AN ANALYSIS OF NONDESTRUCTIVE SENSING OF WATER CONTENT BY MICROWAVES

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PREFACE

This investigation was conducted by Dr. P. Hoekstra, Research Physicist, and SP5 P. Cappillino, of the Research Division, U.S. Army Cold Regions Research and Engineering Laboratory (USACRREL). The work was performed under DA Project 4A061101A91D03, *In House Laboratory Independent Research.*

This study was suggested by Mr. H. Aamot and stimulated by the interest of Mr. R. Berg in microwave moisture sensors.

The experimental measurements on the dielectric properties of sodium kaolinite used in the computations for this study were performed by Mr. A. Delaney.

The report was technically reviewed by Messrs. B. Lyle Hansen and H. Aamot.

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AN ANALYSIS OF NONDESTRUCTIVE SENSING OF WATER CONTENT BY MICROWAVES

by

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Introduction

This investigation was suggested as a result of a study made to find a nondestructive method for detecting wet spots in roof insulations. Since microwave moisture sensors have become commercially available and the specifications given by the manufacturers have proved inadequate, the present investigation was performed.

There are several methods for nondestructively measuring the water content of soils and construction materials such as concrete and wood. Three widely used methods are:

1. transmission of gamma rays
2. scattering of thermal neutrons
3. reflection or transmission of microwaves.

The limitations and capabilities of the two nuclear methods have been well documented. So far, a critical evaluation of the third method has not been published. This report gives an analysis of the use of microwaves in moisture sensing.

Physical basis of microwave moisture sensing

The basis of all microwave moisture sensing is that the dielectric constant and dielectric loss of materials containing water such as soils, concrete and wood are a strong function of water content. In Figure 1 the dielectric constant and loss of water at 25°C are given as a function of frequency. In the entire frequency range of interest in microwave moisture sensors (10^{8} to 2.6×10^{10} Hz), the dielectric constant and loss of water are several times larger than those of dry materials. Additions of small amounts of water, therefore, substantially increase the dielectric constant and loss of a material. A wet material can be considered as a combination of a medium with a very high dielectric constant (water) enclosed in a matrix of low dielectric constant and loss (wood fibers, minerals). Microwave sensors measure a physical quantity (for example, reflection or transmission) that is influenced by the dielectric constants of the materials.

Physics of transmission and reflection

Consider the general case where microwaves are incident normal to a sheet of material (Fig. 2). Part of the incident electromagnetic wave *A* is reflected. The reflected wave *B* and the incident wave *A* form, by interference, a standing wave pattern. Wave *C* traverses the boundary between medium 1 and medium 2 and propagates through medium 2; then part of wave *C* (wave *D*) is reflected

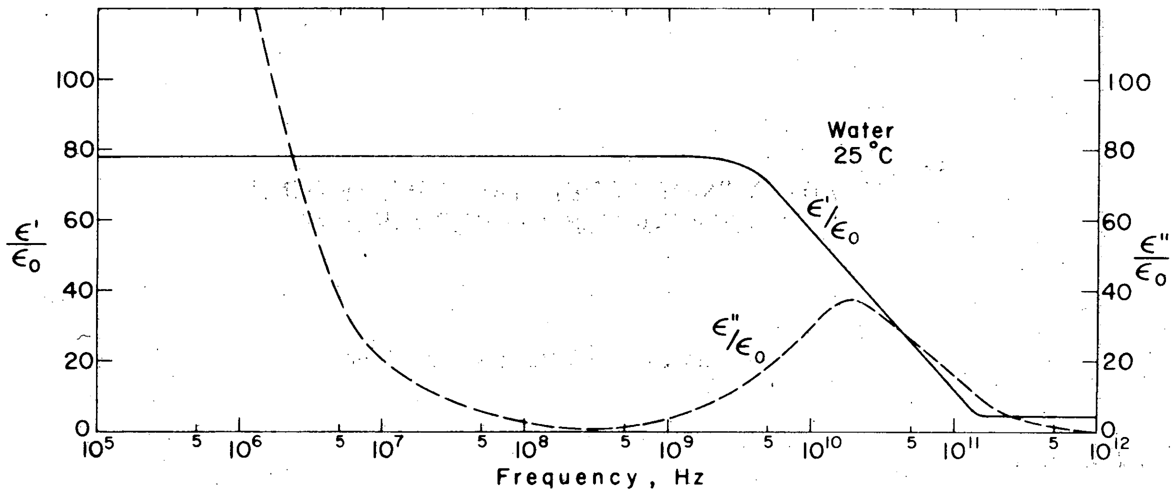


Figure 1. The dielectric constant ϵ'/ϵ_0 and dielectric loss ϵ''/ϵ_0 of water at 25°C as a function of frequency.

at the boundary between medium 2 and medium 3, and the other part (wave E) continues without further reflection in medium 3. The total reflected wave B results from the interference of two partial waves: the reflected wave at the front surface (wave B) and the reflected wave at the back surface (wave D).

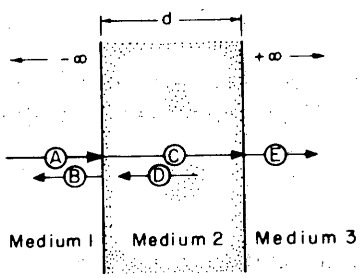


Figure 2. Schematic diagram of electromagnetic waves reflected and transmitted by a dielectric sheet of thickness d.

The reflection coefficient r_0 of the assembly in Figure 2 is defined as the ratio of the reflected amplitude to the incident amplitude at the boundary. Thus, in this case

$$r_0 = \frac{B}{A}. \tag{1}$$

The transmission coefficient t_0 is the ratio of the transmitted amplitude to the incident amplitude. Hence

$$t_0 = \frac{E}{A}. \tag{2}$$

The reflection and transmission coefficients can be calculated from the Fresnel equation and Snell's law of reflection and refraction. To proceed with the calculation several properties of the dielectric medium need to be defined.

