Annotated Bibliography on Wave-Current Interaction

by

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**Annotated Bibliography on Wave-Current Interaction**

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**Abstract:**
This annotated bibliography discusses 60 key publications dealing with wave-current interaction. Each entry includes a bibliographic identification, keywords, a discussion of contents, and a statement of coastal engineering significance. An index of the entries by keywords is provided in an appendix.

The recent growth of the wave-current interaction field is indicated by the fact that more than 30 percent of the selected publications were published in 1978 and 1979.
The report provides coastal engineers an annotated bibliography of the key publications dealing with wave-current interaction, a phenomenon which may affect wave height and wave direction in unexpected ways. Peregrine and Jonsson (1983) presents an overview of wave-current interaction and a comprehensive review of significant references. The work was carried out under the U.S. Army Coastal Engineering Research Center's (CERC) Waves at Entrances work unit, Harbor Entrances and Coastal Channels Program, Coastal Engineering Area of Civil Works Research and Development.

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Comments on this publication are invited.

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TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director
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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

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$^1$To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9) (F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9) (F - 32) + 273.15$. 
I. INTRODUCTION

1. Scope

This report presents brief discussions of selected publications, 55 reports and 5 books, dealing with wave-current interactions. The criteria for the inclusion of reports are as follows:

(a) The report must be published in a recognized and available source.

(b) The report must have evident coastal engineering significance either to direct engineering application or applied research.

(c) The selected reports, taken as a whole, must cover the various subfields of wave-current interaction.

In addition, three reports of historic interest (Unna, 1942; Johnson, 1947; Arthur, 1950) have been included to identify early workers on the subject and to better understand the historical development. The five books (LeBlond and Mysak, 1978; Lighthill, 1978; Phillips, 1977; Shaw, 1979; Whitham, 1974) provide potential users an introduction to necessary mathematical techniques or the current state-of-the-art.

The length of each entry in the bibliography is not necessarily indicative of its importance. Some entries have been lengthened to make available information that has been difficult to obtain because of language or source, e.g., the work of Biesel (1950) which is in French, and Mallory's (1974) excellent analysis of ship damage due to waves on the Agulhas current.

2. Distribution of Publications

a. Format. Each annotated entry is presented in four parts:

(1) Bibliographic identification.

(2) Keywords.
(3) Discussion. This includes a description of the contents, major findings, and in some cases an analysis of the results or a comparison of related works. In some entries, the original abstract of the report is used as part of the discussion. Where this is done, the abstract leads off the discussion and the end of the abstract is indicated by "(author's abstract)."

(4) Coastal Engineering Significance.

Various literature sources are cited in the annotations throughout the bibliography. Those sources included as part of the bibliography have the names of the authors capitalized. Bibliographic information on the other sources is given in Literature Cited.

b. Dates. The publication dates of the entries are relatively recent. More than half (31 of the 55) of the reports were published in the last 5 years (1976-1980); in fact, 31 percent of the 55 were published in 1978 and 1979. Early reports on the subject were included for historic interest.

c. Location. The geographic source of the bibliography can be broken down into three main areas:

(1) English speaking countries outside the United States (approximately 45 percent),

(2) the United States (approximately 34 percent), and

(3) countries where English is not the first language of technical communication (approximately 21 percent).

This geographic distribution is based on the location of the publisher, which does not necessarily indicate the location of the work or the author's residence.

II. USE OF ANNOTATED BIBLIOGRAPHY

Considerable effort went into the selection of appropriate keywords to make this bibliography more accessible to the user. About 150 keywords were identified, but some of these were not used. Appendix A provides a listing of keywords with cross references to related keywords.

Appendix B contains an index of the bibliography by keywords. This appendix should be of the most use to a potential user unfamiliar with the field, since it identifies the publications by subject matter. The best way to use this annotated bibliography is to consult Appendix A for keywords describing the subject of interest, then consult Appendix B to identify the entries on that subject.
III. THE ANNOTATED BIBLIOGRAPHY


Keywords. Current Depth Refraction; Currents, Opposing; Currents, Rip; Historical Interest; Theory; Theory, Ray; Waves, Long; Waves on a Jet.

Discussion. This early paper on wave-current interaction describes a ray solution for shallow-water waves meeting a rip current. The interaction is treated as a refraction problem and rays are drawn for one example.

A beach of uniform slope and an arbitrarily chosen current satisfying mass conservation are specified. The result shows the rays concentrating at the center of the jetlike current for waves directly incident against it; wave crests are also drawn.

The author comments that he discontinues the calculation and diagram in the neighborhood of caustics and that they and wave diffraction modify the solution. He also notes that interaction of currents with a sand bed also modifies the resulting wave pattern because of the greater depth of the current's channel.

The author makes an inconsistent approximation to obtain his equation (4). The correct equation is obtained by adding terms in C in equation (4) to the right-hand side of equation (3). Even so, it is likely that the error in the ray diagram (Fig. 3) is small. The general picture of rays concentrating wave energy toward the center of the current is certainly correct.

Coastal Engineering Significance. Despite the minor error in mathematics and the relatively early date of this work, it is still an interesting paper. The author recognizes the importance of diffraction, and it is likely that his analysis is of little value for rip currents because of these diffraction effects. However, the analysis provides good insight into the general nature of the wave field on larger scale, jetlike currents, such as ebb currents at tidal inlets. There are still no more recently published ray diagrams (at time of writing in 1982) for such a basic pattern of flow.


Keywords. Continental Shelf; Current Depth Refraction; Currents, Large-Scale; Currents, Nearshore; Currents, Tidal; Currents, Unsteady; Wave Observation; Wave Period.
Discussion. Measured swell at Perranporth, Cornwall, has a regular fluctuation in wave period of about 10 percent. It is suggested that this is due to interaction with the tidal streams as the waves traverse the Continental Shelf.

A theory describing the propagation of waves on a time-varying current is derived which provides an expression for the time variation of the period.

A theoretical expression is evaluated for swell traveling from the South Atlantic by determining its refraction over the Continental Shelf, including the effects of hourly changes in the tidal currents and depth encountered by the waves. The results do not agree in detail with all the observations but are of the right order of magnitude.

Coastal Engineering Significance. This paper is one of few papers which consider waves on real unsteady currents. For this reason, it is of present technical importance despite its relatively early publication date. The phenomenon observed at Perranporth is potentially present at the mouth of many large estuaries such as at San Francisco near the Golden Gate.


Keywords. Critical Current Velocity; Currents, Unidirectional; Currents, Vertical Shear; Dispersion Relation; Historical Interest; Theory; Waves, Stationary.

Discussion. This is an early systematic study of the effect of vertical shear in a current on wave motion. It is a theoretical paper which considers small-amplitude waves propagating with or against the current, but it presents mathematical results in relevant physical contexts. In particular, three conditions of practical importance are emphasized: waves propagating up an estuary, wind waves on canals, and stationary waves.

The analysis considers those waves which can be made to appear stationary by a suitable choice of reference frame. The problem is formulated in terms of a stream function and an arbitrary current profile. It reduces to an ordinary differential equation when sinusoidal waves are assumed, although the wave phase velocity is an unknown in the coefficients of the equation. Analytical solutions are found when a simple linear velocity profile is inserted.

The author goes further than Thompson (1949), who had obtained a dispersion relation, by indicating how to find wavelength, wave phase speed, or both, when the current, depth, and one wave property
(wavelength, phase speed, or frequency) are given. In the absence of current, some of the conditions investigated lead to explicit solutions and others to implicit solutions.

This paper also notes the problem of critical current velocities; i.e., when wave phase speed is equal and opposite to current velocity. Solutions then appeared to be unobtainable, but more recent work has developed ways of dealing with this problem (see PEREGRINE, 1976, for more on this topic).

Coastal Engineering Significance. In addition to presenting the mathematical solution, the author discusses physical points of interest to engineers. He emphasizes how vertical shear of the current can have practical importance in changing wave properties. Numerous more recent papers have rediscovered or elaborated on Biesel's (1950) results.


Keywords. Comparison of Theory and Measurement; Currents, Large-Scale; Numerical Model; Refraction-Diffraction; Wave Dissipation; Wave Height.

Discussion. A parabolic approximation of a new wave equation is developed for the practical calculation of wave propagation in an area with slowly varying depth and current.

Using a variational principle, the author derives a water wave equation which is probably the first to include the effect of variable depth and current. The derivation assumes small-amplitude waves, mild bottom slope, slowly varying current, no velocity variation with depth, nearly periodic water motion, and no dissipation. The new equation is a partial differential equation of the hyperbolic type. It has the important restriction that the frequency observed from a fixed point must vary slowly and within a narrow band. For purely periodic waves it transforms to an elliptic-type wave equation which, in the absence of currents, reduces to the mild-slope wave equation developed by Berkhoff (1972).

The new wave equation has two drawbacks: it does not include dissipation, and for areas many wavelengths in size, computer time becomes too long for practical application. Moreover, in irregularly shaped regions, the refraction method tends to leave areas with hardly any rays, where experience indicates appreciable wave heights may occur.

The author addresses these problems by adopting a parabolic approximation (developed mainly in acoustics) which allows for variation of wave height along wave fronts. The parabolic approximation to the steady-state wave equation obtained by the author is used to develop a
finite-difference numerical model. In the model, dissipation terms are included to account for bed friction and breaking. The most important input data in the program are the distribution of depth and currents, the period, direction and amplitude of the incident wave, the boundary conditions along the sides, and data concerning dissipation. Output data are wave amplitudes and directions. The computer program is applied to the entrance of the Oosterschelde estuary in the southwest of the Netherlands, where extensive coastal defense works are undertaken. The graphs indicate that the current (typical value, 0.7 meter per second) does not have a spectacular influence here.

The author also presents solutions to the mild-slope current-wave equation itself for some cases where the wave field is independent of one of the horizontal coordinates. These include waves crossing an undersea slope or gully, propagation along the axis of a channel bounded by vertical sidewalls, and propagation along an undersea gully. The agreement with measurements for the latter case is not very close.

The main results of the report are the hyperbolic-type current-wave equation (3.21), the elliptic version (3.23), and the parabolic approximation (6.19) which is the basis for the numerical model. The following misprints in the first two of these important equations are noted: in (3.21) the sign of the term $\nabla \cdot (\alpha \nabla \phi)$ should be minus instead of plus, and in (3.23) the last term should read $(\sigma_o^2 - \omega^2 - k^2 \alpha) \delta$.

Coastal Engineering Significance. The report is important both for fundamental understanding and practical application. It presents the first published water wave equation taking both depth and current variation into account. Based upon a parabolic approximation of this wave equation, a computer program is developed which allows for refraction-diffraction and dissipation.


Keywords. Currents, Large-Scale; Theory; Wave Action; Waves.

Discussion. The problem of waves propagating in inhomogeneous moving media is discussed. The waves are restricted to small amplitude and the medium is nondissipative and varies only over length or time scales which are much greater than the wavelength or period. Otherwise, considerable generality is achieved.

The discussion centers around the considerable care required to define energy for linearized (as distinct from linear) systems. A medium at rest and in equilibrium is considered first and then results for a moving medium are derived.
The most important result is that the quantity, $(\text{wave energy})/(\text{wave frequency})$, is "conserved," i.e., obeys a conservation equation, where both energy and frequency are measured relative to the moving medium. This quantity is called "wave action."

Hamilton's principle is one example which is considered at length. It is derived for water waves and the results demonstrated.

Coastal Engineering Significance. This rather mathematical and very general paper is of fundamental importance to wave-current interaction. Building on the remarkable advances in the methods of solving problems for nonlinear waves by Whitham (1965, 1967), the authors show that in conservative systems considerable simplification can be made by introducing the concept of wave action. Instead of the relatively complicated energy equation with the physically important radiation stress term, many practical examples can be solved directly by conservation of wave action. This allows for the flow of energy between waves and currents without explicitly calculating the energy exchange.


Keywords. Bottom Friction; Comparison of Theory and Measurement; Current Velocity Profile; Currents, Unidirectional; Experiment; Wave Dissipation; Wave Effect on Current.

Discussion. Properties of mean current and waves were measured in a flume along which regular periodic waves and a current could be sent.

The flume, of length 30 meters and width 1.0 meter, had a piston-type wave generator at one end and a beach which gave reflection coefficients between 5 percent and 10 percent at the other end. The still-water depth was 30 centimeters for the experiments, and the bottom of the flume was covered by corrugated iron plates with transverse ridges of height 1.4 centimeters and wavelength 8 centimeters. A current was generated in the wave direction by a pump which supplied water through an inlet about 2 meters in front of the wave generator. The current outlet was under the beach. At full flume width, the maximum current was about 20 centimeters per second. This was increased to 40 centimeters per second by halving the width of the flume for some runs.

Wave heights and lengths were measured using resistance gages. Horizontal fluid velocities were measured with a 10-millimeter-diameter propeller meter. This meter is unsuitable for measuring reversing flows, so the fluid velocity measurements were performed for the case of strong following currents in the half-width section. Most of the wave measurements were with absolute wave periods of 1.8 and 2.0 seconds, and with mean current velocities of 9.1 and 16.7 centimeters per second.
In still water, the measured wavelength agreed with theory, and the wave attenuation was somewhat greater than that predicted by Jonsson (1980) for small ratios of maximum particle displacement to bed roughness.

For waves on flowing water, the measured wavelength agreed less with a theoretical value obtained by considering an equivalent uniform current. However, the amplitude variation as the waves propagated onto the current was in reasonable agreement with theory.

The wave attenuation on the uniform current was measured and found to be greatest per unit distance for opposing currents. This result is to be expected, since the wave energy is propagating more slowly. The velocity measurements with the strong following current indicated that there was little phase displacement between the free-surface profile and the horizontal velocity component. There was also a reduction of mean velocity in the presence of waves. However, changes in mean water level are not recorded, so it is difficult to judge the full effect of what appears to be increased bed shear.

