ft high, filled with crushed ice. The air entered the bottom of the saturator, passed upward picking up water vapor from the ice, and exited from the top as saturated air. From the saturator, the air passed through a constant-temperature bath and then flowed downward through the snow bed, then through a wet test meter to the atmosphere. Care was taken to see that there were no leaks in the system by sealing all joints and connections with a Dow Corning silicone lubricant and a soft rubber gasket.

To insure that the only heat transfer was parallel to the flow of air and to reduce energy transfer in a radial direction, the bed was contained in a Dewar flask, 2.56 in. I.D. and 6-in. long, made of a double-walled glass tube, with the annular space evacuated and the internal surfaces plated with silver. As indicated in Figure 2, saturated air at constant temperature was introduced into the top of the vertical bed and passed downward counter to the upward flow of heat energy. This heat was transferred into

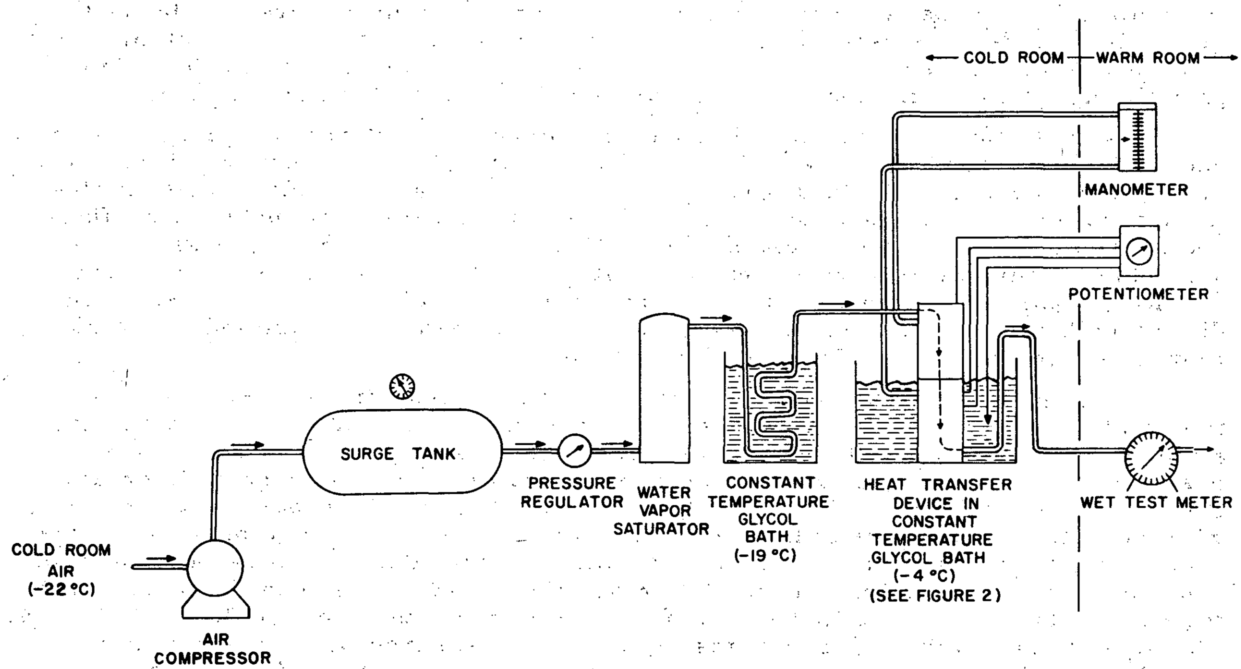

Figure 1. Simplified flow diagram of heat transfer study apparatus.

