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Hydraulic Analog Study of Periodic Heat Flow in Typical Building Walls

U.S. ARMY MATERIEL COMMAND
COLD REGIONS RESEARCH & ENGINEERING LABORATORY
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by Rhoderick Hawk and William Lamb

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PREFACE

Authority for the investigation reported herein is contained in FY 1962 "Instructions and Outline, Military Construction Investigations, Engineering Criteria and Investigation and Studies, Construction Components and Systems: Heat Flow Through Exterior Walls."

The study was conducted for the Office, Chief of Engineers, Technical Development Branch, Engineering Division, by the Thayer School of Engineering, Dartmouth College, Hanover, New Hampshire, under contract with the U. S. Army Cold Regions Research and Engineering Laboratory (USA CRREL).

The principal investigator for this study was Myron Tribus, Dean of the Thayer School of Engineering, Dartmouth College. The report was written by Rhoderick Hawk and William Lamb, Dartmouth students involved in the study.

The final report was prepared under the general direction of Mr. K. A. Linell, Chief, Experimental Engineering Division, USA CRREL, and the immediate direction of Mr. B. L. Hansen, Chief, Technical Services Division, USA CRREL.

Colonel W. L. Nungesser was Commanding Officer of the Cold Regions Research and Engineering Laboratory during the preparation and publication of this report, and Mr. W. K. Boyd was Technical Director.

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SUMMARY

The purpose of this study was to determine transient heat flow through ten exterior building walls for an air-conditioned building having reversible heat flow during a 24-hour period. The building walls were of various construction types; however, they each had essentially the same total heat transmission coefficient of about 0.20 Btu/ft^2-hr-°F. The inside room air temperature was maintained at a constant 70F. The variables involved in this study were the thermal capacitance of the walls and the distribution of capacitance and resistance within the wall sections.

A hydraulic analog computer was programmed to record transient temperature within the walls, and cumulative heat flow and instantaneous heat flux into the room. Three different daily cyclic input temperatures at the outside wall surface, derived from data for September at Washington, D. C., contributed one of the boundary conditions. A photographic technique described herein proved useful in recording results of the computer study. The ability of each wall to dampen load fluctuations was determined as was the time lag of heat flow into the room. A comparison of the cumulative heat flow into the room during office hours was made for each type of exterior wall construction. This report contains: a description of the scaling technique used to program the computer (taking into consideration the time required for cancellation of transient effects), the thermal properties of the ten wall sections, an approximate analytical technique for checking the accuracy of computer results, and results of programming the hydraulic analog computer for both a step and a sinusoidal input function for which analytical results are available.

Results obtained in this study are given on figures and tables contained herein. Walls having high thermal capacitance result in a large thermal time lag, thus permitting the wall to store heat during the day and thereby delay full capacity operation of air conditioners until after working hours. A "peak load factor" is defined which provides a reasonably accurate relative measure of a wall's ability to dampen heat flow variations in walls; it is tentatively concluded that a wall's damping characteristics are improved when the thermal resistance and capacitance are evenly distributed throughout the wall. The results of the study also showed that the greater the thermal capacitance, the greater the damping ability of the wall; this would be expected from theoretical considerations.

It is concluded that the hydraulic analog computer can be successfully applied to solving heat flow problems subject to a few modifications necessary to reduce the average error of results. The influence of a building's location, the number of stories, required window space, and financial considerations governing construction are also factors involved in the selection of building walls in addition to air-conditioning requirements. It is recommended that the effects of direct radiation through windows, orientations of walls other than northerly, heat flow through the roof, and heat sources within the room be investigated more thoroughly before attempting to use the results of this study.

DEFINITION OF SYMBOLS

Symbol	Definition	Measurement unit
A	Cross-sectional area of standpipes	$in.^2$
C	Thermal capacitance	Btu/ft^3-°F
$\overline{C}$	Thermal capacitance for a specified thickness of material	Btu/ft^2-°F
C_p	Specific heat of solid	Btu/lb-°F
h	Height of fluid in standpipes	in.
f	Frequency of input 'temperature'	Cycles/hr
k	Thermal conductivity	Btu/°F-hr-ft
ρ	Density of solid	lb/ft^3
R_a	Analog resistance	$min/in.^2$
R_p	Prototype resistance	°F-hr-ft/Btu
t_a	Analog time	min
t_p	Prototype time	hr
T_s	Time scale = t_a/t_p	min/hr
U_s	Thermal energy scale = $\left(\frac{A}{\overline{C}} \times V_s\right)$	$in.^3$/Btu
V_s	Temperature scale	in./°F
v	Temperature	°F
ϕ	Phase angle of input 'temperature'	hr
a, α	Thermal diffusivity of solid	ft^2/hr
$\dot{Q}$	Heat flow from inside surface to room air	Btu/hr
$\overline{\dot{Q}}$	Average $\dot{Q}$ for one cycle	Btu/hr
P. F.	Peak load factor = $\frac{\dot{Q}max}{\overline{\dot{Q}}}$	unitless
x	Distance through material	ft

HYDRAULIC ANALOG STUDY OF PERIODIC HEAT FLOW IN BUILDING WALLS

by

Rhoderick Hawk and William Lamb

INTRODUCTION AND SCOPE

As building technology progresses and comfortable working conditions become more desirable, accurate calculations of heat flow through building walls become increasingly important in selecting the proper air-conditioning capacity, particularly in office buildings. Determinations of heat gain allow one to choose the most suitable wall, both from economical and thermal considerations.

This investigation was conducted to determine the periodic heat flow through ten typical building wall constructions (Table I) for an air-conditioned building. The problem was approached by using a hydraulic analogy method. However, these results do not represent the full picture of the air-conditioning problem since such important factors as direct radiation through windows, infiltration into the room, heat sources within the room, heat flow through the roof, and orientations other than northerly were not considered.

Although many investigations have been made in this field – by mathematical analysis, analog methods, and direct experimentation – it is felt that this study could be the beginning of a more advanced and complete study of the air-conditioning problem in office buildings.

The model method used in this investigation is based on an analogy between heat conduction in materials and a fluid flowing in a network of small orifices and manometer-like tubes. The analogy between the hydraulic and heat phenomena is shown in the following table:*

Temperature (°F)	Height of fluid in a standpipe (in.)
Thermal energy (Btu)	Volume of fluid ($in.^3$)
Heat flow (Btu/hr)	Fluid flow ($in.^3$/min)
Thermal capacitance (Btu/°F)	Cross-sectional area of standpipe ($in.^2$)
Thermal conductance (Btu/°F-hr)	Volume of fluid passed through small orifice per unit time per inch difference in head of fluid in adjoining standpipes ($in.^2$/min)

POSTULATES OF THE TEST

1) Three inputs were used for each wall. Two are typical of the 24-hour temperature ranges encountered in Washington, D. C., during the air-conditioning month of September.† These are designated as inputs 1 and 2 and have minima at 60F and 75F, respectively. The third input is a temperature variation for one typical day during a heat wave, with a minimum at 82.5F and a maximum at 100F. These inputs are shown in Figure 1.

2) All walls were designed for a total heat transmission coefficient (U) of approximately 0.20 Btu/F-hr. These "U" values were made equal by the addition of selected insulation in multiples of one-tenth of an inch.

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* For the derivation, see Appendix A.

† September data were used for the inputs since data for the complete air-conditioning season (June, July, August, September) had not arrived by the time the computer was ready for operation.

3) The room air was maintained at 70F.

4) All walls were northerly exposures.

5) The problems were solved for one-dimensional heat flow (no edge effects).

6) The walls had no windows; there was no infiltration into the room; and there were no heat sources within the room.

7) Moisture transfer within the walls was zero.

8) The only energy input to the external surface of the wall was by convection from the surrounding air.

9) The outside air was varying in the periodic steady state.

10) All resistances and capacitances were independent of time, temperature, or heat flow.

Table I. Typical exterior building wall construction.

U = .20±

Wall no.	Description
1	4 in. CMU (light)*, 2 in. cavity; 4 in. CMU (light); building paper
2	4 in. brick; 2 in. cavity; 4 in. CMU (heavy)†; insulation
3	8 in. CMU (light); $\frac{3}{4}$ in. furring; $\frac{1}{2}$ in. gypsum board; insulation
4	4 in. brick; 2 in. cavity; 8 in. CMU (light); insulation
5	4 in. brick; 2 in. cavity; 8 in. CMU (heavy); insulation
6	"Sandwich" panel with 2 in. glass fiber core clad in 18-gauge galvanized steel, to be installed in steel frame, primed and painted, inside and outside.
7 & 8	8 in. and 12 in. concrete; $\frac{3}{4}$ in. furring, $\frac{1}{2}$ in. gypsum board; insulation
9	4 in. brick; 1 in. cavity; $\frac{5}{8}$ in. plywood sheathing, 2 x 4 studs, $\frac{1}{2}$ in. gypsum board; insulation if needed
10	Wood siding; $\frac{5}{8}$ in. plywood sheathing, 2 x 4 studs; $\frac{1}{2}$ in. gypsum board; insulation

* Hollow concrete block (shale aggregate).
† Hollow concrete block (gravel aggregate).

PROCEDURE

Figure 2 shows how a thermal network with a finite number of segments was drawn directly from knowledge of the construction and the thermal properties of the materials used in the wall (Table II). The hydraulic analog network used in measuring the desired characteristics of the heat flow through the wall was constructed by appropriately scaling the problem. The procedures for scaling the hydraulic analog network, and the scaling sheets used for the actual calculations are given in Appendix A.

The scaled hydraulic resistances* are set on the computer, and the appropriate standpipes (capacitors) (Fig. 3) are inserted in the places shown in the hydraulic analog network.

* The hydraulic resistances were calibrated by the falling head method, described in reference 1.

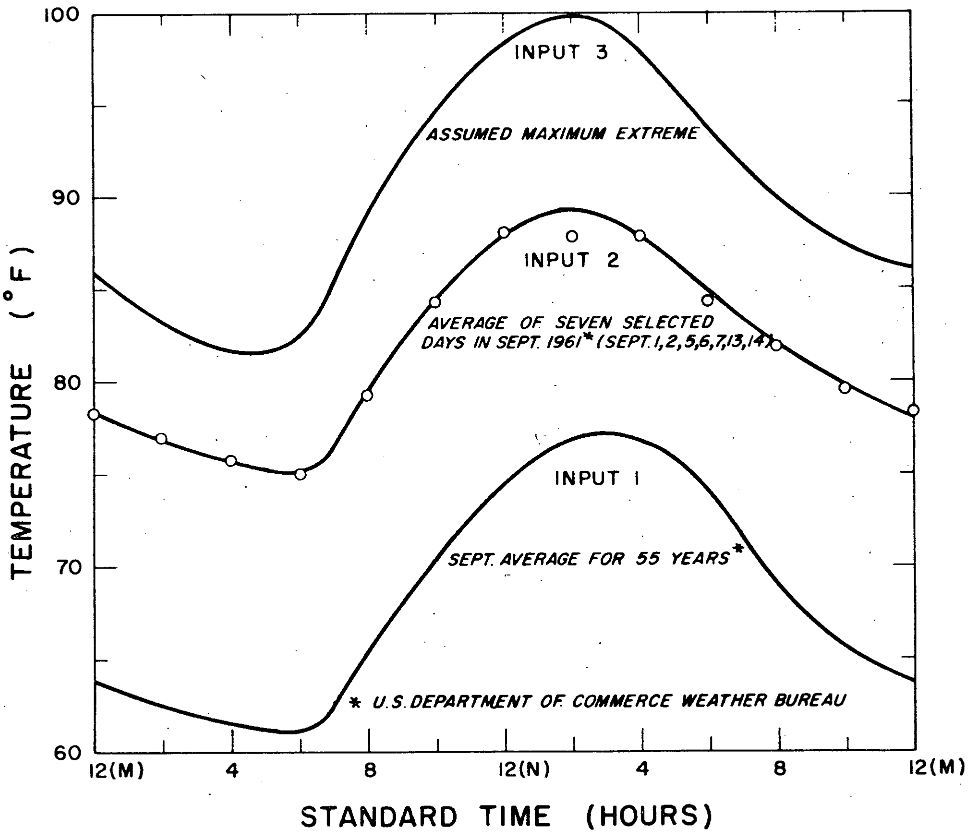

Figure 1. 24-hour temperatures for Washington, D. C. during September.

The input function of time, f(t), is drawn on a sturdy paper to the appropriate scale, cut with scissors and made into a belt, and placed on a kymograph which feeds it at a scaled speed. The water level in a reservoir connected to the computer is made to follow this moving paper cam (by a system composed of contact switches and a motor), thus providing the input "temperature" (Fig. 4).

The hydraulic apparatus representing the air-conditioner (and heater) is provided by a constant head reservoir placed at the end of the computer (Fig. 5). When water is flowing from the computer into the reservoir, it overflows into a calibrated tube. The rate at which the fluid flows is calculated by recording the height of the fluid in the calibrated tube at times representing hours in the thermal network. When the water is flowing from the reservoir into the computer, water is added to the reservoir by a system involving another calibrated tube filled with water, a valve, and a float and contact arrangement. Once again, the rate of flow is calculated by recording the height of fluid in this second calibrated tube.

Although the data necessary for calculating heat gains were recorded manually while the wall sections were being run on the computer, it was decided that a method should be devised to record the temperatures in the various lumps of the walls (height of fluid in the capacitor tubes). A suitable method was obtained by using a photographic technique (App. F).

After the computer components had been calibrated and the computer made ready for operation, two test problems with known solutions were run to test the operation and accuracy of the computer. The results of these two tests were reported previously[2]* and a brief resume is presented in Appendix D.

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* Numbers refer to references.

Table II. Thermal properties of the materials used in the typical wall constructions.

Material	Thermal conductivity $K = 1/R$ (Btu/°F-hr-ft)	Thermal capacitance $C = \rho \times Cp$ (Btu/ft³-°F)	Thermal diffusivity $\alpha = k/C$ (Ft²/hr)	Thermal "wave length" $L = 0.44\sqrt{\alpha/f}$* (ft)	(in.)
Light CMU†	0.33	14.5	0.0228	0.325	3.90
Light CMU + air	0.248	3.6	0.0689	0.565	6.78
Brick	0.75	24.6	0.0305	0.377	4.52
Heavy CMU†	1.00	22.45	0.0445	0.455	5.46
Heavy CMU + air	0.366	5.61	0.0653	0.550	6.60
Furring + air	0.06	2.14	0.0280	0.361	4.33
Furring + fiber glass insulation	0.0225	2.4	0.0094	0.208	2.50
Gypsum board	0.0925	15.55	0.00595	0.166	1.99
Fiber glass insulation	0.021	0.30	0.0700	0.580	6.95
Steel sheet	312.00	58.90	5.3	4.95	59.5
Steel sheet + fiber glass	1.415	0.787	1.114	2.27	27.2
Concrete	1.00	22.45	0.0445	0.471	5.65
Plywood sheathing	0.067	21.5	0.00312	0.116	1.39
2 x 4 studs + air	0.304	2.155	0.141	0.810	9.73
2 x 4 studs + fiber glass	0.0225	2.4	0.0094	0.208	2.50
Wood siding	0.049	17.4	0.00282	0.1145	1.37

* See Appendix B
† Defined in Table I.

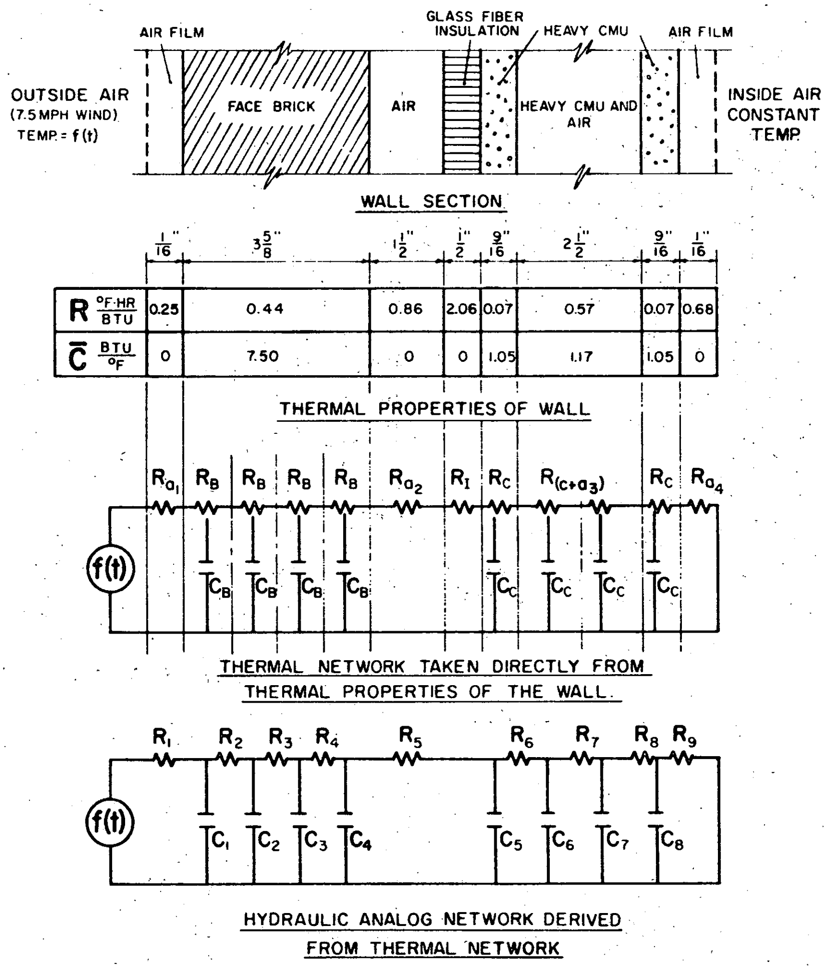


Figure 2. Thermal properties and analog circuits (example using wall 2).

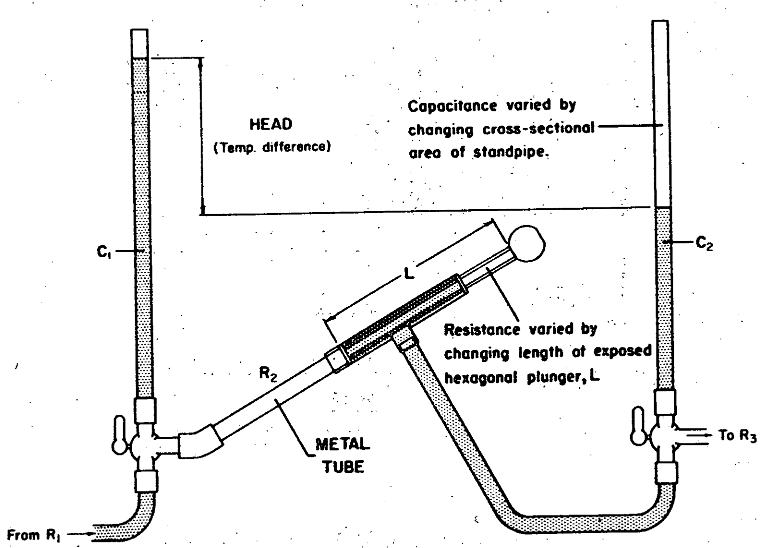


Figure 3. Computer components.

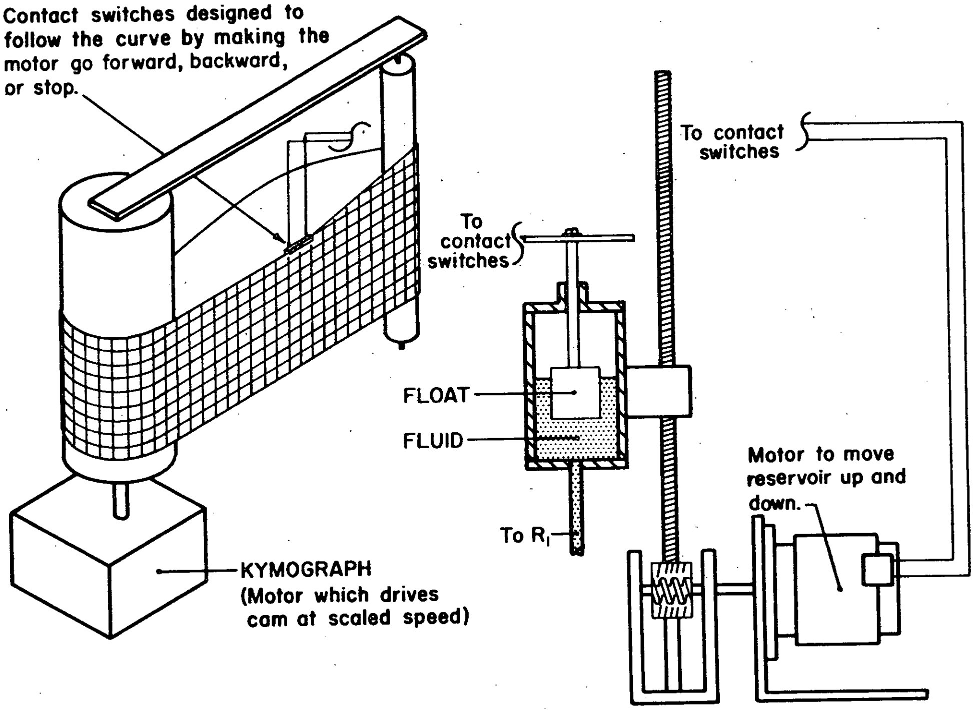


Figure 4. Programming apparatus.

