Interaction of Waves and Currents

by

D. Howell Peregrine and Ivar G. Jonsson

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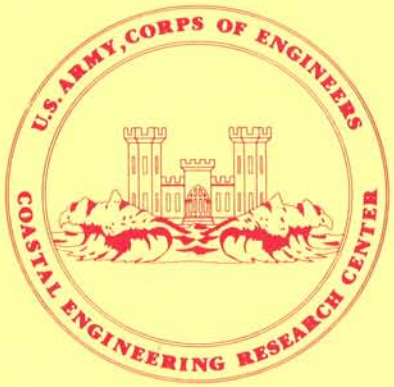

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This report presents an overview of wave-current interaction, including comprehensive review of references to significant U.S. and foreign literature available through December 1981. Specific topics under review are the effects of horizontally and vertically varying currents on waves, wave refraction by currents, dissipation and turbulence, small- and medium-scale currents, caustics and focusing, and wave breaking.

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The results of the review are then examined for engineering applications. The most appropriate general-purpose computer program to include wave-current interaction is the Dutch Rijkswaterstaat program CREDIZ, which is based on a parabolic wave equation. Further applications include wave and current forces on structures and possibly sediment transport. The report concludes with a brief state-of-the-art review of wave-current interaction and a list of topics needing further research and development.

PREFACE

This report reviews wave-current interaction, a phenomenon which may affect wave height and wave direction in unexpected ways. Wave-current interaction has received relatively more attention from Europeans than from Americans because of the greater importance of tides to countries bordering the North Sea. A comprehensive review of the literature, much of it foreign, will increase awareness among U.S. engineers of the important aspects of wave-current interaction. An annotated bibliography on this subject is provided by Peregrine, Jonsson, and Galvin (1983). The work was carried out under the U.S. Army Coastal Engineering Research Center's (CERC) Waves at Entrances work unit, Harbor Entrances and Coastal Channels Program, Coastal Engineering Area of Civil Works Research and Development.

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Comments on this report are invited.

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TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1]To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9)(F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9)(F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

A	wave action (eq. 22) cross-sectional area of cylinder in Morison equation (eq. 28)
a	wave amplitude
$\underset{\sim}{B}$	wave action flux (eq. 25)
C	phase velocity (eq. 7)
C_D	drag coefficient in Morison equation (eq. 28)
C_g	group velocity (see eq. 19)
C_M	inertia coefficient in Morison equation (eq. 28)
C_o	phase velocity of waves in deep water
d	depth of water
D	cylinder diameter (eq. 28)
Diss	rate of dissipation per unit area (eq. 27)
E	energy density (see eq. 22)
$\underset{\sim}{f}$	force per unit length of cylinder in Morison equation (eq. 28)
g	gravitational acceleration
$\underset{\sim}{i}$	unit vector in the positive x direction
$\underset{\sim}{j}$	unit vector in the positive y direction
k	wave number ($2\pi/L$)
$\underset{\sim}{k}$	wave number vector
k_1, k_2	components of the wave number vector in the x_1 and x_2 directions (see eq. 10)
k_α , k_β	components of the wave number vector in the α and β directions (see eqs. 17 and 19)
L	wavelength
L_c	length scale of a current (eq. 2)
p	pressure (see eq. 21)

S	phase of a single progressive wave train (eq. 9)
$S_{\alpha\beta}$	radiation stress tensor (eq. 21)
T	wave period
T_c	time scale of a current (eq. 1)
t	time
u, $\underset{\sim}{u}$	current velocity (magnitude and vector)
u_{max}	amplitude of oscillating current velocity (eq. 1)
$u(x)\underset{\sim}{i}$	unidirectional current in the positive x direction
$u(z)$	current varying in the vertical (z) direction (see eq. 7)
u_α, u_β	oscillatory horizontal (u_α) and vertical (u_β) particle velocities due to wave motion (eqs. 17 and 21)
$V(x)\underset{\sim}{j}$	unidirectional current in the positive y direction
$\underset{\sim}{x}$	position vector (eq. 8)
x_α	distance coordinate in α direction (eqs. 11 and 17)
x_1, x_2	rectangular coordinate directions (eqs. 10 and 12)
z	vertical distance (eq. 7)
α	suffix indicating component in the α direction
δ	phase angle
$\delta_{\alpha\beta}$	Kronecker delta; 1 when $\alpha = \beta$ and 0 when $\alpha \neq \beta$ (eq. 21)
η	free-surface elevation above water level
Θ	angle between u and k
π	constant = 3.14159
ρ	mass density of water
σ	wave radian frequency relative to the current (see eqs. 3 and 4)
$\underset{\sim}{\tau}_b$	mean bottom stress (eq. 27)
∇	gradient of a scalar (first partial derivatives) as in equations (2), (10), etc.
ω	wave radian frequency (eq. 3)

INTERACTION OF WAVES AND CURRENTS

by
D. Howell Peregrine and Ivar G. Jonsson

I. INTRODUCTION

1. Engineering Practice and This Review.

Accounting for the action of waves on structures, vessels, and sediment is a typical task for coastal and ocean engineers; accounting for the combined action of waves and currents is not. There are no widely known design procedures to calculate the effect of wave-current interactions.

Engineering textbooks have almost nothing about wave-current interaction; few even have such a basic and well-known feature as the Doppler effect of a current on wave period. The fluid dynamics-applied mathematics literature has more information (e.g., Whitham, 1974; Phillips, 1977; Lighthill, 1978; and LeBlond and Mysak, 1978), but examples which give direct guidance to the engineer are scarce. The engineering and mathematical research literature has much more information but, apart from the important series of papers by Longuet-Higgins and Stewart (1960, 1961, 1962, 1964), most of the papers are too recent to have affected engineering practice.

In engineering practice, the importance of wave-current interaction has often been poorly understood. In some cases, the fact that both the waves and currents are simultaneously important is not recognized. In other cases, where both waves and currents are understood to be important at the same time, the importance of the **interaction** between the waves and currents is not recognized. Even when both the waves and the currents are known, their interaction may produce a significantly different effect from that obtained by simply adding the effect of the waves and the currents considered separately. This applies particularly to the properties of waves traveling between two points on a current and to the forces resulting from the interaction.

This review provides a guide to and an overview of the subject of wave-current interaction, identifying those areas where enough is known of the subject to be useful in practice and also indicating the many areas where further research or development is needed. This review does not give a textbook account of the better known areas or give detailed guidance for design purposes.

Research into wave-current interaction is pursued today in many countries, and some of it does not appear in English translation. The review, therefore, has a bias toward literature appearing in English.

2. Types and Locations of Currents.

Currents important in wave-current interaction include tidal currents, ocean currents, local wind-generated currents, river currents, and wave-generated currents, including currents associated with wind waves and internal waves. In addition, some special laboratory currents are important for developing insight and testing predictions of theory.

The most regular and predictable currents are the tides, and on most areas of the Continental Shelf, and in many coastal inlets, these are the most significant currents. Regularity of the tides means that in most areas observations already exist for predicting the current regime. Even in a poorly documented area, 1 month's observation can give a reasonable basis for the prediction of currents. The unsteadiness of these currents has a significant effect on waves propagating over them.

Some ocean currents and riverflows are as regular as tides in their behavior; currents generated by local winds are less regular. Surges caused by severe storms have surface elevations and currents similar to tides. For all these cases, reasonably satisfactory estimates of large-scale current fields can be made with numerical models; e.g., Peregrine (1981b) describes work on surges and other currents off Northwest Europe.

It is important to note what information may be required about currents. If the prediction of wave properties is to include refraction through tidal currents, as for example in Barber (1949), then the current field is required in the region across which the waves propagate. A numerical model of the current field is of value in this type of example. On the other hand, if local wave conditions are already known, and it is desired to predict local forces, then only the local current is needed.

The variation of current with depth is important in many applications. A vertical velocity profile arises both from friction at the bottom and from wind stress at the free surface.

Often the most important currents are those local to the site in question. These can include strong nonuniformities, such as thin shear layers and eddies behind headlands, breakwaters or other projecting structures; the flow around a structure such as a pile or floating vessel; or rip currents from a beach. The last example is a wave-generated current; such currents are not discussed in detail here although they are related to the subject of this report.

In almost all currents there is a greater or lesser degree of turbulence. The "turbulence" of oceanic eddies clearly has an effect on waves different from the bottom-induced turbulence of a shallow current because of the large difference in scale.

Internal waves propagating on density variations, such as the thermocline, have their own current field. Surface waves interact with these currents. The interactions provide a surface trace of internal wave motions, and also provide a system convenient for analysis and experiment.

Experiments performed in laboratory flumes give another class of currents which are only represented on a prototype scale by flows in artificial cuts or channels. The level of turbulence and the magnitude of secondary circulations are aspects of these flows which are rarely considered but can affect experimental results.

3. Typical Examples of Wave-Current Interactions.

The bulk of this review considers rather idealized problems such as unidirectional currents, inviscid and laminar flows, etc. This is because a complex natural situation can be interpreted with the help of simple examples, each of which makes some contribution either toward an observer's physical intuition or toward a mathematical model which combines these simple elements to form a more complete picture.

Waves are usually generated by the wind. Currents change the effective wind because the relative velocity between the air and moving water differs from that between the air and the fixed bottom. Once formed and freely propagating, the waves are refracted by currents they meet as well as by variations of water depth. Near coasts, where current gradients often increase, refraction may be stronger. The scale of currents can become so small that refraction may be an inadequate term to describe the interactions. (Diffraction might be a better word but it is not always appropriate.) For example, rip currents are usually no wider than a wavelength, and shear layers shed from obstacles are also relatively thin.

In all such current systems, the ability to predict basic wave properties (period, wavelength, amplitude, and direction) is desired. For design purposes, these properties are the input for estimating stresses or forces; for example, shear stress at the bed to estimate sediment transport or the stability of bed protection, and forces and moments on structures and vessels to establish design criteria.

Neglect of a current can lead to inaccuracies in interpreting field data. This is especially true where measurements near the bed are used to predict surface properties or vice versa.

The stronger currents around headlands or through passages lead to tide rips (tide races with steep irregular waves), a prominent example of wave-current interaction. Navigators have known for centuries that these areas can have extremely rough seas, even in otherwise fair conditions and hence are best avoided. There are numerous recorded examples. An aerial view of the Humboldt Bay Entrance during an ebb current (Johnson, 1947) shows how an opposing current augments wave

height and steepness and increases breaking. An artist's impression of a similar phenomenon is Turner's (1775-1851) "The Junction of the Thames and the Medway" (National Gallery of Art, Washington, D.C.). A radar picture of the Columbia River Entrance in The Quarterly CERCular Information Bulletin, January 1980, displays similar refraction by tidal currents.

Navigators have made other use of wave-current interaction. Polynesian canoeists allowed for currents and identified their existence by the change in shape of the waves (Lewis, 1972, pp. 100-115). Modern nautical experience with waves interacting with currents is reported by Coles (1975), including examples from the Gulf Stream (pp. 102, 214, 216-218) and English Channel (pp. 116, 119, 121, 122, 147, 168, 169, and Plate 14).

Wave-current interaction can have important effects on shoreline position or stability of shore protection. In design for shore maintenance, it is accepted practice to hindcast wave characteristics for deep water and refract these waves into shore without consideration of the currents that the waves may cross in reaching shore. These currents have the potential to change the height and direction of the waves actually reaching shore, and thus the magnitude and possibly direction of longshore transport. Changes in height will also affect the design weight of armor stone used for shore protection.

These effects are to be expected especially where shoreline irregularities, such as projecting headlands or tidal inlets, constrict the flow, or produce large semipermanent eddies. At the present, there is no generally accepted agreement whether such currents render a beach more, or less, exposed to the incident wave.

II. EFFECTS OF CURRENTS ON WAVES

1. Scales.

In interpreting, analyzing and modeling wave-current interactions, it is useful to have a clear appreciation of the relative magnitude of time and length scales for both the waves and the currents. For example, many mathematical techniques and physical concepts are of value only if the scale of the currents is much larger than that of the waves. The dispersion relation is such an example. The most obvious time and length scales of waves are their period, T, and wavelength, L. Thus a large-scale current might be one which varies very little, say, no more than a few percent over a distance of one wavelength or over a time of one wave period. Experience in other fields suggests that in some examples, the shorter length and time scales of the inverse wave number, k^{-1}, where $k = 2\pi/L$, and inverse radian frequency, ω^{-1}, where $\omega = 2\pi/T$, can sometimes be used.

However, for many problems, other considerations lead to different time and length scales. For waves, their coherence could be relevant; i.e., the time scale of a group of waves may be appropriate. In propagation problems, the length of the wave's path and its duration may be more important. For example: the time scale of a current might be represented by:

$$T_c = |u_{max}| / |\partial u/\partial t|_{max} \qquad (1)$$

where u is the current velocity. For semidiurnal tides $T_c = 12/2\pi \cong 2$ hours. Thus, if waves are propagating over tidal currents for more than an hour, the unsteadiness of the current needs to be considered.

A current is large scale if

$$T_c \gg T \text{ and } L_c = |u_{max}| / |\nabla u|_{max} \gg L \qquad (2)$$

This is often the case. The term small-scale currents will be used for the cases $T_c \cong T$ and $L_c \cong L$, as well as $T_c \ll T$ and $L_c \ll L$. Little work has been done on small-scale currents, so the bulk of this review covers large-scale currents.

In detailed applications concerning flow past structures or over bed forms, other scales become important -- in particular the amplitude of water particle excursion due to the wave motion compared with a typical length, or the magnitude of wave-induced water velocities compared with currents.

In some applications, the knowledge of water wave properties in the absence of currents is still inadequate. This is particularly true of sediment transport and wave forces, the applications of greatest concern to coastal engineers. Because of the balance of the present understanding, this review is weighted toward wave properties rather than their effects.

2. Effects of a Horizontally and Vertically Uniform Current.

If a current is perfectly uniform, i.e., if it has the same direction and magnitude over a wide area and at all points from a horizontal bed to the surface, then the current is equivalent to still water viewed from a reference frame moving with the current velocity. If there are water waves on the uniform current, then the apparent speed of the waves will depend on the motion of the observer's reference frame. Proper choice of the reference frame can simplify the analysis and improve interpretation of observations, without changing the physical properties of the waves. As an analogy, the transient passage of a ship viewed from shore suggests a complex series of waves, but when viewed from the deck of the moving ship, the wave pattern becomes stationary and simpler to understand. None of the wave's physical properties are affected, but perception of the wave field changes.

The first and simplest change to be noted is a change in wa period, or frequency. To a shipboard observer in a wind-generated se the wave frequency varies with direction of the ship. If the ship i sailing against the waves, more wave crests are met within a given lengt of time; hence the frequency seems larger. However, if the ship is sailing with the waves, fewer wave crests are met within the same perio of time; hence the frequency seems smaller. At the extreme, a reference frame moving with a wave's phase velocity makes the wave appear stationary, with infinite period or zero frequency. Note that this change in reference frame does not change the wave geometry. All wavelengths and other length scales are unchanged.

The general case is described by the Doppler shift, i.e.,

$$\omega = \sigma + \underset{\sim}{u} \cdot \underset{\sim}{k} \qquad (3)$$

where $\underset{\sim}{u}$ = current velocity

$\underset{\sim}{k}$ = wave number vector (magnitude $k = 2\pi/L$, direction perpendicular to wave crests and troughs, i.e., in their direction of propagation)

ω = waves' radian frequency in the frame of reference in which $\underset{\sim}{u}$ is the current velocity

σ = waves' radian frequency relative to the water moving with the current $\underset{\sim}{u}$

A physical interpretation of this Doppler equation (3), after dividing through by k, is that the phase speed equals the relative phase speed plus the component of current velocity in the direction orthogonal to the wave crests.

The distinction between ω and σ is important. The reference frame of ω is that in which the current $\underset{\sim}{u}$ is defined. Examples of such a reference frame are a fixed bed under the sea or a fixed measuring instrument immersed in the sea. The reference frame of σ is that in which the current is zero. For brevity σ is referred to as frequency relative to the current. These symbols are used consistently in this sense.

As indicated by the distinction between ω and σ, when analyzing the interaction of waves and currents, it is necessary to precisely define the motion of both waves and currents. To do this it is necessary to establish clearly the reference frame in which the motion is considered, and it is often useful to relate this primary reference frame to a second reference frame in which only wave motion is observed. Typically, the primary reference frame is fixed to the earth or a structure imbedded in the earth, but it may be the reference frame of a measuring instrument, or an observer, moving relative to the earth.

Only if the current is perfectly uniform is the second reference frame easy to define. Then it corresponds to that of an observer moving with the current and is the reference frame in which the wave frequency

is σ. Otherwise it is necessary to choose the velocity of the second reference frame relative to the primary. Typical choices are to have the second reference frame moving with either the surface current velocity or the velocity obtained from averaging the current over the depth.

Even in the idealized uniform current case, for which most theoretical work has been done, there is ambiguity in dividing water motion in finite water depths between waves and currents. This ambiguity is most easily recognized by considering how the current should be defined in a situation where both waves and currents are present. Given velocity measurements at one point, the "current" is most naturally defined as the average velocity, and the periodic components that vary around this average are ascribed to the wave motion. This is the most commonly used definition. However, the periodic components, when averaged at a point, are not necessarily zero. Any point which is out of the water for a part of the period, i.e., above trough level, experiences a nonzero mean current in the direction of wave motion. This nonzero average current means that periodic components at fixed points contribute to the total mass flow.

Alternatively, this current can also be described by analyzing the motion of individual fluid particles, rather than the velocity at fixed points. Such an analysis yields a progressive motion of fluid particles diminishing in magnitude with depth. This motion is the current known as the Stokes drift.

Because periodic components contribute to the mass flow, there is a potential ambiguity between "average current" and "wave motion." If the current is defined by requiring the total mass flow due to the waves, integrated over depth, to be zero, it will differ from the current defined by subtracting out the periodic components. Thus, any experimental or analytical work must carefully define what is meant by average current and by wave motion. The basic ambiguity is in defining a rest reference frame for the wave motion itself, as explained by Stokes (1847). As Jonsson (1978a) has pointed out, a large number of papers are not accurate on this point.

