Research Report 172

A THEORETICAL INVESTIGATION ON THE EFFECT OF MELTING ON FORCED CONVECTION HEAT TRANSFER

by

Chi Tien
and
Yin-Chao Yen

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PREFACE

This paper was prepared by Dr. Tien, Expert, and Dr. Yen, Research Chemical Engineer, of the Materials Research Branch, Research Division (James A. Bender, Chief), U. S. Army Cold Regions Research and Engineering Laboratory (USA CRREL).
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SUMMARY

The effect of melting on convective heat transfer between a melting body and surrounding fluid was studied quantitatively from the point of view of boundary layer theory, film theory and penetration theory. These studies indicate that melting retards the rate of heat transfer and the decrease in heat transfer coefficient is found to be a unique function of the parameter $C_p \Delta T / \Delta H_m$, where ΔT is the temperature difference between the fluid and melting body, C_p is the heat capacity of the fluid, and ΔH_m is the enthalpy change due to melting.

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Introduction

In the study of the melting rate of a solid body surrounded by warm fluid, such as a drifting iceberg in sea water, an accurate prediction of the heat transfer coefficient between the solid and fluid is of the utmost importance since the transfer coefficient is the critical factor which determines the rate of energy exchange. Although there have been numerous investigations on the convective heat transfer between a solid and fluid under various conditions, most of the studies do not take into account the interfacial velocity resulting from melting. As demonstrated in the study of mass transfer (Bird et al., 1960; Stewart, 1950), the interfacial velocity could greatly disturb the final velocity and temperature profiles and significantly influence the values of the transfer coefficient.

In an earlier study, the effect of melting on heat transfer was studied (Yen and Tien, 1963) for the Leveque problem. The tangential velocity profile was assumed to be linear. This approximation is valid if one deals with a high Prandtl number fluid so that the significant temperature change takes place only within a thin layer of fluid immediately adjacent to the solid boundary and consequently the velocity profile inside this thin layer can be approximated by a linear segment. Furthermore, the flow was assumed to be laminar. Because of these restrictions, it may not be justified to extend the results to more complicated situations.

The purpose of the present investigation is to extend the previous study on the effect of melting on the convective heat transfer rate (Yen and Tien, 1963). For the case of laminar flow, the two-dimensional equations of motion and energy are solved exactly. In order to extend the study to other conditions such as natural convection heat transfer or forced convective heat transfer under turbulent conditions, the effect of melting on heat transfer was also examined on the basis of two familiar theories (film theory and penetration theory) which have been used extensively in mass transfer processes. Although the quantitative results based on each of these theories are different, they all exhibit the same qualitative trend and indicate that, under certain conditions, the effect of melting can be very significant.

Symbols

A	dimensionless parameter defined as $C_p\,(T_\infty - T_0)/\Delta H_m$
C_p	heat capacity of fluid
E	energy flux vector
f	a quantity defined by eq 9
H	enthalpy of fluid
H_0	enthalpy of fluid at temperature T_0 (or T_m) per unit mass
ΔH_m	enthalpy change due to melting
h_m	heat transfer coefficient with melting
h_{wm}	heat transfer coefficient without melting
k	thermal conductivity of fluid
K	a dimensionless quantity, defined by eq 10

N	mass flux
N_0	mass flux from surface due to melting
Pr	Prandtl number defined as $C_p \mu/k$
q	heat flux due to conduction
R_T	dimensionless quantity defined by eq 25 or 28
t	time
T	temperature
T_m	melting temperature
T_∞	temperature of fluid at infinity
U	internal energy per unit mass of fluid
V	velocity vector
v	velocity
v_∞	velocity of fluid at infinity
x, y	coordinates
α	thermal diffusivity of fluid
Λ	dimensionless parameter, equal to unity (with v as subscript) and to Pr (with T as subscript)
ρ	density of fluid
ϕ	dimensionless quantity defined by eq 38
ϕ_T	dimensionless quantity defined by eq 26
η	dimensionless variable, equal to $(y/2)(v_\infty/\nu x)^{\frac{1}{2}}$
θ_T	ratio of h_m to h_{wm}
Π	dimensionless profile for velocity (with v as subscript) or for temperature (with T as subscript)
Π'	derivation of Π with respect to variable η
π	stress term of fluid
μ	viscosity
ν	kinematic viscosity