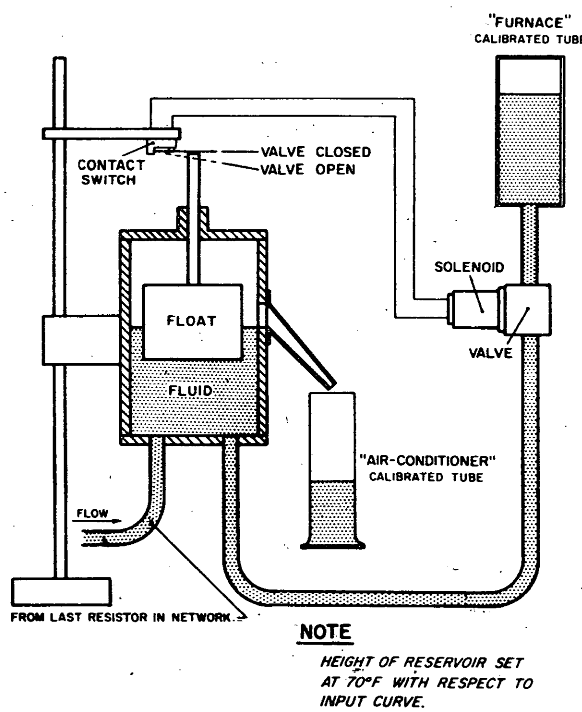


Figure 5. Constant-head reservoir (air conditioner, furnace).

RESULTS

The cumulative heat flowing into the room as a function of time for one cycle for all ten walls and three inputs is plotted in Figures 6-27. The total heat gains for one cycle and for the working hours (7:00 a.m. -3:30 p.m. EST, 8:00 a.m. -4:30 p.m. EDT) are given in Table III. The cumulative heat curves for input 1 are given for only two walls since the heat flow into or out of the room is so small that accurate readings could not be obtained from the computer. This situation could have been remedied by rescaling the problems for input 1, but this would have added greatly to the length of work while increasing its value only slightly.

The instantaneous heat flow to the room as a function of time for one cycle for all ten walls and three inputs is plotted in Figures 28-48. These curves were derived directly from the cumulative heat curves. This method was considered best because the water (representing Btu) did not always flow smoothly from the constant head reservoir (air-conditioner). The time lags of heat flow into the room for the various walls with inputs 2 and 3 are recorded in Table III. The upper values in the table indicate the time lags from when the outside air temperature is at a maximum to when the instantaneous heat flow into the room is at a maximum. The lower values are the time lags for the minima.

Table III. Heat gains and time lags for typical walls.

Wall no.	Heat gain for 1 cycle (Btu/ft^2) input 2	input 3	Heat gain for office hours* (Btu/ft^2) input 2	input 3	Time lag of heat flow to room (hr)† input 2	input 3	Analytical
1	56.8	98.2	16.8	31.6	max 4.42 min 3.00	max 3.75 min 2.00	3.52
2	56.5	99.8	15.1	27.1	5.00 3.00	4.42 4.00	5.56
3	53.5	93.5	15.7	29.6	4.00 2.00	5.42 3.00	5.02
4	49.0	86.9	14.9	29.1	9.00 5.00	7.75 7.00	5.50
5	57.4	107.0	14.0	25.9	4.00 3.00	6.42 4.67	6.61
6	54.3	92.6	24.1	40.2	1.00 1.00	0.00 0.30	0.52
7	53.4	93.0	14.8	28.6	7.00 3.00	6.75 2.75	5.10
8	61.5	108.3	16.9	30.7	9.00 7.00	7.50 6.17	5.81
9	52.4	89.6	14.3	27.7	5.58 3.33	4.75 4.00	5.09
10	54.0	93.6	17.7	32.0	3.75 1.00	1.25 2.50	2.87

* 7:00-3:30 p.m. EST, 8:00-4:30 p.m. EDT.

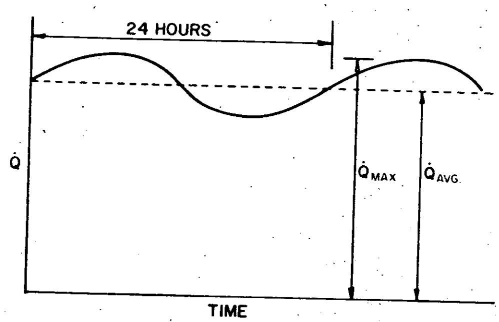
† Upper values indicate time lag between maximum outside air temperatures and maximum heat flow into room; lower values are time lags between minimum outside temperature and minimum heat flow to room.

In Figures 49 and 50, the cumulative heat curves and the instantaneous heat flow curves are given for wall 6, input 3, using three different mesh spacings of the thermal network. A method of determining the necessary mesh size of a certain material to achieve 5% accuracy is described in Appendix B. The mesh sizes of the various wall materials determined by this method are given in the last column of Table II.

In Figures 51, 52, and 53, the temperatures at various distances into wall 6 are given for one cycle for inputs 1, 2, and 3, respectively. In Figure 54, the energy stored in wall 6 throughout one cycle is plotted for all three inputs. These curves were drawn on the assumption that when the wall is at a uniform temperature of 70F, the energy stored in it is zero. This last group of Figures (51-54) is presented as a sample of the information available on film for all ten walls and three inputs.

In order to obtain an approximate check on the results of this study, the walls were divided into two segments, a sine wave was used as the input, and analytical solutions were obtained for the peak load factors and time lags (the procedure used for the analytical solutions is described in Appendix E). The total resistance, capacitance, and density, and the simplified two-segment networks used in the approximation are recorded in Table IV for all of the walls.

The concept of the peak load factor used in this report is a measure of the wall's ability to dampen load fluctuations and is defined in the following manner:

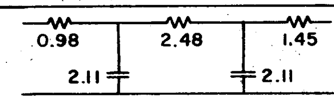
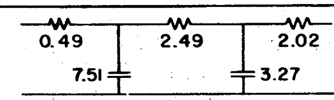
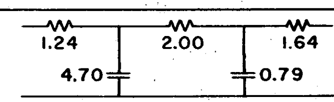
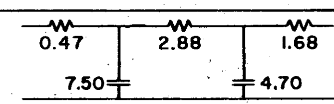
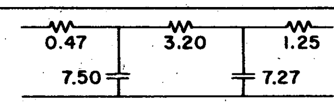
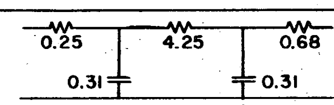
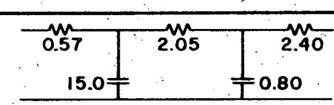
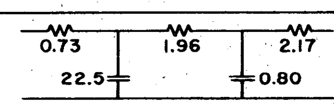
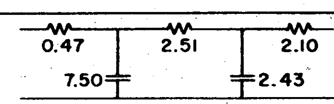
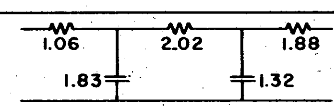


The peak load factor is a ratio of the peak load to the average cooling load on the air-conditioner:

$$\text{P. F.} = \dot{Q}_{max}/\dot{Q}_{avg}.$$

The peak load factors obtained by both the analytical and computer solutions are recorded in Table V. The time lags obtained by both methods are recorded in Table IV.

Table IV. Thermal properties of the walls, and the two-segment simplified networks.

WALL NUMBER	TOTAL P POUNDS*	TOTAL R F, HR/BTU*	TOTAL $\bar{C}$ BTU/F*	"TWO-SEGMENT" NETWORK
1	30.85	4.91	4.22	
2	56.06	5.00	10.78	
3	35.03	4.88	5.49	
4	66.12	5.03	12.20	
5	80.31	4.92	14.77	
6	5.02	5.18	0.62	
7	98.77	5.02	15.80	
8	146.80	4.86	23.30	
9	39.00	5.08	9.93	
10	6.29	4.96	3.15	

* These values are for one square foot of the wall.

DISCUSSION

Total heat gains for one cycle

For any given input, the total heat gains throughout one cycle should be the same for all ten walls. The proof of this is quite simple. The input function (temperature) is of the form A + B f(t), where f(t) is a periodic function. The thermal networks considered are linear circuits, hence the heat flow to the room is of the form A/R + B/Z f(t), where Z is the thermal impedance and R is the total resistance of the network. The integral of the heat flow to the room over one cycle will give the total heat gain. The integral of the periodic component over one cycle will always equal zero, and the integral of the non-time-varying component will equal (A/R) x (period). As A and the period are constants for any given input, and the total resistance, by design, is the same for all of the walls, the total heat gains for a particular input should be the same for all of the wall sections.

An inspection of columns 1 and 2 in Table IV reveals that the heat gains for the walls are not the same, but vary by as much as ± 10%. The total resistance did vary somewhat from wall to wall. (The reasons for this and the other deviations are explained in a subsequent section-"Sources of Error.") Closer agreement is observed if the heat gains are multiplied by the actual resistance of the wall (Table V).

Heat flow to the room

As noted above, the heat flow to the room is of the form A/R + B/Z f(t). Although the integral of this function for a given input will be constant for all of the walls, the amplitude and position of the heat flow curves will vary from wall to wall since the periodic component of the heat flow function has an amplitude and phase angle which depend upon the thermal impedance of the wall. It is in these heat flow curves that the thermal capacity of the walls becomes apparent. For purposes of comparison, a peak load factor, P.F., was defined (page 8). This peak load factor provides a reasonably accurate relative measure of the wall's ability to dampen or attenuate the heat flow variation, since the difference in total heat gains will affect the P.F. values by only a small amount. (The difference in total heat gains is caused by a small shift - up or down - of the complete heat flow curve. This shift will change the P.F. values by a negligible amount).

The ratio P.F., used in this report, or some like measure, would appear to be a useful guide in determining the air-conditioning capacity necessary for a particular building. If the air-conditioning capacity were designed to meet the average heat flow, there would be a time during the day (most likely during the working hours) when the air-conditioner would not be able to keep up with the heat flow into the room. And the larger the P.F. value for that particular wall, the more the temperature in the room would rise when the air-conditioner lagged behind the heat flow. It is obvious, then, that the smaller the P.F. value, the smaller the necessary air-conditioning capacity, and, consequently, the smaller the costs of installing and operating the air-conditioners. It is observed from Table V that there is not always close agreement between the analytical and computed P.F. values. This is to be expected since the input used for the analytical approximation was a sine wave, while the input curve used on the computer is represented by a Fourier series. But what is important is the relative agreement between the various walls. Three of the best walls by both methods are walls 5, 7, and 8. These are also the walls with the largest thermal capacities. This correlation between high damping power and large thermal capacitance is in good agreement with theoretical expectations since the damping characteristic depends upon the impedance, which in turn depends upon the capacitance. However, the P.F. value is only approximately a function of the thermal capacitance and depends to a certain extent on how the resistance and capacitance are distributed throughout the wall.

The dependence of the P.F. value on the distribution of the resistance and capacitance is demonstrated by the results obtained for walls 5 and 7. Wall 7 has a large capacitance, but it is concentrated almost entirely at the outside of the wall. Although wall 5 has a slightly smaller capacitance, it is divided almost exactly in half with the

Table V. Relative heat gains and peak load factors for typical walls.

Wall no.	Product of heat gain for one cycle and total R (°F-hr-ft²) input 2	Δ*(%)	input 3	Δ*(%)	Peak load factor, P.F. = $\left(\frac{\dot{Q}_{max}}{\dot{Q}_{avg}}\right)$ input 2	input 3	Analytical
1	282.0	+4.1	488.0	+2.1	1.49	1.36	1.25
2	282.0	+4.1	499.0	+4.3	1.34	1.27	1.16
3	261.0	-3.7	457.0	-4.3	1.35	1.28	1.18
4	255.0	-5.9	450.0	-5.9	1.18	1.16	1.17
5	271.0	0.0	526.0	+10.	1.16	1.19	1.12
6	282.0	+4.1	480.0	+0.4	1.66	1.47	1.31
7	268.0	-1.1	467.0	-2.3	1.25	1.24	1.14
8	274.0	+1.1	482.0	+0.8	1.17	1.12	1.08
9	266.0	-1.9	456.0	-4.6	1.40	1.26	1.20
10	268.0	-1.1	464.0	-2.9	1.42	1.41	1.27

*Δ = Deviation from average

(Average = 271 for input 2
= 478 for input 3)

bulk of the resistance coming in the central part of the wall (Table III). The P.F. values are lower for wall 5 by both the computer results and the analytical approximation. The conclusion then is that a wall's damping characteristics are improved when its resistance and capacitance are distributed more evenly throughout the wall. But this conclusion is only approximate, and the distribution of resistance and capacitance should be more thoroughly investigated because of its importance in air-conditioning design.

Time lag of heat flow

The time lag is larger at the maximum than at the minimum in almost every case (Table IV). This difference is easily explained by realizing that the time lag is a function of the impedance which is partly dependent upon the frequency of the input. Since the input is not a true sine wave, the frequency of the periodic component varies between the maximum and the minimum and is largest when the input curve is at its maximum (this is borne out by a close inspection of the input curves – Fig. 1). Thus the impedance – and therefore the time lag – is greatest at this point.

If the time lag is large enough that the peak heat flow occurs well after working hours, some of the stored heat will flow back out into the cooling night air instead of into the room. This has an important consequence on the air-conditioning capacity in either one of two ways.

The first case is where the air-conditioners only operate during the working hours, and it is assumed that the room and the walls will cool down naturally during the night. (This principle was first used by the Spanish settlers in Southwestern United States — Albuquerque, N.M., for example – who built adobe houses with walls 2 or 3 feet thick. During the cooler desert nights, they ventilated their homes and cooled the walls from both sides.) In this situation the air-conditioners need only be designed to meet the largest demand during the working hours, which will be in the vicinity of the average heat flow for the cycle, and possibly even lower. This fact has great economic significance, both in the initial investment and in the costs of operation and maintenance.

The second case occurs most frequently in areas where the seasonal temperatures are unusually high. Here the walls and the room are not able to cool naturally during the night, and it is necessary to keep the air-conditioners running constantly throughout the cycle. Although it is necessary to design the air-conditioners to meet the peak load, they will be running at their full capacity only outside of working hours. This consideration is especially important in large industrial areas where the power consumption during the day is already at a point near the upper limit of the power supply.

Table IV shows the analytical and computed values of the time lags. Heat gains for office hours are recorded in the same table. (These values have been adjusted assuming all of the heat gains for one cycle to be equal.) It is observed that walls 7 and 8 have large time lags, but for heat gains during the working hours, wall 7 is one of the best of all the walls, while wall 8 is among the worst. This is the result of a cancellation effect between the peak load factor and the time lag. When the time lag is large – on the order of 6 hours or more – the average heat flow to the room during working hours is less than the average for the cycle. (For a large time lag, the heat flow to the room is near the minimum in its cycle during the working hours. See Figure 4, for example.) If the heat flow to the room during the working hours is less than the average for the cycle, it becomes obvious that the more the heat flow is attenuated the greater the heat gain will be during the office hours. This was the case for wall 8, as can be seen by comparing its relative heat gain and peak load factor with those of wall 7.

It might be pointed out that wall 6 has the smallest time lag and the greatest peak load factor (thereby the least attenuated heat flow) and the greatest heat gain for the office hours. This is a direct consequence of this wall's extremely low thermal capacitance.

<u>Sources of error</u>

As noted earlier, the errors in total heat gains run as high as ±6-7%. Errors of this magnitude were not expected in view of the results of the test problems (App. D). To account for these errors, an appraisal was made of the maximum error contributed by each of the components in the computer.

The hydraulic resistance is directly proportional to the viscosity of the fluid, and the viscosity is dependent upon the fluid temperature[3]. At the time the tests were run, the air temperature in the laboratory varied throughout the day by as much as ±6C. Variations of this magnitude in the fluid temperature during a test would result in errors of 10% or more. But every effort was made during the tests to keep the resistors set within ±1C of the fluid temperature. Variations of 1C contribute an error of ±2% in the overall resistance of the wall.

The period of the input curve varied by about ±0.6% from that used to scale the problems. On wall 5 this error was about ±1%. It is believed that this error is caused partly by the kymograph's speed failing to remain constant, but more by a slight slippage of the cam (input curve). This error in the period affects the resistors directly and changes the time lags by about ±15 minutes.

Both the input reservoir (outside air temperature) and the constant head reservoir (air-conditioner and heater) were set on the computer with respect to the reference lines. An error of approximately ±0.6% is possible from the setting of each of these two components.

An error of roughly ±1% is a reasonable estimate for the calibration and setting of the hydraulic resistors. Also included in this estimate would be the effect of any unaccounted-for resistance in the stopcock valves.

An error of ±1% results from the calibration and selection of the correct capacitor tubes.

As the programmer followed the input curve in small steps up or down, there was a certain amount of induced error. This possibility that a given input curve is not followed in exactly the same manner in two tests accounts for an error of ±1%.

The method used to determine when the effect of transients was insignificant (App. C) should not allow for an error greater than ±0.5%.

The reading of the calibrated tubes for the constant head reservoir, and the method used to record the data from the films (Fig. 51-54) could produce an error of about ±1%.

These sources of error account for a maximum possible error of approximately ±8%. As can be seen in Table V only a few of the deviations approach this magnitude, and most of the errors are much smaller. This is to be expected since a certain amount of cancellation of errors will occur during the normal operation of the computer. The method generally used to calculate the average error consists of taking the square root of the sum of the squares of the maximum possible individual errors. By this method, the predicted average computer error is 3.1%. The average of the computer errors given in Table V is 3.2%.

It was concluded from this agreement between the predicted and the actual errors that the hydraulic model method can be successfully applied to solving heat transfer problems, but that every effort should be made to reduce the average error. Many of the inaccuracies causing this error could be eliminated with a few minor improvements to the computer. Immersion of the computer in a constant temperature bath would eliminate the largest source of error. The design of new resistors with greater resistance ranges would eliminate the necessity of occasionally using several resistors in series, which causes a certain unknown amount of error.

CONCLUSIONS AND RECOMMENDATIONS

Before the results of this investigation, or any other study like it, can be used to select the proper building wall or air-conditioner, it must be remembered that many other factors influence their selection; for instance, the location of the building site, number of stories, window space desired, the financial conditions governing its construction, and the personal preferences of the prospective owner.

We have seen that walls 7 and 8 have very large thermal capacities which gives them low peak load factors and large time lags, desirable characteristics for the walls of an air-conditioned building. But these characteristics are the result of the fact that the exterior of these two walls consists of 8 in. and 12 in. poured concrete, respectively. It is not only obvious that walls constructed in this manner will be very expensive to build, but that they will also be very impracticable walls in a building much higher than two stories. On the other hand, if the building is to be two stories high and air-conditioning is one of the major problems, these walls might be the most suitable – both economically and from a thermal standpoint. These walls might also be satisfactory if they were to be used in the first two stories of a multi-storied skyscraper. 'Heavy' walls such as walls 7 and 8 would provide the strength needed in the base walls of the skyscraper.

Another important factor for consideration is the relative costs of the materials and construction of these walls. Considering complete costs of construction, walls 7 and 8 are the most expensive, wall 10 is by far the least expensive, and the remainder of the walls fall in approximately the same price category. Wall 6 is very easy and inexpensive to erect, but the cost of the metal panel offsets this saving. Although wall 10 is a very inexpensive wall to build, it would very often be considered an undesirable wall for buildings other than residential structures. Reasons for this would include maintenance costs, general appearance, and the fact that it would not be a suitable wall for multi-storied buildings.

Relative ease in framing windows and maintenance costs are two other important factors for consideration. Where a great deal of window space is desired, it would be an easy and inexpensive operation for walls 6 and 10, but quite expensive for the others. Where maintenance costs are considered, it is found that wall 6 is more satisfactory than the others. It would also be a simple procedure to dismantle wall 6 and reassemble it in another location – a characteristic not possessed by the other walls.

In considering relative costs and thermal characteristics of these walls, it should be remembered that this study was subject to many limitations and simplifying assumptions. In the first place, cost comparisons are difficult since many of these walls are unrealistic constructions. The overall transmission coefficient was made a constant value for the various walls by adding successive layers of insulation in thicknesses of one-tenth of an inch. In reality, insulation is seldom used in thicknesses less than one inch. To cut it to a smaller thickness is a difficult and economically unfeasible process.

Factors not included in this study include direct radiation through windows, orientations other than northerly, heat flow through roof, and heat sources within the room. Actual heat conduction through the walls makes up only a small fraction of the total heat gain considering these factors. In particular, direct radiation through windows will have a significant effect on the time lags and peak load factors achieved in this report. A more thorough investigation of these considerations should be made before any attempt is made to use the results of this study in selecting the most suitable wall or air-conditioning capacity for a particular building.

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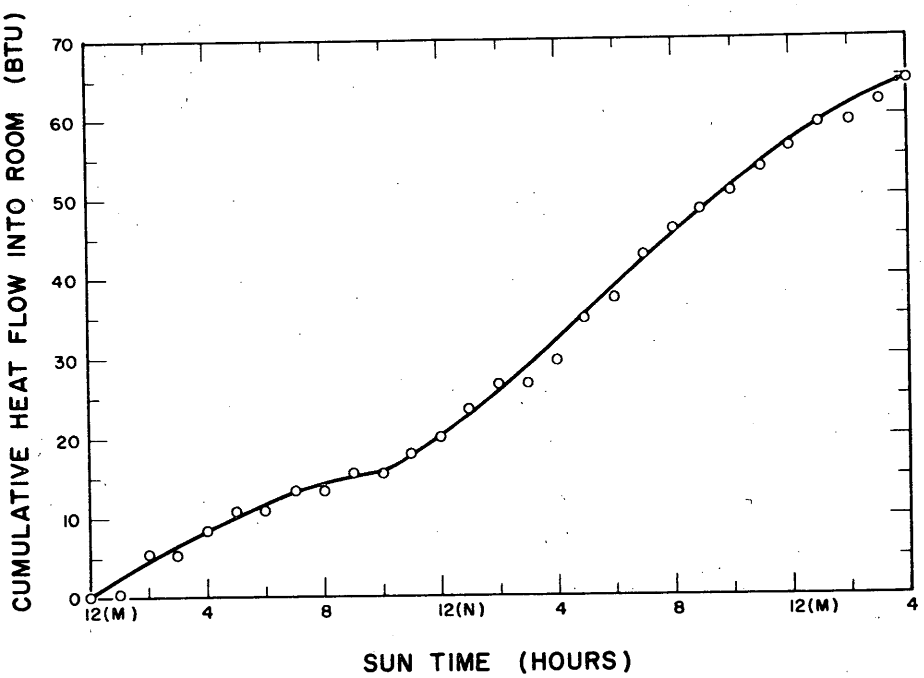


Figure 6. Cumulative heat passed through wall 1, input 2.
Total Btu/day-ft^2 = 56.8.