A closely related problem, particularly in interpreting experiments, is that wave trains are often characterized by only their period, height, and the stillwater depth. This is insufficient; in addition to a properly defined mean current discussed above, the mean water depth is needed. Stillwater depth will usually differ from the mean depth once wave motion commences because the waves redistribute water, causing wave setup or setdown, temporary storage behind the wave generator, or related redistribution of water. For regular periodic nonbreaking irrotational waves, the changes in depth and associated currents tend to be relatively small, as illustrated by Figure 1 for the maximum magnitude of the Stokes drift averaged over depth. However, near the surface, such flow can be strong. As an example, in deepwater irrotational waves of maximum steepness, the surface particles advance

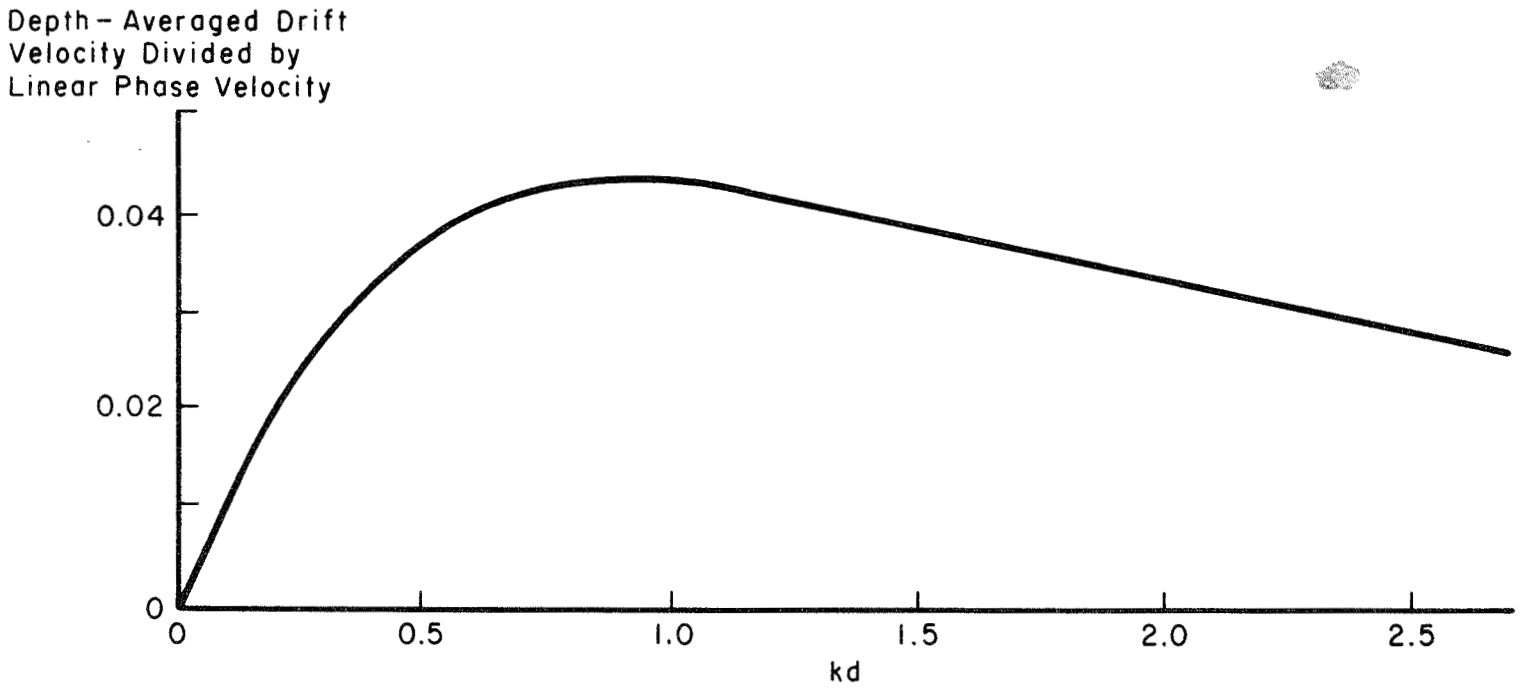

Figure 1. Dimensionless magnitude of Stokes drift averaged over depth for the highest practical wave (assumes Cokelet's (1977) steepness parameter, $\varepsilon^2 = 0.95$).

at a mean speed which is 27% of the phase speed (Longuet-Higgins, 1979). These currents can be important in some applications involving transport of heat, pollution, or sediments. The transfer of momentum from wave to current motion, which occurs when waves break, usually leads to stronger currents.

One of the simplest effects of a current is in convecting a wave field past a measuring instrument. For example, some fair-weather wave measurements in the Bristol Channel showed a systematic variation of significant period between 3 and 5 seconds. This was readily explained once the measurements were plotted along with the tidal currents; a wave field with a significant period of 4 seconds was being convected back and forth by a tidal current of amplitude about 3 knots (1.5 meters per second).

Once details are required of the wave field, the dispersion equation is almost inevitably used since measurements are usually taken as a time series, and information on wave number or wavelength is required. For small-amplitude waves the dispersion relation is

$$\sigma^2 = gk \tanh kd \tag{4}$$

and use of the Doppler relation (eq. 3) leads to

$$(\omega - \underset{\sim}{k} \cdot \underset{\sim}{u})^2 = gk \tanh kd \tag{5}$$

where d is the depth of water, and g the acceleration of gravity. Consideration of equations (4) and (5), from the point of view of solving for $\underset{\sim}{k}$, shows a significant difference. For given σ and d, equation (4) is readily solved numerically for k but gives no information about the direction of $\underset{\sim}{k}$. Equation (5) includes both k and k cos Θ in it, where Θ is the angle between $\underset{\sim}{u}$ and $\underset{\sim}{k}$. The dispersion equation (4) is anisotropic.

Even if the angle between wave and current is known, there may be either two, three, or four solutions for k. Even if waves and currents run parallel, there may be one (if $\omega = 0$), two, three, or four solutions for k, for given values of ω, d, and current speed, $|\underset{\sim}{u}|$. For the parallel case, solutions can be displayed graphically, as in Figure 2, by plotting each side of the reduced Doppler equation

$$\omega - ku = \pm\sigma \tag{6}$$

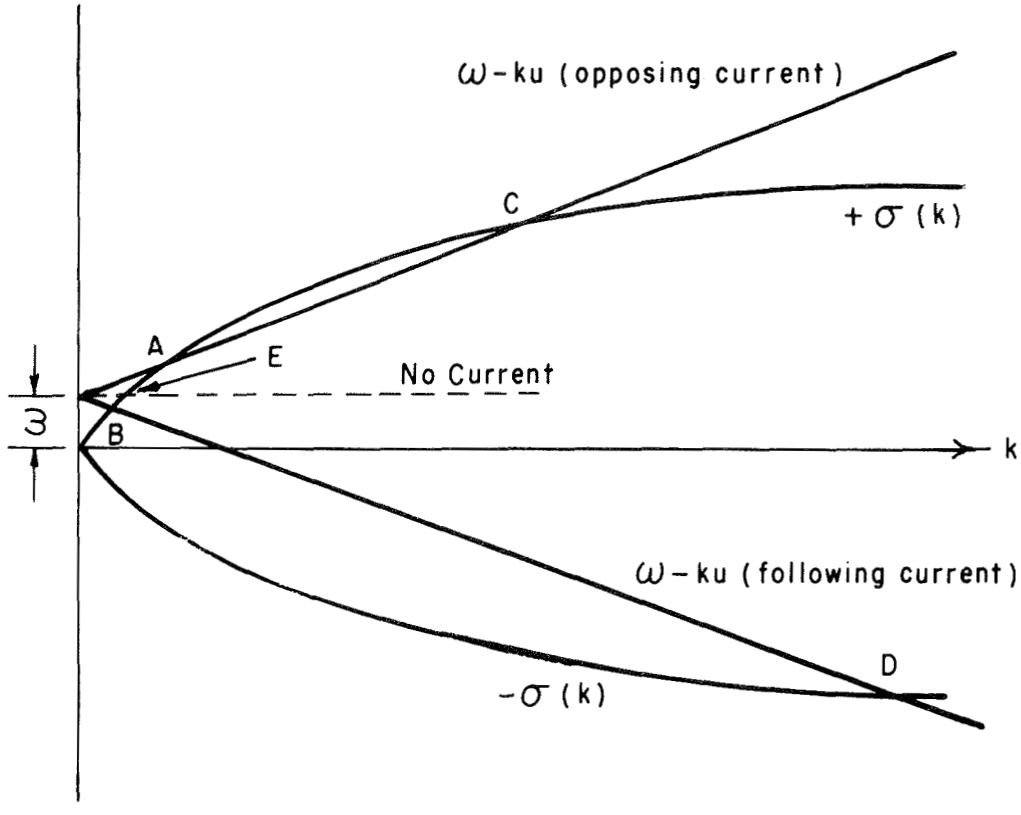

Figure 2. Multiple values of k for solutions of dispersion relation (eq. 6) with collinear waves and currents and given ω, d, and u.

obtained from equation (5) where σ is given by the positive root of equation (4). In Figure 2 only positive values of wave number k have chosen, i.e., the positive direction is by definition that of wave propation. Positive current, u, therefore means a following current (waves moving downstream), and negative u means an opposing current (waves moving upstream).

Solutions to equation (6) are most easily understood by first considering the no-current case, which corresponds to the dashline in Figure 2, parallel to the k-axis. Only one solution is found, corresponding to solution point E from equation (4). The wavelength here is that found in any conventional wave table. When there is a current, the line for ω - ku splits, the two branches corresponding to waves going with or against the current. Then four solutions are possible located graphically at points A, B, C, and D on Figure 2. Points B and D correspond to waves and currents in the same direction (u positive); points A and C correspond to opposite directions (u negative).

In coastal engineering practice, solution points A and B are usually the only ones of interest. This is easily seen by following a wave of constant depth from a no-current environment through a gradual change into a current; simple continuity reasoning shows that either solution point A or B will be met with. It further appears from the figure that everything else being equal, a following current increases wavelength (k is diminished), and an opposing current has the opposite effect. More discussion may be found in Jonsson, Skougaard, and Wang (1970), who also present tables for a direct determination of wavelengths for an arbitrary angle between current and wave direction. These tables can also be found in Jonsson (1978b). A general procedure, including nonlinear terms, has been given by Hedges (1978).

A complete discussion, including solution points C and D, is given by Peregrine (1976, pp. 22-23). Solutions C and D correspond to shorter waves than A and B, and they have no corresponding solution in the no-current case. Solution point D corresponds to waves propagating with the current, and solution point C to waves propagating against it. For cases A and B, energy propagates in the wave direction, while for cases C and D energy is swept downstream by the current. Alternatively, for case A, only, energy propagates against the current.

In stronger currents the two solution points A and C draw closer together until they are coincident; for still stronger currents there are only the B and D solutions. Two coincident solutions (A = C) occur when the current velocity is equal and opposite to the waves' group velocity, $C_g = d\sigma/dk$, relative to the current, that is, total group velocity, $u + C_g$, is zero. In such a case, the energy of the waves is held stationary against the current (their phase velocity indicates upstream travel). They are then "stopped" by the current. For a more complete discussion, see Peregrine (pp. 22-23, 1976).

The idea of stopping waves by an adverse current has been employed in pneumatic and hydraulic breakwaters. Submerged buoyant jets of water or air cause an outflowing surface current which can stop waves up to some limiting wave period. They are ineffective for shallow-water waves. Extensive experiments were conducted by Bulson (1963, 1968). See also Evans (1955) for experiments, Green (1961) for an application, and Taylor (1955) and Brevik (1976) for theory applied to deep and finite-depth water, respectively. In most practical cases, the velocity variation with depth must be accounted for.

Coastal engineering implications of interactions in a uniform current are significant. In general, the result of the interaction is not a simple superposition of the effects of currents and waves, each considered separately. Section III of this report deals with engineering applications, particularly forces on structures (Section III, 2) and sediment transport (Section III, 3). To emphasize the importance of this interaction, a different, and usually neglected, application is considered in the following paragraphs; namely, the effect of the interaction on the basic characteristics of measured waves.

As shown by Figure 2, the wave number k (the horizontal axis) is quite sensitive to the addition of a uniform current (the slope of ω-ku). Ignoring the effect of a current can introduce significant error in coastal engineering analyses, particularly in the use of bottom-mounted pressure gages to measure waves. Reduction of data from such gages requires the transfer of wave properties from bottom to surface. The wave number is used to evaluate terms in the transfer like cosh kd or $e^{\pm kd}$. When kd has value near 1 or greater, errors in k due to neglecting currents are much amplified. This is clearly shown by comparing the results with and without a current, as was first done by Jonsson, Skougaard, and Wang (1970). The point is illustrated graphically in Figure 3, and specific examples are given in the Table, both from Peregrine (p. 25, 1976).

Table. Minimum period of waves for which a current of 0.5 meter per second may be ignored in calculating surface amplitudes from bottom pressure measurements if errors are to be less than 5 and 20 percent (from Peregrine, 1976).

Depth (m)	1	2	5	10	100
Period (s) (with 5 percent error)	4.5	5.4	6.9	8.0	14
Period (s) (with 20 percent error)	2.7	3.2	4.3	5.3	11

The individual components of a wave spectrum are affected in the same way as an individual wave train. Several workers have formally transformed spectra by using the Doppler relation and refraction theory

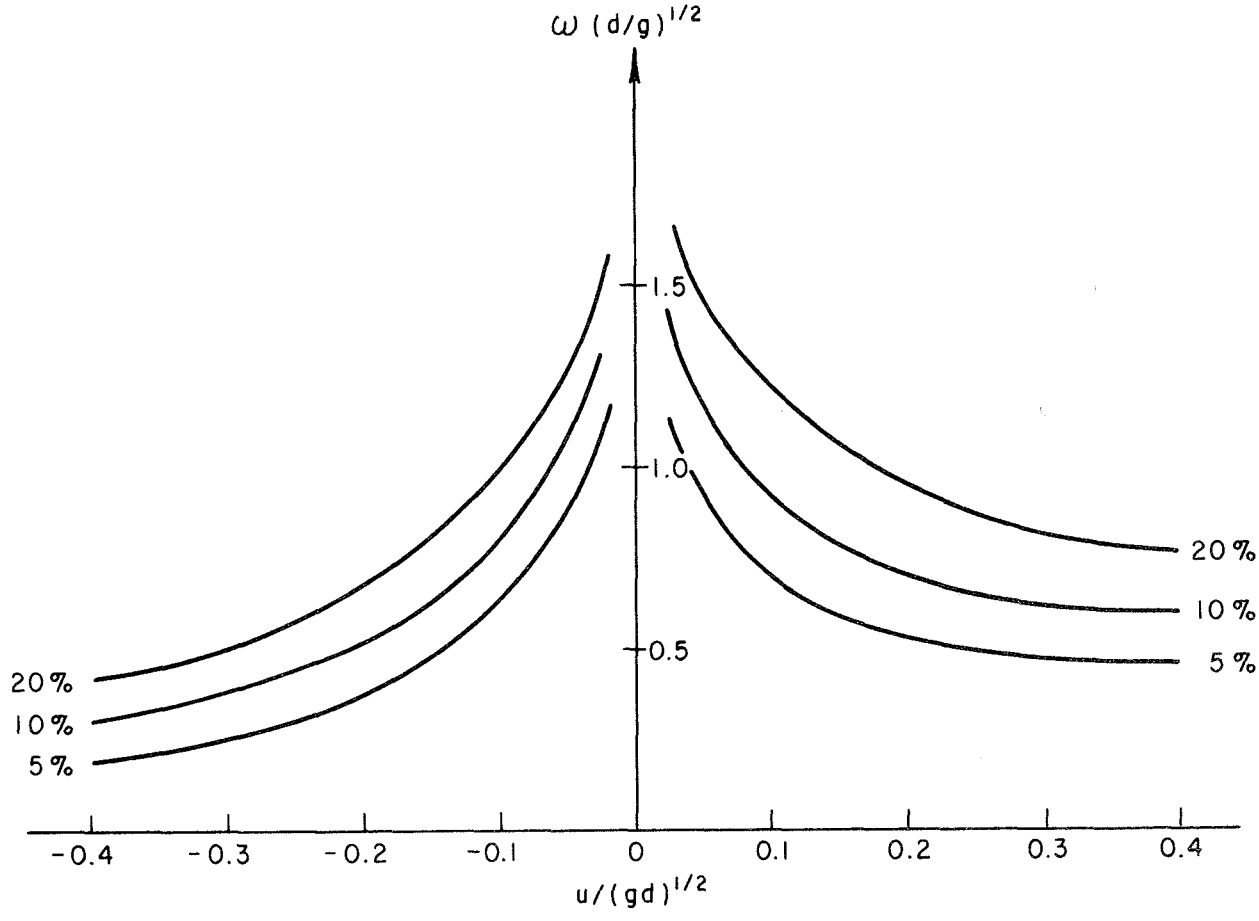

Figure 3. Relative error in surface wave amplitude calculated from bottom pressures due to ignoring a current component, u, parallel with the wave direction.

(see Section II, 7) (e.g., Tung and Huang, 1974; Tayfun, Dalrymple, and Yang, 1976; and Hedges, 1981). These spectra all show singularities at the frequency of zero total group velocity (i.e., $\underset{\sim}{u} + \underset{\sim}{C}_g = 0$). Harper (1980) deals with the mathematical problem that is implied by these singularities. If the total group velocity is zero, then any property related to wave energy remains at the wave sensor and accordingly gives an anomalously high reading. Harper (1980) illustrates this by considering measurements from a moving carriage.

It is only for special circumstances, such as waves in a channel, that simple general spectral calculations are possible. As is indicated in the discussion of refraction (Section II, 4), it is the propagation paths of the wave energy which are important, and these differ for each frequency and direction. Forristall, et al. (1978) found that a detailed hindcast of a directional spectrum, taking account of differing propagation paths, gave good agreement with measurements.

There are two major effects of a current on wave generation by wind. First, the relative velocity between air and water is either increased or decreased; thus a wind has a stronger effect when there is a current opposing it. See, for instance, Kato and Tsuruya (1978a, b). This particular phenomenon has been observed in satellite photographs of the ocean surface by Strong and DeRycke (1973). These photographs show the Gulf Stream quite clearly because of extra sun glitter due to the greater surface roughness on the current. This surface roughness has value in remote sensing, particularly when infrared observations can show no temperature differences between water masses. The authors illustrate this point with a photograph of the major current into the Gulf of Mexico through the Yucatan Channel.

The other major effect is a change in the effective fetch of the wind since the wave energy travels at the vector sum of the current and the group velocity relative to the current. For following currents, the effective fetch is diminished; for opposing currents, it is increased. For example, in a laboratory wind wave flume where wind and current are in the same direction, wave energy reaches the end of the flume quicker than in still water; hence with less duration for growth, the waves do not grow as high as waves with the same relative wind over still water. Waves on an opposing current spend more time under the wind and grow correspondingly larger. Laboratory experiments of this type are described by Kato and Tsuruya (1978a, b).

On the open sea the same effects occur, but in most circumstances, there is the added complication that much of the wave energy will have propagated from other parts of the sea with different currents. In that case, wave refraction, the topic of Sections II, 4 and 5, must be considered.

In practice, real currents are turbulent and are rarely uniform. Turbulence can be sufficiently strong to inhibit wind wave growth (Skoda, 1972). The strongest nonuniformity is usually in the vertical. The two most common causes are "wind drift" and "bottom friction."

