Research Report 186

A STUDY OF A
NEW FOUNDATION MODEL

by

Arnold D. Kerr

MARCH 1966

PREFACE

This paper is a part of work by Dr. Kerr*, Expert, for the Applied Research Branch (A. F. Wuori, Chief), Engineering Division (K. A. Linell, Chief), U. S. Army Cold Regions Research and Engineering Laboratory (USA CRREL). The author wishes to thank Dr. A. Assur, Scientific Advisor, and D. E. Nevel, Applied Research Branch, for reviewing the manuscript.

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CONTENTS

ILLUSTRATIONS

SUMMARY

The characteristics of a new foundation model, consisting of two spring layers interconnected by a shear layer, are studied. The study is conducted on the classical problem of a foundation subjected to a rigid stamp. In order to reduce the number of foundation constants to an absolutely necessary minimum, special attention is given to a possible dependence of the constants of the upper and lower spring layers, particularly to the spring constant ratio three which is suggested by Reissner's foundation model. A comparison of the obtained pressure distributions with relevant experimental data seems to support the adoption of this value, thus reducing the number of foundation constants to two. Advantages of the presented model over other foundation models are pointed out.

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INTRODUCTION

In a recent paper the author discussed a number of foundation models presented in the literature in the past several decades (Kerr, 1964). One conclusion was that the Pasternak foundation (Pasternak, 1954), consisting of a Winkler foundation with shear interactions, is mechanically the most logical extension of the Winkler model and analytically the next higher approximation. The response of the surface of the Pasternak foundation subjected to a distributed load p(x, y) is described by
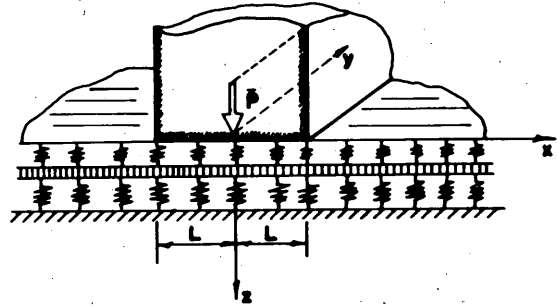

Figure 1. Suggested foundation model.

$$p = k_p w - G_p \nabla^2 w \qquad (1)$$

where k_p and G_p are the foundation parameters, $\underline{w}$ is displacement, and $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$.

In the same paper the Pasternak foundation was formulated in a manner suitable for further generalizations. Using these results a foundation model (Fig. 1) was introduced and formulated. The response of the foundation surface subjected to a continuously distributed load p(x, y) was found to be governed by the differential equation

$$\left(1 + \frac{k}{c}\right)p - \frac{G}{c} \nabla^2 p = kw - G \nabla^2 w \qquad (2)$$

where $\underline{w}$ is the displacement in the z direction, $\underline{c}$ is the spring constant of the upper spring layer, $\underline{k}$ is the spring constant of the lower spring layer, and $\underline{G}$ is the constant of the shear layer.

It was subsequently pointed out that for c = 3k eq 2 coincides with the corresponding equation obtained by E. Reissner (1958) if we set

$$k = \frac{4}{3} \frac{E_f}{H} ; \qquad G = \frac{4}{9} H G_f$$

where E_f is Young's modulus, G_f the shear modulus of the foundation material, and H the thickness of the foundation. Reissner's foundation model consists of an elastic layer resting on a rigid base, subjected to the assumptions that throughout the layer the in-plane stresses $\sigma_x = \sigma_y = \tau_{xy} = 0$ and additionally the horizontal displacements on the upper and lower surface are zero. With c = 3k eq 2 reduces to

$$\frac{4}{3} p - \frac{G}{3k} \nabla^2 p = kw - G \nabla^2 w. \qquad (3)$$

Hence, although the Pasternak foundation covered by a spring layer with constant c = 3k and Reissner's foundation are mechanically two different models, the vertical displacements of their surfaces subjected to p(x, y) are governed essentially by the same differential equation.

