TR 261

Technical Report 261

CUTTING FROZEN GROUND WITH DISC SAWS

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June 1975

CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report 261	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CUTTING FROZEN GROUND WITH DISC SAWS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Malcolm Mellor		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A762719AT30 Task 03, Work Unit 002
11. CONTROLLING OFFICE NAME AND ADDRESS Office, Chief of Engineers Washington, D.C. 20314		12. REPORT DATE June 1975
		13. NUMBER OF PAGES 69
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Cutting tools Excavation Freezing Saws Soils		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The largest commercially available disc saws (7 ft diameter) were tested in frozen ground. Cutting performance was reasonably attractive, specific energy consumption was acceptable, but cutter durability in frozen gravel was judged to be totally inadequate. A new cutter system was developed for the saw that provided best general capabilities, and a dramatic improvement in cutter durability was achieved (wear rate and cutter cost dropped by a factor greater than 10 and possibly by a factor of 100). The modified saw cut slots 3.7 in. wide and 30 to 34 in. deep at rates up to 6.6 ft/min in coarse frozen gravel and up to 16.3 ft/min in frozen silt. Overall values of specific energy for sawing (based on gross machine power) were 4.7×10^3 lbf/in. ² for gravel and 1.8×10^3 lbf/in. ² for		

20. Abstract (cont'd)

silt. Effective specific energy for bulk excavation using the kerf-and-rib technique was projected to be lower than these values by a factor of 5, taking a depth/width ratio for the uncut ribs of 2. Axle forces on the cutter wheel depend on the design of the cutting teeth and on the state of wear. For the test machine, horizontal cutting resistance with well-worn teeth could exceed the tractive capability of the carrier vehicle on some types of running surfaces. All essential data needed to design disc-saw attachments for crawler tractors are now available.

PREFACE

This report was prepared by Dr. Malcolm Mellor, Research Civil Engineer, Applied Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. The work was performed under DA Project 4A762719AT30, *Research for Protective Structures in Theaters of Operations*, Task 03, *Protective Structures for Winter Conditions*, Work Unit 002, *Cutting, Drilling and Excavating Frozen Materials*.

The author gratefully acknowledges the advice and assistance of Paul V. Sellmann and Francis Gagnon of CRREL, who worked actively on the program during the two test seasons. Test sites were made available through the courtesy of T.J. Bascetta of Lebanon Crushed Stone, Inc., and site assistance was rendered by T. Thibodeau and T. Perry of the same company. Machine hire and operation was handled with great efficiency by R. Davis of Ditch Witch North and by Leroy Den Besten and members of his staff from Vermeer Sales and Service. Much information was provided by the technical representatives of various U.S. and British manufacturers of mining tools. The report was reviewed by P.V. Sellmann of CRREL and Dr. I. Hawkes of IRAD.

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CUTTING FROZEN GROUND WITH DISC SAWS

by

Malcolm Mellor

INTRODUCTION

In recent years large diameter drag-bit disc saws have been developed in the U.S. for cutting concrete, asphalt, frozen soils, and the weaker rocks. They have found application in precise cutting of utility trenches in pavements and in seasonally frozen ground, and in sawing of expansion joints in concrete slabs. So far they have not won general acceptance, largely because of rapid wear on the cutting teeth in the harder and more abrasive materials.

In the Soviet Union a number of disc saw attachments have been developed for heavy tractors, and it appears that cutting of frozen ground has been a primary motivation. Mashokov et al. (1973) and Vedenev et al. (1973) describe some of the more recent developments for frozen ground. One model consisted of a 2.2-m-diameter saw mounted on an ETU-354 ditch digger; it had a depth capability of 0.8 m (judged to be too little) and a working rate of 12.5 to 87.5 m/hr (0.7 to 4.8 ft/min) in frozen soil. A bigger type of machine was developed by attaching 3-m-diam saws to ETR-141 and ETR-161 ditchers in place of the usual bucket wheels; two types of mining machine teeth were tried at different spacings, and the better arrangement of the two gave working rates of 54 to 266 m/hr (3 to 14.5 ft/min) in frozen soil with a depth of 1.3 m (4.3 ft) and a slot width of 0.16 m (6.3 in.). Linear tool speed was 8.8 m/sec (1732 ft/min), which is high by comparison with typical drag-bit machines. A double disc saw was mounted on an ETU-353 excavator behind a T-100 tractor: its discs were 2.3 m in diameter and cut two parallel slots, each 0.1 m (4 in.) wide and 1 m (3.3 ft) deep, at a speed of 60 m/hr (3.3 ft/min).

Pobezhaev (1974) gave some mechanical details of a disc saw mounted on a T-100 tractor for work in frozen ground. Maximum cutting depth was 1.2 m (3.9 ft) and cutting width was 0.14 m (5.5 in.). The machine was apparently capable of advancing at 40 m/hr (2.2 ft/min), and cutting cost was estimated as 0.6 ruble/m³.

Volkova (1974) described a cable-layer disc saw consisting of a separate unit towed on rubber tires behind a crawler tractor. Cutting depth was given as 1.3 m (4.3 ft), with a cut width of 0.25 to 0.26 m (10 in.). Cutting rate in frozen soil was 180 m/hr (9.8 ft/min). The cutting wheel appeared (from a photograph) to have relatively few cutters (about 20), but each cutter appeared to be a massive chisel.

If frozen ground can be cut rapidly and economically by large saws, many engineering tasks will be greatly simplified. First of all, there are direct requirements for slots or narrow trenches, e.g. in the laying of cables or small-diameter pipes. Deep, narrow slots can also be used for emplacing linear charges of bulk explosive in granulated, prilled or slurried form,

and they can be used to control explosive excavation, e.g. in avoiding lateral overbreak during trench blasting. For general-purpose excavation, specific energy consumption can be minimized by cutting parallel slots and breaking out the intervening ribs of uncut material with blades, rippers, buckets, or explosive. In the Soviet Union, trenches are sometimes excavated in frozen ground by sawing parallel slots with either disc or chain saws and then breaking the kerf with explosives (Rozhkov 1973).

The object of the CRREL test program was to evaluate U.S. commercial disc saws, to consider exploratory development for better performance, and to investigate operating procedures for use of disc saws in general-purpose excavation of frozen ground.

TEST MACHINES

At the time the test program was initiated there appeared to be only two manufacturers of large disc saws in the U.S., both making fairly small machines adapted from conventional boom-and-chain trenchers. The heaviest model offered by each manufacturer was selected for testing. The heavier of the two machines, the Vermeer T-600, mounted a 7-ft-diam wheel on a crawler carriage, the whole rig weighing about 17,000 lb (Fig. 1). The other machine, the Ditch Witch ES30 Earth Saw (Fig. 2), carried a wheel of approximately 7 ft diameter on a small rubber-tired R60 or R65 tractor, the wheel attachment weighing about 2500 lb. One machine of each type was hired from an authorized dealer after prior discussion with the manufacturer, and a qualified operator was hired with each machine.

The Vermeer operated in the first test season (1972) was a used T-600A machine that was in very good condition. The operator/mechanic was experienced and highly competent, and he was accompanied by an experienced technical representative from the dealership. In the second test season (1973) the Vermeer machine was an almost new T-600B. It was operated by a senior representative of the dealership. Although the Vermeer T-600 is offered with a "frost cutter" wheel, this version was not seriously considered as a contender for excavation of deeply frozen ground, and both Vermeer test machines were equipped with the "rock cutter" wheel. The specifications of the tested machines are given in Table I.

The Ditch Witch, which was used only in the 1972 season, was a brand new machine consisting of the ES30 Earth Saw mounted on the R60 tractor (the R65 is a slightly more powerful optional carrier). It was driven by a new dealer who was highly experienced as a Ditch Witch mechanic, but who had not previously operated large disc saws. Specifications for the machine are given in Table II.

Both the Vermeer and the Ditch Witch were fitted with the standard bullet-type rock bits* in the 1972 tests (Fig. 3, 4, 5), and every effort was made to clean and grease the bit block† so as to permit easy rotation of the bits, which are claimed to be self-sharpening. The Ditch Witch had bits made by a major U.S. manufacturer, while the Vermeer had similar but more competitively priced bits made by a smaller company. In view of the poor durability shown by these bits, a completely new cutter system was developed for the 1973 tests, as described in a subsequent section of this report.

Neither machine was fitted with gauge shoes to control cutting depth and minimize vibration.

* Replaceable rock-cutting tools that work by shearing or chiseling are variously referred to as "bits," "picks," "teeth," or "cutters."

† The holders for replaceable teeth on mining machines are known variously in the industry as "sockets," "pockets," "boxes," "bosses," or "blocks."

Table I. Specifications of Vermeer T-600 Rock Cutter.

Model

T-600A Rock Cutter (changes for T-600B shown in parentheses)

Manufacturer

Vermeer Manufacturing Company, Pella, Iowa 50219

Supplier

Vermeer Sales and Service, Inc., Castleton, New York 12033

General dimensions

Working length	16.7 ft
Working height	8.2 ft
Width	7.1 ft (6.8 ft)
Weight (approx)	17,000 lb (19,500 lb)

Engine

Type	GMC 3-53 diesel, 3 cylinder
Horsepower at governed rpm	78 @ 2325 rpm
Displacement	159 in. ³
Torque	150 lbf-ft @ 2325 rpm
Fuel capacity	28 gal. (30 gal.)

Cutter wheel

Wheel diameter	7 ft
Rated cutting depth	2.6 ft
Cutting width	3.5 to 4.5 in. (4 to 5 in.)
Wheel thickness	1 in.
Cutting teeth	Bullet-type rock bits (special bits used on T-600B)
Number of teeth	130

Drive for cutter wheel

Type	Chain drive from 12- (15-) gpm, 1500- (2000-) psi hydraulic motor
Controls	Two lift cylinders, controls and wheel shift operated from 6-gpm, 1500-psi hydraulic pump
Wheel speeds at full rpm	1st gear 5 rpm (7 rpm)
	2nd gear 15 rpm (14 rpm)
	3rd gear 20 rpm (26 rpm)
	4th gear 35 rpm (44 rpm)
	Reverse 5 rpm (6 rpm)

Track system

Type	Girderized crawler track with semi-cleated pads. Coil spring tension, 9 or 11 rollers per track.
Track pads	12 or 15 in. (15 or 18 in.) wide. 34 (37 or 43) pads per track
Track length	8.1 ft (7.35 or 8.75 ft)
Bearing pressure	7.8 or 6.2 psi (6.0 or 4.2 psi)
Creep speed with no load	0 to 26 ft/min, hydraulic drive
Travel speed (wheel not operating) at full rpm	1st gear 48 ft/min
	2nd gear 100 ft/min
	3rd gear 184 ft/min
	4th gear 310 ft/min
	Reverse 39 ft/min

Table II. Specifications of Ditch Witch ES30 Earth Saw.

Model

ES30 Earth Saw mounted on R60 Trencher.

Manufacturer

Charles Machine Works, Inc., Perry, Oklahoma 73077

Supplier

Ditch Witch North, Bethel, Vermont 05032

General dimensions

Working length	17.3 to 18.0 ft
Working height	7.75 ft
Width	5.3 ft
Approx gross weight	8500 lb
Carrier weight (approx)	6000 lb
Attachment weight (approx)	2500 lb

Engine

Type	Wisconsin V460
Horsepower	60
Fuel capacity	15 gal.

Cutter wheel

Wheel diameter	7 ft
Rated cutting depth	2.7 ft
Cutting width	4 in.
Cutting teeth	Bullet-type rock bits
Number of teeth	110

Drive for cutter wheel

Type	Belt and chain drive, 4 forward speeds plus reverse.
Control	Two lift cylinders driven from 13-gpm, 1750-psi hydraulic pump.