All references made in this report to reflection r and transmission t coefficients refer to *amplitude* coefficients as opposed to *energy* reflection R and transmission T coefficients. At normal incidence, they are related by

$$R = |r^2|$$

and

$$T = |t^2|.$$

Impedance. The intrinsic electromagnetic impedance Z of a dielectric in unbounded space is given by the ratio of the electric field vector E to the magnetic field vector H:

$$Z = \frac{E}{H} \tag{3}$$

and is also equal to

$$Z = \frac{E}{H} = \sqrt{\frac{\mu^*}{\epsilon^*}} \tag{4}$$

where μ^* is the magnetic permeability and ϵ^* is the electric permittivity. The materials under consideration are virtually nonmagnetic so that $\mu^* = \mu_0 = 1.25 \times 10^{-8}$ henry/cm, the magnetic susceptibility of free space.

ϵ^* is complex and has the form $\epsilon^* = \epsilon' - j\epsilon''$. The ratio of ϵ' and ϵ'' to ϵ_0, the electric permittivity of free space, 8.85×10^{-14} farad/cm, gives the dielectric constant K' and the dielectric loss K''. Another common term in use is the loss tangent, which is given by

$$\tan\delta = \frac{\epsilon''}{\epsilon'} = \frac{K''}{K'}. \tag{5}$$

The definition in eq 4 yields for the impedance of free space

$$Z = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{1.257 \times 10^{-8}}{8.85 \times 10^{-14}}} = 376.6 \text{ ohms}. \tag{6}$$

For a lossless dielectric ($\epsilon'' = 0$) with a dielectric constant K', the intrinsic impedance of a medium is

$$Z_d = \sqrt{\frac{\mu_0}{K'\epsilon_0}} = 376.6\sqrt{\frac{1}{K'}}. \tag{7}$$

For a dielectric with loss, the impedance is complex and is given by:

$$Z_L = \sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{1}{\sqrt{K' - jK''}}\right). \tag{8}$$

All media containing water have a significant dielectric loss. The term *dielectric constant* is also misleading in that for water both K' and K'' vary significantly with frequency.

Propagation factor. The propagation factor of an electromagnetic wave in unbounded space is given by:

$$\gamma = jw\sqrt{\epsilon^*\mu_0} \tag{9}$$

where w is the angular frequency $2\pi F$, where F is the frequency in hertz.

In free space

$$\gamma_0 = jw\sqrt{\epsilon_0 \mu_0} . \qquad (10)$$

This can also be written in terms of the velocity of light in free space [$c = (1/\sqrt{\epsilon_0 \mu_0})$] and the free-space wavelength λ_0.

$$\gamma_0 = jw\sqrt{\epsilon_0 \mu_0} = j\frac{w}{c} = j\frac{2\pi F}{c} = j\frac{2\pi}{\lambda_0} . \qquad (11)$$

For a dielectric without loss, the propagation factor is

$$\gamma_d = \gamma_0 \sqrt{K'} \qquad (12)$$

and for a dielectric with loss it takes the form

$$\gamma_d = \gamma_0 \sqrt{K' - jK''} . \qquad (13)$$

The reflection and transmission coefficients of Figure 2 can now be defined in terms of the impedances and propagation factors of air and the media. The reflection coefficient at the boundary of the two semi-infinite media 1 and 2 is given by:

$$r_{12} = \rho e^{i\phi} = \frac{Z_1 - Z_2}{Z_1 + Z_2} . \qquad (14)$$

If both media are lossless dielectrics, the reflection coefficient is real, and $\phi = \pi$; the reflected wave is 180° out of phase with the incident wave. For a medium with loss, both Z and the reflection coefficient are complex.

When dealing with media of finite thickness, one differentiates between the reflection coefficient of the whole assembly r_0, given by eq 1, which results from superposition of front and back reflections, and the reflection coefficients of the partial waves r_{12} and r_{23}. For Figure 2

$$r_{12} = -r_{23} = \frac{Z_{air} - Z_2}{Z_{air} + Z_2} . \qquad (15)$$

To calculate r_0, consider the propagation of an electromagnetic wave in medium 2. The propagation of a TEM wave traveling in the x-direction can be written as

$$E_{y2} = E_2 e^{jwt} e^{-\gamma_2 x} \qquad (16)$$

$$H_{z2} = \frac{E_2}{Z_2} e^{jwt} e^{-\gamma_2 x} . \qquad (17)$$

Since there is a forward and reflected wave in medium 2

$$E_{y2} = E_2 e^{jwt} [e^{-\gamma_2 (x-d)} + r_{23} e^{\gamma_2 (x-d)}] \qquad (18)$$

$$H_{z2} = \frac{E_2}{Z_2} e^{jwt} \left[e^{-\gamma_2(x-d)} - r_{23} e^{\gamma_2(x-d)}\right] . \qquad (19)$$

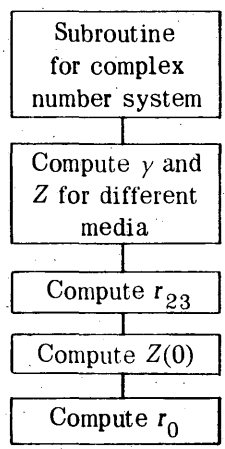
The terminating impedance $Z(0)$, at $x = 0$, defined as the ratio of E/H at $x = 0$, according to eq 18 and 19 is given by

$$Z(0) = \frac{E_2(0)}{H_2(0)} = Z_2 \left(\frac{e^{\gamma_2 d} + r_{23} e^{-\gamma_2 d}}{e^{\gamma_2 d} - r_{23} e^{-\gamma_2 d}} \right) . \qquad (20)$$

The reflection coefficient r_0 of the whole assembly can now be written in terms of the terminating impedance

$$r_0 = \frac{Z(0) - Z_2}{Z(0) + Z_2} . \qquad (21)$$

A computer program (see App. A) to calculate r_0 can be diagrammed as follows:



The transmission coefficient of the electromagnetic vector can also be given in terms of the impedances of the media

$$t_{12} = T_0 e^{jn} = \frac{2Z_2}{Z_2 + Z_1} = 1 - r_{12} . \qquad (22)$$

The best way to derive the transmission coefficient of the whole arrangement in Figure 2 is to superpose the various waves with due regard to phase. So the wave C may be written as superposition of subsequent reflections:

$$C = At_{12} + At_{12}(r_{23} r_{12} e^{-2\gamma_2 d}) + \ldots \qquad (23)$$

which can also be written as

$$C = \frac{At_{12}}{1 - r_{23} r_{12} e^{-2\gamma_2 d}} = \frac{A(1 - r_{12})}{1 - r_{23} r_{12} e^{-2\gamma_2 d}} . \tag{24}$$

Wave C undergoes further reflection at the boundary between medium 2 and medium 3 so that the transmitted wave E at the boundary between medium 2 and medium 3 is equal to:

$$E = \left[\frac{A(1 - r_{12})}{1 - r_{23} r_{12} e^{-\gamma_2 d}}\right] t_{23} = \frac{A(1 - r_{12})(1 - r_{23}) e^{-\gamma_2 d}}{1 - r_{23} r_{12} e^{-2\gamma_2 d}} \tag{25}$$

and the transmission coefficient is equal to

$$\frac{E}{A} = \frac{(1 - r_{12})(1 - r_{23}) e^{-\gamma_2 d}}{1 - r_{23} r_{12} e^{-2\gamma_2 d}} . \tag{26}$$

Equation 26 may be computed in a way similar to that shown for the reflection coefficient (see App. A).

General behavior of reflection and transmission of electromagnetic waves through media of finite thicknesses

The reflection coefficient is a periodic function of thickness. In Figures 3a and b computed values for the amplitude of r_0 as a function of thickness are plotted in terms of the number of wavelengths in the sample. The wavelength in the sample λ_s is related to the free-space wavelength by

$$\frac{\lambda_0}{\lambda_s} = \left[\frac{1}{2} K' \left(\sqrt{1 + \tan^2 \delta} + 1\right)\right]^{1/2} \tag{27}$$

and the free-space wavelength is related to frequency by

$$\lambda_0 = \frac{c}{F} . \tag{28}$$

The reflected wave is the superposition of two partial waves, the reflected wave at the front surface and the reflected wave at the back surface. The electric wave, when reflected from the front surface, undergoes a phase change of π radians. The wave reflected from the back surface stays in phase (since $r_{12} = - r_{23}$) with the incident wave. The total phase difference of the two partial waves is therefore:

$$\Delta = \left(\frac{2\pi}{\lambda_s}\right) 2d \pm \pi . \tag{29}$$

Thus, the phase and amplitude are periodic functions of the number of wavelengths in the sample, d/λ_s. The phase difference between the reflected and incident waves is plotted in Figure 4.

For a dielectric without loss ($\epsilon'' = 0$), the value of r_0 goes to zero at a thickness of $n\lambda_s/2$ and reaches a maximum at a thickness of $(2n - 1)\lambda_s/4$. The partial wave reflected from the front

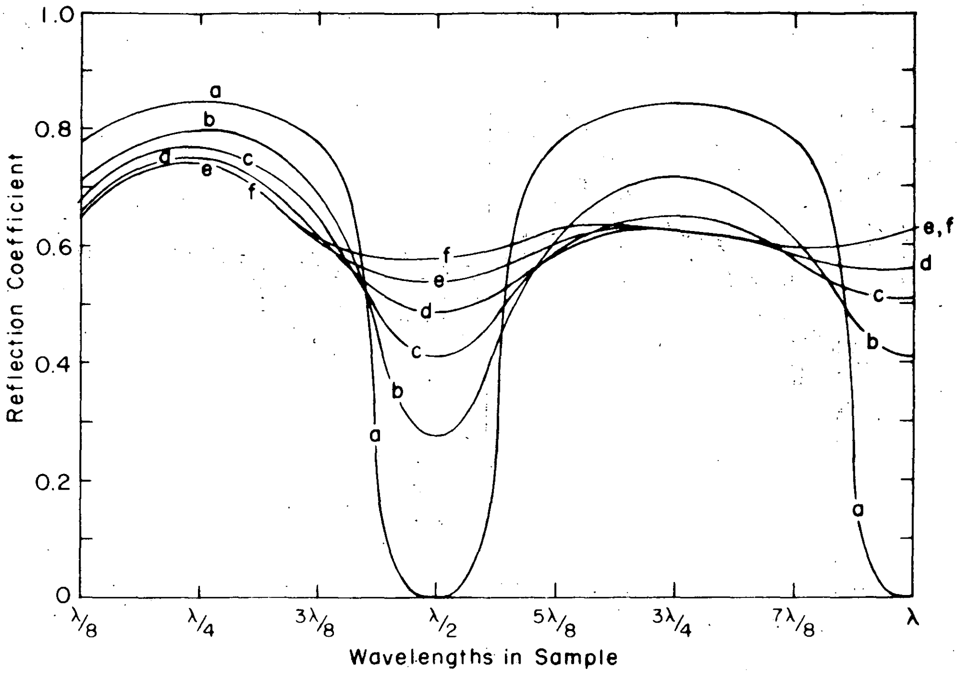


Figure 3a. The amplitude of the reflection coefficients of a TEM wave as a function of the number of wavelengths in the sample. The dielectric constant in curves a, b, c, d, e, and f is 12; and the dielectric losses are 0, 2, 4, 6, 8 and 10 respectively.