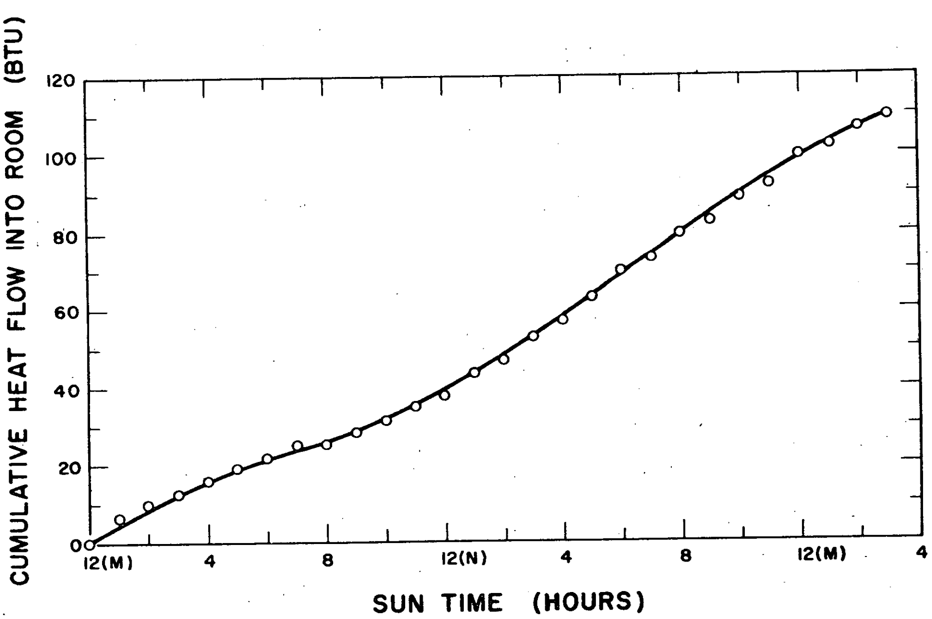


Figure 7. Cumulative heat passed through wall 1, input 3.
Total Btu/day-ft^2 = 98.2.

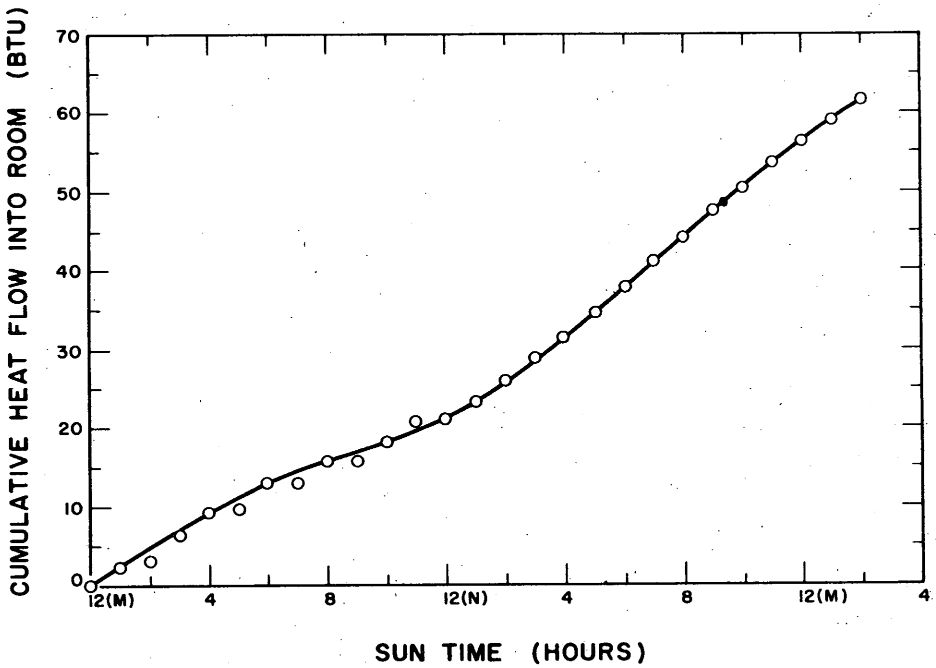


Figure 8. Cumulative heat passed through wall 2, input 2.
Total Btu/day-ft^2 = 56.5.

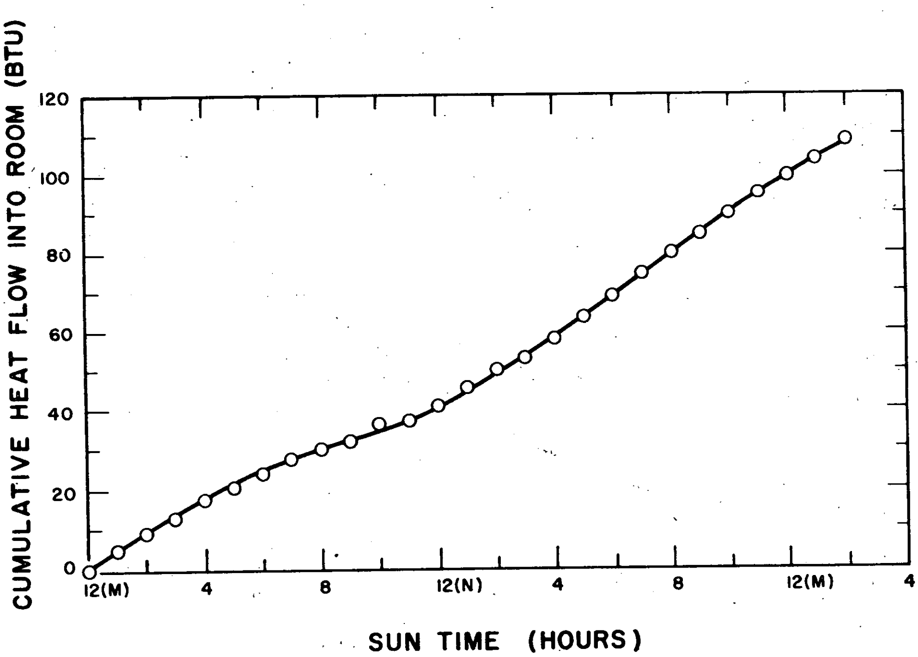


Figure 9. Cumulative heat passed through wall 2, input 3.
Total Btu/day-ft^2 = 99.8.

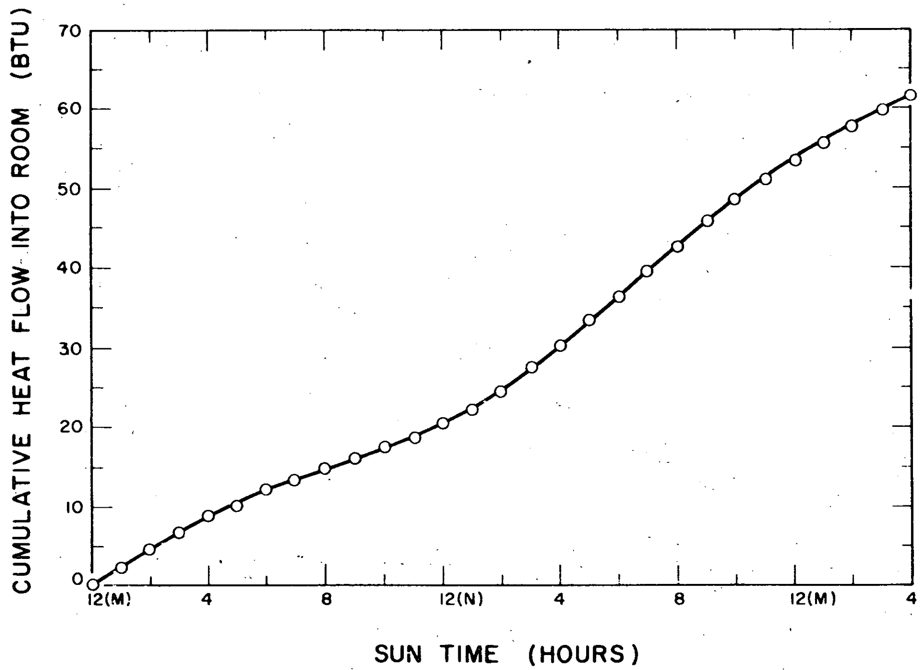


Figure 10. Cumulative heat passed through wall 3, input 2.
Total Btu/day-ft^2 = 56.2.

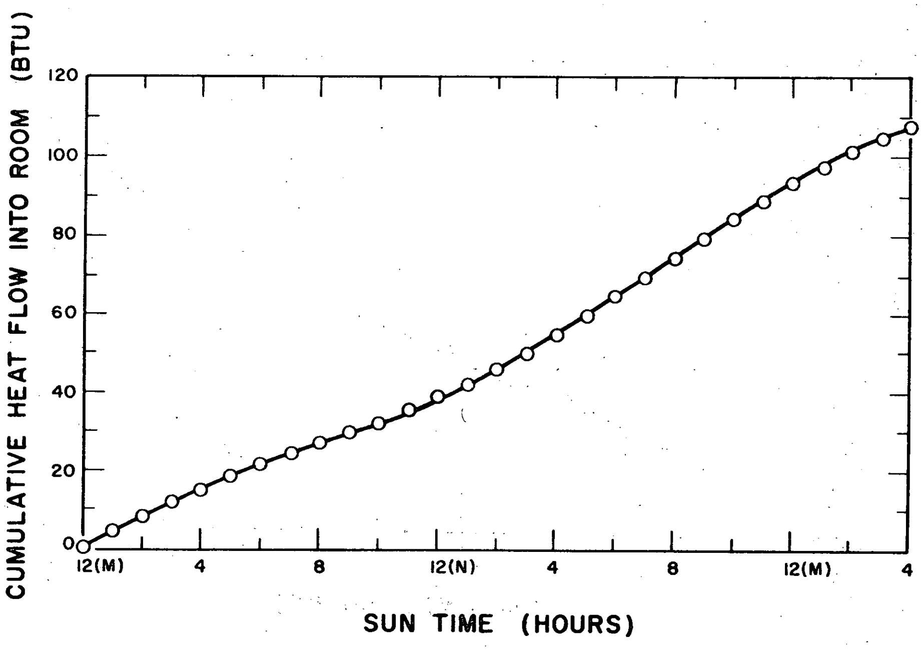


Figure 11. Cumulative heat passed through wall 3, input 3.
Total Btu/day-ft^2 $\cong$93.5.

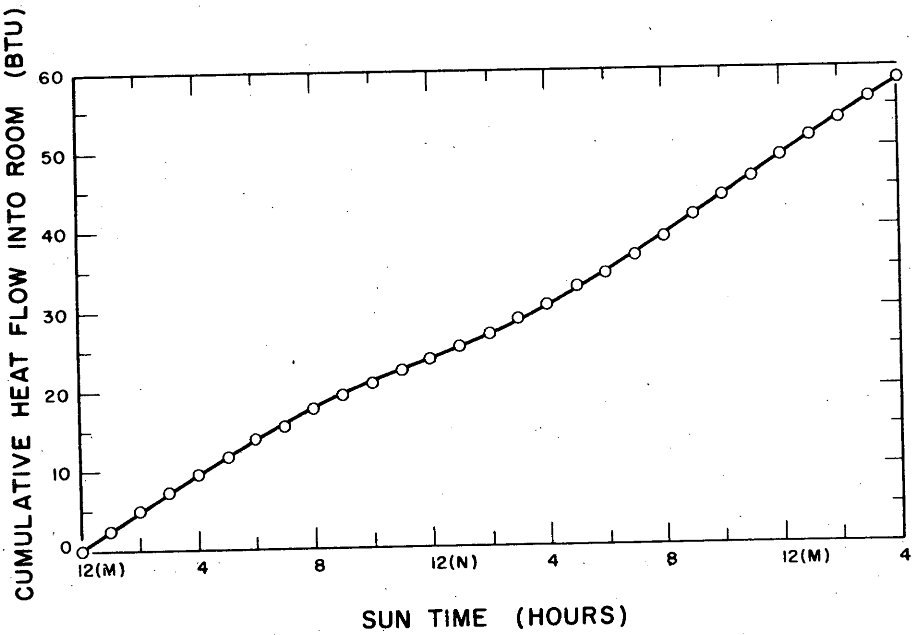


Figure 12. Cumulative heat passed through wall 4, input 2.
Total Btu/day-ft^2 = 49.0

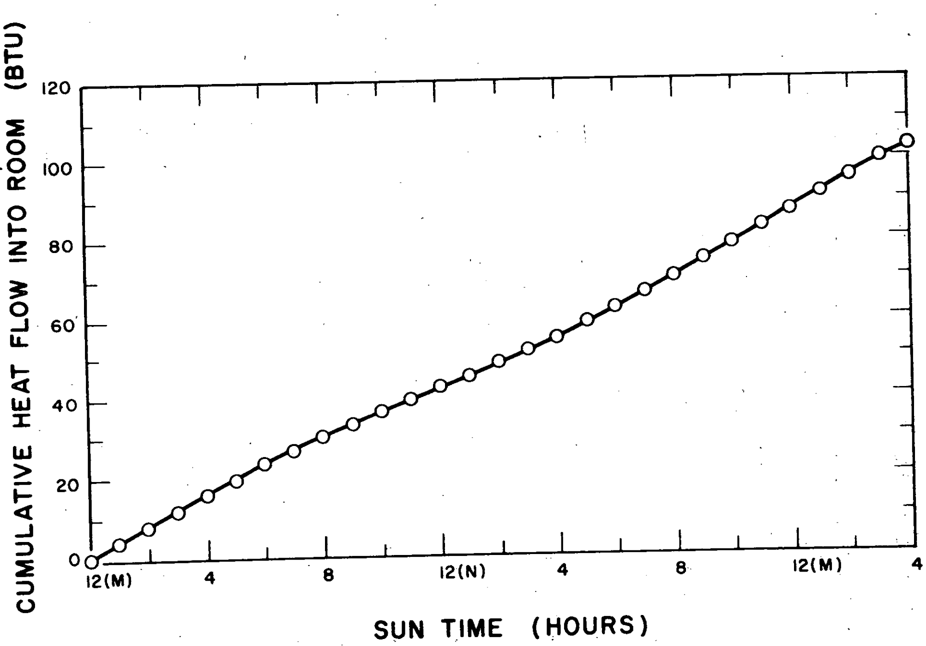


Figure 13. Cumulative heat passed through wall 4, input 3.
Total Btu/day-ft^2 = 86.9.

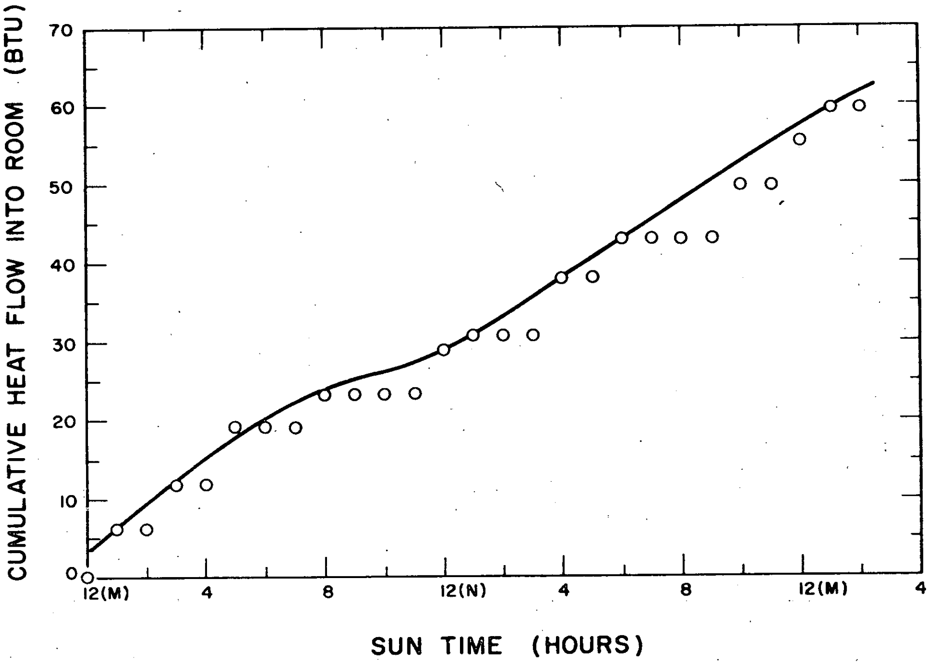


Figure 14. Cumulative heat passed through wall 5, input 2.
Total Btu/day-ft^2 = 57.4.

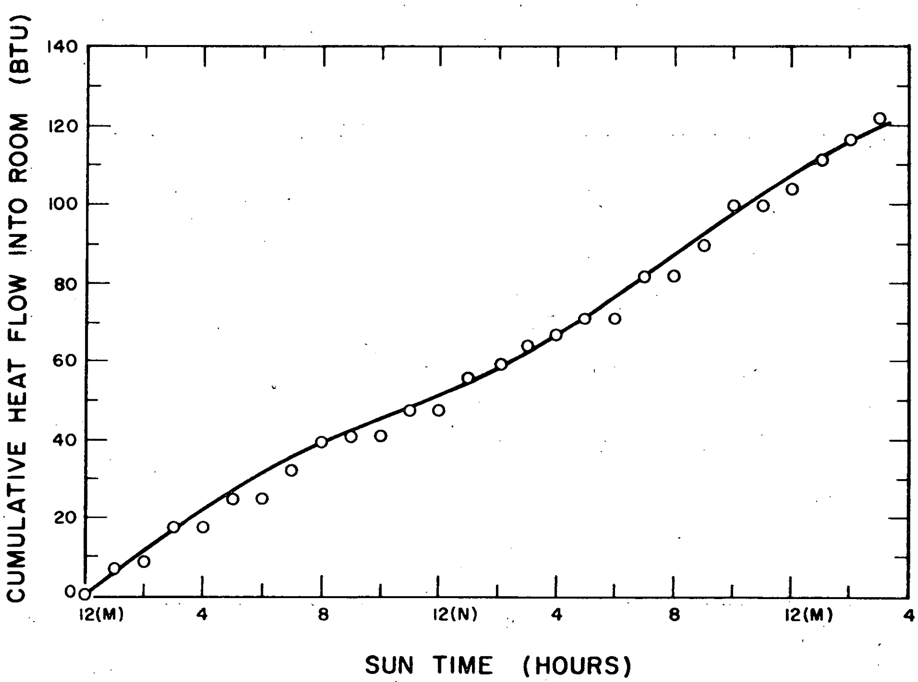


Figure 15. Cumulative heat passed through wall 5, input 3.
Total Btu/day-ft^2 = 107.

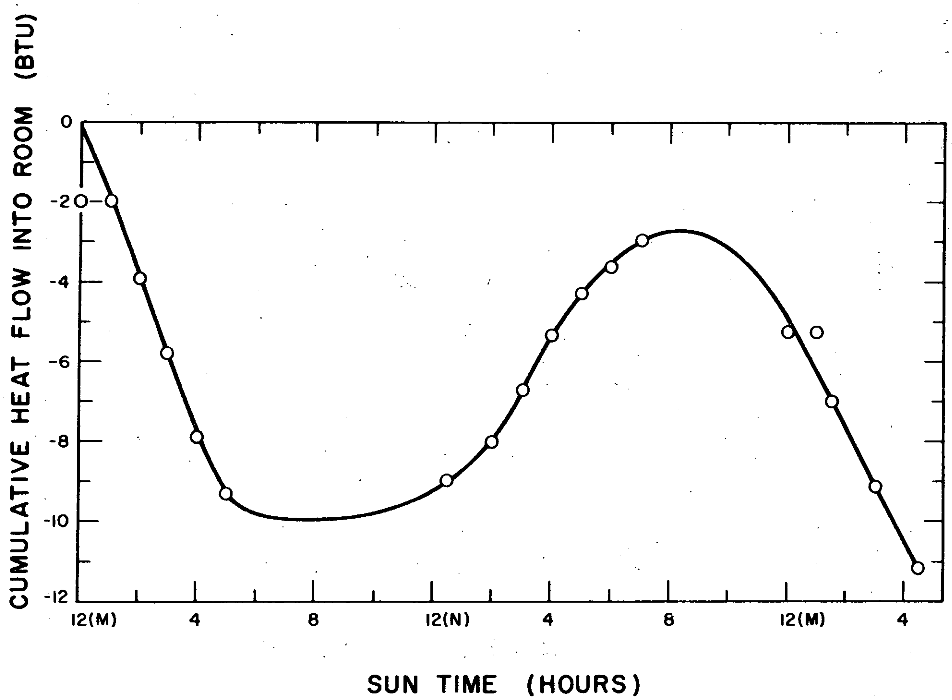


Figure 16. Cumulative heat passed through wall 6, input 1.
Total Btu/day-ft^2 $\cong$ -4.9.

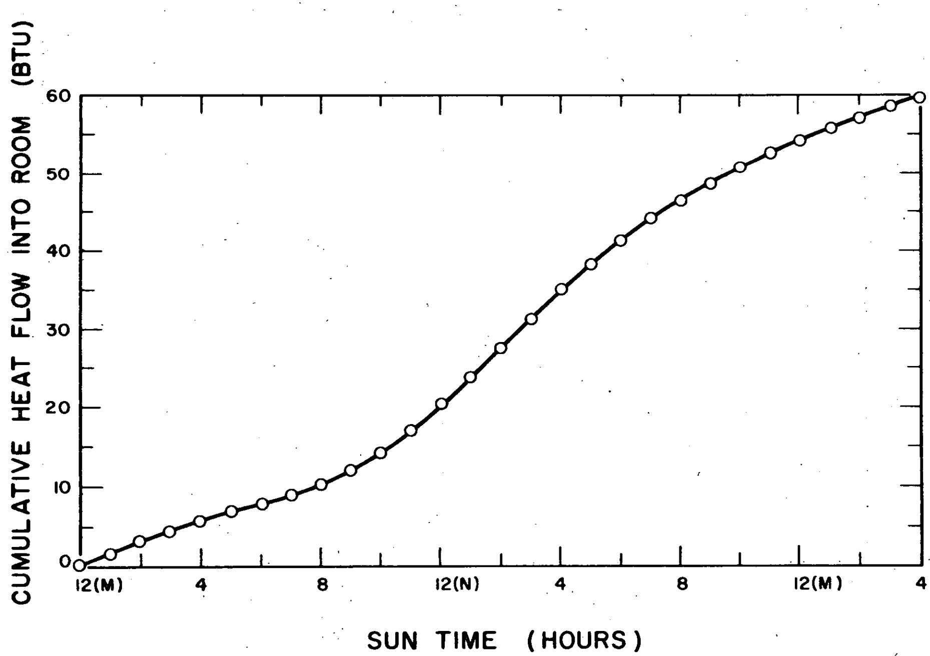


Figure 17. Cumulative heat passed through wall 6, input 2.
Total Btu/day-ft^2 $\cong$ 54.3.