Carriage

Type	Four-wheel rubber-tired tractor with four-wheel drive
Creep speed	0 to 36 ft/min, hydraulic drive
Travel speed	Up to 15 mph

a. T-600A.

b. T-600B.

Figure 1. Vermeer T-600 Rock Cutter.

Figure 2. Ditch Witch ES30 Earth Saw on R60 Trencher.

Figure 3. Cutter wheel of Vermeer T-600A. Note radial scouring bars for cleaning cuttings from slot.

a. Cutter wheel.

b. Replaceable perimeter segment.

Figure 4. Bullet-type teeth on Vermeer T-600. The bits on this machine were fitted to 10 replaceable perimeter segments.

Figure 5. Cutter wheel of Ditch Witch ES30.

TEST SITES

Two types of ground material were required for the basic tests: frozen fine-grained soil, preferably silt, and frozen gravel. Each material had to be well bonded by ice that was at, or near, saturation content. Finally, the depth of the frozen material had to exceed the maximum cutting depth of the machines so as to simulate permafrost.

The selected test area was in West Lebanon, New Hampshire, on the property of Lebanon Crushed Stone, Inc. This area had a variety of natural soil types and reworked materials, it was conveniently located close to CRREL, and support facilities were available through the courtesy of the owning company.

The prime objective was to prepare a site representing the most difficult type of ground condition for excavation, i.e. a compact, coarse, cobbly gravel completely frozen at saturation water content. A suitable site was found at the bottom of the main gravel pit, which had horizontal surfaces on two main levels. At that time the pit was deep and relatively narrow, so that the base received little direct sunlight in winter, and cold air could accumulate with stable thermal stratification in calm weather. The base of the pit was at about the lowest elevation of the entire property, so that it received ample inflow of drainage water.

To accelerate frost penetration, the test site was kept free of snow by bulldozing after each snowfall. The winter of 1971-72 was abnormally warm, and by late January 1972 the gravel was only frozen to about 10 in. depth. However, onset of colder weather improved the situation and by late February the site was frozen to more than 28 in. depth. Test conditions in 1973 were similar, the gravel being frozen to a depth of 30 in. by the end of February.

No special preparations were made for testing in frozen silt, as suitable conditions existed on a haul road where snow was either cleared or compacted by traffic. At this place the soil was a light sandy silt, well compacted at the surface, with a water content that was probably close to saturation (the main freezing period was preceded by rainfalls and intermittent snowmelt in both years). The silt was frozen to a depth of 25 in. in 1972 and to almost 30 in. in 1973.

Two additional sites were used in 1973. A stratum of bouldery gravel in the West Lebanon pit was used for a "torture test," and a bed of fine-to-medium gravel in a pit near CRREL in Hanover, N.H., was used for demonstration cuts. The Hanover site was also used for measurements of traction force.

TEST PROCEDURES IN 1972

Both machines were taken first to the gravel site, where they began operation with new teeth. The operators were invited to familiarize themselves with the conditions before starting the tests.

The tests in gravel were intended to determine the following:

1. Maximum effective sumping depth
2. Maximum traverse speed at different cutting depths
3. Most effective wheel speeds
4. Approximate specific energy consumption
5. Tool wear

After preliminary tests in gravel, both machines were taken to the silt site, where they operated with partly worn teeth. After a few test runs in silt, the Ditch Witch had to withdraw from further testing as its teeth were worn and no replacements were available. The Vermeer had its teeth completely replaced, and then underwent further testing in silt before it was transferred back to the gravel site for final tests there.

TEST RESULTS, 1972

Maximum cutting depths in frozen silt were 33 and 32 in. for the Vermeer and Ditch Witch respectively. In frozen gravel, the maximum depth of cut achieved by the Vermeer was 28 in. and the maximum depth achieved by the Ditch Witch was 12.5 in.

Results of the cutting rate tests are given in Tables III-VI. These tables also give volumetric excavation rates and overall specific energy consumption.

Table III. Performance of Vermeer T-600A in frozen gravel (1972).

Machine: Vermeer T-600A

Engine: 3-53 Detroit diesel rated at 78 hp

Wheel diameter: 7 ft

Engine speed: 2300 rpm

Cut width: 4.25 in.

Depth of frozen layer: > 28 in.

Cutting depth (in.)	Gear	Nominal wheel speed (rpm)	Traverse speed (in./min)	Volumetric excavation rate (ft ³ /min)	Nominal overall specific energy (in.-lbf/in. ³)	Remarks
24	3	20	33	1.95	9.16×10^3	Fine dry cuttings
24	2	15	24	1.42	1.26×10^4	Fine dry cuttings
28	3	20	14	0.965	1.85×10^4	Fine dry cuttings
28	3	20	18	1.24	1.44×10^4	Fine dry cuttings
28	3	20	35	2.41	7.41×10^3	Fine dry cuttings
24	3	20	37	2.19	8.16×10^3	Fine dry cuttings
26	3	20	38	2.43	7.35×10^3	Fine dry cuttings
26	3	20	33	2.11	8.46×10^3	Fine dry cuttings
25.5	3	20	44.5	2.79	6.40×10^3	Fine dry cuttings
25.5	3	20	41	2.57	6.95×10^3	Fine dry cuttings
26	3	20	42	2.69	6.64×10^3	Teeth badly worn by this stage
12	3	20	66	1.95	9.16×10^3	Water running into slot-wheel cutting wet. Bigger cuttings, with some pebbles coming out intact
12	3	20	50	1.48	1.21×10^4	
12	4	35	82	2.42	7.38×10^3	
15.5	4	35	59	2.25	7.94×10^3	Cutting dry again
15.5	4	35	48	1.83	9.76×10^3	Cutting dry again
6	4	35	204	3.01	5.94×10^3	Fresh teeth
28.5	3	20	63	4.42	4.04×10^3	Teeth badly damaged by previous run — 44 replaced
28	3	20	57	3.93	4.55×10^3	

Table IV. Performance of Ditch Witch in frozen gravel.

Machine: Ditch Witch ES30 on R60 trencher

Engine: Wisconsin V460 rated at 60 hp

Cut width: 4.5 in.

Depth of frozen layer: > 28 in.

Cutting depth (in.)	Gear	Traverse speed (in./min)	Volumetric excavation rate (ft ³ /min)	Nominal overall specific energy (in.-lbf/in. ³)	Remarks
12.5	3	30	0.977	1.40×10^4	
12.5	3	37	1.205	1.14×10^4	
12.5	3	36	1.17	1.17×10^4	
12.5	3	36	1.17	1.17×10^4	
12.5	3	38	1.24	1.11×10^4	

Table V. Performance of Vermeer T-600A in frozen silt (1972).

Machine: Vermeer T-600A Engine: 3-53 Detroit diesel rated at 78 hp
 Wheel diameter: 7 ft Engine speed: 2300 rpm
 Cut width: 5 in. Depth of frozen layer: 25 in.

Cutting depth (in.)	Gear	Nominal wheel speed (rpm)	Traverse speed (in./min)	Volumetric excavation rate (ft ³ /min)	Nominal overall specific energy (in.-lbf/in. ³)	Remarks
33.5	4	35	93	9.02	1.98×10^3	Teeth badly worn before test started
33	4	35	67	6.40	2.79×10^3	Teeth badly worn before test started
33	3	20	65	6.21	2.88×10^3	Teeth badly worn before test started
33	4	35	64	6.11	2.92×10^3	Teeth badly worn before test started
33	4	35	65	6.21	2.88×10^3	Teeth badly worn before test started
10	4	35	166	4.81	3.72×10^3	New teeth fitted
10.5	4	35	147	4.47	3.99×10^3	
17.5	4	35	218	11.03	1.62×10^3	
17	4	35	153	7.53	2.37×10^3	
32.5	4	35	144	13.55	1.32×10^3	Rather dry sandy silt
31	4	35	215	19.30	9.25×10^2	Rather dry sandy silt
32.5	4	35	228	21.45	8.32×10^2	
32	4	35	216	20.00	8.93×10^2	

Table VI. Performance of Ditch Witch in frozen silt.

Machine: Ditch Witch ES30 on R60 trencher Engine: Wisconsin V460 rated at 60 hp
 Cut width: 5.4 in. Depth of frozen layer: 25 in.

Cutting depth (in.)	Gear	Traverse speed (in./min)	Volumetric excavation rate (ft ³ /min)	Nominal overall specific energy (in.-lbf/in. ³)	Remarks
20	4	14	0.87	1.58×10^4	
32	4	20	1.99	6.90×10^3	
32	4	14.5	1.44	9.51×10^3	
32	4	18	1.79	7.68×10^3	
32	4	20	1.99	6.90×10^3	
32	3	31	3.09	4.45×10^3	
20	3	91	5.66	2.43×10^3	
20	3	108	6.72	2.05×10^3	
20	3	80	4.98	2.76×10^3	
20	4	27	1.68	8.18×10^3	

Machine operators were soon able to develop a "feel" for the best wheel speed. If the wheel speed was too low, cutting became jerky and the wheel tended to bounce, putting unfavorable loads on the cutters and bearings. This effect was rather more noticeable on the Ditch Witch, which was the lighter rig and which was also mounted on pneumatic tires. At greater cutting depths, both operators selected the lowest wheel speed consistent with smooth running, and this appeared to be 3rd gear for both machines when working in gravel. On the Vermeer, 3rd gear is supposed to give a wheel speed of 20 rpm. The corresponding conversion was not available for the Ditch Witch, but wheel speed looked higher than that of the Vermeer.

For shallow cuts in gravel, the Vermeer operator selected 4th gear, which is supposed to give a wheel speed of 35 rpm. It might be noted that there are kinematic reasons for increasing wheel speed with decreasing cutting depth (Appendix A). The Vermeer operator was able to cut frozen silt to any depth in 4th gear, and this gear was his choice for cutting silt. However, performance of the Ditch Witch in silt improved significantly when it was dropped from 4th to 3rd. Again it should be noted that the Ditch Witch seemed to be somewhat higher geared than the Vermeer.

Tooth wear in frozen gravel was very severe, with damage occurring after just a few revolutions of the wheel. During the first gravel tests, teeth on the Vermeer were for all practical purposes worn out after 45 ft of cutting at 24-28 in. depth, i.e. after about 15 minutes of operation. At the start of the second set of gravel tests, the Vermeer made a fast, shallow (6-in.) cut for one minute, and during the course of this cut struck a boulder, causing the wheel to bounce. As a result of this 44 teeth had to be replaced. The Ditch Witch teeth had bigger tungsten carbide caps on the end, but wear and breakage in gravel were at least as bad as on the Vermeer — many teeth were broken and worn after 17 ft of cutting at 12½ in. depth.

Tooth wear in frozen silt was chiefly by abrasion. The soft outer body of the tooth was ground away, sometimes in curious patterns, and the projecting carbide core was thus left unsupported. Cutting tests in frozen silt were not sufficiently protracted to establish a wear rate, but it seems likely that a set of teeth could go without replacement for at least several hundred feet.

Patterns of tooth wear clearly indicated that some teeth were doing virtually no work, while others bore the brunt of the attack. "Scalloped" abrasion patterns on the side of some teeth seemed to indicate incomplete cutting coverage. Many teeth obviously failed to rotate in their sockets, causing unsymmetrical wear, with little potential for self-sharpening.

The Vermeer men considered the frozen gravel to be a tougher cutting proposition than almost any material they had previously encountered, including reinforced concrete.