Boundary layer theory

If the fluid is flowing laminarly over (or under) the solid, e.g., a piece of ice sheet floating in a river, the melting effect on heat transfer can be studied exactly. For a steady-state, two-dimensional case, one can write the equation of continuity, motion and energy as follows:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{1}$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 v_x}{\partial y^2} \tag{2}$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The boundary conditions are $T = T_0 = T_m$ and $v_x = 0$ at $y = 0$; $T \to T_\infty$, $v_x \to v_\infty$ as $y \to \infty$. The implicit assumptions are (a) boundary layer flow exists, (b) constant property fluid and (c) no pressure gradient. From the continuity equation, we have

$$v_y = v_{y0} - \int_0^y \frac{\partial v_x}{\partial x} dy. \tag{4}$$

The interfacial velocity exists as a result of melting and its magnitude is related to the heat flux transferred from the fluid to the plate as

$$v_{y0} = k\,(\partial T/\partial y)_0 / \rho \Delta H_m \tag{5}$$

or

$$v_y = \frac{k \cdot \left(\frac{\partial T}{\partial y}\right)_0}{\rho \cdot \Delta H_m} - \int_0^y \frac{\partial v_x}{\partial x} dy. \tag{6}$$

By introducing dimensionless quantities $\Pi_V = v_x/v_\infty$, $\Pi_T = (T - T_0)/(T_\infty - T_0)$, $\Lambda v = 1$, and $\Lambda_T = Pr$, eq 2 and 3 can be written into a common form.

$$v_x \frac{\partial \Pi}{\partial x} + \left[\frac{C_p (T_\infty - T_0)}{\Delta H_m} \frac{k}{C_p \rho} \left(\frac{\partial \Pi_T}{\partial y}\right)_0 - \int_0^y \frac{\partial v_x}{\partial x} dy\right] \frac{\partial \Pi}{\partial y} = \frac{\nu}{\Lambda} \frac{\partial^2 \Pi}{\partial y^2} \tag{7}$$

and $\Pi = 0$ at $y = 0$; $\Pi \to 1$ as $y \to \infty$.

Equation 7 together with the boundary conditions can be solved with the familiar similarity transformation. Assume Π is a function of η defined as $\eta = (y/2)(v_\infty/\nu x)^{\frac{1}{2}}$. Equation 7 becomes

$$\Pi'' + f \Lambda \Pi' = 0 \tag{8}$$

where

$$f = \int_0^\eta 2 \Pi_V d\eta - K \tag{9}$$

$$K = \frac{1}{Pr} \frac{C_p (T_\infty - T_0)}{\Delta H_m} \Pi'_T (0, Pr, K) = \left(\frac{1}{Pr}\right) (A)\ \Pi'_T (0, Pr, K). \tag{10}$$

Numerical solutions of Π for various values of $\underline{K}$ and Λ have been given by a number of investigators. For this work, the interest is focused on the effect of the interfacial velocity as a result of melting on the heat transfer rate. Let θ_T be the ratio of the heat transfer coefficient with melting to that without melting; we have

$$\theta_T = \frac{h_m}{h_{wm}} = \frac{\Pi'_T(0, Pr, K)}{\Pi'_T(0, Pr, 0)} . \tag{11}$$

Numerical values of $K \cdot Pr / \Pi'_T (0, Pr, K)$ as a function of θ_T have been compiled by Stewart (1950). From definition of $\underline{K}$ (see eq 10) one can obtain a relationship between θ_T and $C_p (T_\infty - T_0)/\Delta H_m$ since:

$$\frac{K \cdot Pr}{\Pi'_T(0, Pr, 0)} = \frac{1}{Pr} \frac{C_p (T_\infty - T_0)}{\Delta H_m} \Pi'_T (0, Pr, K) \frac{Pr}{\Pi'_T (0, Pr, 0)}$$

$$= \frac{C_p (T_\infty - T_0)}{\Delta H_m} \frac{\Pi'_T (0, Pr, K)}{\Pi'_T (0, Pr, 0)}$$

$$A = \frac{C_p (T_\infty - T_0)}{\Delta H_m} = \left(\frac{K \cdot Pr}{\Pi'_T (0, Pr, 0)} \right) / \theta_T . \tag{12}$$

Film theory

The analysis given in the previous section was obtained on the basis of two-dimensional, forced laminar flow. In a practical situation, in addition to laminar flow, one is also interested in the heat transfer between a melting solid and fluid under natural flow or forced turbulent flow conditions. For these cases, one can carry out the analogous analysis by solving the proper equation of motion and energy. The resulting equation, however, may become too complicated to be solved analytically. To overcome this difficulty, the effect of melting on heat transfer is to be studied by first proposing a mechanism which is assumed to be valid in describing the transport process between a solid boundary and a surrounding fluid. Since these assumed mechanisms are supposed to be valid under all conditions, the results subsequently obtained will be applicable to a wide range of situations.

