Research Report 166

HEAT TRANSFER CHARACTERISTICS OF NATURALLY COMPACTED SNOW

by

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PREFACE

This report was prepared by Dr. Yen, Research Chemical Engineer, Materials Research Branch, Research Division, USA CRREL.

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SUMMARY

The heat transfer characteristics of naturally compacted snow have been determined experimentally. The results are interpreted as effective thermal conductivity k_e and water vapor diffusivity D_e and can be well represented by

$$k_e = 0.0077 (\rho_s)^2 + 0.60 (G)$$

and

$$D_e = 0.65 + 2113 (\rho_s)^{3.20} (G)^{0.615}.$$

In contrast with the results obtained from previous investigations on unconsolidated snow, in the case of naturally compacted snow, values of k_e and D_e are found to be not only a function of air flow rate but also of snow density. From present as well as previous studies, it can be concluded that air flow has considerable effect on the thermal conductivity and water vapor diffusivity of unconsolidated and naturally compacted snow. It is reasonable to state that the essential factor influencing the formation of depth hoar and avalanches is a prolonged process of simultaneous heat and mass transfer due to steep temperature gradients imposed on a snow layer.

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Introduction

The effect of air flow on thermal conductivity k_e and water vapor diffusivity through snow D_e was determined experimentally and reported for the first time by the author (Yen, 1962, 1963). Unconsolidated snow samples with densities ranging from 0.376 to 0.472 g/cm³ were used. Variation of snow density produced no noticeable effect on k_e and D_e. Experimental results were well represented by the expressions:

$$k_e = 0.0014 + 0.58\,(G)$$

and

$$D_e = 0.65 + 133\,(G)^{0.577}$$

in which G represents the mass flow rate of dry air in g/cm²-sec. This proven technique and method of analysis is now extended to the case of naturally compacted snow. The information obtained in these studies is essential for an understanding of the nature of thermodynamic processes occurring in a natural snow cover under the combined effects of daily or seasonal variations in atmospheric temperature and wind currents. The results are needed to interpret the observed phenomena of snow temperature and density distribution profiles near the surface layers of a snow cover. In addition, it is believed also that experiments of this type, involving simultaneous heat and mass transfer, will shed some light on the mechanism underlying the formation of depth hoar and avalanches.

Theory

Consider an element of volume dv of snow through which an air stream and heat are flowing in directions parallel but opposite to each other. The energy and water vapor balance equations can be written respectively as:

$$k_e \frac{d^2t}{dx^2} + Gc_a \frac{dt}{dx} + \frac{GM_w c_w}{M}\left(\frac{p}{\pi - p}\right)\frac{dt}{dx} + \frac{GM_w L_s}{M}\frac{d}{dx}\left(\frac{p}{\pi-p}\right) + \frac{GM_w c_w}{M}\left[(t-t_{ai}) - \frac{d}{dx}(t-t_{ai})\,dx\right]\frac{d}{dx}\left(\frac{p}{\pi-p}\right) = c_s\,(t-t_{ai})\frac{\partial \rho_s}{\partial \theta} \quad (1)$$

and

$$\frac{d}{dx}\left(\frac{D_e}{RT}\frac{dp_s}{dx}\right) + \frac{GM_w}{M}\frac{d}{dx}\left(\frac{p}{\pi-p}\right) = \frac{\partial \rho_s}{\partial \theta} \quad (2)$$

where

k_e	— effective thermal conductivity of snow, cal/cm-sec-°C
D_e	— effective water vapor diffusivity through snow, cm²/sec
p, p_s and π	— partial pressure of water vapor in air, saturation vapor pressure of snow, and total pressure of the system respectively, mm Hg
c_a, c_w and c_s	— specific heats of air, water vapor, and snow, cal/g
M and M_w	— molecular weights of dry air and water

R	— gas constant
T	— temperature, °K
x	— distance from heat source
t and t_{ai}	— temperatures of snow and inlet air respectively, °C
L_s	— latent heat of sublimation, cal/g and
θ	— time, sec.