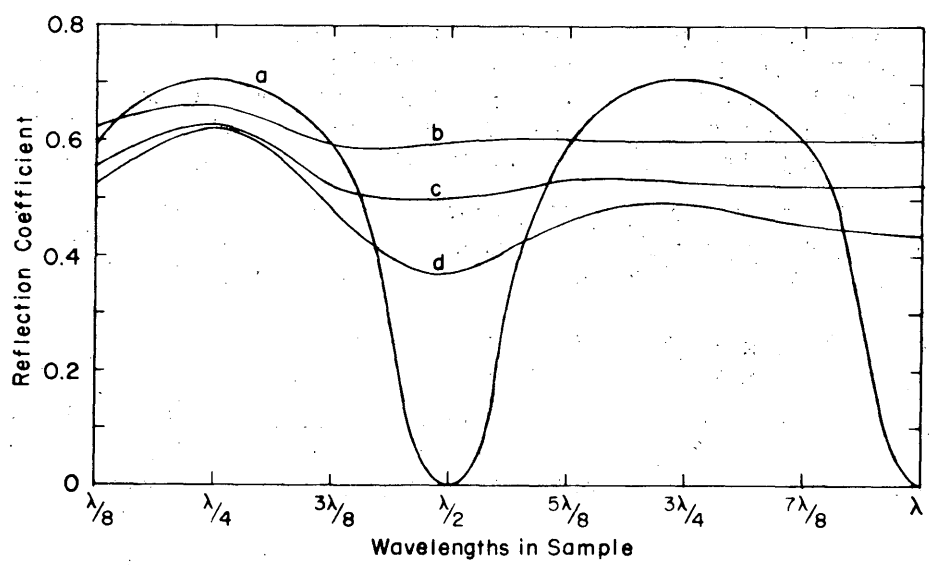


Figure 3b. The amplitude of the reflection coefficients of a TEM wave as a function of the number of wavelengths in the sample. The dielectric constant of medium 2 in curves a, b, c and d is 6 and the dielectric losses are 0, 3, 6 and 10 respectively.

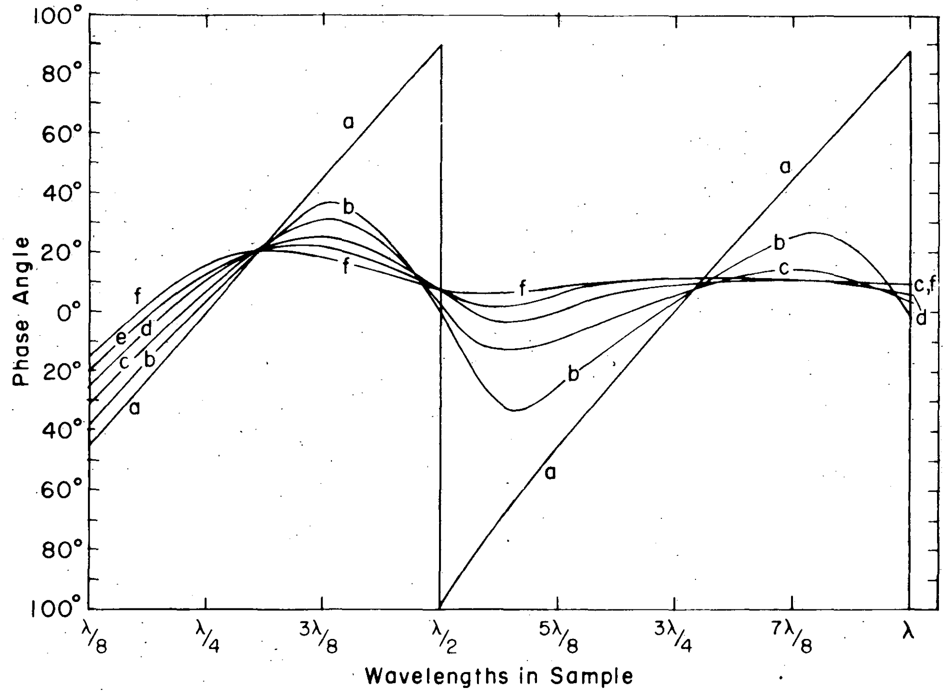


Figure 4. The phase difference between the incident and the reflected TEM wave as a function of the number of wavelengths in the sample. The dielectric constant of medium 2 in curves a, b, c, d, e and f is 12 and the dielectric losses are 0, 2, 4, 6, 8 and 10 respectively.

boundary is of the same amplitude and 180° out of phase with the partial wave reflected from the second boundary, only if $r_{12} = -r_{23}$ (meaning medium 1 is the same as medium 3). When the medium has a dielectric loss the periodic variation is damped and r_0 no longer goes to zero. The variation of r_0 with sample thickness, however, still occurs. The amplitude of the reflected wave, thus, depends on both the thickness d and the dielectric constant of the material.

Similarly, the transmitted wave, and therefore the transmission coefficient, are periodic functions of the number of wavelengths in the samples and lead to similar ambiguities. The amplitude of the transmitted waves as a function of sample thickness is given in Figure 5.

Water-content determination by reflection or transmission measurements on microwaves

The reflection and transmission of microwaves by a medium thickness d were shown to be influenced by d and by the dielectric constants of the medium. The dielectric constants of the medium are a function of the water content, which determines the wavelength of the electromagnetic wave in the sample. The ratio of the wavelength in free space to the wavelength in a dielectric medium is given in eq 27.

If one were to measure water content from 0 to 10% by volume, a change from 4 to 12 in dielectric constant and from 0 to 0.1 in tan δ might be expected. This variation in dielectric constant would be accompanied by a change in the ratio of λ_0/λ_s from approximately 2 to 3.46. Depending on the thickness of the sample, it is evident from Figure 3 that the reflection coefficient could either decrease or increase; the same is true for the transmission coefficient (Fig. 5).