Coastal Engineering Significance. Careful measurements of the type presented here are necessary, yet have rarely been performed. The comparisons with theory are particularly helpful since they give an indication of the adequacy of those theories. This is especially true for any work which involves turbulent flow. The strong influence of the wave motion on the mean current profile is a striking feature which should be studied in the context of the whole flume system. It is ironic that even more detailed measurements of a similar system will soon be published (Kemp and Simons, in preparation, 1983).


Keywords. Bottom Friction; Conservation Equations; Current Depth Refraction; Currents, Large-Scale; Theory; Theory, Ray; Wave Action; Wave Dissipation; Wave Height.

Discussion. Refraction of steady, slowly varying water waves propagating on a steady current over a gently sloping seabed are studied. Dissipation due to bed friction is rigorously incorporated.

The energy equation for the fluctuating motion is formulated in terms of wave action, namely, wave energy divided by relative angular frequency. This results in the classical wave action conservation equation for nondissipative flow, supplemented by a dissipative term, which is strikingly similar to wave action itself. It is simply the total dissipation minus the effect of the current acting on the total mean bed shear stress, divided by the relative frequency. Thus, it is
shown that the concepts of wave action and wave rays are also fundamental for real flows. An inconsistency in PHILLIPS' (1977) book is pointed out in this connection.

Looking at a ray tube, a new formula for the calculation of wave amplitudes on large-scale currents over gently sloping seabeds is described. It gives the amplitude, divided by the initial amplitude, along a wave ray as a product of four factors: a Doppler coefficient, a shoaling coefficient, a refraction coefficient, and a friction coefficient.

This is a natural extension of the pure depth refraction case, and the Doppler coefficient is the new concept. This simple formula is probably not available in other publications.

It is explained how, on the above basis, the total wave field on a current can be calculated, when the following quantities are given: the bathymetry, the absolute frequency, and appropriate boundary conditions in the horizontal plane for the waves and the current.

All expressions are correct to second order in wave amplitude. The current velocity is assumed to be vertically uniform and of zero order.

Coastal Engineering Significance. The theoretical developments which have shown the importance and convenience of the concept of "wave action" were all developed for conservative systems without dissipation (BRETHERTON and GARRETT, 1968; STIASSNIE and PEREGRINE, 1979). This paper extends the value and applicability of the concept to flows with dissipation. The flows and wave fields are assumed to be steady but the extension to unsteady flows is not difficult. There still remains the difficulty of finding reliable and confirmed estimates of dissipation and bed friction.


Keywords. Bottom Friction; Conservation Equations; Current Depth Refraction; Currents, Large-Scale; Depth Refraction; Streamlines; Total Head Line; Wave Dissipation.

Discussion. A well-known principle for the calculation of wave heights in current depth refraction of water waves is the adoption of wave action conservation along wave rays, e.g., see CHRISTOFFERSEN and JONSSON, 1980. This is a natural extension of pure depth refraction, where wave heights are calculated along wave orthogonals.
In this study it is shown that, alternatively, wave heights can be calculated along streamlines, which is a natural analogy to classical steady hydraulics, albeit the superposition of waves makes calculations more complex.

In steady hydraulics the concepts of a total head line and a horizontal energy reference line are known to be useful tools for calculating water surface heights and current velocities in rivers.

Combining the momentum and total energy conservation equations, the above-mentioned energy principle is extended to a steady, rotational, large-scale current wave motion over a gently sloping seabed. Dissipation due to bed friction is included.

The discovery is made that a total head line and a horizontal energy reference line also exist along streamlines in such a combined flow. The horizontal energy reference line demonstrates the existence of a so-called energy reference height above a datum, which is constant along a streamline. It varies, however, from streamline to streamline.

The new energy equation states that this constant height is the sum of the total current wave (energy) head and the current wave dissipation head. The total head is the sum of four terms: the mean water surface height above datum, plus a current velocity head, plus a mean wave velocity head, and minus a current wave interaction term head. The current wave dissipation head is the sum of two terms: a current dissipation head minus a wave dissipation head. This allows the calculation of wave heights along streamlines.

Since the energy reference height varies from streamline to streamline, no horizontal energy reference level exists in general for a combined current wave motion.

The findings are illustrated with four qualitative sketches, corresponding to a strong/weak current, combined with a following/opposing current. A table gives the definitions of the many new concepts.

It is verified that an approach using wave-action conservation leads to the same equations. Various properties of wave refraction by depth changes without current are also described.

Coastal Engineering Significance. The analysis and interpretation of refraction equations given in this paper should assist engineers who desire a better understanding of the subject. The energy integral along streamlines and the reference levels defined here are alternatives to using the conservation of wave action. This may be useful for analysis or in computing simple flows, but in general it will be simpler to use wave-action rather than streamline integrals since rays are computed to find the frequency and wavelength.

Keywords. Currents, Vertical Shear; Numerical Model; Theory; Waves, Finite-Amplitude; Waves, Nonlinear.

Discussion. An iterative finite-difference model is developed to describe two-dimensional periodic gravity waves on the surface of a fluid containing vorticity in the form of a vertical shear current (i.e., a steady horizontal current whose local velocity varies with elevation). A coordinate transformation due to Dubreil-Jacotin has been used to map the fluid domain into a rectangle. The full nonlinear constant pressure free surface boundary condition is used iteratively until convergence is achieved. A comparison is made to an analytical model for a linear shear current, and results are also shown for a mean flow with a seventh power law velocity distribution. (author's abstract)

The key to the finite-difference model is the mapping of the wave domain into a rectangle by posing the problem with $y$ as a function of $x$ and $\psi$. Although the resulting governing equation is more difficult, the domain becomes a rectangle with a base equal to the length of the wave and a height equal to the value of the surface streamline. From symmetry, only half the wavelength need be studied.

The usual boundary conditions pertain: no flow through the bottom, a periodic solution in the direction of wave travel, and a constant pressure streamline at the surface. This last free-surface condition is attacked by iterating the Bernoulli equation along the free-surface streamline. To fix the free surface, three constraints are applied: a mean sea level constraint to ensure that the mean free surface is fixed, a dynamic free-surface boundary condition constrained by the Bernoulli constant, and a wave height constraint to ensure convergence.

The author tests his finite-difference scheme against two cases: a linear velocity distribution and an approximation to the seventh power law. Both tests are for the same wave conditions (height = 0.61 meter; depth = 3.0 meters; period = 10 seconds), and both are for an opposing current with an approximately equal discharge, having peak surface velocities of -0.91 meter per second (linear) and -0.61 meter per second (seventh power). The combined wave-current interaction gives a velocity in the seventh power case that is about 30 percent greater in magnitude than the linear case. The seventh power law case has a relative maximum in the velocity distribution located close to the bottom.

Coastal Engineering Significance. From the example offered by the author, it is clear that the realistic currents approximated by the seventh power law cannot be simply superimposed on the wave particle velocity to get the resulting motion. This is in contrast to currents.
whose velocity varies linearly with elevation where this has been shown possible with reasonable accuracy. The nonlinear finite-difference technique is potentially a useful tool for dealing with currents under combined wave and current flows in real cases. However, the author hints that the rate of convergence needs to be improved.


Keywords. Current Depth Refraction; Currents, Nearshore; Currents, Wave-Induced; Equations of Motion; Theory; Wave Breaking.

Discussion. This technical paper examines analytically the effects of larger angles of incidence and the refraction of waves by the longshore current on planar beaches for the case of no lateral mixing. The longshore current velocity is included in a modified Snell's law, and the momentum equation is derived, based in part on previous work by Liu and Dalrymple (1978) and Iwata (1976). A perturbation analysis is done which depends on the deepwater waves being incident at a small angle to the normal to the beach.

A zero-order result of the analysis is that the longshore current velocity equation of Longuet-Higgins (1970) is obtained. This equation is written as a product of a term $A_D$ and terms in depth, deepwater direction, and deepwater speed. The analysis then shows that there is a critical value of $A_D$ (equal to 0.78), above which the current refraction in the surf zone produces a somewhat higher velocity (above the Longuet-Higgins value). Below the critical value, the "large angle effect" causes a decreased velocity, which in turn, causes the surf zone to widen in order to balance bottom shear. However, the deviations from the Longuet-Higgins value are not significant for the example shown.

Closer examination suggests that the expected values of $A_D$ are much larger than critical. Accepting a breaker height-to-depth ratio of 0.8 used by the author, it appears that $A_D$ must equal 202 m/f, where m is the slope of the planar beach and f is the Darcy-Weisbach friction factor. It is rare for a surf zone to have a slope flatter than 0.02. Thus, $A_D$ probably exceeds 4/f. If this is the case, f would have a value of at least 5 to get $A_D$ below the critical value, but typically f is on the order of 0.01.

Coastal Engineering Significance. The perturbation analysis produces the Longuet-Higgins equation as the zero-order longshore current velocity, which lends further support to the widespread adoption of that equation. Under the interpretation given here, current refraction effects dominate over large-angle effects in practical cases, the effect being to increase the actual current above the predicted Longuet-Higgins value. The amount of increase is not clear, but may be small.

**Keywords.** Currents, Nearshore; Currents, Rip; Currents, Wave-Induced; Shallow Water; Theory.

**Discussion.** The shallow-water equations used by Longuet-Higgins (1970) to discuss longshore currents are used to extend the work of LeBlond and Tang (1974) in attempting to relate the spacing of rip currents to beach and incident wave properties. The bottom is supposed to be flat offshore of the breaker point and to be of uniform slope in the surf zone. The equations are solved for a zero-order solution in which there is no longshore variation. The full equations are then linearized about this solution.

Firstly, it is shown that an assumption of very slight refraction (i.e., variations in wave direction assumed negligible) leads to no periodic solution. An assumption of sinusoidal longshore variation, which is reasonable for these linear equations, leads to a system of ordinary differential equations for which solutions exist and are found numerically. A significant numerical error in LeBlond and Tang (1974) is noted.

**Coastal Engineering Significance.** This paper displays how far away a satisfactory theory is for the details of rip currents, i.e., properties such as their source and spacing. General qualitative concepts of their generation have yet to be translated into really successful theoretical models.


**Keywords.** Currents, Small-Scale; Shear Layer; Theory; Wave Reflection; Wave Transmission.

**Discussion.** The interaction of waves with shear currents which have a length scale of variation much less than a wavelength is modeled in this paper. The region of shear is assumed to be sufficiently narrow that it is modeled by a vertical vortex sheet. The waves are of small amplitude and obliquely incident to the vortex sheet.

The mathematical problem is linearized and possible instabilities of the vortex sheet are excluded from the analysis. An approximate method of solution, which is accurate in other examples where results can be checked, is used to find values for reflection and transmission.
coefficients for various angles of incidence. The latter proves to be remarkably close to the corresponding results for a very wide shear layer which is treated by LONGUET-HIGGINS and STEWART (1961).

Coastal Engineering Significance. This paper has special importance since it is one of the few papers to consider currents with length scales much less than a wavelength. The results allow reflection to be estimated; usually it is very small.


Keywords. Comparison of Theory and Measurement; Currents, Large-Scale; Currents, Ocean; Observation; Spectra; Spectra, Directional; Wave Energy; Waves, Storm; Waves, Wind.

Discussion. Knowledge of the kinematics of the flow beneath surface waves is vital for the design of offshore structures. Due to the technical difficulty of making pertinent measurements in storm conditions, knowledge of the kinematics of storm waves has been based almost entirely on theoretical considerations. Now measurements made with electromagnetic current meters during Tropical Storm Delia have permitted verification of the theories.

There was considerable scatter between the measured velocities and the predictions of unidirectional wave theories, with a clear bias toward overprediction. Use of higher order and irregular unidirectional theories did not substantially improve the comparisons. A good fit with the data could, however, be obtained by using the concept of a directional wave spectrum based on linear wave theory.

The simultaneous wave and particle velocity measurements were used to estimate the directional spectrum through an analysis procedure which took into account the presence of a strong current. The directional spectrum was also hindcast using a numerical model, and the comparison of the hindcast with data was good.

The fact that velocity spectra in confused storm seas can be accurately calculated will be directly important in some design problems. In other cases, it is necessary to know the probability distribution of the extreme events. Using the assumption of a Gaussian sea surface, it was possible to satisfactorily predict the distribution of the magnitudes of velocity. All the comparisons lead to the conclusion that a proper description of storm wave kinematics is dependent on correctly accounting for the directional spreading of the wave energy. (author's abstract)
Coastal Engineering Significance. This is one of the few papers to be based on real field measurements, and it is among several papers whose findings call for increased knowledge of the directional spreading of wave energy. It is also significant in that it shows how unsatisfactory a simplified analysis can be in contrast to a more realistic representation of the current field and wave history.


Keywords. Boundary Layer; Currents, Small-Scale; Experiment; Wake; Wave Breaking.

Discussion. An experiment is described in which surface waves interact with the boundary layer and wake of a vertical flat plate moving through water in its own plane. The waves are generated by identical ship models placed symmetrically on each side of the plate. The bow waves of the ship models intersect in the wake of the plate just behind the plate's trailing edge.

Photographs in the paper contrasting the waves with and without the plate show that there is appreciable interaction. Measurements were made, with and without the plate, of the total drag, the radiated wave field, and the head loss in the water behind the plate and one of the ship models. These measurements are presented in some detail and clearly indicate that the flow around the models and the plate differs from the sum of the flow around a plate on its own plus the flow around the two models.

Coastal Engineering Significance. This is one of the few experiments which have been undertaken to explore the interaction of waves with a small-scale flow field. The interaction in this particular case is strong and involves wave breaking.


Keywords. Observation; Spectra; Waves, Wind.

Discussion. Measurements of wave spectra in the Caspian Sea for depths of water of 6, 12, and 40 centimeters are presented. The data presented are for cases where the current is expected to be within ±30 degrees of
the wind and wave directions. However, currents were not measured at
the time of wave recording but rather by analyzing current measurements
for the same windspeed taken on other occasions.

There is a discussion of the high-frequency characteristics of the
wind-wave spectrum based on Phillips (1966) discussion. The major point
of the paper is that the equilibrium form of the spectrum is modified by
currents.