Under the experimental conditions, described in the following section, terms 3 and 5 of eq 1 are much smaller than terms 2 and 4. Consequently, eq 1 can be simplified to

$$k_e \frac{d^2t}{dx^2} + Gc_a \frac{dt}{dx} + \frac{GM_w L_s}{M} \frac{d}{dx}\left(\frac{p}{\pi - p}\right) = c_s\,(t - t_{ai})\,\frac{\partial \rho_s}{\partial \theta}. \qquad (1a)$$

For experiments of this nature it is believed that p_s can be used as $\underline{p}$. Furthermore, π is much larger than p_s. Also for a small temperature range D_e can be considered constant at a mean system temperature T_m. Based on these assumptions we reduce eq 1a and 2 to

$$k_e \frac{d^2t}{dx^2} + Gc_a \frac{dt}{dx} + \frac{GM_w L_s}{M\pi} \frac{dp_s}{dx} = c_s\,(t - t_{ai})\,\frac{\partial \rho_s}{\partial \theta} \qquad (3)$$

and

$$\frac{D_e}{RT_m} \frac{d^2p_s}{dx^2} + \frac{GM_w}{M\pi} \frac{dp_s}{dx} = \frac{\partial \rho_s}{\partial \theta}. \qquad (4)$$

To solve eq 3 and 4, the function $\partial\rho_s/\partial\theta$ must be known. Any attempt to obtain this function experimentally will distort the steady state condition. However, as shown in Figure 1, the change in snow density is limited to the layer in contact with the heat source. At a distance in excess of 1.27 cm from the heat source the density remains unchanged. Therefore, by shifting the origin a distance of 1.27 cm from the heat source, $\partial\rho_s/\partial\theta$ can be taken as zero in eq 3 and 4. For the temperature range $-12 \leqslant t°C \leqslant -7$ covered in this investigation, p_s can be represented accurately by the linear relationship $p_s = 3.76 + 0.180\,(t)$ from a general expression $p_s = 4.56 \exp(0.0857t)$ given by Yosida (1950). By substituting the above expression in eq 3 and setting $\partial\rho_s/\partial\theta$ equal to zero, eq 3 and 4 become a pair of second order linear differential equations:

$$k_e \frac{d^2t}{dx^2} + \left(Gc_a + 0.180\,\frac{GM_w L_s}{M\pi}\right)\frac{dt}{dx} = 0 \qquad (5)$$

and

$$\frac{D_e}{RT_m} \frac{d^2p_s}{dx^2} + \frac{GM_w}{M\pi} \frac{dp_s}{dx} = 0. \qquad (6)$$

With boundary conditions $p_s = p_{s_0}$, $t = t_0$ at $x = 0$; and $p_s = p_{s_L}$, $t = t_L$ at $x = L$, solutions to eq 5 and 6 are

$$\frac{t - t_L}{t_0 - t_L} = 1 - \frac{1 - \exp(-\alpha x)}{1 - \exp(-\alpha L)} \qquad (7)$$

and

$$\frac{p_s - p_{sL}}{p_{s_0} - p_{sL}} = 1 - \frac{1 - \exp(-\beta x)}{1 - \exp(-\beta L)} \tag{8}$$

