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**Shear Stress Measurements In Situ
of Soils Subjected to Vibratory Loads**

**U.S. ARMY MATERIEL COMMAND
COLD REGIONS RESEARCH & ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE**

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PREFACE

The work described in this report was performed by Dr. Rudolf K. Bernhard, Consultant to USA CRREL for the Applied Research Branch, Dr. Andrew Assur, Chief. Considerable difficulties are experienced in measuring shear stresses generated in snow or frozen ground under dynamic loads. This report is an attempt to improve both measurement and analysis.

This report has been reviewed and approved for publication by Headquarters, U. S. Army Materiel Command.

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Colonel, CE
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SUMMARY

A miniaturized shear cell is described. Results of measurements in situ with this shear cell are compared with those of pressure cells.

Vibratory (sinusoidal) loads were generated by a mechanical oscillator on the surface of the soil and transmitted to both types of cells buried underneath.

A predominantly homogenous and non-cohesive soil was selected so that as a first approximation the theory for biaxial stress distribution could be applied in the analysis.

The standard deviations of the corresponding shear and normal stresses did not exceed 5.3%.

ON SHEAR STRESS MEASUREMENTS IN SITU OF SOILS SUBJECTED TO VIBRATORY LOADS

by

Rudolf K. Bernhard*

INTRODUCTION

Previous studies (Ahlvin, 1954) of shear and pressure cell measurements in situ have been restricted mostly to static loads. Where dynamic loads were applied, the investigations have been confined in general to measurements with pressure cells and seismographs (American Society for Testing Materials, 1953). In these earlier publications, the inherent difficulties resulting from in situ measurements by any technique are discussed as well as some of the advantages of generating dynamic loads as compared with static loads. Principal stress isobars (lines of equal pressure) have been plotted from pressure records; however, the corresponding shear stress isobars could only be derived analytically (Bernhard, 1961).

Experiments have shown that sinusoidal force vectors of constant amplitude, direction, and frequency are reproduced easily in contrast to impact or explosive excitation (ASTM, 1953). Since reproducibility of results is the essence of most experimental research, sinusoidal force generators were used exclusively in the following investigation.

The purposes of this study were to measure shear as well as normal stresses directly, to check the compatibility of the resulting shear stress distribution and the corresponding normal stress pattern, and to correlate the experimental findings with the existing theories for biaxial systems.

The knowledge of maximum shear and principal stress vectors in the soil and in particular the transmission of vibratory loads into the supporting subgrades is of primary importance. Numerous failures of structures subjected to dynamic forces such as highways, railroad tracks, airfields, foundations, etc., have been caused by yielding of the underlying soil.

A detailed knowledge of stress patterns produced in the soil by tremors might facilitate investigations leading to discrimination between nuclear subsurface explosions and earthquakes (macroseismics). Though the following experiments are restricted to short source-detector distances (microseismics, Bernhard, 1957) as well as small exciter forces, certain similarities to blast phenomena exist. For instance, the stresses generated by large distance explosions are also very minute. Furthermore, all stresses are confined to a predominantly elastic response of the surrounding medium. The main difference between blasts or earthquakes and forces investigated in this study is that the disturbing sources of the first have an impact and the second a sinusoidal characteristic.

A measuring device attached to or inserted into a medium will never have physical characteristics identical to its surroundings unless its deformation characteristics are the same. Any discontinuity produced by foreign matter does disturb the stress pattern. For this reason a pick-up unit should be very small, but not so minute that arching or bridging effects, particularly in cohesive soils or material containing large pebbles, prevent an adequate response. These obvious prerequisites are of special importance for in situ measurements of soils.

For any soil investigation, a comparison between shear and the corresponding normal stresses yields a valuable compatibility check. In determining both types of stresses, the fundamental characteristics of the transducers have to be considered.

Shear cells, comprising two parallel plates, respond mainly to their relative displacement, hence tend to compensate for any motion of the shear cell as a unit.

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Figure 1a. Shear cell (top cover removed).

a. Serrated shear plates.	e. Adjustment screw for transformer housing.
b. Leaf springs connecting a.	f. Adjustment screw for transformer core.
c. Differential transformer.	g. Flexible shaft (retractable).
d. Lead wires.	