Film theory was first postulated by Whitman (1923) to describe the interphase transport phenomena. It is assumed that the resistance to a transport process between a solid and fluid is confined to a thin layer of stagnant film immediately adjacent to the solid boundary. The model is assumed to be one-dimensional (perpendicular to the surface) and independent of time (steady-state). For energy transport, we have

$$\nabla \cdot E = 0 \tag{13}$$

where $\underline{E}$ is the energy vector. For the one-dimensional case

$$E = -k \frac{dT}{dy} + NH \tag{14}$$

where $\underline{N}$ is the mass flux and $\underline{H}$ the enthalpy of fluid at the prevailing temperature. Combining eq 13 and 14 it follows

$$-k \frac{dT}{dy} + NH = \text{constant.} \tag{15}$$

Applying eq 15 to y = 0, we have

$$-k\,\frac{dT}{dy} + NH = -k\,\frac{dT}{dy}\Big|_0 + N_0 H_0 \,. \tag{16}$$

Because of the one-dimensional dependence, one can show that $N_0 = N =$ constant, independent of $\underline{y}$. Equation 16, after rearrangement, becomes

$$N_0\,(H - H_0) - k\,\frac{dT}{dy} = -k\,\frac{dT}{dy}\Big|_0 = q_0 \,. \tag{17}$$

Since

$$N_0\,(H - H_0) = N_0\,C_p\,(T - T_0) \tag{18}$$

combining eq 17 and 18 we have

$$N_0\,C_p\,(T - T_0) - k\,\frac{dT}{dy} = q_0 \,. \tag{19}$$

The boundary conditions are $T = T_0$ at $y = 0$, $T = T_\infty$ at $y = \delta_T$. δ_T is the thickness of the film. The solution of eq 19 is

$$1 - \frac{(T - T_0)\,C_p N_0}{q_0} = \exp\,(N_0\,C_p\,y/k) \tag{20}$$

or

$$1 - \frac{(T_\infty - T_0)\,C_p N_0}{q_0} = \exp\,(N_0\,C_p\,\delta_T/k). \tag{21}$$

The right-hand side of eq 21 can be written in terms of Taylor's series as

$$1 - \frac{(T_\infty - T_0)\,C_p\,N_0}{q_0} = 1 + C_p N_0\,\frac{\delta_T}{k} + \frac{1}{2!}\,(C_p N_0\,\frac{\delta_T}{k})^2 + \ldots \tag{22}$$

Consider the limiting case where there is no melting, $N_0 \rightarrow 0$. From eq 22 we have

$$\frac{\delta_T}{k} = \lim_{N_0 \rightarrow 0} \frac{-(T_\infty - T_0)/q_0}{N_0} = \frac{1}{h_{wm}} \tag{23}$$

where h_{wm} is the local heat transfer coefficient without melting. It should be noted that q_0 is inherently negative since the heat is transferred from fluid to plate.

Combining eq 21 and 23, one has

$$1 + \frac{(T_\infty - T_0)\,C_p\,N_0}{-q_0} = \exp\,(C_p N_0/h_{wm}). \tag{24}$$

If one introduces the following quantities

$$R_T = \frac{(T_\infty - T_0)\, C_p N_0}{-q_0} = \frac{C_p N_0}{h_m} \tag{25}$$

$$\phi_T = \frac{C_p N_0}{h_{wm}} \tag{26}$$

$$\theta_T = \frac{h_m}{h_{wm}} = \frac{\phi_T}{R_T} \tag{27}$$

also

$$N_0 = k \left(\frac{dT}{dy}\right)_0 / \Delta H_m = -q_0 / \Delta H_m$$

therefore

$$R_T = \frac{(T_\infty - T_0)\, C_p}{-q_0} \frac{-q_0}{\Delta H_m} = A. \tag{28}$$

Combining eq 24 - 28, we have $1 + R_T = \exp(\phi_T)$, or $\phi_T = \ln(1 + R_T)$

and

$$\theta_T = \frac{\phi_T}{R_T} = \frac{\ln(1 + A)}{A}\,. \tag{29}$$

Equation 29 expresses the effect of melting on the heat transfer coefficient in terms of the parameter A.