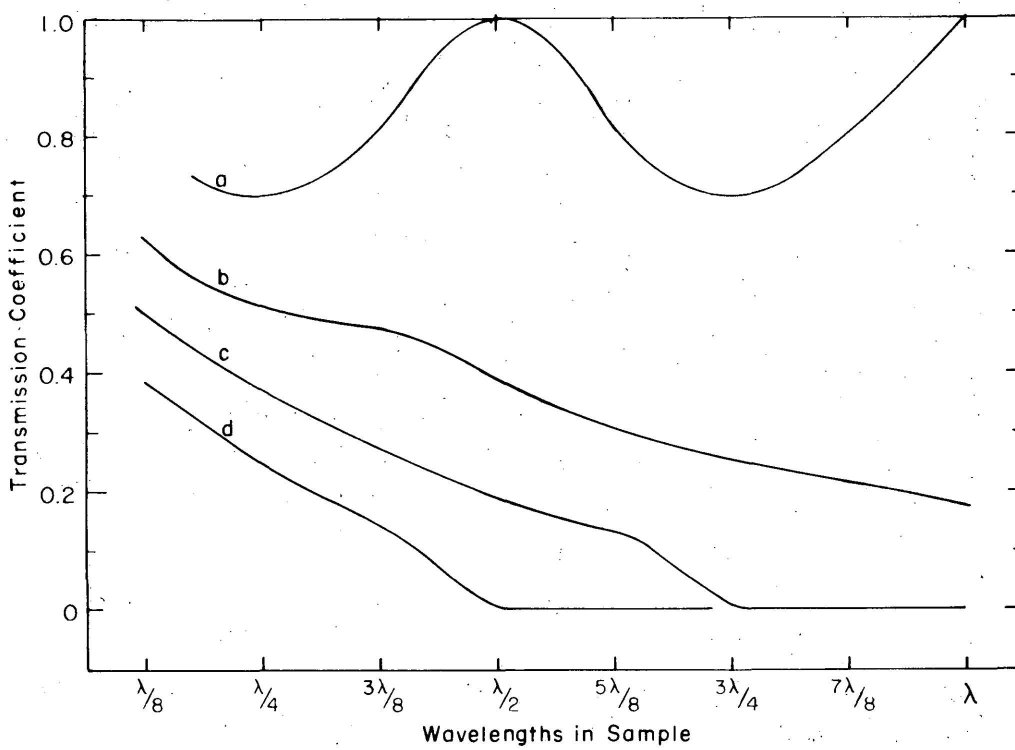


Figure 5. The amplitude of the transmission coefficient as a function of the number of wavelengths in the sample. The dielectric constant of medium 2 in curves a, b, c and d is 6 and the dielectric losses are 0, 3, 6 and 10 respectively.

To illustrate the dependence of reflection and transmission on both the water content and sample thickness the dielectric constants were measured in the frequency range 10^9 to 10^{10} Hz as a function of water content for the clay sodium kaolinite. The results are given in Figures 6a and b. Using these data the reflection and transmission were calculated as a function of water content for various thicknesses and frequencies of medium 2; these data are plotted in Figures 7 and 8 respectively. The result shows that the relation between reflection (or transmission) and water content is not unique, but varies with frequency and thickness of the medium.

The reflection coefficient for a given sample thickness at a given frequency is a function of the number of wavelengths in the sample. The number of wavelengths in the sample, in turn, depends on the dielectric constant $[\lambda_s \sim (\lambda_0/\sqrt{\epsilon^*})]$.

Outlook for microwave moisture sensors

It is evident from the foregoing that microwave sensing of the water content of materials is extremely difficult because of the dependence of reflection and transmission on both the water content (dielectric constants) and thickness of the sample. A calibration made at a particular sample thickness is invalid if the thickness of the sample is changed. However, one can foresee the determination of water content from measurement of the reflection or transmission of microwaves in special situations. These are summarized in the following.

When the number of wavelengths in the sample is adjusted to an odd number of quarter wavelengths. For small changes in water content at a sample thickness of $\lambda_s/4$ the system is least sensitive to perturbations in sample thickness. For example, at a frequency of 10^{10} Hz and a dry

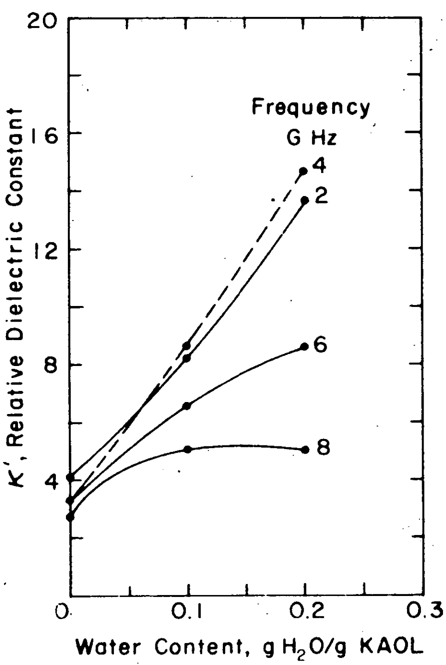


Figure 6a. The dielectric constant of Na-kaolinite as a function of water content at several frequencies.

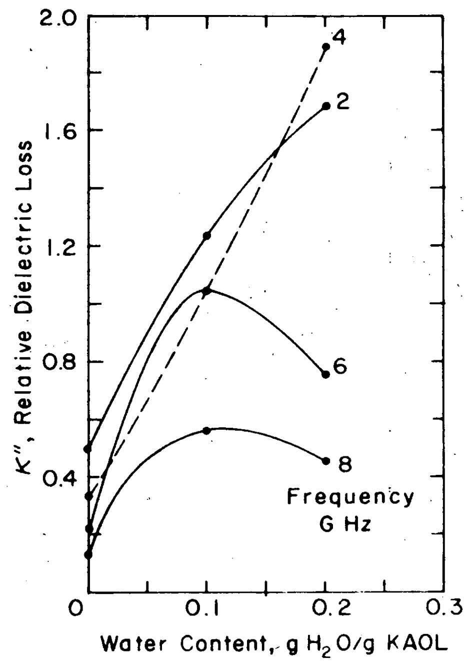


Figure 6b. The dielectric loss of Na-kaolinite as a function of water content at several frequencies.