3. Effects of Vertical Variation of Current Velocity.

There are two particularly strong effects of wave-current interaction which make it important to consider the variation of current with depth. One is the effect of any velocity shear at the surface on the tendency of waves to break. Banner and Phillips (1974) and Phillips and Banner (1974) describe this using an inviscid model. This is discussed in Section II, 12.

The other is the way in which waves in a flume have been shown to modify the current velocity profile. Such experiments have been described by Van Hoften and Karaki (1976), Brevik (1980), Brevik and Aas (1980), Bakker and van Doorn (1978, 1980), and in more detail by Kemp and Simons (1982). Figure 4 shows the mean current profiles and those obtained by adding the experimentally measured current and wave-induced current separately. The difference seems to be best ascribed to turbulent interactions. Note that a simple eddy viscosity would be negative above the maximum of the mean velocity (assuming stress does not change sign). The additional shear stress and transporting capability of velocity maximum are relevant to sediment transport (Section III, 3).

Velocity profiles established by bottom friction and by surface wind stress have attracted most attention. Numerous papers either derive dispersion relations for various simplifications of the profiles or find results numerically. Peregrine (Section IV, 1976) and Jonsson (Section 3.2.7, 1978b) review the subject, and a number of features are noteworthy.

Since water waves are surface waves, they are particularly sensitive to the velocity in the surface layers. In wind wave flumes the velocity profile due to the wind needs to be taken into account in studying wind waves; e.g., Lilly (in an appendix to Hidy and Plate, 1966) calculates a correction to the dispersion equation, Shemdin (1972) gives more detailed numerical and experimental comparisons including the air motion, and Plant and Wright (1980) find that including other effects such as finite-amplitude effects does not improve comparison with experiment.

A sensitivity to surface drift also shows up when wave fields are used to measure surface currents, as is possible by analyzing the Doppler shift in the scattering of high-frequency radio waves. Scattering by water waves of differing wavelengths leads to different values of the "current." Stewart and Joy (1974) give a useful approximate formula for the current so obtained. The phase velocity (after minor correction of their formula) is

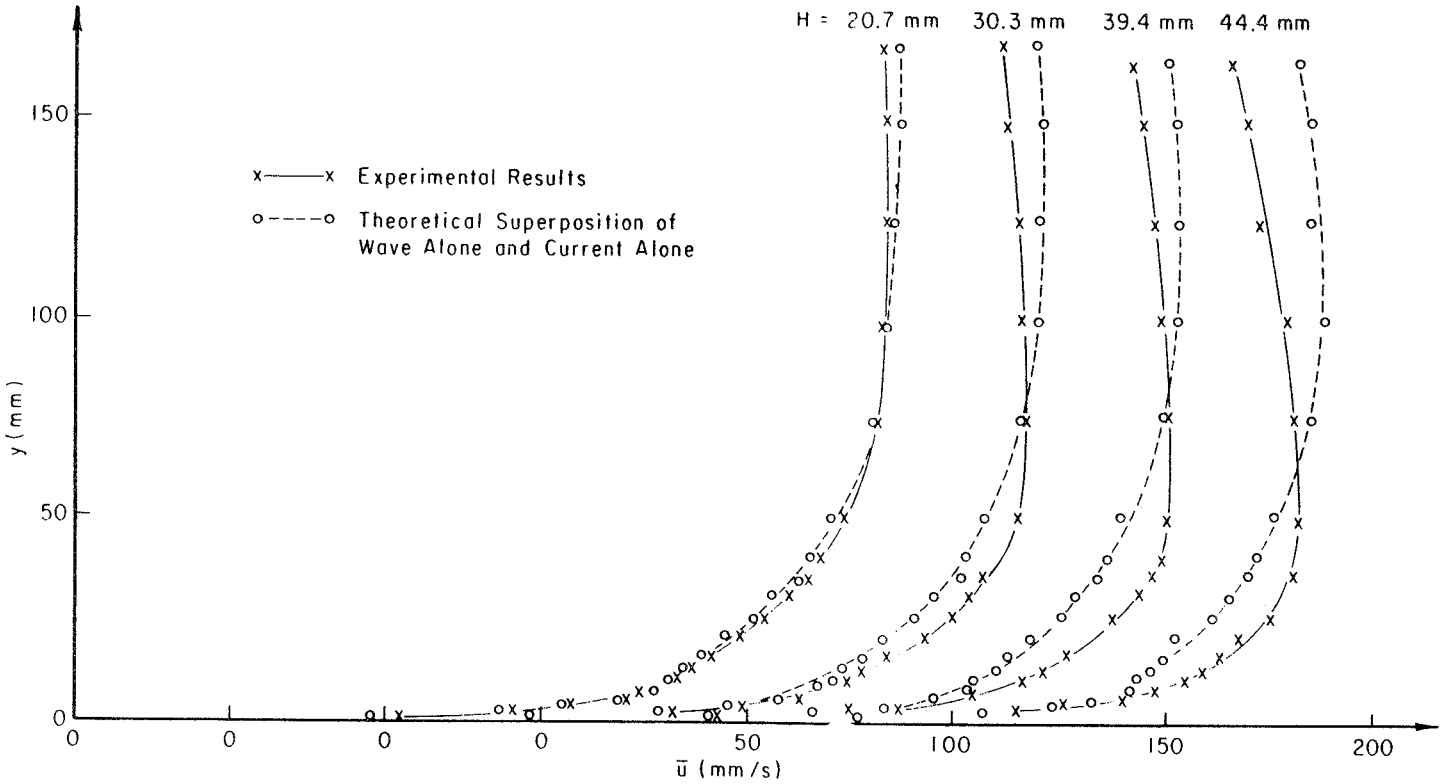

Figure 4. Velocity profiles due to wave-current interaction compared with the profile obtained by superposing measured velocities with waves alone and currents alone (redrawn from Fig. 5 of Kemp and Simons, 1982).

$$C = (g/k)^{1/2} + 2k\int_{-\infty}^{0} u(z) \exp(2kz)\, dz + O(u^2/C^2) \qquad (7)$$

where u(z) is the component of current in the wave direction and deep water is assumed.

Measurements of the lengths of stationary waves in flowing water by Fredsøe (1974) showed some influence of the current vorticity. Jonsson, Brink-Kjær, and Thomas (Fig. 2, 1978) found that a linear current profile would account for most of the deviation from the uniform velocity solution.

Sarpkaya's (1957) experiments indicate another area worthy of investigation. In these, waves propagating upstream in a flume were amplified, if the initial amplitude was big enough, even though the current was uniform along the flume. This is an unexplained phenomenon that has disturbing implications for waves entering inlets and harbors against adverse flows. To date, the experimental results have not been repeated though it is understood (Kemp, University College, London, personal communication, 1981) that experiments in progress may be suitable for verifying the results. Possible mechanisms worth investigating are (a) that flow reversal occurs near the bed and a thickening of the boundary layer acts to amplify the waves, and (b) the waves' interaction with the mean current profile leads to different and nonuniform flow conditions.

For calculations of finite-amplitude waves on a shearing current and useful review, see Dalrymple (1973). Subsequent related work is in Dalrymple (1974a, 1974b, 1977), Dalrymple and Cox (1976), and Brevik (1979).

There have been recent developments in the study of "wind drift" currents which merit notice. Craik and Leibovich (see Craik, 1977) explain how surface waves can interact with the shear due to wind drift and hence cause an instability which leads to a helical type of motion with its axis in the direction of the current and dominant wave direction. Helical motions seen in the field are known as Langmuir vortices. See also Craik (1982). In the development of their theory, an equation known as the Craik-Leibovich equation is derived. This equation describes the effect of Stokes drift in stretching vortex lines. The vorticity of the wind shear is directed perpendicular to the Stokes drift, but any deviation from that direction gives a vorticity component that can be enhanced by stretching.

These results are important for understanding "detailed" currents in the ocean. This is also an area where the theoretical technique of the "generalized Lagrangian mean" developed by Andrews and McIntyre (1978 a, b) can usefully be employed (e.g., see Leibovich and Paolucci, 1981).

4. Refraction of Waves by Currents -- Theory.

Refraction methods are reasonably well established for water waves over water of variable depth and in many other physical applications, especially in acoustics and optics. However, two significant differences occur for water waves on a current. First, the current carries the waves, so that wave energy propagates with the sum, $\underset{\sim}{u} + \underset{\sim}{C}_g$, where $\underset{\sim}{C}_g$ is the group velocity relative to the current, $\underset{\sim}{u}$. Since this sum is usually not perpendicular to wave crests, energy is usually not transmitted in the direction of wave motion. Second, wave energy is **not** conserved in the absence of frictional dissipation since energy is transferred between the waves and the currents.

Refraction theory in general has advanced considerably within the **last 20 years, particularly through recognition of the concept of wave** action. Wave action is important for waves on currents since it, unlike wave energy, is conserved in the absence of wave generation or dissipation. For linear waves, wave action equals (wave energy density)/(wave frequency relative to the current). Much of the recent theory has arisen from the study of nonlinear waves, but the presentation in this review is based on linear theory unless explicitly stated. Some linear results also hold for nonlinear waves, e.g., the Doppler relation (eq. 3), but others are modified. Refraction theory has the **primary assumption of locally plane waves, i.e., at any point waves can be recognized as a train of plane waves on a local scale (on time and length scales corresponding to at least a few wave periods or wavelengths).** This restricts consideration to large-scale currents (defined in Section II, 1) and to large-scale variations in the wave field (but see the discussion of caustics in Section II, 11). For linear waves, separation of waves into two or more superposed wave trains is permissible.

Wherever the primary assumption of locally plane waves holds, the phase of a single progressive wave train can be identified. That is, a description of the form

$$a \cos (\underset{\sim}{k} \cdot \underset{\sim}{x} - \omega t + \delta) \tag{8}$$

is possible for most wave properties such as surface elevation. The phase, S, is

$$S(\underset{\sim}{x},t) = \underset{\sim}{k} \cdot \underset{\sim}{x} - \omega t + \delta \tag{9}$$

On a large scale, $\underset{\sim}{k}$ and ω, and δ are also functions of position, $\underset{\sim}{x}$, and **time, t.** This means it is possible to take the function $S(\underset{\sim}{x},t)$ and say that the partial derivatives,

$$\frac{\partial S}{\partial t} \text{ and } \nabla S = \left(\frac{\partial S}{\partial x_1}, \frac{\partial S}{\partial x_2} \right), \tag{10}$$

are equivalent to $-\omega$ and $\underset{\sim}{k}$, respectively, but that other variations of S are of larger scale and correspond to the waves' refraction. In this notation, the vectors $\underset{\sim}{k}$ and $\underset{\sim}{x}$ are horizontal vectors with components in x_1 and x_2 directions, and following a common convention, a Greek suffix represents the two components, e.g., $\nabla S = \partial S / \partial x_\alpha$.

If the propagation of a wave is to be followed, or predicted, by refraction theory, then $\underset{\sim}{k}$ and ω must be defined and vary smoothly along the wave's path. This means that for any part of the wave field accessible to refraction theory, S, ω, and $\underset{\sim}{k}$ must be smooth functions. Mathematically, these are required to be differentiable. Then, for consistency, the partial derivatives of S must be independent of the order of differentiation, i.e.,

$$\frac{\partial^2 S}{\partial t \partial x_\alpha} = \frac{\partial^2 S}{\partial x_\alpha \partial t} \tag{11}$$

and

$$\frac{\partial^2 S}{\partial x_1 \partial x_2} = \frac{\partial^2 S}{\partial x_2 \partial x_1} \tag{12}$$

In terms of ω and $\underset{\sim}{k}$, these "consistency conditions" become

$$\frac{\partial \underset{\sim}{k}}{\partial t} + \nabla \omega = 0 \tag{13}$$

and

$$\nabla \times \underset{\sim}{k} = 0 \tag{14}$$

Equation (13) can be interpreted as the "conservation of waves" or "conservation of wave number." The description consistency condition is preferred since it helps to emphasize the underlying assumption that the phase, S, is a smooth function.

Equations (13) and (14) are one vector equation and one scalar equation, respectively, for the scalar and vector unknowns, ω and $\underset{\sim}{k}$. However, they are not independent since the curl of equation (13) gives

$$\frac{\partial(\nabla x \underset{\sim}{k})}{\partial t} = 0 \tag{15}$$

hence, equation (14) can be interpreted as just an initial condition for equation (15). This is similar to the irrotational condition, that vorticity is zero, being used for inviscid flow, whereas it may also be considered simply as an initial condition for use with Kelvin's circulation theorem. However, since there are three unknowns, ω, k_1, and k_2, an extra equation is needed. The waves are locally like a plane wave train so they must satisfy the dispersion equation (5) which may be written:

$$\omega = \underset{\sim}{k} \cdot \underset{\sim}{u} + \sigma(k,d) \tag{16}$$

It is in this equation that the space and time dependence of the medium enters in $\underset{\sim}{u}(\underset{\sim}{x},t)$ and $d(\underset{\sim}{x})$.

Now ω can be eliminated by using equation (16) in equation (13) to give

$$[\frac{\partial}{\partial t} + (\underset{\sim}{u} + \underset{\sim}{C}_g) \cdot \nabla]k_\alpha = \frac{\partial \sigma}{\partial d} \frac{\partial d}{\partial x_\alpha} - k_\beta \frac{\partial u_\beta}{\partial x_\alpha} \tag{17}$$

after using equation (14) where $\partial\sigma/\partial d = \sigma k/\sinh 2kd$ from equation (4). Equation (17) is in tensor notation for which the repeated subscript β indicates summation over $\beta = 1, 2$.

Another useful equation to be obtained from the same three equations is

$$[\frac{\partial}{\partial t} + (\underset{\sim}{u} + \underset{\sim}{C}_g) \cdot \nabla]\omega = \underset{\sim}{k} \cdot \frac{\partial \underset{\sim}{u}}{\partial t} + \frac{\partial \sigma}{\partial d} \frac{\partial d}{\partial t}, \tag{18}$$

where d has been allowed time variation for full generality. Here and elsewhere

$$\underset{\sim}{C}_g = C_{g\alpha} = \partial\sigma/\partial k_\alpha \tag{19}$$

is the group velocity of the wave relative to the current.

The structure of equations (17) and (18) is quite simple. The only derivatives of the unknowns $\underset{\sim}{k}$ and ω appear in the bracketed operator on the left-hand side of both equations. The form of this operator indicates that the characteristics of the equation are given by the space-time paths or "rays,"

$$d\underset{\sim}{x}/dt = \underset{\sim}{u} + \underset{\sim}{C}_g \tag{20}$$

Thus, these rays are in the direction of the total group velocity, $\underset{\sim}{u} + \underset{\sim}{C}_g$.

The major difference from the case of zero current is that the ray direction, in general, differs from the direction of the wave number vector. That is, rays are **not** orthogonal to wave crests except in the special case of parallel wave and current directions. Thus, unlike depth refraction, the orthogonals to the crest (i.e., lines parallel to k) do not show the direction of wave travel, but only indicate the local orientation of the wave crest. The difference between the orthogonal, $\underset{\sim}{k}$, direction and the ray direction can be seen in Figure 5 by the geometrical formulation of the vector sum in equation (20). This difference for a particular case is illustrated in Section II, 5.

The mathematical structure of these refraction equations (17) and (18) is unchanged between zero and nonzero current velocity fields. Thus given values of $\underset{\sim}{k}$ along some line, not coincident with a ray, equation (17) can be integrated simultaneously with equation (20) to give rays originating from each point of the original line. For example, the initial line could be a wave maker creating waves in a laboratory basin, or waves incident from deep water on a coastal region.

The only published example directly using these equations in space-time is that of Barber (1949), who traced a single ray across the Continental Shelf to the southwest of Britain in order to explain observed fluctuations in the periods of oceanic swell. The variation of tidal currents in space and time appeared adequate to explain the variation although there was not detailed agreement. With the computational advances now available, such calculations should be tried again.

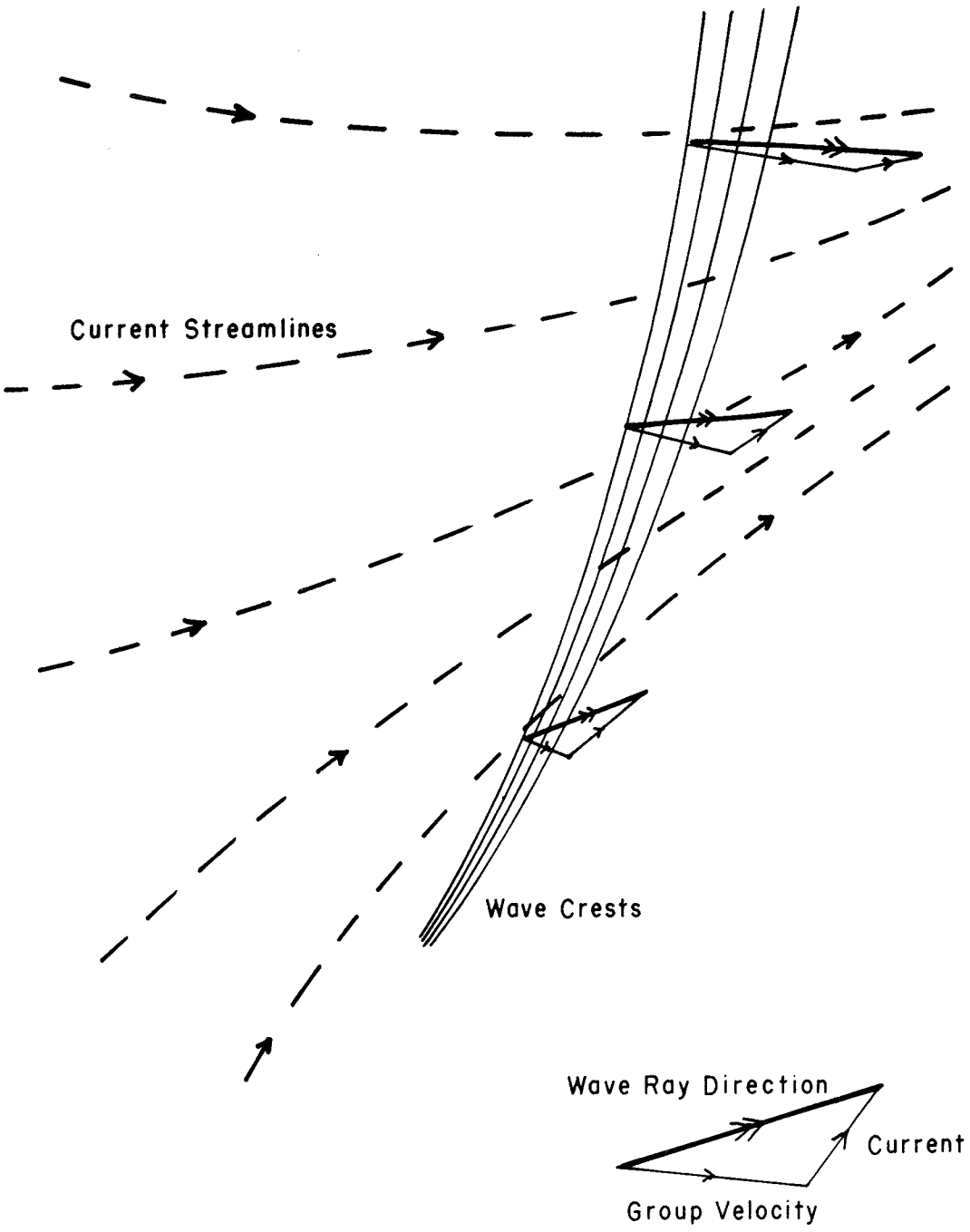

Figure 5. Direction of rays as vector sum of current plus group velocity. Drawing shows sea surface viewed from above.