In an attempt to reduce the constants in a foundation model to an absolutely necessary minimum, one is tempted   by this coincidence to assume the value 3k for the upper spring layer, thus reducing the number of foundation constants to two: k and G.   One advantage of describing the response of the foundation surface by only two constants is that their experimental determination can proceed along lines suggested by Pasternak for his model.   An advantage of the suggested model over other models (such as the isotropic elastic continuum, Reissner's or Pasternak's foundation) is that because of the upper spring layer no concentrated reactions or infinite reaction pressures can appear, even along the edge of a rigid stamp.

An additional advantage of the suggested model over the Pasternak model is that an additional boundary condition is available to specify the restraints on the foundation (see Kerr, 1964) which in some cases may noticeably affect the behavior of the structure.

The present paper is part of an attempt to establish a rational representation (mathematically as well as mechanically) of the response of continuous foundations to structures of various types.   The study herein is conducted on the classical problem of a foundation subjected to a rigid stamp.

FOUNDATION LOADED CENTRALLY BY A RIGID STAMP

A rigid stamp of width 2 L is resting on the foundation and is subjected to a line load $\overline{P}$ as shown in Figure 1.   Since $\overline{P}$ acts centrally, the deflection under the stamp, w_0, is constant.

For the purpose of derivation of the governing differential equations, we assume that the deflection at a point on the foundation surface in $|x| \leq L$ consists of two parts

$$w = w_{ci} + w_{si} \tag{4}$$

where w_{ci} and w_{si} are the contractions or extensions of the upper and lower spring layer respectively.   For the region $|x| \geq L$ the index i is replaced by e.

Denoting the contact pressure between foundation and rigid body by p(x, y), we may write

$$p = cw_{ci} \tag{5}$$

and also

$$p = kw_{si} - G \nabla^2 w_{si}. \tag{6}$$

Performing the operation $\left(\dfrac{k}{c} - \dfrac{G}{c} \nabla^2\right)$ on eq 5, adding the resulting equation to eq 6, and noting eq 4 we obtain

$$\left(1 + \frac{k}{c}\right)p - \frac{G}{c} \nabla^2 p = kw - G \nabla^2 w. \tag{7}$$

This is the relation between the contact pressure and the deflection of foundation surface in $|x| \leq L$.   Noting that in $|x| \leq L$, $w = w_0 = $ const, and in the present case p and w are functions of $\underline{x}$ only, the differential equation for p(x) assumes, with $\underline{c} = nk$, the form

$$\frac{d^2 p}{dx^2} - (1 + n) \frac{k}{G}p = - \frac{nk^2}{G} w_0. \tag{8}$$

Since no boundary conditions can be prescribed for p(x) at x = ±L, the pressure distribution in the contact area cannot be derived directly from eq 8.   In the following this is achieved by formulating the problem in terms of w_{si} and w_{se}, obtaining the expression for w_{si}, and then inserting it into eq 6.

The differential equation for w_{si} is derived by eliminating p from eq 5 and 6 and noting that according to eq 4 $w_{ci} = w - w_{si}$. With c = nk the resulting equation assumes the form

$$\nabla^2 w_{si} - (1 + n) \frac{k}{G} w_{si} = -n \frac{k}{G} w. \tag{9}$$

Since in $|x| \leqq L$, $w = w_0$ = const and $w_{si} = w_{si}(x)$, eq 9 reduces to

$$\frac{d^2 w_{si}}{dx^2} - (1 + n) \frac{k}{G} w_{si} = -n \frac{k}{G} w_0. \tag{10}$$

Its general solution is

$$w_{si} = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + \frac{n}{1+n} w_0 \tag{11}$$

where $\lambda = \sqrt{(1+n) \, k/G}$.

The deflections of the shear layer outside the contact area $|x| \geqq L$ are governed, according to eq 6, by the differential equation

$$\frac{d^2 w_{se}}{dx^2} - \frac{k}{G} w_{se} = 0. \tag{12}$$

Its general solution is

$$w_{se} = C_3 e^{+\mu x} + C_4 e^{-\mu x} \tag{13}$$

where $\mu = \sqrt{k/G}$.