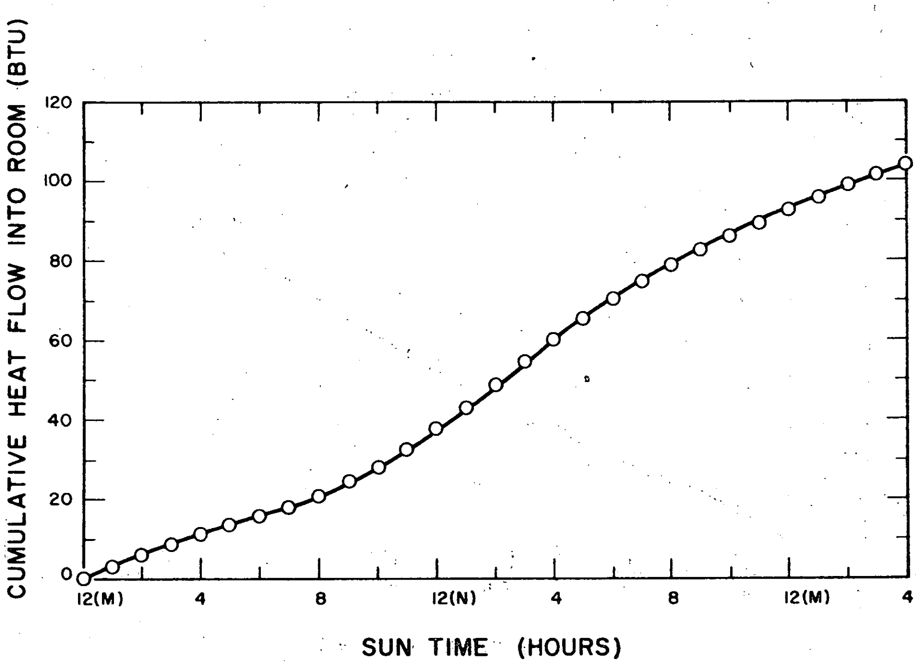


Figure 18. Cumulative heat passed through wall 6, input 3. Total Btu/day-ft^2 $\cong$ 92.6.

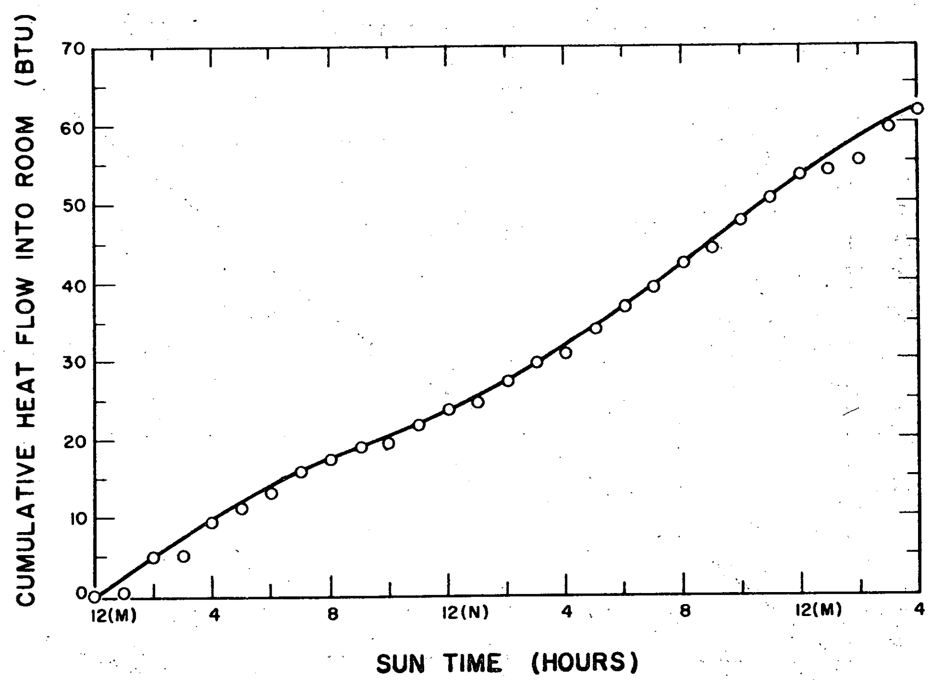


Figure 19. Cumulative heat passed through wall 7, input 2. Total Btu/day-ft^2 $\cong$ 53.4.

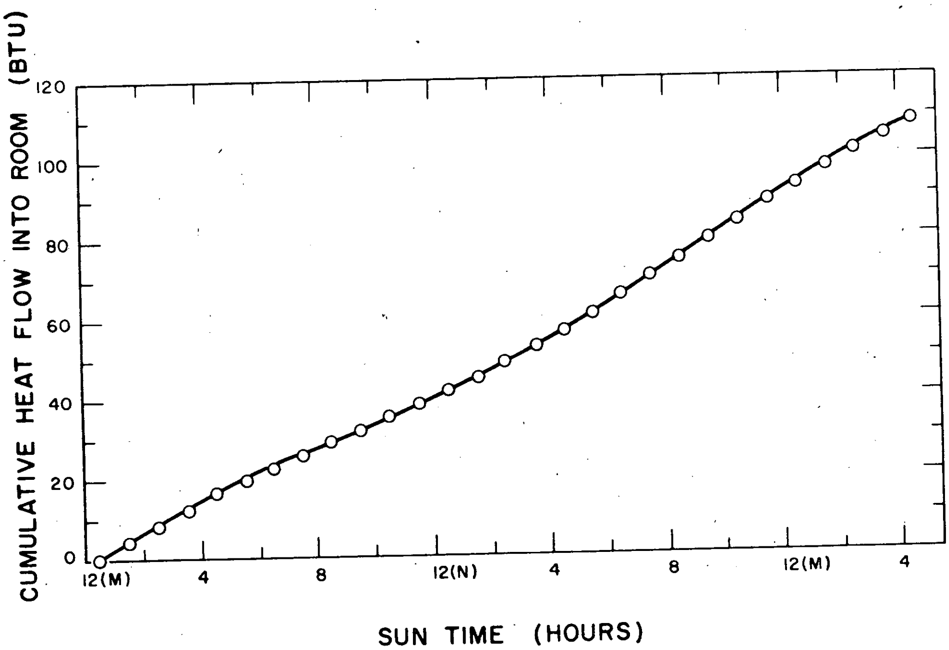


Figure 20. Cumulative heat passed through wall 7, input 3. Total Btu/day-ft^2 $\cong$ 93.0.

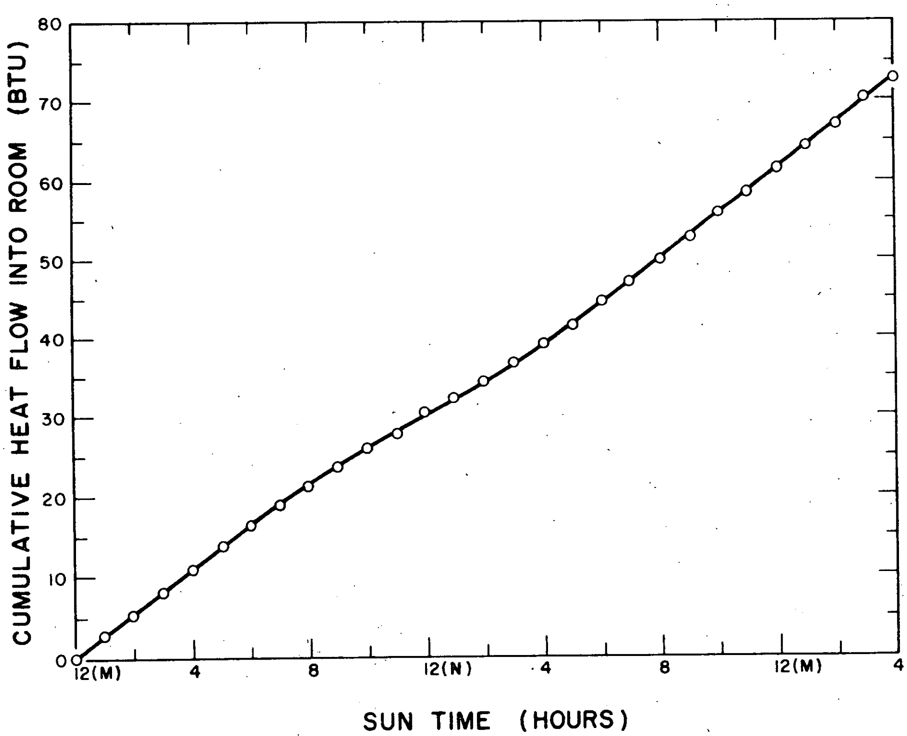


Figure 21. Cumulative heat passed through wall 8, input 2. Total Btu/day-ft^2 $\cong$ 61.5.

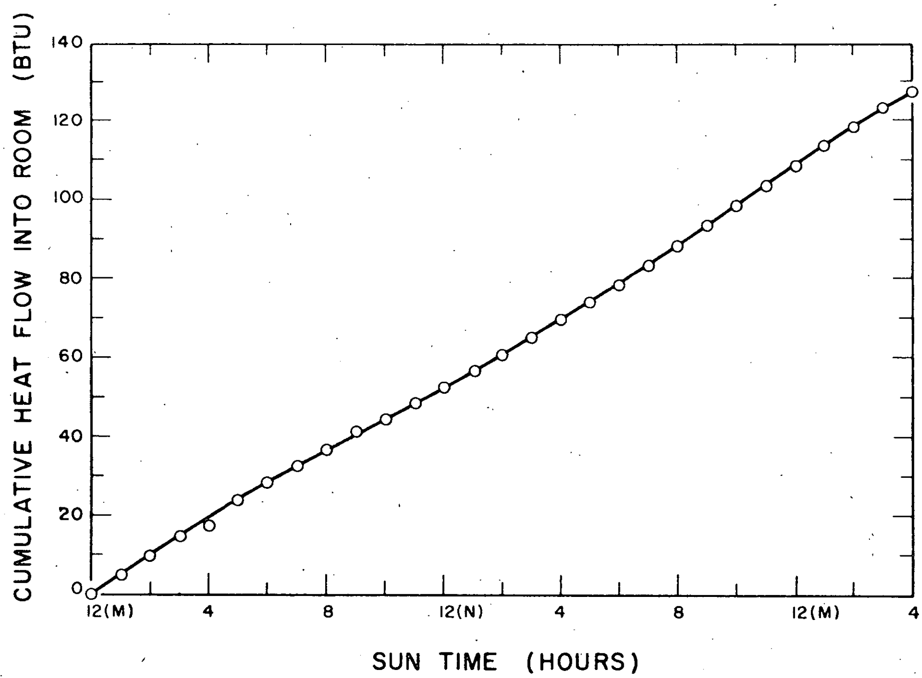


Figure 22. Cumulative heat passed through wall 8, input 3.
Total Btu/day-ft^2 $\cong$ 108.3.

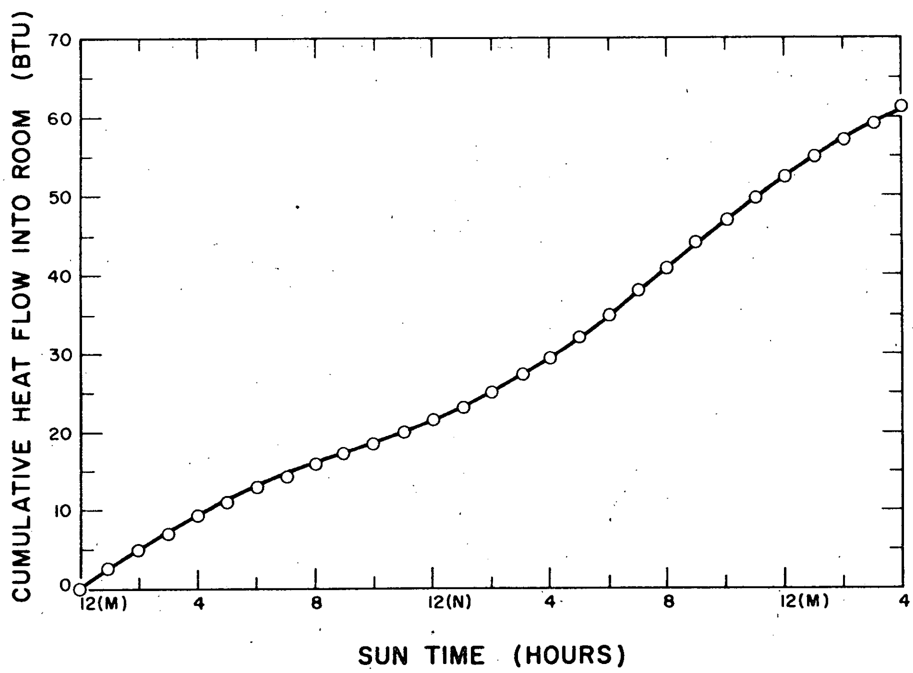


Figure 23. Cumulative heat passed through wall 9, input 2.
Total Btu/day-ft^2 $\cong$ 52.4.

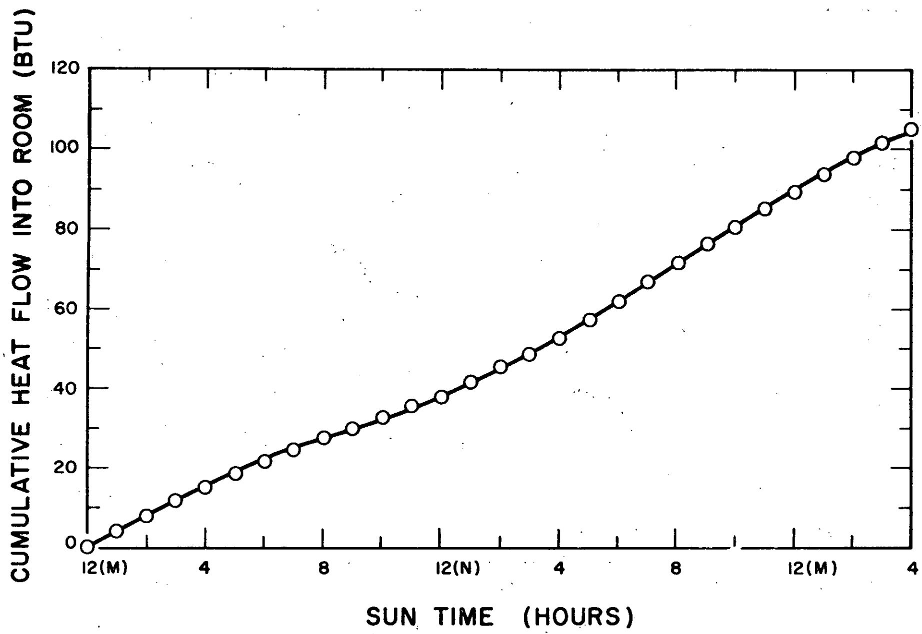


Figure 24. Cumulative heat passed through wall 9, input 3. Total Btu/day-ft^2 $\cong$ 89.6.

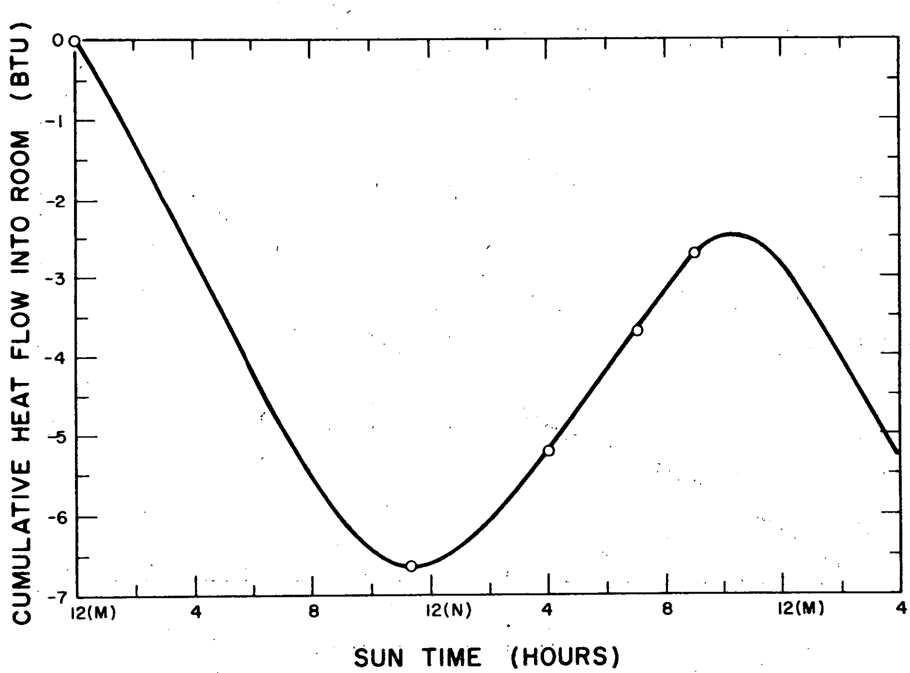


Figure 25. Cumulative heat passed through wall 10, input 1. Total Btu/day-ft^2 $\cong$ -2.85.

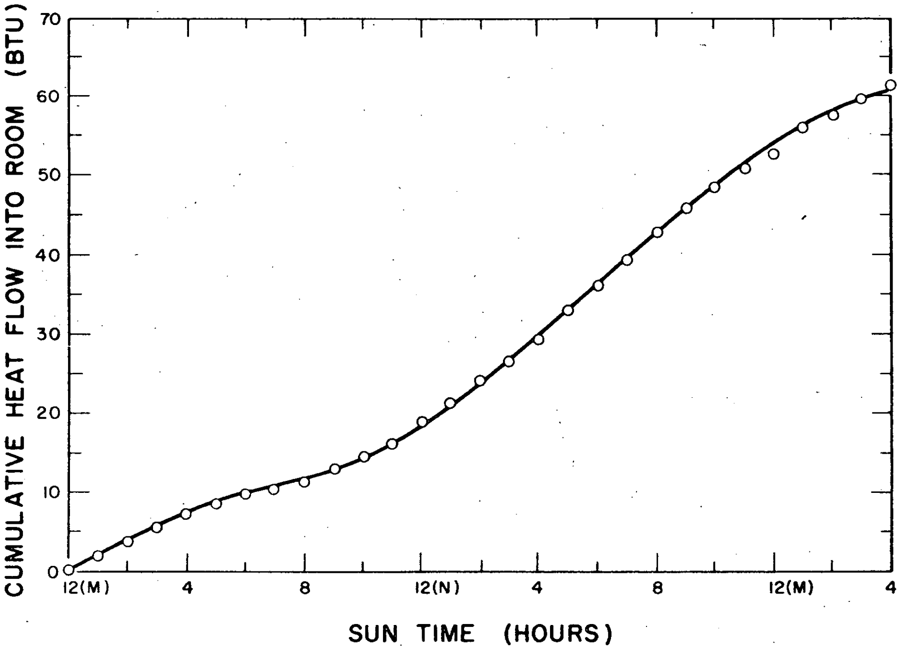


Figure 26. Cumulative heat passed through wall 10, input 2.
Total Btu/day-ft^2 ≅ 54.0.

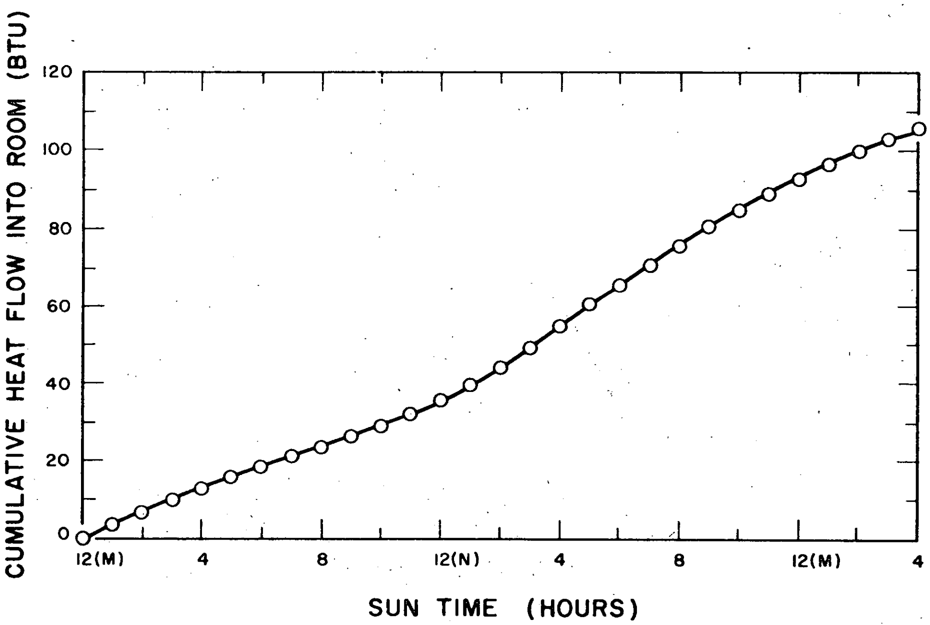


Figure 27. Cumulative heat passed through wall 10, input 3.
Total Btu/day-ft^2 ≅ 93.6.