More studies have been made of the simplified case in which the currents and waves are steady in time. For this case, the right-hand side of equation (18) is zero and ω is constant along rays. Conditions are usually chosen so that ω is equal to the same constant on all rays.

For some applications, information on wave number and frequency is sufficient, but in most cases, wave amplitude is also required. There are several ways of deriving equations for wave amplitude. Early attempts to do so were incorrect (see the pioneering paper by Johnson, 1947) since the fact that energy can be exchanged with the current was not appreciated. The matter was resolved for water waves by Longuet-Higgins and Stewart's (1960, 1961, 1962) papers which are still of interest because of the examples given of various applications. The final paper in the series (Longuet-Higgins and Stewart, 1964) summarizes the important aspects of their work. (For acoustics, the corresponding equations had been correctly formulated by Blokhintzev, 1956.)

The mathematical approach which shows the effect most clearly is to average the equations of motion over the period of the wave motion and examine the resulting terms. This is set out in Phillips (Sec. 3.5, 1977). Averaging the effect of the oscillatory velocity field due to the surface waves leads to an effective stress field in the fluid. Mathematically, this is the same as the Reynolds stresses obtained by averaging a turbulent flow. These effective stresses act on the mean flow. Longuet-Higgins and Stewart call

$$S_{\alpha\beta} = \int_{-d}^{\eta} (\rho u_\alpha u_\beta + p\delta_{\alpha\beta})dz - \tfrac{1}{2}\rho g d^2 \delta_{\alpha\beta} \qquad (21)$$

the radiation stress tensor, in which $\eta(x,t)$ is the free-surface elevation, u_α the oscillatory horizontal particle velocity due to the wave motion, p the pressure, and $\delta_{\alpha\beta}$ is either 1 or 0 (the Kronecker delta). The final term in equation (21) is simply the hydrostatic force corresponding to mean water level. (The sign of $S_{\alpha\beta}$ is opposite to that which is usual for Reynolds stresses.)

Energy is transferred because, as water particles move, they move across a velocity gradient and interact with it. The physical interpretation of radiation stress is best described in Longuet-Higgins and Stewart (1964), and its role in interaction with currents is described in the book by Lighthill (Fig. 78, p. 329, 1978), where it is called a mean momentum flux tensor.

From a different approach, building on Whitham's (1965, 1967) work on averaging nonlinear waves, and using an averaged Lagrangian, Bretherton, and Garrett (1968) drew attention to and showed the importance of a quantity they called wave action. It arises naturally

in Whitham's Lagrangian theory and has proved to be a valuable concept. In moving media, such as the currents being considered here, it is a wave-related property that is conserved in the absence of dissipation.

The first derivation of the concept of wave action was via a Lagrangian. No convenient Lagrangian is available for rotational free-surface flows. However, Stiassnie and Peregrine (1979), using conservation of mass, momentum, and energy, show that wave action is also conserved if the large-scale flow is rotational. (The "local" wave motion is irrotational.) See also Christoffersen and Jonsson (1980) who include dissipation. This result, derived for fully nonlinear waves by Stiassnie and Peregrine, means that wave-action conservation can be applied to any large-scale current system.

For linear waves, wave action is

$$A = E/\sigma = \tfrac{1}{2}\rho g a^2/\sigma \tag{22}$$

where E is the usual energy density of linear wave motion, and a is wave amplitude. As before, σ is the wave frequency relative to the current. (The result (eq. 22) does not extend to nonlinear waves.) The standard refraction analysis for water waves in shoaling water normally has a constant frequency ω, which equals σ for the case of no currents. Thus in the better established case of still water, the conservation of wave action is equivalent to the conservation of wave energy. (Note that this is not so for nonlinear waves, partly due to wave-induced currents.)

In mathematical terms the conservation of wave action is expressed as

$$\partial A/\partial t + \nabla \cdot [(\underset{\sim}{u} + \underset{\sim}{C}_g)A] = 0 \tag{23}$$

For applications equation (23) can be rewritten as

$$[\partial/\partial t + (\underset{\sim}{u} + \underset{\sim}{C}_g) \cdot \nabla]A = -[\nabla \cdot (\underset{\sim}{u} + \underset{\sim}{C}_g)]A \tag{24}$$

The left side of equation (24) is the rate of change of wave action along a ray, and the right side shows that the rate of change varies with the divergence of the rays. The operator on the left side of equation (24) is the same as that in equations (17) and (18). Thus these three equations have the same characteristics and may be integrated along the rays described by equation (20).

An alternative to using wave action is to consider the total energy of the system, which is conserved in the absence of dissipation. For steady flows and wave conditions, total energy is a relatively straightforward quantity to use as is shown by Jonsson (1978a). He also found that under irrotational flow conditions total energy flux is proportional to wave action flux, $\underset{\sim}{B}$.

$$\underset{\sim}{B} = (\underset{\sim}{u} + \underset{\sim}{C}_g)A \tag{25}$$

One feature of this method of solution which is rarely pointed out is that the current field should satisfy the nonlinear shallow-water wave equations. In that approximation, the horizontal flow is uniform with depth. This causes no problem in most cases, but there are some detailed difficulties in reconciling the general equations with the deepwater limit. (For linear waves, the deepwater limit is no problem.) This point is mentioned again in Section II, 10 on nonlinear effects. As noted previously, vertical velocity profile effects do have influence, but except for unidirectional problems, no refraction examples have been examined.

5. Refraction of Waves by Currents -- Simple Examples.

For those cases in which refraction problems can be reduced to problems depending on one coordinate only, being uniform with respect to other coordinates, it is possible to find analytical solutions. For example, the consistency condition (eq. 14) reduces to Snell's law

$$k_2 = \text{constant} \tag{26}$$

if there is no variation in the x_2 direction. An interesting range of problems can be solved in this manner.

One example, which has similarities with waves obliquely incident on a beach, is a current $V(x)\underset{\sim}{j}$, where $\underset{\sim}{j}$ is a unit vector in the positive y direction. That is, consider a current perpendicular to the direction of variation x. Any type of shear of this unidirectional current is possible. A relatively detailed discussion is given in Peregrine (Sec. IIE, 1976) where different axes are chosen.

In this case, it is relatively simple to understand what is happening simply by considering how the current acts to convect the waves. For simplicity, a horizontal bed is assumed. If waves propagate onto steadily stronger currents, wave direction turns toward the slowest part of the current when the waves have any component of propagation in the current direction (compared with depth refraction). See curves for 60° and 240° in Figure 6. On the other hand, if waves have a component

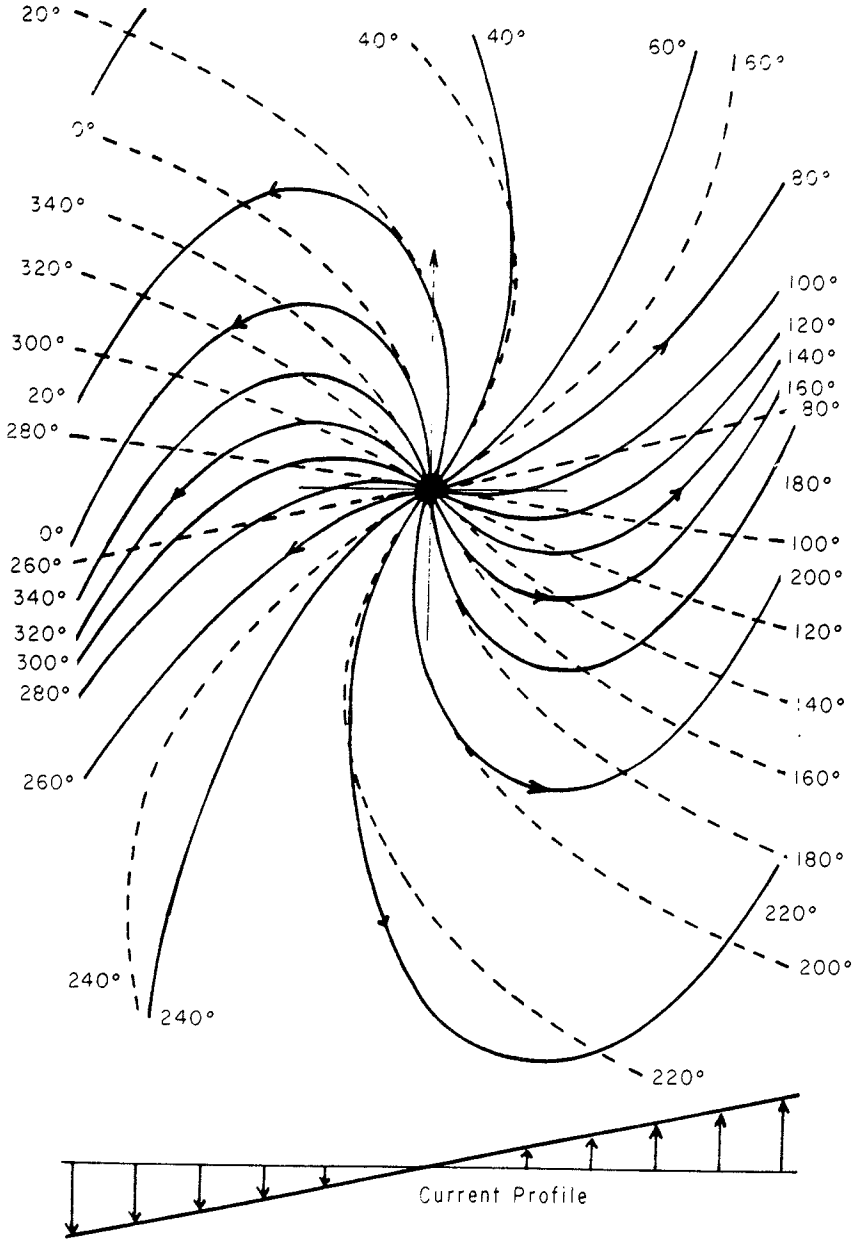

Figure 6. Effect of shearing current on direction of wave energy (solid curves) and wave crests (perpendicular to dashed curves). Angles in degrees indicate the directions of the curves as they pass through the origin, measured clockwise from north. Shear velocity directed north or south. The changed direction varies with magnitude of shear and equals the direction of the curve where intercepted by the line of the velocity vector. Adapted from Kenyon (1971).

of propagation against the current they are turned in the direction of the fastest current (which slow the waves most). See curves for 160° and 340° in Figure 6.

Figure 6 shows the rays calculated for a shear flow with linear variation, $V(x)\underset{\sim}{j}$. The rays (full lines) are shown for every 20° of initial direction from a single point on the line of zero current. Lines parallel to $\underset{\sim}{k}$ are also shown (broken lines), these are orthogonal to wave crests and can be used to deduce what a particular wave field might look like. The two sets of lines differ because of the current.

Certain properties are simply deduced from Snell's law (eq. 26), which shows that the wavelength is proportional to $\cos\theta$ in this case, where θ is the angle between $\underset{\sim}{u}$ and $\underset{\sim}{k}$. Thus, when waves propagate directly with or against the current, $\theta = 0$ or π, they have their maximum or minimum wavelength and relative group velocity, but as they turn to become more nearly perpendicular to the current, $\theta \to \frac{1}{2}\pi$, the wavelength shrinks toward zero, and so does their relative group velocity, $\underset{\sim}{C}_g$. As $\underset{\sim}{C}_g \to 0$, a greater part of the wave propagation is simply due to the current. However, as the wavelength gets smaller, wave steepness increases, the waves break, and the limit $\theta = \frac{1}{2}\pi$ is not attained.

A coastal engineer is accustomed to having an initial region in which waves are uniform. A ray diagram for such a case can be deduced from Figure 6 by choosing a single initial direction along a streamline and repeating the corresponding ray many times by parallel translation up (or down) the diagram (see Fig. 7 for a closely related example).

When wave direction ($\underset{\sim}{k}$) becomes parallel to the current, shown for initial directions 20°, 40°, 200°, and 220° in Figure 6, they are reflected toward weaker currents. The reflection line is called a caustic. Simple refraction theory (ray theory) predicts singular (i.e., infinite) wave amplitudes at such lines, but a better approximation to the full linear theory gives a finite amplitude (Section II, 11). Examples of the rays for two caustics are shown in Figure 7, which is a sketch of waves on a flow in a channel with reflecting walls. The upstream propagating waves are trapped at the center of the channel, the downstream waves near the edge. Only the latter case is comparable with Figure 6 since the waves in midchannel cannot reach the zero current at the edge.

A recent study (Hayes, 1980) models waves propagating across the Gulf Stream near Florida in this way. In agreement with the model, measurements show that the current shelters the coast from certain waves by reflection at a caustic.

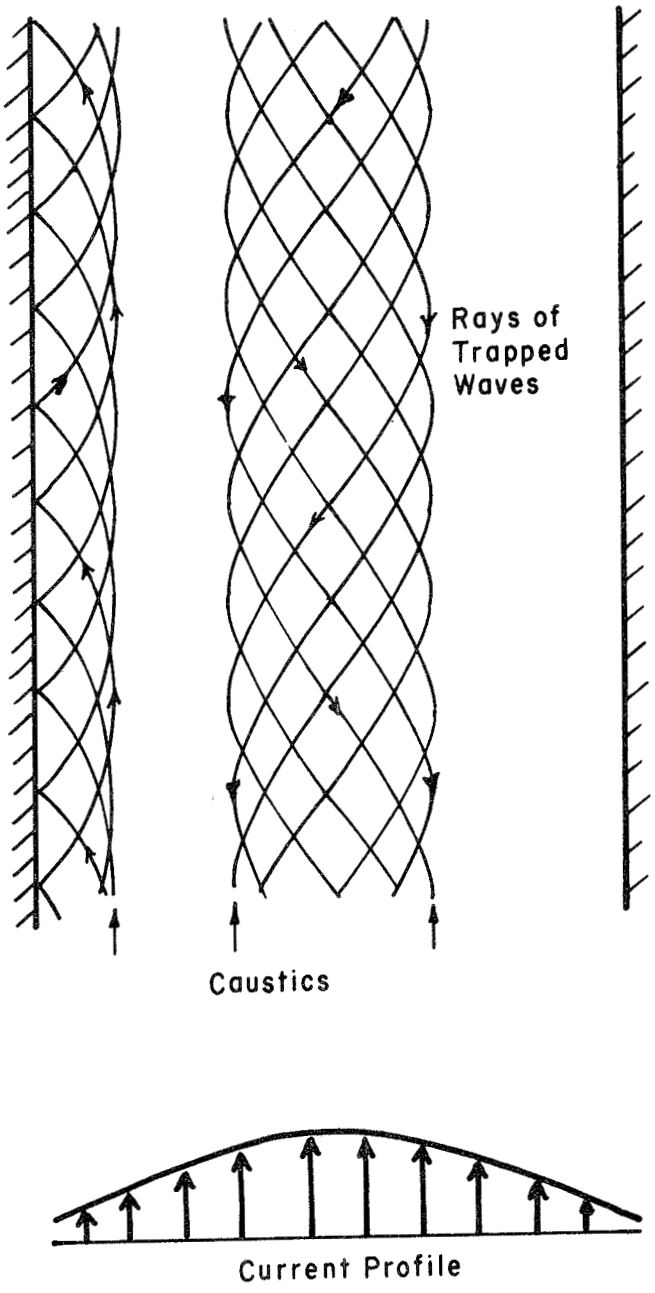

Figure 7. Rays in a nonuniform stream in a channel, showing two sets of trapped waves and their corresponding caustics.

Another basic example is of waves propagating directly against or with a current, i.e., take a current $u(x)\underset{\sim}{i}$, where $\underset{\sim}{i}$ is a unit vector in the x direction. This may occur in a channel of variable depth. More details are in Peregrine (Section IID, 1976) for deep water and Jonsson, Skougaard and Wang (1970) for finite depths.

An effect here is the lengthening of the waves as the current increases in the wave direction and a decrease in wavelength as the current decreases. This corresponds, for a horizontal bed, with the sign of the rate of strain which is simply du/dx in one dimension: $du/dx > 0$ is an extension; $du/dx < 0$ is a compression.

A different influence becomes prominent for sufficiently strong adverse currents. As -u increases and L decreases, a point is reached where the total group velocity $u + C_g = 0$ and the waves are stopped. Since the wave action flux is not zero in such a case, the wave action and wave steepness become infinite in this simple refraction theory. This is the basis of hydraulic and pneumatic breakwaters. The velocity needed to stop deepwater waves is only $\frac{1}{4}C_o$ where C_o is the phase velocity of the waves on still water. When incident waves meet such a strong current they break and lose their energy. Calculations aimed at clarifying the effect of these breakwaters, including a representation of the velocity variation with depth, have been made by Taylor (1955) and Brevik (1976). See also Jonsson, Brink-Kjaer, and Thomas (1978). The stopping point was recognized by Peregrine (1976) and by Stiassnie (1977) as a form of caustic and is mentioned in Section II, 10 and 11.

Further examples resembling these two, such as $u(x)\underset{\sim}{i} + V\underset{\sim}{j}$ where V is constant, can also be examined analytically. These more general examples are discussed in Peregrine (1976) and Peregrine and Smith (1979). In all cases, caustics can arise. This implies that in any general current field there may be areas of particularly steep waves. Sufficiently short waves (i.e., where C is only a few times greater than the maximum current speeds) will meet caustics or propagate into regions where they have much reduced wavelength. In the field, such steep waves often occur where tidal or freshwater flows are constricted. They are common off headlands protruding into tidal currents and may also occur off estuaries with strong tidal or freshwater flows and among islands within an area of significant tidal range.

The examples up to here involve wave-current interaction which makes the surface waves steeper. The complement of these surface areas with steeper waves is the areas of little or no wave activity. Areas of reduced wave steepness are particularly likely with short-period waves which are more easily stopped by adverse currents, or reflected by shear currents, or dissipated when oversteepened. Even following currents can increase wave steepness to the breaking point (Jonsson and Skovgaard, 1978). Shear currents may filter wave spectra, dissipating or reflecting certain components while transmitting others with substantial increase or reduction in amplitude. This has been demonstrated in

detail by Tayfun, Dalrymple, and Yang (1976) in the particular case of one-dimensional variations in currents and bathymetry. This case has a simple analytical solution.