Figure 1b. Shear cell (top cover removed).

Most pressure transducers, however, to a certain extent react as a body to the motion of the surrounding medium. Their output corresponds to the deflection of a sensitive diaphragm with respect to the opposite end of the housing, thus output depends somewhat upon the length of the cell. In addition, part of the energy transmitted from the force generator is diverted into a "sinking-in" effect with resulting friction losses (Berdan and Bernhard, 1950) and possible arching phenomena.

Consequently, shear cells may yield the required parameters more directly than pressure cells, which might be considered an advantage of shear over normal stress measurements.

SHEAR CELL

Construction

The shear cell (Fig. 1) consists primarily of two serrated aluminum plates. A parallel motion of these plates is enforced by two interchangeable steel leaf springs, one on each end. Any relative motion of the two plates is picked up by a linear variable differential transformer (Winters, 1960). The armature of this transformer is connected to one plate, the core to the other. No physical contact between armature and core exists, thus no error is introduced by a variable restraining action of the transformer.

Figure 2. Static calibration of shear cell.

Figure 3. Static calibration of pressure cell. Maximum amplification.

Dimensions: Shear area 2 in.²
 Volume 5 in.³
 Weight 0.3 lb

Measuring ranges: Relative displacement of shear plates: 10^{-4} in. to 10^{-3} in.
 Frequency: 0 to 50 cps.
 Damping ratio: Half aperiodic: $\epsilon = e^{\pi} = 23.1$ (Bernhard, 1943)

Accuracy: $\pm 2\%$ (at lower ranges)

Sensitivity: $\pm 10^{-3}$ lb.

When using gages based on the strain of wires or on materials having piezoelectric properties, small forces caused by deformation of the measuring element are inevitable. Furthermore, piezoelectric substances can be used to measure only pressure changes due to unavoidable leakage losses and do not respond properly to very low frequencies or constant loads. Thus any static calibration by means of stepped-up dead weights is not feasible.

Zero adjustment

A slight displacement of the transformer core with respect to the electric center of its armature will occur, no matter how carefully the soil surrounding the cell is refilled. As explained later, any precompaction of the soil to obtain a stable dynamic equilibrium may add small but unpredictable shear plate displacements. Thus a special device is required to readjust the core to zero output of the transformer. This readjustment is made through a flexible cable and a screw drive attached to the transformer core. The cable can be disconnected before starting the actual recording, so that only the wires leading to the control unit remain.

These lead wires may have a certain disturbing effect upon the output, which could be avoided by incorporating a transistorized, self-contained sender within the shear cell housing. This sender, driven by a button-size mercury battery, telecasts its output to a frequency responsive receiver above ground (Prange, 1960). The development of similar types of instrumentation based on micromodule wafers should be considered for further studies.

Static calibration

The galvanometer deflection versus a concentrated load acting upon one shear plate is plotted in Figure 2. No uniformly distributed load could be applied. This is in contrast to pressure cells where both concentrated and uniformly distributed loads have been used (Fig. 3). All calibration values could be reproduced within an accuracy of $\pm 2\%$ or better.

Figure 4. Experimental setup.

a. Force vector generator.	e. Angular orientation of d.
b. Contact plate of a.	f. Galvanometer oscillograph.
c. Soil surface.	g. Control and amplifier units.
d. Shear and pressure cells.	h. Cathode ray oscillograph.

Dynamic calibration

A check on a vibration table indicated a flat response with the required frequency range and a decay factor equal to 0.7 of critical damping (Bernhard, 1943).

EXPERIMENTS

To reduce the number of variable parameters, the experiments were restricted to one type of soil and to a vertical force vector of one particular frequency and magnitude. The introduction of more variables would make the use of electronic computers almost imperative and is beyond the scope of this study. All studies were confined to biaxial measurements; experiments for triaxial systems are underway.

A sufficient number of values of values to form significant statistical averages was obtained by changing the angular position of shear and pressure cells in 10-deg steps from 0 to 90 deg and repeating this position sequence several times (Fig. 4).

Vibratory loads

The importance of reproducibility of all experiments has been mentioned before. This reproducibility is enhanced by applying vibratory instead of static loads.