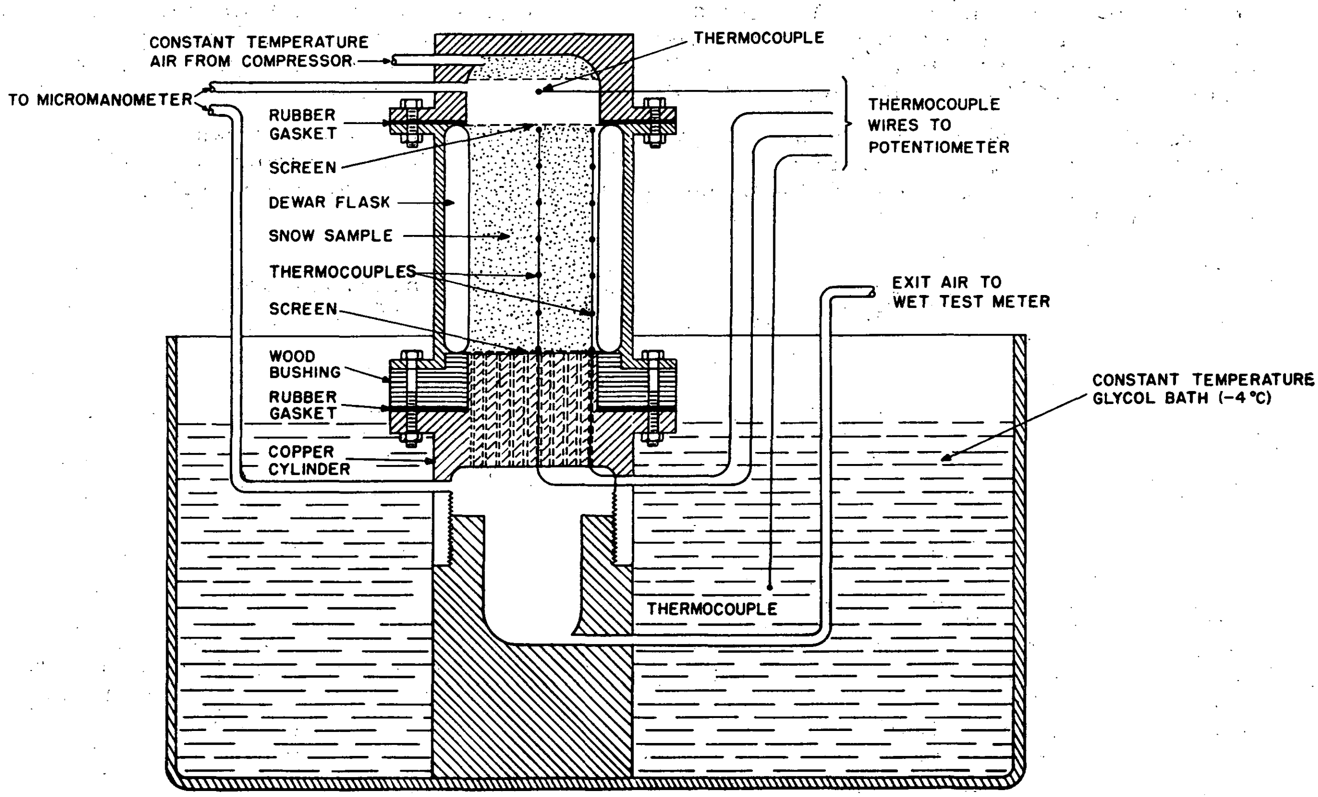

Figure 2. Schematic cut-away view of heat transfer device.

the snow from a copper cylinder immersed in a constant-temperature bath below the bed. The cylinder was used to obtain a uniform temperature across the bottom of the snow bed. Holes 0.125-in. in diam were drilled in the cylinder to allow the air to pass through it and a fine mesh screen was placed on top of it to hold the snow.

The Dewar flask was inclosed in an aluminum casing equipped with flanges at both ends. A wooden bushing between the aluminum casing and the copper cylinder minimized direct conduction of heat between them. The casing was made so that the flask fitted tightly into it and then, to prevent any possible leakage of air between the flask and the casing, the flask was coated with a low-temperature silicone grease before assembly. Temperatures were measured with one set of copper-constantan thermocouples (30 ga) along the wall of the bed and a second set along the center axis. Any difference between corresponding center and wall temperatures would indicate a radial flow of heat through the walls of the Dewar flask.

The temperature at the bottom of the snow bed was maintained by a constant-temperature bath having a maximum temperature variation of ±0.05C. Minor fluctuations were smoothed out by the mass of the large copper cylinder. The air was passed through the snow, and temperature readings were taken at regular intervals until the steady-state condition was achieved. Then air flow rate was measured along with the water temperature of the wet test meter and the atmospheric pressure in the room. Readings were taken again after 30 min to ensure that a steady-state condition had been attained. The time required to reach this steady-state condition varies with the rate of flow and was about 3 to 6 hr for air velocity in the range of 0.2 to 2.7 cm/sec. The water temperatures of the wet test meter were measured with a precision of ± 0.05C by a thermocouple connected to a Rubicon potentiometer. Absolute atmospheric air pressure was read on a mercury barometer during each run; the pressure drop across the snow sample was read on a Trimount micromanometer with an accuracy of ± .01 mm water. A "Precision" wet test meter was used to measure the air flow rate. The meter was checked with the standard displacement method and found to be accurate within 20 cm^3 out of 3000 cm^3. The displacement capacity of the meter is 3000 cm^3 per revolution, with 300 divisions on the dial.

