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SOME OBSERVATIONS ON THE DENSIFICATION OF ALPINE SNOW COVERS

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Charles M. Keeler

JULY 1967

U.S. ARMY MATERIEL COMMAND
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PREFACE

This paper constitutes an interim report accomplished in conjunction with the U. S. Army Cold Regions Research and Engineering Laboratory's (USA CRREL) project on Mountain Snow Research (Snow and Ice Branch, Research Division). This project was supported by the Director's In-House Laboratory Independent Research Program.

The work of Dr. W. F. Weeks and D. L. Alford, who gathered the field data from Goose Lake, Montana, in 1965, is acknowledged. The author would like to thank W. K. Boyd, Chief Engineer, for his support of the project and J. A. Bender, Chief, Research Division, A. J. Gow, R. McGaw, and M. Mellor for their critical comments.

USA CRREL is an Army Materiel Command laboratory.

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Through pit measurements on selected deep seasonal snow covers some observations on the densification rates of dry snows are made. The variation between rates has been compared with such physical characteristics of the snow as temperature, grain size and loading rate. The rate of densification does not appear to be affected by temperature in this temperature range (-1 to -10C) but it is inversely proportional to grain size and sensitive to rates of loading during the formative stage of any particular snow layer.

Values of compressive viscosity vary from 10^6 to 10^9 g cm^{-2} sec which is an order of magnitude less than the lowest values for polar snow.

Plots of specific volume against overburden reveal a sharp discontinuity at a specific volume of about 3.0 cm^3 g^{-1} (density = .33 g cm^{-2}). The persistence of this discontinuity from location to location indicates that it may reflect a real phenomenon. It is suggested that it may be accounted for by extremely high strain rates at low densities.

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INTRODUCTION

A number of papers have been written which attempt to formalize density-depth and density-time relationships in the perennial snows of the polar regions (Anderson and Benson, 1963; Bader, 1963; Kojima, 1964). In the polar regions it is possible to describe these relationships with some degree of accuracy as the necessary assumptions of a constant accumulation rate and a constant annual temperature cycle are reasonably valid. It should be noted, however, even for these snows, that the various coefficients appear to have only local applicability. Density-time relationships in dry seasonal snow covers are considerably more complex due to widely varying accumulation rates, variable diurnal temperature cycles and the effects of metamorphism. While a few generalizations on Alpine snow covers can and have been made (Bader, 1939; Kojima, 1956) the observations presented here point to some of the many variables which prevent a universal solution. The data used have been taken both from the literature and from recent USA CRREL field investigations. Table I shows the location and references for all data. The raw data can be found in Appendix A.

Table I. Sources of data.

Location	Date	Reference
Alta, Utah	1954-55	Atwater and LaChapelle (1956)
Berthoud Pass, Colorado	1957-58	Borland (1958)
Bridger Bowl, Montana	1966	This report
Goose Lake, Montana	1965, 1966	Alford and Weeks (1965) and this report
Hokkaido, Japan	1955	Yoshida _et al_. (1956)
Weissfluhjoch, Switzerland	1936-37	Bader _et al_. (1939)

FIELD RELATIONSHIPS

The relationship between density and time for several Alpine type snow covers is plotted in Figure 1. A least squares analysis of these data indicated that they could be best fitted by an exponential function:

$$(\rho_f - \rho_t) = (\rho_f - \rho_0)e^{-kt} \qquad (1)$$

where ρ_0 is initial density (at t = 0), ρ_f is final density, ρ_t is the density at any specified time, t is time, and k is a coefficient with the dimension of time^{-1}. The values of k varied between 1 and 3 x 10^{-2} time^{-1} and the coefficients of