Because of symmetry with respect to x = 0, it is sufficient to treat the problem only in $x \geq 0$. The constants C_1, C_2, C_3 and C_4 are determined from the following four conditions:

$$w_{se} \text{ is finite at } x = \infty; \qquad\qquad w_{si}(L) = w_{se}(L)$$

$$\left. \frac{dw_{si}}{dx} \right|_{x=L} = \left. \frac{dw_{se}}{dx} \right|_{x=L}; \qquad \left. \frac{dw_{si}}{dx} \right|_{x=0} = 0 \tag{14}$$

Substitution of the resulting w_{si} into eq 6 yields the pressure distribution in $0 \leqq x \leqq L$

$$p_P(x) = \frac{nk}{1+n} w_0 \left\{ 1 + \frac{n \cosh(\lambda x)}{\cosh(\lambda L) + \sqrt{1+n} \, \sinh(\lambda L)} \right\}. \tag{15}$$

The relation between $\overline{P}$ and w_0 is obtained from the equilibrium equation of the stamp in the z-direction

$$\overline{P} = 2 \int_0^{+L} p_P(x) dx. \tag{16}$$

Performing the integration it follows

$$\overline{P} = 2kL \frac{n}{1+n} w_0 \left\{ 1 + \frac{n}{\lambda L \left[\operatorname{ctgh}(\lambda L) + \sqrt{1+n} \, \right]} \right\}. \tag{17}$$

Elimination of w_0 from eq 15 by means of eq 17 results in

$$\frac{2L}{\overline{P}}\, p_P(x) = \frac{\lambda L[\cosh(\lambda L) + \sqrt{1+n}\ \sinh(\lambda L) + n \cosh(\lambda x)]}{\lambda L[\cosh(\lambda L) + \sqrt{1+n}\ \sinh(\lambda L)] + n \sinh(\lambda L)}\ . \qquad (18)$$

The corresponding reactions in the case of a Pasternak foundation consist of a uniform pressure in the contact area

$$\frac{2L}{\overline{P}}\, p_P = \frac{L\sqrt{k_P/G_p}}{L\sqrt{k_P/G_p} + 1} \qquad (19)$$

and concentrated reactions

$$\overline{p} = \frac{\overline{P}}{2}\ \frac{1}{1 + L\sqrt{k_P/G_P}} \qquad (20)$$

along $x = \pm L$ (Vlasov and Leontiev, 1960, p. 101). At this point it should be noted that when $c \to \infty$, i.e. $n \to \infty$, the suggested model reduces to that of a Pasternak foundation.

The distribution of reactions for the case of an elastic (linear) foundation is

$$\frac{2L}{\overline{P}}\, p_P(x) = \frac{2}{\pi\sqrt{1-(x/L)^2}} \qquad (21)$$

and does not depend upon constants of the foundation material (Sadowski, 1928).

The results of the numerical evaluation of eq 18 for various values of the parameters n and $L^2 k/G$ and the evaluation of eq 21 are presented in Figure 2 and Figure 3.

In Figure 2 pressure distributions are shown for $L^2 k/G = 5$ and various values of n. It can be seen that for large values of n (let us say $n = 1000$) the pressure distribution is constant under the stamp except in the vicinity of the edges, where it increases very rapidly and, with further increasing n, approaches the representation of concentrated line reactions. It can be easily verified that the pressure distribution for $n = 1000$ already agrees closely with the distribution obtained from eq 19 assuming $k_P/G_P = k/G$.
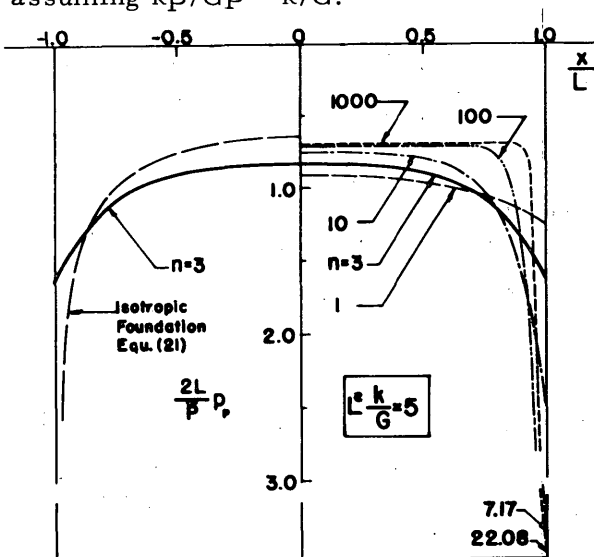