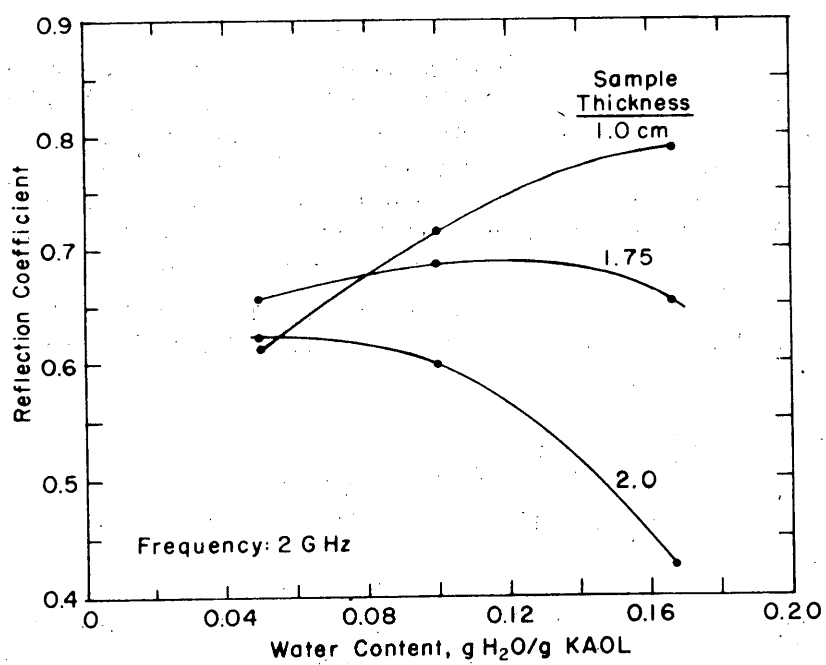


Figure 7. The amplitude of the reflection coefficient of a TEM wave incident on kaolinite samples of different thicknesses.

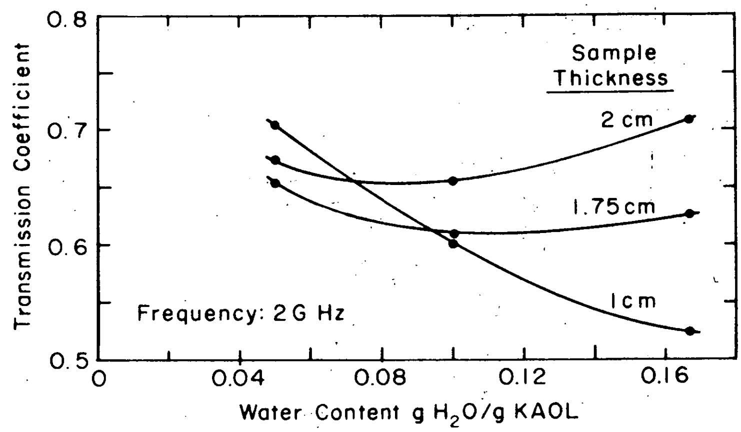


Figure 8. The amplitude of the transmission coefficient of TEM wave incident on kaolinite samples of different thicknesses.

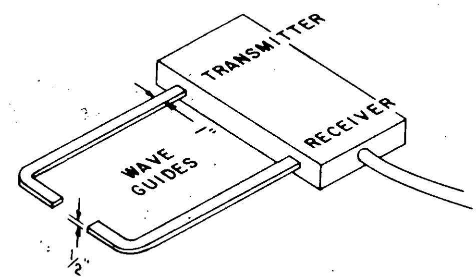


Figure 9. A sketch of a transmission system for measurement of water content in fluids without dielectric loss. The spacing of the sample is fixed.

dielectric constant of 9, the sample thickness at $\lambda_s/4$ would be 0.26 cm. With a water content change of 3% by volume, the number of wavelengths in the sample would change from 0.25 λ_s to perhaps 0.2 λ_s, and a calibration of water content versus transmission or reflection would be unique. Such a setup for transmission is shown in Figure 9 and would be useful for the monitoring of water content in fluids without dielectric loss, such as gasoline, kerosene, and carbon tetrachloride. The transmitter and receiver would be separated by an effective electrical length of $\lambda_s/4$ cm.

When the presence of water must be detected to avoid damage to structures and materials. In detecting wet spots in roof insulation, deviations from the dry system would certainly be found. The only requirement would be that the reflection from the roof should be relatively uniform. Also, if measurements were made at fixed locations periodically, deviations from the average readings would be detected and probably could be attributed to water.

Future studies

For the general problem of moisture measurement in materials of arbitrary (or undefined) thickness and dielectric constant, other avenues are promising for further investigation.

In the analysis set forth, only normal incidence was considered, so that the reflection coefficients for horizontally and vertically polarized waves were identical. In reflection and transmission through media with dielectric loss at oblique incidence, linearly polarized waves become elliptically polarized. Radiating the sample with a horizontally polarized wave and measuring the transmitted vertically polarized component may yield a simple measurement for water content. This configuration is simple in practice: it consists of rotating the receiving wave-guides by 90°.

The dielectric constants of samples containing water are frequency dependent. This generally means that the complex reflection coefficient $re^{j\phi}$ is frequency dependent. When a sample is irradiated with a pulse of a certain spectrum in the frequency domain, the reflected or transmitted pulse has a different spectrum. This is illustrated in Figure 10. The shift in frequency might be related to water content.