All snow samples used for this experiment were taken from a carton which had been stored in the cold room for an unknown period. Snow fractions from sieve analysis were used. Sample densities and snow particles sizes used for this investigation were as follows:

Sample density (g/cm^3)	Snow particle nominal diam (cm)
0.376	0.22
0.387	0.18
0.436	0.13
0.472	0.07

Results and discussion

Figure 3 shows the dimensionless temperature (as defined in eq 3) $(t-t_0)/(t_0-t_L)$ versus α with $\underline{x}$, the distance measured in the direction of heat flow, as the parameter. From the steady-state temperature measurements, temperature profiles of the snow bed were drawn. The two series of temperature measurements, (1) at the center of the sample and (2) at the edge of the sample, were plotted to see that the radial temperature gradients were kept within reasonable limits. Sample plots of these temperature profiles (Fig. 4) show the effect of mass flow rate. Thermocouple readings from

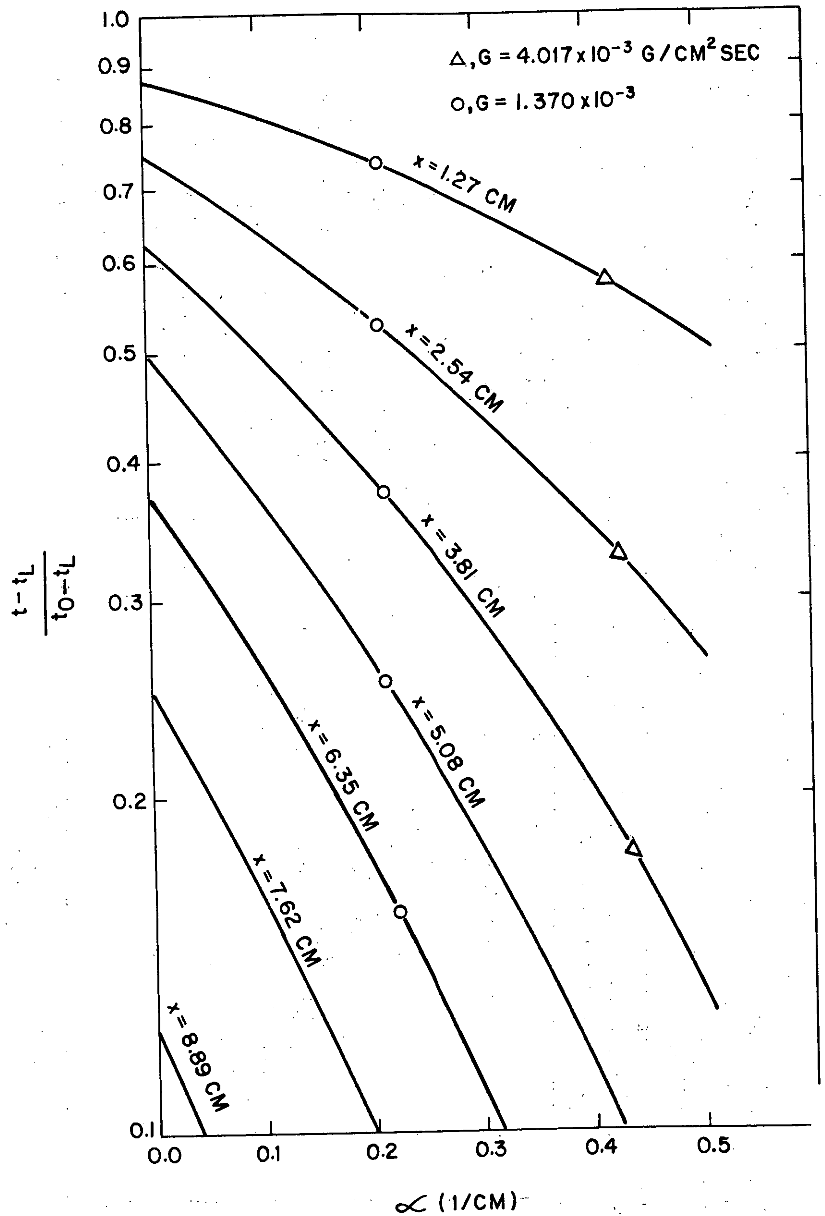

Figure 3. Dimensionless temperature distribution.