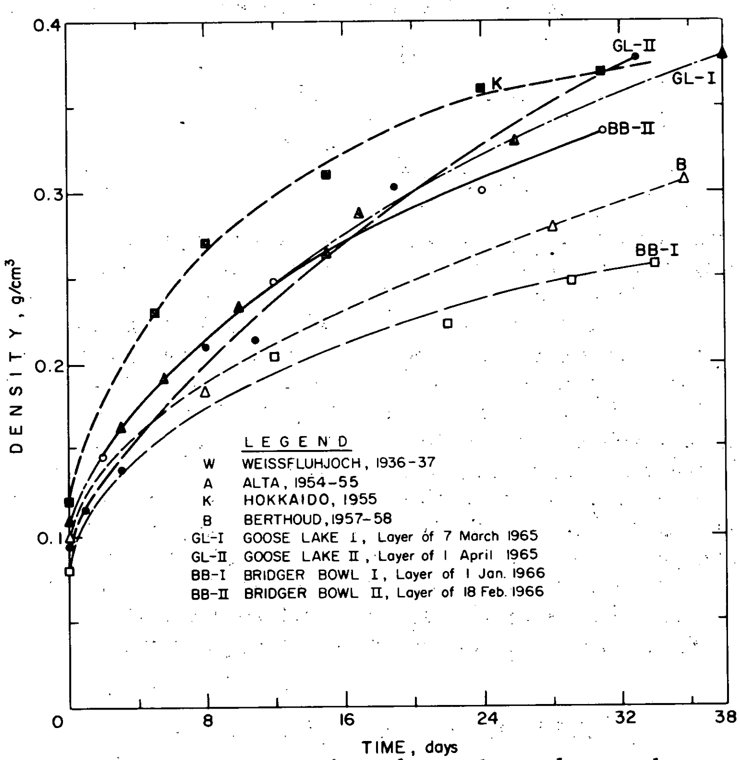

Figure 1. Density vs time for selected snow layers. Symbols and notations for locations will be used in all successive figures.

determination were between .85 and .99. At least one slope was not significantly different from zero (by Student's "t"). The value of ρ_f was chosen to be 0.55 g cm^{-3} which has often been stated to be the limiting density for the initial densification process (Anderson and Benson, 1963; Feldt and Ballard, 1966). The true limiting value of any densification process is that of bubble free ice (ρ = 0.917 g cm^{-3} at 0C) but this is an unrealistic value for a seasonal snow cover. While the choice of ρ_f implies a prior knowledge of process it must be stressed that eq 1 is strictly empirical. This is not to suggest that density-time curves do not reflect processes but that the processes are too complex to be revealed by curve fitting. For the range of densities and times considered, eq 1 appears to provide a workable expression.

Figure 2 shows the relationship between density and load. If low density snow behaved as a noncohesive aggregate of uniform size it might be expected that a single density-load relationship could be found for all snow covers. The fact that this is not the case again indicates that several effects must be operable. In an attempt to state some of these effects explicitly all the available data on the snow covers were gathered (Table II). It can be seen that while there are no obvious correlations, there are some suggestive trends.

Temperature, in general, appears to be of little importance. The fact that the greatest slope occurs with the lowest temperature is somewhat surprising; however, the range of temperatures considered may not be of significant breadth and average snow pack temperature may well not be as meaningful a parameter in this regard as surface or near surface temperature cycles.