There appear to be no published accounts of attempts to calculate the refraction of wave fields for currents of only moderate complexity (e.g., flow around a headland or out of an estuary) except for the early work of Arthur (1950) on rip currents which preceded the full understanding of the relevant equations. In fact, rip currents appear to have too small a scale to be adequately dealt with by refraction theory. Rather more complex computations are described in Noda (1974), Birkemeier and Dalrymple (1975), and Ebersole and Dalrymple (1980) which are concerned with nearshore wave-generated currents. It is difficult to interpret and gain physical understanding from complex models before models of intermediate complexity are understood.

A different set of one-dimensional examples is that of currents caused by wave motions. Examples are the tides, certain currents produced by internal waves, and currents induced by surface waves much longer than the waves riding upon them. The surface velocities due to traveling waves can usually be described as functions of $x - Ct$, where C is the phase velocity of these long waves. By considering the motion in a reference frame moving with the long wave at velocity C, the current field becomes steady and analysis of the shorter wave motion becomes similar to that of the above examples. However, the character of the current field, usually periodic, and the initial conditions for the waves, created on a current of $-C$, are different.

Longuet-Higgins and Stewart (1960) discuss short waves on swell and waves on tidal currents. These are both cases where the "currents" are also surface gravity waves, and in such cases, the vertical acceleration of the surface should also be taken into account. As shown in Peregrine (Section IIF, 1976), this does not make the analysis any more complicated. The major effect of these long gravity waves on shorter gravity waves is to cause the short waves to be steeper on the crests of long waves and gentler in the troughs. Tide-induced variation of this type initiated some of the earliest analysis of waves on currents (Unna, 1941, 1942).

The interaction of short waves with swell led Longuet-Higgins (1969) to suggest a possible interaction with the longer waves causing short waves to grow. Hasselmann (1971) identified more possibilities for energy transfer but disagreed with Longuet-Higgins on the magnitude of any growth. Further work by Garrett and Smith (1976) introduces the possibility of even more interaction when a wind is blowing.

The behavior of water waves on the current field due to internal waves has been studied more intensively than any other aspect of wave-current interaction. This is largely because of the desire to understand how internal waves in the ocean are generated. Interactions with surface water waves are a possible growth mechanism for internal

waves. Currents generated by internal waves lead to patterns on the water surface which, in some cases, are directly due to differing steepnesses of short surface waves. Surface waves with a group velocity close to the phase velocity of the internal waves are those most strongly affected. These have been recorded in the Georgia Strait by Gargett and Hughes (1972). Particularly strong effects have been observed in the Andaman Sea by Osborne and Burch (1980) among others. The most comprehensive field observations are those of Hughes and Grant (1978) who generated internal waves with a ship and then measured surface and internal wave fields. Hughes (1978) follows up these observations with a theoretical analysis in which the general features are reasonably well predicted by simplified analysis in which the current magnitude is assumed small.

There have been different ways of analyzing the interaction of surface and internal waves (e.g., Watson, West, and Cohen, 1976; Rizk and Ko, 1978; Basovich, 1979; Hashizume, 1980). This interaction has also been studied experimentally with some success (Lewis, Lake, and Ko, 1974; Joyce, 1974).

A major effect of the internal wave-current field is to trap sufficiently short surface waves between a pair of caustics each side of the internal wave's crest. The tendency to trap waves and to steepen slightly longer waves leads to clearly visible surface traces of the internal waves. These surface waves are too short to be of any engineering significance except in the interpretation of remote-sensing observations.

The many ways in which the surface-internal wave interaction problem has been studied indicate that it could be a fruitful field for further wave-current interaction research. If all theories and experiments on the topic are set in a framework of both wave-current interaction and current-generating properties of waves, a distinct improvement in understanding is likely.

In practical applications, refraction by varying depth will occur at the same time as refraction by currents. Tayfun, Dalrymple, and Yang (1976) and Jonsson and Wang (1980) have investigated the effect of a shoaling bottom with a shearing current parallel to the bottom contours, another example in which solutions depend on a single coordinate. As might be expected, the effects of current and bed gradients on wave refraction can either reinforce or oppose each other.

Another example which is sufficiently simple for analysis is the case of stationary waves on a current. Peregrine and Smith (1975) investigate a number of examples including variations of velocity with depth as well as horizontally.

6. Dissipation and Turbulence.

Dissipation of ocean waves is often considered negligible, but where waves propagate for appreciable distances over shallow water, or in any depth under significant wind action, dissipation may be significant. In the absence of currents, dissipation occurs by two mechanisms. The most effective mechanism is wave breaking. This usually occurs relatively close to shore, but for the largest ocean waves with heights of 30 meters, some breaking can be expected on the Continental Shelf. And while waves are being generated by the wind, some of the steeper waves are breaking, whatever the depth of water. Breaking related to currents is discussed further in Section II, 12.

The other major mechanism for dissipation is bottom friction. This becomes important when waves travel a long distance in shallow water. For ocean waves, the bottom boundary layer is turbulent and hence only a semiempirical approach is possible, e.g., see Knight (1978) and Jonsson (1980). More recently, Brevik (1981) has presented a simple two-layer model, which yields good agreements with Danish measurements.

The introduction of currents adds to the difficulty of estimating bed shear and dissipation. First attempts were made by Jonsson (1966) and Bijker (1966, 1967). More elaborate approaches to the problem have been made: Lundgren (1972), Bakker (1974), Smith (1977), Grant and Madsen (1979), Bakker and van Doorn (1978, 1980), Engelund (1979), Greulich (1980), Christoffersen (1980a, b), Christoffersen, Skovgaard and Jonsson (1981), Fredsøe (1981), Bakker and van Kesteren (unpublished, 1981). The investigations use a variety of eddy viscosity assumptions and mixing length hypotheses. Most models assume that the problem of interest is the interaction of waves and currents in a wave-dominated environment. Some consider the reverse case: a dominant current with waves on it (Smith, 1977; Engelund, 1979). Some of the papers give detailed advice as how to calculate the bed shear and velocity variation near the bed for the current at an arbitrary angle to the wave motion, e.g., Grant and Madsen (1979), and Christoffersen (1980b). Soulsby and Dyer's (1981) results deduced from boundary-layer measurements in accelerating tidal flows may help improve the modeling of wave-current boundary layers.

For experimental results see van Hoften and Karaki (1976), Bakker and van Doorn (1978, 1980), George and Sleath (1979) (oscillating bed), Iwagaki and Asano (1980), Brevik and Aas (1980), Brevik (1980), van Doorn (unpublished, 1981), Kemp and Simons (in preparation, 1982), all limited to plane flume flow. Bijker (1967), and Rasmussen and Fredsøe (1981), measured sediment transport in a basin, with the waves perpendicular to the current, see Section III, 3. Kemp and Simons' measurements are probably the most comprehensive; they include both mean velocities and turbulent fluctuations, as do those of Iwagaki and Asano.

A number of qualitative findings have come out of these measurements: the waves give added resistance to the mean flow; in the region outside the wave boundary layer, a logarithmic velocity distribution is often found with a greater roughness than the physical one; for a smooth bed there is a distinct "overshooting" (Figure 4) in the velocity profiles near the bed which increases with wave height (over a rough bed there is a more complex change); there is an enhanced upstream sediment transport when a weak current is superimposed on the waves over a rough bottom; mean velocities near a smooth bed are increased by the addition of waves whereas near the rough bed they are reduced.

It is difficult to assess properly the different theoretical approaches since the number of very accurate measurements is limited, and the range of wave and current parameters that have been investigated is very limited. Where comparisons are made between experiments and theories some agreement is found (Brevik, 1980; Brevik and Aas, 1980; Fredsøe, 1981; Christoffersen, in preparation, 1982).

There is no doubt that the interaction of waves, current, and bottom friction is nonlinear. The simplest nonlinear friction law is a quadratic "Chezy" friction term, $k\underset{\sim}{u}|\underset{\sim}{u}|$, in shallow-water equations. A detailed investigation of the effect of this type of term in an interaction problem has been made by Prandle and Wolf (1978).

The problem considered by Prandle and Wolf is the interaction of two long waves--the tide and a storm surge--both propagating in the North Sea and up the Thames Estuary. They analyze terms such as $u\partial u/\partial x$ and $k\underset{\sim}{u}|\underset{\sim}{u}|$ in a mathematical model. The model gives a good representation of the observed behavior of the surge wave on top of the tide. The nonlinear effects, which had been previously identified in statistical analysis of surges, are quite appreciable. Prandle and Wolf find that the friction term is a more important contribution to the interaction than is the nonlinear advection term $u\partial u/\partial x$.

If appropriate expressions for bed shear and dissipation are found, it is straightforward to add appropriate terms to the momentum and energy equations. But a more convenient equation is the wave action conservation equation. Clearly, dissipation leads to a loss of wave action. Christoffersen and Jonsson (1980) show that dissipation can be included in the wave action equation (23) by adding a term

$$\frac{\text{Diss}}{\sigma} = -\frac{\underset{\sim}{\tau}_b \cdot \underset{\sim}{u}}{\sigma} \tag{27}$$

to its left-hand side. Diss is the rate of dissipation per unit area and $\underset{\sim}{\tau}_b$ the mean bottom stress.

Even where waves are too short to affect or be affected by bottom friction, they can propagate through the turbulence generated by bottom friction of a current. In other cases, the turbulence associated with wind stress, which is in part transmitted to currents by wave breaking, may be more relevant. There is dissipation of waves in turbulence, but there is insufficient information to confidently assess its magnitude. Skoda (1972) describes experiments on waves propagating through turbulence, and Peregrine (Sec. V, 1976) reviews some of the few other papers on the topic. Experiments are difficult to interpret, as Skoda indicates, since turbulence may scatter the waves as well as increase dissipation (Phillips, 1959), and since it is difficult to generate turbulence, or waves, without setting up some small mean currents. Even if these currents are measured, their effect can easily be as great as the dissipation being sought.

7. General Solution Methods.

Only in simpler cases such as those described in Section II, 5, can explicit analytical solutions to refraction problems be found. In some respects, wave-current refraction is similar to wave-depth refraction and in practice the two are combined, i.e., in the dispersion equation (5) both $\underset{\sim}{u}$ and d are functions of position.

The natural way to approach any large-scale problem is to use ray methods following the wave. For steady currents, the most significant difference from stillwater refraction is that the wave orthogonal direction, i.e., the direction of $\underset{\sim}{k}$, differs from the ray direction, $\underset{\sim}{u} + \underset{\sim}{C}_g$, and hence an extra variable, the ray direction, must be accounted for. No extra equations are required, but this extra variable and the directional dependence of the dispersion equation (5) interfere with some simplifications that are possible when $\underset{\sim}{u} = 0$. $\underset{\sim}{C}_g$ is parallel to $\underset{\sim}{k}$.

In the process of calculating rays both σ and $\underset{\sim}{k}$ are also found, and singularities of the method such as focuses or caustics may be identified. If wave conditions are required only at a point, it may be more efficient to trace the rays backward from the point as is done for depth refraction.

Once rays have been found, the conservation of wave action between rays gives the wave amplitude. In depth refraction, it is usually efficient to use a differential equation as first described by Munk and Arthur (1952); see also Skovgaard, Jonsson, and Bertelsen (1975). A similar equation can be derived for the more general case involving currents (Skovgaard and Jonsson, 1976). For the steady flow case, Christoffersen and Jonsson (1981) find an integral of the energy equation along the streamlines of the current. This cannot replace the equations necessary to obtain $\underset{\sim}{k}$ and σ, but it may be useful in some circumstances for finding wave amplitude and water depth.

For discussions of some detailed aspects of the use of ray theory for wave-current interaction, see Skovgaard and Jonsson (1976), Iwagaki, et al. (1977), Christoffersen and Jonsson (1980), and Christoffersen (in preparation, 1982). For a mathematical review of depth refraction see Meyer (1979). Dingemans (1978, 1980) reviews both refraction and diffraction.

If a whole spectrum of waves is to be considered, then there is at present no alternative to considering each frequency and direction band separately, as is the case for depth refraction. Forristall, et al. (1978) describe an example of current refraction where this has been done with some success in an investigation of waves due to a tropical storm. Tayfun, Dalrymple, and Yang (1976) consider the transformation of a wave spectrum across a simple current shear combined with a depth transition. Several authors have considered the effect of currents parallel to the waves ($\theta = 0 \sim \pi$) on spectra, e.g., Huang, et al. (1972), Hedges (1979, 1981), and Burrows, Hedges, and Mason (1981).

An alternative to matching local solutions to a ray solution at any focus or caustic is to use a "parabolic" approximation. This corresponds to a second approximation in the modulation rate of the waves and usually implies a restriction to waves traveling within a small angle of some given direction. The method is described well in the context of ocean acoustics by Tappert (1977). It has advantages in that it includes diffractive effects which is valuable if a ray solution leaves an area too sparsely covered with rays. A parabolic equation for water wave-depth refraction is derived and applied in Radder (1979). Booij (1981) derives an equivalent of Berkhoff's (1972) linear wave equation (see Meyer, equation 3.6, 1979) from which a parabolic equation for wave-current interactions is found.

It is possible to consider computation from the basic equations of fluid motion. For short waves, this would be prohibitively expensive at present, if possible at all. For long waves, with the usual long wave approximations which effectively eliminate dependence on a vertical coordinate, it is possible. As already mentioned, tide-surge interactions can be modeled this way. The present state of this type of calculation is discussed in Peregrine (1981b) which deals with computational models of the North Sea and other seas around the British Isles to the edge of the Continental Shelf. One such model is being used on an operational basis with direct input from U.K. Meteorological Office's computer forecasts to forecast storm surges on British coasts. These computations include long wave-current interactions. The model described by Davies (1981) goes further to include the variation of velocity with depth. However, it too is a long wave model since pressure is taken to be hydrostatic.

A more versatile numerical model is that described by Abbott, Petersen, and Skovgaard (1978) which uses Boussinesq's equations. Although a long wave approximation is made in this model, some dispersive effects are also included, and hence somewhat shorter waves

may be accurately represented. The model is capable of dealing directly with currents and waves, but no reports of such combined wave and current calculations have been published.

Once any nonlinear effects are included, whether they involve dispersion, dissipation or wave breaking, there is interaction with the current field, and most of the above approaches become inapplicable because Fourier wave components may not be superposed. Boundary conditions around the whole region become important, and it is advisable to solve the differential equations over some spatial grid, using either a finite-difference or finite-element method, even if linear theory is used in part. Noda (1974) used finite differences in his investigation of nearshore circulation. Skovgaard and Jonsson (1976) indicate how finite elements may be used. See Section II, 10.

8. Small- and Medium-Scale Currents.

When the length or time scales of a current are comparable with those of waves upon it, it is not appropriate to use refraction theory. The example which has been most studied is the flow caused by a moving ship. This flow interacts with the waves generated by the ship, and work to date has been more successful in showing how difficult the problem is rather than in providing solutions; see Peregrine (Sec. VI, 1976). Photographs in Gadd (1975) show just how strong this interaction can be.

Coastal engineers are more familiar with the case of rip currents. These are often identified visually by the relatively low amplitude of waves advancing toward a beach over a rip current. This low amplitude, as discussed in early papers such as Shepard, et al. (1941), is contrary to what is expected from refraction theory (Arthur, 1950), which is that wave energy would become concentrated over the strongest current. In wave theory terms, this is probably a case where diffraction is important. Other cases in which small-scale currents exist include thin shear layers, e.g., where a current goes straight past a headland, or similar circumstances where a current comes out of an inlet. Photographs in Hales and Herbich (1972) show a jetlike current from an inlet with complex wave formations.

Theoretical results for small-scale currents are sparse. Evans (1975) successfully modeled deepwater waves incident on a vortex sheet. Except for the reflection, the results for the transmitted waves were very similar to those obtained from refraction theory (Longuet-Higgins and Stewart, 1961) for a slow change of current velocity. McKee (1977) found the reflection coefficient for waves entering a following jet and gave numerical examples for surface waves in deep water.

Evans' work is extended by Smith (1980) to two vortex sheets modeling a jetlike current. Among the many results presented by Smith there is a tendency for wave amplitudes on the current to be weaker than elsewhere. However, it is worth noting in this context that for waves

whose incident direction is almost parallel to the current, linear theory is valid only for waves of strictly zero amplitude. Smith notes this. Peregrine (1982) considers nonlinear effects, and shows that certain diffractive effects, which are usually not considered, are important in the case of near-parallel incidence for linear and nonlinear waves.

Another case that may be relevant to this problem is wave-wave interaction, some examples of which might be interpreted as wave-current interaction (see discussion of internal waves in Sec. II, 5). Another isolated example is Taylor's (1962) study of standing waves on a current.

9. Experimental and Field Observations.

Most of the experimental work has been in flumes, with or without wind, which constrain waves and current to travel in the same or opposite directions. The most detailed work of this type is by Kemp and Simons (1982). They confirm observations by Van Hoften and Karaki (1976), Bakker and van Doorn (1978, 1980), George and Sleath (1979), Brevik and Aas (1980), and Brevik (1980) that waves have a significant effect on the mean flow and hence on the bed friction. This then raises questions about the equivalent flow without waves, and means that it is difficult to assess any measurements which do not include observations of the mean level and mean flow in a flume. (An exception can be made for waves of very small amplitude where any reaction on the flow is negligible; see also Sec. II, 2.) As discussed previously, Sarpkaya's (1957) experiments which show wave amplification are among the most interesting.

Experiments which have set out to check the linear wave dispersion equations, such as Shemdin (1972) and Thomas (1981), have shown good agreement with theory when the variation of current velocity in the vertical has been taken into account. Shemdin also included the airflow since he was considering wind waves. However, Plant and Wright (1980) find that inclusion of finite-amplitude effects does not improve comparisons between theory and experiment. Fredsøe (1974) finds satisfactory agreement for stationary waves. Observations and experiments of wind-generated waves on currents were briefly discussed in Section II, 3.

Measurements of water waves in regions of appreciable tidal currents normally show fluctuations which correlate with the tides. Examples of such observations are given by Vincent (1979) and Vincent and Smith (1976).

One of the problems in correlating measurements in the open sea is that wave energy arrives from different directions with different frequencies, and all components have different time histories of wind and current. However, since there are wave-forecasting numerical models

in regular use, it would be desirable to see if inclusion of current refraction improves accuracy.

Many observations are anecdotal, e.g., Isaacs (1948). One large class of such events concerns "giant" or "freak" waves when waves and currents are in opposing directions. One area where severe wave damage to large ships has been reported is in the Agulhas Current. Mallory (1974) has studied some incidents in detail and reports cases of waves subject to strong winds over a fetch of 2000 kilometers before meeting the adverse current. Mallory suggests that interference with locally generated shorter waves may also contribute to particularly steep-fronted waves. See also the discussion and anecdotes of Draper (1975).