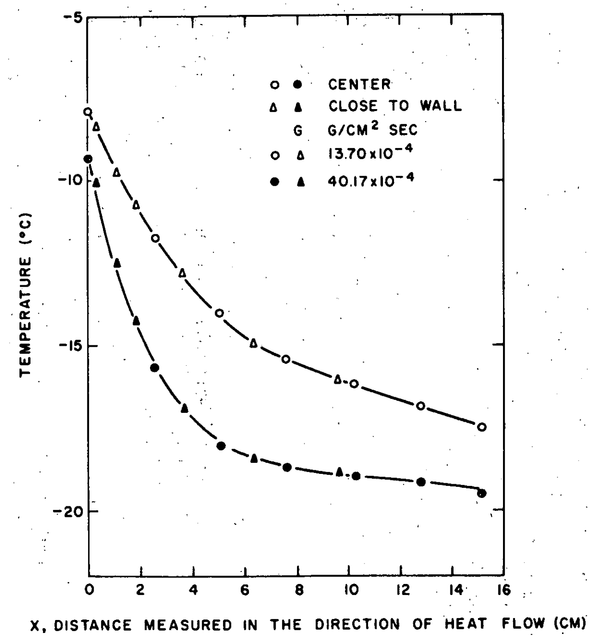

Figure 4. Typical temperature distribution in packed bed of unconsolidated snow particles.

the center and edge of the sample fit on a smooth curve, indicating that radial transfer is completely eliminated with this experimental set-up. For each run, $(t-t_L)/(t_0-t_L)$ was evaluated for several points along the snow bed. By combining these dimensionless temperature values with Figure 3, 3 or 4 values of α were calculated for one set of steady-state temperature measurements. An average value of α was then obtained for each experimental run. Theoretically, these α values must be the same; actually there was some small deviation. Errors in reading the temperature would give a random variation in the α values. Furthermore, any inhomogeneity in snow samples would give a consistent difference. If the steady-state condition had not been reached, there would have been an abnormal variation. Two typical sets of α values are shown in Figure 3. It can be seen that more or less identical α values were obtained for each experimental run.

The average α values obtained in the above manner were then used to obtain $\underline{k}_e$ values from eq 4. In eq 4, $\underline{G}$ is the mass flow rate determined by the wet test meter, and $\underline{c}_p$ is the average specific heat of air at the mean snow-bed temperature. A sample calculation of α and $\underline{k}_e$ is shown in the Appendix. In the preparation of Figure 3, the value of $\underline{L}$ was taken as 4 in. (10.16 cm) measured from the hot end of the sample. Though the snow samples used throughout for this investigation were 6 in. long, it was decided to use 4 in. in order to eliminate the end effect in the calculation of α and $\underline{k}_e$ values. Table I a-d summarizes all the experimental data and calculated results. Figure 5 shows $\underline{k}_e$, effective thermal conductivity of snow, plotted against $\underline{G}$, mass flow rate. In the experimental range covered in this investigation, the values of $\underline{k}_e$ can be correlated very well by the following least squares equation:

$$k_e = 0.0014 + 0.58\,G \tag{10}$$

From Figure 5, it is noted that, for snow densities ranging from 0.376 to 0.472 g/cm³ and corresponding snow particles from 0.07 to 0.22 cm nominal diameter, there is no noticeable effect on thermal conductivity. It is suspected that snow density and grain size have opposite effects on the effective thermal conductivity because of the flow of fluid in snow. For the same mass flow rate $\underline{G}$, it is believed that less turbulence will prevail in snow samples of larger grain size, which usually have a lower density; consequently, $\underline{G}$ has less effect on thermal conductivity of larger-grained snow. However, larger snow particles will have more area of contact; this might increase the thermal conductivity of the system.