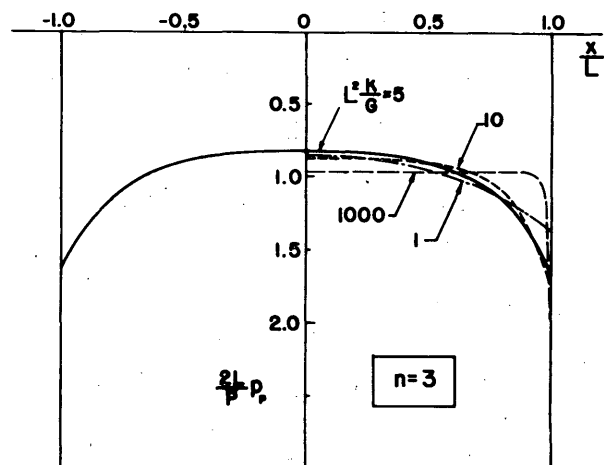Figure 2. Pressure distribution under centrally loaded stamp for various values of $\underline{n}$.

Figure 3. Pressure distribution under centrally loaded stamp for various values of $L^2 k/G$.

In this connection it should be pointed out that V. Z. Vlasov and N. N. Leontiev (1956; 1960) obtained an expression similar to eq 18, considering a two-layered foundation and assuming that the upper layer is very thin and hence its shear interactions are negligible. They concluded that when $c \to \infty$ the pressure distribution approaches that of a semi-infinite elastic solid; in view of the above findings, this is incorrect.

In Figure 3 pressure distributions are shown for $n = 3$ and different values of $L^2 k/G$. It can be seen that when $k/G \to \infty$, i.e. $G \to 0$, the pressure distribution approaches the constant distribution corresponding to a Winkler model with an equivalent spring constant $kc/(k+c)$.

Comparing the pressure distributions presented in Figure 2 and Figure 3 as well as the corresponding ratios $p(L)/p(o)$ with some pressure distributions obtained from experiments, it seems that for a number of foundation materials, such as clay (immediately after load application) (Faber, 1933) and well compacted granular media (Siemonsen, 1948), $c = 3k$ is indeed a reasonable assumption.

FOUNDATION LOADED ECCENTRICALLY BY A RIGID STAMP

In this case the load acting on the rigid stamp consists of a centrally acting load $\overline{P}$ and a moment $\overline{M}$. Because of the linearity of the formulation, the displacements $\underline{w}$ may be considered to consist of a translatory displacement and a rotation, each part treated separately, and the effects added as shown symbolically in Figure 4. Since the centrally loaded case was treated in the previous section, the rigid stamp subjected to a moment is investigated in the following.

The procedure is as before. The governing differential equations are eq 10 and 12, noting that in the present case, assuming relatively small angles of rotation α,

$$w = w_0 = \alpha x \tag{22}$$

in $|x| \leq L$.

The corresponding general solutions are

$$w_{si} = B_1 e^{+\lambda x} + B_2 e^{-\lambda x} + \frac{n}{1+n} \alpha x \tag{23}$$

and

$$w_{se} = B_3 e^{+\mu x} + B_4 e^{-\mu x}. \tag{24}$$

Since the problem is anti-symmetric with respect to $x = 0$, it is sufficient to treat it only for $x \geq 0$. The constants B_1, B_2, B_3 and B_4 are determined from the following four conditions:

$$\left. \begin{array}{ll} w_{se} \text{ is finite at } x = \infty \ ; & w_{si}(L) = w_{se}(L) \\[2mm] \dfrac{dw_{si}}{dx}\bigg|_{x=L} = \dfrac{dw_{se}}{dx}\bigg|_{x=L} \ ; & w_{si}(0) = 0 \end{array} \right\} . \tag{25}$$

Substituting the resulting w_{si} into eq 6 we obtain the contact pressure in $0 \leq x \leq L$

$$p_M(x) = \frac{n^2 k(\alpha L)}{1+n} \left\{ \frac{(1 + \mu L)\, \sinh(\lambda x)}{(\mu L)[\,\sinh(\lambda L) + \sqrt{1+n}\, \cosh(\lambda L)]} + \frac{1}{n}\, \frac{x}{L} \right\} . \tag{26}$$