TEST PROGRAM FOR 1973

The 1972 tests showed that existing disc saws had useful capabilities for cutting frozen ground, but the durability of standard cutting teeth appeared to be completely inadequate for heavy work in frozen gravels. Thus the goal for work in 1973 was development of better cutting teeth.

Figure 6. Heavy drag bits fitted to Vermeer T-600 in second test season. Pick boxes were welded to 10 replaceable perimeter segments.

Figure 7. Detail of Hoy parrot-beak pick as fitted to Vermeer T-600B for 1973 tests.

The mining research literature was surveyed, U.S. and foreign manufacturers of mining tools were canvassed, and the problem was studied analytically. This resulted in the selection of a particular style of heavy-duty parrot-beak pick, the Hoy bayonet pick number B200/2/H3144, and in the design of an appropriate tooth-setting pattern. A set of perimeter segments for the Vermeer T-600 were made up with the new teeth, and tests were arranged.

The first test for durability was made on two sections of road near Catskill, New York, where the local traprock aggregate had previously made sawing of concrete economically unattractive. Tests were then made on frozen gravel and frozen silt at Hanover and West Lebanon, New Hampshire. Test procedures were broadly similar to those employed in 1972, but additional measurements were made to determine the tractive thrust, or drawbar pull, of the carrier vehicle.

The machine was also taken to a lake north of Hanover for a demonstration of ice cutting (Fig. 10).

Figure 8. Vermeer disc saw operating in frozen gravel.

Figure 9. Vermeer T-600B cutting concrete and asphalt. Note that a cable layer is fitted to the wheel.

Figure 10. Vermeer T-600B cutting floating ice. The machine was able to cut 18-in.-thick ice at maximum crawl speed (26 ft/min) with no perceptible effort. A contractor recently made long cuts in lake ice with a Ditch Witch saw, averaging 6.7 ft/min.

TEST RESULTS, 1973

The cutting performance of the Vermeer T-600B with the new teeth is given in Tables VII-IX. These results show that the production rates were not much affected by the change of teeth, but the machine was able to cut to greater depth in frozen gravel with the new teeth. However, the durability tests showed a dramatic improvement in tooth life.

In the road-material tests at Catskill, the saw sumped-in to its maximum possible depth (3 in. greater than maximum rated cut depth) and began cutting very hard material. The machine operator and the dealer estimated (on the basis of past work in the same material) that the standard bullet-bits would last for a linear distance of 15 ft $\pm$ 3 ft under such conditions. After cutting for 28 ft, the machine was stopped and the new teeth were inspected for wear and damage; no significant deterioration had occurred by that stage, but the relief faces of some of the carbides were showing wear. After 150 linear feet of cutting the teeth had developed definite wear-flats on the relief faces; the machine was still performing well, but maximum cutting speed had dropped by 30%. The machine was then moved to another section of road, where it continued to make demonstration cuts at shallower depths, slicing through the asphalt and concrete pavement but not penetrating the gravel base. After about 1600 linear feet of cutting, four teeth had been broken and the remaining teeth were well worn, although far from the stage at which a contractor would discard them.

In the frozen gravel tests at West Lebanon the machine was deliberately pushed at maximum speed through a bed of bouldery material, and under these conditions 13 teeth were broken over a linear distance of about 25 ft. However, with the bullet-bits used in 1972,

Table VII. Performance of modified Vermeer T-600B in road materials (1973).

Machine: Vermeer T-600B with Hoy parrot-beak picks			Engine: 3-53 Detroit diesel rated at 78 hp			
Wheel diameter: 7.2 ft			Engine speed: 2300 rpm			
Cut width: 3.7 in.			Material: Road pavement consisting of 4.5-in. surface layer of asphalt, 7.5-in. layer of lightly reinforced concrete (trap- rock aggregate, $\frac{3}{8}$ in. rebar), deep base course of coarse frozen gravel.			
Cutting depth (in.)	Gear	Nominal wheel speed (rpm)	Traverse speed (in./min)	Volumetric excavation rate (ft ³ /min)	Nominal overall specific energy (in.-lbf/in. ³)	Remarks
34	4	44	52	3.79	4.72×10^3	New picks
34	4	44	49	3.57	5.01×10^3	
34	3	26	45	3.28	5.45×10^3	Machine jerking, tracks scuffing
34	2	14	not measurable			Very jerky, frequent halts
34	4	44	35	2.55	7.01×10^3	Steady rate after 30 ft of deep cutting

Table VIII. Performance of modified Vermeer T-600B in frozen gravel (1973).

Machine: Vermeer T-600B with Hoy parrot-beak picks Engine: 3-53 Detroit diesel rated at 78 hp
 Wheel diameter: 7.2 ft Engine speed: 2300 rpm
 Cut width: 3.7 in. Depth of frozen layer: Approximately 30 in.

Cutting depth (in.)	Gear	Nominal wheel speed (rpm)	Traverse speed (in./min)	Volumetric excavation rate (ft ³ /min)	Nominal overall specific energy (in.-lbf/in. ³)	Remarks
31	4	44	79	5.24	3.41×10^3	All-out test in compact gravel with boulders of 10 in. diameter or more. Almost equivalent to cutting solid rock. 13 teeth broken in a few minutes of operation.
30 total (25 gravel, 5 ice and fine soil)	4	44	55	3.53	5.06×10^3	Operating with broken teeth in cobbly gravel. Some track slip.
30 total (25 gravel, 5 ice and fine soil)	4	44	55	3.53	5.06×10^3	Second test under same conditions produced identical results.
30	4	44	61	3.92	4.56×10^3	Broken teeth replaced. Cobbly gravel.
34	4	44	48	3.50	5.11×10^3	Good teeth. Fine gravel. Some slip in drive belts.
34	4	44	60	4.37	4.09×10^3	After tightening belts.
34	4	44	42	3.06	5.85×10^3	Fine gravel with ice topping. Some track slip.

Table IX. Performance of modified Vermeer T-600B in frozen silt (1973).

Machine: Vermeer T-600B with Hoy parrot-beak picks Engine: 3-53 Detroit diesel rated at 78 hp
 Wheel diameter: 7.2 ft Engine speed: 2300 rpm
 Cut width: 3.7 in. Depth of frozen layer: Approximately 30 in.

Cutting depth (in.)	Gear	Nominal wheel speed (rpm)	Traverse speed (in./min)	Volumetric excavation rate (ft ³ /min)	Nominal overall specific energy (in.-lbf/in. ³)	Remarks
35.5	4	44	151	11.5	1.55×10^3	
35.5	4	44	168	12.8	1.40×10^3	
31	4	44	195	12.9	1.39×10^3	
31	3	26	120	7.97	2.24×10^3	
31	4	44	129	8.56	2.09×10^3	
30	4	44	164	10.5	1.70×10^3	
30	4	44	138	8.86	2.02×10^3	Some crushed stone in top foot of soil.

Table X. Drawbar pull of Vermeer T-600B on various working surfaces.

Machine: Vermeer T-600B (short track) Engine: 3-53 Detroit diesel rated at 78 hp
Machine weight: 19,500 lb Engine speed: 2300 rpm
Dynamometer: BLH electrical resistance load cell with Keithley digital voltmeter readout Track speed: 0 to 15 ft/min

Ground surface	Drawbar pull (lbf)	Drawbar coefficient (drawbar pull/gross wt)	Remarks
Thin snow cover over frozen sand	5,910	0.303	
Thin snow cover over frozen sand	6,570	0.337	
Thin snow cover over frozen sand	8,760	0.449	
Thin snow cover over frozen sand	10,960	0.562	
Thin snow cover over ice	8,540	0.438	
Thin snow cover over ice	7,450	0.382	Tracks slipping
Thin snow cover over ice	7,450	0.382	
Thin snow cover over ice	7,670	0.393	Tracks slipping
Thin snow cover over ice	6,350	0.326	
Mixture of snow and dirt	5,260	0.270	
Mixture of snow and dirt	5,690	0.292	
Mixture of snow and dirt	5,910	0.303	
Slush over thawing ice	7,670	0.393	
Slush over thawing ice	9,200	0.472	
Slush over thawing ice	9,640	0.494	
Slush over thawing ice	7,450	0.382	
Slush over thawing ice	8,110	0.416	
Slush over thawing ice	6,790	0.348	
Frozen sandy silt lightly thawed on surface	10,080	0.517	
Frozen sandy silt lightly thawed on surface	10,960	0.562	
Frozen sandy silt lightly thawed on surface	12,050	0.618	
Frozen sandy silt lightly thawed on surface	9,860	0.506	Tracks slipping
Frozen sandy silt lightly thawed on surface	9,640	0.494	

44 teeth had been destroyed by striking a single boulder. Under less severe cutting conditions in frozen gravel there was no serious deterioration after 50 ft of cutting, although the carbides showed some wear and minor chipping.

In frozen silt there was no significant wear, and it appeared that the teeth would last indefinitely.

Results of the traction tests are given in Table X.

GENERAL PERFORMANCE

Both of the saws tested in 1972 had good capabilities for cutting frozen fine-grained soils (silt, sand, clay), and tooth durability seemed adequate for these materials. Both machines were capable of cutting frozen gravel at useful rates, but cutter durability was judged to be completely inadequate in this material.

On the basis of the limited tests that were carried out, the heavier and more expensive machine, the T-600, demonstrated significantly higher capabilities than the lighter ES30. It was able to cut to more than the rated maximum depth in both frozen silt and frozen

gravel, whereas the ES30 could only achieve about 40% of its rated cutting depth in frozen gravel. When cutting at the same depth, the T-600 progressed about 80% faster than the ES30 in frozen gravel, and it was more than twice as fast in frozen silt. The installed power of the ES30 was only 80% of the installed power of the T-600, but this did not appear to be the limiting factor. The limitation seemed to be more a matter of force and stability. The ES30 wheel, having open segments cut in it, was obviously lighter than the T-600 wheel, and while this probably made it more compatible with a light carrier vehicle it did reduce the desirable flywheel effect.

The performance of the T-600 can be used to establish an interim standard of comparison for sawing of frozen ground. At approximately 33 in. cut depth, traverse speeds range from 5 to 19 ft/min in frozen silt and from 3.5 to 6.6 ft/min in frozen gravel, variations occurring with tooth condition and the strength of the frozen ground. In terms of volumetric excavation rate (i.e. cut width $\times$ cut depth $\times$ traverse speed), the range is from about 5 to 21 ft³/min in frozen silt and 2 to 5 ft³/min in frozen gravel.

CUTTER DURABILITY

The bullet bits fitted to the 1972 machines did not have adequate durability. In frozen gravel the T-600 wore out a set of 130 teeth in 45 linear feet of cutting at 80% of maximum depth; this is roughly equivalent to one tooth per square foot of sawcut, or nearly four teeth per cubic foot of excavated material. According to operator reports, the wear rate in hard-aggregate concrete can be as high as three teeth per square foot of sawcut, or about nine teeth per cubic foot of excavated material. Depending on the amount of tungsten carbide in the tip, teeth of this type typically cost \$1.50 to \$4 each (1973 prices) when bought in bulk, so that the cost of sawing frozen gravel can be prohibitive.

A bullet bit may be likened to a pencil, with the hard but brittle carbide analogous to the lead of a pencil and the soft steel body corresponding to the wood of the pencil. The bit works well as long as the resultant cutting force is directed along its central axis, but when the cutting force is angled away from its axis the steel abrades and allows the carbide to be snapped off in flexure. In other words, it is well adapted to "stabbing," but vulnerable to side forces. Much of the research on rock-cutting tools (which tends to be done with ideal sharp tools) indicates that the normal and tangential components of cutting force are approximately equal, so that the resultant cutting force makes an angle of roughly 45° with the tangent to the tooth-tip trajectory, and bullet bits tend to be set accordingly. However, more obscure research investigations that take into account realistic tool wear indicate that the ratio of normal to tangential components of cutting force increases as the bit wears, reaching values as high as 4 after a modest amount of cutting. Bullet bits are designed to avoid development of wear flats, their self-sharpening characteristics being provided by rotational symmetry and rotational freedom of the bit in its socket. However, in actual operation bits often fail to rotate, even when the sockets are cleaned and greased frequently.