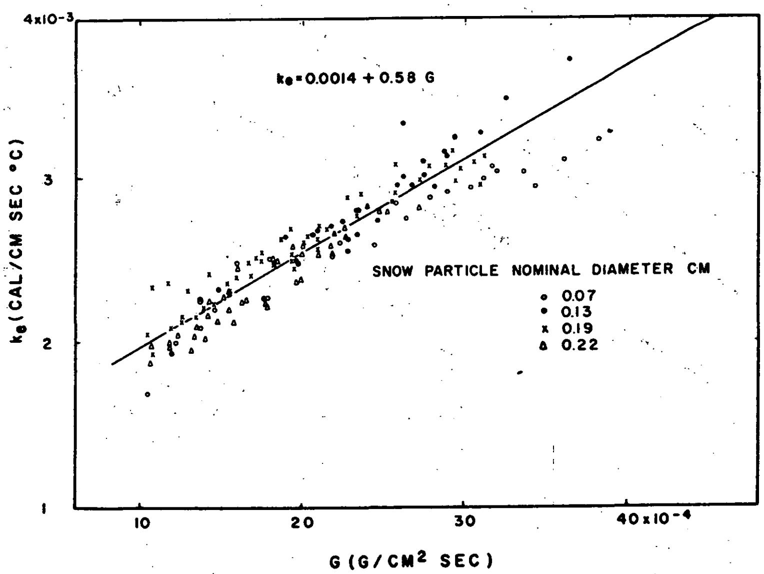

Figure 5. Relationship between effective thermal conductivity and mass flow rate.

When there is no flow, $G = 0$ in eq 5, $\underline{k}_e$ reduces to a constant value of 0.0014 cal/cm-sec-C, equivalent to the thermal conductivity of snow with stagnant fluid within it, $\underline{k}_e^o$. For the average value of snow densities used for this investigation, 0.426 g/cm³, the value of 0.0014 cal/cm-sec-C is in good agreement with the data reported by Abel's (1893) and Kondrat'eva (1945).

This correlation is interesting in the light of eq 11, obtained by rearrangement of eq 7

$$k_e = \frac{G\left(c_p + 0.195\,\frac{M_w L_s}{M\,\pi}\right)\frac{dt}{dx}}{\frac{d^2t}{dx^2}}. \tag{11}$$

Equation 11 indicates that the value of k_e should be a function of G and the ratio of longitudinal temperature gradients, which is understood to be a function of snow density and grain size in addition to the mass flow rate of dry air, G, because the total pressure of the system π is nearly a constant value for G varying from approximately 10 to 40 x 10^{-4} g/cm²-sec.

Figure 4 clearly indicates that steeper temperature gradients prevailed in the warmer sections of the snow sample. Therefore, more water vapor was lost to the air stream, resulting in lower density in the warmer regions. This is also the case for a shallow snow cover under static condition and subjected to a temperature gradient over a period of time. Under these conditions, water vapor will diffuse from snow of higher to lower temperatures. The extent of this diffusion phenomena has been clearly demonstrated in studies by Kondrat'eva in 1945 and by de Quervain (1958). In Kondrat'eva's experiment, snow samples, initially not uniform in density and about 41 cm in depth, were exposed to temperature differentials of approximately 11 C. She started with a snow sample of density 0.45 g/cm³ at the warmest layer and 0.32 g/cm³ at the coolest layer. After five days, 0.135 g/cm³ of snow had been transferred to lower temperature regions. Murcray and Echols (1960) reported the same temperature patterns found in this study and shown in Figure 4, which were in contrast to those reported by Kondrat'eva in 1945. In their study, a snow pack about 30 cm deep was observed under the following conditions:

1. No wind was apparent,
2. The temperature was nearly 30 C higher at the ground than at the surface,
3. The snow bed had existed for some time.

Murcray and Echols expected higher densities near the ground or warmer parts of the snow pack, which would have resulted in temperature profiles similar to those observed by Kondrat'eva in 1945. Hence, they concluded that the principal factor in producing the observed temperature pattern was heat transfer caused by radiation. In doing so, they disregarded the effect of the prolonged vapor transfer caused by the imposed temperature differential. Because the snow pack had existed under the observed conditions for some time, it is more reasonable to conclude that the difference between the temperature patterns observed by Kondrat'eva and Murcray and Echols was caused by contrasting density distributions in the two cases. In view of the above considerations, the effect of mass transfer caused by temperature gradients must be considered for any heat transfer studies on snow under static or air flow conditions.