During the last year, time domain reflectometry has become commercially available at a reasonable price. It allows the reflection coefficient of the partial waves r_{12} and r_{23}, which are directly

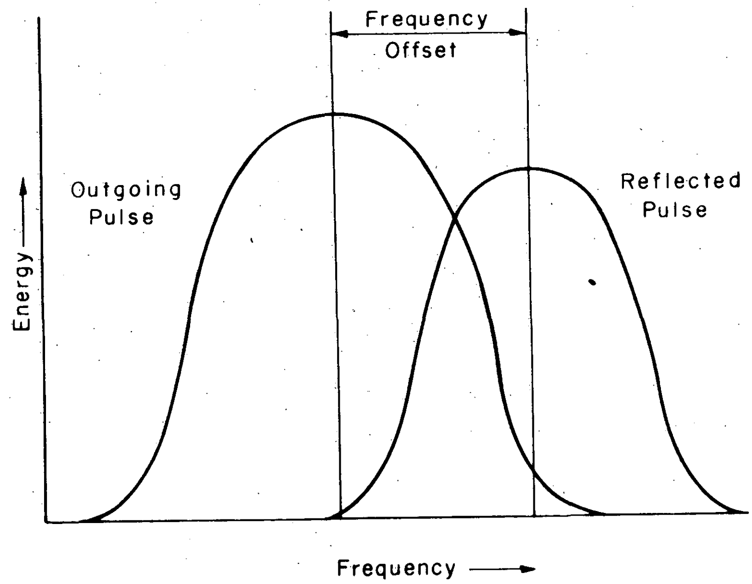


Figure 10. An illustration of how the spectrum of the incident pulse and the reflected pulse may differ in a medium for which the complex dielectric constant is a function of frequency.

related to the dielectric constant (water content), to be measured without being influenced by the thickness of the sample. Furthermore, it allows the layering in the sample and the water content in the layers to be determined. At this time, it seems to be the most promising way to pursue this research.

Selected bibliography

Hippel, A. von (1954) *Dielectrics and waves.* New York: John Wiley and Sons, Inc.

Stratton, J.A. (1941) *Electromagnetic theory.* New York: McGraw-Hill Book Co.

APPENDIX A. COMPUTER PROGRAM

The following program was written in basic computer language for use on the Dartmouth College Time Sharing System. This program is designed to compute the reflection and transmission coefficients of a system of three media and two boundaries. The theoretical derivation was adapted from Stratton's electromagnetic theory.

The first section of the program defines the complex arithmetic needed to evaluate the coefficients.

The propagation factor K is defined as

$$K = w\sqrt{\mu_0 \epsilon^*} .$$

For medium 1, $K = K_1 + jK_2$

$$K = K_3 + jK_4$$

$$K = K_5 + jK_6 .$$

The impedance Z for each boundary is defined as

$$Z_{jk} = \frac{Z_j}{Z_k} = \frac{\mu_j K_k}{\mu_k K_j} .$$

For nonmagnetic media

$$\mu_j = \mu_k = \mu_0 .$$

Therefore

$$Z_{jk} = \frac{K_k}{K_j} .$$

For example, the impedance at the boundary between medium 1 and 2 is

$$Z_{12} = \frac{K_2}{K_1} .$$

E_5 and E_6 are the real and imaginary components of the reflected wave, respectively. E_7 and E_8 are the real and imaginary components of the transmitted wave. R_1 denotes the amplitude reflection coefficient and is defined as the amplitude of the reflected wave (a unit incident wave is assumed). T_1 denotes the transmission coefficient and is defined as the amplitude of the transmitted wave. U_3 and U_4 are the relative phase angles of the reflected and transmitted components respectively