Penetration theory

We now proceed to examine the effect of melting on heat transfer on the basis of penetration theory which was first proposed by Higbie (1935) and modified and expanded by many other investigators (Danckwerts, 1951; Hanratty, 1956; Toor and Marchello, 1958; Perlmutter, 1961) in recent years. Based upon experimental evidence on mass transfer, it appears that this theory provides a more plausible explanation for interphase transport processes (energy and mass). It assumes that the transport process is affected by the sweeping of small eddies of the fluid in a turbulent field into contact with the interfaces. Since the contact time, for most cases, is small, the eddy may be treated as a semi-infinite solid and a transient state heat conduction equation can be used in describing the energy transfer. Although one cannot predict the transfer coefficient based on this model unless one has complete knowledge of the surface renewal mechanism (or in other words, the residence-time distribution of eddy), it is possible to study the effect of interfacial velocity resulting from melting on heat transfer if one is to assume that the surface renewal mechanism is not affected by melting.

The unsteady state energy equation is written to describe the eddy. Namely,

$$\frac{\partial}{\partial t} \rho\,(U + \tfrac{1}{2} v^2) = -\nabla \cdot E \tag{30}$$

$$E = \rho (U + \tfrac{1}{2} v^2) V - k \nabla T + (\pi \cdot V). \tag{31}$$

For a one-dimensional case, neglect the kinetic energy term and $\pi \cdot V$. For constant property, we have

$$\rho \frac{\partial U}{\partial t} = - \frac{\partial}{\partial t} (NU) + k \frac{\partial^2 T}{\partial y^2} . \tag{32}$$

As in the case of the film model, penetration also presupposes a unidirectional dependence; this implies that

$$N = \text{constant} = N_0 = k \left(\frac{\partial T}{\partial y}\right)_0 / \Delta H_m \tag{33}$$

and furthermore

$$U = C_p (T - T_r) \tag{34}$$

T_r being any reference temperature. Combining eq 32 - 34, it follows

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{\alpha C_p \left(\frac{\partial T}{\partial y}\right)_0}{\Delta H_m} \frac{\partial T}{\partial y} . \tag{35}$$

The boundary and initial conditions are $T = T_\infty$ at $y \geqslant 0$, $t \leqslant 0$; $T = T_0$ at $y = 0$, $t \geqslant 0$ and $T \to T_\infty$ as $y \to \infty$. Solution to eq 35 is

$$T^+ = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{1 - \operatorname{erf} (\beta + \phi)}{1 - \operatorname{erf} \phi} \tag{36}$$

where

$$\beta = y/(4 \alpha t)^{\frac{1}{2}} \tag{37}$$

$$\phi = (A/2)(T^+)_0' . \tag{38}$$

Differentiating eq 36 and substituting it into eq 38, it follows

$$(1 - \operatorname{erf} \phi) \sqrt{\pi}\, \phi \exp (\phi^2) = -A. \tag{39}$$

The heat flux q_0 is

$$q_0 = -k \left(\frac{\partial T}{\partial y}\right)_0 = -k (T_0 - T_\infty) \left(\frac{\partial T^+}{\partial y}\right)_0$$

$$= k (T_\infty - T_0)(T^+)_0' \left[1/(4 \alpha t)^{\frac{1}{2}}\right]$$

$$= k (T_\infty - T_0) \left(\frac{-1}{1 - \operatorname{erf} \phi}\right) \frac{2}{\sqrt{\pi}} \frac{1}{\sqrt{4 \alpha t}} \exp (-\phi^2). \tag{40}$$

The transfer coefficient, h_m, is defined as

$$h_m = \frac{-q_0}{T_\infty - T_0} = \frac{k \exp(-\phi^2)}{(1 - \operatorname{erf} \phi) \sqrt{4 \alpha t}} \tag{41}$$

and the limiting value of h_m for no melting is obtained by taking the limit of h_m for $\phi \to 0$. This gives

$$h_{wm} = \lim_{\phi \to 0} h_m = \frac{k}{\sqrt{4 \alpha t}} . \tag{42}$$

The ratio of h_m/h_{wm} is thus

$$\theta_T = \frac{h_m}{h_{wm}} = \frac{1}{(1 - \operatorname{erf} \phi) \exp(\phi^2)} . \tag{43}$$

The effect of melting on the heat transfer coefficient can be seen by combining eq 39 and 43.