For non-cohesive dry soils, vibratory loads change part of the friction between individual soil particles from friction at rest into the smaller friction of motion. After a short transition period during dynamic precompaction, a quasidynamic equilibrium is reached and a predominantly elastic response of the stabilized soil prevails (Bernhard, 1961). Consequently all experiments started only after vibratory compaction, generated by a constant sinusoidal force vector, produced this equilibrium. This stable

state is indicated by a steady output of the transducers. In other words, the sinusoidal force vector must correspond to a similar response of the pickup units without any further shift in zero line of their sinusoidal output. Even if the stabilization effect of repetitive loads were independent of the frequency, the time consumed for static as compared with dynamic loads becomes prohibitive. Assuming a dynamic precompaction at a frequency of 30 cps for a duration of 2 min, that is 3600 load changes, the equivalent time for static loads repeated at one per minute would require 60 hr.

Soil

Dry beach sand with approximately 3% moisture content was available. The grain size distribution is shown in Figure 5. The absence of very fine particles, which would predominate in clays and silts, promotes noncohesive properties, thus minimizing arching effects. The lack of the large particles which would prevail in gravels excludes bridging effects. Both effects are detrimental from the experimental point of view. This soil appears to approach more nearly than any other type the conditions of an almost elastic, isotropic, and homogeneous medium. These characteristics are desirable to permit a preliminary analysis according to elastic theory.

Setup

The soil was placed into a concrete-lined basin, 24 ft long, 11 ft wide, and 5 ft deep. A three-mass oscillator located on top of the soil generated a vertical sinusoidal force vector of ± 500 lb at 23 cps. Both shear cells and pressure cells were located 6 in. below the soil surface and vertically under the center of the contact plate supporting the oscillator (Fig. 4). The shear cell is described above. The pressure cell and oscillator are described in ASTM Special Technical Publication No. 156 (1953).

The output of both types of cells could be fed into a cathode ray oscillograph for visual observation and into a galvanometer oscillograph for recording purposes.

Preliminary experiments were made to determine the effect of the walls and floor of the basin. Load generator and pick-up units were arranged at different distances from the walls. As long as the distance between the units and the tank's borders exceeded approximately 3 ft, no change in output of the transducers due to reflection or restraining effects of the walls could be observed. Thus all further tests were performed in the central part of the reservoir.

Records

The resulting image of a shear and a pressure cell are traced in Figure 6 in form of galvanometer oscillograph records. The angular orientation of the shear cell is 45 deg and of the pressure cell 90 deg, approaching the maximum values for both. The almost sinusoidal shape of each image is clearly visible, corresponding to the force vector generated by the exciter.

Figure 5. Grain size distribution, beach sand, void ratio 0.3-0.4.

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Figure 6. Pressure and shear cell records.

Figure 7. Shear stress distribution.

Figure 8. Normal stress distribution.

Stress distribution

In Figures 7 and 8 the MSR-values of the measured stress vectors are plotted for one quadrant. An extension over all four quadrants would show the expected four-leaved rose for the shear stress distribution and the corresponding type of double cardioids for the normal stresses (Bernhard, 1961). Both curves coincide reasonably well with the computed stress distribution as detailed in the following analysis.

ANALYSIS

Theory

Three parameters are required to determine the biaxial stress distribution. These parameters may be any combination of three of the following six values (Fig. 9):

Four normal stresses: $\sigma_x, \sigma_y, \sigma_u, \sigma_v$

and two shear stresses: τ_{xy}, τ_{uv} .

For convenience's sake a start is made by using the following three normal stresses (Fig. 8):

$$\sigma_y = 0.14 \text{ lb/in}^2, \quad \sigma_x = 0.06 \text{ lb/in}^2 \quad \text{and} \quad \sigma_u = 0.11 \text{ lb/in}^2$$

$$\text{Thus the invariant: } I_{xy} = \sigma_y + \sigma_x = 0.20 \text{ lb/in}^2 \quad (1)$$

$$\text{and } \sigma_v = I_{xy} - \sigma_u = 0.09 \text{ lb/in}^2 \quad (2)$$

$$\text{The shear stresses are: } \tau_{xy} = \frac{1}{2}(\sigma_u - \sigma_v) = 0.01 \text{ lb/in}^2 \quad (3)$$

$$\text{and } \tau_{uv} = \frac{1}{2}(\sigma_y - \sigma_x) = 0.04 \text{ lb/in}^2 \quad (4)$$