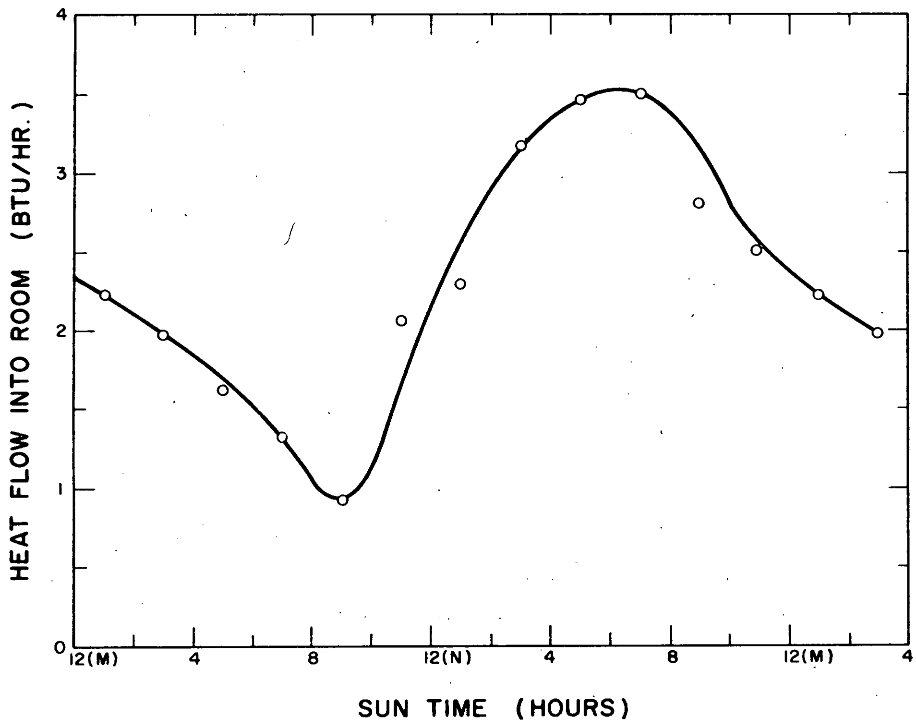


Figure 28. Heat flow through wall 1, input 2.

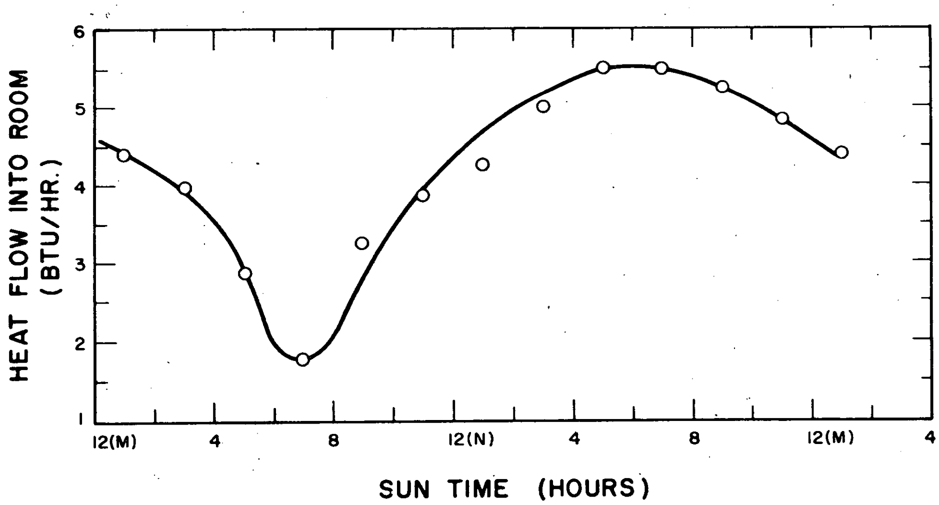


Figure 29. Heat flow through wall 1, input 3.

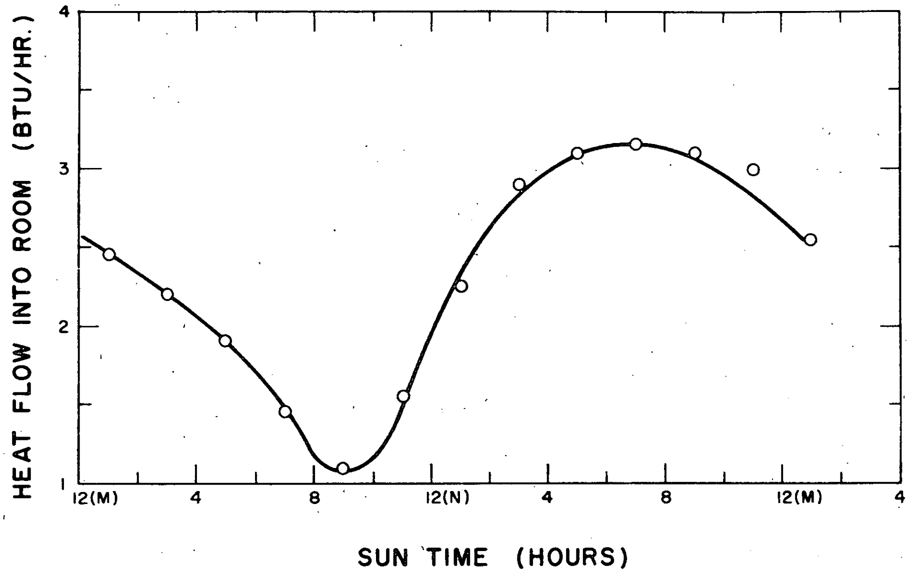


Figure 30. Heat flow through wall 2, input 2.

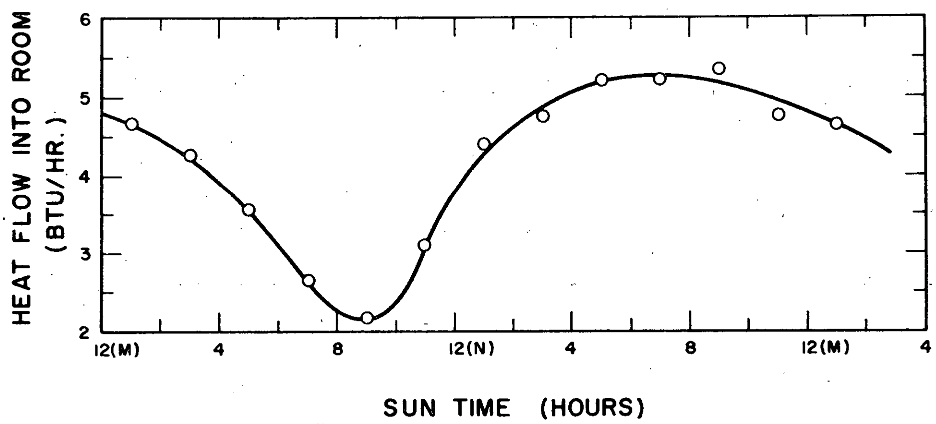


Figure 31. Heat flow through wall 2, input 3.

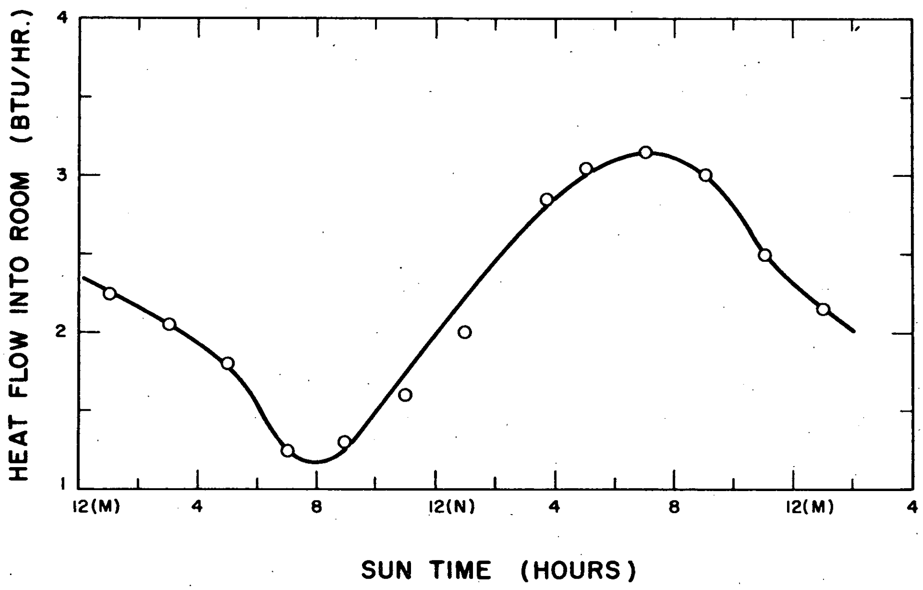


Figure 32. Heat flow through wall 3, input 2.

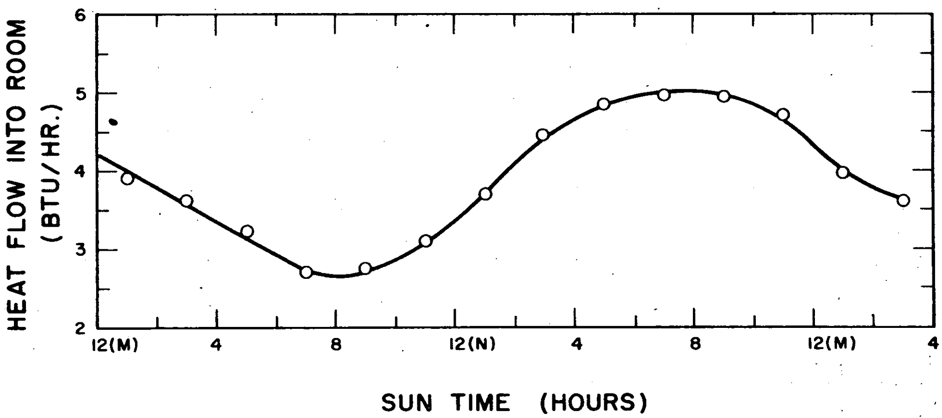


Figure 33. Heat flow through wall 3, input 3.

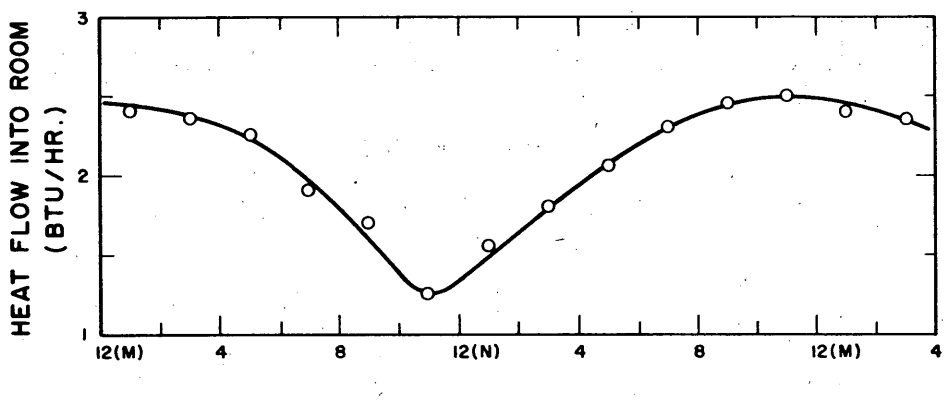


Figure 34. Heat flow through wall 4, input 2.

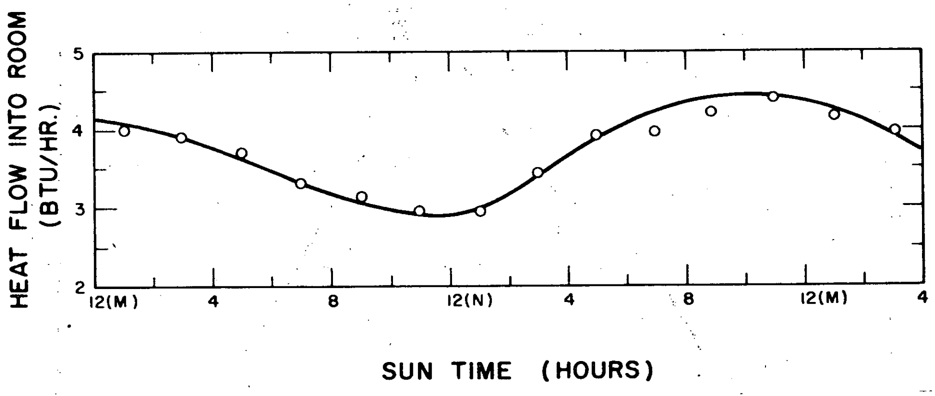


Figure 35. Heat flow through wall 4, input 3.

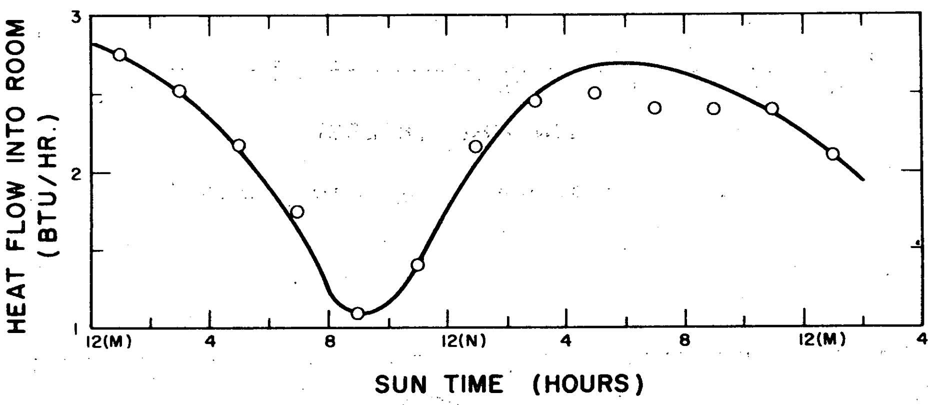


Figure 36. Heat flow through wall 5, input 2.

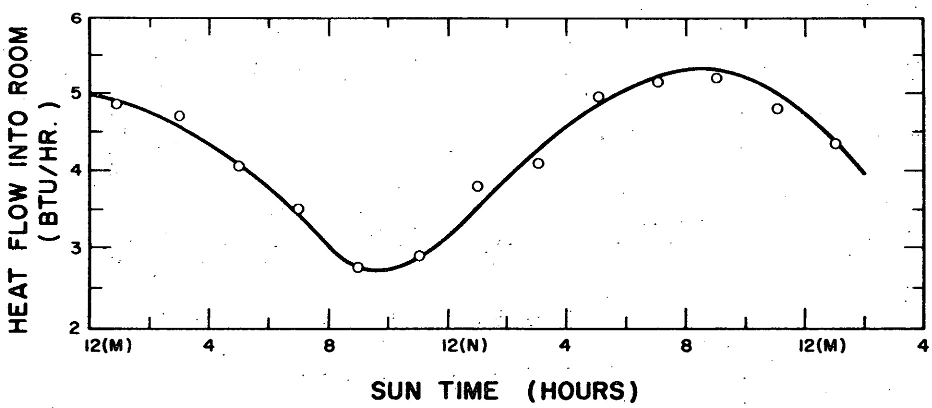


Figure 37. Heat flow through wall 5, input 3.

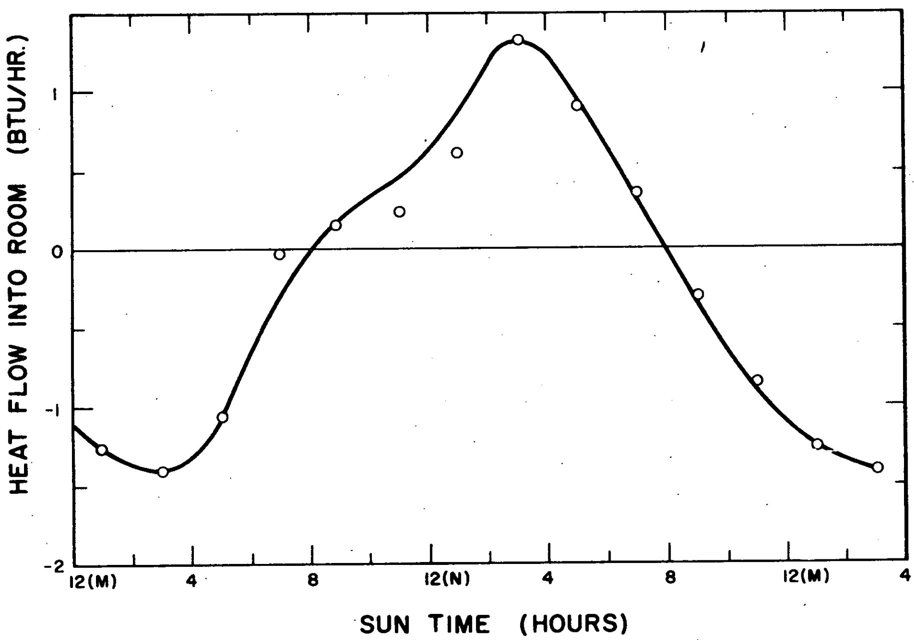


Figure 38. Heat flow through wall 6, input 1.

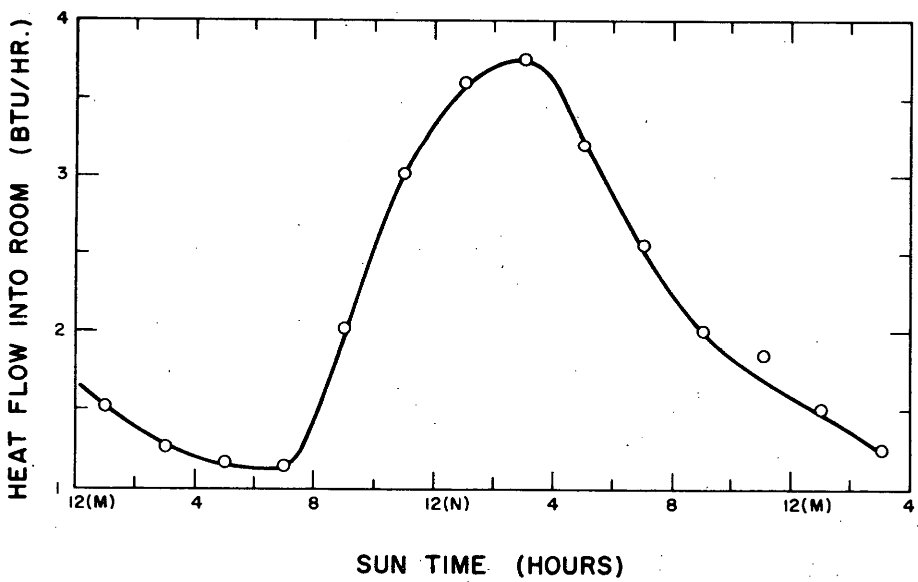


Figure 39. Heat flow through wall 6, input 2.

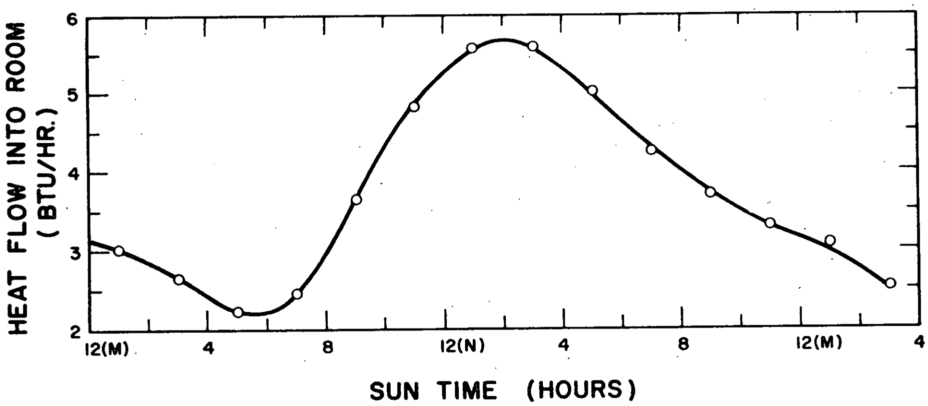


Figure 40. Heat flow through wall 6, input 3.

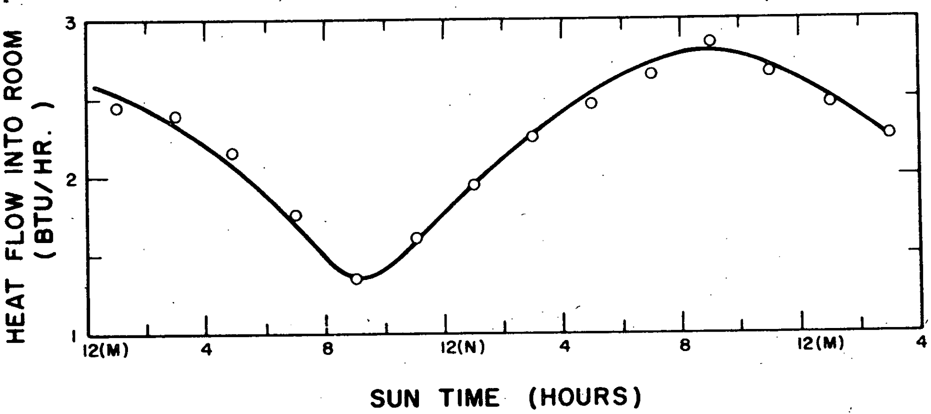


Figure 41. Heat flow through wall 7, input 2.

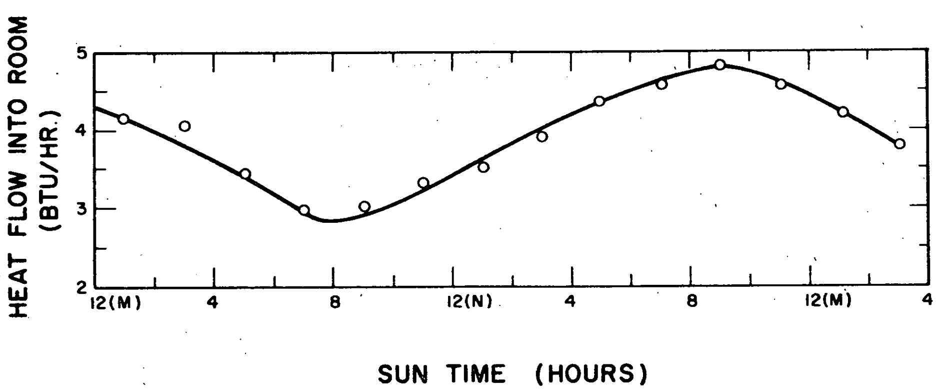


Figure 42. Heat flow through wall 7, input 3.