Conclusion

The effect of melting on heat transfer as expressed in terms of $\theta_T = h_m/h_{wm}$ versus the parameter $A = C_p \Delta T / \Delta H_m$ describing the thermal state has been studied in the investigation for three different models. The results are given in Table I and also shown graphically in Figure 1. Also included are the results obtained earlier by Yen and Tien (1963) on the model of the Leveque problem. Merk (1954) studied analytically the effect of melting on heat transfer by solving the equations of motion and energy using the approximate integral method. The final expression in our notation is

$$\theta_T = \frac{h_m}{h_{wm}} = \left[\frac{1 - \frac{89}{217} s + \frac{19}{434} s^2}{1 - \frac{1}{2} s + \frac{1}{4} s^2} (1 + \tfrac{1}{2} s)^5 \right]^{\frac{1}{4}}$$

in which s is given in terms of A by

$$s = -2 - \frac{3}{A} + \frac{3}{A} (1 + \tfrac{4}{3} A)^{\frac{1}{2}} .$$

Values of θ_T as a function of A computed from the above two expressions are also shown in Table I and Figure 1 for comparison. Although the quantitative results differ, they all show the same qualitative trend and indicate that melting inhibits the heat transfer rate. The results based on the Leveque solution agree with those based on boundary layer theory for small values of A but deviate from each other as A increases. This is expected since the solution obtained by Yen and Tien, in a sense, is an asymptotic solution of that based on boundary layer theory for small A. The difference in results based on these three models makes it necessary to exercise caution in the selection among these results for practical application. This will, to a large extent, depend upon the flow condition. For example, if the flow is laminar, the results based on boundary layer theory should certainly be used. On the other hand, if the flow is turbulent (such as the case where either the longitudinal dimensions of the melting solid are large or the velocity of the fluid is high) or in cases where natural flow or a combination of forced and natural flow exists, no definite conclusion can be made as to which of these results should be used. The results based on penetration theory should provide a better picture because this theory has been proven experimentally to be more adequate in explaining the related transport process of mass transfer. This, however, should be strictly considered as a speculation until more experimental results become available.

Table I. Effect of parameter A on heat transfer rate.

$A = \frac{C_p (T_\infty - T_0)}{\Delta H_m}$	$\theta_T = h_m / h_{wm}$				
	Boundary layer theory	Film theory	Penetration theory	Merk (1954)	Yen and Tien (1963)
0.01006	0.9940	0.9950	0.9943	0.9956	0.990
0.02022	0.9890	0.990	0.9887	0.9915	0.986
0.05144	0.9720	0.9751	0.9713	0.9785	0.966
0.10593	0.9440	0.9505	0.9428	0.9573	0.927
0.22472	0.8900	0.9021	0.8879	0.9156	0.858
0.67659	0.7390	0.7638	0.7219	0.7964	0.668
1.90840	0.5240	0.5594	0.5009	0.6177	-
8.69565	0.2300	0.2612	0.2053	0.3395	-
36.14457	0.0830	0.1000	0.0703	0.1637	-
815.66089	0.0061	0.0082	0.0040	0.0263	-