The measurements for 0.15 to 1.2 hertz show considerable deviation
from Phillips' equilibrium form. There is no real attempt to discuss
experiments and theory, and it is suggested that the large deviation in
shallow depths is due to unrecorded swell.

Coastal Engineering Significance. This paper indicates that the
importance of wave-current interaction is recognized in the Soviet
Union. It is included as a starting point for someone interested in
Russian work on the subject.

16. GARRETT, C., and SMITH, J., "Interaction Between Long and Short
Surface Waves," Journal of Physical Oceanography, Vol. 6, No. 6,

Keywords. Interactions, Short Wave-Long Wave; Radiation Stress; Theory;
Waves, Wind.

Discussion. The effect of long waves on short waves riding over them is
discussed theoretically. This topic has been treated extensively by
other authors, yet this paper includes another possibly significant effect.

The basic mathematics of the problem is first described, since it
is much easier to formulate, in the linear approximation, using wave
action conservation which was not used in earlier work. The various
interaction terms are identified and their order of magnitude is
ascertained.

It is suggested that when the wind is actively generating short
waves, the work done on the long waves by such transfer of momentum is a
dominant term in the energy equation for long waves.

Coastal Engineering Significance. The authors conclude that the
mechanism they describe contributes a significant part of long wave
momentum. Thus, this paper is potentially significant in understanding
the development of wind wave spectra.

This paper is representative of a whole series of papers on short
wave-long wave interaction. The various means of momentum transfer
could also be relevant in other wave-current interactions, especially
for wave-generated currents.

Keywords. Bottom Friction; Comparison of Theory and Measurement; Current Velocity Profile; Eddy Viscosity; Momentum Equation; Theory; Turbulence; Wave Boundary Layer; Wave Effect on Current.

Discussion. Authors present an analytical theory for the bottom friction under combined waves and currents over a rough seabed. The two-layer model is based on time-invariant eddy viscosities increasing linearly with height over the bottom. Inside the wave boundary layer the eddy viscosity is somewhat arbitrarily related to the maximum bed shear stress, while outside this layer the eddy viscosity is related to the mean bed shear stress. Thus, there is a discontinuity in eddy viscosity at the top of the wave boundary layer. The steady current velocity profile, however, is assumed continuous over the boundary layer interface.

The influence of the wave on the current is clearly shown, and for large waves relative to the current this influence is seen to be significant.

The paper starts out with a short review of phenomena, where current wave interaction with a rough bottom is important, namely, the influence on sediment transport and circulation on the Continental Shelf. In addition, the problem of loading on pipelines for gas and oil at the seabed can be mentioned.

The linearized governing equations -- momentum equations in the two horizontal directions neglecting convective accelerations -- are solved for the wave and current kinematics both inside and outside the wave boundary layer. It is stated that the analysis is valid for values of the current of the same order of magnitude as the wave orbital speed. It is found that the current outside the wave boundary layer experiences an increased near-bottom turbulence intensity associated with the waves. The presence of the wave motion tends to retard the current velocity over that expected for a pure current.

This increased bottom resistance leads to the introduction of an apparent roughness parameter, which is the roughness that must be introduced into the conventional logarithmic velocity distribution to give the correct current profile outside the wave boundary layer in a combined current wave motion. Thus the apparent roughness is always larger than the physical (Nikuradse) roughness. It depends on the physical roughness as well as on flow characteristics.
A current wave friction factor, \( f_{cw} \), is introduced, relating the instantaneous bed shear stress to the square of an instantaneous near-bed particle velocity, in analogy with Lundgren and Jonsson (1961) and Jonsson (1966). The friction factor is assumed independent of time.

The apparent roughness and the friction factor are found as functions of a near-bed current speed, \( u_a \), over near-bed orbital speed, \( u_b \), the near-bed orbital amplitude over Nikuradse roughness, and angle between current and wave. It should be observed that \( u_a \) is not known a priori but is itself a part of the solution to the problem.

Numerous illustrative graphs are given. The reasonable magnitude of the apparent roughness is demonstrated by comparison with field observations of very large bottom roughnesses by previous investigators. The paper ends with a discussion giving instructions for the use of the proposed set of formulas. For many cases the ratio \( u_a/u_b \) is small. This is convenient, since here the procedure to calculate the bed shear stress and velocity profiles is greatly simplified.

It should be noted that the quantity \( A_b = u_b/\omega_a \), where \( \omega_a \) is the absolute angular frequency, is not the wave particle amplitude \( a_b \) at the bottom (relative to the current). These quantities are related by \( A_b = (\omega I / \omega_a) a_b \), where \( \omega_I \) is the relative frequency, given by the normal dispersion relation for linear waves.

There is an error in the first term on the right-hand side of equation (54): in the numerator, \( \alpha^{3/4} \) should be \( \alpha^{3/2} \). In this context it should be observed that (54) is in fact a quadratic equation, and so can be solved explicitly.

The paper has some weaknesses. Since the theory is a combination of linear theory (eddy viscosity) and nonlinear theory (quadratic friction), the near-bed current velocity \( \bar{u}_a \) is somewhat fictitious, and a physical interpretation is not feasible. Furthermore, since this reference current velocity is not known beforehand, it is of little use as a parameter in the figures. Also the use of the near-bed orbital speed as a reference velocity in the mean bed shear stress/friction factor equation (15) is not very illuminating, since it forces the dimensionless factor \( V_2 \) to diverge for the case of a vanishing current. If the reference velocity had been the average-over-depth current velocity, then the corresponding dimensionless factor times \( f_{cw} \) would directly show the influence of the waves on the current bed shear stress.

The greatest weakness in the analytical model lies in the estimation of the wave boundary layer thickness. According to equation (38) the somewhat arbitrary choice of \( \delta_w = 2\zeta \) has been made, where length scale \( \zeta \) is given by equation (29). A closer investigation demonstrates that by choosing \( \delta_w = 2\zeta \) instead, one can easily get results that are 20 percent smaller for the maximum shear stress and 30 percent smaller for the mean shear stress. Further inspection makes it likely
that the ratio $\omega/\omega_r$ is somewhat smaller than one (see Christoffersen, 1980, in preparation, 1983). At least one accurate laboratory experiment is needed to settle this problem.

Another matter is that for small values of wave particle amplitude over roughness, a constant eddy viscosity in the wave boundary layer is more likely than a linearly varying one. Using this concept, a simpler set of formulas emerges for friction factors, etc. (see the above references).

Coastal Engineering Significance. This is a serious attempt to determine the bottom shear stress and the current profile in a three-dimensional current wave motion. It gives detailed instructions for the calculation procedure. Analytically it is quite complicated, involving Kelvin functions and iterations. For many cases of practical interest, however, the near-bed current speed is small compared with the orbital speed and this allows approximations which greatly simplify the calculations. (In the above-mentioned references it is shown that the Grant/Madsen friction factor formula, which involves Kelvin functions, generally can be approximated by a logarithmic friction factor expression. This is formally analogous to Jonsson's friction factor formula, adjusted to take the current into account.) Some of the major assumptions are open to discussion, but the model can explain some of the very large (apparent) roughnesses found in field experiments by other investigators.


Keywords. Interactions, Internal Waves; Theory; Wave Effect on Current.

Discussion. The problem is treated theoretically starting with Laplace's equation for a liquid of two layers of different density and the exact inviscid boundary conditions. A perturbation solution for the combination of surface waves and internal waves is sought for two different cases. The group velocity of the surface waves is assumed to be close to the phase velocity of the long internal wave in order that there be resonant interaction.

The first case considered is where the particle velocities due to surface waves are much greater than those due to the internal wave. Then modulation of the surface waves induces a mean flow which affects the internal wave. Interaction equations are derived, and solutions which propagate unchanged are derived.
In the second case the velocity fields of the two waves are of the same order of magnitude. Interaction equations are derived, and it is noted that they are identical to equations for capillary-gravity wave interaction derived by Kawahara, et al. (1975). The equations are not interpreted physically.

Coastal Engineering Significance. This type of problem is important in the wave-current context since it demonstrates the fact that the current field is itself affected by the waves. Analysis of this type of problem has proceeded much farther than that of any other wave-current interaction problem. Further study of work such as this may increase understanding of the general problem of wave-current interaction.


Keywords. Currents, Unidirectional; Forces on Cylinders; Forces on Structures; Review; Spectra; Statistics; Theory.

Discussion. The report starts with a simplified review of wave-current interaction and waves propagating in a single direction onto a current with the same or opposite direction. Attention is drawn to the high-frequency limits of a spectrum and its "equilibrium range" due to breaking. Several examples are given of transformed spectra and of waves generated on a current.

The above spectra are then used in a variety of calculations for the probability distributions of horizontal forces on vertical cylinders. These calculations use the results of Borgman (1965, 1967), which are based on the Morison equation for the drag. A further section then uses these results and considerations of wave climate to make long-term force distribution estimates.

Coastal Engineering Significance. This report gives a convenient up-to-date summary of methods that are used to estimate forces on structures in the presence of waves and currents. There are a considerable number of simplifications in the analysis, e.g., unidirectional currents and waves, Gaussian statistics, and Morison's equation. However, in considering general cases it would not be possible to progress far without them. The variety of examples may be useful for getting an appreciation of the subject.

Discussion. Interactions between steady nonuniform currents and gravity waves are generalized to include the case of a random gravity wave field. The Kitaigorodskii-Pierson-Moskowitz frequency spectrum is used as the basic spectral form for zero current condition. Modified spectral functions in both wave number and frequency spaces under the influence of current are found by using energy conservation and kinematic wave conservation laws. The relative importance of the current-wave interaction was measured by the nondimensional parameter $U/C_0$, with $U$ as the current speed and $C_0$ the phase speed of a wave under no current. As a result of the current-wave interaction, the magnitude and the location of the energy peak in the spectrum are altered.

Since the phase speed of gravity waves is a monotonically decreasing function of wave number and frequency, the influence of current will be predominant at the higher wave number range. Furthermore, the contribution from the higher wave number range dominates the surface slope spectrum; the current conditions change the surface slope pattern drastically. This phenomenon is studied by use of Phillips' equilibrium range spectrum in wave number space. It was found that the root-mean-square (rms) surface slope is extremely sensitive to the change of current conditions especially for the case of adverse current, but eventually becomes saturated at a high positive value. The surface slope data together with a generalized dispersion relation offer a possible current measurement technique which appears ideally suited for remote-sensing devices such as stereoscopic photography and radar scattering. (author's abstract)

Coastal Engineering Significance. This is one of the few investigations of the effect of current on the generation of wave spectra (see also GADZHIYEV, KITAYGORODSKII, and KRASITSKIY, 1978). The findings that opposing currents produce a high surface slope and that this slope approaches an equilibrium value are in agreement with intuitive expectations. The authors suggested that slope and dispersion relations can be used in satellite observation of waves. This suggestion has been the object of many subsequent studies.


Discussion. The paper describes a combination of experiment and field observation. Initially a research vessel generated internal waves by proceeding slowly through an area with a layer of relatively freshwater at the sea surface. Then, after about 5 minutes, the ship traveled through the group of internal waves with measuring instruments in position.
Among the measurements made were those of currents at various depths, the wind, wave height and slope. Photographs included in the paper clearly show how gentle wind waves are modulated in wave steepness.

Interpretation of results includes a good correlation between the total surface slope variance and the current, optimally shifted in phase. The phase shift was somewhat scattered between 0 and -90 degrees.

The authors comment that their visual and photographic observations are better indicators of internal waves than measurements of the surface along a single line.

This paper is followed by another (Hughes, 1978) which provides a good theoretical discussion and comparison of the experiments.

Coastal Engineering Significance. The authors provide clear documentation that an interaction exists between the currents induced by internal waves and the surface wind waves. From an engineering point of view, the results of this paper should alert wave gage users to the fact that such interaction can exist and affect wave gage statistics, particularly at the higher frequency end of the spectrum.


Keywords. Current Refraction; Currents, Shearing; Currents, Unidirectional; Historical Interest; Theory, Ray; Wave Height; Wavelength; Waves, Deepwater.

Discussion. When ocean waves, moving through deep still water, encounter a current, moving at an angle with the wave direction, the waves undergo a change in length, steepness and direction of travel. A theoretical development is given for these factors in terms of initial wavelength and direction, and the magnitude of current. Discussion is given of the action of a coastal current in affording protection against short period waves. (author's abstract)

The early pioneers in current wave studies did not have at their disposal the correct energy principle for waves on large-scale currents: wave action conservation between rays. The author's equations (6), (7) and (8) and Figure 4, are therefore, not correct. (A closer inspection shows that the correct result, still disregarding reflection at the discontinuity, is obtained by deleting the denominator \((1 + m \sin \alpha)\) in the last bracket in the expression just above equation (8)).
The values of wave direction and wavelength that are presented for refraction by shear currents are correct. The paper contains photographs of the entrance to Humboldt Bay which gives an indication of the increase in wavelength with a following current, and the increase in steepness on an adverse current under natural conditions.

Coastal Engineering Significance. This is one of three papers included in the annotated bibliography for its historical significance. Although superseded and corrected by later work, it has a useful simplicity and directness.

23. JONSSON, I.G., "The Friction Factor for a Current Superimposed by Waves," Progress Report No. 11, Coastal Engineering Laboratory and Hydraulic Laboratory, Technical University of Denmark, Copenhagen, Denmark, Apr. 1966, pp. 2-12.

Keywords. Bottom Friction; Comparison of Theory and Measurement; Conservation Equations; Currents, Unidirectional; Radiation Stress; Setdown; Setup; Theory; Turbulence; Wave Dissipation; Wave Effect on Current.

Discussion. Plane turbulent flow over a horizontal, rough bottom is considered. The current velocity is assumed uniformly distributed over depth and not to exceed the bottom particle velocity in the wave motion. Further the Froude number is assumed small. Both waves and current are steady.

For a pure wave motion over a horizontal bed, the reduction in radiation stress in the direction of wave travel will cause a small setup of the mean water surface. From the momentum equation for the combined current wave motion it is demonstrated, however, that even a very small current velocity -- order of magnitude Froude number 0.01 -- will produce a negative tilt of the mean water surface, a setdown. This setdown occurs once the downward slope of the surface needed to drive the current exceeds the setup due to the waves.