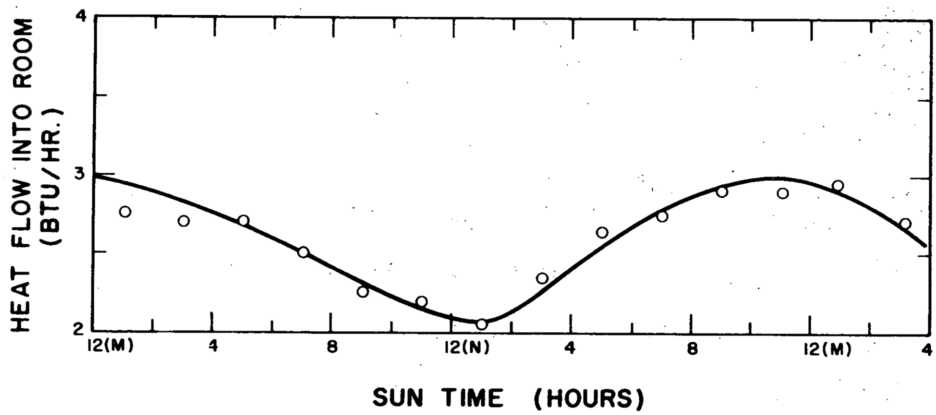


Figure 43. Heat flow through wall 8, input 2.

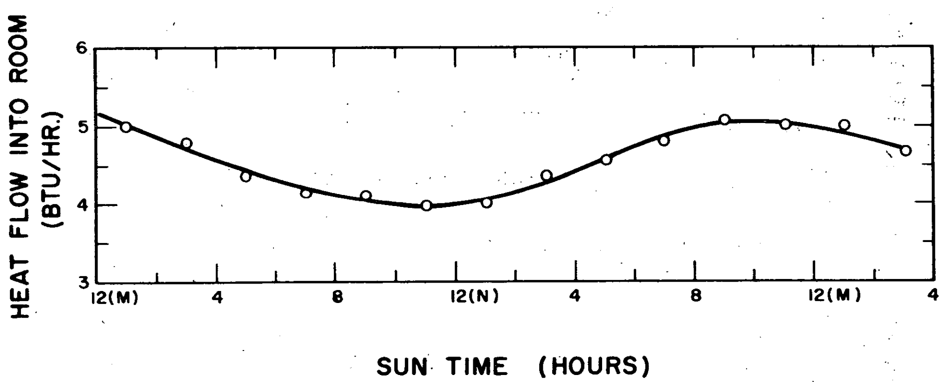


Figure 44. Heat flow through wall 8, input 3.

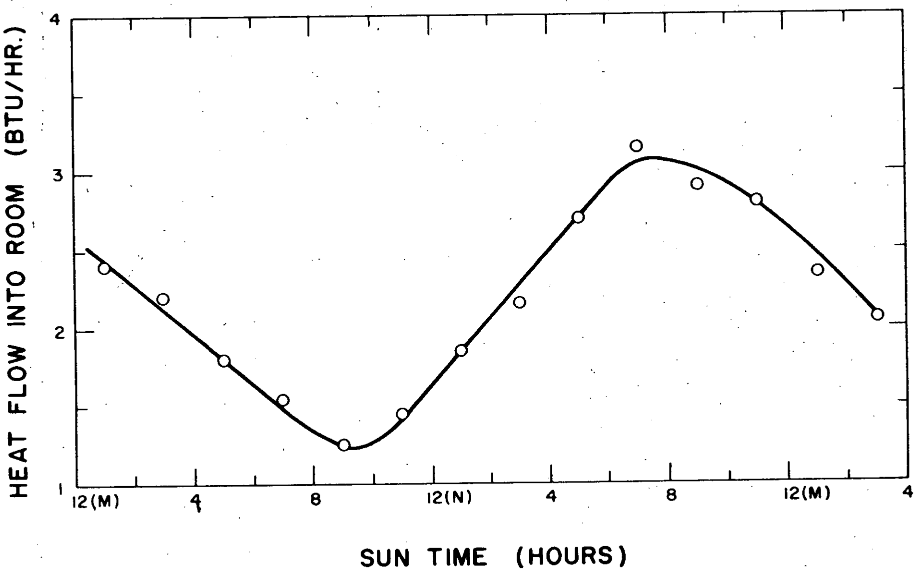


Figure 45. Heat flow through wall 9, input 2.

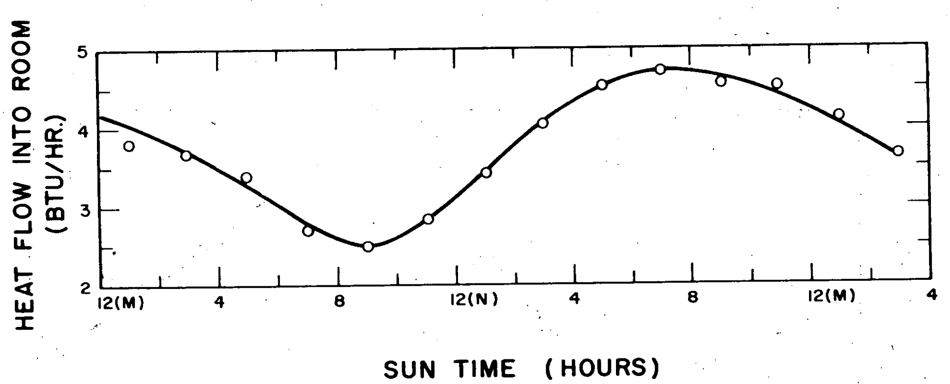


Figure 46. Heat flow through wall 9, input 3.

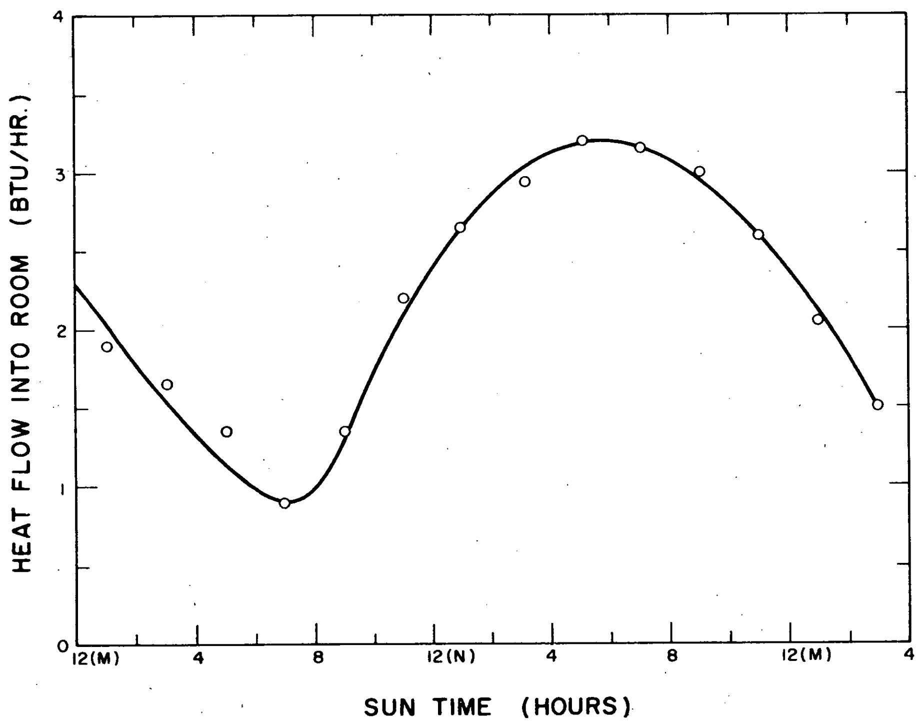


Figure 47. Heat flow through wall 10, input 2.

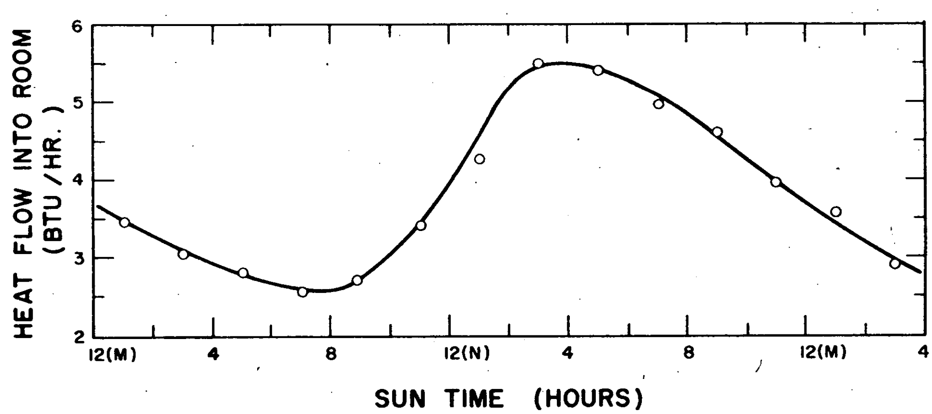


Figure 48. Heat flow through wall 10, input 3.

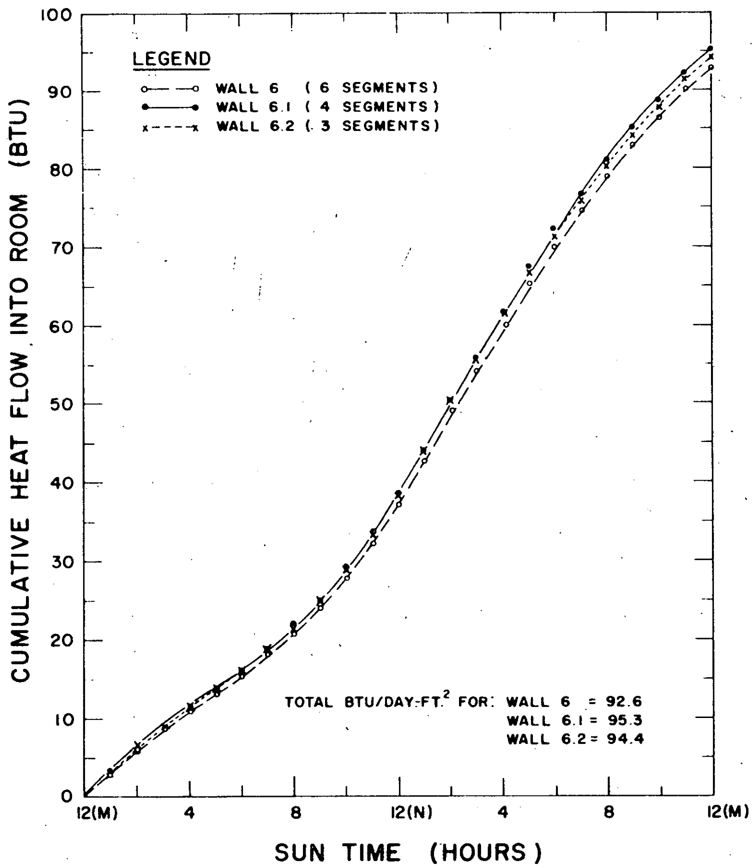


Figure 49. Cumulative heat curves for wall 6 with 3 mesh sizes

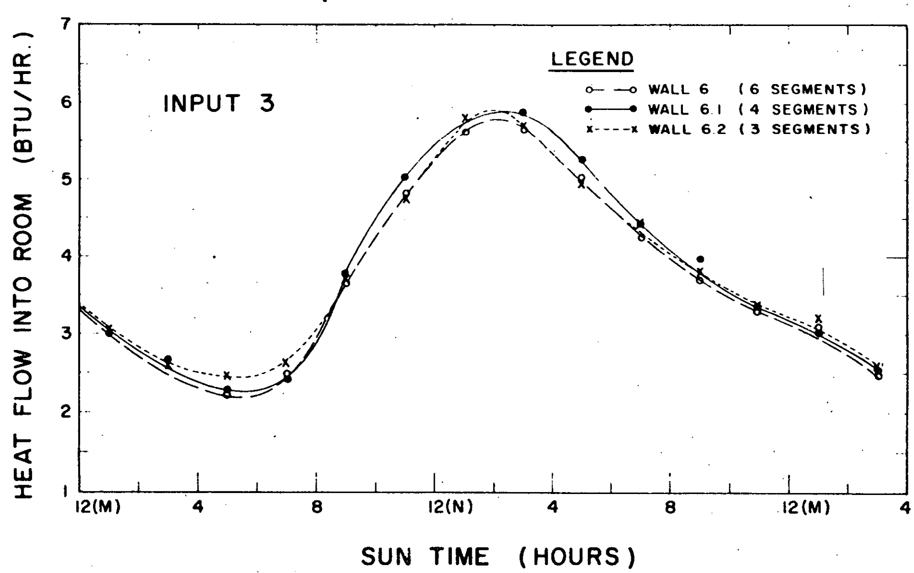


Figure 50. Heat flow curves for wall 6 and 3 mesh sizes.

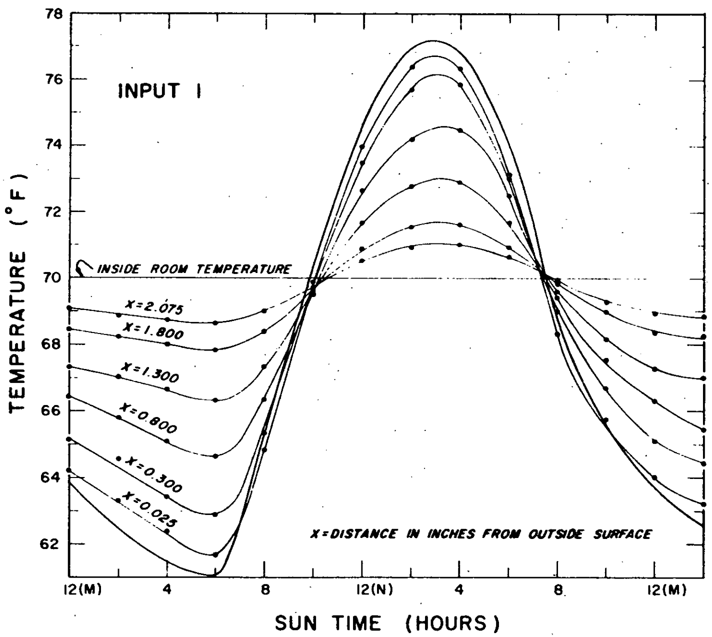


Figure 51. Temperature through day at various points in wall 6, input 1.

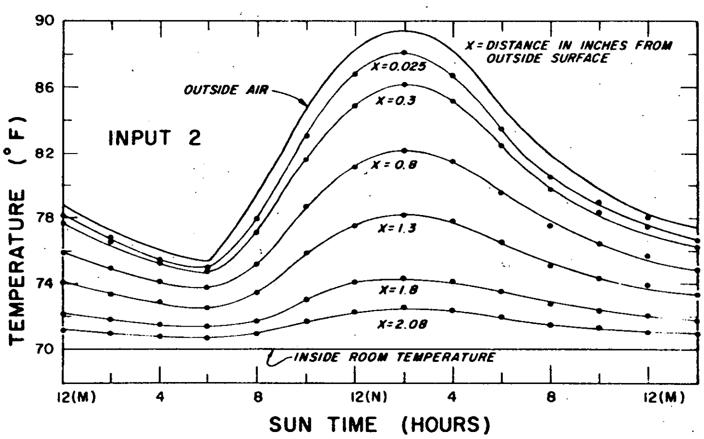


Figure 52. Temperature through day at various points in wall 6, input 2.

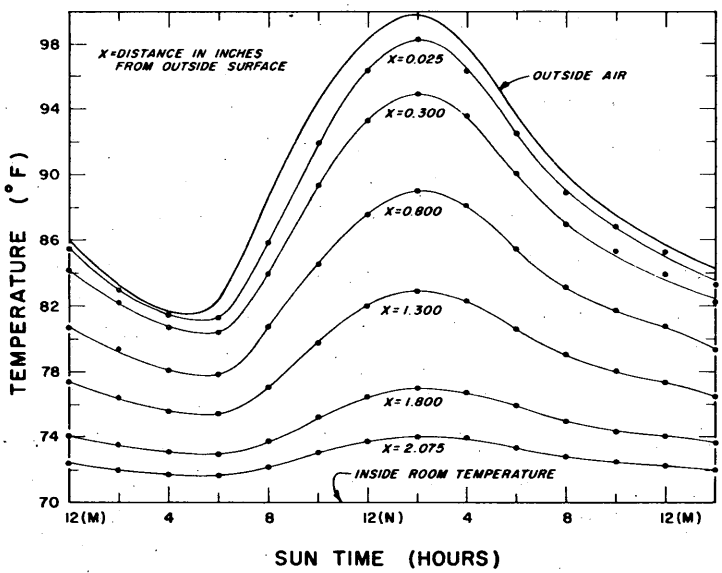


Figure 53. Temperature through day at various points in wall 6, input 3.

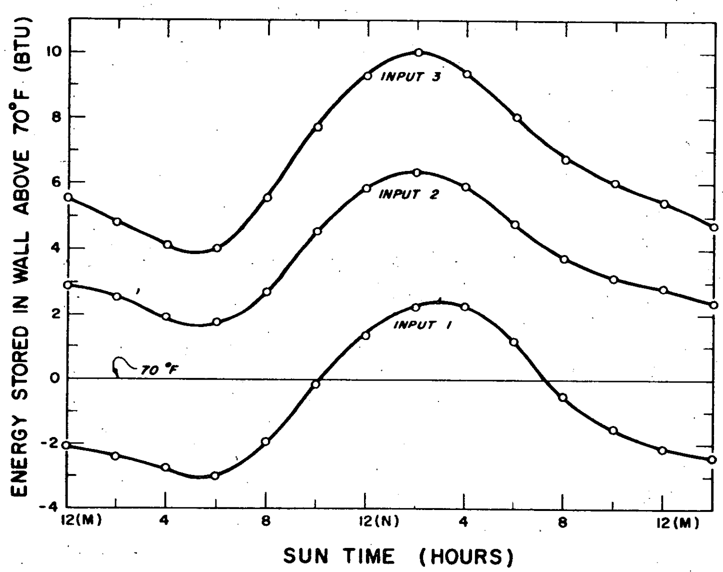


Figure 54. Energy stored in wall 6 during day. Energy in wall at 70F taken as zero.

APPENDIX A.

SCALING THERMAL NETWORK FOR THE HYDRAULIC ANALOG NETWORK

If we consider heat transfer through any material in one direction only, the field equation is given by:

$$\frac{\partial^2 v}{\partial x^2} = \frac{\rho C_p}{k} \frac{\partial v}{\partial t_p} \tag{A1}$$

where, v = temperature difference, °F
x = distance through material, ft
ρ = density, lb/ft³
C_p = specific heat, Btu/lb°F
k = thermal conductivity, Btu/°F-hr-ft
t_p = prototype time, hr

A finite difference equation can be derived from the field equation as follows:

$$\frac{v(x+\Delta x) - v(x)}{(\Delta x)^2 / k} - \frac{v(x) - v(x-\Delta x)}{(\Delta x)^2 / k} = \rho C_p \frac{\partial v(x)}{\partial t_p}.$$

If we note that $(\Delta x / k)$ is the thermal resistance (multiplied by unit area) of a specified thickness of material, and that $\Delta x \cdot \rho C_p$ is the thermal capacitance (per unit area) of the same specified thickness of material, we have

$$\frac{v_1 - v_0}{R_p} - \frac{v_0 - v_{-1}}{R_p} = \bar{C} \frac{\partial v_0}{\partial t_p}. \tag{A2}$$

Equation A2 is an approximate solution for v_0 in the thermal network shown in Figure A1. The finite difference equation used for the solution of h_0 on the hydraulic analog network (Fig. A2) is given by:

$$\frac{h_1 - h_0}{R_a} - \frac{h_0 - h_{-1}}{R_a} = A \frac{\partial h_0}{\partial t_a} \tag{A3}$$

where, A = area of standpipe, in.²
h = height of fluid in standpipe, in.
R_a = analog resistance, min/in.²
t_a = analog time, min.

It is immediately obvious that equations A2 and A3 are analogous, and that we are now ready to scale the thermal network for the hydraulic analog network.

The first step is to divide the wall into a finite number of segments, taking into account the number of available positions on the computer. In general, the thickness of the segments should be smaller where the temperature gradient is large. (In this study, the temperature gradient was largest in the outer portion of the wall.) Once the thicknesses of the segments have been determined, the thermal capacitance for each segment is determined (see Figure A3 for a sample problem):

Thermal capacitance = $\Delta x \rho \, Cp = \bar{C}, \frac{\text{Btu}}{°\text{F}}$ (unit area).

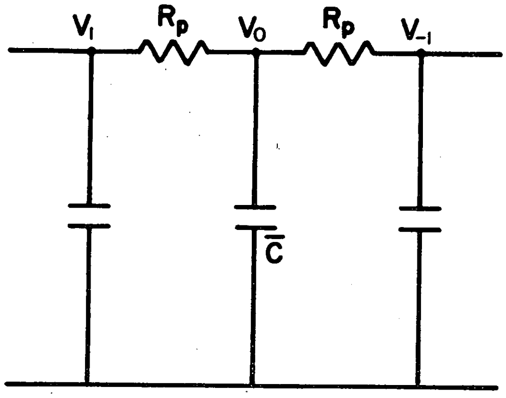


Figure A1. Thermal network.