In seeking an alternative to the bullet bit for the disc saw and for other experimental machines, six U.S. manufacturers of carbide-tipped rock-cutting tools were contacted and their product ranges were considered. However, the industry is heavily committed to the production of bullet bits in varying shapes and sizes, and the available fixed drag bits appeared to be mainly very small tools that lacked any kind of design sophistication.

Technical representatives of bit makers concentrated exclusively on bullet bits, and were reluctant to consider alternative designs. Other developers of specialized machines have apparently experienced similar industry response. Since there was no domestic source for suitable manufactured teeth, and since special fabrication of experimental teeth would have been prohibitively expensive, new bits were purchased from a firm in England, where there are three manufacturers of mining tools with highly varied product lines.

The new bit was a Hoy bayonet pick, type 200/2/H3144 (Fig. 7). This tool is a sturdy parrot-beak pick with a high-impact carbide. The gauge length (projecting length) is 2 in. and the width overall is 1 in. The design relief angle is approximately 12° , but the tools supplied had a primary relief angle of approximately 8° and a secondary relief angle on the carbide of approximately 18° . The design rake angle was approximately $+6^\circ$, but on the tools supplied the rake angle was zero or very slightly negative. The carbide had a maximum width of 1 in., and maximum dimensions in the normal and tangential directions of 0.75 in. and 0.5 in. respectively. A total of 60 bits were fitted to the wheel, replacing the 130 bullet bits previously used. At time of purchase each bit cost \$2.87, so that the cost of a complete replacement set of teeth for the wheel was about equal to the lowest known cost for a set of bullet teeth, and about one-third of the cost for a set of high-priced bullet teeth.

The cutting tests in road materials suggested that a set of the new teeth might have a working life two orders of magnitude higher than a set of bullet teeth, and the Vermeer dealer immediately reacted by obtaining a U.S. franchise for these tools. The tests in frozen gravel were not sufficiently protracted to establish wear rates, but they did demonstrate the superior impact resistance of the new teeth and they suggested that wear life would exceed that of the bullet teeth by more than a factor of 10.

Frozen ground is highly variable, but in frozen gravel that has few cobbles and no large boulders it might be expected that the T-600 wheel with the parrot-beak picks would have a wear rate of no more than 0.1 tooth per square foot of sawcut, or 0.4 tooth per cubic foot of excavated material. In monetary terms, this is equivalent to about 30 ¢/ft² (sawcut area) and \$1.10/ft³ (excavated volume). In 1 hour of operation with actual cutting going on for 60% of the time, tooth costs might be around \$140 in frozen gravel, so that total operating cost for the machine would be dominated by tooth cost.

TOOTH PATTERNS

Tooth layout in 1972

On both of the machines tested in 1972 the layout and spacing of the cutting teeth (Fig. 4, 5) seemed less than ideal. Apart from the question of bit angle, which was touched on above, some teeth were set directly in the wake of others so that they were doing no work, while some teeth were having their sides scraped by uncut material. There was clearly uneven coverage of the cutting face, and the bits were being loaded unevenly. In considering the last point, it is instructive to consider the chipping depth of the teeth, i.e. the depth to which each tooth penetrates into uncut material during the course of its sweep through the work.

Chipping depth for 1972 Vermeer teeth

In considering the effective depth of bite taken by the teeth, it is somewhat difficult to decide on the number of tracking cutters (n). The 1972 Vermeer wheel was fitted with 10 identical perimeter segments, and on each segment every tooth has a different setting. However, on each side of each segment there were five gauge cutters, and there was very little difference in the setting of individual teeth for each of these groups. There were also three center cutters on each segment, and again there was little difference in the setting of each individual tooth in these groups. The minimum value that can be taken for n is 10, but this is too low for most of the cutters. It would be more realistic to say $n = 50$ for the gauge cutters if they were all uniformly spaced, but in fact there was a gap between each of the 10 groups. In the same way $n = 30$ would be realistic for the center cutters if they were uniformly spaced, but they were actually clumped at the leading end of each segment.

This tooth arrangement means that the lead tooth in each group took a relatively large bite, while the ones following in its wake took smaller bites. Approximate calculation of the bite taken by each type of tooth can be made by assuming the following values.

<i>Center-cutting teeth:</i>	Lead tooth	$n \approx 13$
	Follower teeth	$n \approx 100$
<i>Gauge-cutting teeth:</i>	Lead tooth	$n \approx 40$
	Follower teeth	$n \approx 65$

Calculations will be made here for two cutting conditions:

Cutting depth d	28 in.	(32 in.)
Wheel radius R	42 in.	(42 in.)
Wheel speed f	20 rpm	(35 rpm)
Traverse speed U	60 in./min	(200 in./min)

The first set of values is supposed to represent cutting in frozen gravel, and the second set of values in parentheses represents cutting in frozen silt.

The maximum chipping depth $\ell_{\max}$ is given by*

$$\ell_{\max} = \frac{U}{fn} \times \frac{d}{R} \left(\frac{2R}{d} - 1 \right)^{\frac{1}{2}}$$

and the calculated values are

<i>Center-cutting teeth:</i>	Lead tooth	0.22 in. (0.43 in.)
	Follower teeth	0.028 in. (0.056 in.)
<i>Gauge-cutting teeth:</i>	Lead tooth	0.071 in. (0.14 in.)
	Follower teeth	0.044 in. (0.085 in.)

These calculations lead to a number of questions. First of all, it is hard to see why the teeth were set with an irregular spacing when this results in uneven loading or working of the teeth. Secondly, when the machine was cutting frozen gravel or other strong materials

* See Appendix A for derivation of this and other equations used in the discussion.

Figure 11. Cutter arrangement for 1973 tests with Vermeer saw.

the chipping depth was very small for all teeth except the lead teeth of the center-cutting groups. In principle, bite could be increased by dropping the wheel speed, but this procedure on the Vermeer would result in unduly large bites by the widely spaced lead cutters of the center teeth, with consequent vibration. There is also a matter of vibration arising from inhomogeneity of the material when a "soft," or compliant, device is run at low speed. If the bite is too small, the machine is grinding the rock instead of chipping it. Even in frozen silt, where the machine was traversing at fairly high speed, the bite taken by the follower teeth was small.

Tooth layout in 1973

Original plans for layout of the new teeth in 1973 called for a three-track arrangement with only center cutters and left and right gauge cutters. This seemed reasonable in light of research findings concerning cutter spacing, but after discussion with the Vermeer dealer it seemed safer to adopt the six-track arrangement shown in Figure 11. Nominal gauge was

Figure 12. Modified cutter arrangement adopted after 1973 test season.

reduced to 3.5 in., with an initial gauge (new picks) of 3.7 in. Actually, a number of errors were made during the welding of pick blocks to the wheel rim sector plates, and some teeth were set out of sequence or at angles other than the planned ones.

The test wheel and a copy were used by the machine dealer subsequent to the frozen ground tests, and on the basis of his experience in cutting concrete he recommended an increase of gauge and a change from a 10-segment to a 12-segment arrangement (the latter change being largely for reasons of economy and convenience in stocking spares). The five-track design shown in Figure 12 was then drawn up for the use of the dealer, and it is believed that it proved satisfactory in concrete-cutting operations.

Chipping depth for 1973 experimental teeth

Ignoring the errors that were made in setting some teeth, the 1973 test wheel had 10 tracking cutters ($n = 10$). Taking the same two sets of cutting conditions used in the

calculation of maximum chipping depth for the 1972 teeth, the calculated values for the 1973 arrangement are:

All teeth	0.28 in.	(0.055 in.)
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However, the 1973 machine was geared higher than the 1972 machine, and it was operated most effectively at the highest wheel speed, i.e. 44 rpm. It was also capable of somewhat greater operating depth. The cutting conditions for calculation can therefore be revised to:

Cutting depth d	31 in.	(33 in.)
Wheel radius R	43 in.	(43 in.)
Wheel speed f	44 rpm	(44 rpm)
Traverse speed U	60 in./min	(200 in./min)
No. of tracking cutters n	10	(10)

With these changes the calculated values for maximum chipping depth in the two materials become:

All teeth	0.13 in.	(0.44 in.)
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For the five-track arrangement shown in Figure 12 the chipping depths are 20% lower, i.e. 0.11 and 0.37 in.

POWER AND SPECIFIC ENERGY

Specific energy for sawing

Power output of the machine divided by volumetric cutting rate gives the overall specific energy of the saw, i.e. the energy expended per unit volume of material cut. With power expressed in in.-lbf/min and volumetric cutting rate expressed in in.³/min, specific energy is given in in.-lbf/in.³, or lbf/in.². Averages of the overall specific energy values measured in 1972 for the Vermeer are 8.8×10^3 lbf/in.² for gravel and 2.2×10^3 lbf/in.² for silt. Corresponding values from the 1973 tests are 4.7×10^3 lbf/in.² for gravel and 1.8×10^3 lbf/in.² for silt, i.e. improvements over the 1972 results by about 50% and 20%.

The above values give overall specific energy for the machine. The process specific energy for the saw itself is the actual net power consumed by the saw divided by the volumetric cutting rate. Perhaps the simplest way of defining net power is to take the difference between the total power consumption for cutting and the "windmilling" power for whirling the disc and propelling the machine. Windmilling power was not measured, but it can be roughly estimated: the belt and chain drive to the wheel probably dissipated about 10% of the available power, while the track drive system may have accounted for another 10%. Thus process specific energy values ought to be 20% or more lower than the values given above for overall specific energy.

A dimensionless performance index for the cutting process can be obtained by normalizing specific energy with respect to the uniaxial compressive strength of the material being cut (Mellor 1972). Taking average values of bulk compressive strength as 1500 and 1000 lbf/in.² for the gravel and silt respectively, and taking estimated values of net process specific energy, the performance indices for the 1973 wheel are 2.5 for gravel and 1.4 for

silt. These are not particularly good values, as very efficient rock drills can get down to about 0.3, while the best modern tunnel boring machines can achieve values as low as 0.1 under ideal conditions. However, it should be recognized that the value for frozen gravel is deceptive, in that it really represents a certain amount of rock cutting in material that could have compressive strength in the range 15,000 to 20,000 lbf/in.²

Once a realistic value of specific energy has been determined for a disc saw in a certain kind of material it becomes possible to calculate the power requirements for any new saw after the performance specifications have been decided. The required power is simply the specific energy multiplied by the volumetric cutting rate.

Specific energy for bulk excavation

While the specific energy for sawing is not outstandingly attractive, a saw has the capability of achieving much better specific energy values for certain types of bulk excavation. If a saw is used to cut suitably spaced parallel kerfs, the intervening ribs of uncut material can usually be broken out with almost negligible energy consumption. If the width of the sawcut is w and the center-to-center spacing of adjacent kerfs is W , the effective specific energy for bulk excavation E_e is related to the specific energy for sawing E_s by

$$E_e/E_s \approx w/W$$

in which the energy required for breaking off the uncut rib is neglected. There is, however, a practical limit to W , as the uncut rib has to be sufficiently narrow to snap off easily and reliably at its base. To help determine an optimum value of W , some model tests were made with frozen silt (see Appendix B), and it was decided that a working value of $(W - w)/d = 0.5$ could be adopted, where d is the depth of the sawcut.