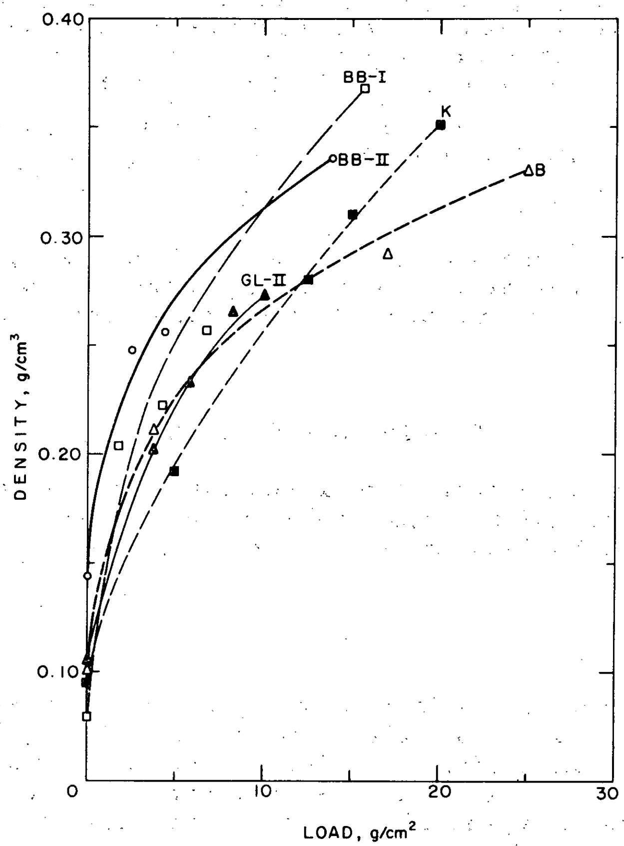

Figure 2. Density vs load for selected snow layers.

Table II. Physical characteristics of selected snow layers.

Location	-k ($time^{-1}$)	Temp (°C)	Rate of loading $\Delta\sigma/\Delta t$ ($g\ cm^{-2}\ day^{-1}$)	Rate of loading $\Delta\sigma/\Delta t$ (first 5 days) ($g\ cm^{-2}\ day^{-1}$)	Avg grain diam (mm)
Bridger Bowl I	.0128	-2.0	.54	.3	1.5
Berthoud	.0170	-5.0	.3	.25	1.0
Bridger Bowl II	.0226	-3.0	.54	.32	1.0
Goose Lake II	.0241	-4.5	1.0	1.0	0.5
Goose Lake I	.0287	-5.0	1.0	1.2	0.5
Hokkaido	.0288	-7.0	.7		

Field estimates of grain size by visual inspection are probably quite subjective, but assuming the data to be comparable there is some suggestion that the rate of densification is inversely proportional to grain size. This is in agreement with earlier work (Bucher, 1948; Bader, 1962).

While the average rate of loading ($\Delta\sigma/\Delta t$) does not appear to have a systematic effect on densification rate the initial rate of loading ($\Delta\sigma/\Delta t$) for time = $0 \rightarrow 5$ days) shows a definite trend. The effect here may be to increase the initial strain rates.

In order to bring these relationships out (if they exist) much more careful data gathering is necessary. The collecting of data on grain size, shape and structure appears to be a particularly desirable object of future field and laboratory investigations.

COMPRESSIVE VISCOSITY

A concept which has been used extensively in theories of snow densification is that of compressive (or compactive) viscosity, η, which is an expression of the linear relationship between stress σ and strain rate $\dot{\varepsilon}$:

$$\eta = \frac{\sigma}{\dot{\varepsilon}} . \tag{2}$$

The assumption implicit in this relationship is that compression is a creep process which is Newtonian in the range of interest. In theory (Feldt and Ballard, 1966) and in experiment (Mellor and Smith, 1966) it has been suggested that this assumption is valid only at low stresses. Certainly one would intuitively expect that Newtonian viscosity would be a poor approximation for such a porous medium as low density snow. Although the true linearity of eq 3 is extremely difficult to test for in a field situation since η is a strong function of density, Kojima (1966) has found in field experiments that it holds down to quite low densities. In any event it is of interest to compare values of η for various areas as the differences could be indicative of fundamental differences in snow properties.

The calculation of η is readily made for the field situation by the repeated measurements of density and overburden changes for selected stratigraphic horizons in snow pits.

$$\sigma = \int_0^z \rho \, dz$$

and

$$\dot{\varepsilon} = \frac{1}{\rho}\left(\frac{d\rho}{dt}\right) \sim \frac{1}{\rho}\frac{\Delta\rho}{\Delta t} .$$

Values of η for the data studied ranged from 10^6 to 10^9 $\mathrm{g\,cm^{-2}\,sec}$ which is about one order of magnitude less than those obtained for polar snows of similar density (Ramseier and Pavlak, 1964) and two orders of magnitude less than that for coarse grained snow found in the St. Elias Range of Canada (Grew and Mellor, 1966). Grew and Mellor suggest that grain size is the controlling factor here which is in agreement with Bader's (1962) suggested relation:

$$\eta \sim D^3$$

where D = grain diameter.