The relation between $\overline{M}$ and α is derived from the moment equilibrium equation of the stamp

$$\overline{M} = 2 \int_0^L p_M x \, dx \tag{27}$$

which yields

$$\overline{M} = \frac{2nk}{3\lambda^2(1+n)} (\lambda L)^2 (\alpha L) \left\{ 1 + \frac{3n(\mu L+1)[(\lambda L)\cosh(\lambda L) - \sinh(\lambda L)]}{(\mu L)(\lambda L)^2 [\sinh(\lambda L) + \sqrt{1+n} \cosh(\lambda L)]} \right\} . \tag{28}$$

Elimination of α from eq 26 by means of eq 28 and noting that $\overline{M} = \overline{P}e$ results in

$$\frac{2L}{\overline{P}} p_M(x) = \frac{3\frac{e}{L}(\lambda L)^2 \{n(1+\mu L)\sinh(\lambda x) + [\sinh(\lambda L) + \sqrt{1+n} \cosh(\lambda L)](\mu x)\}}{(\mu L)(\lambda L)^2 [\sinh(\lambda L) + \sqrt{1+n} \cosh(\lambda L)] + 3n(1+\mu L)[(\lambda L)\cosh(\lambda L) - \sinh(\lambda L)]} . \tag{29}$$

The corresponding reactions in the case of a Pasternak foundation (Vlasov and Leontiev, 1960, p. 102) consist of a linearly varying pressure in the contact area

$$\frac{2L}{\overline{P}} p_M(x) = \frac{3e}{L} \frac{(\mu L)(\mu x)}{[(\mu L)^2 + 3(\mu L) + 3]} \tag{30}$$

and concentrated reactions

$$\overline{p} = \frac{3\overline{P}e(1+\mu L)}{2L[(\mu L)^2 + 3(\mu L) + 3]} \tag{31}$$

along $x = \pm L$. The distribution of reactions for the case of an elastic (linear) solid is (Florin, 1936)

$$\frac{2L}{\overline{P}} p_M(x) = \frac{4e}{L} \frac{(x/L)}{\pi\sqrt{1-(x/L)^2}} \tag{32}$$

and does not depend upon constants of the foundation material.

In Figure 5 pressure distributions are shown for $L^2 k/G = 5$ and various values of n. It can be seen that, for large values of n, the pressure distribution varies linearly under the stamp, except in the vicinity of the edges where it increases very rapidly, and with further increase in n approaches the representation of concentrated line reactions. It can be verified that, for example, the pressure distribution for n = 1000 already agrees closely with the distribution obtained from eq 30, assuming $kp/Gp = k/G$.

In Figure 6 pressure distributions are shown for n = 3 and different values of $L^2 k/G$. It can be seen that when $k/G \to \infty$, i.e. $G \to 0$, the pressure distribution approaches the linearly varying distribution corresponding to a Winkler model with an equivalent spring constant $kc/(k+c)$.
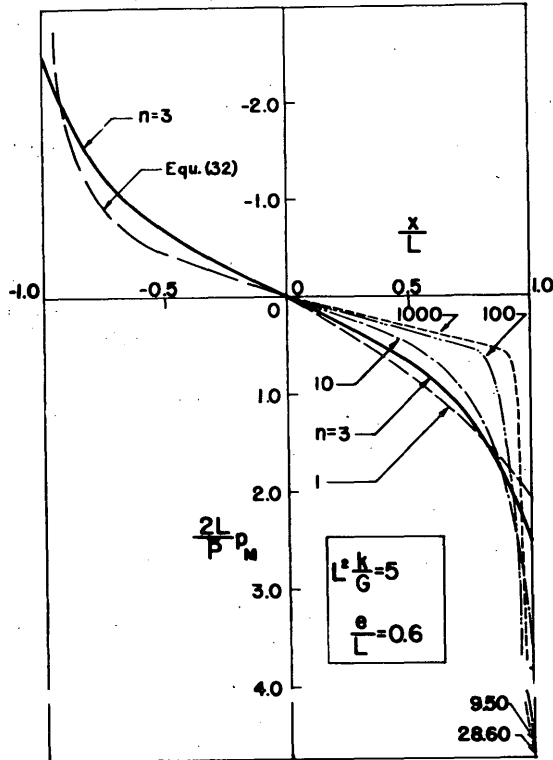

Figure 4. Stamp subjected to an eccentric load P.