Several authors have suggested that wave caustics may enhance wave magnitude. It is commonly observed that over the Continental Shelf, inside the Agulhas Current, wave conditions are significantly less severe; Schumann (1976) provides wave observations from this area. Sugimori (1973) also gives wave observations on a strong current, the Kuroshio, which show different wave conditions on and off the current. Such strong changes in wave conditions are best interpreted as wave caustics, and it is likely that in some places the pattern of caustics forms a cusp with a focus of wave energy that increases the chance of a "freak" wave.

Remote sensing in the form of satellite photography (e.g., Perry and Shimke, 1965; Strong and DeRyke, 1973; Osborne and Burch, 1980), short wave radar scanning, and high-frequency radio scattering can all provide wave information on large sea areas. Some verification of wave refraction is possible as suggested by work of Mattie, Lichy, and Beal (1980). The traces of internal waves show up particularly well in aerial photography. It is very likely that the bands of rough water which occur at their crests may be the same as the "tide rips" described in Maury (Secs. 751 to 755, 1861).

A number of experiments have been performed with capillary waves on currents (e.g., Hughes and Stewart, 1961; Wu, 1979; Plant and Wright, 1980). These may appear to be of little interest to engineers. However, they are important in that surface roughness (gravity-capillary waves) is measured by some remote techniques, such as short wave radar. An appreciable fraction of the wind stress can be supported, and momentum transferred, by such surface roughness in moderate winds. These short waves are strongly affected by the currents of the larger scale wave motion.

10. Nonlinear Effects in Refraction.

For refraction calculations with finite-amplitude waves, it is necessary to allow the currents and depth of water to be determined as part of the solution. That means appropriate boundary conditions must

be used--a problem in itself in some cases. The equations given by Stiassnie and Peregrine (1979) are a good starting point, but there is no simplification to a ray theory such as there is for linear waves.

Simple examples, depending on a single coordinate, and using accurate periodic wave solutions, are given by Stiassnie and Peregrine (1980) and Ryrie and Peregrine (1982). These are for waves incident on a beach with zero mass flow toward the beach; they show that current and depth variations are quite small in the absence of dissipation. This suggests that for some engineering purposes an inconsistent approximation neglecting current and depth changes due to finite-amplitude irrotational effects might be adequate. However, a wider class of problems should be examined before reaching such a conclusion, since nonlinear effects from bed friction can be important; for example, in the surge-tide interaction (Sec. II, 6).

The major differences from linear theory are, as may be expected, for the steepest waves. An advantage of using accurate nonlinear solutions in a refraction calculation is that where the steepest possible progressing wave is predicted, it can reasonably be assumed that real waves will break within a few wavelengths of that position. See Stiassnie and Peregrine (1980) for more details. Sakai, et al. (1981) present experimental information on the effect of nonlinearity for waves breaking on an opposing current with three different bed slopes.

For deep water, with currents defined independently of the waves, Peregrine and Thomas (1979) give the results of refraction calculations for the two current distributions $u(x)\underset{\sim}{i}$ and $V\underset{\sim}{j}$. The major interesting results concern the neighborhood of caustics, as discussed in Section II, 11. Although it is not strictly consistent to define the current (e.g., see McIntyre, 1981), in these cases no significant error is likely.

Both Peregrine and Thomas (1979) and Ryrie and Peregrine (1982) show that for waves nearly perpendicular to the gradient of the medium, e.g., for waves whose direction is at glancing incidence to a beach, the refraction differs from linear theory in a qualitative manner. This is discussed in more detail in Peregrine (unpublished, 1982) where it is shown that diffraction becomes the dominant influence, suggesting that linear ray theory may be unreliable even for quite gentle waves in some circumstances.

It is difficult to extend refraction theory for nonlinear waves to complex realistic examples. The above-mentioned cases are all dependent on a single coordinate so that much of the necessary work can be done analytically. For nonlinear waves, there is no direct equivalent of group velocity. The differential equations have several characteristic velocities. For one-dimensional problems, there are two velocities corresponding to the single group velocity of linear waves plus another long wave velocity corresponding to depth and current changes (see

Whitham, 1962; Hayes, 1973; Whitham, Ch. 15, 1974). A discussion of the problem is given in Peregrine and Thomas (Sec. 5, 1979). This means that a ray approach is not directly available.

One effect of wave dissipation, whether it is by bed shear, interaction with turbulence, or breaking, is for momentum to be transferred from the wave motion to current motion. This is particularly important in the surf zone where such wave-induced currents include longshore currents and rip currents and cause wave setup.

An effect of wave motion on flowing currents, described in Section II, 3, is to change the bottom shear stress. This change can lead to significant changes in the magnitude of the current depending on how the current is maintained. Numerical examples are given by Christoffersen, Skovgaard, and Jonsson (1981).

These last two effects can only be modeled by simultaneously solving for both the current and wave fields; see Noda (1974), Birkemeier and Dalrymple (1975), Dalrymple (1980), and Ebersole and Dalrymple (1980).

11. Caustics and Focusing.

The concept of a caustic arises out of ray theory. In a family of rays, successive rays may cross. In the limit of a full, infinitely dense, set of rays the successive crossing points define curves by their envelopes. These curves are known as caustics; straight-line examples are shown in Figure 7. However, in a smoothly varying refractive medium, a single caustic does not initiate by itself. Rather two caustics initiate from a cusp, which is like a focus (Fig. 8).

Ray theory gives singular amplitudes at caustics and their cusps; this is a shortcoming of that theory. Better approximations have been known since the work of Airy (1838) in optics and are discussed in the water wave literature (e.g., Pierson, 1951; Chao, 1971). The wave behavior near a caustic is described by the Airy function, and all the major wave field properties can be found directly from ray theory solutions and the Airy function. Peregrine and Smith (1979) give some appropriate formulas. Similar results hold at cusps of caustics where the Pearcey (1946) function can be used (see also Peregrine and Smith, 1979). McKee (1974, 1975, 1977) has also studied linear caustics on currents (see Smith, 1981).

Caustics of two types occur in refraction patterns of rays (this is based on depth refraction patterns because of the lack of real examples of current refraction): (a) numerous crossings of a few rays corresponding to weak focusing and forming weak caustics (sometimes called "spaghetti" diagrams); (b) clear, well-defined focuses and caustics caused by major topographic features, such as the edge of a dredged channel or a lens-shaped shoal.

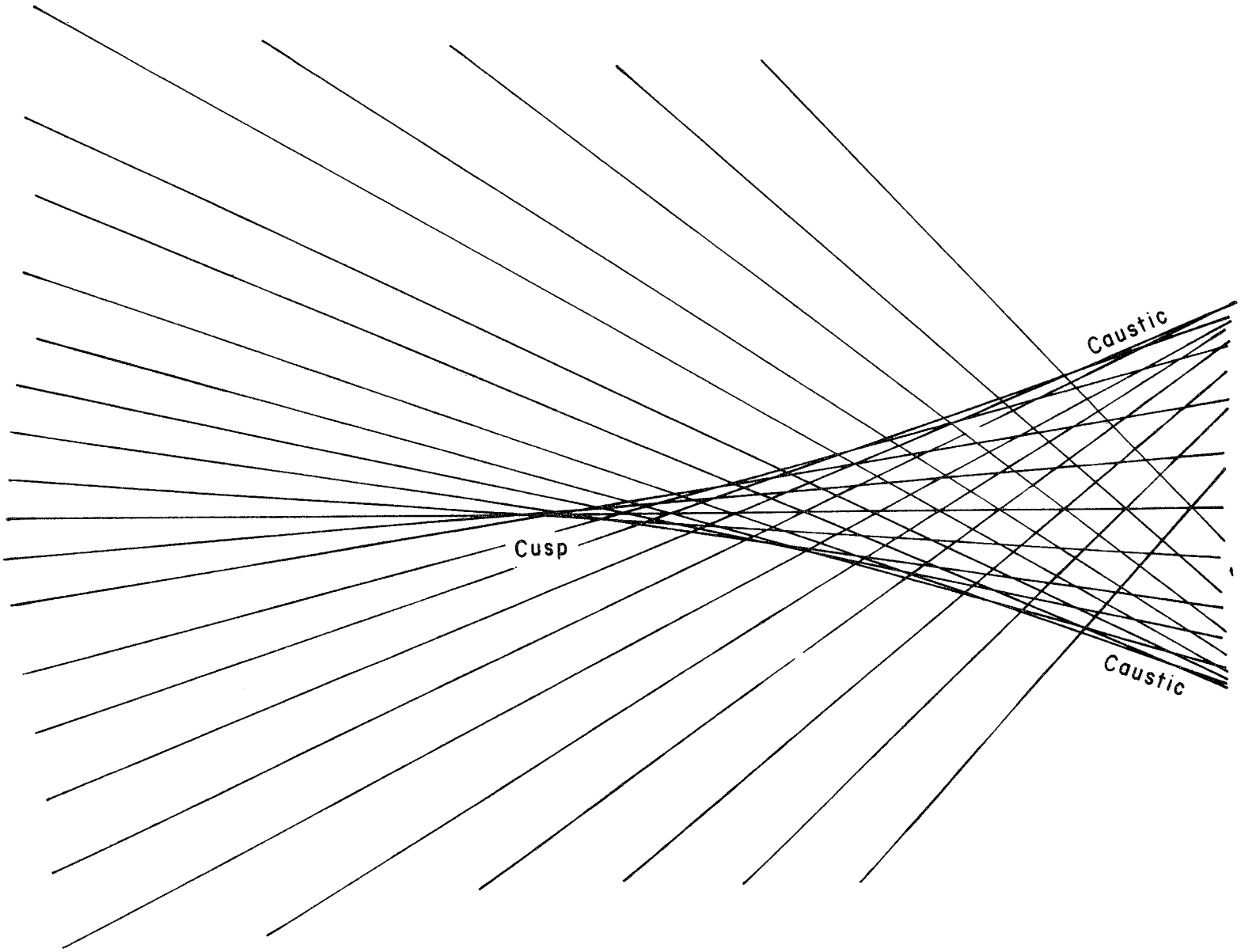

Figure 8. Rays focusing at a cusp. The cusp is formed where two caustics meet.

Case (a) can be dealt with adequately by using the parabolic approximation to simplify analysis; see for instance Radder (1979) and Booij (1981). Results of calculations for weak focuses using this approximation are given by Buckley (1975). For coastal engineering purposes, Buckley's results require an assessment in the context of water wave refraction.

Significant reflection from caustics (case b) seems to be particularly important in navigation; see for instance Pierson (1972) and Smith (1976). The former mentions waves reflected from a caustic formed by a submarine canyon as the possible cause of shipwreck, the latter waves from a caustic formed at the side of the Agulhas Current, giving rise to frequent ship damage.

Strong caustics can easily be calculated on the basis of linear theory from refraction results, as indicated above. However, a caustic or focus region is also a region of relatively large amplitudes so that nonlinear effects might be important.

Nonlinear behavior of water waves near caustics has been discussed by Peregrine and Thomas (1979) for waves on currents, by Peregrine (1981a) for circular caustics, and in a more general context by Peregrine and Smith (1979). These works show that there are two types of caustics, R and S. At R-type caustics, which are the only type of water wave-depth refraction, waves are unlikely to break unless there is too much convergence of wave energy before the caustic region is reached. Peregrine (1981a) gives a "caustic parameter" which can be used in estimating the likelihood of breaking, and the case of a perfect focus is also analyzed.

S-type caustics have nonlinear solutions which suggest that waves will normally break at such caustics unless their incident amplitude is very small. For water waves, some caustics on currents are type S; these include the "stopping current" caustic. Peregrine and Smith (1979) analyzed the different types of caustics for deepwater waves on currents to indicate which are S- and R-type caustics.

At R-type caustics, which are the most common for water waves, the above-mentioned papers give an incomplete analysis since they do not include interaction between incident and reflected waves. The importance of this interaction is shown by Yue and Mei (1980) who describe reflection of nonlinear water waves from a small-angle wedge. They find that reflection leads to a "wave jump," i.e., there is a jump in wave properties, amplitude, direction and length, along a line starting at the apex of the wedge. Reflection at R-type caustics may lead to similar jumps. Peregrine (1982) shows that caustics must be considered from their initiation, i.e., at cusps, focuses or discontinuities. It appears that nonlinear effects oppose the convergence of wave energy, and even a refraction calculation for a caustic cusp may have no singularity. If it does have a singularity then one, or two, wave jumps form. These jumps partially reflect the

incident waves and a completely different wave field from that predicted by linear theory can arise. A sketch of the lines parallel to $\underset{\sim}{k}$ at such a focus, with jumps, is given in Figure 9. Additional details, including the waves reflected, are given in Peregrine (1982).

The nonlinear results can be interpreted as a tendency to smooth the wave field; hence, for the case of weak focuses, it may be justifiable to smooth linear computations so that they do not arise. Further work on this topic is desirable.

12. Wave Breaking.

There is still much that needs to be understood about the breaking of both wind-generated waves on deep water and waves approaching a beach. Currents affect the likelihood of breaking, producing, for example, the less steep waves over rip currents already mentioned.

Refraction of water waves can cause breaking whether that refraction is due to variation of currents or depth. Breaking may be due to a convergence of wave action or to a slowing down of wave action transport when the flux is constant. Both phenomena can occur on simple shear flows (Isaacs, Fig.1, 1948; Jonsson and Skovgaard, 1978).

The focusing described in Section II, 11 is another description of the convergence of wave action by current. As indicated, nonlinear effects can delay or eliminate any singularity, but if Peregrine's (1981a) caustic parameter is small enough, the waves will break. Features of wave breaking for this case, which is essentially three dimensional, have not been studied.

A slowing down of wave action transport occurs as waves approach beaches; the total group velocity decreases, causing waves to steepen and break. Although a slowing down also occurs for waves on opposing currents (see discussion of Fig. 2, Sec. II, 2), the wave breaking that occurs on opposing currents usually has a different character from that on beaches. The waves are generally more complex and the surface motion appears to be more oscillatory than translatory. In part, this is because water depth is important on beaches and is less so on opposing currents. Also the general refraction pattern is often quite simple on beaches and is probably complex on currents.

In both beach and current cases, linear theory predicts reflection if waves are gentle enough, but the details are very different. On a beach, linear theory predicts reflection, and most of the reflection occurs within one wavelength of the shore. In practice, waves often break before this point.

Against a current reflection can be assumed to occur in the first "wavelength" of the Airy function, but the Airy function modulates the wave in this case (see Peregrine (eq. 2.109, 1976) for the approximate linear solution). Thus reflection takes place over a number of

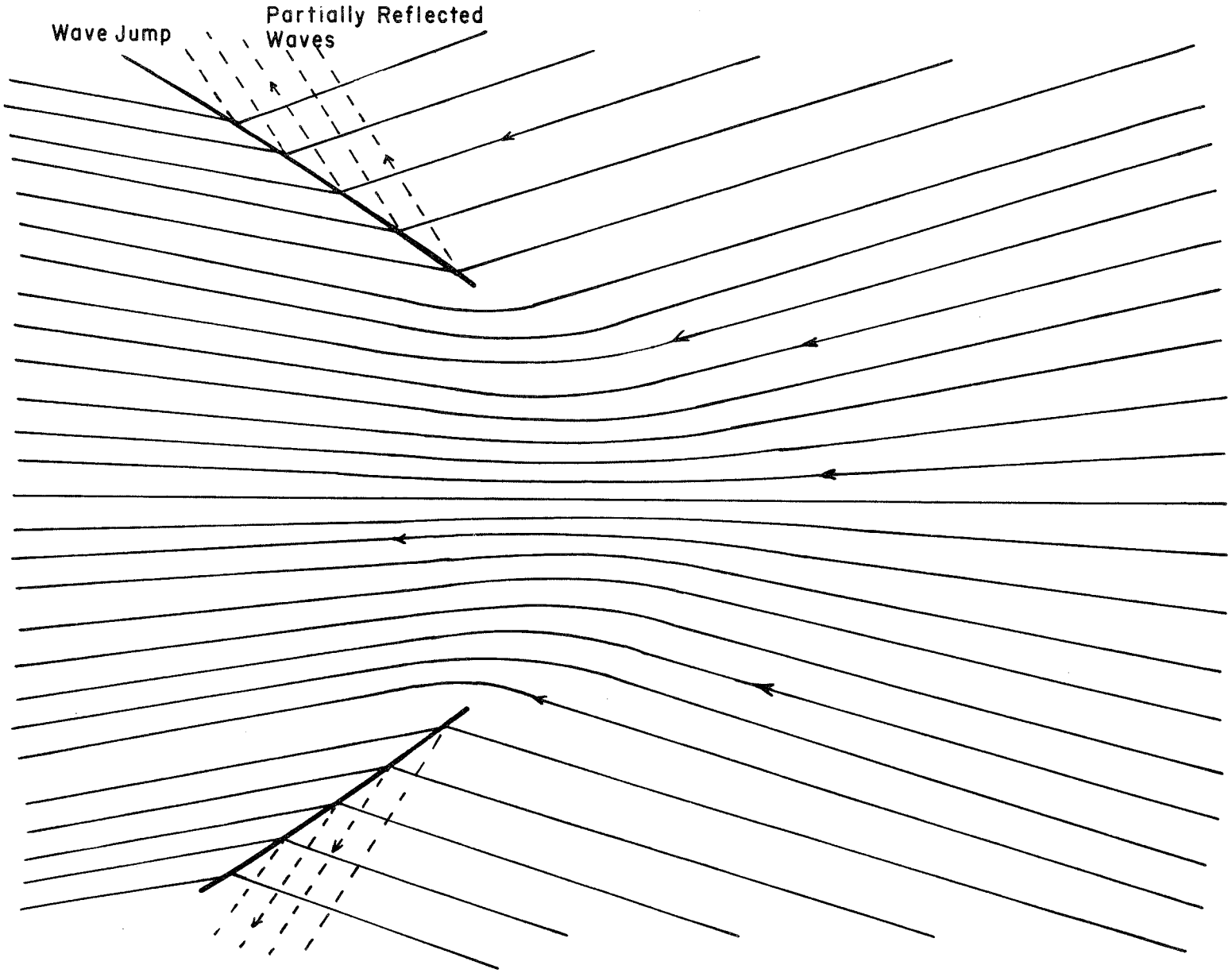

Figure 9. Nonlinear analog of rays focusing (compare Fig. 8). Nonlinearities smooth out the cusp found in linear case. Lines here are everywhere perpendicular to wave crests, rather than rays as in Figure 8.

wavelengths, the distance depending on the velocity gradient. It is thus likely that circumstances in which currents are stopping waves, some appreciable reflection may occur. Waves breaking under conditions in which a partially reflected wave exists tend to have steeper slopes on the back side as breaking occurs and to project water with a greater vertical component of velocity.