where

$$\alpha = \frac{G\left(c_a + 0.180\,\frac{M_w L_s}{M\pi}\right)}{k_e} \tag{9}$$

and

$$\beta = \frac{GRT_m M_w}{M\pi D_e}\,, \tag{10}$$

Experimental apparatus and procedures

The experimental apparatus and procedures employed are described in earlier papers concerning studies of unconsolidated snow (Yen, 1962, 1963). Only slight modification of the apparatus was made. The experiments were carried out in the field. Snow samples were obtained by drilling into a snow mass with a 7.62-cm auger. The snow container is a Dewar flask which is especially made to fit the snow sample tightly. Small holes were drilled perpendicular to and terminated at the axis of the sample for thermocouple installation. Ten 40-gage copper-constantan thermocouples were used; their leads were directed out along the periphery by cutting a small groove in the sample parallel to its axis. Vacuum grease was applied to the inner wall of the Dewar flask before the sample was pushed into position. The Dewar flask with its snow sample and thermocouples was then fitted into an aluminum casing equipped with flanges which connected to a constant temperature bath at the bottom and an air inlet device at the top. Temperature readings, to ± 0.05C, were taken with a Leeds and Northrup potentiometer at regular intervals as soon as the air stream, saturated with water vapor at its inlet temperature, started to flow downward through the sample. When the temperature readings of all thermocouples were no longer a function of time, the air flow rate was measured with a wet test meter installed in the exit line of the air stream (to prevent freezing, the meter was filled with ethyl alcohol). Alcohol temperature of the meter and barometric pressure were recorded simultaneously. Snow samples 15 cm in length and 7.34 cm in diameter with densities ranging from 0.50 to 0.59 g/cm^3 were investigated.

Results and discussion

In preliminary studies, separate samples were used to determine the initial and the final density. The initial density was determined by cutting the snow sample into six equal sections and weighing each with an analytical balance. The sample used for experiment was cut into sections and the density distribution determined after the test. Figure 1 shows typical results of these determinations. It is evident that only a thin layer of snow near the heat source decreased in density. Therefore, by shifting the heat source origin up 1.27 cm, we can consider $\partial\rho_s/\partial\theta = 0$ in eq 3 and 4.

For the subsequent experiments, only the bulk density before the test was measured. Since the air is believed to be saturated at each point as it passes downward, the pressure data can be calculated from the relation given by Yosida (1950). Temperature readings of a typical experiment are shown in Figure 2. Figure 3 clearly indicates that the ratios $(d^2t/dx^2)/(dt/dx) = -\alpha$ and $(d^2p_s/dx^2)/(dp_s/dx) = -\beta$ remain constant, as predicted from eq 7 and 8. To expedite the calculations of all α and β values, a chart was prepared relating the dimensionless temperatures and pressures as defined in eq 7 and 8 to α and β with $\underline{x}$, the distance from heat source, as parameter.

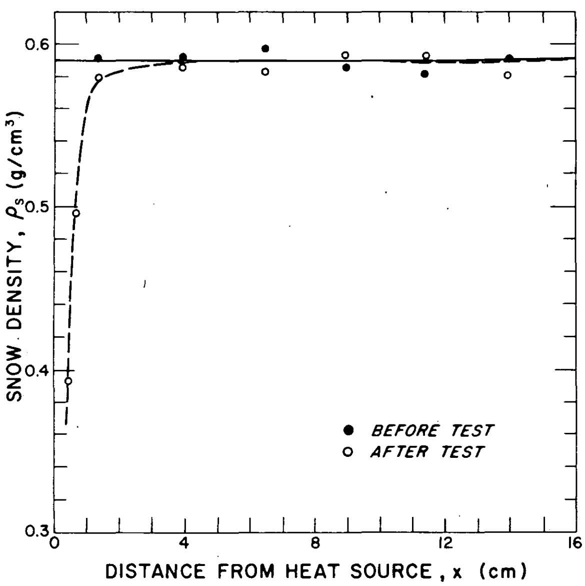

Figure 1. Snow density distribution as a function of distance from heat source.