Figure 9. Nomenclature.

s = Principal stresses, σ = Normal stresses (+sign=compression)

τ = Shear stresses, $\tau_{\max}$ = Maximum shear stresses

α = Angle between s and X-Y axis

β = Angle between $\tau_{\max}$ and X-Y axis

and the maximum shear stress is:

$$\tau_{\max} = \sqrt{\frac{1}{4}(\sigma_y - \sigma_x)^2 + \tau_{xy}^2} = 0.0412 \text{ lb/in}^2 \quad (5)$$

The principal stresses follow from:

$$\left. \begin{matrix} s_1 \\ s_2 \end{matrix} \right\} = \frac{1}{2}(\sigma_y + \sigma_x) \pm \tau_{\max} = \begin{cases} 0.1412 \text{ lb/in}^2 \\ 0.0588 \text{ lb/in}^2 \end{cases} \quad (6)$$

Table I. Normal and shear stresses.

ϕ°	$\sigma(\text{lb/in.}^2)$			$\tau(\text{lb/in.}^2)$		
	Computed from Eq. 8	Measured from Fig. 8	d Deviation %	Computed from Eq. 9	Measured from Fig. 7	d Deviation %
0	0.141, 2(max)	0.14	0.85	0	0	0
10	0.141	0.143	1.42	0.014, 1	0.017, 1	11.7
20	0.139	0.142	2.1	0.026, 5	0.029	8.62
30	0.132	0.137	3.65	0.035, 8	0.036, 8	2.72
40	0.121	0.131	7.64	0.040, 6	0.041, 1	1.21
45	-	-	-	0.041, 2(max)	0.041, 5	0.72
50	0.107	0.114	6.14	0.040, 6	0.041, 3	1.69
60	0.093	0.103	9.7	0.035, 8	0.036, 3	1.37
70	0.079	0.085	7.06	0.026, 5	0.027	1.85
80	0.068	0.07	2.86	0.014, 1	0.014, 7	4.25
90	0.061(min)	0.06	1.64	0	0	0

Standard deviation: $(\frac{1}{n} \sum_{i=1}^n d^2)^{1/2} \sim 5.22\%$

$\sim 4.75\%$

Compatibility check

Table II lists the measured and computed pertinent stress vectors. The deviation of the maximum shear stress is 1% and of the minimum principal stress 2%.

Though these small percentages are restricted to the above two stresses, it seems to indicate that the shear as well as pressure cells are responding satisfactorily. The somewhat smaller deviation of the shear values shows that in this particular case the shear cell yielded slightly more reliable results than the pressure cell.

Table II. Comparison between computed and measured stresses.

	Stresses (lb/in. ²)		Deviation ~%
	Computed	Measured	
σ_y	0.139	0.14	1
σ_x	0.061	0.06	1.6
σ_u	0.11	0.115	4
s_1	0.141, 2	0.14	1
s_2	0.058, 8	0.06	2
τ_{xy}	0.01	0.010, 5	1
τ_{uv}	0.04	0.041	2
$\tau_{\max}$	0.041, 2	0.041, 5	1

$$I = \sigma_x + \sigma_y = s_1 + s_2 = 0.20$$

CONCLUSIONS

From the few pilot experiments, the following tentative conclusions may be drawn. They are restricted to an elastic response and a steady-state dynamic equilibrium of the soil.

1. Shear and normal stress vectors can be measured in situ provided that the necessary precautions are adhered to, particularly when inserting the cells into the soil.
2. Vibratory (sinusoidal) excitation facilitates the reproducibility of the results.
3. The standard deviation of the experimentally obtained shear stress values from values computed by the theory for a biaxial system did not exceed 4.75%.
4. To avoid the restraining effect of any lead wires between transducer and recorder, the development of self-contained miniaturized pick-up senders, telecasting their output to a receiver, is being considered.
5. Before definite conclusions can be drawn substantially, more experiments are recommended with soils of different characteristics and using triaxial systems as reference frames.

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