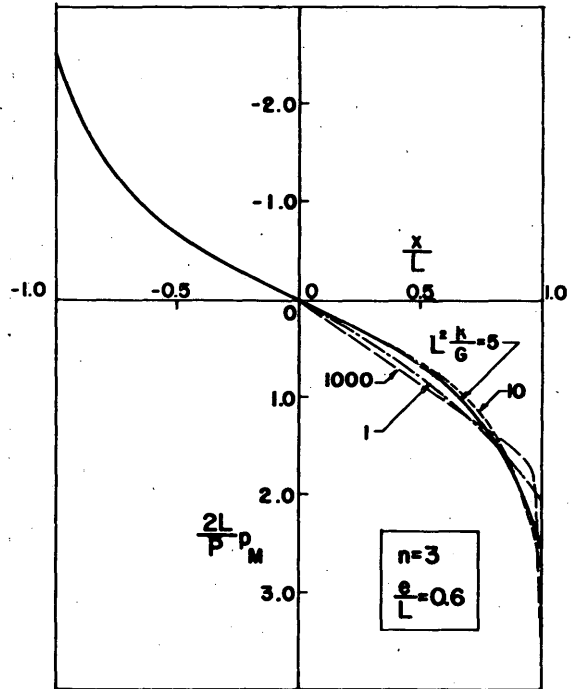Figure 5. Pressure distribution under stamp subjected to moment M for various values of n.

Figure 6. Pressure distribution under stamp subjected to moment M for various values of $L^2 k/G$.

The pressure distribution for the case of an eccentrically loaded rigid stamp is obtained by superposing the corresponding pressures from eq 18 and eq 29. In this connection it should be noted that for large eccentricities tension will occur in the contact area. For situations where no tension can take place (for example when a soil foundation is loaded by a stamp) for large e-values the above analysis will have to be modified.

SOME ADDITIONAL CHARACTERISTICS OF THE SUGGESTED FOUNDATION MODEL

In a recent paper M. I. Gorbunov-Posadov (1963) reviewed some shortcomings of the weightless isotropic elastic continuum when used as a soil foundation model, particularly when subjected to structures with large bearing areas. He demonstrated his major point—the effect of the bearing area upon the vertical displacements—by an example of a semi-infinite foundation subjected to a rigid stamp of square cross section of area a^2.

Based on experimental data of H. Press, F. Kögler, D. E. Polshin, and others, he presents for soil foundations a typical stamp size versus vertical displacement curve for a constant average bearing pressure $p_{av} = P/a^2$ (Fig. 7). For very small bearing areas this curve is monotonically decreasing which is due to the diminishing predominance of "plastic" deformations in the base along the stamp edge; with increasing bearing area the curve rises linearly which agrees with the solution of an isotropic elastic solid, but then with further increase of the bearing area the curve levels off which contradicts the solution of the isotropic elastic semi-infinite solid, but agrees with the results for the Winkler foundation or the isotropic foundation of finite depth.

After discussing various approaches to bring into agreement the analytical results of the weightless semi-infinite elastic base and the experimental data, Gorbunov-Posadov finally suggests as a solution the treatment of the semi-infinite elastic continuum subjected simultaneously to structure and gravity forces—generally a rather complex problem.

In this connection it is of interest to investigate the corresponding behavior of the foundation model discussed in the present paper. For this purpose eq 17 is rewritten as follows:

Figure 7. Typical stamp size $\bar{a}$ versus displacement $\bar{w}$ graph for soils. (From Gorbunov-Posadov, 1963.)