Taking $w = 3.7$ in. and $d = 32$ in. as representative values for the 1973 tests, $W = 16$ in. and effective values of specific energy for bulk excavation E_e could therefore have been:

$$E_e \approx \frac{3.7}{19.7} E_s$$

Taking overall values of E_s based on gross machine power

$$E_e \approx \frac{3.7}{19.7} \times 4.7 \times 10^3 = 8.83 \times 10^2 \text{ lbf/in.}^2 \quad \text{for gravel}$$

$$E_e \approx \frac{3.7}{19.7} \times 1.8 \times 10^3 = 3.38 \times 10^2 \text{ lbf/in.}^2 \quad \text{for silt.}$$

Process values based on net cutting power could be taken as about 20% lower than the above values.

These values of E_e are fairly attractive, although they are not yet competitive with the best values of specific energy that have been attained by large rippers working in favorable conditions.

TOOTH AND WHEEL FORCES

Forces acting on individual cutting teeth and on the cutting wheel as a whole have previously been analyzed (Appendix A), and values for the Vermeer saw can be estimated from the resulting equations.

Resultant tangential force acting on the rim of the disc

Wheel torque can be estimated from the shaft power and the rotational speed, and from the torque a tangential rim force F_t can be obtained. If shaft power is taken as 80% of the rated power at the governed engine speed, F_t for a wheel speed of 44 rpm is 2070 lbf.

Tangential tooth forces

The gross rim force F_t is distributed among the working cutting teeth according to the layout of the teeth on the wheel and to the positions of individual teeth relative to the work. The maximum-time-averaged value of the tangential tooth force, $f'_{t\max}$, is reached as the individual tooth takes its deepest bite on exit from the work.* Neglecting differences of loading between gauge cutters and center cutters, $f'_{t\max}$ for the 1973 wheel can be estimated as

$$f'_{t\max} = 0.133 F_t = 275 \text{ lbf} \quad \text{for } d/R = 0.75$$

$$f'_{t\max} = 0.336 F_t = 696 \text{ lbf} \quad \text{for } d/R = 0.15.$$

It should be noted that tooth force increases as cutting depth decreases when full power is being supplied to the wheel (F_t has to be shared among fewer teeth at smaller cutting depths), so that there is more danger of breaking teeth when the wheel is making shallow cuts.† Caution might dictate a reduction of power when the saw is sumping-in for the start of a run.

Radial tooth forces

Within the normal range of operating conditions, the radial component of tooth force can be taken as proportional to the tangential component at any given time. The ratio of radial to tangential force components K is typically about 1 for a sharp new pick with adequate relief angle, but it can rise to values as high as 4 when the pick has become blunted by wear.

Wheel axle forces

The axle forces on the wheel can be expressed as horizontal and vertical components H and V respectively. For a given torque level, H and V depend on the relative cutting depth d/R and the tooth characteristic K . Some calculated values of H and V for "upmilling" are given in Table XI.

* Time-averaged values are probably realistic for a very compliant cutting system, but with a "rigid" system peak forces could be almost an order of magnitude higher than the mean values.

† This assumes a compliant system and the potential to draw full torque.

Table XI. Estimated values of axle forces for various cutting conditions.

Relative cutting depth d/R	Tooth sharpness factor K	Dimensionless horizontal force component H/F_t	Dimensionless vertical force component V/F_t	H for $F_t = 2070$ (lbf)	V for $F_t = 2070$ (lbf)
0.75	1.0	1.34	-0.09	2780	-191
0.75	2.0	2.06	0.53	4260	1100
0.75	4.0	3.49	1.78	7230	3690
0.15	1.0	1.28	0.57	2650	1180
0.15	2.0	1.64	1.49	3390	3090
0.15	4.0	2.35	3.34	4870	6920

The important things to note here are the change in relative importance of horizontal and vertical components with cutting depth, and the general increase of force levels as the bits wear. When cutting deep with worn bits, the wheel has to exert a high downthrust, and at the same time the tracks have to provide high horizontal force. According to Table X, track slip occurred on some surfaces (snow-covered ice) at a drawbar coefficient of 0.38. Taking a somewhat simplistic approach and subtracting the vertical axle force V from the vehicle weight to obtain an effective weight (i.e. neglecting the moment), this means that with $V = 3690$ lbf the vehicle could reach the limit of its tractive thrust at 6000 lbf, which is below the required horizontal thrust $H = 7230$ lbf. Thus the tractive capabilities of the vehicle could set a performance limit under some circumstances.

It might be noted that the operating practice of increasing wheel speed (selecting higher gear) to smooth out jerky cutting amounts to decreasing the value of F_t at full engine power, thereby lowering tooth and axle forces.

CONCLUSIONS

A large disc saw of adequate strength and rigidity is capable of cutting most types of frozen soils at reasonably high rates when fitted with suitable teeth. This capability could be useful where precise control of excavations is necessary or desirable. Some existing commercial disc saws are satisfactory in general mechanical design, but the cutting teeth normally fitted are unsuitable for work in coarse frozen gravels, concrete with hard aggregate, or other high strength materials. The latter problem has been fully overcome by developments at CRREL, although further optimization seems possible. In terms of performance, energetics and economics, development of disc saw attachments for engineer tractors might be justifiable.

On the basis of tests performed so far, process specific energy for sawing is around 4×10^3 lbf/in.² for well-bonded coarse frozen gravel, and around 1.5×10^3 lbf/in.² for well-bonded compact frozen silt. Overall specific energy based on gross power depends on the details of machine design, but for a reasonably efficient machine using a mechanical transmission, overall values would probably be 10% to 20% higher than the numbers given above. For bulk excavation by the kerf-and-rib technique, effective specific energy can be reduced appreciably; with a sawcut depth/width ratio of 8, effective specific energy is lower than the sawing energy by a factor of 5.

Existing commercial saws are capable of cutting slots 30 in. or more deep at linear speeds around 15 ft/min in frozen silt, and around 5 to 6 ft/min in frozen gravel. Performance limits on cutting rate tend to be set by force limitations rather than by power inadequacies, and force levels are in turn strongly influenced by the design and condition of the cutting teeth. As the teeth are blunted by wear, the horizontal force requirements may exceed the tractive capabilities of the carrier vehicle on certain kinds of running surfaces.

The standard cutting teeth used by all makers of commercial saws at the time of the CRREL tests gave acceptable cutting performance in terms of rate and specific energy, but their durability was judged to be completely unacceptable for work in frozen gravel. Wear rate in well-bonded coarse frozen gravel appeared to be equivalent to loss of a complete set of teeth in 40 ft³ of cutting. Modifications developed at CRREL during the course of the test program produced a dramatic improvement in cutter durability, wear rate for the wheel dropping by a factor greater than 10, and possibly by a factor of 100, without any increase in cost. This development, which might be considered the major achievement of the project, brings cutter costs to a tolerable level, although cutter cost could still dominate the total machine operating cost for continuous working in coarse frozen gravel.

If necessary, disc saw attachments for military engineer tractors could be developed. Operating characteristics, power requirements, and force levels are probably compatible with an attachment mounting one or two discs and driving through a mechanical transmission from the power takeoff of a crawler tractor that has hydraulic track drive.

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**APPENDIX A. REPRINT OF CRREL TECHNICAL NOTE
“MECHANICS OF TRANSVERSE ROTATION CUTTING DEVICES”**

The following document was produced during the course of tests on disc saws and milling drums in order to facilitate data analysis and to provide a scheme for systematic design of new experimental machines. It has now been superseded by a more detailed and comprehensive mechanical analysis, but the latter is not yet ready for publication, and therefore the original work is reprinted to illustrate the thinking underlying the disc saw program.

APPENDIX A

CRREL TECHNICAL NOTE

MECHANICS OF TRANSVERSE-ROTATION CUTTING DEVICES

by

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June 1972

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MECHANICS OF TRANSVERSE-ROTATION CUTTING DEVICES

by

Malcolm Mellor

Introduction. This note explores simple basic relationships for the mechanics of rotary cutting devices that advance, or traverse, in a direction that is normal to the axis of rotation. The immediate motivation for the work arises from interest in the design and operation of disc saws and rotary excavators for cutting rock and frozen ground, but the same principles apply to a wide range of cutting and milling tools, including certain types of rotary snowplows. The analysis does not attempt to cover the mechanics of devices that advance in a direction parallel to the axis of rotation, as in typical drills and full-face tunnel boring machines.

In the first section, kinematic relations derived from tool geometry and drum velocities are considered. In the second section some simple dynamic relations are derived. In the third section some relevant energy and power considerations are outlined.

I. KINEMATIC RELATIONS

Performance limits for rotary excavators and tunnel boring machines are commonly set by dynamic or energetic limitations, but when such machines are working in relatively weak materials it is quite possible for performance limits to be set by kinematic factors. For example, at a certain mine visited by the writer a boring machine was achieving high driving rates in a rather weak rock, but the cutters of the machine were suffering excessive wear; a simple calculation showed that the inherent geometric limits of the machine were being exceeded, and consequently the whole body of each cutter was being forced into uncut rock. The primary purpose of this section is to examine kinematic relations and limits for transverse-rotation cutting machines. The secondary goal is to explore the effects of various tool arrangements on the cutting process.

This analysis is applicable to machines that have a cutter drum, bucket wheel, or disc saw that rotates about a horizontal axis set normal to the direction of travel. The direction of rotation is usually such that the cutters move upward on the forward face of the drum. In normal operation the base of the drum is set at some fixed distance below base level of the running wheels or tracks, and the machine travels forward as the drum rotates. The first problem is to develop relationships between machine travel speed, excavation rate, drum speed, drum diameter, cutter height, cutter spacing, and cutting depth.

Chip thickness or "depth of bite"

Consider a cutting tool (drag bit) at position A in Figure 1. The drum is rotating at f revolutions per unit time, and if there are n cutters evenly spaced around the periphery at a given cross section, then the cutter at A will be replaced by the following cutter at the same angular position after a time interval of $1/fn$. The whole drum is moving forward horizontally at velocity U , and therefore in the time interval $1/fn$ it will move a horizontal distance U/fn . Thus a cutter newly arriving at A will extend into uncut material a horizontal distance U/fn . The radial penetration into uncut material, i.e. the theoretical radial chip thickness ℓ , is

$$\ell = \frac{U}{fn} \sin \theta \quad (1)$$

and the maximum value of chip thickness for a given set of drum conditions is

$$\begin{aligned} \ell_{\max} &= \frac{U}{fn} \sin \theta_{\max} = \frac{U}{fn} \sin \left\{ \cos^{-1} (1 - d/R) \right\} \\ &= \frac{U}{fn} \cdot \frac{d}{R} \left(\frac{2R}{d} - 1 \right)^{\frac{1}{2}} \end{aligned} \quad (2)$$

where d is cutting depth and R is overall drum radius (measured to cutter tips).*

In practice, maximum chipping depth $\ell_{\max}$ is limited geometrically by the projecting length of the cutters (from the drum face or from broad shanks),

* This is really a simplification for a rapidly rotating multi-cutter drum. A more exact analysis is obtained from consideration of tooth trajectories.

by the radial extent of carbide tips or hardfacing, or by some similar factor. If the maximum working extent of the cutting tool in the radial direction is h , then a kinematic limit is set by the condition

$$\ell_{\max} \leq h$$

i.e.

$$\begin{aligned} \text{or } h &\geq \frac{U}{fn} \sin \left\{ \cos^{-1} (1 - d/R) \right\} \\ h &\geq \frac{U}{fn} \cdot \frac{d}{R} \cdot \left(\frac{2R}{d} - 1 \right)^{\frac{1}{2}} \end{aligned} \quad (3)$$

Condition (3) is shown in dimensionless form in Figure 2, where $\ell_{\max}/h$ is plotted against d/R with (U/fnh) as parameter. Since in normal practice $d/R < 1.0$, it can be seen that the kinematic limit for machine performance is only likely to be reached when $U/fnh > 1.0$. For example, if a machine is being run with its drum set to a cutting depth equal to 25% of the drum diameter, $d/R = 0.5$ and from condition (3) the limiting value of U/fnh (for $\ell_{\max}/h = 1.0$) is 1.15. If $f = 30$ rev/min, $n = 24$ tracking cutters, $h = 0.1$ ft, the limit of forward speed set by kinematic factors is 83 ft/min.