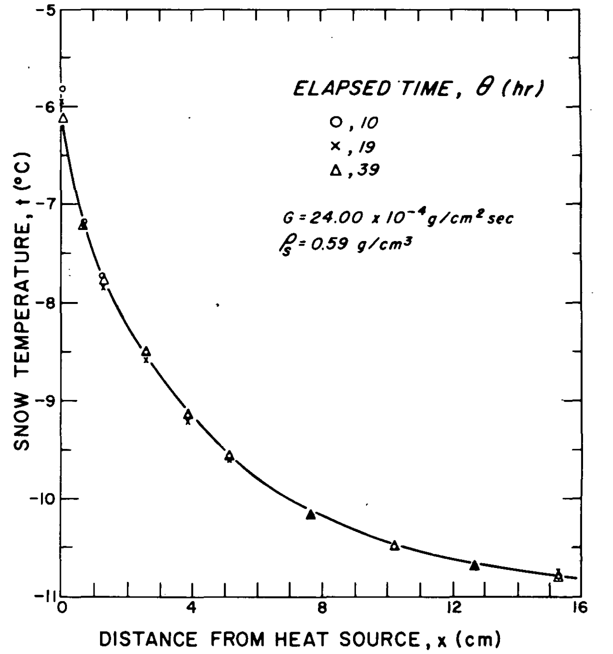

Figure 2. Typical steady-state temperature distribution in snow.

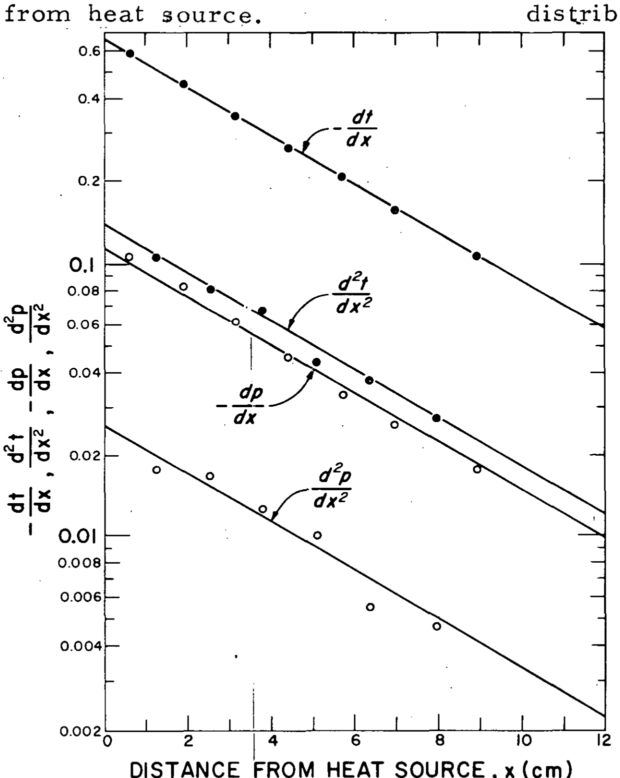

Figure 3. Typical values of $-dt/dx$, d^2t/dx^2; $-d\rho_s/dx$, $d^2\rho_s/dx^2$ as a function of distance from heat source ($G = 24.00 \times 10^{-4}$ g/cm^2-sec, $\rho_s = 0.59$ g/cm^3).

The necessary measurement for each test is the steady state temperature distribution, from which the dimensionless temperaturesand pressures can be evaluated for several points along the path of heat flow. Theoretically, the values of α and β should be more or less constant. Figure 3 demonstrates this. After values for α and β are determined in the above manner, k_e and D_e are calculated from relationships 9 and 10 in which all the other variables are known. In contrast to the results obtained from previous investigations on unconsolidated snow it is observed that, in the case of naturally compacted snow, values of k_e and D_e are found to be not only a function of air flow rate but also a function of snow density. Table I summarizes the calculated values of k_e and D_e. Figure 4 graphically indicates the relationship between k_e and $\underline{G}$. Figure 5 represents the functional relation between D_e and $\underline{G}$. Values of k_e can be empirically expressed as

$$k_e = k_e^\circ + b(G) \tag{11}$$

while values of D_e can be related to ρ_s and $\underline{G}$ by

$$D_e = D_e^\circ + C(\rho_s)^m(G)^n \tag{12}$$

for ρ_s from 0.50 to 0.59 g/cm^3 and $\underline{G}$ from 5.0 to 32 x 10^{-4} g/cm^2-sec.