$$\frac{w_0 k}{p_{av}} = \frac{1+n}{n} \frac{1}{1 + \dfrac{n}{(\lambda L)[\operatorname{ctgh}(\lambda L) + \sqrt{1+n}]}} \tag{33}$$

where $p_{av} = \overline{P}/2L$ is the average bearing pressure. It can be seen that as $L \to 0$, eq 33 reduces to

$$\left(\frac{w_0 k}{p_{av}}\right)_{L=0} = \frac{1}{n} \tag{34}$$

as $L \to \infty$ it reduces to

$$\left(\frac{w_0 k}{p_{av}}\right)_{L=\infty} = \frac{1+n}{n} \tag{35}$$

and that

$$\left(\frac{w_0 k}{p_{av}}\right)_{L=\infty} - \left(\frac{w_0 k}{p_{av}}\right)_{L=0} = 1. \tag{36}$$

The numerical evaluation of eq 33 for $n = 3$ and $\overline{k}/G = 5$ is shown in Figure 8. Comparing Figure 7 and Figure 8, it can be concluded that, except for very small bearing areas where plastic effects predominate, the response of the presented foundation model is similar to that observed in experiments with soil foundations. This result is not unexpected since the suggested model, like the Winkler and Pasternak foundations, is essentially a model of finite depth.

In this connection it should be noted that k and G, like the foundation modulus of the Winkler foundation, depend upon the depth of the foundation layer H.

In the present paper the analytical results were compared with data of only a few experiments found in the literature on soil mechanics. In order to establish the range of validity of the suggested model for a number of foundation materials, a comparative study with systematic carefully conducted tests is in order. Preliminary tests are presently being conducted which should contribute to the clarification of some of the problems mentioned above.
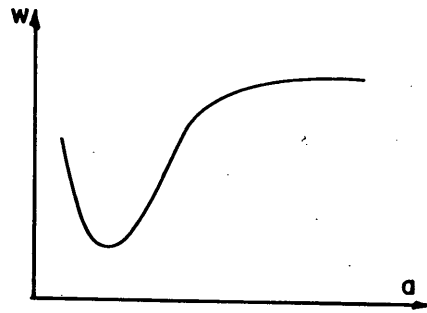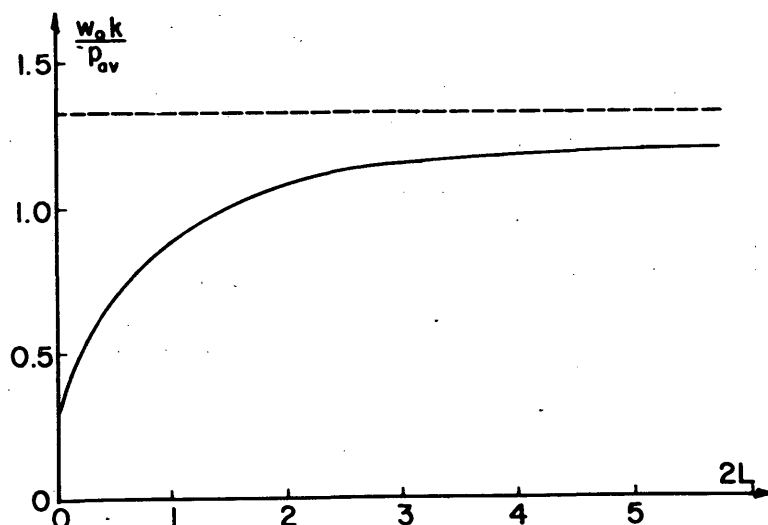
Figure 8. Stamp size versus displacement graph of suggested
model for n = 3 and k/G = 5.

In conclusion it should be mentioned that the suggested model may be easily extended into the viscoelastic range (Kerr, 1961).

Because of the above features and the fact that the response of the suggested foundation model is described by a relatively simple mathematical expression, the model also seems to be very suitable for the analytical treatment of continuously supported flexible beams, plates, and shells of various shapes.

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13. ABSTRACT

The characteristics of a new foundation model, consisting of 2 spring layers interconnected by a shear layer, are studied. The study is conducted on the classical problem of a foundation subjected to a rigid stamp. In order to reduce the number of foundation constants to an absolutely necessary minimum, special attention is given to a possible dependence of the constants of the upper and lower spring layers, particularly to the spring constant ratio 3 which is suggested by Reisaner's foundation model. A comparison of the obtained pressure distributions with relevant experimental data seems to support the adoption of this value, thus reducing the number of foundation constants to 2. Advantages of the presented model over other foundation models are pointed out.

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