For one machine of current interest (the UMM Mark III planer) the following values might apply: $f_{\max} = 90$ rev/min, $R = 19.9$ in., $d_{\max} = 9$ in., $n = 3$, $h_{\max} = 0.5$ in. or 1.0 in. (depending on the cutters used). Thus, if there is adequate drum power, the maximum forward speed at maximum cutting depth is 16 ft/min or 32 ft/min, depending on which cutters are used.

Excavation rate as a function of cutting depth

It is also interesting to consider the relation between maximum excavation rate and cutting depth. Excavation rate per unit drum width, $\dot{V}$, is

$$\dot{V} = U d \quad (4)$$

The maximum excavation rate at any given cutting depth is given in dimensionless form by

$$\begin{aligned} \frac{\dot{V}}{\dot{V}_1} &= \left(\frac{U}{f n h} \right)_{\max} \frac{d}{R} = \frac{1}{\sin \left\{ \cos^{-1} (1 - d/R) \right\}} \cdot \frac{d}{R} \\ &= \left(\frac{2R}{d} - 1 \right)^{\frac{1}{2}} \end{aligned} \quad (5)$$

where $\dot{V}_1$ is the volumetric excavation rate at $d = R$. Eq (5) is plotted in Figure 3.

Lateral tool settings

On wide cutter drums, the teeth are not usually set in straight lines along generators like the vanes of a paddle wheel, but instead they are set in helical patterns. One important reason for this is that serious vibrations would be set up by simultaneous impact of a row of teeth. It can also be seen from a consideration of tooth trajectories that vibrations are caused by intermittent action. Another reason is that helical setting can provide scrolls for lateral transport of cuttings. However, from the standpoint of cutting efficiency it seems that one of the most important reasons for staggering the teeth is formation of an extra free face for breakage.

Suppose that m teeth are spaced uniformly across the width of a drum along one single wrap helix, and assume that the effective cutting width of one tooth is $1/m$ times the drum width. Since each tooth lags behind its neighbor, and the drum is moving forward horizontally, the face of the cut will be stepped, with $m-1$ "steps," or new free faces, in the vertical plane and along the direction of travel. If there is more than one single wrap helix on the drum, the first steps cut by one helix will be chipped away by the following helix while the last steps are still being cut, and thus the number of steps in existence at any given instant will be less than $m-1$.

The time interval t_1 between successive passes of tracking cutters through a given angular position θ is:

$$t_1 = 1/fn \quad (6)$$

(note that n can now be identified with the number of helices). The time interval t_2 between passes of diagonally adjacent teeth through angular position θ is:

$$t_2 = \frac{p}{2\pi R f} = \frac{s \tan \alpha}{2\pi R f} \quad (7)$$

where p is the peripheral distance between diagonally adjacent teeth, s is the lateral spacing between teeth, and α is the helix angle (measured from a generator of the drum surface). If $t_1 = t_2$, no step will be formed; in general, the number of steps in existence at any given time, N , is:

$$N = t_1/t_2 - 1 = \frac{2\pi R}{ns \tan \alpha} - 1 \quad (8)$$

A fractional step means one smaller than the full step depth for a single helix, while a negative result means that the steps form in the opposite direction from the steps formed by a single helix. If the helices are "single wrap," i.e. each helix makes one complete revolution in the width of the drum, then

$$(m - 1) s \tan \alpha \approx 2\pi R \quad (9)$$

and

$$N \approx \frac{(m-1)}{n} \quad (10)$$

More generally, if each helix makes M revolutions in the width of the drum

$$N \approx \frac{(m-1)}{Mn} \quad (11)$$

The approximation in these equalities relates to the exact disposition of the outermost cutters.

It seems important to design the drum so that there is always at least one full step in the positive or negative direction, i.e. so that

$$\text{or } \left. \begin{array}{l} \left| \frac{(m-1)}{Mn} \right| \geq 1 \\ \left| \frac{2R}{ns \tan \alpha} \right| \geq 1 \end{array} \right\} \quad (12)$$

If one or more steps are formed, the radial height of the step between the tracks of diagonally adjacent cutters l_s is

$$l_s = \frac{s U}{2\pi R f} \tan \alpha \sin \theta \quad (13)$$

It is probably desirable to have the step height at least equal to the chipping depth for the following tool (as given by eq. 1), i.e.

$$l_s / l \geq 1$$

or

$$\frac{s n \tan \alpha}{2\pi R} \geq 1 \quad (14)$$

or

$$\frac{Mn}{m} \geq 1$$

It is also likely that there is an optimum ratio of step height to step width (step width is the effective cutting width for one tool, assumed here to be equal to the lateral tool spacing s). However, while step width is a constant for a given drum, step height varies with angular position of the tool, so that the ratio must vary from zero at $\theta = 0$ to a maximum value at $\theta = \cos^{-1} (1 - d/R)$. Since brittle materials tend to break into equant fragments, an intuitive guess might be that step height should be approximately equal to step width, i.e. $l_s \approx s$. This condition satisfies the requirement for minimum specific surface if s is allowed to vary (in the design stage) so as to keep the swept volume for each tooth constant (the consideration changes if s is fixed). If it is assumed that $l_s = l$, and also that the maximum values of l_s or l should be greater than or equal to s , then eq.(2) gives the condition

$$s \leq \frac{U}{fn} \sin \left\{ \cos^{-1} (1 - d/R) \right\}$$

or

$$s \leq \frac{U}{fn} \cdot \frac{d}{R} \left(\frac{2R}{d} - 1 \right)^{\frac{1}{2}} \quad (15)$$

Comparing condition (15) with condition (3), and recognizing that it is unlikely that h would be greater than s , it appears that compatibility is most easily established by taking $h = s$ (a result that can be deduced directly from the assumption that step height equals step width). Thus it seems that, for a suitably designed drum, maximum traverse speed is also optimum traverse speed for most efficient cutting, and the applicable criterion is

$$s = h = \frac{U}{f_n} \sin \left\{ \cos^{-1} (1-d/R) \right\} = \frac{U}{f_n} \cdot \frac{d}{R} \cdot \left(\frac{2R}{d} - 1 \right)^{\frac{1}{2}} \quad (16)$$

Eq. (16) is plotted in dimensionless form in Figure 4.

When cutting tools are arranged on a drum in helical patterns, one-way wrapping of the helices will almost certainly give rise to a net lateral force on the drum, and this will have to be resisted by the drum mountings and the carrying machine. The cutting tools will also tend to wear preferentially on one side. If there is a continuous web between pick boxes, the helices will transport cuttings to one side of the drum and there will be a concentration gradient across the width of the drum. For all of these reasons it seems desirable to consider setting the cutting tools in chevron patterns, i.e. along two sets of helices wrapping in opposite directions from the center section of the drum. In this way there would be zero lateral force, cutters could be interchanged periodically to balance the wear, and material could be transported laterally over a shorter distance, either to both sides of the drum or to the center.

Tooth trajectories

In calculating chipping depth as a function of angular position it is convenient to use an origin of coordinates that moves horizontally with the wheel at velocity U . If the trajectory of each tooth relative to the rock is of interest, then the origin must be fixed relative to the rock.

Consider the motion of a single tooth after it enters the work at a point directly beneath the axle of the wheel. Take as origin the point on the rock where the tooth starts its sweep through the work. After the wheel has rotated through an angular distance θ , the tooth has progressed vertically through a distance $R(1 - \cos \theta)$, and has progressed horizontally a distance $(U \theta / \omega + R \sin \theta)$, where $\omega (= 2\pi f)$ is the angular velocity of the rotor. If the upmilling rotor is more than axle deep in the work, i.e. $d/R > 1$ and $\theta > 90^\circ$, the value of $R \sin \theta$ decreases progressively as θ increases in the second quadrant. Horizontal extent of the tooth trajectory reaches a maximum when $\cos \theta = -U/\omega R$, and the trajectory starts to loop back against the machine's traverse direction when $\omega R \cos \theta$ is numerically greater than U . If the wheel is climb milling, i.e. rotating in the opposite sense to that shown in Fig. 1, and $d/R < 1$, then the tooth trajectory is the same as the last part of the trajectory for an upmilling rotor with $d/R = 2$ (slot milling); if the point of exit is taken as origin, the upmilling expressions hold with $(180^\circ - \theta)$ substituted for θ . The equation of the locus for the tooth tip on an upmilling rotor (taking point of entry as origin) is

$$\left. \begin{aligned} x &= \frac{U \theta}{\omega} + R \sin \theta \\ y &= R(1 - \cos \theta) \end{aligned} \right\} \quad (17)$$

Fig. 5 shows tooth trajectories for a range of values of $U/2\pi fR$, for both upmilling and climb milling. When the value of $U/2\pi f$ is high, the tooth tends to make a long forward sweep, but when $U/2\pi f$ is small the tooth comes close to sweeping through a circular arc. Typical values of $U/2\pi fR$ for rock-cutting machines are in the range 0.01 to 0.1, so that the tooth sweep is almost circular.

If $\theta_{\max} \geq 2\pi/n$ a complete tooth trajectory can be traced out by a working cut, but if $\theta_{\max} < 2\pi/n$ the trace left by one tooth is being cut by the next tooth before the complete sweep is finished. It is easy to see that serious vibrations would be set up with $\theta_{\max} \geq 2\pi/n$ if some damping arrangements were not made. Probably the simplest way of smoothing out these potential vibrations is to set laterally adjacent cutters along helical paths.

Tooth relief angles

When a tooth is cutting the surface left by the previous tooth pass it follows a path that is not perfectly parallel with the surface to be cut. This effect can be seen by taking two identical tooth trajectories from Fig. 5 and setting them apart by a horizontal distance equivalent to the horizontal travel of the machine in the time taken for successive passes of tracking teeth through the same angular position.* If the shoulder of the cutting tool behind the cutting edge is exactly tangential, it will grind against uncut material and impede penetration of the cutting edge. Thus the shoulder

* This exercise illustrates a theoretical shortcoming of eq (1), as it shows that $\mathcal{L}$ is finite at $\theta = 0$ and $\theta = 180^\circ$.

behind the cutting edge is usually cut back to give a relief angle, or clearance angle, that keeps it clear of uncut material.