Table II is a summary of some numerical data from statistical analysis of k_e values. The average value of $\underline{b}$ in relation 11 is found to be 0.60, which is essentially equal to the value 0.58 obtained from studies on unconsolidated snow with mean snow density of 0.42 g/cm^3. When there is no artificial flow, i.e. $G = 0$, k_e and D_e reduce to k_e° and D_e° respectively. As shown in Figure 6, k_e° can be well represented by

$$k_e^\circ = 0.0077\ (\rho_s)^2 \tag{13}$$

for ρ_s from 0.42 to 0.59 g/cm^3, which is in close agreement with reported values by numerous investigators (Yen, 1962). Therefore, the effective thermal conductivity of snow can be written as

$$k_e = 0.0077\ (\rho_s)^2 + 0.60\ (G). \tag{14}$$

The first term on the right side of relation 14 represents the influence of density while the second term represents the effect of air flow on the thermal conductivity of snow.

A value of 0.65 cm^2/sec was obtained for D_e° by extrapolation as indicated in a previous study (Yen, 1963). This is about three times higher than the widely accepted value of 0.22 cm^2/sec at 0C and atmospheric pressure. Yosida (1950) reported that D_e° does not vary with snow density and has a value of 0.85 cm^2/sec for density 0.08 to 0.51 g/cm^3. As shown in Figure 5, the effect of ρ_s on D_e is obvious. The constant $\underline{C}$ and exponents $\underline{m}$ and $\underline{n}$ in relation 12 are found to be 2113, 3.20 and 0.615 respectively. Finally, D_e is represented by

$$D_e = 0.65 + 2113\ (\rho_s)^{3.20}\,(G)^{0.615}. \tag{15}$$

From present as well as previous studies, it can be concluded that air flow has a considerable effect on the thermal conductivity and water vapor diffusivity of unconsolidated and naturally compacted snow. Turbulence introduced by the air stream apparently has reduced the resistance to the simultaneous transfer processes of heat and mass. The increase in thermal conductivity and vapor diffusivity due to air ventilation is held responsible for the flat snow density distribution profile and small temperature gradients near the surface layers of a snow cover. It is reasonable to conclude that the essential factor influencing the formation of depth hoar and avalanches is a prolonged process of simultaneous heat conduction and vapor diffusion due to steep temperature gradients imposed on the snow layer.

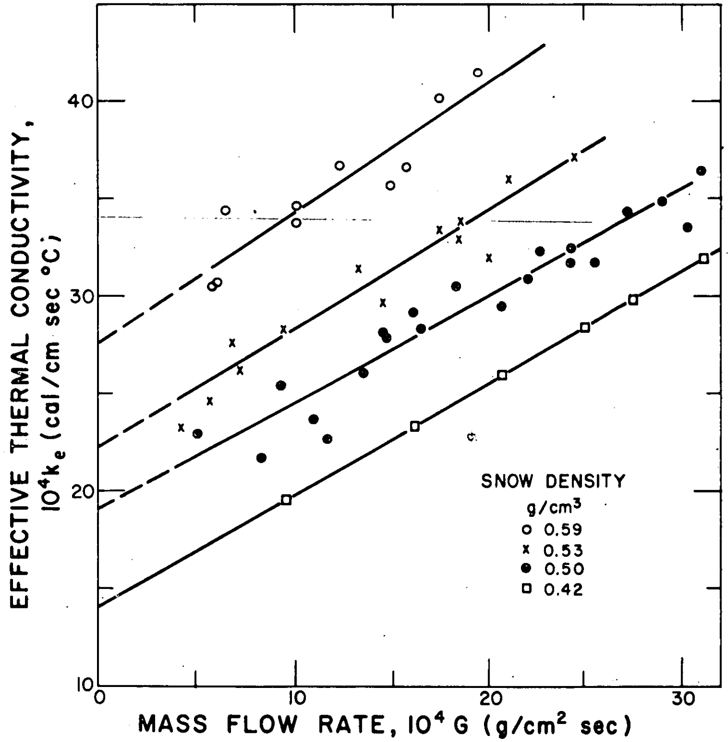

Figure 4. Dependence of k_e on G. Square points are calculated from the least square equation for k_e from studies on unconsolidated snow with mean snow density of 0.42 g/cm³ (0.376 to 0.476 g/cm³).