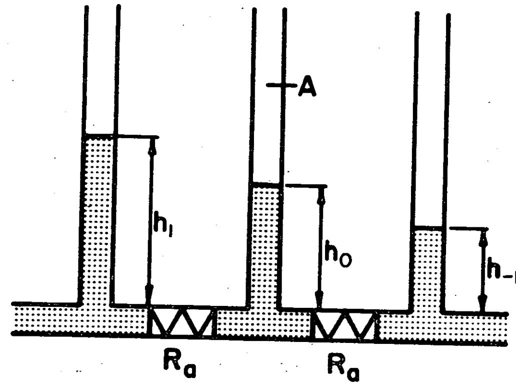


Figure A2. Hydraulic network.

As seen from equations A2 and A3, the hydraulic analog of thermal capacitance is A, the cross-sectional area of the standpipes. A ratio of the standpipe area to the thermal capacitance is chosen:

$$\frac{A}{\overline{C}}, \ \frac{\text{in.}^2\ {}^\circ\text{F}}{\text{Btu}} = \text{constant.} \tag{A4}$$

The value of this ratio is determined by the areas of the available standpipes, and by the fact that this ratio is a constant for all of the segments in any one wall problem.

From the two finite difference equations, it is observed that the hydraulic analog of temperature difference is the hydraulic head. A temperature scale V_s, which is a ratio of the hydraulic head to the temperature difference, is chosen taking into account the limits of the computer (i.e., the temperature range of the input curve and the height of the standpipes):

$$V_s = h/v, \ \text{in.}/{}^\circ\text{F}. \tag{A5}$$

A time scale T_s, the ratio between the analog time and the prototype time, must be selected, remembering the limits of the input curve and the kymograph speed, and checking to see that it does not go above or below the limits of the linear range of the hydraulic resistors:

$$T_s = t_a/t_p, \ \text{min/hr.} \tag{A6}$$

The thermal network resistances must be determined next. The procedure for making this calculation is demonstrated in the sample problem in Figure A3. The analog resistances are then scaled from the thermal resistances according to the following equation:

$$R_a = T_s\,(\overline{C}/A)\,R_p, \ \text{min/in.}^2.$$

This last equation is derived by substituting the three scaling ratios already obtained — eq A4, A5, and A6 — into the finite difference equation for the thermal network:

Starting with $\dfrac{v_1 - v_0}{R_p} - \dfrac{v_0 - v_{-1}}{R_p} = \overline{C}\,\dfrac{\partial v_0}{\partial t_p}$

we multiply through by V_s:

$$\frac{h_1 - h_0}{R_p} - \frac{h_0 - h_{-1}}{R_p} = \overline{C}\,\frac{\partial h_0}{\partial t_p}$$

divide by T_s:

$$\frac{h_1 - h_0}{T_s R_p} - \frac{h_0 - h_{-1}}{T_s R_p} = \overline{C}\,\frac{\partial h_0}{\partial t_a}$$

and multiply by $A/\overline{C}$:

$$\frac{h_1 - h_0}{T_s(\overline{C}/A)R_p} - \frac{h_0 - h_{-1}}{T_s(\overline{C}/A)R_p} = A\frac{\partial h_0}{\partial t_a}.$$

A comparison of this last equation with equation A3 reveals that,

$$R_a = T_s(\overline{C}/A)\,R_p, \text{ min/in.}^2.$$

The ratios A4 and A5 determine U_s, the energy scale, which is a ratio between the volume of water and the thermal energy:

$$U_s = (A/\overline{C})\,V_s, \text{ in.}^3/\text{Btu}.$$

The input curve is then scaled in accordance with T_s, V_s and the speed of the kymograph. The constant head reservoir (air-conditioner and heater) is set at the proper height on the computer with respect to the input curve, and the computer is ready for operation.

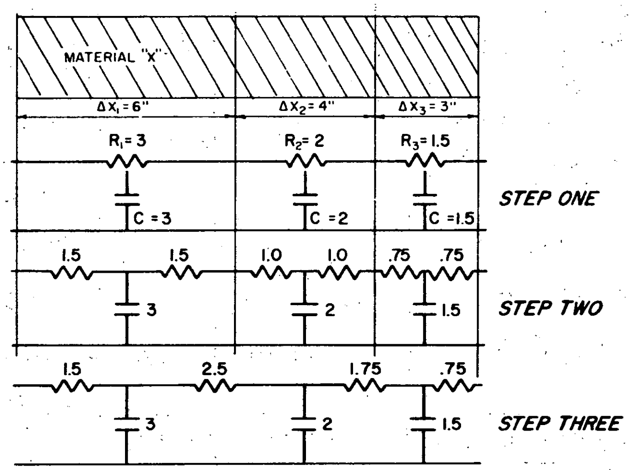


Figure A3. Sample problem of resistance and capacitance scaling procedures.

$K_X = 1/6$ Btu/°F-hr-ft.

$R_1 = \Delta x_1/k_x \cdot 1\ \text{ft}^2 = 3$°F-hr/Btu

$R_2 = \Delta x_2/k_x \cdot 1\ \text{ft}^2 = 2$°F-hr/Btu

$R_3 = \Delta x_3/k_x \cdot 1\ \text{ft}^2 = 1.5$°F-hr/Btu

$C_X = 6$ Btu/ft³-°F.

$\overline{C}_1 = \Delta x \cdot 1\ \text{ft}^2\ C_X = 3$°F/Btu

$\overline{C}_2 = \Delta x_2 \cdot 1\ \text{ft}^2\ C_X = 2$°F/Btu

$\overline{C}_3 = \Delta x_3 \cdot 1\ \text{ft}^2\ C_X = 1.5$°F/Btu

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALLS

TEST NO.:* 1

DATE: 21 MARCH 1962

COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR 1 °F = 0.4 INCH 1 BTU = 0.0436 INCH³ 1 °F HR/BTU = 9.17 MIN/IN²

WALL PROFILE	$\bar{C}$ BTU/°F	$\bar{R}$ °F HR/BTU	
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25	
SECTION 1 LIGHT CMU 9/16"	0.68	0.33	
AIR SPACE / SECTION 2 LIGHT CMU 2-1/2" x 3" / AIR SPACE	0.75	0.84	1.50
SECTION 3 LIGHT CMU 9/16"	0.68	0.33	
AIR SPACE ~ 2"	0.000262	0.86	
BUILDING PAPER 2 LAYERS	0	0.12	
SECTION 1 LIGHT CMU 9/16"	0.68	0.33	
AIR SPACE / SECTION 2 LIGHT CMU 2-1/2" x 3" / AIR SPACE	0.75	0.84	1.50
SECTION 3 LIGHT CMU 9/16"	0.68	0.33	
INSIDE SURFACE (STILL AIR)	0	0.68	

STANDPIPE AREA (INCH²)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN²	R_a SETTING (INCHES)
0.0740	0.563		0.425	3.89	8.50
		0.906	0.385	3.53	9.05
0.0407	1.250				
		1.250	0.420	3.85	8.00
0.0411	1.250				
		0.906	0.385	3.53	8.70
0.0741	0.563				
0	2.000	2.563	1.310	12.00	6.10
0.0411	0.563				
		0.906	0.385	3.53	13.15
0.0408	1.250				
		1.250	0.420	3.85	11.75
0.0410	1.250				
		0.906	0.385	3.53	9.05
0.0411	0.563				
			0.855	7.84	7.84

* Test no. indicates wall no.

ΣR = 4.91 U ≅ 0.204 ROOM AIR TEMPERATURE = 26.3 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALLS

TEST NO.: 2
DATE: 21 MARCH 1962
COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR   1 °F = 4 INCH   1 BTU = 0.0275 INCH^3 1 °F HR/BTU = 14.55 MIN/IN^2

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU	
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25	
4" FACE BRICK (3-5/8")	7.50	0.44	
AIR SPACE 1.5"; BUILDING PAPER	0.000262	0.86	
GLASS FIBER INSULATION 0.5"	0.013	2.06	
SECTION 1 HEAVY CMU 9/16"	1.053	0.07	
AIR SPACE / SECTION 2 HEAVY CMU (2-1/2" x 3")	1.170	0.57	0.71
SECTION 3 HEAVY CMU 9/16"	1.053	0.07	
INSIDE SURFACE (STILL AIR)	0	0.68	

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN^2	R_a SETTING (INCHES)
			0.305	4.44	7.23
0.1292	0.906				
		0.906	0.11	1.60	12.4
0.1287	0.906				
		0.906	0.11	1.60	12.45
0.1289	0.906				
		0.906	0.11	1.60	12.45
0.1285	0.906				
				10	7.53
				10	4.72
0	1.50	2.734	3.010		
				13.8	6.9
0	0.5				
				10	4.9
0.0722	0.563				
		0.906	0.178	2.59	13.82
0.0400	1.25				
		1.25	0.285	4.15	11.3
0.0400	1.25				
		0.906	0.178	2.59	10.75
0.0723	0.563				
			0.715	10.40	11.6

$\Sigma R = 5.00$ $U \cong .200$ ROOM AIR TEMPERATURE = 27.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALL

TEST NO.: 3

DATE: 21 MARCH 1962

COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR   1 °F = 0.4 INCH   1 BTU = 0.0748 INCH³   1 °F HR/BTU = 5.72 MIN/IN²

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25
SECTION 1 LIGHT CMU 1-5/16"	1.59	0.50
AIR SPACE / SECTION 2 LIGHT CMU (5" x 3") / AIR SPACE	1.52	1.00
SECTION 3 LIGHT CMU 1-5/16"	1.59	0.50
PAPER COVERED; AIR / FURRING 3/4" x 1.2" / SPACE	0.098	0.76
GLASS FIBER INSULATION 0.2"	0.040	0.74
GYPSUM BOARD 1/2"	0.65	0.45
INSIDE SURFACE (STILL AIR)	0	0.68

Sections 1–3: 2.00

STANDPIPE AREA (INCH²)	LUMP THICKNESS (INCHES)
0.300	1.313
0.0702	1.25
0.0708	1.25
0.0710	1.25
0.0716	1.25
0.297	1.313
0.0261	0.75
0.1184	0.50

INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN²	R_a SETTING (INCHES)
	0.50	2.86	10.34
1.282	0.375	2.12	11.43
1.25	0.25	1.43	12.77
1.25	0.25	1.43	12.76
1.25	0.25	1.43	14.6
1.282	0.375	2.12	14.15
1.032	1.005	5.75	9.45
.625	0.970	5.55	4.95
	0.903	5.16	10.6

$\Sigma R = 4.88$ $U \cong .204$ ROOM AIR TEMPERATURE = 27.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: **HEAT FLOW THROUGH WALL**

TEST NO.: 4
DATE: 20 MARCH 1962
COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR   1 °F = 0.4 INCH   1 BTU = 0.0258 INCH^3 1 °F HR/BTU = 15.90 MIN/IN^2

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25
4" FACE BRICK (3-5/8")	7.50	0.44
AIR SPACE 1.8"	0.000262	0.86
GLASS FIBER INSULATION 0.2"	0.005	0.80
SECTION 1 LIGHT CMU (1-5/16)	1.59	0.50
5"x9"x12" AIR SPACE / SECTION 2 LIGHT CMU (5"x3") / AIR SPACE	1.62	1.00
SECTION 3 LIGHT CMU (1-5/16)	1.59	0.50
INSIDE SURFACE (STILL AIR)	0	0.68

$\}$ 2.00 (SECTIONS 1–3)

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN^2	R_a SETTING (INCHES)
			0.305	6.14	8.15 / 10.95
0.1166	0.906				
		0.906	0.11	2.21	10.9
0.1170	0.906				
		0.906	0.11	2.21	11.18
0.1169	0.906				
		0.906	0.11	2.21	10.50
0.1167	0.906				
				12.00	5.85
0	1.80	3.109	1.965	9.00	5.95
0	0.20			10.20	10.87
0.1001	1.313				
		1.283	0.375	5.96	11.1
0.0238	1.25				
		1.25	0.25	3.97	11.47
0.0244	1.25				
		1.25	0.25	3.97	8.05
0.0241	1.25				
		1.25	0.25	3.97	11.87
0.0240	1.25				
		1.283	0.375	5.96	15.8
0.1001	1.313				
			0.93	14.80	5.00

$\Sigma R \cong 5.03$ $U \cong .199$ ROOM AIR TEMPERATURE = 27.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: **HEAT FLOW THROUGH WALL**

TEST NO.: 5
DATE: 20 MARCH 1962
COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR 1 °F = 0.4 INCH 1 BTU = 0.01785 INCH^3 1 °F HR/BTU = 22.5 MIN/IN^2

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25
4" FACE BRICK (3-5/8")	7.50	0.44
1.9" AIR SPACE	0.000262	2.04
0.1" GLASS FIBER INSUL. W/POLISHED ALUM COATED PAPER BOTH SIDES	0.0025	0.40
SECTION 1 HEAVY CMU 1-5/16"	2.46	0.18
9"x5"x12" AIR SPACE / SECTION 2 HEAVY CMU (5"x3") / AIR SPACE	2.35	0.75
SECTION 3 HEAVY CMU 1-5/16"	2.46	0.18
INSIDE SURFACE (STILL AIR)	0	0.68

(Sections 1–3 together: 1.11)

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN^2	R_a SETTING (INCHES)
			0.305	6.86	~~8.55~~ 8.7
0.839	0.906				
		0.906	0.11	2.48	11.13
0.837	0.906				
		0.906	0.11	2.48	10.95
0.838	0.906				
		0.906	0.11	2.48	10.95
0.840	0.906				
		3.11	2.585	58.20	4.85
					4.78
0	1.90				4.8
					4.52
0	0.10				9.7
0.1102	1.313				
		1.54	0.222	5.00	12.14
0.0368	1.76				
		1.47	0.220	4.95	10.8
0.0245	1.17				
		1.18	0.176	3.96	8.55
0.0247	1.18				
		1.03	0.155	3.48	10.2
0.0183	0.88				
		1.10	0.156	3.51	13.4
0.1107	1.313				
		.656	0.774	17.40	4.4

$\Sigma R = 4.92$ $U \cong .203$ ROOM AIR TEMPERATURE = 24.0 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALLS

TEST NO.: 6

DATE:

COMPUTED BY:

TIME SCALE: 1 MIN = .391 HR   1 °F = 0.4 INCH   1 BTU = .296 INCH³   1 °F HR/BTU = 3.46 MIN/IN²

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
OUTSIDE SURFACE (7.5 MPH WIND) PAINTED	0	0.25
0.05	0.245	0.0016
GLASS FIBER INSULATION 2" — 18 GAGE SHEEL STEEL 0.10" — GLASS FIBER INSULATION 2"	0.1312	4.242
0.05	0.245	0.0016
PAINTED INSIDE SURFACE (STILL AIR)	0	0.68

STANDPIPE AREA (INCH²)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN²	R_a SETTING (INCHES)
		.025	0.251	.868	13.82
0.1815	0.05				
		0.275	0.531	1.84	12.1
.0241	0.50				
		0.50	1.061	3.67	8.52
.0240	0.50				
		0.50	1.061	3.67	8.57
.0244	0.50				
		0.50	1.061	3.67	12.42
.0245	0.50				
		0.275	.531	1.84	14.15
.1812	0.05				
		.025	0.681	2.36	13.55

$\Sigma R = 5.175$ $U \cong 0.1966$ ROOM AIR TEMPERATURE = 25.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: **HEAT FLOW THROUGH WALLS**

TEST NO.: **6.1**
DATE: **10 AUGUST 1962**
COMPUTED BY: **R.H.**

TIME SCALE: 1 MIN = **1** HR 1 °F = **0.4** INCH 1 BTU = **0.111** INCH^3 1 °F HR/BTU = **3.6** MIN/IN^2

WALL PROFILE	$\bar{C}$ $\frac{BTU}{°F}$	R $\frac{°F\ HR}{BTU}$
OUTSIDE SURFACE (7.5 MPH WIND) PAINTED	0	0.25
0.05"	0.245	0.0016
GLASS FIBER INSULATION 2" / 18 GAGE SHEET STEEL 0.10" / GLASS FIBER INSULATION 2"	0.1312	4.242
0.05	0.245	0.0016
PAINTED INSIDE SURFACE (STILL AIR)	0	0.68

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p $\frac{°F\ HR}{BTU}$	R_a $\frac{MIN}{IN^2}$	R_a SETTING (INCHES)
.0675	0.05	0.025	0.251	.905	13.76
		0.525	1.069	3.85	8.87
.0182	1.00				
					10.05
		1.00	2.121	7.64	
					11.15
.0183	1.00				
		0.525	1.069	3.85	8.5
.0684	0.05	0.025	0.681	2.45	13.77

$\Sigma R = 5.175$ $U \cong 0.1966$ ROOM AIR TEMPERATURE = 24.0 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: **HEAT FLOW THROUGH WALLS**

TEST NO.: 6.2
DATE: 10 AUGUST 1962
COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR   1 °F = 0.04 INCH   1 BTU = .123 INCH^3 1 °F HR/BTU = 3.245 MIN/IN^2

WALL PROFILE	$\bar{C}$ $\frac{BTU}{°F}$	R $\frac{°F\ HR}{BTU}$	STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p $\frac{°F\ HR}{BTU}$	R_a $\frac{MIN}{IN^2}$	R_a SETTING (INCHES)
OUTSIDE SURFACE (7.5 MPH WIND) PAINTED	0	0.25						
					0.025	0.251	.815	14.0
0.05"	0.245	0.0016	.0752	0.05				
								8.45
					1.025	2.122	6.89	
								10.55
GLASS FIBER INSULATION 2" / 18 GAGE SHEET STEEL (0.10") / GLASS FIBER INSULATION 2"	0.1312	4.242	.0404					
								10.0
					1.025	2.122	6.89	
								11.86
0.05	0.245	0.0016	.0756	0.05				
					0.025	0.681	2.21	11.5
PAINTED INSIDE SURFACE (STILL AIR)								

$\Sigma R = 5.175$ $U \cong 0.1966$ ROOM AIR TEMPERATURE = 24.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: **HEAT FLOW THROUGH WALLS**

TEST NO.: 7
DATE: 20 MARCH 1962
COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR 1 °F = 0.4 INCH 1 BTU = 0.01635 INCH^3 1 °F HR/BTU = 26.10 MIN/IN^2

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25
8" CONCRETE (SAND AND GRAVEL OR STONE AGGREGATE)	15.0	0.64
AIR SPACE; FURRING 1.2" x 3/4"	0.045	1.32
GLASS FIBER INSULATION 0.5"	0.101	1.68
GYPSUM BOARD 1/2"	0.65	0.45
INSIDE SURFACE (STILL AIR)	0	0.68

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN^2	R_a SETTING (INCHES)
0.0716	1.00				
		0.50	0.29	7.57	5.95 9.85
		1.00	0.08	2.09	11.40
0.0717	1.00				
		1.00	0.08	2.09	11.25
0.0720	1.00				
		1.00	0.08	2.09	11.30
0.0720	1.00				
		1.00	0.08	2.09	13.90
0.0722	1.00				
		1.00	0.08	2.09	13.80
0.0723	1.00				
		1.00	0.08	2.09	13.70
0.0728	1.00				
		1.00	0.08	2.09	11.70
0.0729	1.00				
		1.00	2.200	57.50	5.50 11.30 4.98 5.87 7.40
0.0811	1.25				
			1.970	51.40	6.20 5.68 7.85 5.55

$\Sigma R = 5.02$ $U \cong .199$ ROOM AIR TEMPERATURE = 28.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALLS

TEST NO.: 8

DATE: 20 MARCH 1962

COMPUTED BY: R. H.

TIME SCALE: 1 MIN = 1 HR   1 °F = 0.4 INCH   1 BTU = 0.01555 INCH^3 1 °F HR/BTU = 26.10 MIN/IN^2

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25
12" CONCRETE (SAND AND GRAVEL OR STONE AGGREGATE)	22.5	0.96
3/4" FURRING 1.2" FIBER GLASS INSUL.	0.151	2.52
GYPSUM BOARD 1/2"	0.65	0.45
	0	0.68

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN^2	R_a SETTING (INCHES)
			0.29	7.57	6.05 10.50
0.0716	1.00				
		1.00	0.08	2.09	11.57
0.0717	1.00				
		1.00	0.08	2.09	11.55
0.0720	1.00				
		1.00	0.08	2.09	11.55
0.0720	1.00				
		1.00	0.08	2.09	13.98
0.0722	1.00				
		1.00	0.08	2.09	13.88
0.0723	1.00				
		1.00	0.08	2.09	13.77
0.0728	1.00				
		1.00	0.08	2.09	11.82
0.0729	1.00				
		1.00	0.08	2.09	11.95
0.0710	1.00				
		1.00	0.08	2.09	11.15
0.0730	1.00				
		1.00	0.08	2.09	14.02
0.0730	1.00				
		1.00	0.08	2.09	14.16
0.0719	1.00				
		1.375	2.408	62.80	4.86 6.95 3.85 6.65 15.15
0.0311	1.25				
			0.859	22.40	6.20 15.5

$\Sigma R = 4.86$ $U \cong .206$ ROOM AIR TEMPERATURE = 26.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALLS

TEST NO.: 9
DATE: 20 MARCH. 1962
COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR 1 °F = 0.4 INCH 1 BTU = 0.0468 INCH³ 1 °F HR/BTU = 8.55 MIN/IN²

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25
4" FACE BRICK (3-5/8")	7.50	0.44
1" AIR SPACE	0.000131	0.86
DOUG. FIR. PLYWOOD (5/8")	1.12	0.78
AIR SPACE (3.425") / WIDE DOUG. FIR. STUD (1.2") / AIR SPACE	0.615	0.94
GLASS FIBER INSULATION 0.2"	0.040	0.68
GYPSUM BOARD (1/2")	0.65	0.45
INSIDE SURFACE (STILL AIR)	0	0.68

STANDPIPE AREA (INCH²)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_q MIN/IN²	R_q SETTING (INCHES)
			0.305	2.61	10.83
0.2156	0.906				
		0.906	0.11	0.94	13.56
0.2156	0.906				
		0.906	0.11	0.94	13.83
0.2158	0.906				
		0.906	0.11	0.94	13.73
0.2155	0.906				
0	1.00	1.766	1.305	11.15	6.60
0.1289	0.625				
		0.867	0.547	4.67	12.13
0.0238	1.108				
		1.108	0.313	2.68	12.96
0.0240	1.108				
		1.108	0.313	2.68	10.6
0.0241	1.108				
		1.004	1.062	9.08	6.30
0.0704	0.75				
			0.905	7.73	13.95

ΣR = 5.08 U ≅ .197 ROOM AIR TEMPERATURE = 27.5 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALLS

TEST NO.: 10
DATE: 20 MARCH 1962
COMPUTED BY: R. H.