The minimum or theoretical relief angle β is given by the difference between the slope of the tooth trajectory and the slope of the tangent to the rotor. The slope of the tooth trajectory is obtained from eq. (17):

$$\frac{dy}{dx} = \frac{\sin \theta}{U/R\omega + \cos \theta} \quad (18)$$

and therefore

$$\beta = \theta - \tan^{-1} \left[\frac{\sin \theta}{U/R\omega + \cos \theta} \right] \quad (19)$$

The critical value of θ where β reaches its maximum value for the swing, given by $\partial\beta/\partial\theta = 0$, is $\cos^{-1}(-U/R\omega)$. This is in the second quadrant, and is never reached when $d < R$. If $d \leq R$, the maximum value of β occurs at $\theta_{\max}$, i.e. $\theta = \cos^{-1}(1 - d/R)$. The following are maximum values of β :

$$\text{Maximum for complete } 180^\circ \text{ cut} \quad \beta = \cos^{-1}(-U/R\omega) - \pi/2 \quad (20a)$$

$$\text{Maximum when } d \leq R \quad \beta = \theta_{\max} - \tan^{-1} \left[\frac{\sin \theta_{\max}}{U/R\omega + \cos \theta_{\max}} \right] \quad (20b)$$

in which $\theta_{\max} = \cos^{-1}(1 - d/R)$.

An example can be worked for the cutting drum considered in the discussion of chip thickness, i.e. taking $f = 90$ rev/min, $R = 19.9$ in., $d_{\max} = 9$ in., and $U = 32$ ft/min. With these values the minimum required clearance angle is 1.6° , to which should be added a further allowance for irregular chipping and intermittent tooth penetration. The machine is actually built with a $7\frac{1}{2}^\circ$ clearance angle.

II. SIMPLE DYNAMIC RELATIONS

In the preceding section, kinematic relations pertaining to design and operation of drum cutters and disc saws were developed. It is also important to consider the dynamics of these rotary tools, since tool force usually sets the upper limit for the strength of material that can be cut economically. Instrumentation of cutting teeth to provide force data is awkward and expensive, since some telemetry is usually required. However, a few useful relations can be developed from very simple dynamic considerations.*

Torque Resistance

The main requirement is for estimates of time-averaged tool forces, and their variation with cutting depth, tooth spacing, etc. Other requirements include investigation of the forces acting on the axle of the drum or wheel.

The torque of a wheel or drum has an upper limit, the stall torque, $T_{\max}$, which may be preset by some torque-limiting device on the machine. The absolute upper limit of $T_{\max}$ is set by the peak power of the drive, $P_{\max}$, and the angular velocity of the wheel or drum, :

$$T_{\max} = \frac{P_{\max}}{\omega} \quad (21)$$

$T_{\max}$ can also be expressed in terms of the maximum time-averaged value of

* A paper on the cutting of rock or frozen soil with disc saws has been published by Nalezny (1971), but since its results are based on unconfirmed assumptions concerning the fundamental cutting mechanism a simpler approach is followed here.

the resultant tangential force acting on the active segment of the periphery of the wheel or drum (F_t):

$$T_{\max} = F_t R \quad (22)$$

where R is the radius of the wheel or drum (see Fig. I-1). Hence, F_t can be written as:

$$F_t = \frac{K P_{\max}}{\omega R} = \frac{K P_{\max}}{2\pi N R} \quad (23)$$

where N is the wheel or drum speed in revolutions per unit time and K is a factor which gives the percentage of peak power at which stall occurs.

To get an ideas of actual magnitudes, calculations can be made for some machines that have been used in recent tests. For the Vermeer saw, $P_{\max}$ can be taken as 78 horsepower, N as 20 rev/min (3rd gear), and R as 3.5 ft. For purposes of illustration, K can be taken as 0.6, i.e. stall at 60% of installed power, since some of the installed power is used for vehicle propulsion and there are losses in drive trains. Thus,

$$F_t = \frac{0.6 \times 78 \times 3.3 \times 10^4}{2\pi \times 20 \times 3.5} = 3,510 \text{ lbf}$$

On some tests of the UMM Mark III planer, limiting drum torque was measured from the hydraulic circuits as 9,400 lbf-ft with a drum of 1.64 ft radius. This indicates a value of $F_t = 5,730$ lbf.

The next step is to estimate how F_t is shared among the teeth that are cutting.

Tooth Forces and Axle Forces

As a first approximation, assume that the active teeth are all equally loaded, i.e. the tangential loading of each tooth remains constant throughout its active sweep. The length of the active perimeter that is involved in cutting, S , is determined by the drum radius R and the cutting depth d :

$$S = R \cos^{-1} (1 - d/R) \quad (24)$$

If all active teeth are equally loaded, the distribution of F_t over the active segment is

$$\frac{F_t}{S} = \frac{K P_{\max}}{2\pi N R^2 \cos^{-1} (1 - d/R)} \quad (25)$$

If there are n tracking cutters on the drum or wheel, and tangential force is uniformly distributed across the width, the maximum value of the time-averaged tangential force per cutter, f_t , is

$$f_t = \frac{2\pi F_t}{m n \cos^{-1} (1 - d/R)} = \frac{K P_{\max}}{m n R N \cos^{-1} (1 - d/R)} \quad (26)$$

where m is the number of rings of tracking cutters set across the width of the drum or wheel (i.e. mn is usually equal to the total number of useful cutters on the drum or wheel). There is a restriction on eq. (26) for wide cutter spacing and shallow cutting depth, i.e. $2\pi/mn \leq \cos^{-1} (1 - d/R)$.

From eq. (26) it can be seen that the peak (stall) load per cutter decreases as the cutting depth increases, and also decreases as the number of cutters on the wheel or drum increases. Actually, it is somewhat unrealistic to assume that all cutters are uniformly loaded, but the resulting

calculation is so simple that it provides a reliable estimate of the general magnitude of tooth forces. The next step in calculation is to account for systematic variation of tangential tooth force through the length of the working stroke.

For the second approximation, ignore the high frequency repetitive force fluctuations that correspond to discrete chipping stages in the cutting sequence, and consider only the systematic variation of tangential force with position in the active sector. It will be assumed that tangential tooth force is directly proportional to chip thickness at any given cutting stage. Since there is virtually no radial, or normal, stress on the uncut material, this is equivalent to assuming that the area of shear surface for each chip is proportional to chip thickness, which is merely an expression of geometric similitude for a two-dimensional situation. The assumption is supported by test data by Barker (1964), who tested full size picks in sandstone.

In the preceding section, where kinematic relations were analyzed, it was shown that an "upcutting" wheel or drum takes a bite that increases the chip thickness from zero at point of entry to a maximum at point of exit (provided that cutting depth is less than the wheel radius, as is usually the case). Thus, under the current assumption, tangential tooth force f_t' would vary from zero at point of entry to a maximum at point of exit, with f_t' proportional to $\sin \theta$ at intervening positions. When tangential force is averaged across the width of the drum or wheel, tangential force per unit angle, p_t , is proportional to $\sin \theta$, and the proportionality constant is

$p_{t_{\max}}/\sin \theta_{\max}$. The value of $p_{t_{\max}}$ can be determined by integrating p_t with respect to θ from 0 to $\theta_{\max}$ and setting the result equal to F_t :

$$\begin{aligned} F_t &= p_{t_{\max}}/\sin \theta_{\max} \int_0^{\theta_{\max}} \sin \theta \, d\theta \\ &= \frac{p_{t_{\max}}}{\sin \theta_{\max}} (1 - \cos \theta_{\max}) = p_{t_{\max}} (2R/d - 1)^{-\frac{1}{2}} \quad (27) \end{aligned}$$

In considering the distribution of p_t among the cutting teeth, the tooth pattern has to be taken into account. If adjacent rings of tracking cutters are staggered relative to each other, they can be considered as equivalent to a single tracking ring provided that there is uniform angular spacing between the teeth. Rings of tracking cutters that run parallel with synchronous teeth (i.e. teeth in different rings lie along common generators) can be assumed to share the tangential force equally. For these reasons, F_t is assumed to be partitioned between m' equivalent rings of parallel cutters, where m' is an integer representing the number of lateral repetitions of synchronous teeth. If there are n cutters in each equivalent ring, one cutter accounts for an angular distance of $2\pi/n$, and the maximum value of f_t' is achieved during the final cut from $(\theta_{\max} - 2\pi/n)$ to $\theta_{\max}$:

$$\begin{aligned} f_{t_{\max}}' &= \frac{p_{t_{\max}}}{m' \sin \theta_{\max}} \int_{(\theta_{\max} - 2\pi/n)}^{\theta_{\max}} \sin \theta \, d\theta = \frac{p_{t_{\max}}}{m'} \left[\cos \theta_{\max} (\cos 2\pi/n - 1) + \sin 2\pi/n \right] \\ &= \frac{F_t}{m'} \left[(R/d - 1)(\cos 2\pi/n - 1) + (2R/d - 1)^{\frac{1}{2}} \sin 2\pi/n \right] \quad (28) \end{aligned}$$

Eq. (28) can only be applied when $\theta_{\max} \geq 2\pi/n$, i.e. when $d/R \geq (1 - \cos 2\pi/n)$.

This is because the equation does not account for intermittent tooth contact. Figure 6 gives the trends of eq. (28) in dimensionless form.

Even before any numerical results are calculated, eq. (26) and eq. (28) have some interesting implications. Of particular interest is the indication that teeth are most vulnerable at shallow cutting depths, where the entire stall torque can be thrown on to one tooth if the material being worked is sufficiently resistant. By contrast, when the drum or wheel is cutting deep, the torque resistance is shared by many teeth. This is perhaps only a common-sense deduction, but it does conflict with the intuition of some equipment operators, who are afraid that deep cutting will be more likely to damage the equipment. While there could be other practical considerations, such as bearing problems or drive-train weaknesses, practical experience tends to support the view that cutting teeth are most vulnerable in shallow cuts (in tests with the Vermeer saw, tooth breakage was most severe in the shallowest cut). The practical lesson is that a drum or wheel working in strong material should "sump-in" under reduced power; full power can be applied when the wheel has been sunk to adequate depth.

Actual tooth loadings depend on the resistance of the material, the radius of the drum, the cutting depth, and the number of teeth. Some idea of relative magnitudes can be gained from Figure 6. To get some idea of absolute values, assume that a disc saw of 3.5 ft radius has 100 teeth arranged in a staggered pattern that effectively forms a single ring ($m' = 1$). If the saw is cutting at 6 in. depth, $R/d = 7$ and so the maximum tangential force on a tooth, $f_{t_{\max}}'$, is 21.5% of the tangential stall force, F_t , which

might be about 3500 lbf for a saw like the Vermeer. Thus $f'_{t_{\max}}$ could be about 750 lbf.

As another refinement to the analysis, it would be desirable to consider the high frequency force fluctuations that occur as discrete chips are formed in brittle material. Barker (1964) looked at this question experimentally, and found that when cutting sandstone at constant chipping depth with heavy picks the ratio of peak tangential force to mean tangential force had values of 6.2 and 7.7 for two different pick designs. Thus, in the numerical example given above, the transient peak force reached during chipping pulsations might be about 7 times higher than the mean maximum value, i.e. the absolute peak force could be 5250 lbf.

Other forces that are of interest are the radial components of tooth forces, and the horizontal and vertical components of the force on the axle of the wheel or drum. To make a simple investigation of these forces, another assumption is introduced. This new assumption is that the ratio of radial and tangential components of tooth force remains constant throughout the working stroke, i.e. the ratio is independent of chip depth for the range of chip depths considered. From the results of Barker (1964), this assumption appears to be justified for chip depths up to 0.5 in. when V-face picks and chisel picks are cutting rock.