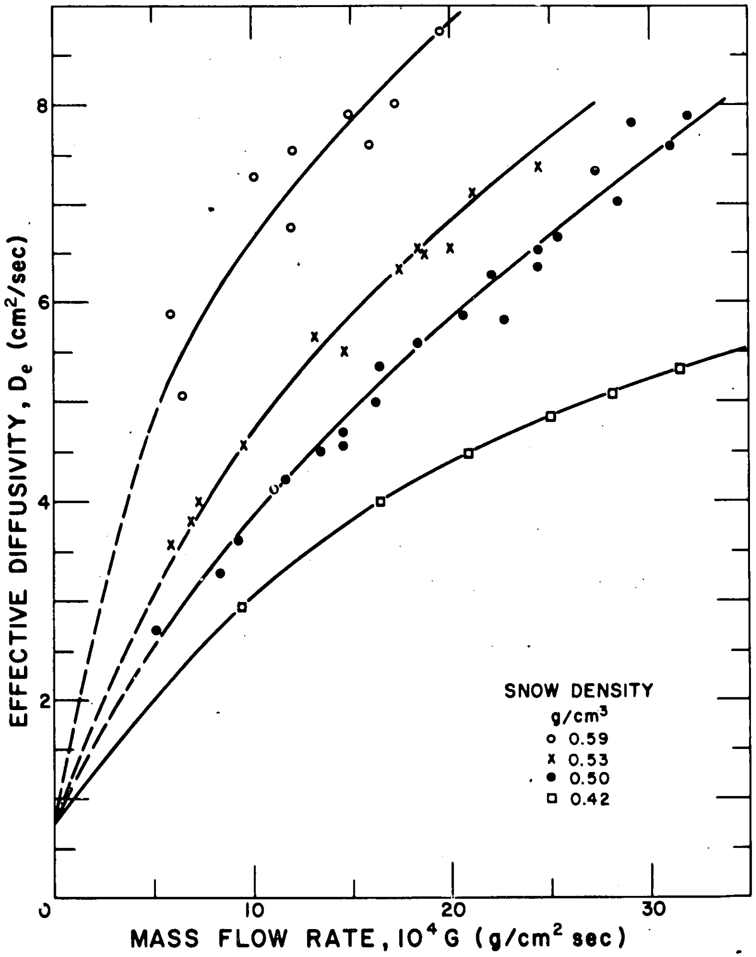

Figure 5. Dependence of D_e on G. Square points are calculated from the correlation equation for D_e from studies on unconsolidated snow with mean snow density 0.42 g/cm³ (0.376 to 0.476 g/cm³).

Table I. Effective thermal conductivity k_e and water vapor diffusivity D_e of naturally compacted snow as function of air flow rate $\underline{G}$.

ρ_s(g/cm³)	G(g/cm²-sec) x 10^4	k_e(cal/cm-sec-°C) x 10^4	D_e(cm²/sec)
0.50	16.17	29.22	5.00
	31.00	36.41	8.10
	28.41	33.51	7.06
	24.33	31.68	6.36
	20.68	29.43	5.88
	16.47	28.31	5.35
	11.69	22.73	4.24
	11.01	23.66	4.13
	8.38	21.75	3.28
	22.64	32.26	5.81
	14.56	28.07	4.70
	9.25	25.47	3.60
	13.46	26.06	4.50
	25.41	31.77	6.68
	18.33	30.47	5.58
	27.25	34.36	7.33
	41.42	-	8.66
	41.80	-	9.16
	45.04	-	8.95
	29.02	34.86	7.83
	24.32	32.37	6.54
	14.61	27.81	4.57
	22.09	30.87	6.28
	31.86	-	7.90
	5.16	22.86	2.71
0.53	7.31	26.22	4.00
	5.78	24.64	3.56
	20.00	31.93	6.55
	18.60	33.88	6.50
	18.41	32.86	6.56
	14.57	29.65	5.50
	9.55	28.29	4.57
	13.22	31.42	5.65
	4.26	33.42	-
	21.09	35.98	7.10
	24.41	37.23	7.40
	17.48	33.45	6.35
	6.86	27.57	3.80
0.59	12.03	33.68	6.75
	19.47	41.56	8.75
	14.88	35.62	7.90
	17.43	40.17	8.00
	15.78	36.66	7.65
	5.87	30.59	5.86
	10.03	34.56	7.26
	12.37	36.62	7.55
	6.46	34.37	5.07
	6.13	30.77	5.38
	24.00	44.20	9.68