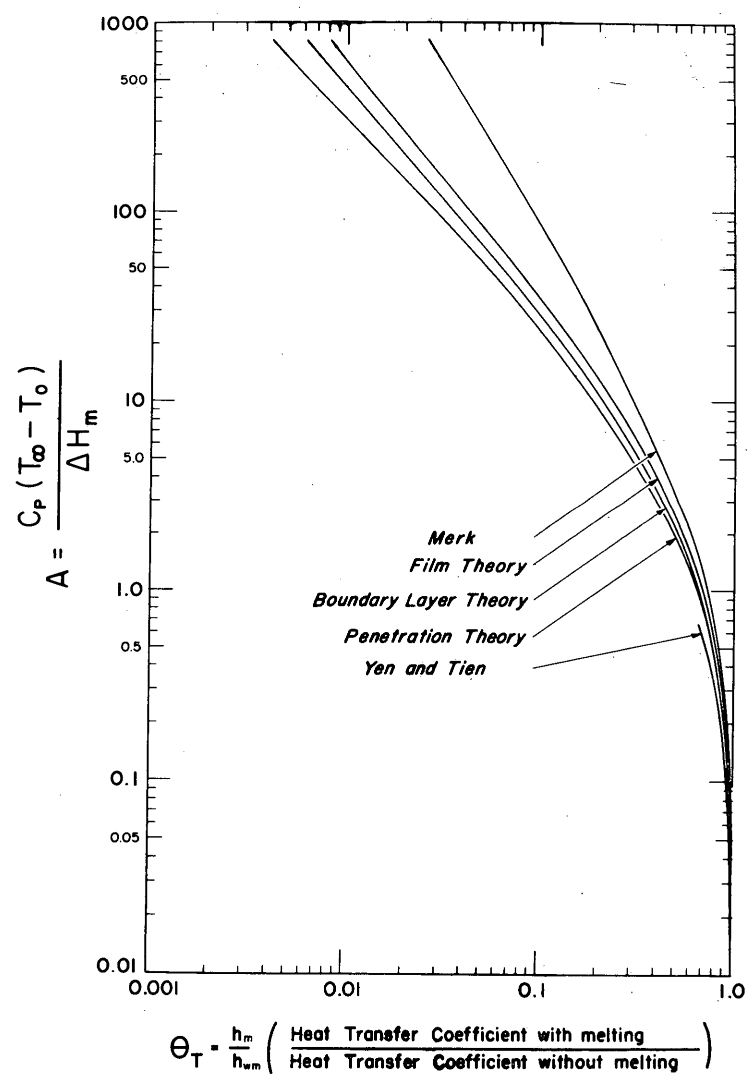

Figure 1. Relationship between parameters A and θ_T.

LITERATURE CITED

Bird, R. B., Stewart, W. E. and Lightfoot, E. N. (1960) Transport phenomena. New York: John Wiley and Sons, Inc., p. 613.

Danckwerts, P. V. (1951) Significance of liquid film coefficients in gas absorption, Industrial and Engineering Chemistry, vol. 2, p. 1460-1467.

Hanratty, T. J. (1956) Turbulent exchange of mass and momentum with a boundary, Journal of the Amercian Institute of Chemical Engineers, vol. 2, p.359-362.

Higbie, R. (1935) The rate of absorption of a pure gas into a still liquid during short periods of exposure, Transactions, American Institute of Chemical Engineers, vol. 31, p. 365-389.

Merk, J. H. (1954) The influence of melting and anomalous expansion on the thermal convection in laminar boundary layers, Applied Scientific Research, vol. 4, section A, p. 435-452.

Perlmutter, D. D. (1961) Surface renewal models in mass transfer, Chemical Engineering Science, vol. 16, p. 287-296.

Stewart, W. E. (1950) Interaction of heat, mass and momentum transfer, Massachusetts Institute of Technology, D. Sc. Thesis.

Toor, H. L. and Marchello, J. M. (1958) Film penetration model for mass and heat transfer, Journal of the American Institute of Chemical Engineers, vol. 4, p. 97-101.

Whitman, W. G. (1923) Chemical and Metallurgical Engineering, vol. 24, p. 146.

Yen, Y. C. and Tien, C. (1963) Laminar heat transfer over a melting plate, the modified Leveque problem, Journal of Geophysical Research, vol. 68, no. 12, p. 3673-3678. Also U. S. Army Cold Regions Research and Engineering Laboratory, Research Report 125 (1964).

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13. ABSTRACT

The effect of melting on convective heat transfer between a melting body and surrounding fluid was studied quantitatively from the point of view of boundary layer theory, film theory, and penetration theory. These studies indicate that melting retards the rate of heat transfer, and the decrease in heat transfer coefficient is found to be a unique function of the parameter $C_p\Delta T/\Delta H_m$, where ΔT is the temperature difference between the fluid and melting body, C_p is the heat capacity of the fluid, and ΔH_m is the enthalpy change due to melting.

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It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.