The instantaneous bed shear stress is assumed proportional with the instantaneous total particle velocity (wave plus current) squared, and phase differences are neglected.

The result of the momentum equation is that measurement of wave height gradient and mean water surface slope will give the current friction factor. The energy equation, however, makes it possible to eliminate either of the two quantities.

In the formulation of the energy equation, it is pointed out that the reference level for the potential energy must be horizontal. This adds an extra term to the conventional expression for the energy flux (at the time of the paper, wave action had not yet been introduced in water wave dynamics). A strict physical interpretation of the new
energy equation is given. (It should be noted that Inman and Bowen (1962) missed the just-mentioned correction term in their analysis.) The dissipation per unit area is assumed to be the product of instantaneous bed shear and corresponding particle velocity.

Also the energy equation results in an expression which determines the current friction factor, knowing wave height gradient and mean water surface slope. Eliminating either of these two quantities from the two conservation equations results in two expressions for the current wave friction factor, which again determines the current friction factor through a factor containing the ratio between current velocity and maximum wave particle velocity at the bed.

Measuring the slope of the mean water surface is difficult; however, measurements of the decrement of wave height have been performed by Inman and Bowen (1962), in a wave flume with a rippled bed, and with currents in the direction of wave travel of up to 6 centimeters per second. A run with waves of period 1.4 seconds, wave height 15.4 centimeters and water depth 50.3 centimeters is analyzed.

For no current the wave friction factor is 0.30 and the current wave friction factor (i.e., the factor giving the instantaneous bed shear from total bed velocity squared) was almost constant, ranging from 0.26 to 0.31, i.e., close to the wave friction factor. This is expected, since current velocity over maximum wave particle velocity at the bed is small, at maximum about 0.28.

A further result of the analysis is that "large waves" (wave particle velocity much larger than the current velocity) produce a current friction factor (i.e., the factor giving the mean bed shear stress from mean velocity squared) which is larger -- by order of magnitude -- than the friction factor for a pure current. In the experiments the friction velocity was, in fact, of the order of magnitude of the current velocity. So the superposition of the waves on the current drastically increased the mean bed shear stress, as expected.

A simple interpolation formula for the current wave friction factor is introduced, giving rather good agreement with the experiments that predict mean bed shear stress. It should be observed, though, that since the current velocities in these experiments are quite weak, this agreement does not really verify the interpolation chosen. Rather it indicates that the idea of a constant friction factor on the instantaneous velocity squared is reasonable when calculating the instantaneous shear stress. (The interpolation formula has later been used with some success by Brevik (1980) and BREVIK and AAS (1980).)

In the present adaptation of the Inman and Bowen (1962) data there is a small error, since wave phase speed etc. is calculated without correction for the "Doppler shift." Because of the small current velocities, however, the maximum error on the wavelength is only about 3 percent.
Considering the high friction velocities, the assumption of a uniform velocity distribution can be questioned.

Coastal Engineering Significance. This paper is important in demonstrating that waves and currents together can result in drastically higher bottom shear stress than for currents alone. A procedure is presented for experimental measurement of the friction factor and tested against data from Inman and Bowen (1962).


Keywords. Currents, Large-Scale; Flow, Irrotational; Mass Transport; Setdown; Theory; Wave Action.

Discussion. Small-amplitude waves are considered, propagating with slow variations of depth and current in both horizontal directions. In many respects it is a generalization to three-dimensional flow of the results obtained in JONSSON, SKOUGAARD, and WANG (1970).

The currents are steady, the flow irrotational, and all results are taken to second order in wave height. Dissipation is neglected.

A significant new result is that wave action flux is proportional to the total energy flux with the mean energy level as a datum. This leads directly to the concept of the wave ray and the wave action conservation principle.

The setdown of the mean water surface is easily found as the mean velocity head at the bottom, using a result for progressive waves from the above reference. The result shows the importance of distinguishing between the average-over-depth current velocity and the Eulerian mean velocity below wave trough level.

It is also shown that the so-called mass transport in water waves is a boundary problem rather than a physical necessity.

Coastal Engineering Significance. The most important finding is the simple physical interpretation of wave action flux. This physical interpretation in hydraulic terms is familiar to engineers.

Amsterdam, The Netherlands, 1978(b), pp. 162-203. (See also Report No. 515, Danish Center of Applied Mathematics and Mechanics (DCAMM), May 1979.)

**Keywords.** Conservation Equations; Current Depth Refraction; Currents, Large-Scale; Mass Transport; Review; Theory; Theory, Ray; Wave Action; Wavelength.

**Discussion.** The scope of this survey is to provide the necessary engineering background for calculating the lengths, directions and heights of sea waves propagating over a water area with an arbitrary large-scale current distribution and bottom topography. So the emphasis is on so-called current depth refraction, and a general procedure for solving this problem is outlined. The survey begins by presenting a number of wave phenomena, where interaction with currents is important. Large-scale currents are considered, i.e., currents which only vary significantly over many wavelengths. The same is assumed for water depths. Only regular and nonbreaking waves are studied.

Wavelengths are determined by wave crest conservation, and directions of wave travel by the orthogonal equation, which is presented. The flux of wave action between neighboring wave rays, properly reduced because of dissipation, then determines the wave heights. It is stressed throughout that there are three sets of characteristic curves in the general case of current depth refraction: streamlines, wave orthogonals, and wave rays. It should have been emphasized, however, that the wave rays are the primary curves. They can be calculated one at a time from initial conditions and will then determine the orthogonal field. In general, the opposite cannot be done.

The complete set of depth-averaged conservation equations for mass, momentum, and energy are given in three dimensions, including bottom friction and the corresponding energy dissipation. It is further shown how wind stress and horizontal turbulent shear forces can be included in vertical sections.

The energy equation is given both in the traditional form with the radiation stress appearing, and in wave action conservation form, thus introducing the wave ray. In the latter case the dissipation term takes a special simple form, similar to wave action.

Three special cases are discussed in detail: pure current refraction, straight coastline, and inflow from the sides.

Normally, the wave transformation is calculated on a given current. Here an attempt has been made to find criteria when the wave feedback on the undisturbed current is important. The discussion is incomplete, though, since the often significant increase in current bed shear, due to the wave motion, is not mentioned explicitly.
The existence of the so-called Stokes mass transport is discussed in detail.

A steady situation is considered throughout. It is explained, though, how the energy equations transform if there is a time-varying current.

Shallow- and deep-water approximations are normally presented after the general expressions. The effect of the current profile not being vertically uniform is discussed in a special section, where linear current profiles are considered. The tricky problem of allowable boundary conditions is not treated. For irrotational flow this has been discussed in Skovgaard and Jonsson (1976).

Coastal Engineering Significance. This article is important because it is published in an engineering text that will introduce the wave-current interaction considerations to engineers not previously acquainted with them. Tables and graphs are included to show wavelength changes due to interaction with current. In particular, the "stretching" due to a following current and the "compression" due to an opposing current are demonstrated.


Keywords. Current Refraction; Currents, Large-Scale; Currents, Shearing; Currents, Unidirectional; Setdown; Theory, Theory, Ray; Wave Filtering; Wave Height, Wavelength.

Discussion. The paper deals with the transformation of plane, monochromatic waves, as they cross a shearing current, where the current velocity changes from one value in region 1 to another in region 2.

The object of the study is to determine the direction of propagation, and the length and height of the wave motion in region 2. This is done by applying Snell's law, and the conservation equations for wave crests and wave action. Input parameters are water depth (assumed constant), absolute wave period, angle of incidence, initial wave height, and current velocities in the two regions. Amplitude effects are disregarded, and the current gradient is assumed small.

This is an extension of the work by LONGUET-HIGGINS and STEWART (1961), who considered the special case of deepwater waves progressing from still water into a region with a uniform current. Here the depth is arbitrary, and also current velocities can be arbitrary on both sides of the shear layer.
The results of the study are given in a number of dimensionless graphs, between which interpolation can be made. Since the quantities in the graphs may be difficult to read, and also because interpolation is necessary, the calculation procedure is illustrated in an example. It is shown to be quite simple.

A physical discussion of the transforming effect of the shear layer is given on the basis of a sequence of dimensional graphs. It is shown that everything else being equal, wave height (and steepness) in region 2 has a minimum for a certain value of the current velocity in that region. This is because a large positive current results in a refraction angle equal to 90 degrees, and so the wave rays tend to lie infinitely close, resulting in (theoretically) infinite wave heights. The waves are swept along the streamlines here.

For a large negative current, the absolute group speed becomes small, and since wave action flux (wave action times absolute group speed) is constant, the result is high waves here also. This is the first "filtering effect" shown: both a large positive current and a large negative current in region 2 can cause wave breaking.

Another filtering effect is due to the fact that wave steepening across the shear current is most pronounced for the short waves, and therefore, the long waves pass through more easily. Isaacs (1948) demonstrated this effect with a photo in which the breaking short waves show up as a foam line.

Physically possible solution domains are also given, as well as an analytical expression for the (small) change in mean water level across the shear layer. A horizontal bottom is assumed throughout. The calculations are easily extended, however, to the case of depth contours running parallel to the streamlines.

Dissipation due to the bottom friction and vortex formation in the shear layer is neglected; however, the wave action conservation principle for this situation is presented.

An appendix provides general expressions for the determination of wave orthogonals and rays, emphasizing the important difference between these quantities.

Coastal Engineering Significance. This paper is probably the first to give an engineering formula for wave transformation across a shearing current.

Discussion. The refraction of small surface gravity waves on large-scale currents over a gently sloping seabed are studied. Some results are a generalization to three dimensions of those obtained in JONSSON, SKOUGAARD, and WANG (1970).

Assuming irrotational flow, the complete set of conservation equations for combined current depth refraction is presented, correct to second order in wave height, and solved in two special cases. Dissipation is neglected.

One case is two-dimensional flow, pure current depth shoaling, studied previously in the above reference. Similar graphs for variation in wavelength and height are given.

The other is a straight coast with the current, irrotational or rotational, running parallel with the straight and parallel depth contours. In the former case it is shown that the current effects on the orthogonals and rays are opposite: If one bends more, the other bends less.

As a start on wider classes of problems, a general solution procedure for the refraction equations is sketched. Two equations for the current wave setdown are derived. One which is general, and one which only applies to progressive waves on a current. The velocity potential for a three-dimensional current wave motion is given.

Coastal Engineering Significance. The equations for current depth refraction are solved in two situations, current depth shoaling, and refraction at a straight coast with the current parallel to the bottom contours. The "bending" by the current of wave orthogonals and rays is illustrated and discussed. New formulas for the current wave setdown are derived. The neglect of dissipation means that wave height variations are only qualitatively correct and limit the applicability of the setdown expressions.

Discussion. Regular waves propagating on a steady current over a gently sloping bed are analyzed. The current varies linearly with depth, and so has constant vorticity. The analysis is two-dimensional, and dissipation is neglected.

Definitions and expressions correct to second order in wave amplitude are given for the radiation stress, wave energy density, and total energy flux.

Applying average Lagrangian techniques, the authors generalize BRETHERTON and GARRETT's (1968) results for waves on a current with uniform velocity profile, namely that wave action density is equal to the wave energy density divided by the intrinsic (i.e., relative) angular frequency. In the extended theory the relative frequency is that relative to a frame of reference moving with the average-over-depth current velocity. This determines the wave amplitude variation explicitly.

Also an analytical expression for the current wave setdown is found. Graphs illustrate the effect of the vorticity on wavelength, wave amplitude, and setdown.

Coastal Engineering Significance. This paper demonstrates that a linear current profile over depth is a good first approximation to real flows with waves superimposed. It further shows that in the wave action conservation equation, the relevant frequency is that relative to the average-over-depth current velocity for linear current profiles.


Keywords. Conservation Equations; Current Wave Shoaling; Currents, Large-Scale; Currents, Unidirectional; Flow, Irrotational; Mean Energy Level; Setdown; Theory.

Discussion. This paper considers interaction between regular gravity waves and a steady current over a gently sloping bed. Flow is irrotational and a second-order Stokes wave expansion is used. The general concept of a mean energy level is introduced, and is shown to be a constant horizontal level for periodic, irrotational free-surface flow.

The complete set of conservation equations for a two-dimensional current wave motion is presented, leading to practical equations for wavelength and height, and current wave setdown. Dissipation is disregarded, and there is no comparison with measurement.
A simple graphical method for the determination of the wavelength is introduced. Graphs and tables are presented for direct determination of wavelength and height for a given dimensionless water depth and current flux. Explicit wavelength expressions are given for deep and shallow water.

The velocity potential is produced for two-dimensional flow, and this leads to an expression for the depression of the mean water surface below the mean energy level, the so-called setdown. The necessity of using a horizontal datum when calculating the energy flux is shown in this connection.

It is further demonstrated that the energy flux equation with the mean energy level as a datum is a special case of Garrett's (1967) adiabatic invariant expression (i.e., introducing the new concepts, wave action and flux).

The findings are generalized to three-dimensional flow in JONSSON (1978a) and JONSSON and WANG (1980).

Coastal Engineering Significance. Significant to coastal engineering is the convenient calculation of the wavelength in a three-dimensional current wave situation, and in the formula given for solving the conservation equations for two-dimensional flow. A numerical example illustrates this, and a final example demonstrates the importance of taking the effect of a possible current into consideration when calculating wave heights from bottom pressure cells.


Keywords. Currents, Unidirectional; Currents, Vertical Shear; Currents, Wind-Drift; Experiment; Fetch; Spectra; Waves, Wind; Wind Shear; Wind Velocity Profile.

Discussion. Experimental results are presented of wind waves generated on currents in a wind-wave tunnel with a water circulating pump system.

The uniform test section was 1.5 meters wide, 1.3 meters high, and 28.5 meters long. Water depth was 0.50 meter. Average windspeeds were nearly 5.6, 8.2, and 11.0 meters per second. The average crosssectional current velocity ranged from about +30 to -20 centimeters per second.