From this investigation, it can be concluded that air flow in snow has considerable effect on the thermal conductivity of snow. For instance, when $G = 12 \times 10^{-4}$ g/cm²-sec or bulk air velocity approximately 1 cm/sec, k_e is 0.0020 cal/cm-sec-C compared to 0.0014 when there is no air flow, an increase of about 30% in the thermal conductivity of snow. The pressure difference needed to produce such a velocity, for snow density of 0.376 g/cm³, is about 0.0025 mb/cm of snow. Since such a small pressure difference is required to cause this air flow, it is very likely that convective heat transfer and the accompanying mass transfer must occur to a great extent in the top layers of a natural snow cover. These results are useful in connection with the idea of cooling an undersnow camp by circulating warm air into the lower snow pack of much lower temperature.

Table I. Experimental results and calculated data.

a) ρ_s, avg = 0.376 g/cm³; Snow particle nominal diam = 0.219 cm

$G(g/cm^2\text{-}sec) \times 10^{-4}$	α (1/cm)	$k_e(cal/cm\text{-}sec\text{-}C) \times 10^{-3}$
13.112	0.221	1.947
14.022	0.228	2.022
15.089	0.218	2.275
16.203	0.238	2.242
17.795	0.264	2.214
19.521	0.272	2.358
24.703	0.292	2.785
27.089	0.317	2.812
25.089	0.297	2.779
22.679	0.282	2.643
21.475	0.274	2.576
19.975	0.259	2.533
11.635	0.194	1.978
14.135	0.215	2.165
15.818	0.246	2.116
19.771	0.274	2.374
21.748	0.282	2.533
24.021	0.279	2.834
21.908	0.271	2.664
10.613	0.187	1.864
16.601	0.253	2.248
17.702	0.259	2.248
20.930	0.272	2.527
22.567	0.276	2.692
16.021	0.215	2.450
14.703	0.228	2.122
13.294	0.215	2.034
18.362	0.243	2.489
10.704	0.177	1.984
12.341	0.198	2.045
15.453	0.220	2.314
11.705	0.194	1.990
13.363	0.212	2.078
15.544	0.233	2.193
18.203	0.239	2.500
19.388	0.248	2.576
19.998	0.254	2.587
14.158	0.207	2.253

Table I (Cont'd)

b) ρ_s, avg = 0.387 g/cm³; Snow particle nominal diam = 0.182 cm

G(g/cm²-sec) x 10^{-4}	α (1/cm)	k_e(cal/cm-sec-C) x 10^{-3}
10.687	0.148	2.374
12.924	0.184	2.307
28.828	0.308	3.076
29.429	0.325	2.984
30.482	0.323	3.103
23.488	0.266	2.904
13.889	0.207	2.214
16.727	0.221	2.480
19.430	0.256	2.560
20.941	0.256	2.692
29.202	0.303	3.169
25.504	0.289	2.909
31.156	0.326	3.143
20.922	0.262	2.625
15.645	0.223	2.307
17.088	0.225	2.506
18.222	0.244	2.453
25.651	0.274	3.076
12.613	0.192	2.161
13.671	0.198	2.267
10.731	0.184	1.923
12.593	0.194	2.135
30.931	0.344	2.957
29.716	0.318	3.063
21.500	0.262	2.692
20.260	0.251	2.652
19.463	0.261	2.453
19.330	0.253	2.519
18.330	0.236	2.559
16.840	0.231	2.400
15.942	0.220	2.387
27.800	0.297	3.076
22.719	0.297	2.877
18.627	0.259	2.625
14.547	0.208	2.294
20.091	0.254	2.599
19.319	0.236	2.692
17.499	0.226	2.543
20.930	0.269	2.561
23.271	0.277	2.762
25.543	0.294	2.862
27.271	0.300	2.987
11.749	0.164	2.358
14.239	0.194	2.417
15.408	0.215	2.358
17.521	0.231	2.489
18.203	0.243	2.466
10.431	0.167	2.050
11.885	0.187	2.088
13.408	0.205	2.148

Table I (Cont'd)

c) ρ_s, avg = .436 g/cm³; Snow particle nominal diam = 0.129 cm

G(g/cm²-sec) x 10^{-4}	α (1/cm)	k_e(cal/cm-sec-C) x 10^{-3}
14.925	0.269	2.324
22.678	0.292	2.554
11.911	0.202	1.940
19.771	0.262	2.478
21.740	0.262	2.725
27.532	0.299	3.026
22.462	0.269	2.746
23.409	0.275	2.801
26.102	0.284	3.021
27.370	0.290	3.108
28.641	0.298	3.163
29.307	0.289	3.262
32.424	0.306	3.486
36.375	0.321	3.727
26.095	0.256	3.355
20.614	0.256	2.648
18.973	0.236	2.648
20.601	0.255	2.659
23.355	0.385	2.692
17.681	0.256	2.275
25.682	0.285	2.960
26.731	0.297	2.960
28.089	0.313	2.949
28.820	0.302	3.141
30.938	0.310	3.283
24.601	0.295	2.741
19.730	0.261	2.488
20.896	0.256	2.686
22.698	0.284	2.631