Wave breaking has a major effect on currents. All the momentum that is lost from the wave motion is gained by the mean current. This can be described more precisely for a quasi-steady breaking wave like a spilling breaker (Peregrine and Svendsen, 1978). The momentum lost by the wave in the breaking region spreads like a turbulent wake behind the wave. This has been confirmed by measurements (Stive 1980; Battjes and Sakai, 1981). In relatively shallow water, this soon becomes a uniform change to any preexisting current. In deeper water, or where the breaking is weak, it leads to a velocity shear in the upper layers (see Fig. 10). Shear in the surface layers of water has a strong influence on wave breaking.

Phillips and Banner (1974) and Banner and Phillips (1974) examine the effects of a surface wind drift layer on wave breaking. Wind drift is referred to as that part of the wave drift which is not Stokes drift due to the wave motion. Much of this wind drift is due to wave breaking from capillary waves up to the largest waves. Phillips and Banner show that if the drift is in the same direction as wave propagation, as is usually the case, then the surface layer cannot ride smoothly over large waves. There must be some breaking, at least on a small scale, well before waves reach the maximum height predicted by irrotational theory.

It is worth noting the results of calculating a nonlinear solution for periodic traveling waves on a current with a uniform shear in the vertical (Tsao, 1959; Dalrymple, 1974a; Brink-Kjaer, 1976; Brevik, 1979). If the shear is positive, i.e., the greatest current in the wave direction at the surface, then the waves have sharper crests and flatter troughs than the corresponding waves on a uniform current. This is the case for the wind drift example above and might well indicate a lower maximum amplitude and greater tendency to break (see Fig. 11).

If the shear is in the opposite direction, then the waves become more rounded than waves on a uniform current. Such waves may grow larger and are less likely to break (see Fig. 12). An extreme example is the large-amplitude "surface shear wave" described by Peregrine (1974), which occurs on fast flowing streams, in a form also known as a "wave" hydraulic jump, and in backwash on beaches.

The extrapolation to breaking properties of these waves is tentative, but the work of Banner and Phillips and the example of the surface shear wave, which can have a vertical face without breaking, support these suggestions. A possible partial explanation for the relative lack of breaking over rip currents is provided. The velocity shear delays breaking while the onshore drift and breaker wakes along the rest of

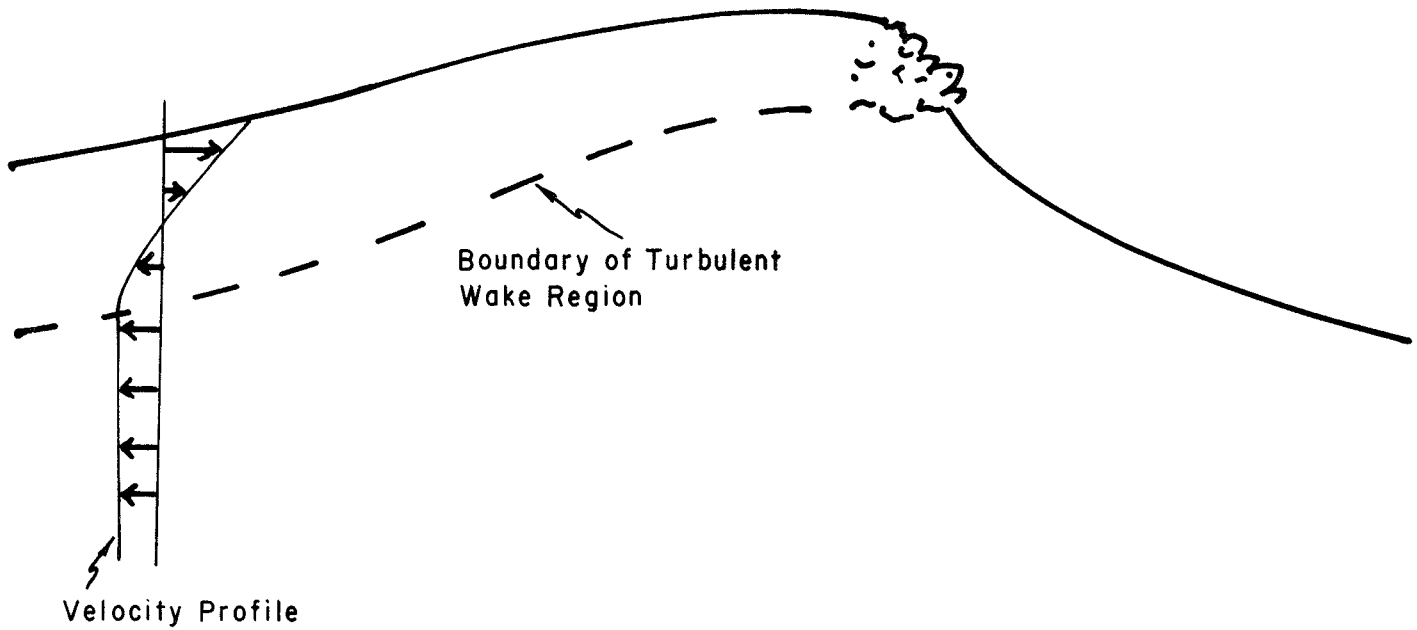

Figure 10. Mean current induced in wake of spilling breaker (current profile diagrammatic).

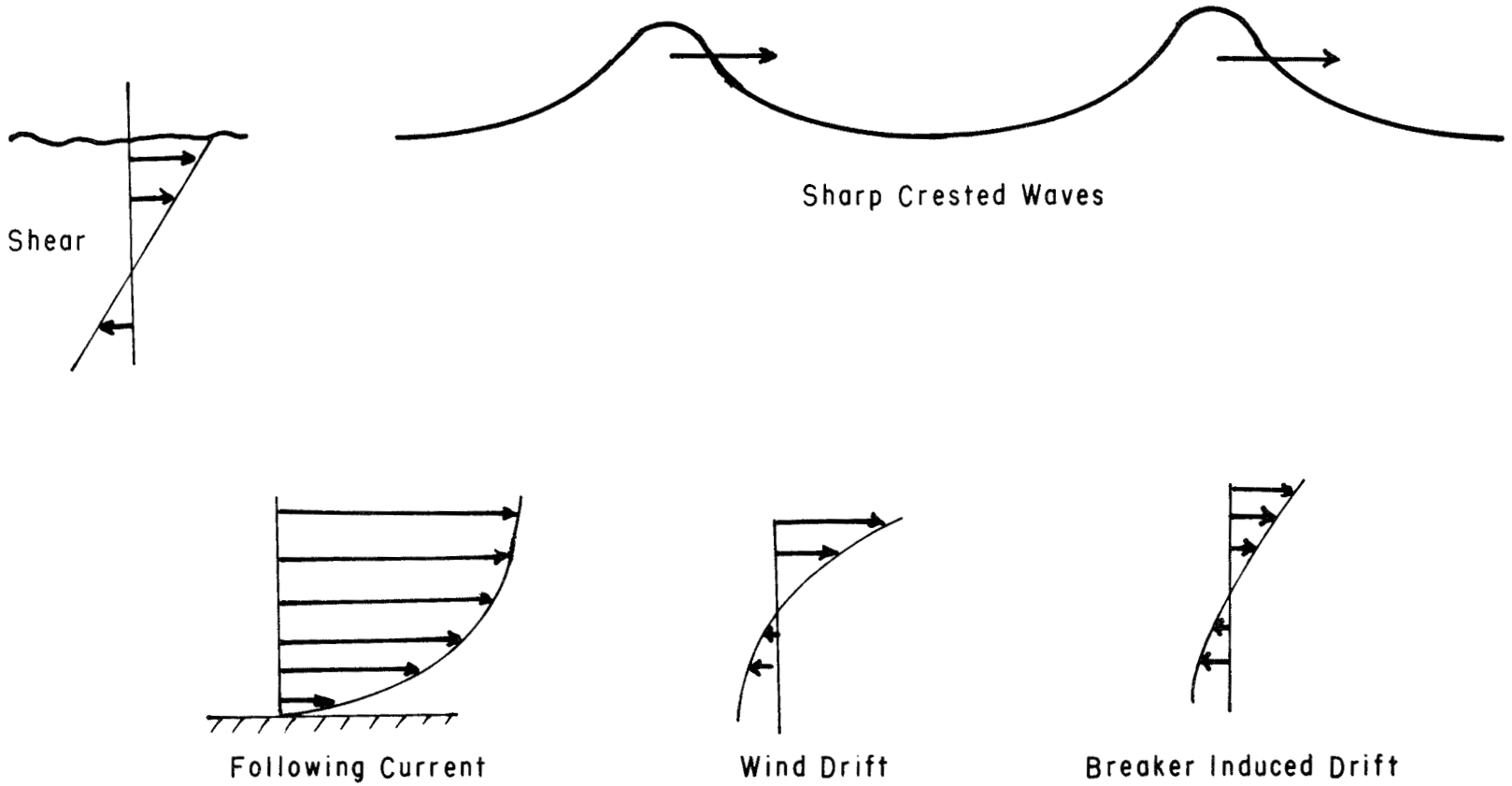

Figure 11. Examples of surface shear in the wave direction, and resulting effect on wave profile.

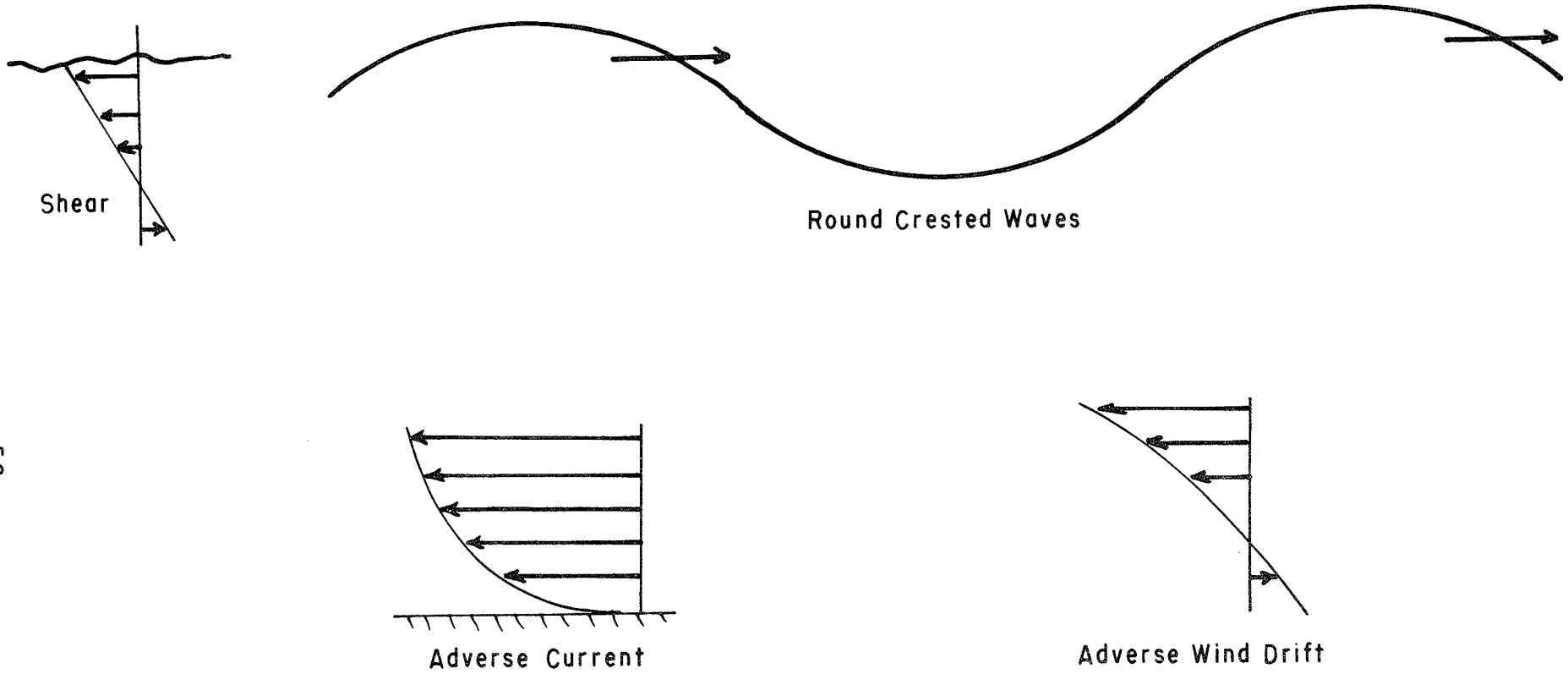

Figure 12. Examples of surface shear against the wave direction, and resulting effect on wave profile.

the beach accelerate breaking. The surface shear caused by winds may explain why onshore winds are said to promote breaking and offshore winds to "hold up" the waves.

III. ENGINEERING APPLICATIONS

The preceding sections deal either with the interaction between waves and currents, or with the effect of depth changes on that interaction. However, for engineering applications, interest lies in the effect of the wave-current interaction on its surroundings: structures, vessels, and seabed. The wave and current motions are assumed given -- by forecasting, tide tables, field or experimental studies, simple prediction methods, or experienced judgment. The fact that the waves and currents are not independent of each other, as emphasized in preceding sections of this report, must be considered in determining these given conditions.

In addition to the interaction between waves and currents, there may be an interaction between the flow and the structure itself. This additional interaction is often overlooked, and even if observed, may lie beyond the practical capability of present-day analysis. Cases where the triple (wave-current-structure) interaction is important include flow-induced vibrations, forces on large structures where diffraction effects are important, and flow over loose beds, where the flow determines the bed roughness which, in turn, affects the flow itself. Fortunately, however, there are many cases where the added interaction of the structure is of minor importance, and it suffices to take only the wave-current interaction into consideration.

1. Computer Programs for Wave-Current Interaction.

In Section II, 7, general methods of solution of wave-current interaction problems are described, but in most cases, the referenced literature deal only with the depth refraction solution. Even where wave-current interaction is covered, the reference usually concerns either theoretical examples, e.g. Ebersole and Dalrymple (1980), or an interpretation of one particular set of observations, e.g., Forristall, et al. (1978).

To date, there are only two general-purpose computer programs which are potentially suitable for computing wave propagation over currents in an engineering project, and neither involve the ray theory approach. These programs are the Dutch Rijkswaterstaat program based on recent work by Booij (1981), and the Danish Hydraulic Institute's short wave program (Abbott, Petersen, and Skovgaard, 1978). Both programs use finite differences to approximate differential equations.

The Dutch program (CREDIZ) is based on the parabolic approximation of Booij's (1981) linear wave-current equation. It takes waves of one frequency, and accounts for both refraction, by depth and current, and

diffraction, subject to the assumption that there are no large changes in wave direction. Dissipation terms are included to account for bottom friction and breaking. The program has been applied to the entrance of an estuary in the Netherlands, and is said to be capable of covering areas a few hundred wavelengths across. Required input includes bathymetry and currents; height, period, and direction of incident wave; data concerning dissipation; and certain boundary conditions and computational options. The output includes wave height, phase, and direction.

The Danish Hydraulic Institute's "short wave" program is a second possibility, but there is still no experience regarding its use in wave current situations. This program uses the long wave Boussinesq equations to describe the water motion. These equations include the nonlinear shallow-water terms, plus dispersive terms which permit description of somewhat shorter waves than simple long wave equations. These equations could be used for harbor studies with partial reflection at boundaries. Terms partially accounting for bottom friction and breaking can be included. Although wave and current motions are not separated in the computation, it would be relatively straightforward to separate them in solutions so that pictures of mean current and wave height could be extracted. If a sea surface with dimensions greater than a few tens of wavelengths is to be studied, considerable computer resources would be required with present-day technique.

At the time of writing, computational and theoretical developments in wave refraction mean that the state-of-the-art is changing rapidly. The engineer seeking to calculate wave-current refraction may choose between a ray theory approach, using a parabolic equation, or more direct solution of the equations. Ray theory is well established for depth refraction, but inclusion of currents is not a simple modification to most existing programs (e.g., see Skovgaard and Jonsson, 1976; Christoffersen and Jonsson, 1980; Christoffersen, 1982). Parabolic methods can be useful in just those cases where ray theory is poor: sparse coverage by rays, multiple crossing of adjacent rays. However, apart from the Dutch work mentioned above, there is no experience in the use of parabolic equations for water-wave refraction. The pace of development is such that an individual wishing to compute wave-current refraction should make an effort to find up-to-date information.

2. Forces on Structures in Waves and Currents.

Structures can be divided into two classes based on relative size: (a) those that are larger than about one-fifth of the wavelength and (b) those that are smaller than one-fifth of the wavelength. The factor one-fifth is a frequently adopted value, e.g., Hogben (1976). Examples of the larger class (a) include: submerged oil storage and ballast tanks, semisubmersibles, large drill ships, and large offshore breakwaters. Examples of the smaller class (b) include: structural members

of open-work piers and steel platforms, offshore pile-supported platforms, and steel risers for drilling rigs or oil production platforms.

Hogben (Fig. 1, 1976) describes the force regime in which some offshore structures may be found for steep, deepwater waves. The larger structures are in a wave diffraction regime in the absence of currents. In the presence of currents, both incident and reflected waves are refracted, and perhaps diffracted, by the current variations caused by the structure. This is very much like the problem of a moving ship meeting waves. For a proper analysis full account should be taken of all solutions of the dispersion equation described in Section II, 2. This has only recently been achieved to any extent in the ship hydrodynamics field by Newman and Sclavounos (1980). However, in most circumstances, solutions C and D of Figure 2 (the shortest wavelength solutions) may be neglected.

There are a number of approximations, many known to be inconsistent, used in the field of naval architecture and ocean engineering which might be adapted to coastal engineering problems. A measure of the difficulty of dealing with the wave forces on a moving structure is indicated by the number of papers on the forces due to waves on ships at "zero forward speed." A review of this large and active field is not considered appropriate, but much of the work can be found in the Journal of Ship Research, Proceedings of Naval Hydrodynamics Symposia, and Transactions of the Royal Institution of Naval Architects.

For smaller cylindrical structures, class (b), present engineering practice is to use Morison's formula for the force on each element of a structure:

$$\underset{\sim}{f} = \tfrac{1}{2}\rho C_D D \underset{\sim}{u}|\underset{\sim}{u}| + \rho C_M A \, (d\underset{\sim}{u}/dt) \tag{28}$$

where

$\underset{\sim}{f}$ = force per unit length of cylinder
ρ = mass density of water
C_D = drag coefficient
D = cylinder diameter
$\underset{\sim}{u}$ = instantaneous velocity of the water in the absence of the cylinder
C_M = inertia coefficient
A = cross-sectional area of the cylinder

The first term in equation (28) is a drag force per unit length and the second term is an inertial force due to the water's acceleration. More

details on present practice may be found in Hallam, Heaf, and Wootton (1978). When the waves and currents have different directions, the scalar coefficients in equation (28) may not be appropriate.

Lacking experiments measuring forces in the presence of waves and currents, it is of some value to note discussion of Morison's formula in closely related flows. Shaw (1979) is a valuable collection of papers on this topic, among which Pearcey (1979) gives a good survey of the problems that arise in estimating forces on cylinders. The book by Sarpkaya and Isaacson (1981) should also be consulted.