The waves were measured with a resistance-type wave gage, and wind velocity profiles by a pitot static tube and a differential pressure transducer. Current velocities were measured with a small propeller-
type current meter, and the surface current using paraffin flakes. The drift current profile near the surface was checked with a hot-film anemometer.

The directly obtained wave data are the apparent spectra, where frequencies correspond to absolute phase speeds. To calculate these the wave speed solution for a logarithmic drift current obtained by Kato (1974) was used. By further assuming the conventional dispersion relation for the relative phase speed (corresponding to a uniform current velocity distribution), the true spectra, corrected for the Doppler effect, can be calculated.

Wind friction velocities found from the wind profiles were larger for adverse than for favorable currents. In the former case the lateral current velocity distribution was almost uniform, while in the latter velocities were largest in the central part.

The significant wave height, determined as $4 < \eta^2 >$ with $\eta$ being the surface displacement, was chosen to represent wave heights. Further, a "dominant" wavelength was introduced, corresponding to the peak frequency in the true energy spectrum. Everything else being equal, significant wave heights and dominant wavelengths were largest for adverse currents. (The variation in wavelength is contrary to what happens when waves move from still water into a current region.)

For a given windspeed the high-frequency part of the true spectra almost coincide, regardless of the current magnitude. This leads authors to conclude that the most prominent effect of a water current on the development of wind waves is a change in the effective fetch length.

Coastal Engineering Significance. This is probably the first attempt to evaluate quantitatively the effect of a current in the prediction of wind waves.


Keywords. Current Refraction; Currents, Large-Scale; Currents, Ocean; Currents, Shearing; Theory, Ray; Wave Reflection; Waves, Ocean; Waves, Wind.

Discussion. The paths or "rays" of packets of water-wave energy propagating on a current with uniform transverse shear are computed. Diagrams are given showing that they differ markedly from lines which are everywhere perpendicular to the wave crests. Results are also obtained for the curvature of rays.
The propagation of wind-generated waves onto and in ocean currents such as the Gulf Stream is then discussed in a quantitative manner making use of the above results. The reflection and trapping of waves are particularly noted.

The effect of the circumpolar Antarctic Current on waves propagating from the Indian Ocean to the Pacific Ocean is estimated and found to be of the order of magnitude necessary to explain observations by Munk, et al. (1963) and Snodgrass, et al. (1966). These observations indicated that swell in the Pacific Ocean had traveled greater distances than were possible on the basis of great-circle propagation. The refraction of the ocean currents is adequate to curve the propagation path of the swell into the geometric "shadow" of landmasses for these examples.

Coastal Engineering Significance. This paper is important since it deals with the refraction of waves by ocean currents in a quantitative manner. The estimates of wave behavior are based on measured currents and waves. It also provides interesting, accurate diagrams showing how rays differ from curves parallel to the wave number vector on a shearing current.


Keywords. Experiment; Forces on Structures.

Discussion. A relatively full report is given of force and moment measurements on a model caisson. The study was made in connection with the Delta Plan (established in 1953 by the Rijkswaterstaat), and the caisson was one of a group of four closely spaced piers (three in a preliminary investigation). Currents and waves were directed at the structure. Various angles of incidence and angles between waves and currents were used. Some experiments were with periodic waves, others with wave spectra.

Conclusions drawn from the experiments are that the wave-current interaction is especially important for the transverse force component, while less so for the longitudinal force component provided the appropriate Doppler correction to frequency is made. The average force component is always increased over that due to the current alone, whatever the wave direction relative to the current. There are indications from the torsion measurements that there is appreciable interaction between the waves and the flow due to the presence of the cylinder.
Coastal Engineering Significance. This is an experimental study on large structures subject to wave and current action, showing that the force on the structure due to the combination of waves and currents differs from their separate effects. The paper presents numerous graphs of transfer functions for various force components and numerous tables of measurements. A study of these detailed results provides an indication of the magnitudes of these forces. No other measurements of this type have been published.


Keywords. Review, Waves.

Discussion. This book gives an up-to-date account of those water motions in the ocean which may be described in terms of waves in, on, or on, an incompressible fluid. For the most part, these are waves involving gravity (surface and internal waves) and rotation. The simpler, more basic properties of the waves alone are considered first. Then the influence of lateral boundaries, statistical methods, wave interactions, wave-current interactions, wave generation and dissipation is treated. Each type of wave is discussed in the different contexts. The book is written with a view to bringing together some of the widespread literature on the subject. It is a research level book with numerous references to research papers, and a bias toward more recent results.

Coastal Engineering Significance. The book is valuable for reference on a wide range of topics concerning surface waves and their interaction with currents. It also sets these wave motions in the wider context of all waves in the oceans. Similarly, some of the currents that influence surface waves are simply the local water motion of waves (e.g., tides) on much longer scales.


Keywords. Radiation Stress; Theory.

Discussion. The book gives a comprehensive introduction to wave motion in fluids. It emphasizes the fundamental ideas of wave motion and wave propagation.

In particular, the theory of rays for general wave systems is given in Section 4.4, followed by the effect of mean flows in Section 4.6, which includes a detailed physical interpretation of radiation stress (on page 329, the term radiation stress is not used, but the corresponding momentum flux and the more general notion of Reynolds stress are applied) and the introduction of wave action. An annotated bibliography is a distinctive feature of the book.
Coastal Engineering Significance. Although much of the presentation of the book is in terms of sound waves and internal waves, it is at present the best approach to understanding the interaction of water waves with large-scale currents.


Keywords. Currents, Tidal; Currents, Unidirectional; Dispersion Relation; Equations of Motion; Interactions, Short Wave-Long Wave; Interactions, Wave-Wave; Radiation Stress; Theory; Wave Energy; Waves, Deep-water.

Discussion. This is the first paper that deals correctly with the interaction between water waves and currents. (The corresponding problem for sound waves was correctly treated by Blokhintzev, 1946.)

The problem of interaction between two wave trains is treated as a perturbation problem; that is, the interaction is assumed to be quadratic in wave amplitudes. Examples considered include tidal streams and standing waves.

There is a detailed physical discussion of the results with particular emphasis on the energy of the short wave motion and its transfer. In particular, the term "radiation stress" is introduced for the Reynolds stress of the wave motion. This leads to an equation for the energy of the short-wave motion explicitly including the loss, or gain, of energy transferred to or from the long-wave motion.

Although most of the paper is in terms of wave-wave interaction, the important section on radiation stress is written from the viewpoint of waves on a steady current.

Coastal Engineering Significance. The existence of energy transfer between water waves and current, or longer waves, is recognized for the first time in this paper. It means that it is possible for the amplitudes of waves on currents to be deduced correctly. Earlier work by UNNA (1942) and JOHNSON (1947) had failed to calculate amplitudes correctly, though the general trend of their results is correct.

The interaction between short waves and tides described in this paper is a valuable theoretical account of a commonly observed variation of wave energy which needs to be allowed for both in interpretation of observations and in design.

Keywords. Conservation Equations; Current Refraction; Currents, Non-uniform; Currents, Opposing; Currents, Shearing; Currents, Slowly Varying; Radiation Stress; Theory; Theory, Ray; Wave Energy.

Discussion. A formal perturbation analysis gives the second-order interaction between gravity waves and a current which has a small linear variation in magnitude. The result is used to resolve an ambiguity in a wave-energy conservation equation deduced directly from the expression for energy flux. This leads to the correct, unsteady two-dimensional energy conservation equation.

As well as extensive physical discussion, the following particular examples are examined: waves on currents varying (a) in the direction of flow, (b) across the direction of flow, (c) sinusoidally, as in tidal currents, and (d) along the "centerline" of a flow convergence.

Coastal Engineering Significance. This important paper gives the first major theoretical results for the interaction of surface gravity waves with large-scale currents. The examples given are the primary simple examples to which these equations can be applied. The results are of significance to all wave-current interactions.


Keywords. Currents, Wave-Induced; Mass Transport; Radiation Stress; Setdown; Setup; Surf Beat; Theory; Wave Breaking; Wave Groups.

Discussion. The concept of radiation stress is used to determine the mass transport associated with groups of surface gravity waves. Three methods of derivation are given (a) a direct perturbation approach, (b) an indirect perturbation approach, and (c) a consideration of conservation of mass and momentum following WHITHAM (1962) and indicating the limitation of Whitham's results to wave groups which are much longer than the depth. Two simple examples are discussed in detail; wave groups long compared with the depth and groups formed by only two sinusoidal components. It is found, contrary to all expectation, that high waves are associated with a negative mass flux.

Steady wave trains in water of variable depth are considered and the mean water level is shown to decrease as the depth decreases and wave height increases, that is "setdown." It is indicated that the
response of the mean level to wave groups can be much larger, but it is pointed out that it is a resonant response which would need time to build up. The effects of wave dissipation and breaking are shown to lead to "setup."

These ideas are compared with observations of reflected long waves from swell (Tucker, 1950). They give the sign of elevation in the long waves (depression rather than elevation) but the variation with incident wave amplitude is unconvincing.

Coastal Engineering Significance. This paper presents the first theoretical analysis and identification of wave setup and setdown. It is of great significance in providing the theoretical basis for the study of wave-induced currents and is still an appropriate starting point for their study.


Keywords. Averaged Equations; Current Refraction; Currents, Large-Scale; Currents, Opposing; Currents, Shearing; Group Velocity; Interactions, Wave-Wave; Radiation Stress; Setdown; Setup; Surf Beat; Theory; Wave Groups.

Discussion. The results of LONGUET-HIGGINS and STEWART (1960, 1961, 1962) are brought together in this paper, which omits many of their mathematical details and yet presents a plausible physical argument.

The initial section gives a very detailed account of all three components of radiation stress $s_{xx}$, $s_{xy}$, $s_{yy}$ for a sinusoidal traveling gravity wave. Standing waves and capillary-gravity waves are then considered in slightly less detail.

Applications that are described are (a) wave setup on a beach, which includes some observational data, (b) the setdown under groups of waves in deep water, (c) wave groups in shallow water and their relevance to "surf beat," (d) the interaction of waves and steady currents including irrotational plane strains, laterally converging currents and shear flows, (e) nonlinear interactions between waves of disparate lengths, and (f) the damping of gravity waves by capillary waves.

Coastal Engineering Significance. This paper is intended for non-theoreticians interested in the physical applications of the results described in more detail in earlier papers (LONGUET-HIGGINS and STEWART, 1960, 1961, 1962). Coastal engineers may prefer to have that detail; however, this paper does present some further extension of their work. It was of great value in extending knowledge of their basic advances.

Keywords. Case Study; Continental Shelf; Current Refraction; Currents, Nearshore; Currents, Ocean; Currents, Opposing; Fetch; Forces on Structures; Interactions, Short Wave-Long Wave; Meteorology; Observation; Waves, Deepwater; Waves, Wind.

Discussion. This paper gives a detailed account of the conditions in which unusually large waves have caused damage to several vessels when proceeding in a southwesterly direction of the southeast coast of South Africa. Eleven cases of ships encountering such conditions, all on the southwesterly flowing Agulhas Current, are recounted. Severe structural damage was sustained by some of the vessels; in particular, the tanker "World Glory" broke in half in June 1968.

The abnormally high waves occur in winter when wind waves coming from the southwest meet the Agulhas Current which is directly against them. They are reported to last for only a few minutes, but heights in excess of 18 meters have been reported. There are consistent reports of a long, deep trough occurring in front of the most severe wave. This configuration has led to cases where the ship's forepart has failed to rise as the crest advances and causes damage by its impact on the deck.

Detailed descriptions of the environment in which these waves are generated are given. They occur off the narrow Continental Shelf; wave conditions are often markedly more severe just outside the 200-meter contour. The possibility that the numerous submarine canyons refract the waves is also mentioned.

The Agulhas Current is at its strongest just off the Continental Shelf, attaining a depth of 300 meters and a maximum speed of 2 to 2.5 meters per second just seaward of the shelf edge. A counter-current is observed inshore after the passage of a cold front.

In all cases of "freak" waves, a southwesterly wind had prevailed for a long time over a long fetch. Specific details are given for one incident, and it is concluded that waves could have a strong wind generating them for as much as 1932 kilometers. It is suggested that other locally-generated waves are present at the same time, and that the combination of these waves, amplified by the current, produces the really dangerous waves.

Coastal Engineering Significance. This is an admirably well-documented study of abnormal wave behavior in a region where high waves meet a strong current. It provides stimulating information, and it can be used as both a basis and an inspiration for much theoretical work in this highly important field for navigators.
Although this paper concerns waves in deep water, the ways in which current systems such as this increase the probability of extreme waves is of direct relevance to any structural design for waters in which strong currents occur.


Keywords. Comparison of Theory and Measurement; Currents, Unidirectional; Currents, Wind-Drift; Experiment; Wind Shear; Wind Velocity Profile.

Discussion. These are detailed measurements of the phase speed of mechanically generated surface waves at different periods and strengths of an opposing wind. The experiments were carried out in a wind-wave tunnel 0.6 meter wide and 0.8 meter high. The glass test section was 13.4 meters long, and the maximum wind fetch about 8.5 meters. Water depth throughout the experiment was 0.35 meter. Wave periods were 0.8, 1.0, and 1.2 seconds, and windspeeds ranged from 2.5 to 12.5 meters per second.

Wave measurements were made simultaneously at six locations with resistance-type gages. The complex Fourier coefficients at the fundamental frequency were used for calculating the amplitude and phase of the waves. The phase speed of the waves was then determined from the phase difference between any two stations. Wave heights ranged from 1.5 to 4.0 centimeters. Vertical wind profiles were measured with a standard Pitot tube. The reference airspeed was that of the free stream over the rough upstream transition plate.

The surface-drift current was measured using small paper floats, and the magnitude was about 3 percent of the reference airspeed, as found by many other investigators. The hydrogen bubble technique provided the current profile. In all experiments, the current became zero about 7 centimeters below the water surface.

Friction velocity and surface roughness were determined from the wind profiles which were logarithmic. Friction velocity was somewhat larger in the presence of the mechanically generated waves, except at the downstream end of the tunnel.