The relationship between η and density is shown in Figure 3. The line from Ramseier and Pavlak has been extrapolated from their lowest value of 5×10^9 for snow at Camp Century, Greenland. These data can be expressed by:

$$\eta = \eta_0 \exp [k \rho]. \tag{3}$$

k varies from about 25 $cm^3\ g^{-1}$ to 100 $cm^3\ g^{-1}$, in all cases higher than Kojima's and Yoshida's value of 21, despite the fact that all the data are presumably for snow of similar temperature and structure.

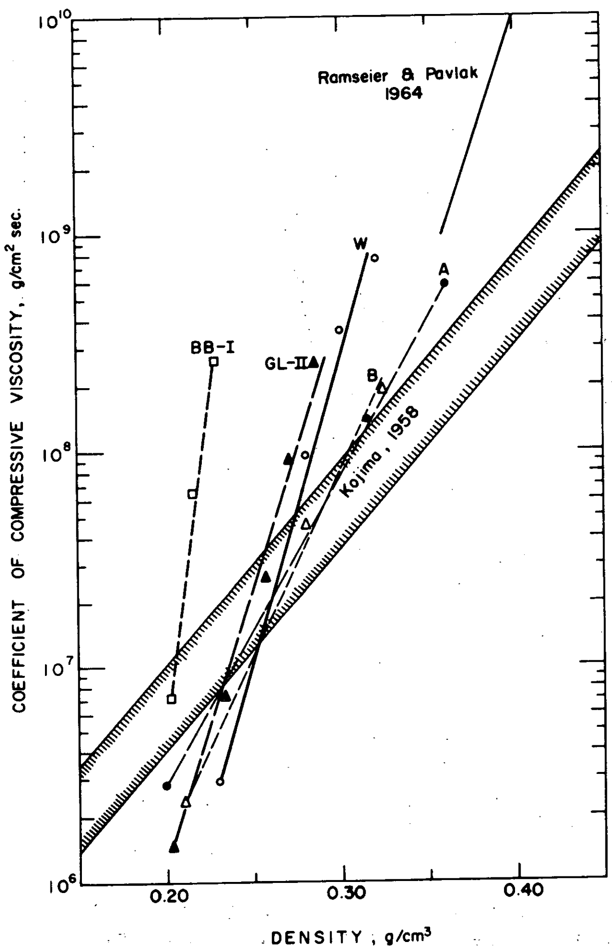

Figure 3. Relationship between the log of compressive viscosity and density. Kojima's data are for several snow layers and are plotted as an area.

The relationship between η and time shown in Figure 4 can be expressed by:

$$\eta = \eta_0 \exp[k\sqrt{t}]. \qquad (4)$$

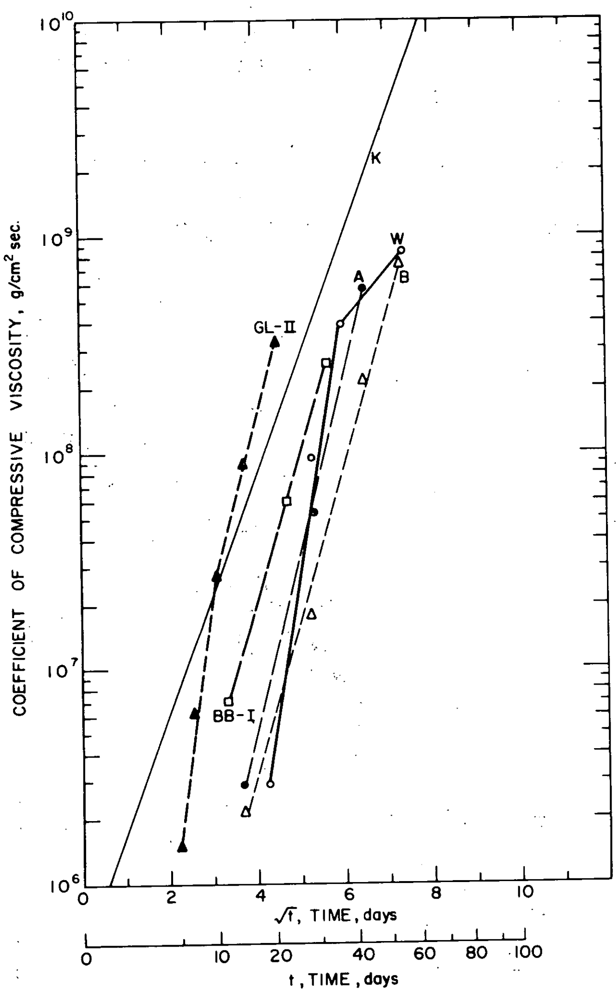

Figure 4. Relationship between the log of compressive viscosity and the square root of time.

DENSIFICATION

Yoshida et al. (1956, 1958) have formalized density-time relationships for snow covers in Japan by assuming that the process is Newtonian viscous even into the low density range. Objections to this have been raised and the fact that divergence of the curves of Figure 1 cannot be related solely to accumulation rates suggests that factors other than purely mechanical compaction are of considerable importance.

Anderson and Benson (1963) have used a purely descriptive treatment, involving no assumptions regarding process, to describe the densification of polar snow. This treatment is open to criticism, namely that their basic assumption probably has no connection with physical reality; however, its use provides some interesting points of comparison between data from different density ranges.

The Anderson-Benson thesis is roughly as follows: Assume that, under steady-state conditions (i.e., accumulation at a constant rate, low enough for the stresses in the snow cover to maintain equilibrium conditions, and constant temperature) the rate of pore space elimination with respect to loading is proportional to the amount of pore space present:

$$\frac{dV_p}{d\sigma} = -mV_p \quad \text{or} \quad \frac{dv}{d\sigma} = -m(v - v_i)$$

where

V_p = volume of pores

v = specific volume = $1/\rho$

v_i = the specific volume of ice

$\sigma = \int_0^z \rho\, \delta z$

m = a function of the mechanics of compaction.

Solving for v gives:

$$v = v_i + (v_0 - v_i)e^{-m\sigma} \tag{5}$$

where

v_0 is specific volume at $\sigma = 0$.

Equation 5 has been used to fit data from Goose Lake for three depth-specific volume situations (pits of 28 March and 10 May 1965 and 28 March 1966). The data for 10 May 1965 and 28 March 1966 are plotted in Figure 5. The 28 March 1965 data are not included because they were almost exactly identical to those of 10 May.

The pertinent parameters of eq 5 are tabulated in Table III. The parameter v_0 for the Goose Lake data is not at zero load but rather at the load where eq 5 became operable.

The values of the parameter m for Goose Lake are approximately 4 times that reported for Greenland dry snow and approximately 1.5 times that reported

Table III. Characteristic parameters for fitting the Anderson-Benson equation.