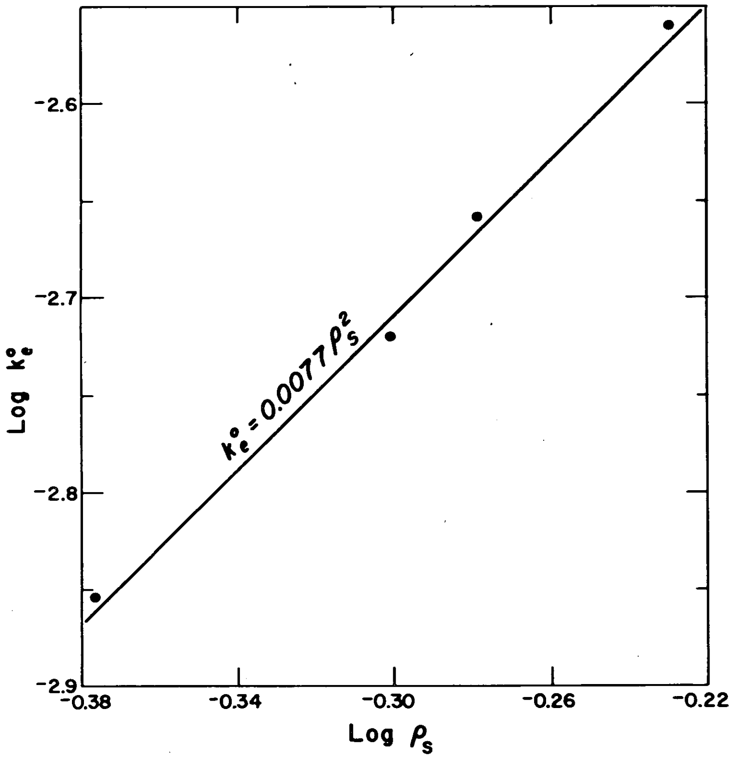

Figure 6. Log k_e° vs log ρ_s for snow densities 0.42, 0.50, 0.53 and 0.59 g/cm³.

Table II. Numerical data from statistical analysis of k_e values.

Snow density, g/cm³	0.50	0.53	0.59
Constant $k_e^\circ \times 10^4$ cal/cm-sec-°C	18.977	22.178	27.505
Constant $\underline{b}$	0.546	0.603	0.660
Standard deviation $S \times 10^4$	1.194	1.170	1.560
Correlation coefficient $\underline{r}$	0.934	0.935	0.910
Interval estimate of $k_e^\circ \times 10^4$ for 95% confidence level	$17.476<k_e^\circ<20.478$	$20.180<k_e^\circ<24.176$	$24.330<k_e^\circ<30.680$
Interval estimate of $\underline{b}$	$0.472<b<0.621$	$0.473<b<0.733$	$0.415<b<0.905$
$k_e \times 10^4$ predicted at $G = 10 \times 10^{-4}$ g/cm²-sec	24.437	28.208	34.105
Interval estimate of $k_e \times 10^4$ at $G = 10 \times 10^{-4}$ and for 95% confidence level	$23.586<k_e<25.288$	$27.242<k_e<29.174$	$32.862<k_e<35.348$

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