The phase speed of the waves decreased markedly with windspeed, and the rate of decrease increased with decreasing wave period. The latter observation is easily explained, since the smaller the period, the more the wave motion takes part in the upper region, where the wind drift is strongest.
A theory is given for the phase speed in the presence of an adverse wind. Using a stream-function approach in the inviscid momentum equations leads to a well-known form of the Orr-Sommerfeld equation. Solving these for the air and water flow separately, and combining them by the dynamic (pressure) and kinematical boundary condition at the interface, a perturbation technique yields the following expression for the phase speed \( C \):

\[
\frac{C}{C_0} = 1 + \frac{C_w}{C_0} + \frac{C_a}{C_0}
\]

in which \( C_0 \) is the phase speed in the absence of the wind, \( C_w \) the correction for the surface current, and \( C_a \) the correction for the wave-induced pressure fluctuations in the wind at the water surface. In the solution for water flow, a parabolic profile going down to zero was employed (Kato, 1972), and for airflow, a logarithmic profile was used up to a uniform free stream. For the wave-induced aerodynamic pressure at the water surface, Miles' (1957) expression was introduced.

For a fixed windspeed, \( -\frac{C_w}{C_0} \) grows rapidly with wave frequency, while for \( -\frac{C_a}{C_0} \) the increase is much more gradual. In the example given, the two curves follow each other up to about 1 hertz.

The first correction term in the phase speed expression is proportional to the surface current and thus to windspeed; it is, therefore, positive or negative according to a favorable or adverse wind. The aerodynamic term \( \frac{C_a}{C_0} \), however, is always negative. This means that for waves of the same period, the wave speed will decrease more significantly for an adverse wind than it will increase in a favorable wind. Measurements by Shemdin (1972) with a favorable wind, compared with the authors' results, confirm this theoretical prediction.

The results of the theoretical analysis were compared with the experimental results. There was fair agreement for periods 0.8 and 1.0 second, but some discrepancy for 1.2 seconds. According to the authors, this discrepancy may be attributed to the chosen parabolic current profile. They refer to measurements by Dobrokloenskiy and Lesnikov which show a logarithmic profile. Since the parabolic profile implies laminar flow, and the Reynolds number for the drift current is quite large, it seems that the flow was in fact turbulent in the water also, and thus a logarithmic profile is expected. (The wave speed solution for a logarithmic drift current was later obtained by Kato, 1974.)

Coastal Engineering Significance. Apart from the weakness in the chosen drift current profile, the investigation shows that for the wave characteristics in question, it is important to include the aerodynamic term \( \frac{C_a}{C_0} \) in the expression for the phase velocity of a wave over which a wind is blowing. Lilly's (1966) solution considers only the drift current correction.

Keywords. Current Depth Refraction; Currents, Nearshore; Currents, Wave-Induced; Numerical Model; Surf Zone.

Discussion. A numerical model of waves approaching a beach is extended to include wave-current interaction. The beach topography, which is relatively complicated, and the wave conditions are chosen to represent a field example which had been previously studied.

The approximations used include linearized equations of motion and a "ray theory" approach to the wave-current and wave-bed interaction. However, the calculations used a finite-difference approximation since this proved more efficient than computing rays. A steady state was assumed and calculated.

The results show appreciable differences from earlier computations with no wave-current interaction. However, there is room for improvement. Indeed, it is stated that "the nearshore circulation system is basically a nonsteady pulsating system."

Coastal Engineering Significance. This is an early attempt to study wave-current interaction under realistic surf zone conditions. The system that is modeled is too complicated for interpreting the various physical effects that are included. An improved development of this model is used in a simpler context by Ebersole and Dalrymple (1980).


Keywords. Averaged Lagrangian; Caustics, Current Refraction; Current Velocity Profile; Currents, Large-Scale; Currents, Opposing; Currents, Shearing; Currents, Slowly Varying; Currents, Small-Scale; Currents, Unidirectional; Dispersion Relation; Equations of Motion; Group Velocity; Interactions, Short Wave-Long Wave; Review; Theory; Theory, Ray; Turbulence, Wave Action; Wave Energy; Waves, Finite-Amplitude.

Discussion. This substantial review paper commences with a description of the various situations in which waves interact with preexisting currents. Wave-generated currents are not discussed in this work.

More than half the paper is devoted to currents which vary on a scale much larger than the waves. The dispersion equation for waves on a constant current yields a wider range of solutions for a given
frequency than for still water. The discussion of slowly varying currents is based on the conservation of wave-action equation, and three different examples are discussed in detail.

The first such example is for waves on a current which is unidirectional and only varies in that direction. This is treated more thoroughly than earlier work, including a discussion of possible caustics at a "stopping point," nonlinear effects, and the behavior of waves propagating at an angle to the current. Waves on a shearing, unidirectional current are also given a similar thorough treatment. The nonlinear theory for both cases is carried further in PEREGRINE and THOMAS (1979) and PEREGRINE and SMITH (1979).

Another substantial section deals with waves on currents which vary with depth. A number of results are drawn together, and present deficiencies in the theory are noted. Unlike most of the other topics in the paper, there are some experimental results available, but these raise further questions.

Shorter sections deal with currents that are smaller in scale than the waves, with turbulence, and with the influence of the boundary layer and the wake of a ship on ship-generated waves.

Coastal Engineering Significance. This book-length review provides an integrated overview of wave-current interactions. It gathers together almost all the work in the field and discusses many topics in such a way that both the results and limitations of present knowledge are exposed.


Keywords. Caustics; Jetlike Streams; Waves, Stationary.

Discussion. A variety of mathematical methods are employed in order to describe stationary waves on currents. The currents are taken to be "jetlike," i.e., uniform in direction but decaying in magnitude away from some central line. They may or may not decay with depth or may be symmetrical in the transverse horizontal direction. A feature of stationary waves in these circumstances is that they are "trapped." They cannot propagate off the current. In the case of waves considered short when compared with the current scale, this means that they are trapped between caustics.

The problem is first considered generally for linearized waves and shown to be an eigenvalue solution. An exact solution is given for a uniform jet bounded by vortex sheets at the sides and below (a "top hat" velocity profile). It simply illustrates the type of results to be obtained.
For these short waves, a considerable amount is deduced by using asymptotic methods. These include the case of waves trapped between caustics and the low-mode number cases where the caustics are too close together to be treated individually.

No exact solutions were found for the above cases, so corresponding results for both short and long waves are deduced for the case where the current only varies with depth. A number of exact linear solutions for special velocity profiles are given and compared with approximations.

Coastal Engineering Significance. These solutions can be used in several other contexts when a change of reference frame can make the waves stationary.


Keywords. Averaged Lagrangian; Caustics; Current Refraction; Currents, Large-Scale; Currents, Shearing; Dispersion Relation; Theory; Waves, Nonlinear.

Discussion. The usual linear theory of waves in a slowly varying medium indicates that wave amplitudes are especially large at caustics. The effects of nonlinearity are considered by taking a general averaged Lagrangian for near-linear waves. In the first approximation to a slowly varying medium this shows that caustics are of two types: an $R$ type, which has a singularity of the approximation at small amplitude and hence is probably regular, and an $S$ type, which has no singularity but has solutions growing without bound near caustics and hence, if there is a limit to wave growth, such as breaking, the waves may reach it.

The theory is discussed for straight and curved caustics and a detailed example of water waves on currents is given. A wide set of current distributions is considered. They have the form of any current field which depends on position through a single coordinate,

\[ e.g., U(x) i + V(x) j \]

The character of the caustics that can arise is examined, and it is found that both types of caustics occur. Further properties of these are illustrated by PEREGRINE and THOMAS (1979).

A second approximation improving the representation of the "slow variations" leads to uniform solutions involving Airy functions for linear waves. Details of their use are given. For nonlinear waves, corresponding solutions involve a Painlevé transcendent function.
Coastal Engineering Significance. This paper treats the question of waves near caustics, a question that has often worried coastal engineers. The results indicate two types of caustics, only one of which leads to breaking. They also emphasize the wide range of conditions in which caustics may be formed.


Keywords. Averaged Lagrangian; Caustics; Current Refraction; Currents, Large-Scale; Currents, Opposing; Currents, Shearing; Group Wave Breaking; Wave Height; Waves, Finite-Amplitude.

Discussion. Longuet-Higgins' (1975) accurate solution for periodic deepwater waves of any steepness up to the highest is used with Whitham's averaged Lagrangian method. Longuet-Higgins' numerical values are fitted by simple rational functions, and solutions are found for two current distributions.

Waves on a current which has a shear in the horizontal direction perpendicular to the current have a singularity in their solution for small amplitudes, and no solution for stronger currents. There is also a second, steeper solution for an appreciable range of currents before this singularity is reached. This singularity corresponds to the neighborhood of a caustic in the linear theory. The near-linear theory of PEREGRINE and SMITH (1979) describes it as an R-type caustic at which it is unlikely that waves will break. There is always a reflected solution.

For waves progressing directly against an adverse current, there are solutions which progress smoothly up to the highest waves as the magnitude of the current increases. More remarkably, the solution for reflected waves ceases to exist for moderate initial waves. This is an S-type caustic. It seems likely that water waves normally break in the region of an S-type caustic.

There is a section discussing how the concept of group velocity may be extended to finite-amplitude waves. There are several different possible definitions, and it is shown that most have different values for a given train of water waves. No firm conclusions are drawn, though some new stability results are presented.

Coastal Engineering Significance. This paper shows how finite-amplitude effects alter the amplitude of waves in the neighborhood of caustics. In particular, it shows how the type of caustic that occurs on an adverse current is more likely to lead to wave breaking.
Keywords. Equations of Motion; Waves, Surface; Waves, Wind.

Discussion. This book is a standard reference on surface and internal waves in the ocean. The first substantial chapter (Ch. 2) develops the basic equations of motion and outlines equations applicable to general wave trains, such as the conservation of wave action and weakly nonlinear interactions between waves.

Chapter 3 is concerned with deterministic aspects of surface waves. It includes detailed properties such as water particle motion and dissipation due to molecular viscosity. Then the properties of wave trains on variable depth and currents are treated in detail by averaging mass, momentum and energy conservation equations. Applications to the waves advancing into still water, onto a beach, against a variable current, and over internal waves are given. (Much of this was newly presented in the first edition.) Surface wave interactions and wave breaking are also included.

Chapter 4 deals with specifying a natural wave field, its generation by wind, and subsequent development. It is mostly written in terms of a spectral description and has been considerably revised since the first edition, which is indicative of the amount of activity in this field. The discussion of several points clearly indicates that there is more to be learned.

Chapter 5 is on internal waves and Chapter 6 is about oceanic turbulence, neither includes much reference to surface waves.

Coastal Engineering Significance. For the study of wave-current interactions, Chapters 2 and 3 provide an orderly background and introduction.

Keywords. Boundary Layer; Comparison of Theory and Measurement; Current Velocity Profile; Currents, Vertical Shear; Currents, Wind-Drift; Experiment; Flow, Rotational; Phase Velocity; Spectra; Waves, Wind; Wind Shear; Wind Velocity Profile.

Discussion. Phase speeds of wind-generated waves, traveling both upwind and downwind in a wind-wave flume, were measured for wavelengths up to 36 centimeters. The measurements were from the Doppler shift in first-order Bragg scattering which gives an accuracy of about +3 percent even for cases involving wave breaking.
This type of measurement gives values of the current due to the wind. Linear perturbation theory indicates that the advection of the wave by wind-drift is odd (depends on wind direction) while the initial effects of the air are even (independent of wind direction). Thus, it is possible to separate these effects at first order by forming the sum and difference of the speeds of downwind- and upwind-traveling waves.

The advection effect of wind drift is found to increase less rapidly than linearly with the air friction velocity $u_*$. Since Wu (1975) showed that surface drift is proportional to $u_*$, this implies that wind drift becomes thinner for high winds.

The effect of inertial pressure on the speeds of 9.8-centimeter waves, particularly windspeed dependence, was well predicted by using values of the air roughness length deduced from several measurements. For longer waves it was much smaller, possibly due to a reduction of the mean air flow near the surface by wave drag.

The interaction between wave components due to finite-amplitude effects leads to an increase in phase speed. However, measurements show that at constant windspeed each component increased in magnitude with fetch, but its phase speed decreased.

Coastal Engineering Significance. These experimental measurements indicate the importance of allowing for current and wind profiles in work with surface waves being actively generated by wind. The decrease of phase velocity with fetch is opposite to all present theoretical results. These results are most relevant to interpreted "remote sensing" measurements.


Keywords. Bottom Friction; Comparison of Theory and Measurement; Currents, Tidal; Equations of Motion; Interactions, Long Waves; Numerical Model; Observation; Statistics; Wave Height; Waves, Long.

Discussion. Tide surge interaction is examined in three ways. The first is a statistical analysis and uses surges which can be identified propagating along the whole east coast of the British Isles (approximately 1000 kilometers). There is clear indication of an interaction when the rate of amplification is plotted for positive and negative surges occurring at different phases of the tide. Also, a comparison of measurements at Lowestoft and London gives an indication that amplification is linearly dependent on surge and amplitude.
A one-dimensional numerical model of the Thames, used with a complete range of surges added to the tide, gives maximum amplitudes of surge occurring before high tide and of the order 25 percent larger than without tide. The maximum outer levels occur close to high tide.

A "parallel" numerical model, with tide and surge calculated separately but interactions occurring between them, gives an indication that quadratic friction terms are more important to the interaction than the nonlinear advective terms, $\partial^2 u / \partial x^2$.

Coastal Engineering Significance. This paper shows from field data that there is a definite interaction between storm surge and tides, such that the surge in an estuary can be significantly amplified by tidal currents.

The detailed modeling of the interaction terms is of wider interest. In particular, the importance of the friction term should be considered in other types of wave-current interaction.


Keywords. Comparison of Theory and Measurement; Current Refraction; Currents, Nonuniform; Currents, Ocean; Observation; Wave Energy; Wave Observation; Waves, Ocean; Waves, Wind.