Location and date		σ (g cm^{-2})	v_0 (cm^3 g^{-1})	m (cm^2 g^{-1})
Goose Lake	- 10 May 1965	$10<\sigma<150$	2.70	5.3×10^{-3}
	- 28 March 1965	roughly the same		
	- 28 March 1966	not well fitted by equation 5		
Greenland (from Anderson and Benson)		$0<\sigma<450$	3.20	1.6×10^{-3}

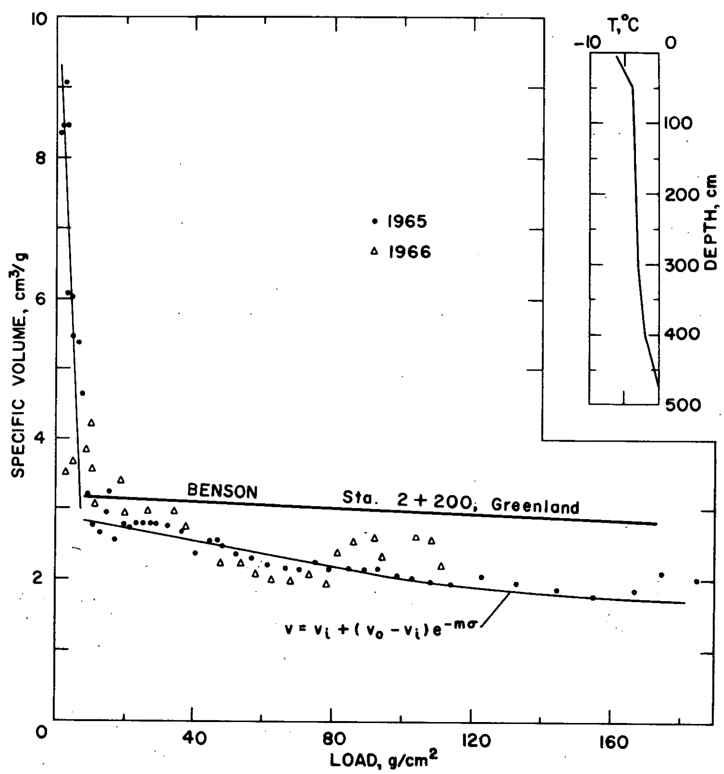

Figure 5. Specific volume vs load for the pits of 10 May 1965 and 28 March 1966. Benson's data (from Benson, 1962) are indicated as a line. The temperature profile is that of the pit of 10 May 1965.

for the soaked facies of the Upper Seward and Blue Glaciers ($m = 3.7 \times 10^{-3}$ cm^2 g^{-1}, Anderson and Benson, 1963). As m is an index of the relative rate of increase of density with respect to load this latter fact is somewhat surprising. It may be due in part to the fact that the Goose Lake data are for snows of higher specific volume.

Of particular interest in Figure 5 is the point of inflection at $v_0 = 2.70$. For higher specific volumes the v vs load relationship appears linear. For lower specific volumes eq 5 holds with some degree of accuracy. This point of inflection is not to be confused with the critical point (occurring at the critical load and critical density) referred to by Anderson and Benson (1963) which exists at considerably lower specific volumes ($v = 2.00$). Presumably if the Goose Lake data extended to these low specific volumes the critical point would also appear and similarly if Benson's Greenland data extended to specific volumes above 3.0 the upper discontinuity (for convenience termed the "discontinuity" and occurring at v_d and σ_d) would appear on his plots. Plots of specific volume vs load were made for several localities and in each case the "discontinuity" was observed. Table IV gives values of v_d and σ_d for these plots. While in each case the severity of the "discontinuity" varied it was always possible to pick out a definite point of inflection. The remarkable similarity of the values of v_d and σ_d from several locations indicates that a real physical control may be reflected by the "discontinuity."

Anderson and Benson (1963) reasoned that their critical point marked the transition between purely mechanical packing at densities lower than 0.55 g cm^{-3} and some mechanism such as viscous flow or molecular diffusion at high densities. Other investigators have felt that viscous creep was effective at densities lower than 0.55 g cm^{-3}. Feldt and Ballard (1966), for example, felt that, since at densities as low as 0.41 g cm^{-3} consolidation must be accompanied by the deformation of intergranular bonds, viscous flow is effective in this range. Kojima and Yoshida derived depth density curves for densities down to 0.1 g cm^{-3} on the basis of Newtonian viscosity. The existence of the "discontinuity," however, suggests additional complications in the density range of less than 0.35 g cm^{-3} ($v = 2.9$).