On each angular increment of the cutting perimeter there acts a force that can be resolved tangentially and radially and expressed as:

$$\left. \begin{array}{ll} \text{Tangential} & p_t d\theta = K_1 \sin\theta d\theta \\ \text{Radial} & p_r d\theta = K_2 \sin\theta d\theta \end{array} \right\} \quad (29)$$

where $K_1 = p_{t_{\max}} / \sin \theta_{\max}$, and K_2/K_1 is the ratio of radial to tangential force. The incremental forces can also be resolved horizontally and vertically and summed to give the horizontal and vertical components of the force on the axle of the wheel or drum:

$$\left. \begin{aligned} H &= \int_0^{\theta_{\max}} (K_1 \sin \theta \cos \theta + K_2 \sin^2 \theta) d\theta \\ V &= \int_0^{\theta_{\max}} (-K_1 \sin^2 \theta + K_2 \sin \theta \cos \theta) d\theta \end{aligned} \right\} \quad (30)$$

Evaluating the integrals, substituting for K_1 from eq. (27), and denoting K_2/K_1 by K_3 :

$$\left. \begin{aligned} H &= \frac{F_t}{2} \left[2 - d/R + K_3 \frac{R}{d} \theta_m - K_3 (1-d/R) (2R/d-1)^{\frac{1}{2}} \right] \\ V &= \frac{F_t}{2} \left[(1-d/R) (2R/d-1)^{\frac{1}{2}} - \frac{R}{d} \theta_m + K_3 (2-d/R) \right] \end{aligned} \right\} \quad (31)$$

in which $\theta_m = \cos^{-1}(1-d/R)$. The trends of eq. (31) are shown in Figure 7 for two different values of K_3 .

It appears that for an upmilling wheel or drum the horizontal axle force H varies within fairly narrow limits as d/R varies. The vertical axle force V decreases continuously as cutting depth increases, and it becomes negative when a certain cutting depth is reached. This is a very interesting feature, as it means that, for a given value of K_3 , there is a value of d/R at which no vertical thrust or reaction is required. It might be mentioned that in

some tests with the UMM Mark III Planer, the operator reported that he had reversed thrust on the vertical travel actuators when the 19.5 in. diameter drum was set at a depth of 9 in. (i.e. $d/R = 0.46$).

For a wheel or drum that is climb milling, the horizontal force is a forward driving force rather than a resistance when d/R is less than about 0.64. At greater drum depths the horizontal force is a resistance to forward motion. In climb milling the vertical force is always positive, and comparable in magnitude to the horizontal forces that are experienced in upmilling.

According to the assumptions made here, the time-averaged values of radial tooth forces are proportional to the corresponding tangential forces, K_3 being the proportionality constant. Barker (1964) measured peak radial* forces relative to time-averaged radial forces, finding ratios of 4.7 and 6.2 for different pick designs.

The tooth force calculations for the disc saw example considered in the discussion of tangential tooth forces can now be completed. If the maximum value of time-averaged tangential force is 750 lbf, the corresponding radial component is $K_3 \times 750$ lbf, where K_3 might have values between 0.7 and 1.0. Taking 5.5 as the ratio of peak fluctuating force to time-averaged force for the radial direction, the absolute peak values for radial force would be $K_3 \times 5.5 \times 750$ lbf, i.e. $K_3 \times 4125$ lbf. The time-averaged resultant force is 1060 lbf for $K_3 = 1$, and 915 lbf for $K_3 = 0.7$. The absolute peak resultant

* Due to a difference in terminology, these were called normal forces in Barker's paper.

force is 6680 lbf for $K_3 = 1$, and 5990 lbf for $K_3 = 0.7$, assuming that peak values of radial and tangential components coincide. These are very substantial forces, but they are well within the range of forces that heavy picks withstood during Barker's experiments.

The value of the horizontal force H is important in determining the traction requirements for the carrier vehicle. On a firm surface, where the tracks or wheels of the carrier vehicle do not sink appreciably, most of the external tractive resistance is imposed by the cutter unit. According to the analysis made here, the maximum horizontal cutter resistance for a machine like the Vermeer T-600 might be around 5000 lbf (assuming stall at 60% of installed power). This machine weighs nearly 17,000 lbf, and on favorable ground (firm but penetrable by grousers) it might have a maximum drawbar coefficient of about 0.4, i.e. it might be able to overcome a horizontal resistance of 6800 lbf. However, on icy, muddy, or other unfavorable surfaces the drawbar coefficient would drop considerably, and performance would then necessarily be limited by vehicle traction. This kind of problem is likely to be more acute with carriers that run on wheels rather than tracks.

III. ENERGY CONSIDERATIONS

Power Distribution

A rotary cutter such as a disc saw or a milling drum consumes energy in overcoming torque resistance, and the power required to supply this energy, P_R , is

$$P_R = T\omega = F_t R \omega = 2\pi N R F_t = U_R F_t \quad (32)$$

where T is the torque on the wheel or drum, ω is angular velocity, R is the radius of drum or wheel, N is the number of revolutions per unit time, F_t is the resultant tangential force on the perimeter of the wheel or drum, and U_R is linear tool speed. If there is an independent drive on the wheel or drum, P_R can be measured or estimated from the power of the drive.

Energy is also required to move the rotary unit forward horizontally against a resistance. The power required to furnish this energy, P_H , is

$$P_H = H U_H = U_H F_t \phi(d/R) \quad (33)$$

where H , the horizontal resistance, is the horizontal component of force acting on the axle of the wheel or drum, U_H is the horizontal advance speed of the machine. In the previous section it was shown that $H = F_t \phi(d/R)$, and the function $\phi(d/R)$ was derived and plotted.

It is interesting to know how power is partitioned between the rotary drive and the horizontal thrust:

$$P_H/P_R = \frac{U_H}{U_R} \phi(d/R) \quad (34)$$

Values of $\phi(d/R)$ are given in Fig. 7 of Section II; representative values for $d/R < 1$ might lie in the range 1.15 to 1.35% ^{for an upmilling rotor.} Tool speed U_R is typically in the range 300 to 900 ft/min for drag bits, while traverse speed U_H is typically in the range 2 to 20 ft/min. Thus P_H is likely to be less than 9% of P_R .

There are other resistance forces that have to be overcome by the carrier vehicle, whether the rotary unit is cutting or not. The first is the internal rolling resistance of the vehicle, i.e. the resistance that would be measured in towing the vehicle on a hard surface. For a tracked vehicle in ordinary working condition (some dirt in the tracks), a resistance coefficient of 0.1 is probably realistic, while for a wheeled vehicle the corresponding value is probably about 0.02. The power needed to meet this resistance, P_1 , is

$$P_1 = C_r W U_H \quad (35)$$

where C_r is the resistance and W is the gross weight of the vehicle. For a 17,000 lbf weight tracked vehicle traveling at 20 ft/min, 1 horsepower is needed to overcome the rolling resistance, and this is not likely to be much over 1% of the installed power.

If the vehicle carrying the rotary unit has to climb, or to overcome sinkage resistance in soft ground, then additional power demands are made. The power required for slope climbing can be calculated by substituting into Eq. (35) the sine of the slope angle for C_r . For the example just worked, the same result would be given by having the machine climb a 6° slope.

The effective resistance created by soft ground is much more variable and complicated to assess, but if cutting equipment is being used on the surface layers it seems reasonable to assume that the ground will be firm.

Efficiency

The specific energy of a rotary cutting unit, E_s , can be expressed as the energy used to cut unit volume of material, which is equivalent to power consumption P_R divided by volumetric excavation $\dot{V}$:

$$E_s = \frac{P_R}{\dot{V}} \quad (36)$$

When consistent units are adopted, e.g. P_R in ft-lbf/min and $\dot{V}$ in ft³/min, the resulting value of E_s has the dimension of a stress, e.g. ft-lbf/ft³ = lbf/ft².

It is to be expected that E_s will vary with the properties of the material that is being cut, and some method for taking account of material properties is required. In the fields of rock drilling and tunnel boring it has been found that there is a good correlation between specific energy consumption for a given cutting process and the uniaxial compressive strength of the material being cut, and so there has arisen a practice of normalizing specific energy with respect to compressive strength. A rationale for this empirical procedure has been given by Mellor (1972). Thus a dimensionless performance index I_p can be expressed as

$$I_p = E_s / \sigma_c \quad (37)$$

where σ_c is the uniaxial strength of the material being cut. It should be

understood that I_p is not a highly exact number that can be used for close critical comparisons of machines and processes, but rather a broad index that is useful in making performance estimates and general comparisons.

Low values of I_p indicate high efficiency in a machine or a cutting process. In rock drilling and in laboratory rock cutting tests, values of I_p of the order of 0.3 are regarded as very good. Modern tunnel boring machines sometimes achieve slightly better values that approach 0.1.

Some care has to be taken in calculating and interpreting values of I_p . In eq. (36), only P_R is used as input power, whereas P_H should also be used for an exact calculation or in cases where high traverse rates prevail. Sometimes, values of E_s are calculated on the basis of total machine power rather than power utilized for cutting, and this inflates the values of E_s and I_p . Major problems can arise if inappropriate values are taken for σ_c . Poor testing technique and inadequate sampling procedures often result in incorrect values for σ_c . Furthermore, the bulk strength of some materials is an unsuitable strength index; e.g. when small cutters are working in a coarse conglomerate they tend to be affected more by the strength of the cobbles than the overall strength of the rock, which may be only weakly cemented.

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- Nalezny, C. L. (1971) Cutting rock or frozen soil with a circular saw, Journal of Terramechanics, vol. 8, no. 1, p. 23-40.

Figure 1. Symbols used in Section I.

Figure 2. Dimensionless plot of chipping depth as a function of drum depth.

Figure 3. Dimensionless plot of maximum excavation rate as a function of drum depth.

Figure 4. Dimensionless plot giving optimum ratio of traverse speed to drum speed as a function of drum depth.

Figure 5. Tooth-tip trajectories for upmilling and climb milling rotors.

Figure 6. Dimensionless plot of maximum tangential tooth force as a function of drum depth.

Figure 7. Dimensionless plot of horizontal and vertical components of axle force.

- A - Upmilling, $K_3 = 1.0$, H/F_t
- B - Upmilling, $K_3 = 1.0$, V/F_t
- C - Upmilling, $K_3 = 0.7$, H/F_t
- D - Upmilling, $K_3 = 0.7$, V/F_t
- E - Climb milling, $K_3 = 1.0$, H/F_t
- F - Climb milling, $K_3 = 1.0$, V/F_t
- G - Climb milling, $K_3 = 0.7$, H/F_t
- H - Climb milling, $K_3 = 0.7$, V/F_t

H is positive when thrust applied by machine is in direction of travel. V is positive when thrust applied by machine is downward.

APPENDIX B. MODEL TESTS ON KERF-AND-RIB EXCAVATION

There are a number of rule-of-thumb guides covering the depth and spacing of adjacent cuts when kerf-and-rib breakage is being used in rock, but some simple model tests were made in order to check the required geometry.

The models were made by casting blocks of frozen silt in mould-boxes, using spacer strips to form slots at various spacings. The silt was compacted by vibration while still dry, distilled water was added to bring it to saturation water content, and the block was frozen at -10°C . Each of the slots was 3 in. deep and 0.375 in. wide, giving a 1/10 scale model of a typical sawcut. Ribs between the slots had widths of 1.0, 1.5, 2.0 and 3.0 in. The blocks were brought temporarily to -1°C to relax any possible freezing strains, and were then cooled slowly back to -10°C for testing.

Each rib was broken by applying an impulsive force along one top edge in a direction approximately parallel to the surface plane. A strip of aluminum, 0.5 in. X 3 in. in cross section, was set in contact with the edge of the rib, and the mid-point of the aluminum bar was then struck by a swinging hammer.

The 1.0- and 1.5-in.-wide ribs broke consistently at the base. The 2.0-in.-wide ribs sometimes broke at the base, but sometimes broke along planes inclined up from the base. The 3-in.-wide ribs never broke cleanly at the base.

These crude tests indicated that a width/depth ratio of 0.5 was about the maximum value for consistent breakage along the base of the rib.