Discussion. After a brief discussion of the theory, details of the method of measuring wave spectra, wind, and current are given. The spectra are frequency spectra measured by a shipborne wave recorder, but visual estimates of wave direction are also given. The character of the sea state, both in and outside the Agulhas Current, is determined using data from 15 pairs of stations, occupied daily over a 7-day period.

The energy density, $E$, at the station with weakest current was extrapolated to zero current, $E_0$, by assuming a linear variation and comparing with other "weak current" measurements. $E/E_0$ could then be evaluated at a station in a strong current (1.1 to 1.6 meters per second).

A comparison is made with results from LONGUET-HIGGINS and STEWART (1961, 1964) for waves propagating onto a directly adverse current, for the two cases where the current variation is due to vertical and to horizontal inflow. The experimental points show quite reasonable agreement considering the uncertainties involved.
Coastal Engineering Significance. Field data from and near an important ocean current are examined and show reasonable agreement with theory. Wave-current interaction in the Agulhas Current has been suspected as the cause of giant waves that endanger shipping (see MALLORY, 1974).


Keywords. Forces on Structures; Review.

Discussion. This book of conference proceedings provides an overview of the current state-of-the-art in predicting forces on cylinders due to water-wave motion. There are some papers solely on water waves reviewing aspects such as wave-current interactions, wave breaking, and wave generation in the laboratory.

The bulk of the papers describe experiments, some giving details of flow fields about cylinders, others on the effect of cylinders on waves, but the majority are concerned with forces on cylinders. These cylinders are mostly circular in shape and are usually either vertical or horizontal.

The variety of objects that come under the heading of "cylinders" includes cables and pontoons as well as the more obvious pipes and elements of large structures.

Coastal Engineering Significance. This book provides a collection of recent work on wave-induced forces, including forces due to wave-current interactions, by the leading workers on the subject.


Keywords. Experiment; Spectra; Turbulence; Waves, Wind.

Discussion. The report investigates a system in which water waves are generated and propagated in a turbulent flow field. The growth of wind wave spectra and the decay of monochromatic waves are considered. For monochromatic waves the turbulence in the water can greatly increase the rate of wave energy dissipation and the data can be fitted by an equation with an eddy viscosity term proportional to the wave height, the phase speed of the waves, and the intensity of the large-scale turbulence. Growth of wind waves in turbulent water is faster than in still water; however, maximum wave height in turbulent water is always lower than in still water. Wave energy spectra at the longer fetches in turbulent water show more wave energy at low frequency and less at high frequency than those for still water. Turbulence in water alters the
rate of transfer of energy from the wind to the waves, the maximum height allowed, and the distribution of wave energy among the various frequencies. (author's abstract)

Coastal Engineering Significance. The interaction between turbulence and waves has been reported anecdotally by several observers and can easily be appreciated by observing the wake of a powerboat moving perpendicular to the wind in a light chop. However, there have been few reports on the subject other than the one referenced here.


Keywords. Depth Refraction; Wave Energy; Wave Height; Waves; Ocean; Waves, Surface.

Discussion. The equations for the wave orthogonals and the wave heights are presented for depth refraction of regular, long-crested, small-amplitude surface gravity waves over an arbitrary bottom, using time as the independent variable. The effect of turbulent bottom friction on the wave height is included. The computer outputs are in the form of automatically plotted wave orthogonals with the wave heights written at discrete points along these paths. Systematic tests are presented for water areas with straight and parallel bottom contours. The errors inherently connected with the introduction of a grid plus a set of formulas for the differentiation and interpolation are examined for a horizontal bottom with an infinitely long sinusoidal threshold. For one plane sloping bottom, information is given for the influence of the initial curvature of the wave front, bottom roughness, and initial wave height. (authors' abstract)

Coastal Engineering Significance. This is one of the relatively few engineering papers in the United States directly tied to the European tradition of wave-current interaction. The paper is an attempt at a numerical approach to the subject, which is the important long-term objective according to the several investigators.


Keywords. Caustics; Current Refraction; Currents, Large-Scale; Currents, Opposing; Flow, Irrotational; Stability; Theory; Wave Profiles; Waves, Nonlinear.

Discussion. Waves propagating against adverse currents, particularly near the boundaries of such currents, have given cause for concern. In this paper a perturbation analysis is carried out to find an equation
for a slowly varying wave train on deep water which includes both terms due to the current variation and terms arising because the waves are not infinitesimal (i.e., a near-linear approximation is used for the waves).

Two derivations for such an equation are given, one systematic, the other heuristic. The equation obtained is similar to the nonlinear Schrödinger equation. The solutions of the equation are investigated for the neighborhood of a caustic where it is expected that waves may be at their steepest. A radiation condition is determined for matching with simpler approximations. A stability analysis indicates that wave trains in such circumstances do not suffer the usual modulational instability of deepwater waves.

The systematic perturbation approach also gives information on the asymmetry of the wave profile. PEREGRINE and SMITH (1979) build on some aspects of this work, but their work does not supersede this paper.

Coastal Engineering Significance. Giant waves have long posed a threat to navigation, and the interaction between waves and currents has been suspected as a cause. This paper indicates that large stable waves are possible on currents in deep water. The analysis does not provide symmetric wave profiles, as is usually the case.


Keywords. Averaged Equations; Averaged Lagrangian; Conservation Equations; Current Depth Refraction; Currents, Large-Scale; Flow, Rotational; Momentum Equation; Theory; Wave Action; Waves, Finite-Amplitude.

Discussion. Equations for slowly varying wave trains in inhomogeneous, moving media can be derived either from an averaged Lagrangian or from averaging the equations of motion. In the former case a wave action conservation equation is found for nondissipative flows. For surface water waves such equations have been derived from averaged Lagrangians for potential flow and directly from the equations of motion.

In this paper the equations obtained from averaging the equations of motion are manipulated into the same form as the equations derived from an averaged Lagrangian. The motion on the scale of the waves is assumed to be irrotational in both cases. It is found that wave action is still conserved and that the only differences arise in the equations which correspond to the consistency conditions of a pseudophase. One of these is the irrotationality condition for the large-scale current. The more complete equations permitting large-scale vorticity can be written in various forms. Some include the vorticity explicitly; alternatively, the equations clearly show the assumption of a shallow-water approximation for large-scale flows.
Particular simple examples are briefly indicated, and it is noticed that assumed properties of the current are also constrained by the shallow-water approximation.

**Coastal Engineering Significance.** This paper compares and shows the relation between equations derived from an averaged Lagrangian and from averaging equations of motion. The relations for wave properties are for exact finite-amplitude periodic solutions. The equations presented here form a good starting point for simplifying assumptions, or for constructing numerical approximation schemes for current and depth refraction.


**Keywords.** Current Depth Refraction; Currents, Large-Scale; Currents, Shearing; Currents, Unidirectional; Spectra; Spectra, Directional; Theory; Theory, Ray; Wave Action.

**Discussion.** The properties of wave rays in depth and current varying conditions are reviewed and applied to a representation of wave directional spectrum. There is discussion of the transformation of direction, wave action, and wave energy as well as how rays may terminate due to wave breaking or reflection. The analysis is restricted to linear theory.

Details of analytical solutions are given for cases with unidirectional currents, including deepwater waves on a shearing current and a combination of depth and current refraction in shallow water. The details are plotted in a frequency-direction diagram. This procedure has advantages since the absolute frequency of each wave component remains constant on the steady currents considered. This is a straightforward extension of well-established results obtained by adding the results for each Fourier component. However, the discussion and presentation place the theory in a slightly different context.

**Coastal Engineering Significance.** The initial discussion makes this paper a reasonable starting point for engineering applications. The examples could be used for verifying computer programs.


**Keywords.** Forces on Cylinders; Spectra; Statistics.
Discussion. Fluid forces on cylinders just below water level are evaluated for the combined occurrence of random waves and current. The waves are deepwater, zero mean, stationary random waves. The current is steady and uniform in depth. Morison's formula is used to predict forces.

The emphasis is on fatigue failure, as measured by the probability density function of the peaks of the forces induced in the structure. The paper examines how this probability density function is affected by currents and wave-current interactions. The approach is statistical, with evaluation of specific cases clearly presented in plotted form.

The analysis identifies a statistical quantity as a measure of the potential fatigue damage. This quantity is a rate of change of the expected value of the peak forces above a threshold with respect to the change in threshold.

The effect of assuming a Gaussian distribution of forces (rather than a distribution of forces resulting from Gaussian waves) is examined. From the examples shown, it is clear that the Gaussian force assumption significantly underestimates the initial forces. The examples also show that the wave-current interaction serves to reduce critical forces when the current and waves move in the same direction, and significantly affects the force in the wave trough when the current is opposite the wave direction.

Coastal Engineering Significance. The assumption of a Gaussian distribution of forces is shown to underestimate the forces most critical in inducing fatigue failure. The presence of wave-current interactions significantly changes the critical forces, usually reducing them.


Keywords. Currents, Tidal; Historical Interest; Phase Velocity.

Discussion. This is the first paper to consider the change of phase velocity (and hence of frequency) as waves propagate over varying currents. It follows an earlier brief note (Unna, 1941) in which the author considered the contraction and expansion of the water as a cause of the steepening of short waves at the crest of long waves.

The change in linear phase velocity for currents parallel to the direction of wave propagation is given correctly.

The implications for waves entering an estuary are discussed, and it is indicated that there is likely to be a bunching of wave energy at the end of the flood which agrees with experiences recorded by Bristol Channel pilots.
Coastal Engineering Significance. UNNA (1942) is the oldest paper included in the annotated bibliography. It is included solely for historical reasons. Observations of wave-current interaction, especially in tide races, are familiar to many navigators and are briefly mentioned by Rayleigh (1911), but no attempt to analyze the interaction has been found to predate this paper.


Keywords. Comparison of Theory and Measurement; Currents, Large-Scale; Currents; Tidal; Fetch; Observation; Wave Height; Waves, Wind.

Discussion. A relatively simple one-dimensional theory is given for waves propagating upon a tidal stream. Results are given for the cases where the tide is just a propagating long wave and where it includes a component propagating in the opposite direction, e.g., where the tide is a standing wave.

The 16-day periods of wave records from gas production platforms on the North Sea were analyzed. These showed wave heights to be modulated with a tidal period. The tides were modeled by a simple expression of the above type and the theoretical wave modulation was estimated and compared with observation. Good correspondence was found with the phase of the modulations, but the actual amplitude of the modulations was much larger than estimated.

A discussion of the limitations of the approach includes mention of the true tidal current field and depth refraction of the waves.

Coastal Engineering Significance. Field data at the North Sea platforms show that height is affected by tidal currents, and the observed height is higher than predicted by the simple theory used. The data shows the type of height variation to be expected in areas with significant tidal currents.


Keywords. Averaged Equations; Conservation Equations; Currents, Unidirectional; Equations of Motion; Group Velocity; Wave Energy; Waves, Finite-Amplitude.

Discussion. It is noted that the energy of a current with waves upon it cannot be uniquely divided into wave energy and current energy. The energy equation can be modified in a consistent manner by subtracting
suitable multiples of the momentum conservation equation. This is illustrated by considering the energy of a system of particles in a moving system.

Stokes' second-order wave solution is given, and particular attention is given to a term proportional to amplitude squared which can appear in any of three places: (a) in Bernoulli's equation, (b) in the velocity potential, and (c) as a change in mean level.

The conservation equations for total mass, momentum and energy are derived to second order for waves on a current. The equation for "conservation of waves" (a consistency equation for the existence of a phase function) is then added to these equations. Three examples are considered in detail to demonstrate that currents and depths cannot in general be specified in advance since they depend on wave conditions.

The four unsteady equations for the case of unidirectional waves on initially undisturbed flow are a hyperbolic system. It is shown to have four characteristics with velocities equal to the waves' group velocity and the long wave velocity in both directions.

The paper was stimulated by Longuet-Higgins and Stewart (1960, 1961) and broadens their discussion of radiation stress.

Coastal Engineering Significance. This paper shows some of the difficulties of dealing with the refraction of finite-amplitude water waves. Water depths and currents may not be specified in advance but should come from initial and boundary conditions. Unlike linear wave theory, wave energy does not travel with a single group velocity. The group velocity splits into two distinct velocities for perturbation (see also Hayes, 1973 and Peregrine and Thomas, 1979).


Keywords. Waves, Nonlinear.

Discussion. This book draws together much of the research of the previous 25 years. It is divided into two large sections. The first section, "Hyperbolic Waves," deals with waves described by hyperbolic equations, such as shallow-water waves, sound waves, and shock waves. Five chapters discuss unidirectional propagation and are primarily concerned with nonlinear effects. The remaining four chapters of this section discuss propagation in two or three dimensions including geometrical optics approximations for linear waves and work on shock waves.

The second section discusses dispersive waves with particular emphasis on water waves. There are discussion and derivation of the
various nonlinear solutions. Modulations, group velocities, instabilities, averaging methods, and solutions are all described in adequate detail.

Coastal Engineering Significance. The book is a definitive account of recent work to which the author is one of the more important contributors. In 1981 it is still a valuable up-to-date textbook and reference for the basic theory of wave propagation.
LITERATURE CITED IN ANNOTATIONS


LUNDGREN, H., and JONSSON, I.G., "Bed Shear Stresses Induced by Waves and a Current," Progress Report No. 1, Coastal Engineering Laboratory, Technical University of Denmark, Copenhagen, Denmark, Jan. 1961, pp. 6-7.


APPENDIX A

LISTING OF KEYWORDS

The following keywords are used with the annotated bibliography. There is some overlap and redundancy to include related keywords that may be more familiar to groups such as theoreticians, experimentalists and engineers.