Table IV. Data at the "discontinuity."

Locality	v_d (cm^3 g^{-1})	σ_d (g x cm^{-2})
Bridger Bowl	3.4	22
Mosiri (from Yoshida)	2.9	20
Berthoud	2.9	21
Alta	3.0	19
Goose Lake	2.7	18
Weissfluhjoch	3.1	22

The possible explanations for very high strain rates in the low density range are both numerous and, in actual fact, probably interconnected. While failure is probably not due to collapse in compression as the stresses involved are nearly an order of magnitude less than laboratory yield stresses it may well be that extremely high stress concentrations exist at the points of dendritic crystals and bring the stress strain-rate relationship out of the Newtonian range despite the fact that the overburden stress is low. Secondly, there are undoubtedly intense thermodynamic stresses imposed on the upper snow layers by both the large diurnal temperature fluctuations and by vapor pressure gradients within pore spaces which are created by the extreme curvatures of the irregularly shaped new snow. Thirdly, intergranular bonds are both few and small in newly fallen snow and shear strengths are quite low (on the order of 250 g cm^{-2} for dry snow with a density of .35 g cm^{-3}).

The fact that the discontinuity lies at about the same point irrespective of location, with its implication of varying rates of loading, varying temperature regimes and varying snow structural properties, is not explained by present theory. From a purely mechanical point of view the lack of a known finite yield strength for ice would suggest that for any given load there should be an infinite number of densities whereas, in point of fact, there is not. The implication is that, in low density dry snow, the stresses are highly local and not greatly dependent on the outside environment.

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DENSITY, LOAD, AND ELAPSED TIME (SINCE DEPOSITION) FOR SELECTED SNOW LAYERS

Notation: ρ, density (g cm^{-3}); σ, overburden stress or load (g cm^{-2}); t, time since deposition (days).

Hokkaido - 1956			Bridger Bowl I - 1966		
ρ	σ	t	ρ	σ	t
.21	0	0	.08	0	0
.23	6	5	.21	2	12
.27	7	8	.22	4.5	22
.31	14	15	.24	6	29
.37	20	31			
Berthoud Pass - 1957-58			**Bridger Bowl II - 1966**		
.10	0	0	.11	0	0
.18	4.5	8	.14	.5	2
.28	17	28	.25	2	12
.31	25	36	.30	4.5	24
			.33	14	30
Goose Lake I - 1965			**Weissfluhjoch - 1936-37**		
.11	0	0	.10	0	0
.16	6	3	.23	2	19
.19	7	6	.26	7	47
.23	10	10	.30	24	83
.26	10	15	.32	46	98
.28	13	17			
.33	26	26			
.38	27	38			
Goose Lake II - 1965			**Alta - 1954-55**		
.09	0	0	.10	0	0
.12	2	1	.27	7	8
.14	8	3	.30	26	38
.21	9	8	.33	31	53
.22	15	11	.35	70	86
.31	18	18			
.37	31	33			

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13. ABSTRACT

Through pit measurements on selected deep seasonal snow covers, observations have been made on the densification rates of dry snows. The variation between rates has been compared with such physical characteristics of the snow as temperature, grain size, and loading rate. The rate of densification does not appear to be affected by temperature in the -1 to -10°C range but it is inversely proportional to grain size and sensitive to rates of loading during the formative stage of any particular snow layer. Values of compressive viscosity vary from 10^6 to 10^9 gm/cm^2 per second which is an order of magnitude less than the lowest values for polar snow. Plots of specific volume against overburden reveal a sharp discontinuity at a specific volume of about 3.0 cm^3/gm. The persistence of this discontinuity from location to location indicates that it may reflect a real phenomenon. It is suggested that it may be accounted for by extremely high strain rates at low densities.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Snow--Density Snow cover--Subsidence--Mathematical analysis Snow--Metamorphism Snow cover--Physical properties						