For cylinders in waves, Morison's formula gives the right order of magnitude for forces, but for detailed comparisons C_D and C_M must be allowed to vary with the Keulegan-Carpenter number, the Reynolds number, and the ratio of roughness to cylinder diameter. The Keulegan-Carpenter number ($u_{max}T/D$ where u_{max} is the amplitude of the oscillating velocity) is a measure of the water particle displacement divided by cylinder diameter. When this number is in the range 6 to 20, drag and inertia terms are of approximately equal importance, increasing the uncertainty in using Morison's equation (eq. 28). Sarpkaya's (1976) results are often used, but papers in Shaw (1979) give results that differ by 10 to 20 percent. It is clear that more experiments over physically representative ranges of parameters are needed.

There is also a transverse oscillating force, not predicted by Morison's formula. Usually this is not important on free-standing members unless resonance is a possibility. For pipelines laid against the bottom, one-sided transverse lift forces may be significant when currents are added, but the requirement to design for drag and bottom erosion usually insures stability against transverse forces in this case. Other phenomena to be considered are the interaction between eddies shed from different members of the structure, and the effects of sidewalls on experiments. In wave-current experiments, eddies that move perpendicular to the flow direction are confined by the channel walls and may give misleading results when they interact with test members.

Cylinders oscillating in a current have been compared with cylinders subject to combined wave and current motion. Such experiments are described by Ottesen Hansen, Jacobsen, and Lundgren (1979), Verley and Moe (1979), and Matten, et al. (1979). In particular, Matten, et al. (1979) report very substantial changes in drag and lift coefficients depending on whether or not their cylinder had end plates to inhibit three-dimensional flow. Oscillating cylinders differ physically in some ways from cylinders in waves, since the pressure fields are different due to the different acceleration field relative to the cylinders (see Batchelor, 1967, p. 409). These differences must be considered before applying the results in engineering design.

Theoretical works on statistical distribution of hydrodynamic forces on small structures in waves and currents are Tung and Huang (1973a, 1973b, 1974, 1976), Tung (1974), Moe and Crandall (1978), Hedges, Burrows, and Mason (1979), Vinje (1980), and Burrows, Hedges, and Mason (1981). These all start with unidirectional waves and currents, make certain statistical assumptions, and assume that a wave spectrum propagates from still water onto the current. Burrows, Hedges, and Mason (1981) point out that some earlier spectral studies did not use the Doppler-shifted frequency.

Sekita (1975) measured forces on a model of a deep-sea platform, both with waves and in waves and currents. The current was obtained by moving the model with uniform velocity. Sekita found that there is appreciable sheltering of structural members by other upstream members in both waves and currents, leading to relatively smaller drag coefficients for the whole structure. There is no discussion of wave-current interaction in this paper.

Kruijt and van Oorschot (1979) report experiments in a large wave basin with both waves and currents incident on a model of concrete supports for a storm surge barrier. The wave direction, relative to both the current and the structure, was varied. The authors conclude from their force measurements that interactions between waves and currents must be taken into account. Longitudinal force components are reasonably well predicted if the Doppler effect on the waves' frequency is taken into account. However, Doppler effects do not explain measured transverse forces. Forces resulting from the wave-current interaction are greater, and moments about a vertical axis have larger fluctuations than those due to current alone. The increased fluctuations are attributed to the effect of currents in exaggerating the phase difference between wave crests on either side of the structure.

Dalrymple (1975) gives a report of the analysis of measurements taken on an instrumented platform in 30 meters of water during Hurricane Carla in 1963. He discusses the interpretation in terms of waves and currents, and concludes that drag coefficients which had been obtained from the same data without considering the presence of a current are too large. The data cannot be interpreted as due to one wave train and a uniform current. More than one wave train was present, with different directions of propagation, and the current could have varied with depth. Force direction did vary with depth.

3. Sediment Transport.

Sediment transport due to the action of a current alone is reasonably well documented. Although detailed mechanisms are not fully understood, reasonable estimates can be made. Transport by waves is more difficult to estimate.

Waves may transport large volumes of sediment. In some circumstances, especially for longshore transport, the direction of sediment transport can be predicted, but estimates of the transport rate can be an order of magnitude wrong. For transport perpendicular to the coast, it is often difficult to predict even the direction of net sediment transport. Given the uncertainty about sediment transport by currents alone or by waves alone, it is very difficult to predict the effects of waves and currents together on sediment transport.

A difficulty inherent in any sediment transport problem is the appropriate description of the sediment. Even though sediment size and density can be surprisingly uniform, the variation encountered adds complexity to any computation. (On the other hand, the remarkable ability of oscillating flows to sort materials by size and occasionally by density suggests that the techniques of minerals processing should be examined for improving fundamental understanding of sediment transport.)

Next, consider the difficult distinction between bedload and suspended load in wave-induced sediment transport. First, it is difficult to define bedload in a way that has practical meaning in interpreting measurements, even when only currents are considered. Second, the addition of waves to even small currents is likely to increase both bedload and suspended-load transport. Bedload transport due to waves and currents has been reported from laboratory experiments by Inman and Bowen (1962), Abou-Seida (1964), and Tanaka, Ozasa, and Ogasawara (1973). The difficulty of interpreting these experiments is seen from results for coal by Tanaka, Ozasa, and Ogasawara (1973). Seven out of eight of their measurements show the net bedload transport to be **opposite** to the direction of the current. Bijker, van Hijum, and Vellinga (1976) include a few measurements of wave and current sand transport, and they note that in the absence of current the direction of sand transport depends upon wave shape. See also van de Graaf and Tilmans (1980).

Another difficulty in predicting sediment transport is the influence of bed form. First, it is difficult to predict the bed form that will result, and second, it is more difficult to predict the effect of bed forms on sediment transport rates. For currents without waves, the shape of the bed form depends on depth, velocity, and sand size, but available information on the relations among these variables has, to date, come largely from laboratory facilities (Southard, 1971; Southard and Boguchwal, 1973). Extension of these relations to prototype scale is uncertain for the current only case; considerably less is known for the wave and current case, particularly for storm conditions on a prototype scale when most of the transport occurs.

For sediment in suspension, the turbulence and the current are two important factors. Measurements of turbulence with good spatial distribution under a combination of waves and currents are reported by Iwagaki and Asano (1980) and Kemp and Simons (1982). Kemp and Simons

used a laser Doppler anemometer. Their interpretation of the measurements, as they apply to sediment transport, is repeated verbatim:

> The entrainment of sediment under flat bed conditions can be related to the predicted instantaneous shear stress. However, although the entrainment of material from the bed can be considered to show a considerable increase under the combined action of waves and currents, the distribution of turbulence intensities suggests that the zone of diffusion would not increase. In fact, the results indicate a reduction in boundary layer thickness [when waves are added to a current.] One might expect, therefore, that there would be an increase in sediment concentration in the near bed region. In the light of Nielsen's (1979) observations, this distribution would change dramatically under spilling breakers, the material rapidly dispersing over the whole depth of flow.
>
> The greater turbulent stresses found when waves are superimposed on a current are likely to result in a considerable increase in sediment pickup from a rippled bed. While the increase in turbulence is limited to a region within 6 or 7 roughness heights of the bed with a tendency for this zone to decrease with wave height for a constant wave period, it is to be expected that sediment brought into suspension by the nearbed vortex action will be diffused over the zone of the current-induced turbulence. This could result in significantly higher transport rates as long as the increased bed shear stress is not such as to prevent the formation of high bed ripples.
>
> In the case of waves alone, the shear stresses at the bed are of the same order as for combined wave and current flow, but the vortex-dominated layer extends only approximately four roughness heights above the bed, and the only means of transporting sediment is by relatively weak wave-induced mean velocities. The limited thickness of the wave-induced vortex layer over a rippled bed has previously been noted by Tunstall and Inman (1975). This suggests that sediment would be concentrated in this near-bed layer.

It has long been recognized that sediment transport by waves is enhanced by currents generated by these waves. The interaction of the waves with a mobile bed can lead to complicated systems of currents, and

the prediction of these currents has been a major difficulty. Since there is abundant wave energy to suspend the sediment, even weak currents may lead to significant sediment transport.

In the field there may not be a constant mean current throughout the water column at each point. The surface layer of the sea has a mean motion dominated by the wind and Stokes drift. In the case of breaking or near breaking the latter can be a substantial fraction of the wave speed, e.g., when breakers pass over a sandbar and the return flow is in a channel. Nielsen's (1979) observations of the effect of gently spilling breaking waves should be noted. Turbulence from wave breaking may affect sediment transport once it has diffused to the lower boundary of the fluid.

The midlayer of water will largely move under its own inertia and pressure gradients from the mean surface and mean wave stresses. The bottom boundary layer not only responds to the stress from the mean flow and waves but also to pressure gradients so that, in a general three-dimensional situation, all three layers have different directions. Even such a well-defined problem as the mean current profile for currents and waves in the same direction has no satisfactory solution yet. Brevik (1980) and Kemp and Simons (1982) both find that an increase in wave height "steepens" the velocity profile outside the wave boundary layer. However, this effect does not seem to be found by Bakker and Van Doorn (1978) in their Figure 3b.

Different modes of transport occur depending on the bed features, or lack of them. These features are well documented for currents or waves alone (Allen, 1968; Nielsen, 1979), but for the combination of waves and currents there is little except for Tanaka, Ozasa, and Ogasawara's (1973) measurements of ripples. Theoretical models (based on extensive experiments) for sediment movement in oscillatory flow, with or without a mean current, have been developed by Nielsen, Svendsen, and Staub (1978) and Nielsen (1979). As is the case for structures, it is a triple interaction (waves, current, and bed) which requires study.

Three-dimensional laboratory experiments with a controlled flow have been performed by Bijker (1967) and Rasmussen and Fredsøe (1981). Bijker's measurements were for waves at right angles and at oblique incidence to a current; the author notes that the measurements of sediment transport were only partly successful in his current-dominated regime. Rasmussen and Fredsøe made more accurate measurements in a wave-dominated regime with waves at right angles to the current. They find reasonable agreement with a theory in which bedload is calculated using their recent theory (Fredsøe and Rasmussen, 1980), and suspended load is found from concentration and velocity profiles based on Nielsen (1979) and Fredsøe (1981).

Field studies of sediment motion under waves can significantly improve understanding. In particular, repetitive observations made consistently at one site (e.g., Short, 1978) can provide insight into the important processes which the engineer will actually meet, free of possible scale effects.

To summarize this section on sediment transport, it is evident that wave-current interaction is a fundamental aspect of most littoral sediment transport. Yet it is equally evident that a physically-based prediction of this transport is lacking. Experimental results are contradictory, and may even appear inconsistent with physical insight. Model effects and experimental error are possible explanations. Some confusion may be due to lack of common definition for the phenomena under investigation, and it is always possible that appropriate phenomena to be measured are yet undiscovered. Differences between sediment transport in steady, uniform flow (the standard textbook condition) and in quasi-periodic, unsteady flow under waves are incompletely appreciated up to now. More well-defined data from field and laboratory are needed.

4. Field Data.

Observations of current and wave properties suitable for comparison with theoretical predictions of wave-current interaction are desirable. Some observations have been reviewed in Section II, 9. The major limitation in available data is the lack of detail for describing the current field over which waves have propagated. Indeed, unless new remote-sensing techniques are developed to give such information, the best available data will be from point measurements of current. However, for many applications a computed current field is likely to be more satisfactory, especially if it can be verified by comparison with observed currents.

Numerous current and wave observations have been made in coastal waters. Many are from areas with strong, predictable tidal currents and hence are suitable for comparison with theory. The Marine Information Advisory Service, at the Institute of Oceanographic Sciences, Wormley, England, maintains a worldwide register of such data. Data such as these should be adequate for a large-scale study (i.e., where wave propagation over distances of about 100 kilometers are considered). A study on a scale of 10 kilometers or less would require special measurements.

The problems of comparison between field data and theory are partly revealed by Cardone, et al. (1981) who analyzed ocean waves measured in the North Atlantic in connection with the GATE program. In that case waves arrive from different directions and sources, and not all can be identified. Comparisons involving current fields are made more difficult by the directional dependence of the dispersion equation (Sec. II, 2). This makes it particularly important for some sort of directional measurements to be made.

5. State-of-the-Art for Coastal Engineers.

This review shows gaps in the knowledge of wave-current interaction, but there are significant areas of understanding, particularly with regard to refraction.

For engineering design, the wave height is usually the most important wave variable. For sites with sandy beaches or small-craft harbors, wave direction may be equally important. If the currents are known seaward of a particular shore, it is not possible to say for the general case whether wave heights reaching that shore will be increased or decreased by their interaction with the currents. There is limited experience in calculating wave refraction across any but the simplest unidirectional currents.

To the extent that field conditions satisfy the assumed conditions of large-scale, parallel, shearing flows treated in Section II, 5, the corresponding solutions in Section II, 5 can be used to predict wave behavior. For the nonshearing case where current velocity is constant across the flow and the waves travel in the same direction as, or opposite to, the currents, the effect is straightforward: following currents reduce wave heights, opposing currents increase them. But for shearing currents over constant depth with waves oblique to the flow, the wave will have a minimum height while on the current when the incident angle between wave direction and current direction is approximately 45°. Any refraction away from that direction increases wave height. Minimum wave steepness occurs when this angle is approximately 30°. Refraction away from this 30° direction increases steepness. This increase in steepness may be sufficient to cause the wave to break while it is on the current, in which cases the wave will emerge from the other side of the current with a lower height. However, if it does not break, the wave will emerge from the other side of the current with the same height and direction it had on entering the current, only displaced downsteam from the path it had on entering the current. For a synoptic picture of what happens to a wave entering obliquely to the current, see Figures 7 and 8 of Peregrine (1976).

In most cases, the currents of engineering interest in wave-current interaction are due to the tides. Tidal currents are usually relatively uniform with depth, which simplifies somewhat the complicated possibilities in the interaction. Since tides are usually reversing currents, it is likely that one direction will be more critical than the other for the engineering project. Conservative design requires that this more critical case be identified and used to establish design criteria, but there is not now any general rule on establishing this critical direction. In this context, it would be useful to examine time series of wave measurements from nearshore sites adjacent to substantial reversing tidal flows.

Perhaps of equal importance to predicting wave behavior is the need to be careful that the effect of currents does not lead to significant errors in interpreting and using wave measurements. As discussed in Section II, 2, the correct dispersion equation for waves on a current should be used in reducing wave data recorded by pressure sensors submerged under significant currents. If that is not possible, an engineer should be satisfied that currents are below some appropriate threshold value (see Table).

The effects of a vertical variation of mean current velocity are as yet poorly understood. Such vertical variation is rarely measured so it is difficult to give guidance on these effects, but they are briefly mentioned in Section II, 3, and should be considered where field conditions warrant.

Application of wave-current knowledge to practical problems of sediment transport and wave forces is in its infancy. The experimental evidence indicating the accuracy of existing theories is discussed for sediment transport in Section III, 3. Limits on the use of the Morison equation (eq. 28) for wave force calculations are discussed in Section III, 2.

6. Development of Capability and Understanding.

Mathematical books such as Phillips (1977), Lighthill (1978), and Whitham (1974) and research papers are not the most effective means of communicating with practicing engineers. Case histories and summaries in technical handbooks are more likely to be effective. The significant new knowledge gained over the last decade must be converted to a format usable in engineering practice by appropriate applied research.

Perhaps the easiest applied research to accomplish is the computation of refraction across idealized models of selected realistic current fields. For example, flow around a headland, with and without a separation eddy behind it, should show the tide race effect and whether there is any correspondingly sheltered shoreline. As another example, the flood and ebb flows through a tidal inlet are quite different, and the interaction of these different flows with incoming waves should be examined.

Field research to verify whether refraction theory does describe ocean wave propagation onto coasts is desirable. There are so few examples of field observations, even for ordinary depth refraction, that its basis might be questioned. Comprehensive field observations require much time and manpower but are valuable to a proper understanding. Prominent examples of such field studies are Mallory (1974) on wind, wave, and current interaction, and Short's (1978) long-term studies of beach development. A number of remote-sensing techniques are now available and could be used in an area with strong currents.

Experiments on nonuniform currents are desirable. They would be particularly valuable in showing how eddies and turbulence accompanying the currents modify refraction by the currents. If in some circumstances the eddies and turbulence dominate the waves, prevailing views on the relevance of refraction will need to be revised.

Detailed measurements of the type being done by Kemp and Simons (University College London, personal communication, 1981) are very important for applications. There should be more attempts to measure waves and currents which are not collinear. Such measurements should be possible in existing large wave basins also equipped for generating currents.

There are many other fields of science and engineering employing wave theory (e.g., quantum physics, radio propagation, plasma physics, internal waves in atmosphere and ocean, acoustics and seismology). It could be very valuable if workshops were arranged to enable practitioners in these fields to see what can be learned from their different disciplines. For example, Blokhintsev (1956) was the first (in 1946) to correctly deal with radiation stress in acoustics, and later Longuet-Higgins and Stewart (1960, 1961, 1962, 1964) and others independently introduced it to water waves.

IV. CONCLUSIONS

This review of wave-current interaction shows that there is a good theoretical framework for the refraction of water waves by large-scale currents which are uniform in depth. However, with minor exceptions, the theory has only been applied to very simple current distributions and has not been subject to rigorous experimental or field verification. Even so, it is clear that simple current fields can strongly refract waves, and both observations and experiments show changes in wave properties qualitatively consistent with theory. A start has been made on treating refraction and diffraction caused by both depth and currents, using a computer.

Effects caused by varying velocity with depth is given some emphasis. The major effects of interaction with waves in this case are the way in which the change in the velocity field affects the mean bottom stress and hence influences the mean current, and the strong effect of surface shear on wave breaking.

Once wave and current properties are known, their major engineering applications are to the prediction of forces on structures and sediment transport. Both of these applications involve a triple interaction problem: waves, current, and structure; or waves, current and sediment. Yet even the double interaction cases (waves and structure; or waves and sediment) still require fundamental research.

Recommendations are:

(a) Educate practicing coastal engineers to be aware that currents may affect wave data measured on submerged pressure gages, increase or decrease wave heights, and alter the direction of waves reaching the shoreline.

(b) Apply recent advances in understanding to wave refraction in moderately complex realistic current fields. Summarize results in a format usable by engineers.

(c) Develop, test, and document a practical refraction program to cover depth and current changes. Ultimately, this program should treat irregular waves, but the initial objective should be to adequately treat regular waves.

(d) Measure the propagation of waves across nonuniform currents in the laboratory. The current and turbulence characteristics should be measured in detail, especially the changes in current distribution due to waves. Also measure wave-current interaction effects on simple structures and sediment beds.

(e) Evaluate field evidence for wave-current interaction, initially with computed current fields and existing wave and bathymetric data. Collect wave data at sites inshore of strong, reversing tidal flows.

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