Table I (Cont'd)

d) ρ_s, avg = 0.472 g/cm³; Snow particle nominal diam = 0.065 cm

G(g/cm²-sec) x 10^{-4}	α (1/cm)	k_e(cal/cm-sec-C) x 10^{-3}
13.683	0.198	2.269
24.299	0.308	2.592
13.700	0.216	2.083
22.308	0.282	2.598
14.598	0.218	2.199
36.022	0.380	3.113
33.460	0.362	3.037
31.936	0.344	3.047
28.866	0.325	2.921
27.820	0.318	2.877
26.430	0.317	2.746
25.725	0.297	2.851
23.287	0.274	2.795
12.123	0.198	2.006
14.578	0.218	2.199
16.059	0.213	2.478
17.963	0.236	2.500
21.783	0.284	2.527
34.311	0.382	2.954
38.138	0.387	3.239
10.405	0.203	1.683
30.264	0.338	2.944
31.099	0.339	3.015
31.582	0.338	3.075
17.896	0.249	2.270
40.170	0.426	3.103
40.749	0.413	3.239

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APPENDIX

Sample calculations of α and k_e: The following is the procedure used for determining the effective thermal conductivity of snow reported in this paper. The snow density and snow grain size for this particular calculation are 0.472 g/cm^3 and 0.07 cm respectively.

Table A1. Steady-state temperature distribution in a snow bed.

Thermocouple position x, measured in the direction of heat flow, (cm)	Temperature (C)	
	Center of sample	Edge of sample container
0.00	-9.35	
0.32		-10.60
0.99		-12.65
1.75		-14.35
2.54	-15.65	
3.53		-16.95
5.12	-18.05	
6.35		-18.40
7.66	-18.07	
9.72		-18.80
10.32	-18.93	
12.94	-19.08	
15.28	-19.50	

From a plot of the above temperature vs x, temperatures at 1.27 cm intervals were taken from the smooth curve shown in Figure 4 and Table A2 was then prepared.

Table A2. Evaluation of dimensionless temperatures and α values.

Position x (cm)	Temp. taken from the curve t (C)	$t - t_L$	$\frac{t - t_L}{t_0 - t_L}$	α From Fig. 3 (cm^{-1})
0.00	- 9.10(t_0)	9.8	1.0	
1.27	-13.15	5.65	0.577	0.417
2.54	-15.70	3.2	0.327	0.428
3.81	-17.15	1.75	0.179	0.432
5.08	-18.00	0.90	0.092	
6.35	-18.40	0.50	0.051	
7.62	-18.65	0.25	0.026	
10.16	-18.90(t_L)			

Mean value of $\alpha = 0.426\ cm^{-1}$

$$= \frac{G(c_p + 0.195\,\frac{M_w L_s}{M\pi})}{k_e}$$

Flow rate = 472 ℓ in 68.82 min measured when the wet test meter was at 15.7 C and the atmospheric pressure was at 1007.7 mb

Partial pressure of water at 15.7 C = 17.8 mb

Partial pressure of air 1007.7 - 17.8 = 989.9 mb

Cross-sectional area of the snow container = 33.91 cm^2

Mass flow rate of dry air, $G = 472 \times 1000 \times 0.0012 \times \frac{989.9}{1012.3} \times \frac{293}{288.7} \times \frac{1}{68.82 \times 60}$

$\times \frac{1}{33.91} = 0.004017\ g/cm^2\text{-sec}$

Effective thermal conductivity, $k_e = \frac{G(c_p + 0.195\,\frac{M_w L_s}{M\pi})}{\alpha} =$

$$\frac{0.004017(0.248 + 0.195\,\frac{18 \times 675}{29 \times 1007.7})}{0.426} = 3.10 \times 10^{-3}\ cal/cm\text{-sec-C.}$$

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DA Task 8X99-27-001-03, CRREL Subtask 5010.03136
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