TIME SCALE: 1 MIN = 1 HR 1 °F = 0.4 INCH 1 BTU = 0.0480 INCH^3 1 °F HR/BTU = 8.34 MIN/IN^2

WALL PROFILE	$\bar{C}$ $\frac{BTU}{°F}$	R $\frac{°F\ HR}{BTU}$
OUTSIDE SURFACE (7.5 MPH WIND)	0	0.25
REDWOOD SIDING 1/2" x 8" LAPPED	0.726	0.85
FIR. PLYWOOD SHEATING (5/8")	1.12	0.78
AIR SPACE / WIDE STUD (1.2") / AIR SPACE	0.595	0.94
GLASS FIBER INSULATION 0.3"	0.061	1.01
GYPSUM BOARD (1/2")	0.65	0.45
INSIDE SURFACE (STILL AIR)	0	0.68

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)
0.0869	0.50
0.1335	0.625
0.0238	1.108
0.0240	1.108
0.0241	1.108
0.0851	0.30
	0.50

INCHES TO CENTERS	R_p $\frac{°F\ HR}{BTU}$	R_a $\frac{MIN}{IN^2}$	R_a SETTING (INCHES)
	0.675	5.63	4.70
0.563	0.815	6.80	9.8
			8.2
0.867	0.547	4.56	6.10
1.108	0.313	2.61	10.5
1.108	0.313	2.61	13.1
0.744	1.322	11.02	6.6
	0.975	8.13	6.48

$\Sigma R = 4.96$ $U \cong .201$ ROOM AIR TEMPERATURE = 29.0 °C

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: **HEAT FLOW THROUGH WALLS**

TEST NO.: **PROBLEM 1**

DATE: **12 JUNE 1962**

COMPUTED BY: **B. L.**

TIME SCALE: 1 MIN = .666 HR   1°F = 1 INCH   1 BTU = .0267 INCH^3 1°F HR/BTU = .01775 MIN/IN^2

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
CONCRETE	22.5	0.96
INSULATION	0	8

STANDPIPE AREA (INCH^2)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN^2	R_a SETTING (INCHES)
.0400	.80	.4	.032	1.8	12.5
		.804	.0643	3.62	13.05
.0404	.808				
		.814	.0651	3.66	8.45
.0410	.82				
		.828	.0663	3.72	8.55
.0418	.836				
		.838	.067	3.76	12.07
.0420	.84				
		.848	.0678	3.81	13.04
.0428	.856				
		1.103	.0883	4.96	12.77
.0675	1.35				
		1.371	.1098	6.17	4.6
.0696	1.392				
		1.406	.1124	6.32	16.3
.0710	1.42				
		1.43	.1144	6.43	11.1
.0720	1.44				
		1.44	.1151	6.48	15.13
.0720	1.44				
		.72	.0576	3.24	

HYDRAULIC ANALOGUE COMPUTER SCALING SHEET

PROBLEM: HEAT FLOW THROUGH WALLS

TEST NO.: PROBLEM 2
DATE: 21 JULY 1962
COMPUTED BY: R.H.

TIME SCALE: 1 MIN = 1 HR 1 °F = 5/8 INCH 1 BTU = ______ $INCH^3$ 1 °F HR/BTU = 26.05 MIN/IN^2

WALL PROFILE	$\bar{C}$ BTU/°F	R °F HR/BTU
MATERIAL " X "	11.25	0.96
INSULATION		

STANDPIPE AREA ($INCH^2$)	LUMP THICKNESS (INCHES)	INCHES TO CENTERS	R_p °F HR/BTU	R_a MIN/IN^2	R_a SETTING (INCHES)
		0.50	0.08	2.08	11.05
0.0720	1.00				
		1.00	0.16	4.16	7.73
0.0722	1.00				
		1.00	0.16	4.16	7.05
0.0720	1.00				
		1.00	0.16	4.16	7.17
0.0723	1.00				
		1.00	0.16	4.16	12.5
0.0717	1.00				
		1.00	0.16	4.16	11.2
0.0716	1.00				
		0.50	0.08	2.08	

ROOM AIR TEMPERATURE = 28.0 °C

APPENDIX B.

DETERMINING PARAMETERS IN THE THERMAL NETWORK

It has been shown that the replacement of a field problem by a network approximation, such as the thermal network used in this investigation, leads to an inevitable error in the final result. This error will be 5% or less if the thickness of the section is chosen such that it is equal to or less than one-eighth of a thermal wavelength[4]:

$$\lambda = 2\sqrt{\frac{\pi a}{f}} \text{ ft}$$

$$\Delta x \leq L = 0.44 \sqrt{a/_f} \text{ ft} \qquad \text{(B1)}$$

where L = one-eighth thermal wavelength ($\lambda/8$), ft.

a = thermal diffusivity $\left(\frac{k}{\rho C_p}\right)$, ft^2/hr.

f = frequency, cycles/hr.

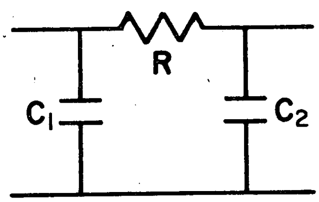
In Table II the maximum mesh sizes for an error of 5% or less are recorded for the construction materials used in the ten walls, taking $f = 1/24$ cycles/hr. It is observed that all of the mesh sizes used in the ten walls tested were considerably smaller than the values listed in this table.

From eq B1 above, it is seen that, if the frequency is increased, the mesh size necessary for the same degree of accuracy is decreased. Since the input curve is composed of a constant plus a diurnal sine wave and several higher frequencies significant to the fourth harmonic, sections about half the size given in Table II are needed to hold the 5% tolerance. Each section should contain only one material, where possible, for this estimate of the error to be accurate.

APPENDIX C.

METHOD USED IN DETERMINING THE TIME WHEN THE INPUT WAS CYCLING PERIODICALLY IN THE STEADY STATE

In any ladder network, like the thermal network pictured in Figure 2, the input must cycle for a certain period of time before the system is in the steady state and the transients have died out. A glance at the thermal network reveals that it is composed of several loops with several different time constants. The time necessary for the system to reach equilibrium is dependent upon the loop wherein the time constant is largest:



Time constant = RC = hr.

where $C = C_1 + C_2$.

For every time constant elapsed, 63% of the transients still in the loop will have disappeared, or 99.5% will have disappeared after the input has been cycling for five time constants. This time was converted to computer time — the computer was allowed to run for this length of time plus half as much again before data were taken. This time rarely exceeded three cycles and was usually much less.

APPENDIX D.

CHECK ON HYDRAULIC ANALOG COMPUTER ACCURACY

The hydraulic analog computer was tested in the first phase of the investigation by application of a sine wave and step input to the outside surfaces of a homogeneous slab of width 2ℓ (see test problems 1 & 2). The results from the computer were then checked against the analytical solutions[5] (Fig. D1, D2). In Figure D1 (the step input), the difference between the analog and analytical solutions is less than 1% for

$\theta^{*}\left(\frac{at}{\ell 2}\right) > 0.10$. When $\theta^{*} = 0.005$, the computer value is about 10% lower than the

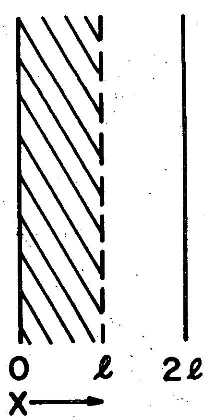
analytical value. The latter deviation can be satisfactorily explained by realizing that the flow in the hydraulic resistors is turbulent whenever the head is much larger than 3 in. (The head for the first few seconds of the step problem was almost 5 in.) Turbulence increases the hydraulic resistance, which results in a decrease of current and potential. This problem was not encountered in testing any of the ten wall sections.

The sine wave problem (Fig. D2) shows reasonable agreement with the analytical solution, although the error in the time lag is of the order of 4 or 5%. This error was found to be due largely to the input being fed by the kymograph at a speed slower (3-4%) than expected. This error in speed had the effect of increasing the effective scale of resistance. The net result is to lower the amplitude and change the time lag.

The accuracy of the computer was therefore considered reasonable and the sources of error in these two test problems were corrected as much as possible before studying the actual wall problems.

DEFINITION OF TERMS

v — temperature
P — period of the function
ω — $2\pi/P$
ϵ — phase angle sine wave input temperature
ϕ — phase angle of v within the solid
k — thermal conductivity of solid
Cp — specific heat of solid
ρ — density of solid
a, α — thermal diffusivity of solid
t — time



Test problem 1. Step function.

$$0 \le x \le 2\ell$$

$$v = v\,(x,\ t)$$

Governing equation:

$$\frac{k}{\rho Cp} \cdot \frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$$

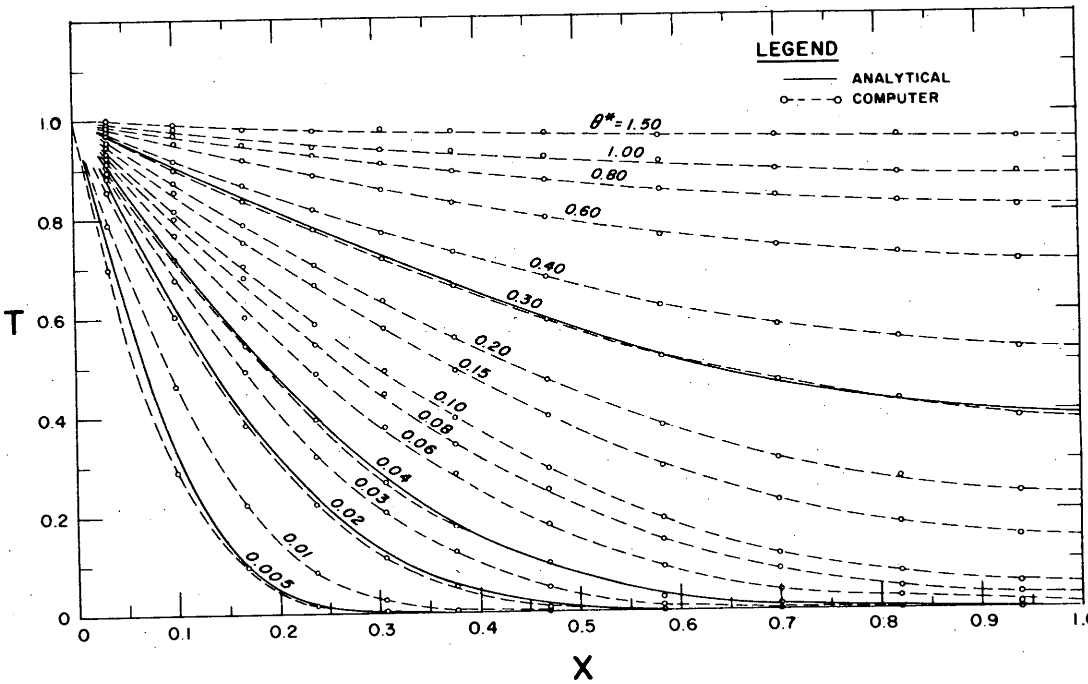


Figure D1. Temperature variations in a uniform slab.

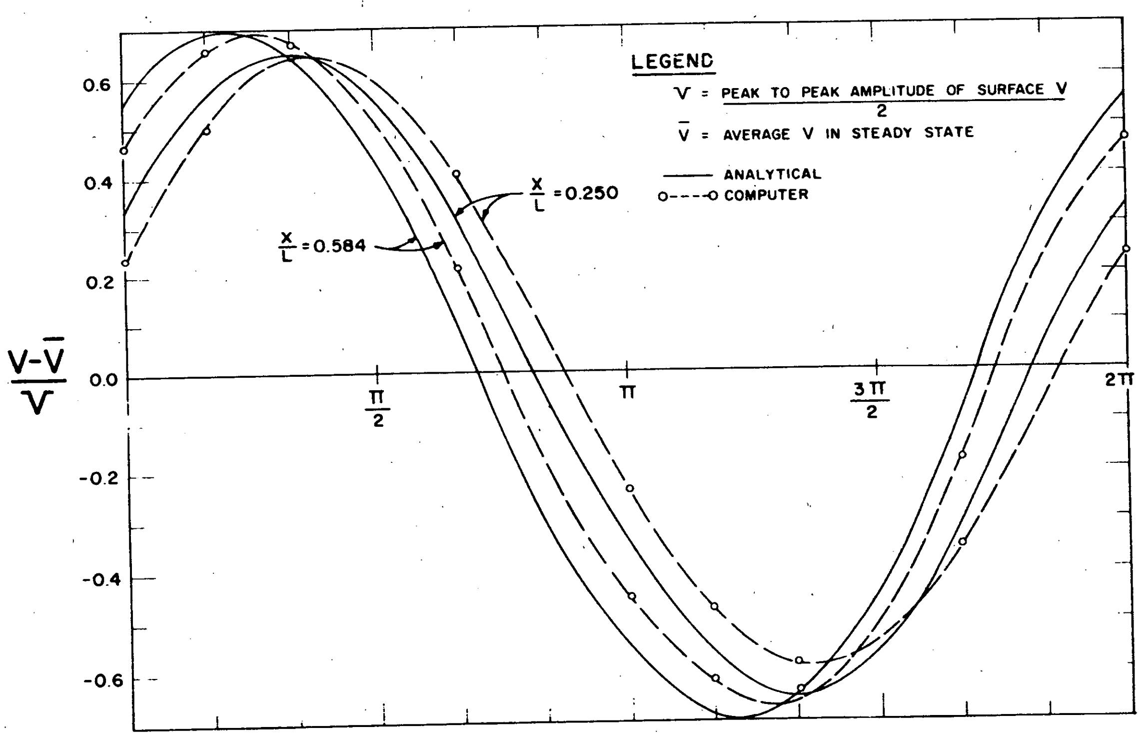


Figure D2. Temperature history at two depths in a homogeneous solid with harmonic surface temperature.

Boundary conditions:

$v\,(2\ell,\ t) = v\,(o,\ t) = V \qquad t \geq 0$

$v\,(x,\ t) \ = 0 \qquad t < 0$

$v\,(x,\ 0) \ = 0 \qquad \text{for } 0 < x < 2\ell$

$\frac{\partial v}{\partial x} = 0 \qquad \text{at } x = \ell$

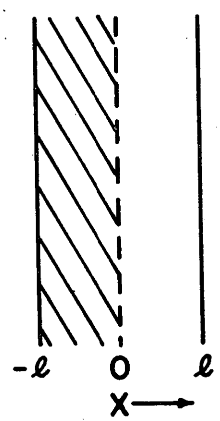
Define:

$X = \frac{x}{\ell};\ T = \frac{v}{V};\ a = \frac{k}{\rho C p};\ \theta^* = \frac{at}{\ell^2}$

$\therefore\ \frac{\partial^2 T}{\partial X^2} = \frac{\partial T}{\partial \theta^*}$

Solution:

$$T = 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\, e^{-(2n+1)^2\, \pi^2\, \theta^*/4} \cos \frac{(2n+1)\pi X}{2}\ ^\dagger$$



Test problem 2. Sine wave.

$v = 0 \qquad t < 0 \qquad -\ell < x < \ell$

$v = \sin(\omega t + \epsilon) \qquad x > |\ell|$

$t > 0$

$$V = A \sin(\omega t + \epsilon + \phi) \qquad \text{(steady state)}$$

$$+ 4\pi k \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)\,[4\ell^2 \omega \cos\epsilon - k(2n+1)^2 \pi^2 \sin\epsilon]}{16\ell^4 \omega^2 + k^2 \pi^4 (2n+1)^4}\,.$$

$$e^{-k(2n+1)^2 \pi^2 t/4\ell^2} \cos \frac{(2n+1)\pi x}{2\ell} \qquad \text{(transient)}^{**}$$

† Carslaw & Jaeger, Conduction of heat in solids, Second Edition, Clarendon Press, 1959, p. 100.

** Ibid., p. 105

Assume steady state reached:

$$V = A\sin(\omega t + \epsilon + \phi)$$

$$A = \left|\frac{\cosh \gamma x\,(1+i)}{\cosh \gamma \ell\,(1+i)}\right| = \left(\frac{\cosh 2\gamma x + \cos 2\gamma x}{\cosh 2\gamma\ell + \cos 2\gamma\ell}\right)^{\frac{1}{2}}$$

$$\phi = \mathrm{ARG}\left\{\frac{\cosh \gamma x\,(1+i)}{\cosh \gamma\ell\,(1+i)}\right\}$$

$$\gamma = (\omega/2\alpha)^{\frac{1}{2}}$$

$$\alpha = k/\rho Cp$$

$$P = 2\pi/\omega$$

<u>Calculations</u>

$x = 0.292$ ft $\qquad \ell = 0.500$ ft $\qquad \omega = 0.262$/hr

$k = 0.521$ Btu/hr-ft-°F

$\alpha = k/\rho Cp = 0.0231$ ft^2/hr

$\rho Cp = 22.50$ Btu/ft^3-°F

$$\gamma = (\omega/2\alpha)^{\frac{1}{2}} = 2.38/\mathrm{ft}$$

$$\epsilon = 90° \qquad \therefore\ v = A\cos(\omega t + \phi)$$

$$A = \left(\frac{\cosh 1.39 + \cos 78.6}{\cosh 2.38 + \cos 136.3}\right)^{\frac{1}{2}} = 0.696$$

$$\phi = \mathrm{ARG}\left\{\frac{\cosh \gamma x\,(1+i)}{\cosh \gamma\ell\,(1+i)}\right\}$$

$$\cosh \gamma x\,(1+i) = \tfrac{1}{2}\left[e^{\gamma x(1+i)} + e^{-\gamma x(1+i)}\right]$$

$$= \tfrac{1}{2}\left[e^{\gamma x}(\cos\gamma x + i\sin\gamma x) + e^{-\gamma x}(\cos\gamma x - i\sin\gamma x)\right]$$

$$= \tfrac{1}{2}\left[\cos\gamma x\,(2\cosh\gamma x) + i\sin\gamma x\,(2\sinh\gamma x)\right]$$

$$\theta_N = \tan^{-1}\left\{\tan\gamma x \tanh\gamma x\right\} = 26°37'$$

$$\theta_D = \tan^{-1}\left\{\tan\gamma\ell \tanh\gamma\ell\right\} = 64°20'$$

$$\phi = \theta_N - \theta_D = -37°43'$$

$x = 0.125$ ft (other parameters same)

$$A = \left(\frac{\cosh 0.595 + \cos 34.1°}{\cosh 2.38 + \cos 136.3°}\right)^{\frac{1}{2}} = 0.650$$

$\theta_N = 5°5'$

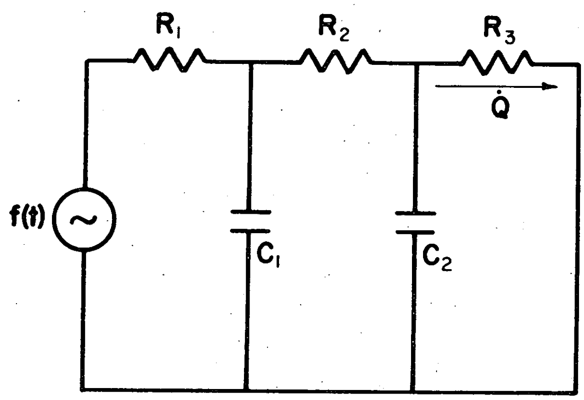
$\theta_D = 64°20'$

$\phi = -59°15'$.

APPENDIX E.

CHECK ON HEAT FLOW RESULTS

As an approximate check on the results obtained for the heat flow through the ten walls, the thermal circuit was divided into two sections (Table III) and a sine wave with a frequency of 1/24 cycles/hr was used as the input:



$$\dot{Q} = \frac{A}{R} + \frac{B}{|Z|} \sin(2\pi f(t) - \emptyset)$$

where

$R = R_1 + R_2 + R_3$

Z = network impedance

$f(t) = A + B \sin(2\pi ft)$

$A = 20°F$

$B = 8°F$

P. F. = Peak load factor

$$= \frac{A/R + B/Z}{A/R}$$

$\emptyset$ = Time lag, hrs.

Values for Z and $\emptyset$ for the ten walls were obtained by a method outlined by Guillemin[6]. The resulting time lags and peak load factors provided by this analytical approximation were recorded in Tables IV and V, respectively.

APPENDIX F.

PHOTOGRAPHIC TECHNIQUE FOR RECORDING DATA

Most of the wall sections tested in this investigation were divided into 8 to 14 sections. It was decided that to record the height of the fluid manually in ten or more standpipes every minute (analog equivalent of one hour in the thermal system) would be both impracticable and inaccurate. Consequently, the authors chose a method of recording data which consists of photographing these heights.

A 35 millimeter camera was set up in front of the computer, at a distance just great enough to encompass all of the standpipes. A string of five flood lights was placed about 2 ft behind the standpipes. A large sheet of onion skin paper was placed directly in back of the standpipes to diffuse the light. To make the fluid show up against the light background, ordinary food coloring was added to the distilled, de-aired water. (A check was made to see whether the food coloring affected the viscosity of the water. The change in viscosity was negligible.) The standpipes were coated with Drifilm* to eliminate the meniscus and 0.01% mercuric chloride solution was added to the water to prevent the formation of bacteria. In order to read the heights accurately from the film, two horizontal reference lines placed 15 in. apart were set up directly in front of the standpipes.

Pictures were taken at one-minute intervals over the duration of one cycle. The films were developed in the laboratory. To read the heights, the film was projected on a large screen which had already been calibrated to tenths of an inch.

In the two test problems, the error produced by this method of recording data was found to be less than 1%. The photographic method was subsequently used in the investigations of the wall sections.

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*Drifilm 0087. General Electric Organosilicon Products