- Averaged Equations
- Averaged Lagrangian
- Bottom Friction
- Boundary Layer (see also Wave Boundary Layer)
- Case Study
- Caustics
- Comparison of Theory and Measurement
- Conservation Equations
- Continental Shelf
- Critical Current Velocity
- Current Depth Refraction
- Current Refraction
- Current Velocity Profile
- Current Wave Shoaling
- Currents, Large-Scale
- Currents, Nearshore
- Currents, Nonuniform (see other entries under Currents)
- Currents, Ocean
- Currents, Opposing
- Currents, Rip
- Currents, Shearing
- Currents, Slowly Varying
- Currents, Small-Scale
- Currents, Tidal
- Currents, Unidirectional
- Currents, Unsteady (see Currents, Rip)
- Currents, Vertical Shear
- Currents, Wave-Induced
- Currents, Wind-Drift
- Depth Refraction
- Dispersion Relation
- Eddy Viscosity
- Equations of Motion
- Experiment
- Fetch
- Flow, Irrotational
- Flow, Rotational
- Forces on Cylinders
- Forces on Structures
- Group Velocity
Historical Interest
Interactions, Internal Waves
Interactions, Long Waves
Interactions, Short Wave-Long Wave Interactions, Wave-Wave
Jetlike Streams
Mass Transport
Mean Energy Level
Meteorology
Momentum Equation
Numerical Model
Observation
Phase Velocity
Radiation Stress
Refraction-Diffraction
Review
Setdown
Setup
Shallow Water
Shear Layer
Spectra
Spectra, Directional
Stability
Statistics
Stream Lines
Surf Beat
Surf Zone
Theory
Theory, Ray
Total Head Line
Turbulence
Wake
Wave Action
Wave Boundary Layer
Wave Breaking
Wave Dissipation
Wave Effect on Current
Wave Energy
Wave Filtering
Wave Groups
Wave Height
Wavelength
Wave Observation
Wave Period
Wave Profiles
Wave Reflection
Wave Transmission
Waves
Waves, Deepwater
Waves, Finite-Amplitude (see also Waves, Nonlinear)
Waves, Long
Waves, Nonlinear
Waves, Ocean
Waves on a Jet (see Currents, Rip and Currents, Shearing)
Waves, Stationary
Waves, Storm
Waves, Surface
Waves, Wind
Wind Shear
Wind Velocity Profile
APPENDIX B

KEYWORD INDEX OF ENTRIES

Averaged Equations
38. Longuet-Higgins and Stewart (1964)
54. Stiassnie and Peregrine (1979)
59. Whitham (1962)

Averaged Lagrangian
42. Peregrine (1976)
44. Peregrine and Smith (1979)
45. Peregrine and Thomas (1979)
54. Stiassnie and Peregrine (1979)

Bottom Friction
17. Grant and Madsen (1979)
48. Prandle and Wolf (1978)

Boundary Layer
47. Plant and Wright (1980)

Case Study

Caustics
42. Peregrine (1976)
43. Peregrine and Smith (1975)
44. Peregrine and Smith (1979)
45. Peregrine and Thomas (1979)
53. Smith (1976)

Comparison of Theory and Measurement
17. Grant and Madsen (1979)
40. Mizuno and Mitsuyasu (1973)
47. Plant and Wright (1980)
48. Prandle and Wolf (1978)
49. Schumann (1976)
58. Vincent (1979)

Conservation Equations

25. Jonsson (1978b)
54. Stiassnie and Peregrine (1979)
59. Whitham (1962)

Continental Shelf

2. Barber (1949)

Critical Current Velocity

3. Biesel (1950)

Current Depth Refraction

1. Arthur (1950)
2. Barber (1949)
25. Jonsson (1978b)
54. Stiassnie and Peregrine (1979)
55. Tayfun, Dalrymple, and Yang (1976)

Current Refraction

22. Johnson (1947)
31. Kenyon (1971)
38. Longuet-Higgins and Stewart (1964)
42. Peregrine (1976)
44. Peregrine and Smith (1979)
45. Peregrine and Thomas (1979)
49. Schumann (1976)
53. Smith (1976)

Current Velocity Profile

17. Grant and Madsen (1979)
42. Peregrine (1976)
47. Plant and Wright (1980)

Current Wave Shoaling


Currents, Large-Scale

2. Barber (1949)
5. Bretherton and Garrett (1968)
24. Jonsson (1978a)
25. Jonsson (1978b)
31. Kenyon (1971)
38. Longuet-Higgins and Stewart (1964)
42. Peregrine (1976)
44. Peregrine and Smith (1979)
45. Peregrine and Thomas (1979)
53. Smith (1976)
54. Stiassnie and Peregrine (1979)
55. Tayfun, Dalrymple, and Yang (1976)
58. Vincent (1979)

Currents, Nearshore

2. Barber (1949)
11. Dalrymple and Lozano (1978)
Currents, Nonuniform

49. Schumann (1976)

Currents, Ocean

31. Kenyon (1971)
49. Schumann (1976)

Currents, Opposing

1. Arthur (1950)
38. Longuet-Higgins and Stewart (1964)
42. Peregrine (1976)
45. Peregrine and Thomas (1979)
53. Smith (1976)

Currents, Rip

1. Arthur (1950)
11. Dalrymple and Lozano (1978)

Currents, Shearing

22. Johnson (1947)
31. Kenyon (1971)
38. Longuet-Higgins and Stewart (1964)
42. Peregrine (1976)
44. Peregrine and Smith (1979)
45. Peregrine and Thomas (1979)
55. Tayfun, Dalrymple, and Yang (1976)

Currents, Slowly Varying

42. Peregrine (1976)

Currents, Small-Scale

12. Evans (1975)
42. Peregrine (1976)
Currents, Tidal

2. Barber (1949)
35. Longuet-Higgins and Stewart (1960)
48. Prandle and Wolf (1978)
57. Unna (1942)

Currents, Unidirectional

3. Biesel (1950)
19. Hedges, Burrows, and Mason (1979)
20. Huang, et al. (1972)
22. Johnson (1947)
35. Longuet-Higgins and Stewart (1960)
40. Mizuno and Mitsuyasu (1973)
42. Peregrine (1976)
55. Tayfun, Dalrymple, and Yang (1976)
59. Whitham (1962)

Currents, Unsteady

2. Barber (1949)

Currents, Vertical Shear

3. Biesel (1950)
9. Dalrymple (1977)
47. Plant and Wright (1980)

Currents, Wave Induced

21. Hughes and Grant (1978)
37. Longuet-Higgins and Stewart (1962)

Currents, Wind-Drift

40. Mizuno and Mitsuyasu (1973)
47. Plant and Wright (1980)
Depth Refraction

52. Skovgaard, Jonsson, and Bertelsen (1975)

Dispersion Relation

3. Biesel (1950)
35. Longuet-Higgins and Stewart (1960)
42. Peregrine (1976)
44. Peregrine and Smith (1979)

Eddy Viscosity

17. Grant and Madsen (1979)

Equations of Motion

35. Longuet-Higgins and Stewart (1960)
42. Peregrine (1976)
46. Phillips (1977)
48. Prandle and Wolf (1978)
59. Whitham (1962)

Experiment

21. Hughes and Grant (1978)
32. Kruijt and Van Oorschot (1979)
40. Mizuno and Mitsuyasu (1973)
47. Plant and Wright (1980)
51. Skoda (1973)

Fetch

58. Vincent (1979)

Flow, Irrotational

24. Jonsson (1978a)
53. Smith (1976)
Flow, Rotational

47. Plant and Wright (1980)
54. Stiassnie and Peregrine (1979)

Forces on Cylinders

19. Hedges, Burrows, and Mason (1979)
56. Tung (1974)

Forces on Structures

19. Hedges, Burrows, and Mason (1979)
32. Kruijt and Van Oorshot (1979)
50. Shaw (1979)

Group Velocity

38. Longuet-Higgins and Stewart (1964)
42. Peregrine (1976)
45. Peregrine and Thomas (1979)
59. Whitham (1962)

Historical Interest

1. Arthur (1950)
3. Biesel (1950)
22. Johnson (1947)
57. Unna (1942)

Interactions, Internal Waves


Interactions, Long Waves

48. Prandle and Wolf (1978)

Interactions, Short Wave-Long Wave

35. Longuet-Higgins and Stewart (1960)
42. Peregrine (1976)

Interactions, Wave-Wave

35. Longuet-Higgins and Stewart (1960)
38. Longuet-Higgins and Stewart (1964)
Jetlike Streams

43. Peregrine and Smith (1975)

Mass Transport

24. Jonsson (1978a)
25. Jonsson (1978b)
37. Longuet-Higgins and Stewart (1962)

Mean Energy Level


Meteorology


Momentum Equation

17. Grant and Madsen (1979)
54. Stiassnie and Peregrine (1979)

Numerical Model

9. Dalrymple (1977)
48. Prandle and Wolf (1978)

Observation

21. Hughes and Grant (1978)
48. Prandle and Wolf (1978)
49. Schumann (1976)
58. Vincent (1979)

Phase Velocity

40. Mizuno and Mitsuyasu (1973)
47. Plant and Wright (1980)
57. Unna (1942)
Radiation Stress

34. Lighthill (1978)
35. Longuet-Higgins and Stewart (1960)
37. Longuet-Higgins and Stewart (1962)
38. Longuet-Higgins and Stewart (1964)

Refraction-Diffraction


Review

19. Hedges, Burrows, and Mason (1979)
25. Jonsson (1978b)
33. LeBlond and Mysak (1978)
42. Peregrine (1976)
50. Shaw (1979)

Setdown

24. Jonsson (1978a)
37. Longuet-Higgins and Stewart (1962)
38. Longuet-Higgins and Stewart (1964)

Setup

37. Longuet-Higgins and Stewart (1962)
38. Longuet-Higgins and Stewart (1964)

Shallow Water

11. Dalrymple and Lozano (1978)

Shear Layer

12. Evans (1975)

Spectra

19. Hedges, Burrows, and Mason (1979)
20. Huang, et al. (1972)
47. Plant and Wright (1980)
51. Skoda (1973)
55. Tayfun, Dalrymple, and Yang (1976)
56. Tung (1974)

Spectra, Directional

55. Tayfun, Dalrymple, and Yang (1976)

Stability

53. Smith (1976)

Statistics

19. Hedges, Burrows, and Mason (1979)
48. Prandle and Wolf (1978)
56. Tung (1974)

Stream Lines


Surf Beat

37. Longuet-Higgins and Stewart (1962)
38. Longuet-Higgins and Stewart (1964)

Surf Zone


Theory

1. Arthur (1950)
3. Biesel (1950)
5. Bretherton and Garrett (1968)
9. Dalrymple (1977)
11. Dalrymple and Lozano (1978)
12. Evans (1975)
17. Grant and Madsen (1979)
19. Hedges, Burrows, and Mason (1979)
20. Huang, et al. (1972)
24. Jonsson (1978a)
25. Jonsson (1978b)
34. Lighthill (1978)
35. Longuet-Higgins and Stewart (1960)
37. Longuet-Higgins and Stewart (1962)
38. Longuet-Higgins and Stewart (1964)
42. Peregrine (1976)
44. Peregrine and Smith (1979)
53. Smith (1976)
54. Stiassnie and Peregrine (1979)
55. Tayfun, Dalrymple, and Yang (1976)

Theory, Ray

1. Arthur (1950)
22. Johnson (1947)
25. Jonsson (1978b)
31. Kenyon (1971)
42. Peregrine (1976)
55. Tayfun, Dalrymple, and Yang (1976)

Total Head Line


Turbulence

17. Grant and Madsen (1979)
42. Peregrine (1976)
51. Skoda (1973)

Wake


Wave Action

5. Bretherton and Garrett (1968)
24. Jonsson (1978a)
25. Jonsson (1978b)
42. Peregrine (1976)
54. Stiassnie and Peregrine (1979)
55. Tayfun, Dalrymple, and Yang (1976)

Wave Boundary Layer
17. Grant and Madsen (1979)

Wave Breaking
37. Longuet-Higgins and Stewart (1962)
45. Peregrine and Thomas (1979)

Wave Dissipation

Wave Effect on Current
17. Grant and Madsen (1979)
59. Whitham (1962)

Wave Energy
35. Longuet-Higgins and Stewart (1960)
42. Peregrine (1976)
49. Schumann (1976)
52. Skovgaard, Jonsson, and Bertelsen (1975)
59. Whitham (1962)

Wave Filtering
### Wave Groups

37. Longuet-Higgins and Stewart (1962)  
38. Longuet-Higgins and Stewart (1964)

### Wave Height

22. Johnson (1947)  
45. Peregrine and Thomas (1979)  
48. Prandle and Wolf (1978)  
52. Skovgaard, Jonsson, and Bertelsen (1975)  
58. Vincent (1979)

### Wavelength

22. Johnson (1947)  
25. Jonsson (1978b)  

### Wave Observation

2. Barber (1949)  
49. Schumann (1976)

### Wave Period

2. Barber (1949)

### Wave Profiles

53. Smith (1976)

### Wave Reflection

12. Evans (1975)  
31. Kenyon (1971)

### Wave Transmission

12. Evans (1975)

### Waves

5. Bretherton and Garrett (1968)  
33. LeBlond and Mysak (1978)  
52. Skovgaard, Jonsson, and Bertelsen (1975)
Waves, Deepwater

22. Johnson (1947)
35. Longuet-Higgins and Stewart (1960)

Waves, Finite-Amplitude

9. Dalrymple (1977)
42. Peregrine (1976)
45. Peregrine and Thomas (1979)
54. Stiassnie and Peregrine (1979)
59. Whitham (1962)

Waves, Long

1. Arthur (1950)
48. Prandle and Wolf (1978)

Waves, Nonlinear

9. Dalrymple (1977)
44. Peregrine and Smith (1979)
53. Smith (1976)
60. Whitham (1974)

Waves, Ocean

31. Kenyon (1971)
49. Schumann (1976)
52. Skovgaard, Jonsson, and Bertelsen (1975)

Waves on a Jet

1. Arthur (1950)

Waves, Stationary

3. Biesel (1950)
43. Peregrine and Smith (1975)

Waves, Storm


Waves, Surface

46. Phillips (1977)
52. Skovgaard, Jonsson, and Bertelsen (1975)
Waves, Wind

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21. Hughes and Grant (1978)
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51. Skoda (1973)
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Wind Velocity Profile

40. Mizuno and Mitsuyasu (1973)
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Peregrine, D. Howell
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