$$R_1 = (E_5^2 + E_6^2)^{1/2}$$

$$T_1 = (E_7^2 + E_8^2)^{1/2}$$

$$U_3 = \text{Arctan}\left(\frac{E_6}{E_5}\right)$$

$$U_4 = \text{Arctan}\left(\frac{E_8}{E_7}\right).$$

```
PUNCH
100 READ L1,L2,L5,L6
110 REM L1=E1,L2=E1',L3=E2,L4=E2',L5=E3,L6=E3'
120 LET L1 = 8.854E-14*L1
130 LET L2 = -(8.85E-14*L2)
140 LET L5 = 8.854E-14*L5
150 LET L6 = -(8.85E-14*L6)
160 READ WO
170 LET L3 = 12*8.854E-14
180 FOR L4 = -10*8.854E-14 TO 0 STEP 2*8.854E-14
190 PRINT "E1 =";L1/8.854E-14,TAB(15);"E2 =";L3/8.854E-14,TAB(30);
200 PRINT "E3 =";L5/8.854E-14
210 PRINT"E1'=";L2/8.854E-14,TAB(15);"E2'=";L4/8.854E-14,TAB(30);
220 PRINT "E3'=";L6/8.854E-14
230 PRINT "FREQUENCY";TAB(16);"THICKNESS";TAB(33);"REF COEF";TAB(46);
240 PRINT"TRANS COEF";
250 PRINT TAB(40);"PHASE ANGS(T,R)"
260 LET S2 = 3E10/(WO*(L3/8.854E-14)↑.5)
270 LET W = WO*6.28
280 FOR D = .125*S2 TO S2 STEP .125*S2
290'*********COMPLEX ARITHMETIC**********
300 REM ADDITION
310 DEF FNC(X1,X2,Y1,Y2) = X1 +X2
320 DEF FNB(X1,X2,Y1,Y2) = Y1+ Y2
330 REM SUBTRACTION
340 DEF FNU(X1,X2,Y1,Y2) = X1 -X2
350 DEF FNT(X1,X2,Y1,Y2) = Y1 - Y2
360 REM MULTIPLICATION
370 DEF FNO(X1,X2,Y1,Y2) = X1*X2 - Y1*Y2
380 DEF FNN(X1,X2,Y1,Y2) = X1*Y2 + X2*Y1
390 REM DIVISION
400 DEF FNF(X1,X2,Y1,Y2) = (X1*X2+Y1*Y2)/(X2↑2+Y2↑2)
410 DEF FNE(X1,X2,Y1,Y2) = (Y1*X2-X1*Y2)/(X2↑2+Y2↑2)
420 LET U = (1.257E-8)↑.5
430 LET K1 = W*U*(L1↑2+L2↑2)↑.25*COS(.5*ATN(-L2/L1))
```

```
440 LET K2 = W*U*(L1↑2+L2↑2)↑.25*SIN(.5*ATN(-L2/L1))
450 LET M1 = EXP(-2*D*K2)*COS(2*D*K1)
460 LET M2 = EXP(-2*D*K2)*SIN(2*D*K1)
470 LET K3 = W*U*(L3↑2+L4↑2)↑.25*COS(.5*ATN(-L4/L3))
480 LET K4=  W*U*(L3↑2+L4↑2)↑.25*SIN(.5*ATN(-L4/L3))
490 LET M3 = EXP(-2*D*K4)*COS(2*D*K3)
500 LET M4 = EXP(-2*D*K4)*SIN(2*D*K3)
510 LET K5 = W*U*(L5↑2+L6↑2)↑.25*COS(.5*ATN(-L6/L5))
520 LET K6 = W*U*(L5↑2+L6↑2)↑.25*SIN(.5*ATN(-L6/L5))
530 LET M5 = EXP(D*K6)*COS(-D*K5)
540 LET M6 = EXP(D*K6)*SIN(-D*K5)
550 LET M7 = EXP(D*K4)*COS(-D*K3)
560 LET M8 = EXP(D*K4)*SIN(-D*K3)
570 LET M9 = EXP(-D*K4)* COS(D*K3)
580 LET M0 = EXP(-D*K4)*SIN(D*K3)
590 REM **********NUMERATOR OF E1**********
600 LET P9 = FNF(K3,K1,K4,K2)
610 LET P0 = FNE(K3,K1,K4,K2)
620 LET A1 = FNU(1,P9,0,P0)
630 LET P7 = FNF(K5,K3,K6,K4)
640 LET P8 = FNE(K5,K3,K6,K4)
650 REM IMAGINARY
660 LET A2 = FNT(1,P9,0,P0)
670 REM REAL
680 LET B1 = FNC(1,P7,0,P8)
690 REM IMAGINARY
700 LET B2 = FNB(1,P7,0,P8)
710 REM REAL
720 LET C1 = FNO(A1,B1,A2,B2)
730 REM IMAGINARY
740 LET C2 = FNN(A1,B1,A2,B2)
750 REM REAL
760 LET A3 = FNC(1,P9,0,P0)
770 REM IMAGINARY
780 LET A4 = FNB(1,P9,0,P0)
790 REM REAL
800 LET B3 = FNU(1,P7,0,P8)
810 REM IMAGINARY
820 LET B4 = FNT(1,P7,0,P8)
830 REM REAL
840 LET C3 = FNO(A3,B3,A4,B4)
850 REM IMAGINARY
860 LET C4 = FNN(A3,B3,A4,B4)
870 REM REAL
880 LET D1 = FNO(C3,M3,C4,M4)
890 LET D2 = FNN(C3,M3,C4,M4)
900 REM REAL
910 LET N1 = FNC(C1,D1,C2,D2)
920 REM IMAGINARY
930 LET N2 = FNB(C1,D1,C2,D2)
940 REM **********DENOMINATOR OF E1**********
950 REM REAL
960 LET A5 = FNO(A3,B1,A4,B2)
970 REM IMAGINARY
980 LET A6 = FNN(A3,B1,A4,B2)
990 REM REAL
```

```
1000 LET A7 = FNO(A1,B3,A2,B4)
1010 LET A8 = FNN(A1,B3,A2,B4)
1020 LET C5 = FNO(A7,M3,A8,M4)
1030 LET C6 = FNN(A7,M3,A8,M4)
1040 REM REAL
1050 LET N3 = FNC(A5,C5,A6,C6)
1060 REM IMAGINARY
1070 LET N4 = FNB(A5,C5,A6,B6)
1080 REM REAL
1090 LET E5 = FNF(N1,N3,N2,N4)
1100 REM IMAGINARY
1110 LET E6 = FNE(N1,N3,N2,N4)
1120 LET F1 = FNO(4,M5,0,M6)
1130 LET F2 = FNN(4,M5,0,M6)
1140 LET F3 = FNO(A7,M9,A8,M0)
1150 LET F4 = FNN(A7,M9,A8,M0)
1160 LET F5 = FNO(A5,M7,A6,M8)
1170 LET F6 = FNN(A5,M7,A6,M8)
1180 REM **********DENOMINATOR OF E3**********
1190 REM REAL
1200 LET F7 = FNC(F3,F5,F4,F6)
1210 REM IMAGINARY
1220 LET F8 = FNB(F3,F5,F4,F6)
1230 REM REAL
1240 LET E7 = FNF(F1,F7,F2,F8)
1250 REM IMAGINARY
1260 LET E8 = FNE(F1,F7,F2,F8)
1270 REM E3 = E7+E8I
1280 GO TO 1330
1290 PRINT "REFLECTED WAVE",TAB(30);"TRANSMITTED WAVE"
1300 PRINT TAB(3);E5;"+";E6;"I",TAB(33);E7;"+";E8;"I"
1310 PRINT
1320 PRINT
1330 REM IGNORE
1340 LET R = E5↑2 + E6↑2
1350 LET R1 = SQR(R)
1360 LET T = E7↑2 + E8↑2
1370 LET T1 = SQR(T)
1380 LET U3 = ATN(E6/E5)*180/3.14159
1390 LET U4 = ATN(E8/E7)*180/3.14159
1400 PRINT W/6.28;
1410 PRINT TAB(13);D;"CM";
1420 PRINT TAB(23);R1;
1430 PRINT TAB(33);T1;
1440 PRINT TAB(43);U3"**"U4
1450 NEXT D
1460 PRINT
1470 NEXT L4
1480 GO TO 160
1490 GO TO 100
1500 DATA 1,0,1,0
1510 DATA 1E8
1520END
READY
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13. ABSTRACT
Microwave instrumentation is used for nondestructive measurement of the water content of materials. The basis of all microwave moisture sensors is that the dielectric constants of material that contains water are a strong function of water content. The microwave moisture sensors based on a reflection or transmission principle are shown to have the disadvantage of requiring that a calibration be made for each sample thickness. Several alternative routes for developing reliable microwave moisture sensors are discussed.

14. Key Words

Microwave equipment
Moisture content detection of soils and materials